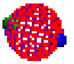
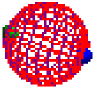
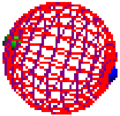
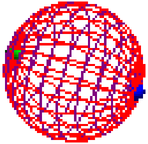
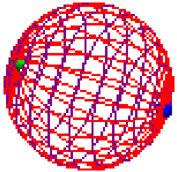
Not an overview!

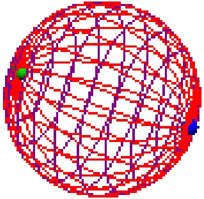


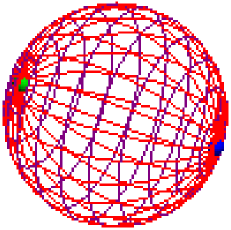


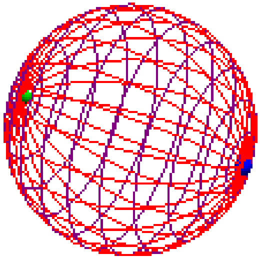


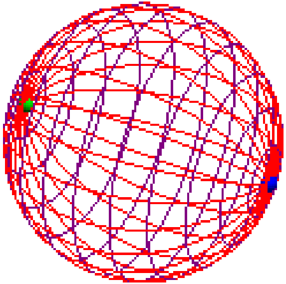


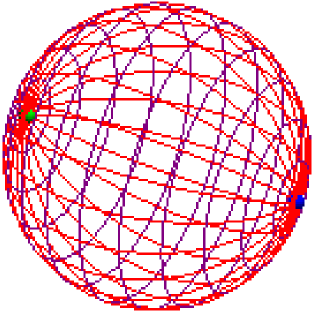


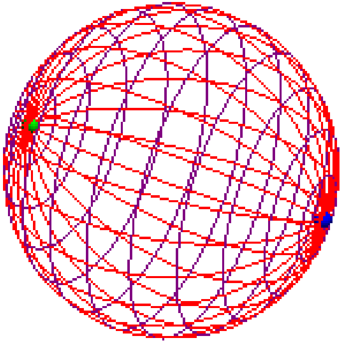


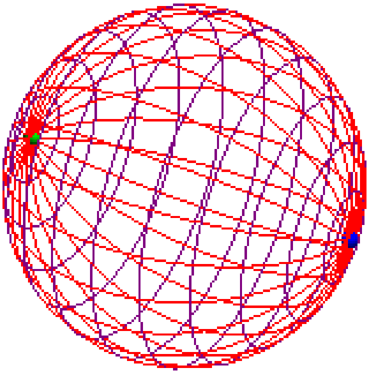


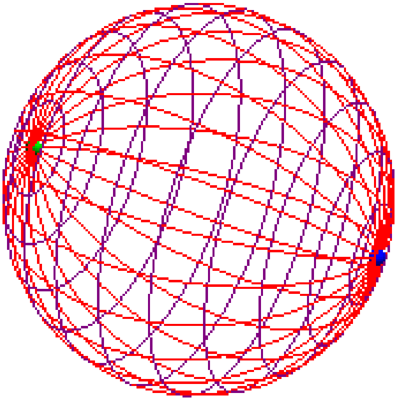


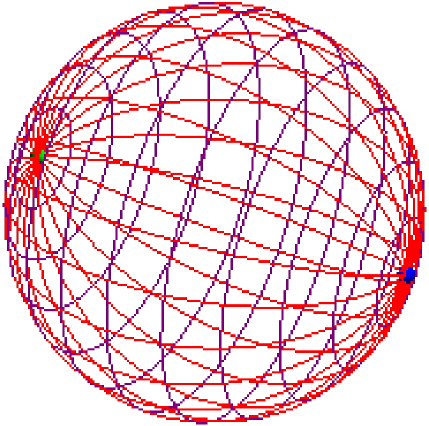


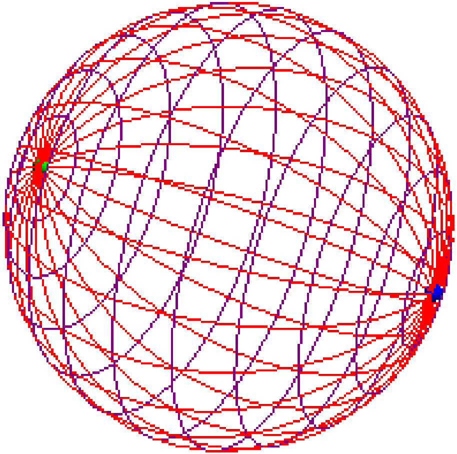


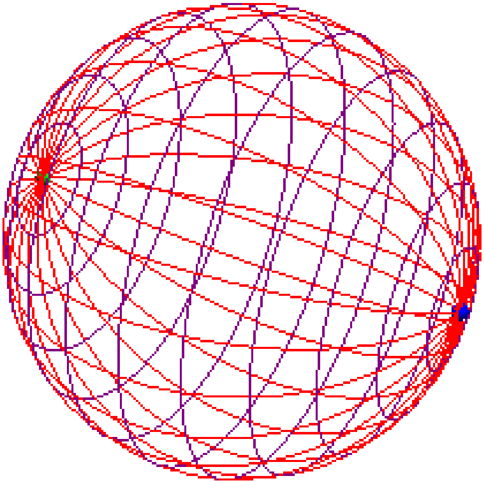


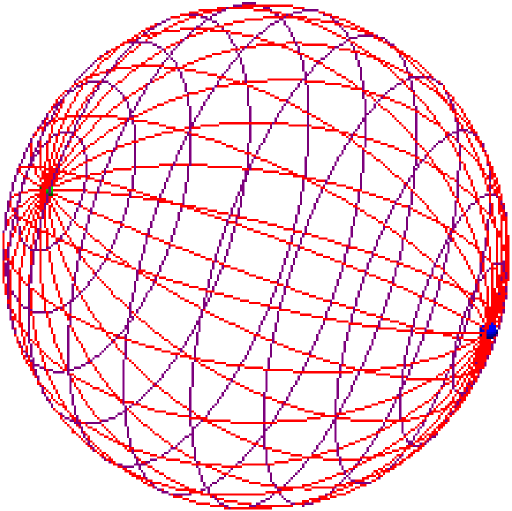


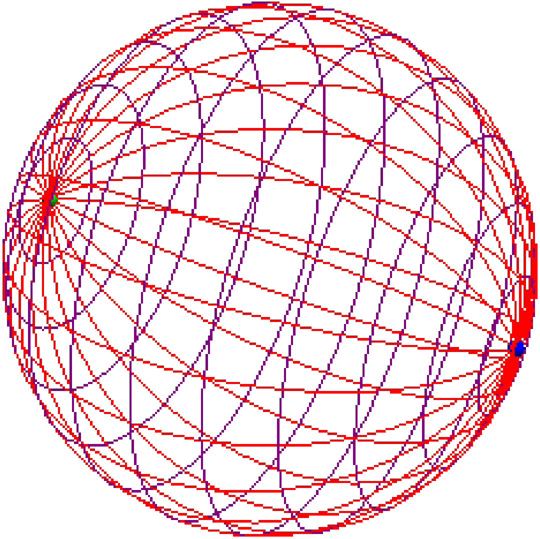












Charged particles and fields produced from very high energy scattering move away from the interaction point at the speed of light.

Classical calculation — charged particles created very quicky at $t = 0$ and moving outward produce a pulse of electromagnetic radiation in a thin shell expanding at the speed of light. As the energy of the particles goes to infinity, the particles keep up with the shell of radiation and all the interesting physics is happening on the **"Light Shell" — at $r = t$.**

It was an obvious thing to try to do, I thought, to add quantum field theory and construct a 2D effective field theory on the light shell — maybe shed additional light on Sudakov effects, SCET, factorization and the rest of the long list of things that I don't understand very well about field theory at high energies. Maybe even be useful at the LHC.

For simplicity, I decided to start by thinking about 0-flavor E&M.

Joined by graduate students — Greg Kilcup and Aqil Sajjad — I thought this would be a nice easy warm-up project —

Almost 4 years later, the students are graduating having solved a number of interesting technical problems, but we are not done.

I will start by telling you how I thought/think this theory should behave and motivate (I hope) some of the technical problems we have solved. I am proud of these things, but I will discuss them only very briefly, because until we put this all together, you have every right to say that this just mathematics and I should talk to you again when I have done some physics.

Finally I will say what I think is missing in the hopes that one of you can help me out.

Physical picture including quantum effects

time $t = 0$ — a gauge invariant source at the origin of space time — ie $\phi(0)\,\phi(0)^\dagger$ — produces charged particle-antiparticle pairs with all possible energies and momenta —

this is our idealization of the “hard scattering” in a parton model description — but here anything can happen because we start not with an energy-momentum eigenstate, but a point interaction.

after $t = 0$ the fields are spread out on the light-shell and give rise to a quantum amplitude that describes all the classical processes, splitting into more and more incoherent outcomes as the system evolves. These fields on the light-shell play the role in the source for $t > 0$ and the what we want is an effective field theory that describes that source.

Light-Shell Gauge — in our analysis of the classical case, we realized early on that there are special gauges in which not only the $\vec{E}$ and $\vec{B}$ are confined to the light shell in the high energy limit, but the vector potential was as well. These are the gauge in which

$$v_\mu A^\mu(\vec{r}, t) = 0$$

where v^μ is the light-like vector

$$(1, \hat{r})$$

pointing outward from the origin in space.

We reasoned that we should construct our quantum theory with this gauge condition.

The derivation of the photon propagator in LSG is highly nontrivial. The reason is the dependence of the radial unit vector $\hat{r}$ — while the projection operator onto the radial direction is very simple, it does not commute with $\Box$.

We could analyze this using vector spherical harmonics, but this obscures the physics of the light shell, so we developed an operator treatment of vector functions on the sphere that was very convenient for this problem. Here is what the propagator looks like in this notation. A^0 is fixed to be $\hat{r}\cdot\vec{A}$, by the LSG condition. Then the propagator for $\vec{A}$ is

$$T^{\dagger^{-1}}\left(\hat{R}\,(\partial_t+\vec{\nabla}\cdot\hat{R})^{-1}\,(\partial_t+\hat{R}\cdot\vec{\nabla})^{-1}\hat{R}^T+C\right)T^{-1}\quad\text{where}$$

$$T^{-1}\equiv I-\vec{\nabla}_\perp\,(\partial_t+\hat{R}\cdot\vec{\nabla})^{-1}\,\hat{R}\qquad T^{\dagger^{-1}}\equiv I-\hat{R}\,(\partial_t+\vec{\nabla}\cdot\hat{R})^{-1}\,\vec{\nabla}_\perp^T$$

$$C=\vec{L}\,\Box^{-1}\,\Delta\vec{L}^T+\mathcal{S}\vec{L}\,\Box^{-1}\,\Delta\vec{L}^T\mathcal{S}$$

$$\Delta(\vec{r},t,\vec{r}',t')=\frac{\delta(t-t')\,\delta(r-r')}{4\pi}\lim_{\epsilon\to 0}\left(\log\frac{1}{(1+\epsilon-\hat{r}\cdot\hat{r}')}+\text{constant}\right)$$

All of the objects here are interpreted as linear operators in acting on vector functions and $\vec{b}^\perp\equiv\vec{b}-\hat{R}(\hat{R}\cdot\vec{b})$

This is actually less complicated that it looks and it simplifies considerably on the light-shell

One of the keys to any effect field theory is understanding what is left out in going from the full theory to the effective field theory. We believe that we have part of the answer. We are interested only in particles moving away from the origin, and in the high energy limit, so it was natural to try something we call a Large Radial Energy expansion analogous to the $1/m$ expansion is heavy quark effective field theory. We do this in two steps (1st like HQET)

$$\phi = \phi_s + \sum_{E>\mathcal{E}} \left(\frac{e^{-iE(t-r)}}{\sqrt{2E}} \Phi_{E,+q} + \frac{e^{iE(t-r)}}{\sqrt{2E}} \Phi^*_{E,-q} \right)$$

where $\mathcal{E}$ is a scale that divides high energy from low energy. As usual in such an effective field theory decomposition, the x dependence of the EFT field is assumed to be slow compared to the t and r dependence of the exponential factor $e^{iE(t-r)}$, and derivatives of Φ_E are assumed to be small compared to $\mathcal{E}$ in the effective theory. This is not defined at the origin - but we are OK with that - our origin is very singular and LSG is also not defined there.

$$\mathcal{L} = i\,\Phi_E^*\,(\partial_t + (\hat{r}\cdot\vec{\nabla} + \vec{\nabla}\cdot\hat{r})/2)\Phi_E + \frac{1}{2Er^2}\Phi_E^*\,\tilde{L}^2\,\Phi_E$$

$$\tilde{L}^2 = -r^2\,(\vec{\nabla}_\perp^T - iq\vec{A}_\perp)\cdot(\vec{\nabla}_\perp - iq\vec{A}_\perp)$$

$$\tilde{\Phi}_E(x) \equiv \exp\left[i\frac{\tilde{L}^2}{2Er}\right]\,\Phi_E(x)$$

$$\left\langle 0 \left| T\,\tilde{\Phi}_E(x)\,\tilde{\Phi}_E^*(x') \right| 0 \right\rangle$$

$$= \frac{1}{rr'}\,\theta(t-t')\,\delta(t-r-t'+r')\,\delta(z-z')\delta(\phi-\phi')$$

(where $z = \cos\theta$). This says that that the propagation of the LRE fields is directional - away from the origin at the speed of light at fixed θ and ϕ. This is a key to a surprising aspect of the theory — the position-space — momentum-space correspondence — angles on the light shell give directions of particle propagation at large distances (as in a diffraction problem)

Another bonus of LRE fields in LSG!

$$\mathcal{L} = -i\frac{1}{2}[(\partial^\mu \Phi_E^\dagger) v_\mu \Phi_E + v^\mu \Phi_E^\dagger \partial_\mu \Phi_E] + v_\mu A^\mu \Phi_E^\dagger \Phi_E$$

$$+ \frac{1}{2E}(D^\mu \Phi_E)^\dagger D_\mu \Phi_E$$

Interactions vanish to leading order

Interactions appear only through the fields on the source.

So the quantum mechanical light-shell for $t > 0$ describes all the physics of the original source, and it must describe infinitely many different sectors associated with different configurations of LRE fields, describing high energy particles moving outward with various energies and angles. In leading order, the fields on the light shell are determined by gauge invariance. I argued that the light-shell is the image at $t > 0$ of the gauge invariant source at the origin of space-time. It should also be gauge invariant.

$$\exp\left(i\frac{e}{2\pi}\int\left(\sum_j \ell(\hat{r},\hat{r}_j)\right)\,\partial_\mu A^\mu(x)\,dS\right)$$

$$dS = \theta(t)\,r\,\delta(r-s)\,\delta(r^2-t^2)\,d^4x$$

and assuming 0 net charge $\quad \ell(\hat{r},r_j) = q_j\log(1-\hat{r}_j\cdot\hat{r})$

These pieces put together give a consistent description of an effective theory for the LRE fields at tree level. At zeroth order in e, a source at the origin matches correctly onto LRE fields on the light-shell. At next order, to first order in e, the gauge fields on the light shell correctly produce photon emission.

Beyond leading order, when loop diagrams are important, the situation is more complicated. We do not have a fully consistent calculation.

I suspect that part of the problem is that we have not understood how to properly combine our LRE fields into jets, which means that we have not finished the job of understanding what to leave out of the full theory to the effective theory. We continue to hope that this picture will give us clues to a natural physical construction of these jets, but either it hasn't happened so far, or we have stubbornly ignored the clues.