One loop structure of effective non-linear Lagrangian with light dynamical Higgs

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in preparation

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Chiral perturbation theory at one loop

Expansion of terms of energy

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4$$

(physical) amplitudes are renormalizable order by order

$\mathcal{L}_0 + \mathcal{L}_2$ used at loop level, $\mathcal{L}_4$ as counterterm source

Renormalization of off-shell processes:
-Define running of physical parameters
-Consistency check of the $\mathcal{L}_4$

Setup

1.Only longitudinal components of gauge fields ("pions")

2.Boson sector only, including light higgs

3.No custodial breaking terms

4.Renormalization of processes with up to 4 external legs

5. General U-matrix parameterization

6. Off-shell amplitudes renormalization, div parts only.

General U-matrix parameterization

Three would-be goldstones analogous to pions in QCD

$$\boldsymbol{\pi} = \{\pi_1, \pi_2, \pi_3\}$$

Represented by matrix $\mathbf{U}(\boldsymbol{\pi})$, transforming linearly

$$\mathbf{U}' = L\mathbf{U}R^\dagger \quad \text{under} \quad SU(2)_L \times SU(2)_R$$

We define a general parameterization of $\mathbf{U}(\boldsymbol{\pi})$,

$$\mathbf{U} = 1 - \frac{\boldsymbol{\pi}^2}{2v^2} - \left(\eta + \frac{1}{8}\right)\frac{\boldsymbol{\pi}^4}{v^4} + \frac{i\boldsymbol{\tau}\boldsymbol{\pi}}{v}\left(1 + \eta\frac{\boldsymbol{\pi}^2}{v^2}\right) + O(\boldsymbol{\pi}^5)$$

$$\eta = 0 \;\Rightarrow\; \mathbf{U} = \sqrt{1 - \boldsymbol{\pi}^2/v^2} + i\boldsymbol{\tau}\boldsymbol{\pi}/v$$

$$\eta = -1/6 \;\Rightarrow\; \mathbf{U} = e^{i\boldsymbol{\tau}\boldsymbol{\pi}/v}$$

All the physical quantities are η independent

Weinberg ‘68

Lagrangian set

$$\mathcal{L}_0 \;=\; -\lambda_1 v^3 h - \frac{1}{2} m_h^2 h^2 - \frac{\lambda_3}{3!} v h^3 - \frac{\lambda_4}{4!} h^4$$

$$\mathcal{L}_2 = \frac{v^2}{4} \mathrm{Tr}[\partial_\mu \mathbf{U} \partial^\mu \mathbf{U}^\dagger] \mathcal{F}_C(h) + \frac{1}{2} \partial_\mu h \partial^\mu h \; \mathcal{F}_H(h)$$

$$\begin{aligned}
\mathcal{L}_4 = & \left(\mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu]\right)^2 c_6 \mathcal{F}_6(h) + \left(\mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}_\nu]\right)^2 c_{11} \mathcal{F}_{11}(h) + \mathrm{Tr}[\mathcal{D}_\mu \mathbf{V}^\mu \mathcal{D}_\nu \mathbf{V}^\nu] c_9 \mathcal{F}_9(h) + \\
& + \mathrm{Tr}[\mathbf{V}_\nu \mathcal{D}_\mu \mathbf{V}^\mu] \frac{\partial^\nu h}{v} c_{10} \mathcal{F}_{10}(h) + \\
& + \mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \left(\frac{\Box h}{v} c_7 \mathcal{F}_7(h) + \frac{\partial_\nu h \partial^\nu h}{v^2} c_{20} \mathcal{F}_{20}(h) \right) + \mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \frac{\partial^\mu h \partial^\nu h}{v^2} c_8 \mathcal{F}_8(h) + \\
& + \frac{\Box h \Box h}{v^2} c_{\Box H} \mathcal{F}_{\Box H}(h) + \frac{\Box h \partial_\mu h \partial^\mu h}{v^3} c_{h2} \mathcal{F}_{h2}(h) + \frac{(\partial_\mu h \partial^\mu h)^2}{v^4} c_{DH} \mathcal{F}_{DH}(h)
\end{aligned}$$

$$\mathbf{V}_\mu \;=\; (D_\mu \mathbf{U})\, \mathbf{U}^\dagger$$

$$\mathcal{F}_i(h) = 1 + 2a_i h/v + b_i h^2/v^2$$

Contino et al. '10

Alonso, Gavela, Merlo, Rigolin, and Yepes '12

Higgs/pion scatterings renormalization off mass shell

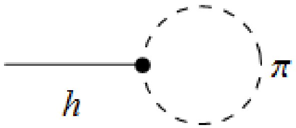
Pion scattering amplitudes in non linear sigma model has been extensively studied in the past.

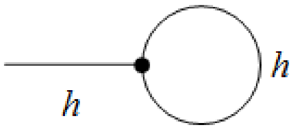
What happens if singlet scalar is added?

$$\mathcal{F}_i(h) = 1 + 2a_i h/v + b_i h^2/v^2$$

3 point functions are new and are possible because higgs is not exact goldstone boson of global symmetry group breakdown

Higgs tadpoles



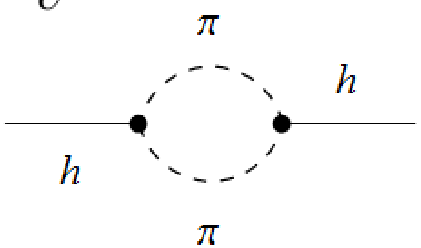


$\lambda_1^{\text{bare}} = \delta\lambda_1$ to cancel divergences

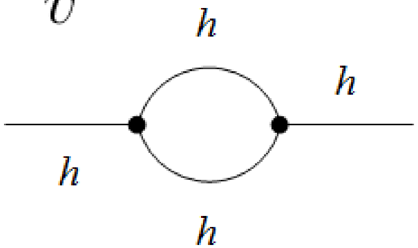
$\lambda_1^{\text{ren}} = 0$ for vacuum stability

Self energies

$$\frac{a_C}{v}\, h\, \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}$$



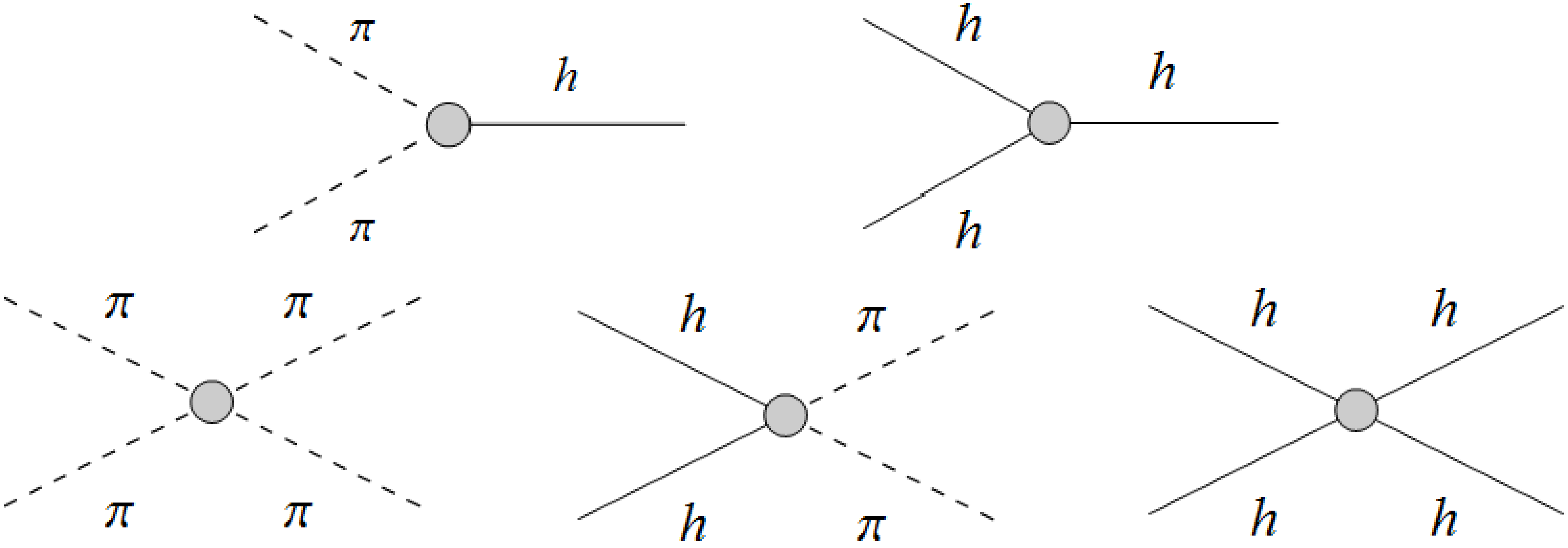
$$\frac{a_H}{v}\, h\, \partial_\mu h \partial^\mu h$$



$$\Pi^{hh}_{\text{div}}(q^2) \sim \frac{(3a_C^2 - a_H^2)\, q^4}{2v^2} \Delta_\varepsilon + \ldots$$

renormalization of $\Box h \Box h$ is induced

3,4-point functions



On shell calculations of these types:

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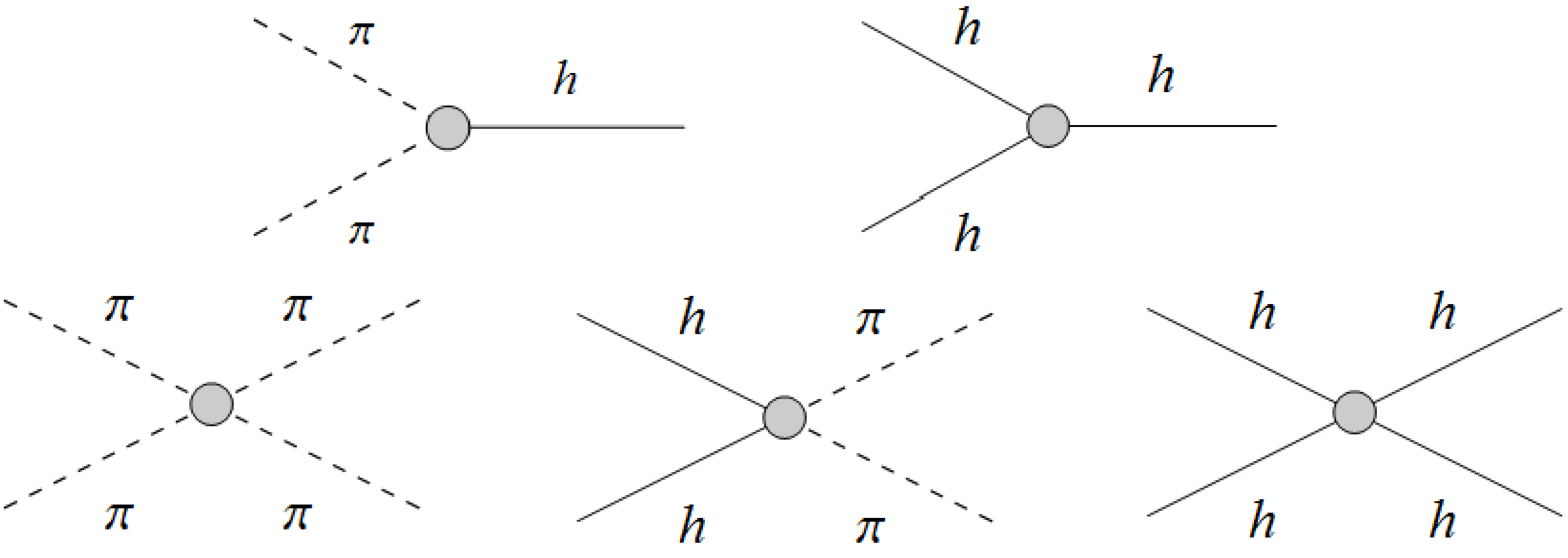
Delgado, Dobado, and Llanes-Estrada ‘13

Espriu, Mescia, Yencho ’13

Delgado, Dobado, Herrero, and Sanz-Cillero ’14

...



1. $L_{0,2,4}$ parameters renormalized as $x_b = x_{ren} + \delta x$ and δx is independent on **η**.
2. Off-shell **non-chiral-invariant divergences** are generated!!!
3. They cannot be absorbed into $L_{0,2,4}$ parameters.
4. No impact on-shell.

Non-invariant divergences

In nonlinear sigma model:

1. Using modified Feynman rules (non-covariant)

Gerstein, Jackiw, Weinberg, and Lee ’71

2. Modified background field method

Honerkamp ‘72
Kazakov, Pervushin, and Pushkin ‘76

3. Field redefinition Appelquist, Bernard ‘81

Field redefinition and NID

$$\pi_i \to \pi_i \left(1 + \frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\tilde{\gamma}_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) +$$

$$+ \frac{\alpha_3}{v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

Field redefinition and NID

$$\pi_i \to \pi_i \left(1 + \frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\tilde{\gamma}_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) + $$

$$+ \frac{\alpha_3}{v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

$$\Delta\mathcal{L} = -\boldsymbol{\pi}\Box\boldsymbol{\pi}\left(\frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\gamma_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) -$$

$$- \frac{\alpha_3}{v^4}(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})(\boldsymbol{\pi}\boldsymbol{\pi}) - \frac{\alpha_4}{v^4}(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}) - \frac{2a_C\beta}{v^4}\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\partial^\mu h\Box h,$$

Field redefinition and NID

$$\pi_i \to \pi_i \left(1 + \frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\tilde{\gamma}_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) + \\ + \frac{\alpha_3}{v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

$$\Delta\mathcal{L} = -\boldsymbol{\pi}\Box\boldsymbol{\pi}\left(\frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\gamma_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) - \\ - \frac{\alpha_3}{v^4}(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})(\boldsymbol{\pi}\boldsymbol{\pi}) - \frac{\alpha_4}{v^4}(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}) - \frac{2a_C\beta}{v^4}\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\partial^\mu h\Box h,$$

$$\boldsymbol{\pi} \to \boldsymbol{\pi} + \delta\boldsymbol{\pi}, \qquad \mathcal{L} \to \mathcal{L} + \delta\boldsymbol{\pi}\left(\frac{\delta\mathcal{L}}{\delta\boldsymbol{\pi}} - \partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\boldsymbol{\pi}}\right)$$

Field redifenition generate additional piece in Lagrangian, which is proportional to EOM.

It vanish if EOM is satisfied.

Ostrogradskiy 1850; Grosse-Knetter '94; Scherer, Fearing '94; Arzt '95

Field redefinition and NID

$$\pi_i \to \pi_i \left(1 + \frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\tilde{\gamma}_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) +$$

$$+ \frac{\alpha_3}{v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

$$\alpha_1 = \left(9\eta^2 + 5\eta + \frac{3}{4}\right)\Delta_\varepsilon$$

$$\alpha_2 = \left[\left(a_C^2 + 4\right)\eta + \frac{a_C^2}{2} + 1\right]\Delta_\varepsilon$$

$$\alpha_3 = 2\eta^2\Delta_\varepsilon$$

$$\alpha_4 = 2(a_C^2 - 1)\eta\Delta_\varepsilon$$

$$\gamma_1 = \left(5\eta + \frac{3}{2}\right)\left(2a_C^2 - b_C\right)\Delta_\varepsilon$$

$$\gamma_2 = \left(5\eta + \frac{3}{2}\right)\left(a_C^2 - b_C\right)\Delta_\varepsilon$$

$$\beta = -\left(5\eta + \frac{3}{2}\right)a_C\Delta_\varepsilon$$

Only non physical divergences depend on η.

Field redefinition and NID

$$\pi_i \to \pi_i \left(1 + \frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\tilde{\gamma}_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right) + \\ + \frac{\alpha_3}{v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

$$\alpha_1 = \left(9\eta^2 + 5\eta + \frac{3}{4}\right)\Delta_\varepsilon$$

$$\alpha_2 = \left[\left(a_C^2 + 4\right)\eta + \frac{a_C^2}{2} + 1\right]\Delta_\varepsilon$$

$$\alpha_3 = 2\eta^2\Delta_\varepsilon$$

$$\alpha_4 = 2(a_C^2 - 1)\eta\Delta_\varepsilon$$

$$\gamma_1 = \left(5\eta + \frac{3}{2}\right)\left(2a_C^2 - b_C\right)\Delta_\varepsilon$$

$$\gamma_2 = \left(5\eta + \frac{3}{2}\right)\left(a_C^2 - b_C\right)\Delta_\varepsilon$$

$$\beta = -\left(5\eta + \frac{3}{2}\right)a_C\Delta_\varepsilon$$

$\eta = -3/10$ eliminate parameters of pion-through-higgs field redefinition. (to our knowledge) it does not correspond to any known U-matrix parameterization.

Summary

• off shell 1 loop structure of non-linear effective theory for dynamical higgs is analyzed
• complete set of operators needed is identified e.g.

• Some of parameters may have significantly large coefficients of beta-functions -> RGE

• Non-chiral-invariant divergences have been found. It has been shown that they do not have impact on on-shell quantities and can be eliminated by proper field redefinition