

# Some Aspects of Leptogenesis

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Invisibles Workshop, Paris, 2014

# I. Leptogenesis & Neutrino Masses

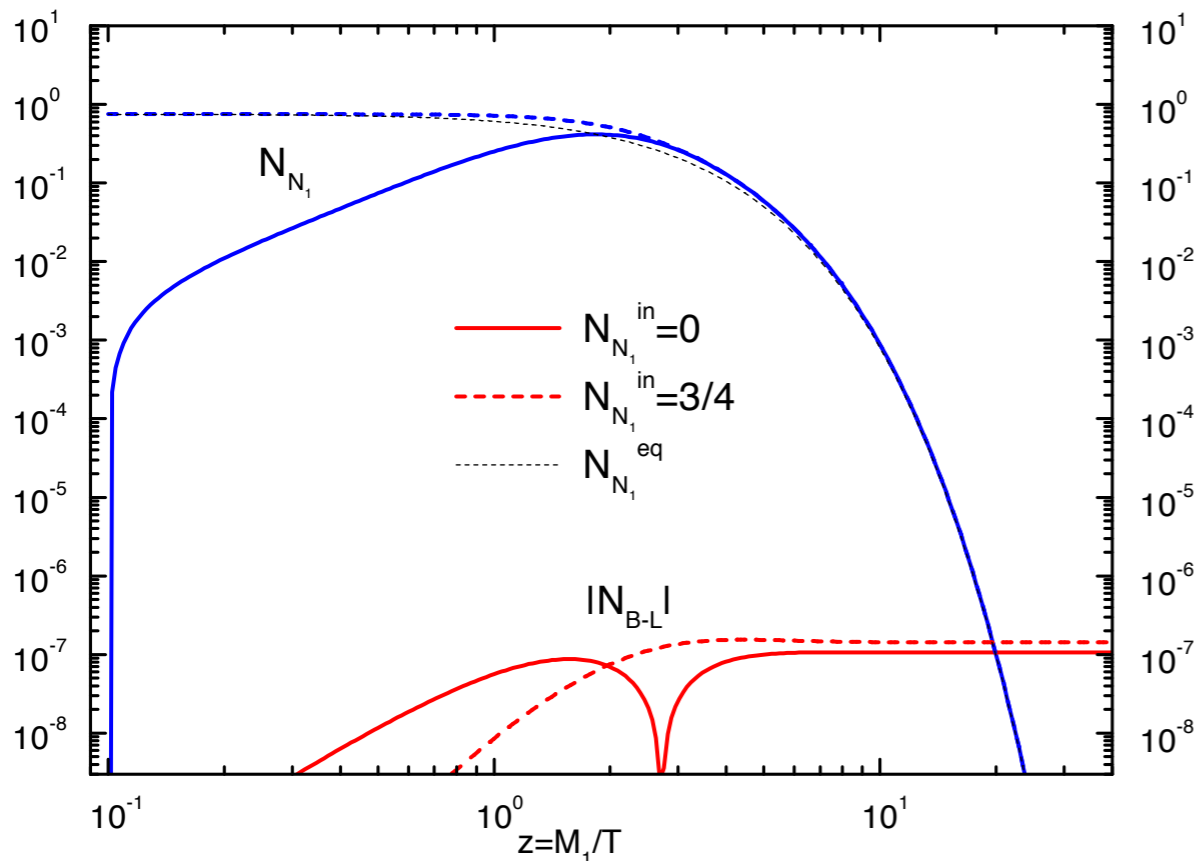
Cosmological baryon asymmetry successfully explained by leptogenesis [Fukugita & Yanagida '86], direct consequence of seesaw mechanism (in the following: hierarchical right-handed neutrinos); lepton asymmetry from CP violating interactions of heavy Majorana neutrinos in thermal bath of standard model particles, partial conversion to baryon asymmetry via sphaleron processes; nonequilibrium process via Boltzmann equations:

$$\frac{dN_1}{dz} = -(N_1 - N_1^{\text{eq}})(D + S),$$
$$\frac{dN_{B-L}}{dz} = -(N_1 - N_1^{\text{eq}})\varepsilon_1 D - N_{B-L}W$$

CP violation and baryon asymmetry:

$$\eta_B \sim 10^{-4} \varepsilon_1, \quad \varepsilon \leq \varepsilon_{\text{max}} \sim 10^{-6} \left( \frac{M_1}{10^{10} \text{GeV}} \right)$$

# Nonequilibrium process



Rough estimate for CP asymmetry (hierarchical heavy neutrinos) and baryon asymmetry:

$$\varepsilon_1 \sim 0.1 \frac{M_1 m_3}{v_F^2}$$

$$\sim 0.1 \frac{M_1}{M_3} \sim 10^{-5} \dots 10^{-6}$$

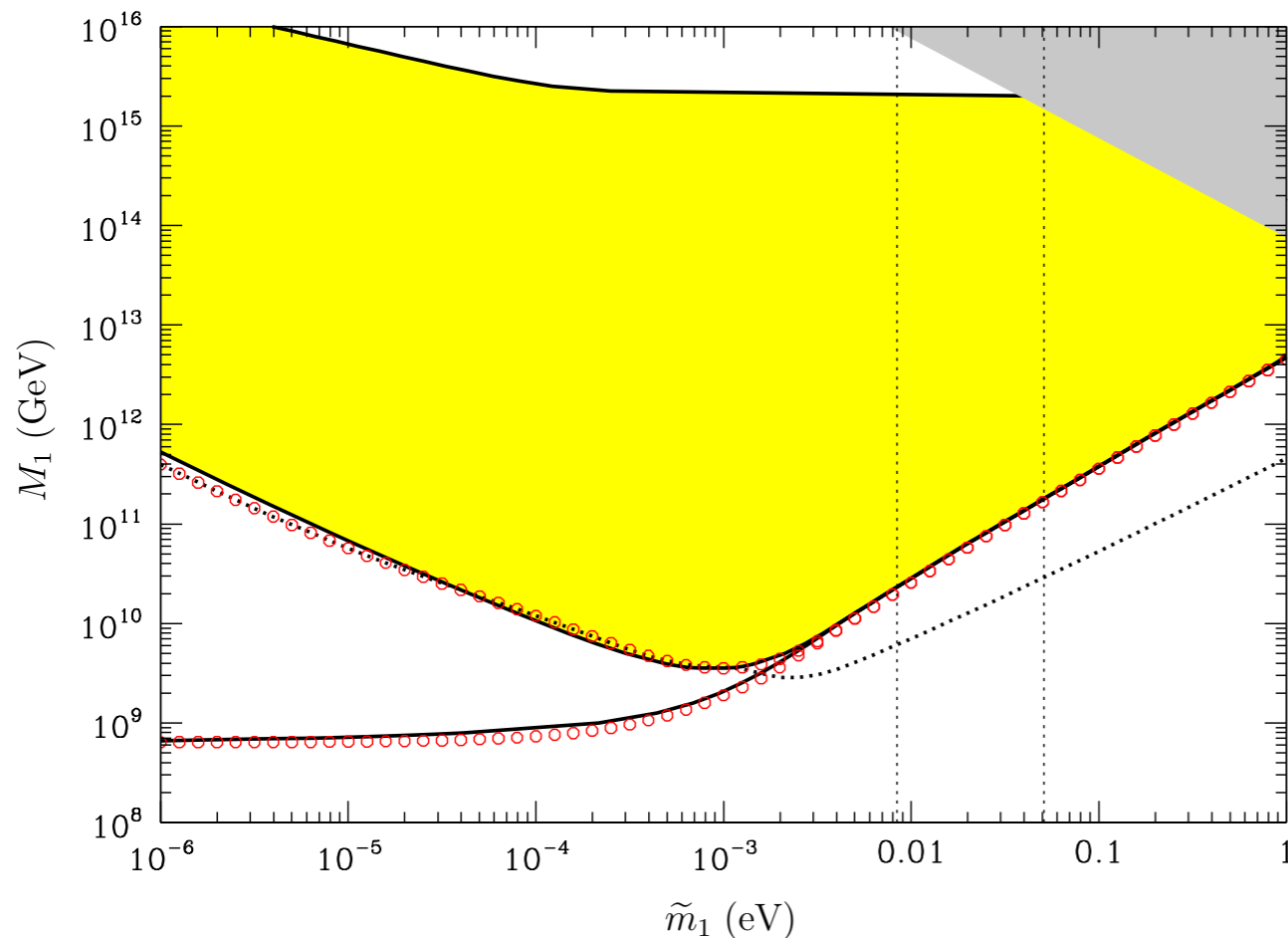
$$\eta_B \simeq -\frac{c_s}{f} N_{B-L} \simeq 10^{-2} \varepsilon_1 \kappa_f$$

Neutrino masses suggest that leptogenesis is process close to thermal equilibrium, i.e.  $\Gamma_1 \sim H|_{T=M_1}$ ; in terms of neutrino masses:

$$\tilde{m} = \frac{(m_D m_D^\dagger)_{11}}{M_1} \sim m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v_F^2}{M_P} \simeq 10^{-3} \text{ eV}$$

confirmed by solution of Boltzmann equations.

# Constraints on neutrino masses



Detailed study of Boltzmann equations leads to bound on light and heavy neutrino masses (and reheating temperature); in simplest approximation (sum over lepton flavours):

$$m_i < 0.1 \text{ eV} , \quad M_1 > 4 \times 10^8 \text{ GeV}$$

Preferred neutrino mass range (“strong washout regime”, independence of initial conditions):

$$10^{-3} \text{ eV} < m_i < 0.1 \text{ eV}$$

*modifications:* lepton flavour effects, neutrino mass degeneracies; *wanted:* more information on neutrino mass matrix!

# How robust is leptogenesis?

Instructive example: leptogenesis by Monte Carlo [[Lu & Murayama, 1405.0547](#)];  
input: anarchy & U(1) flavour symmetry:

$$\Delta\mathcal{L} \supset -\epsilon^{ab} \bar{L}_a H_b^\dagger y_\nu \nu_R - \frac{1}{2} \bar{\nu}_R^c m_R \nu_R + h.c.$$

$$y_\nu \sim O(1), \quad m_R \sim \mathcal{M} \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

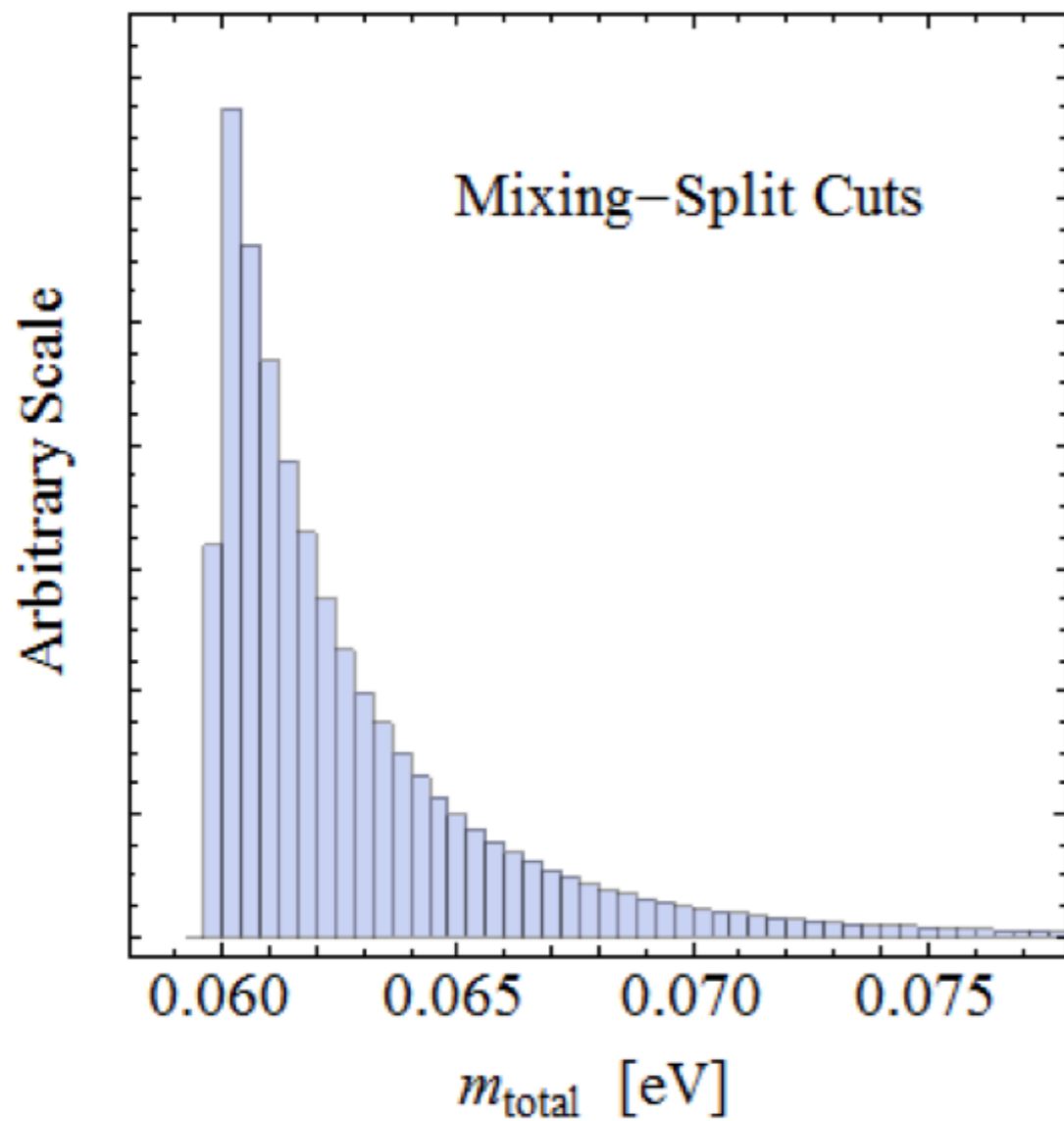
with  $\epsilon \simeq 0.1$  and  $\mathcal{M}$  fixed by  $\Delta m_l^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ; random coefficients (Gaussian measure); data (mixing-split cuts):

$$\sin^2 2\theta_{23} = 1.0$$

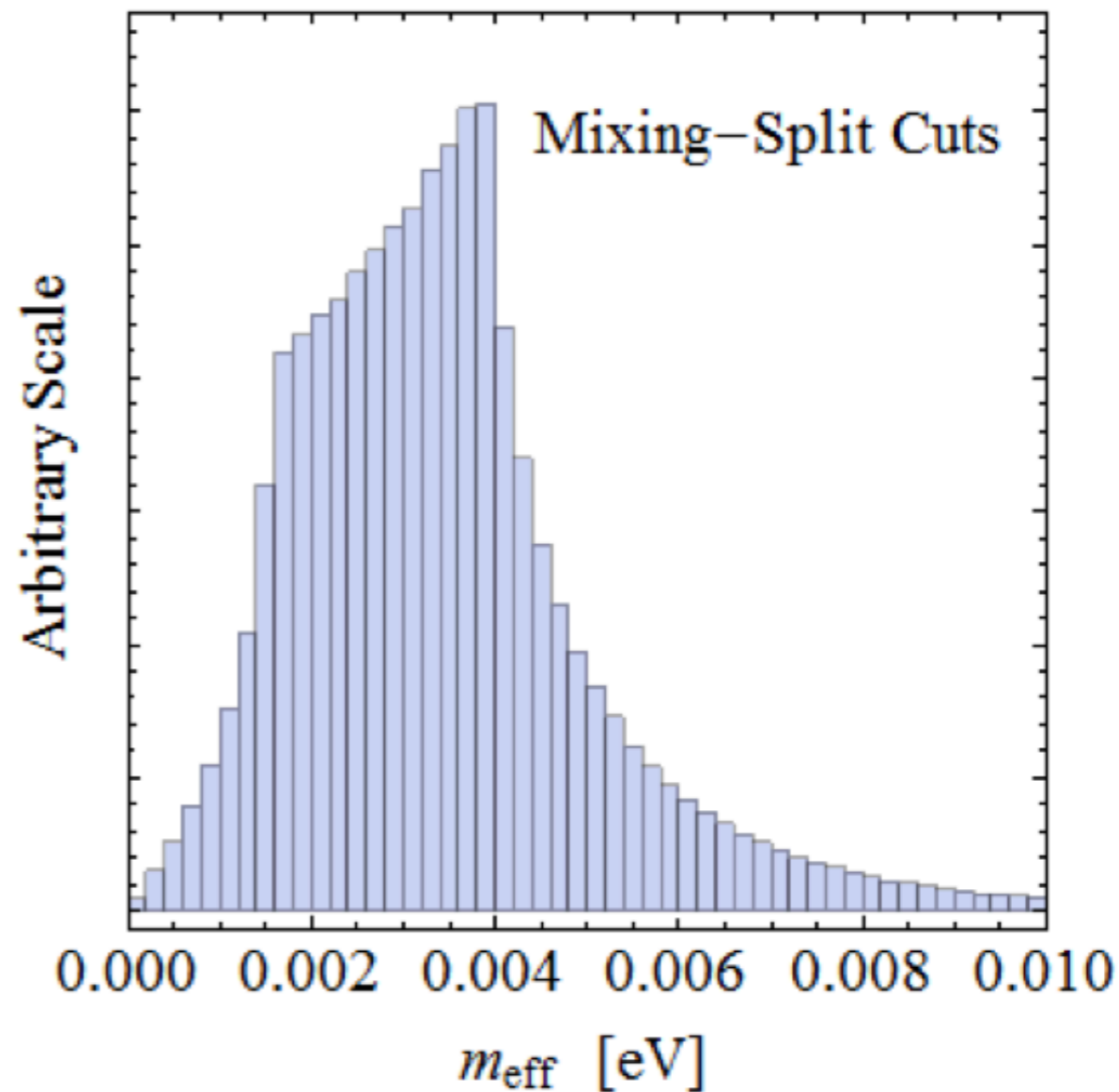
$$\sin^2 2\theta_{12} = 0.857$$

$$\sin^2 2\theta_{13} = 0.095$$

$$R = \frac{\Delta m_s^2}{\Delta m_l^2} \in R_{\text{exp}} \times (1 - 0.05, 1 + 0.05)$$

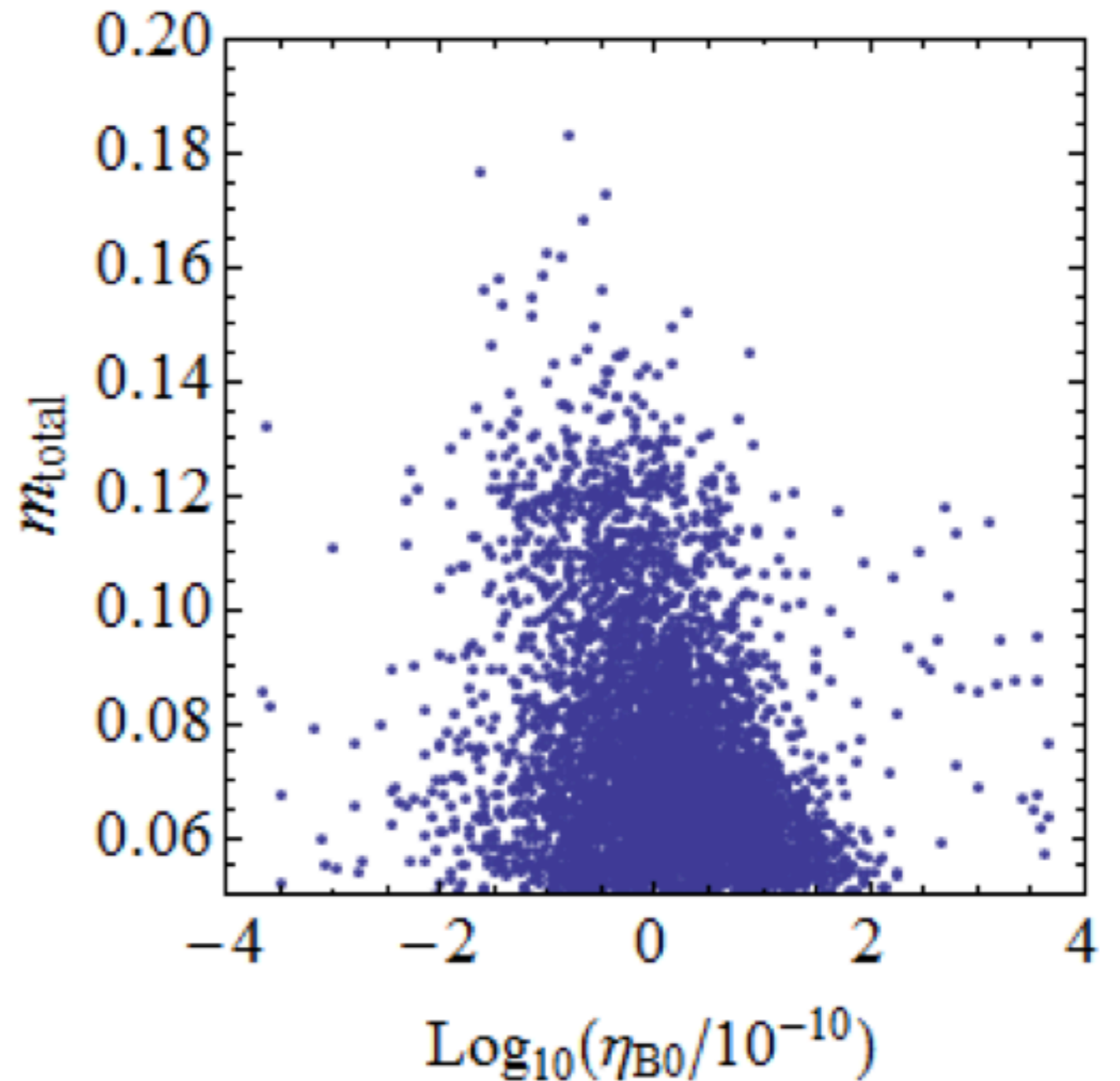
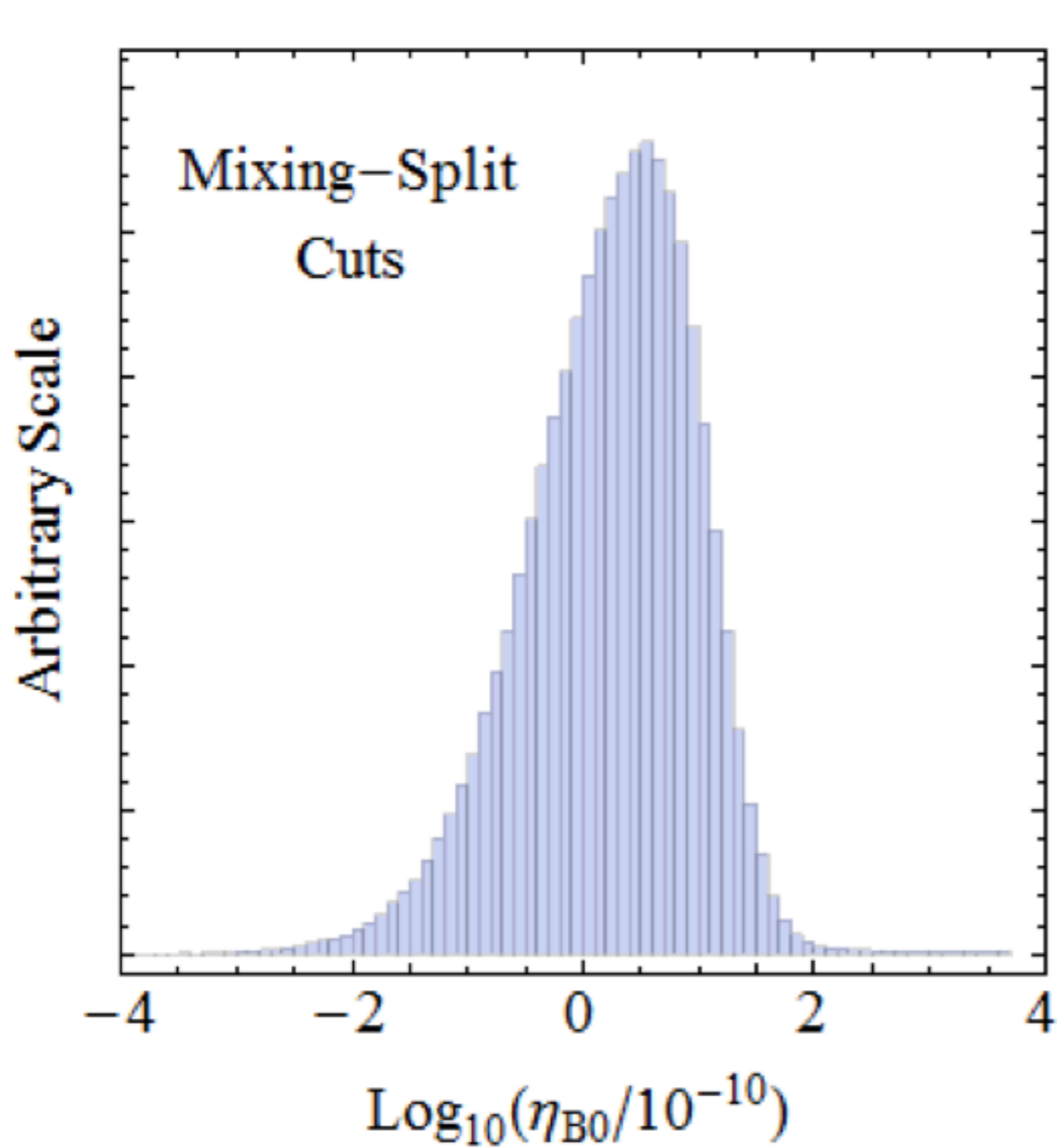


$$m_{\text{total}} \equiv m_1 + m_2 + m_3$$



$$m_{\text{eff}} \equiv \left| \sum_i m_i U_{v,ei}^2 \right|$$

(normal hierarchy dominant; inverted hierarchy only 0.1%! )



Baryon asymmetry peaks at observed value! Leptogenesis likes small neutrino masses (consistent with upper bound on effective neutrino mass)

For comparison: semi-anarchy & U(1) flavour symmetry

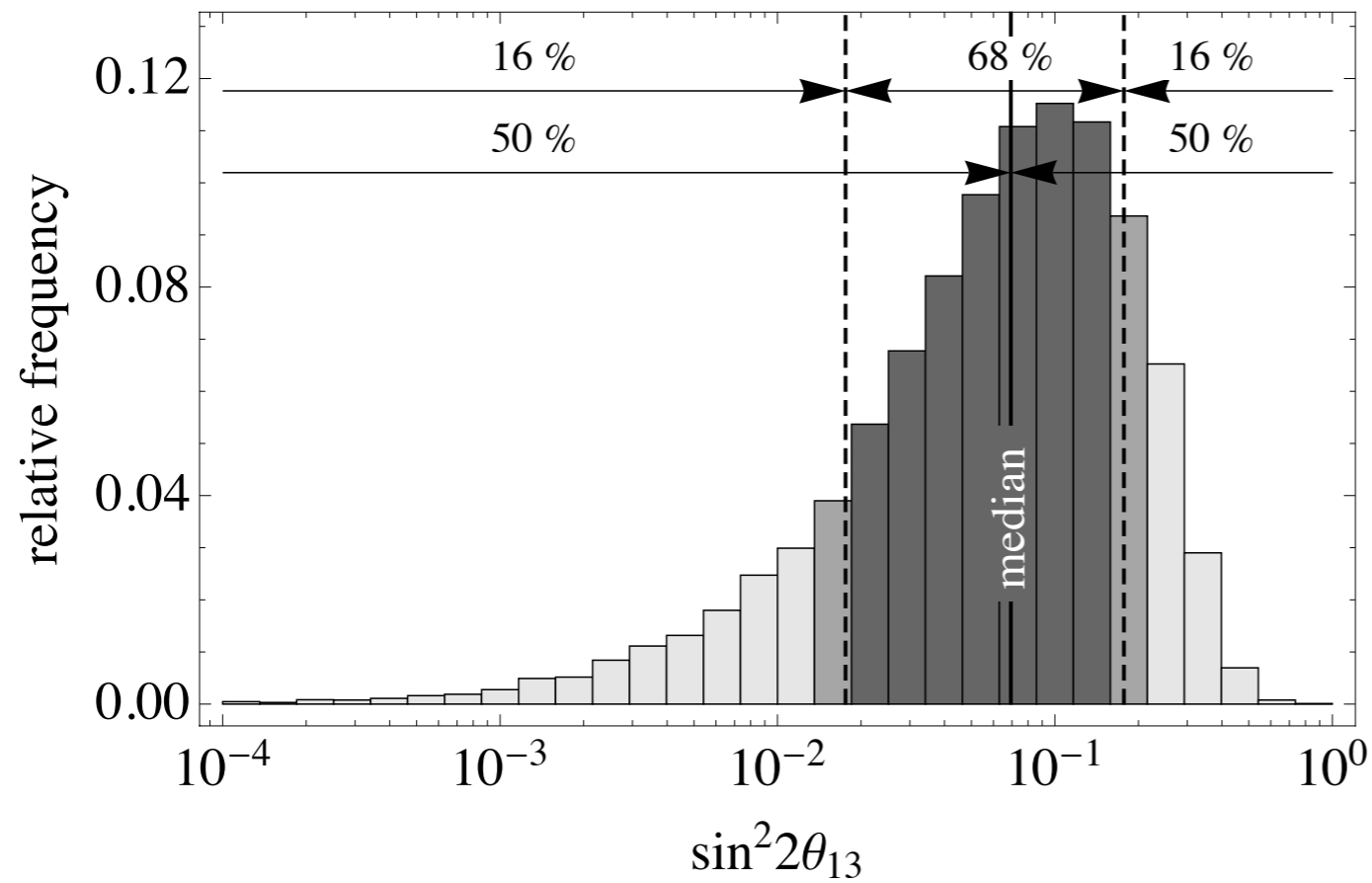
[WB, Domcke, Schmitz | | | 1.3872]; input:

$$h^{(\nu)} \sim \eta^a \begin{pmatrix} \eta^{d+1} & \eta^{c+1} & \eta^{b+1} \\ \eta^d & \eta^c & \eta^b \\ \eta^d & \eta^c & \eta^b \end{pmatrix}, \quad M_R \sim \begin{pmatrix} \eta^{2d} & 0 & 0 \\ 0 & \eta^{2c} & 0 \\ 0 & 0 & \eta^{2b} \end{pmatrix},$$
$$\rightarrow m_\nu \sim \eta^{2a} \begin{pmatrix} \eta^2 & \eta & \eta \\ \eta & 1 & 1 \\ \eta & 1 & 1 \end{pmatrix}, \quad h^{(e)} \sim \eta^a \begin{pmatrix} \eta^3 & \eta^2 & \eta \\ \eta^2 & \eta & 1 \\ \eta^2 & \eta & 1 \end{pmatrix}$$

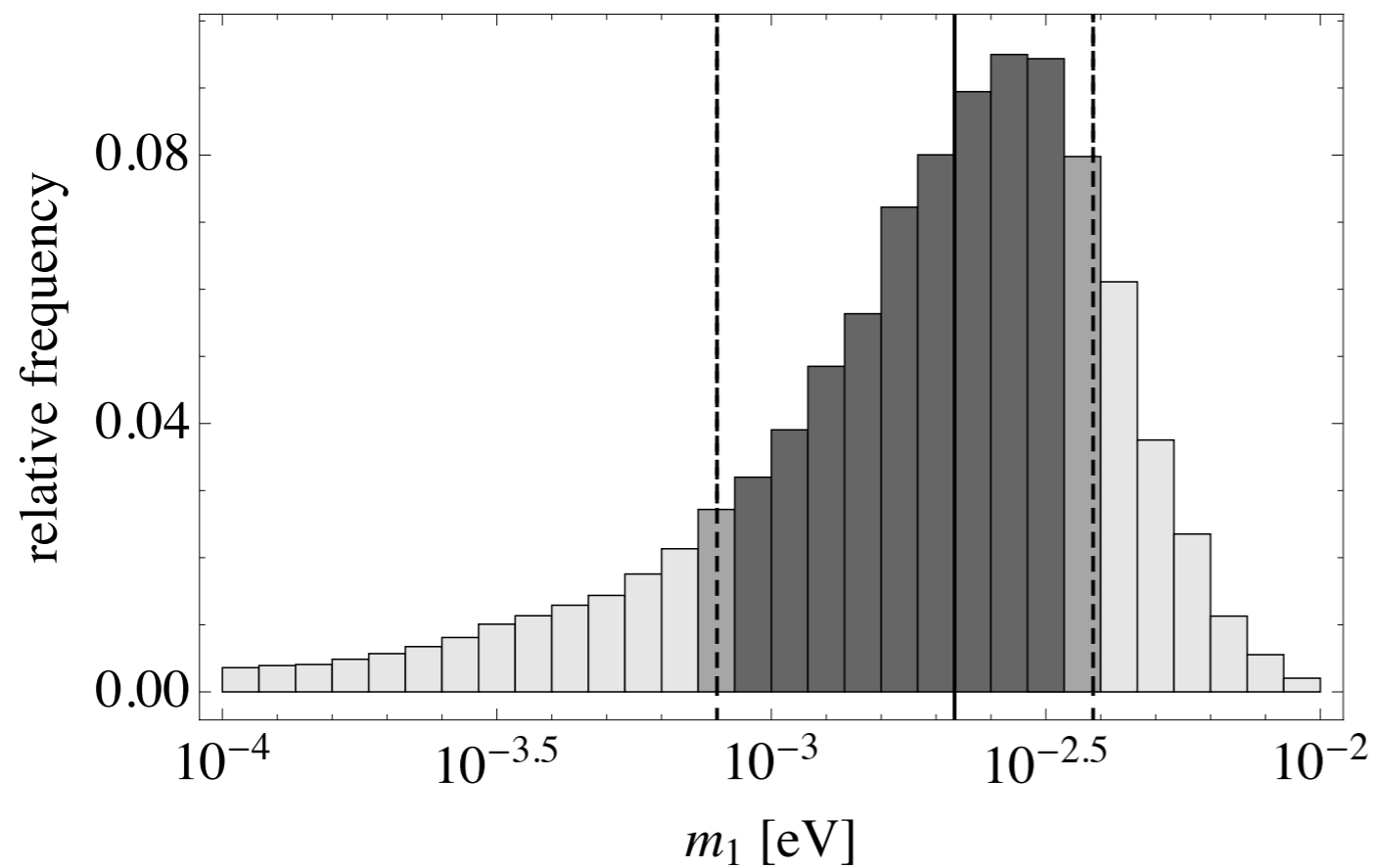
with  $0 \leq a \leq 1$  ( $\tan \beta$ );  $b \leq c \leq d$ ;  $a + d = 2$ ; **39** real parameters, random numbers  $\mathcal{O}(1)$ , uniform on logarithmic scale; data:

$$2.07 \times 10^{-3} \text{ eV}^2 \leq |\Delta m_{\text{atm}}^2| \leq 2.75 \times 10^{-3} \text{ eV}^2,$$
$$7.05 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{\text{sol}}^2 \leq 8.34 \times 10^{-5} \text{ eV}^2,$$
$$0.75 \leq \sin^2(2\theta_{12}) \leq 0.93,$$
$$0.88 \leq \sin^2(2\theta_{23}) \leq 1$$





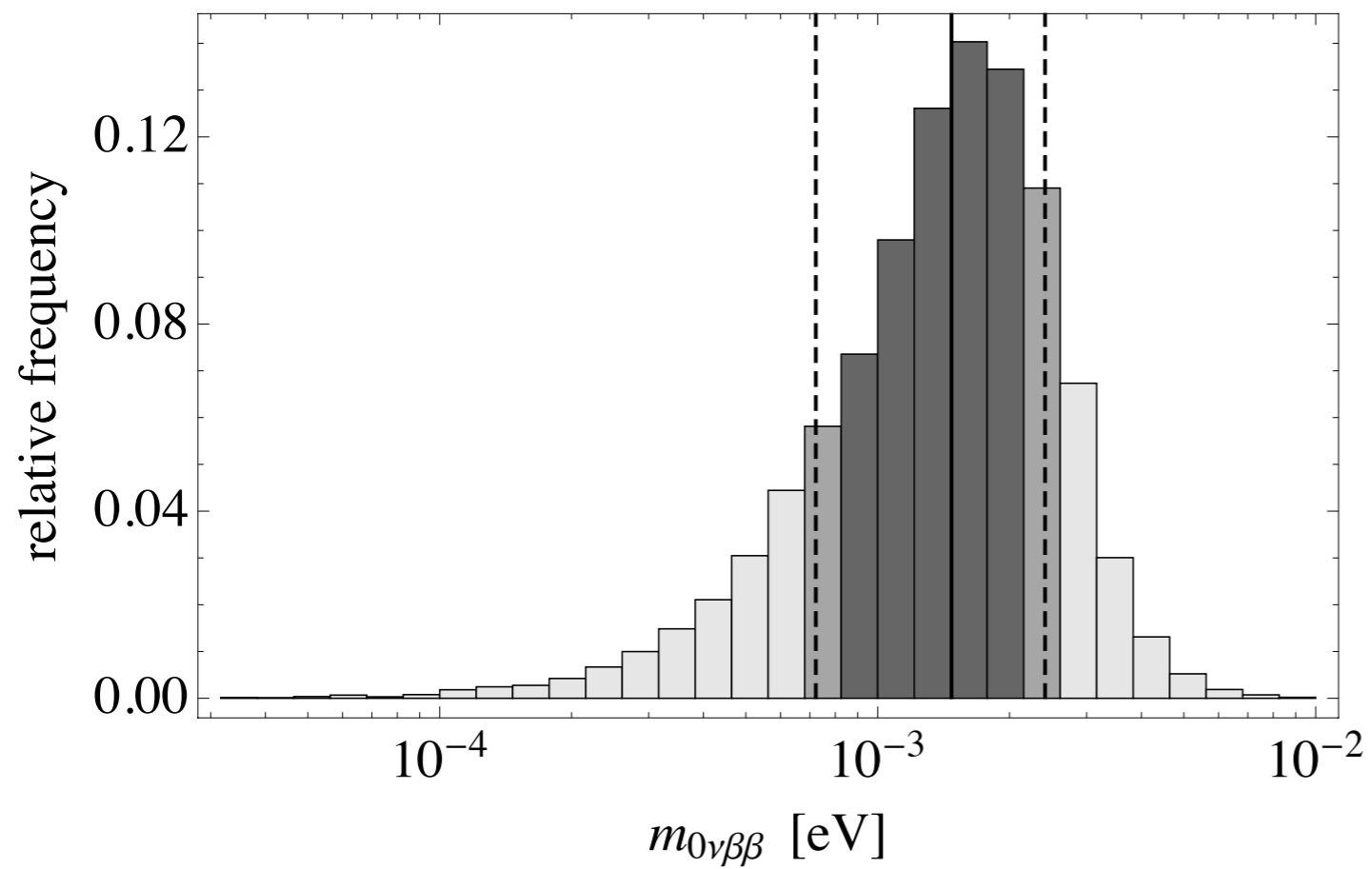
successful “prediction”!



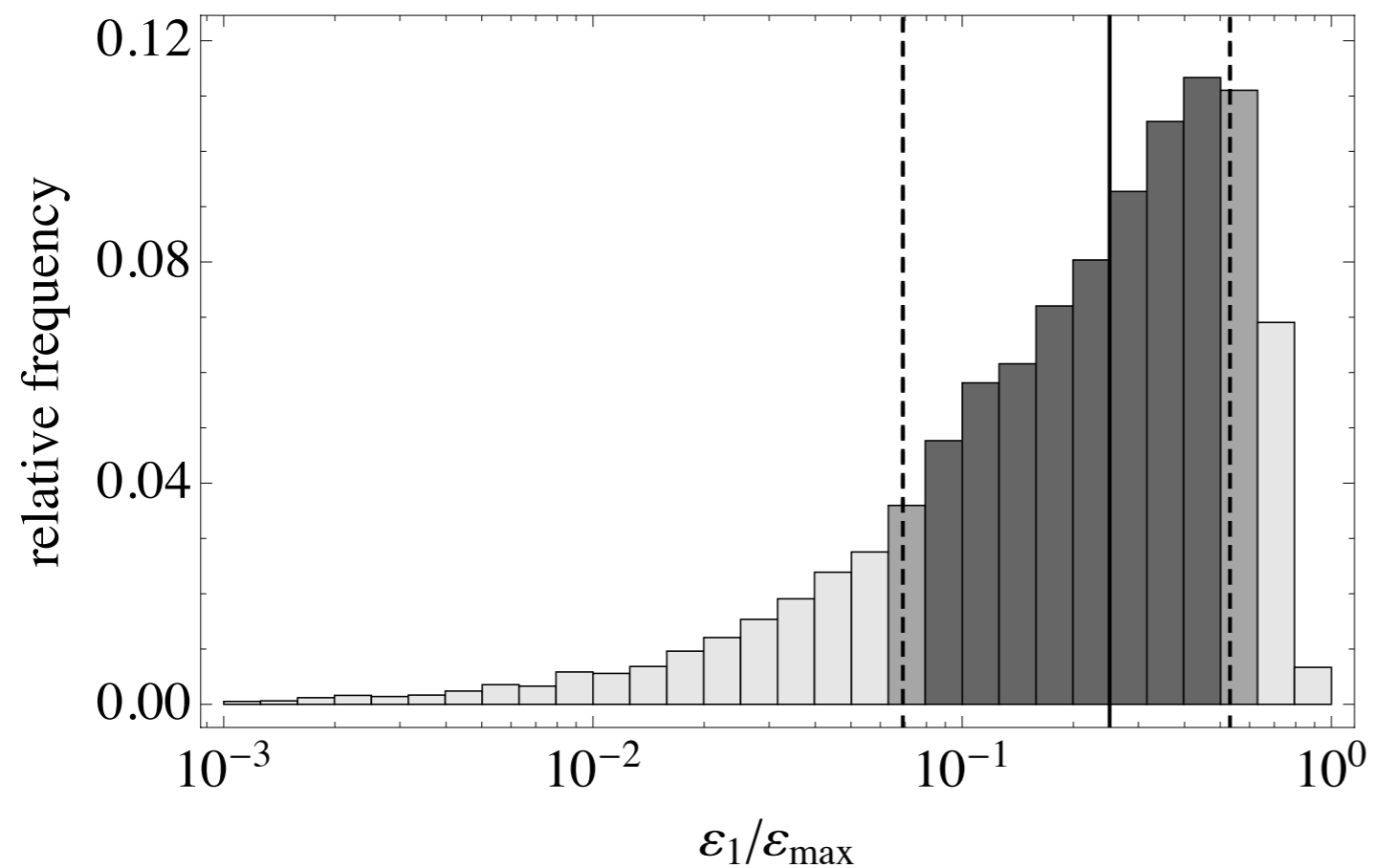
corresponds to

$$m_{\text{tot}} = 6.0^{+0.3}_{-0.3} \times 10^{-2} \text{ eV}$$

consistent with Lu/  
Murayama



consistent with  
Lu/Murayama



mean corresponds to

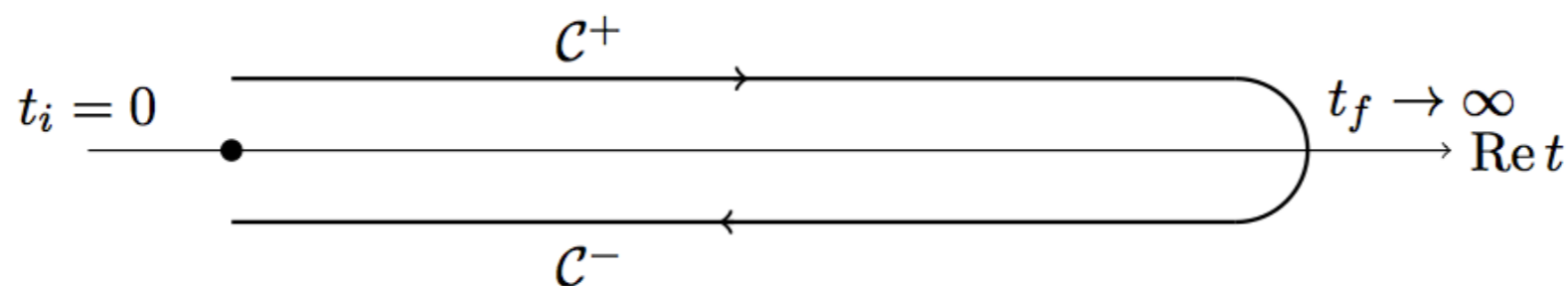
$$\eta_B \sim 5 \times 10^{-10}$$

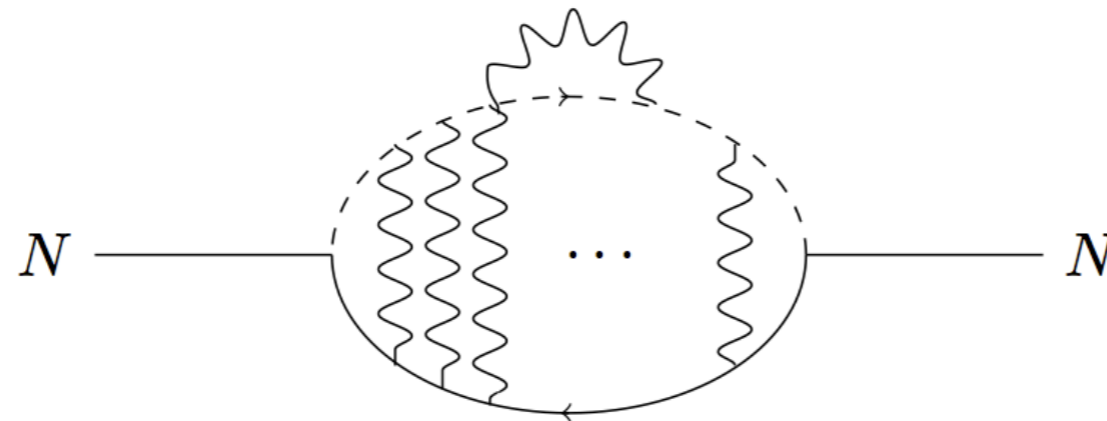
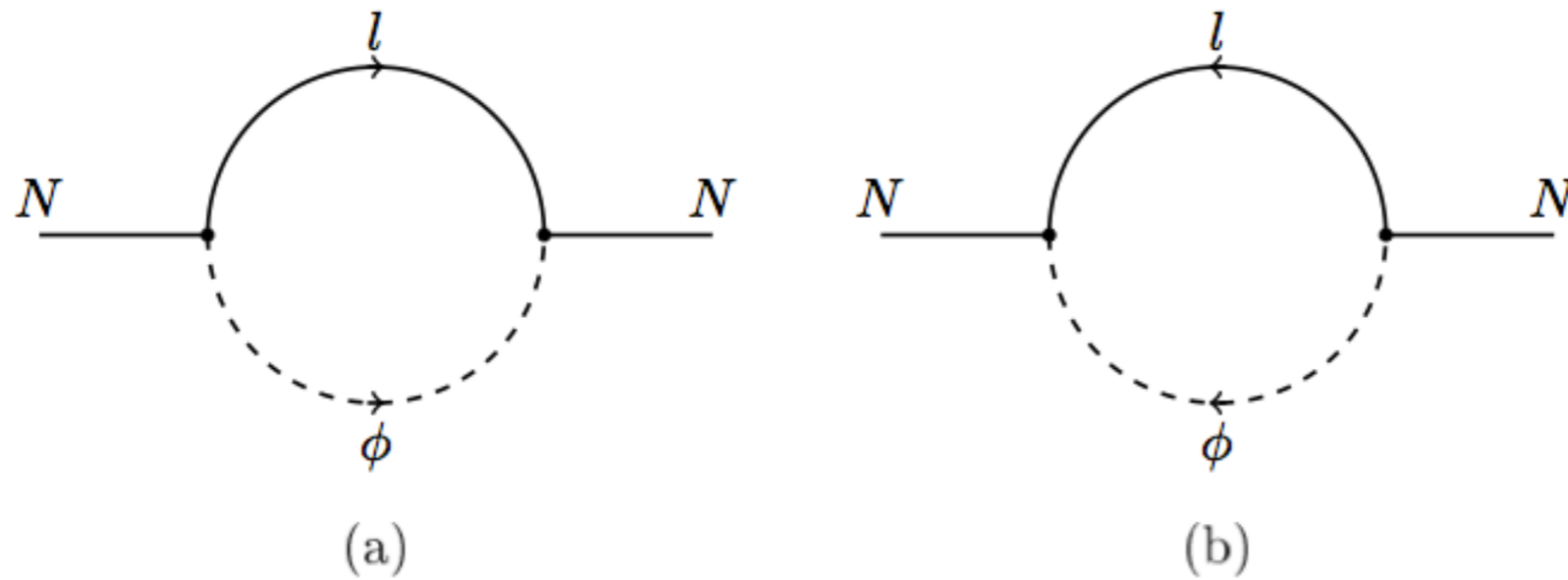
observed baryon asymmetry  
“likely” for leptogenesis with  
hierarchical heavy neutrinos!

# II. Leptogenesis in Thermal QFT

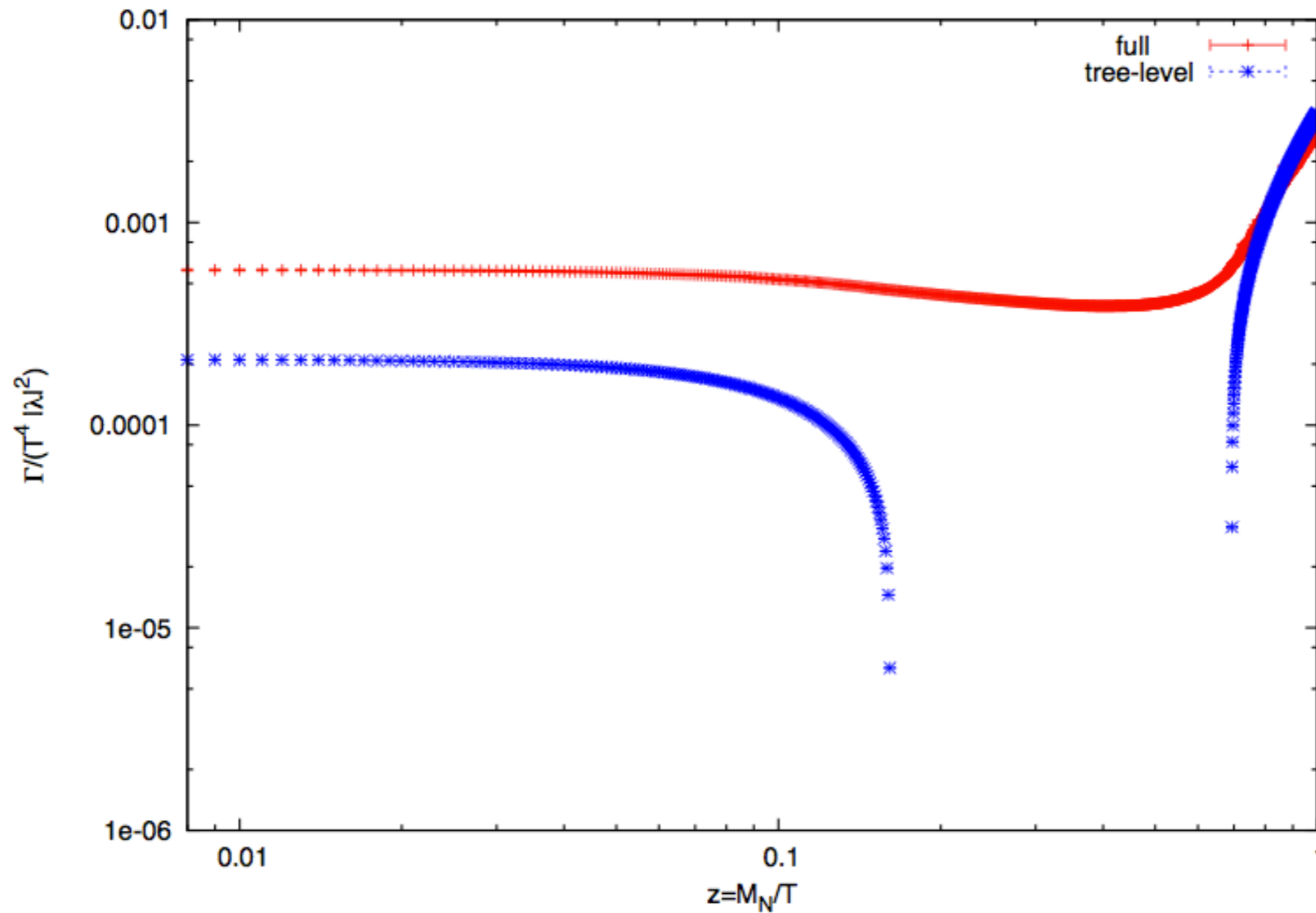
Leptogenesis is “simple” nonequilibrium process: weakly coupled heavy neutrino in large SM thermal bath, close to thermal equilibrium; rigorous treatment in nonequilibrium QFT? [Hutig, Mendizabal, Philipsen (thesis, Frankfurt, 2013, paper 14xx.yyyy)]; Schwinger-Keldysh formalism yields Kadanoff-Baym equations for spectral functions and statistical propagators of lepton doublets and heavy Majorana neutrino:

$$\begin{aligned}
 (\partial_{t_1}^2 + \omega_{\mathbf{q}}^2) \Delta_{\mathbf{q}}^-(t_1 - t_2) &= - \int_{t_2}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1 - t') \Delta_{\mathbf{q}}^-(t' - t_2) \\
 (\partial_{t_1}^2 + \omega_{\mathbf{q}}^2) \Delta_{\mathbf{q}}^+(t_1, t_2) &= \int_{t_i}^{t_2} dt' \Pi_{\mathbf{q}}^+(t_1 - t') \Delta_{\mathbf{q}}^-(t' - t_2) \\
 &\quad - \int_{t_i}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1 - t') \Delta_{\mathbf{q}}^+(t', t_2),
 \end{aligned}$$

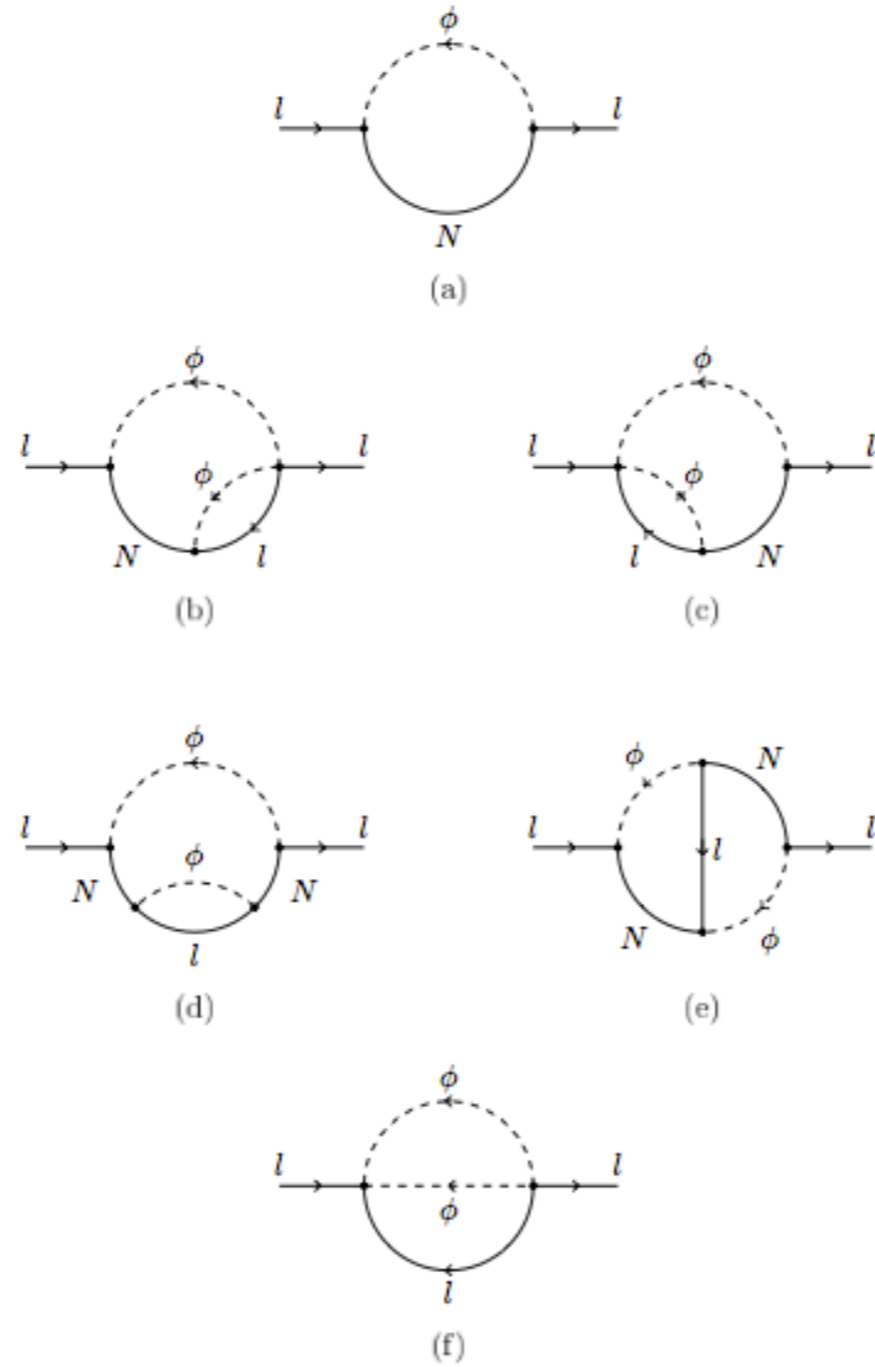




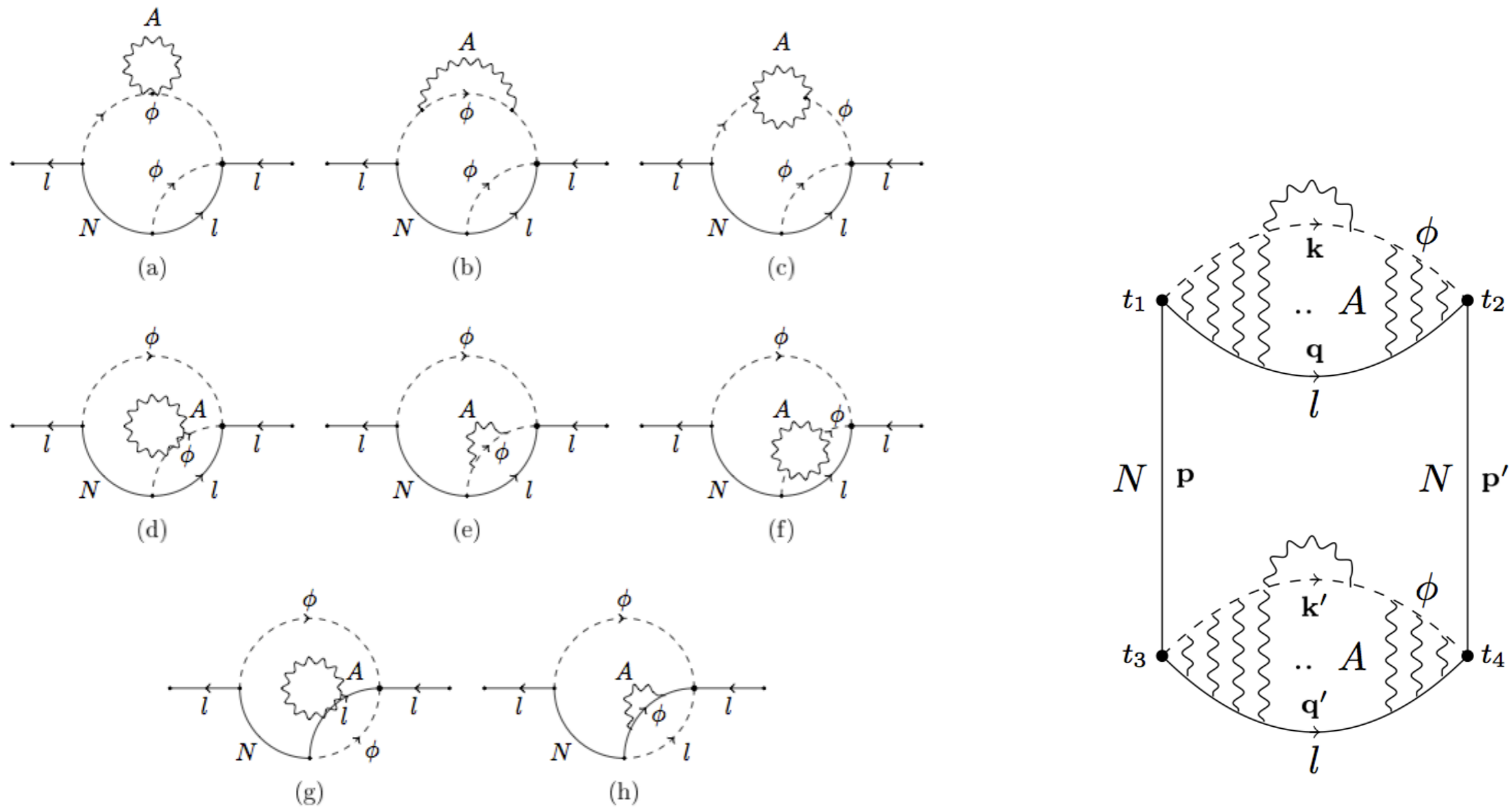
production and decay of heavy neutrino from/into lepton-Higgs pairs in thermal bath; resummation of soft gauge bosons (collinear thermal loops)



resummation of gauge bosons leads to smooth behaviour of heavy neutrino production rate as function of temperature



generation of lepton asymmetry (CP violation!) starts at two loops;  
 cut: interference of tree-level and one-loop diagrams



difficult problem: resummation of soft gauge bosons; compact analytical expressions obtained, numerical analysis in progress; expected result: thermal damping in the bath leads to local (in time) equation for lepton density matrix, including corrections from quantum statistics and propagator effects; systematic expansion in couplings...

# III. Leptogenesis & supersymmetry

see WB, Domcke, Kamada, Schmitz '12, '13, '14

Potential gravitino problem (standard neutralino WIMP excluded!). Thermal production can yield observed amount of dark matter:

$$\Omega_{3/2} h^2 = C \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2, \quad C \sim 0.5$$

$\Omega_{3/2} h^2 \sim 0.1$  is natural value; but why is reheating temperature close to minimal LG temperature?

*Simple observation:* heavy neutrino decay width (for typical LG parameters)

$$\Gamma_{N_1} = \frac{\tilde{m}_1}{8\pi} \left( \frac{M_1}{v_F} \right)^2 \sim 10^3 \text{ GeV}, \quad \tilde{m}_1 \sim 0.01 \text{ eV}, \quad M_1 \sim 10^{10} \text{ GeV}$$

yields reheating temperature (for gas of decaying heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma_{N_1}^0 M_P} \sim 10^{10} \text{ GeV}$$

wanted for gravitino DM. *Intriguing hint or misleading coincidence?*



# Spontaneous B-L breaking & false vacuum decay

Supersymmetric SM with right-handed neutrinos:

$$W_M = h_{ij}^u \mathbf{10}_i \mathbf{10}_j H_u + h_{ij}^d \mathbf{5}_i^* \mathbf{10}_j H_d + h_{ij}^\nu \mathbf{5}_i^* n_j^c H_u + h_i^n n_i^c n_i^c S_1$$

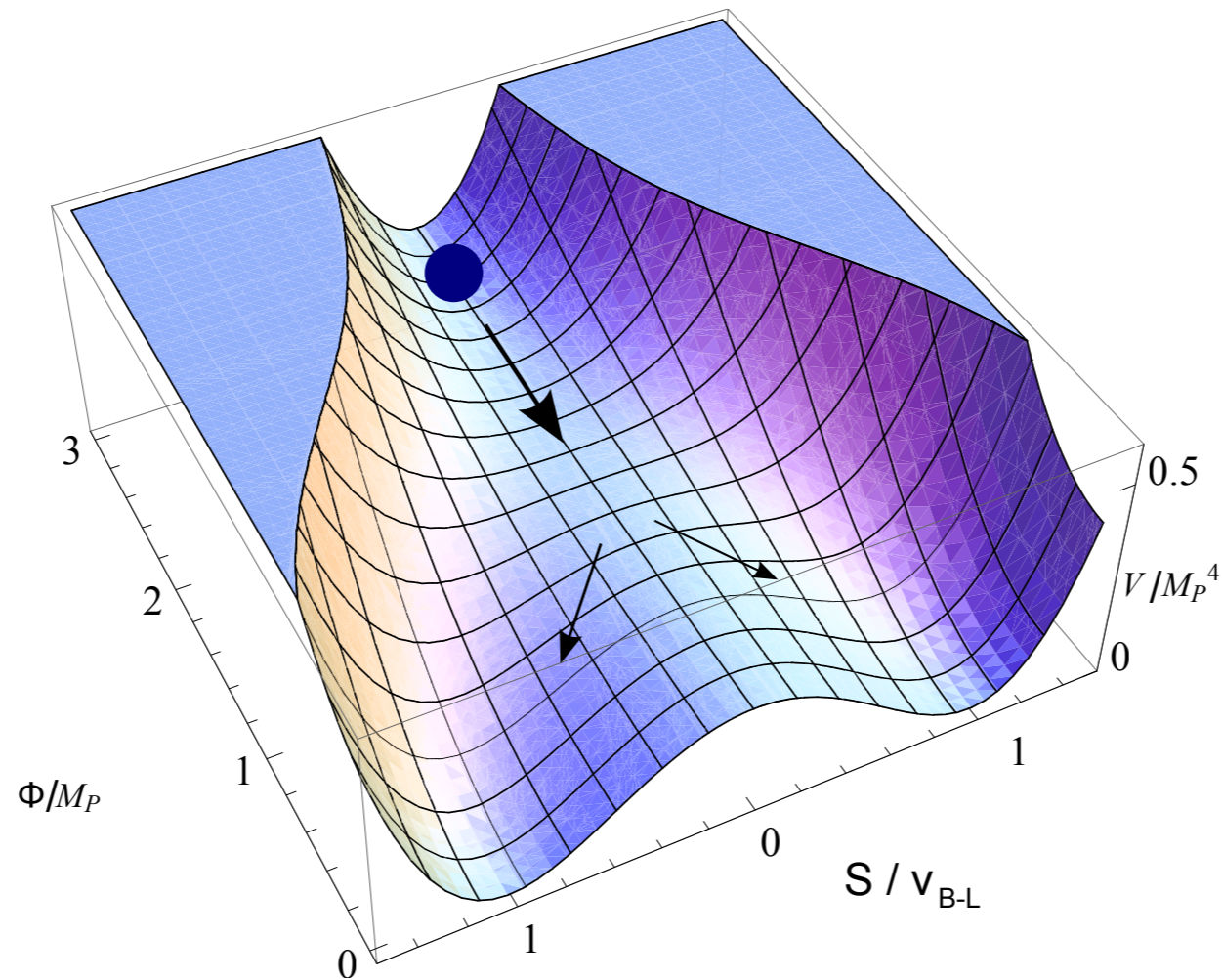
in SU(5) notation:  $\mathbf{10} \supset (q, u^c, e^c)$ ,  $\mathbf{5}^* \supset (d^c, l)$ ,  $n^c \supset (\nu^c)$ ; B-L breaking:

$$W_{B-L} = \lambda \Phi \left( \frac{1}{2} v_{B-L}^2 - S_1 S_2 \right)$$

$\langle S_{1,2} \rangle = v_{B-L} / \sqrt{2}$  yields heavy neutrino masses.

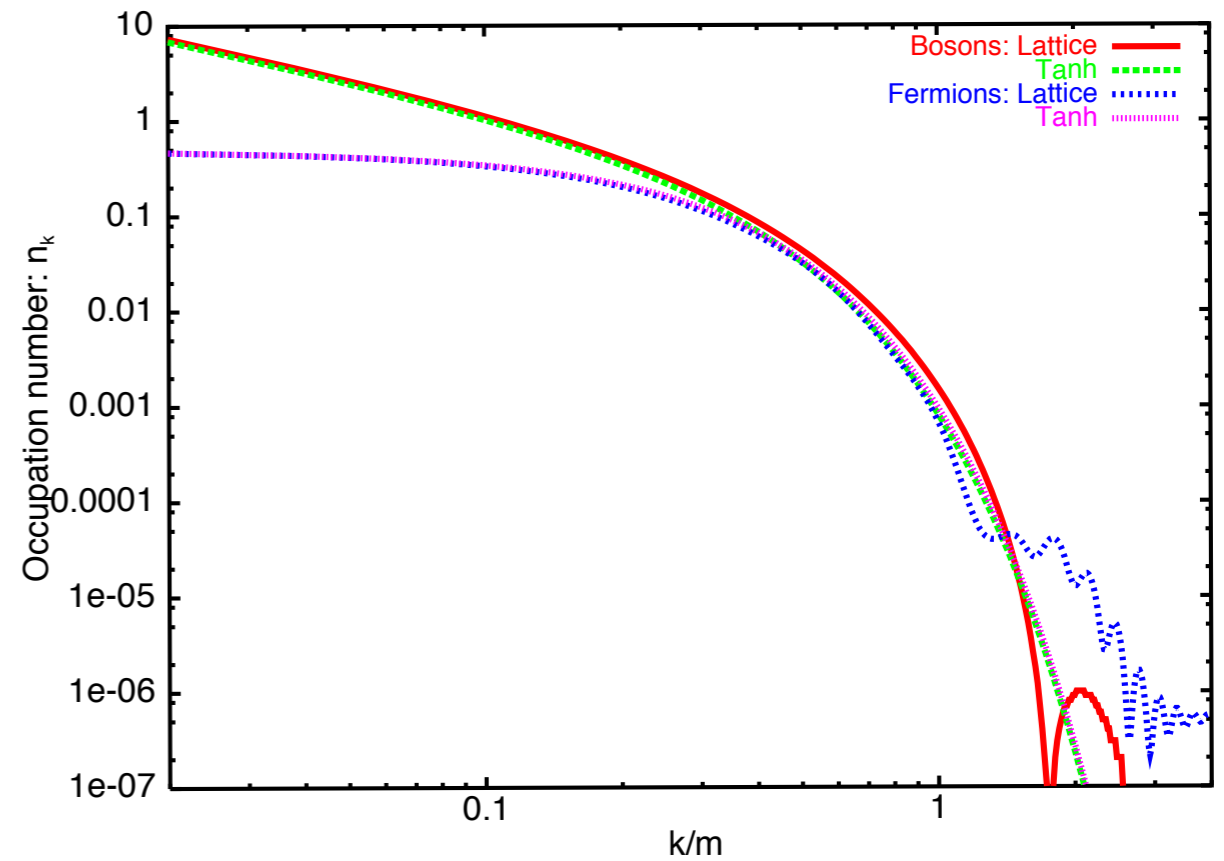
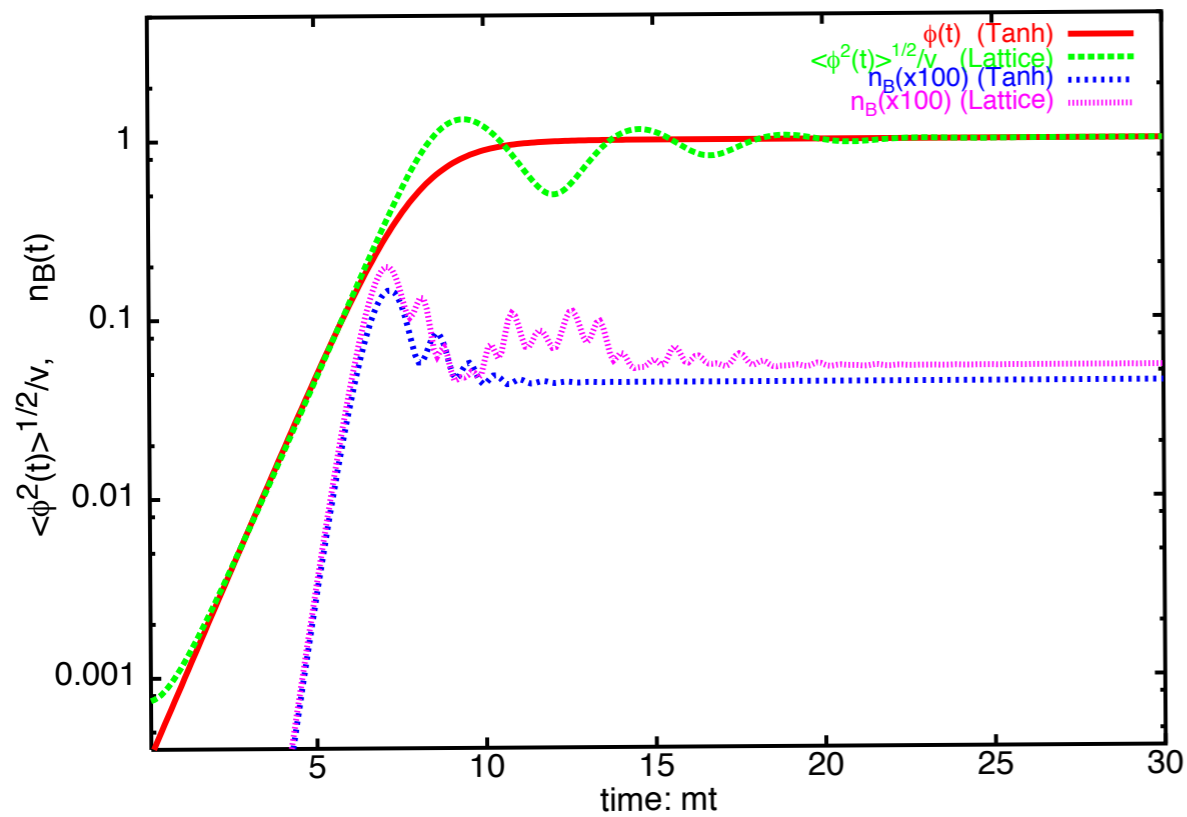
Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it *contains all ingredients wanted in cosmology*: inflation, leptogenesis, dark matter, ..., all related! [hybrid inflation: Copeland et al. '94, Dvali et al. '94].

*Technically*: Abelian Higgs model in unitary gauge; inflation ends with phase transition (“tachyonic preheating”, “spinodal decomposition”)

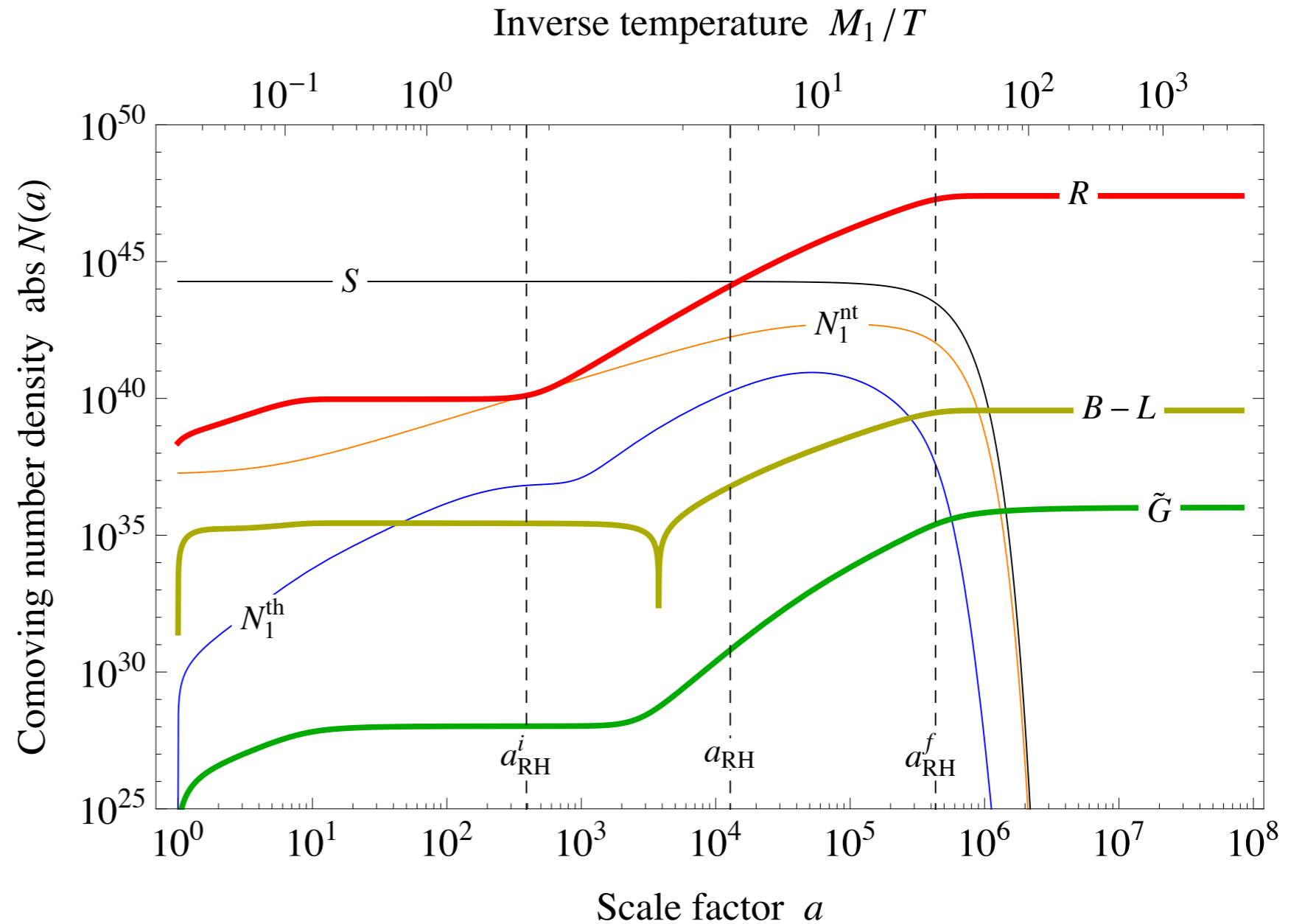
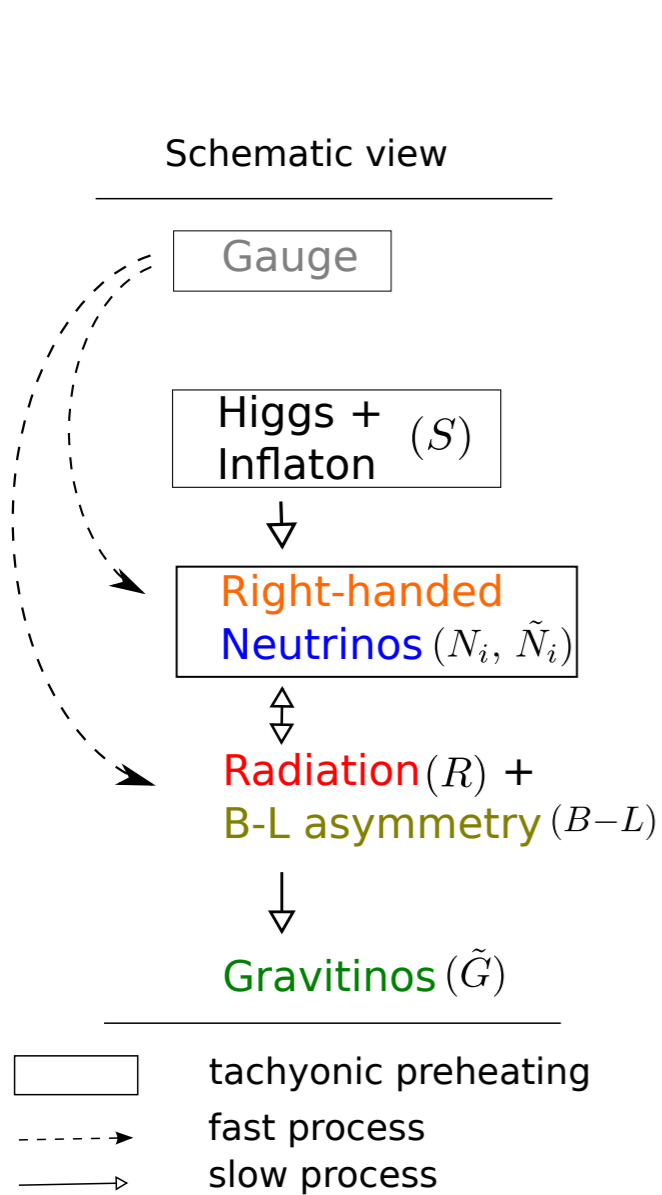


time-dependent masses of B-L Higgs, inflaton, heavy neutrinos ... (bosons and fermions):

$$m_\sigma^2 = \frac{1}{2}\lambda(3v^2(t) - v_{B-L}^2) , \quad m_\phi^2 = \lambda v^2(t) , \quad M_i^2 = (h_i^n)^2 v^2(t) \dots$$



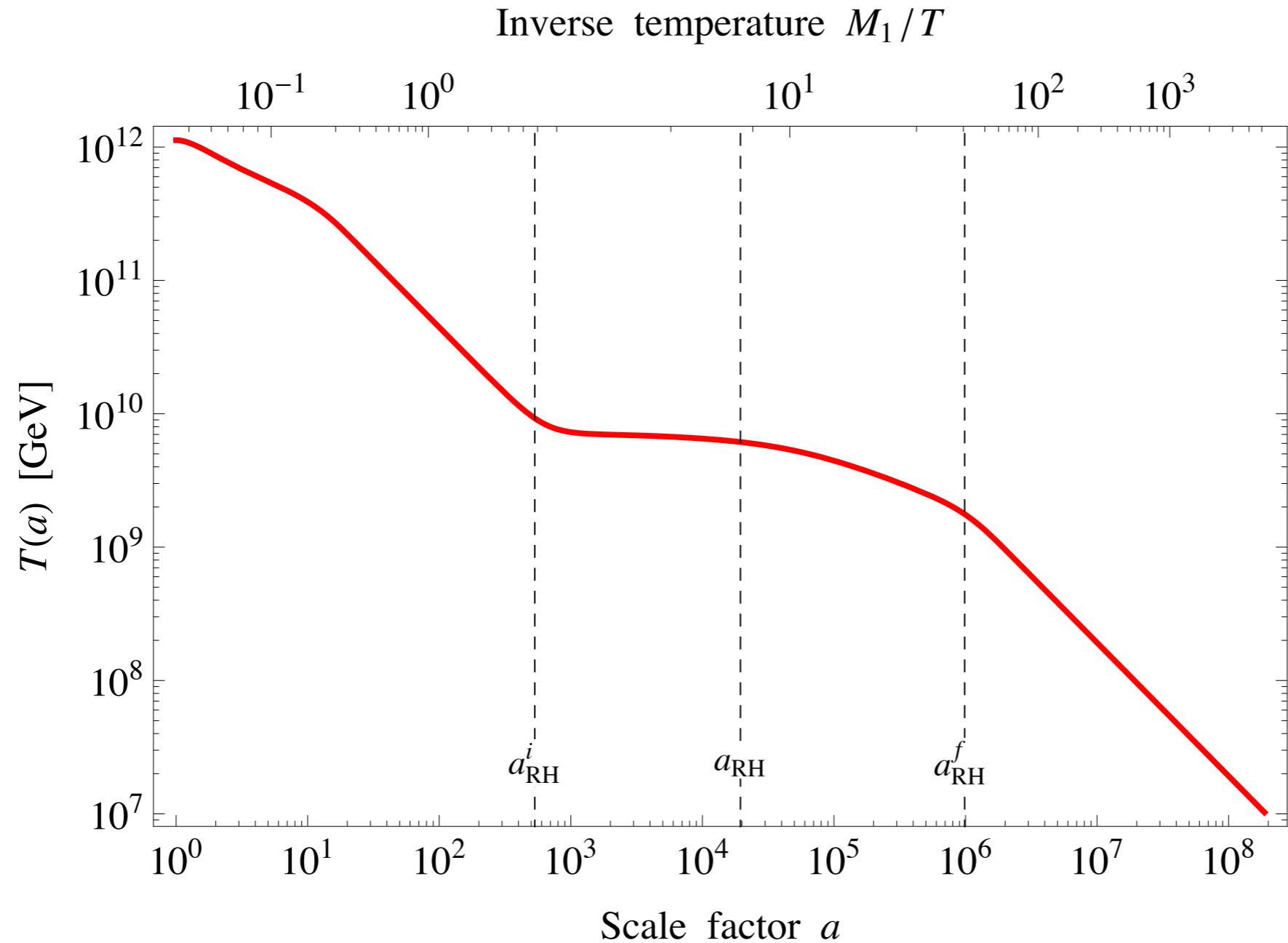
Rapid transition from false to true vacuum by fluctuations of ‘waterfall’ B-L Higgs field; production of low momentum Higgs bosons (contain most energy), also other bosons and fermions coupled to B-L Higgs field [Garcia-Bellido, Morales '02], production of cosmic strings: initial conditions for reheating



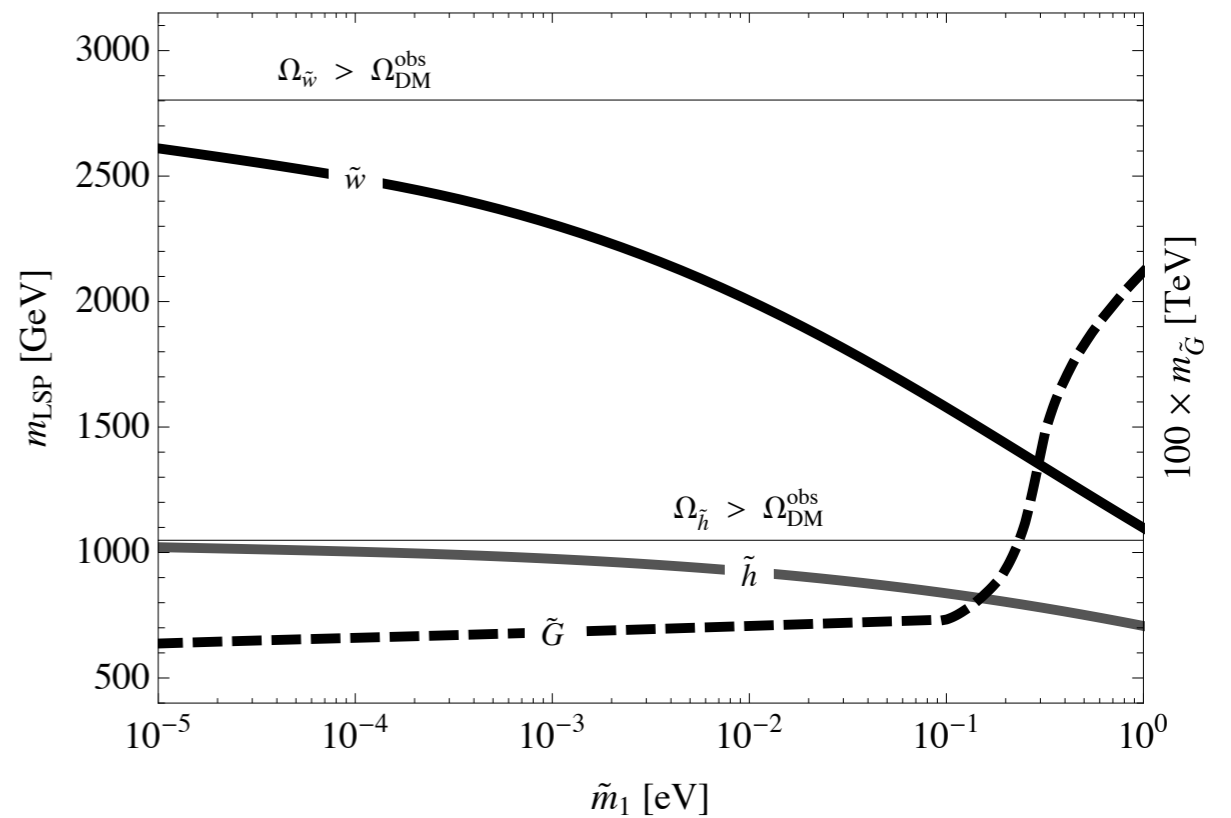
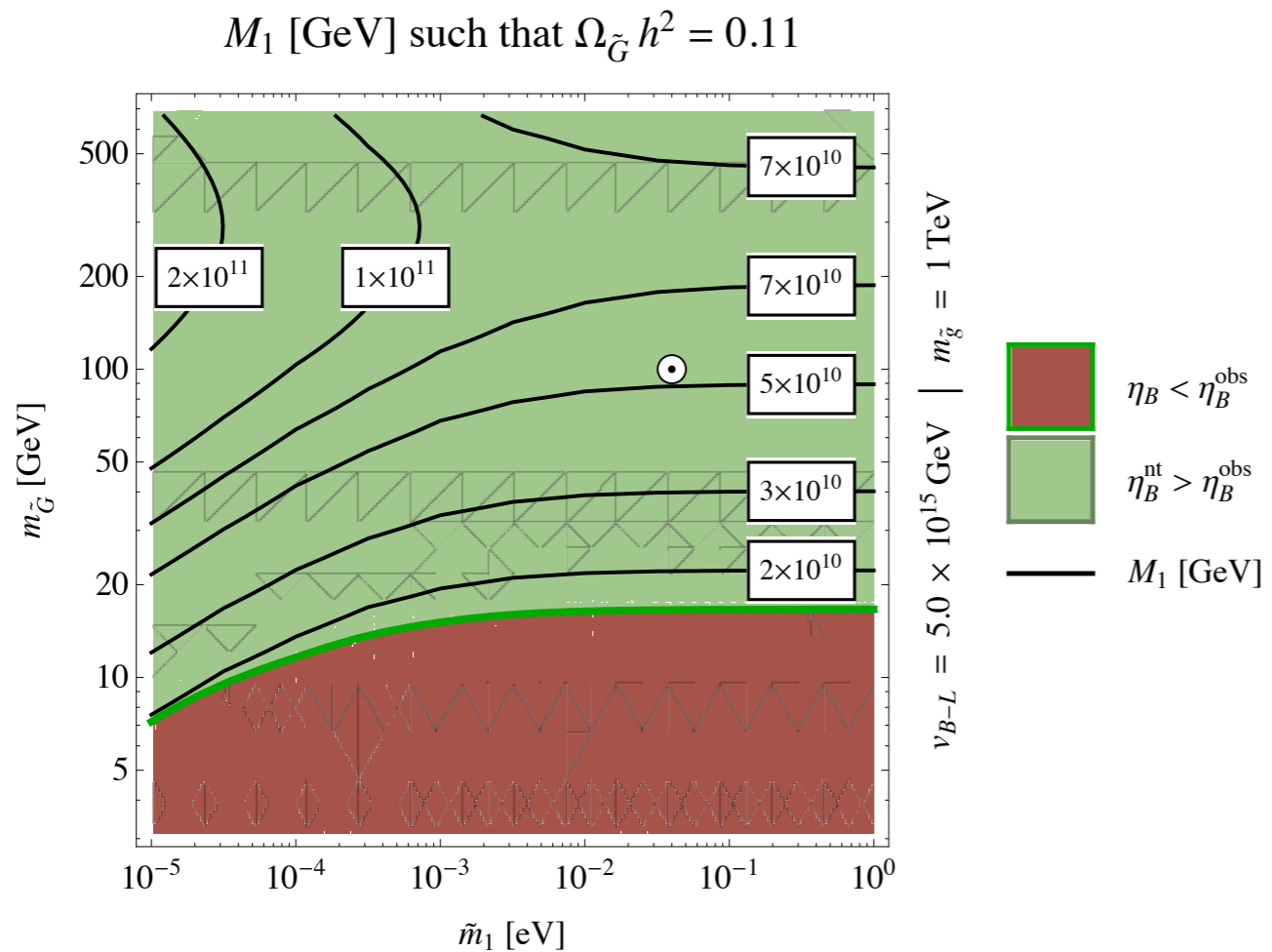
Transition from end of inflation to hot early universe (typical parameters), calculated by means of Boltzmann equations:

$$E \left( \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right) f_X(t, p) = \sum_{i' j' \dots} \sum_{i j \dots} C_X(X i' j' \dots \leftrightarrow i j \dots)$$

yields correct baryon asymmetry and dark matter abundance



Time evolution of temperature: intermediate plateau (“maximal temperature”), determined by neutrino properties! Yields gravitino abundance when combined with ‘standard formula’



parameter scans: successful leptogenesis and **gravitino DM** (left) or **neutralino DM** (right) [non-thermally produced in decays of thermally produced gravitinos] constrains neutrino and superparticle masses; can be tested at LHC!

# Conclusions

For hierarchical heavy neutrinos, seesaw mechanism predicts observed order of magnitude of baryon asymmetry via leptogenesis (include flavour effects; also other scenarios: resonant leptogenesis...)

Theoretical challenge: theory of leptogenesis based on non-equilibrium QFT (basis Schwinger-Keldysh formalism; applications: resonant leptogenesis, flavour effects...)

In supersymmetric models potential gravitino problem: standard neutralino DM excluded; gravitinos/higgsinos OK

Interesting implications: hybrid inflation, cosmic strings & gravitational waves

# Backup Slides



# Gravitational Waves

Relic gravitational waves are window to very early universe; contributions from inflation, preheating & cosmic strings (see Maggiore '07, Dufaux et al '10, Hindmarsh '11); cosmological B-L breaking: prediction of GW spectrum with all contributions! Perturbations in flat FRW background:

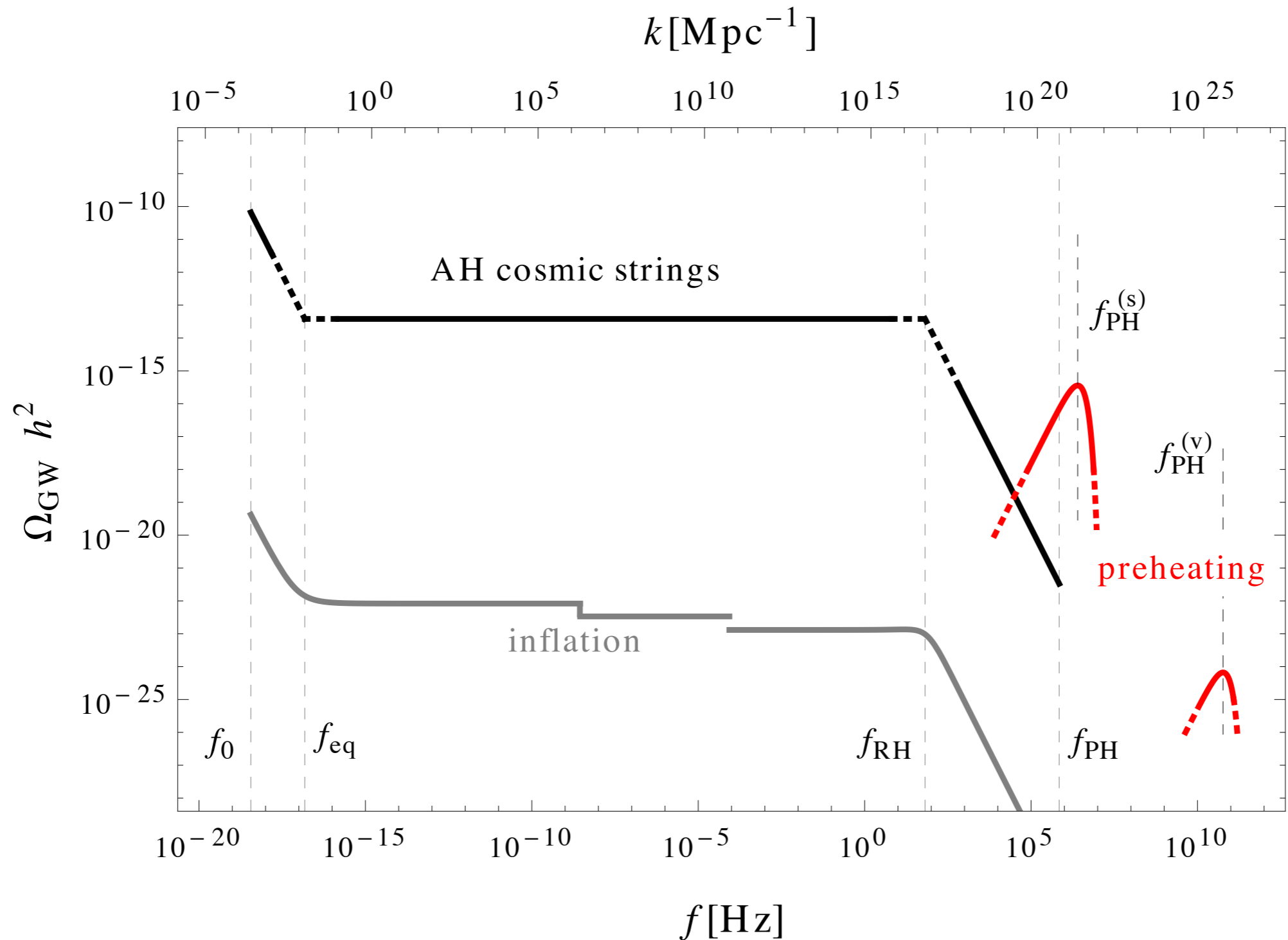
$$ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\rho{}_\rho$$

determined by linearized Einstein equations,

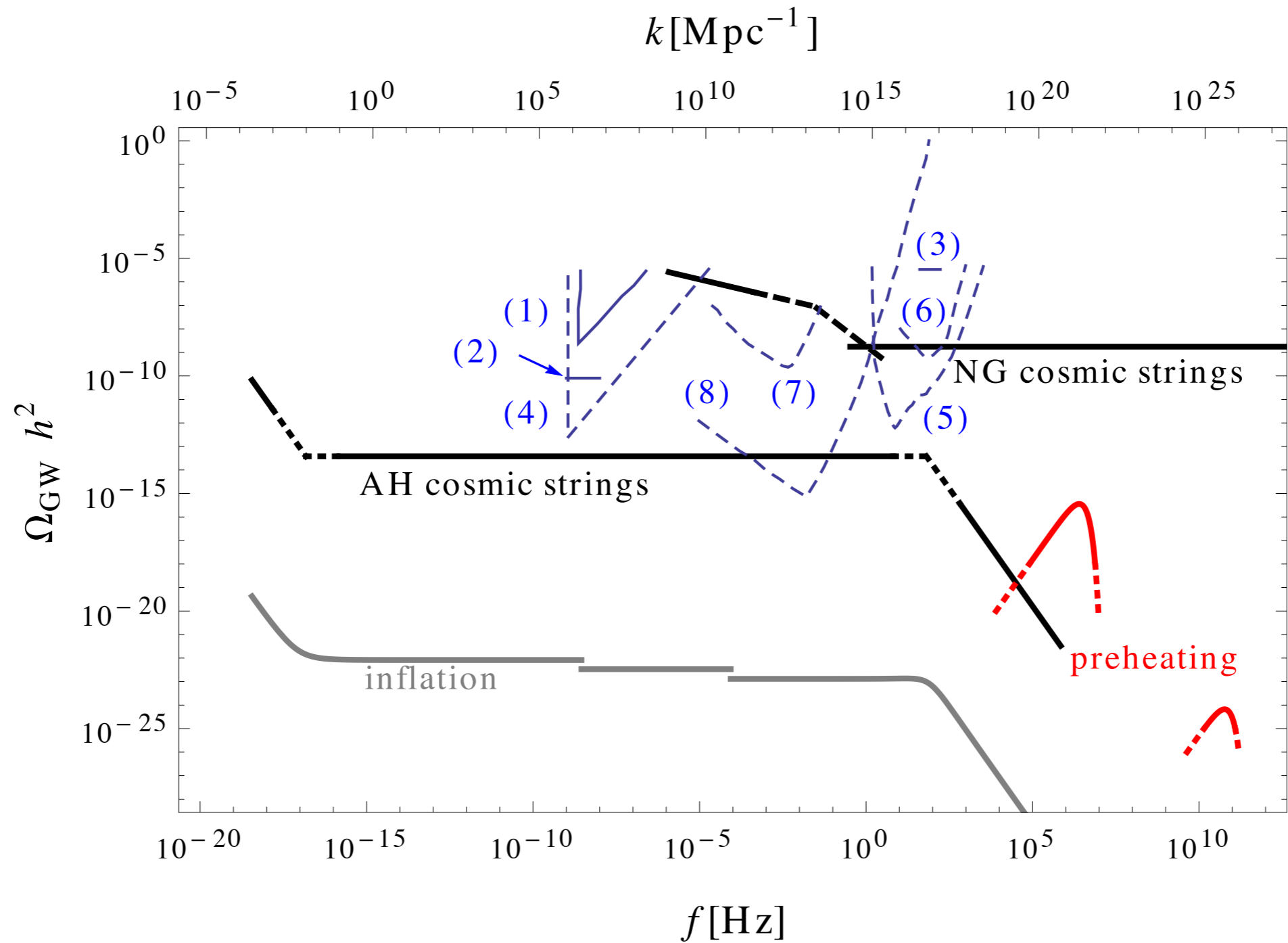
$$\bar{h}''_{\mu\nu}(\mathbf{x}, \tau) + 2\frac{a'}{a}\bar{h}'_{\mu\nu}(\mathbf{x}, \tau) - \nabla_{\mathbf{x}}^2\bar{h}_{\mu\nu}(\mathbf{x}, \tau) = 16\pi GT_{\mu\nu}(\mathbf{x}, \tau)$$

yields spectrum of GW background:

$$\Omega_{GW}(k, \tau) = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k},$$
$$\int_{-\infty}^{\infty} d \ln k \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle$$



Result: GWs from inflation, preheating and cosmic strings (Abelian Higgs), for typical parameters consistent with leptogenesis and dark matter; similar spectrum from inflation and cosmic strings, but different normalization!



Observational prospects (for typical parameters); 'soon': Advanced Ligo (6) [100 Hz], eLISA [0.01 Hz]; 'later': Einstein Telescope [100 Hz], BBO/DECIGO [0.01 Hz]; eventually determination of reheating temperature (leptogenesis)?!

# Hybrid Inflation in the Complex Plane

Inflationary potential can be strongly affected by supersymmetry breaking (supergravity correction; WB, Covi, Delepine '00):

$$V(\phi) = V_0 + V_{\text{CW}}(\phi) + V_{\text{SUGRA}}(\phi) + V_{3/2}(\phi) ,$$

$$V_0 = \frac{\lambda^2 v^4}{4} ,$$

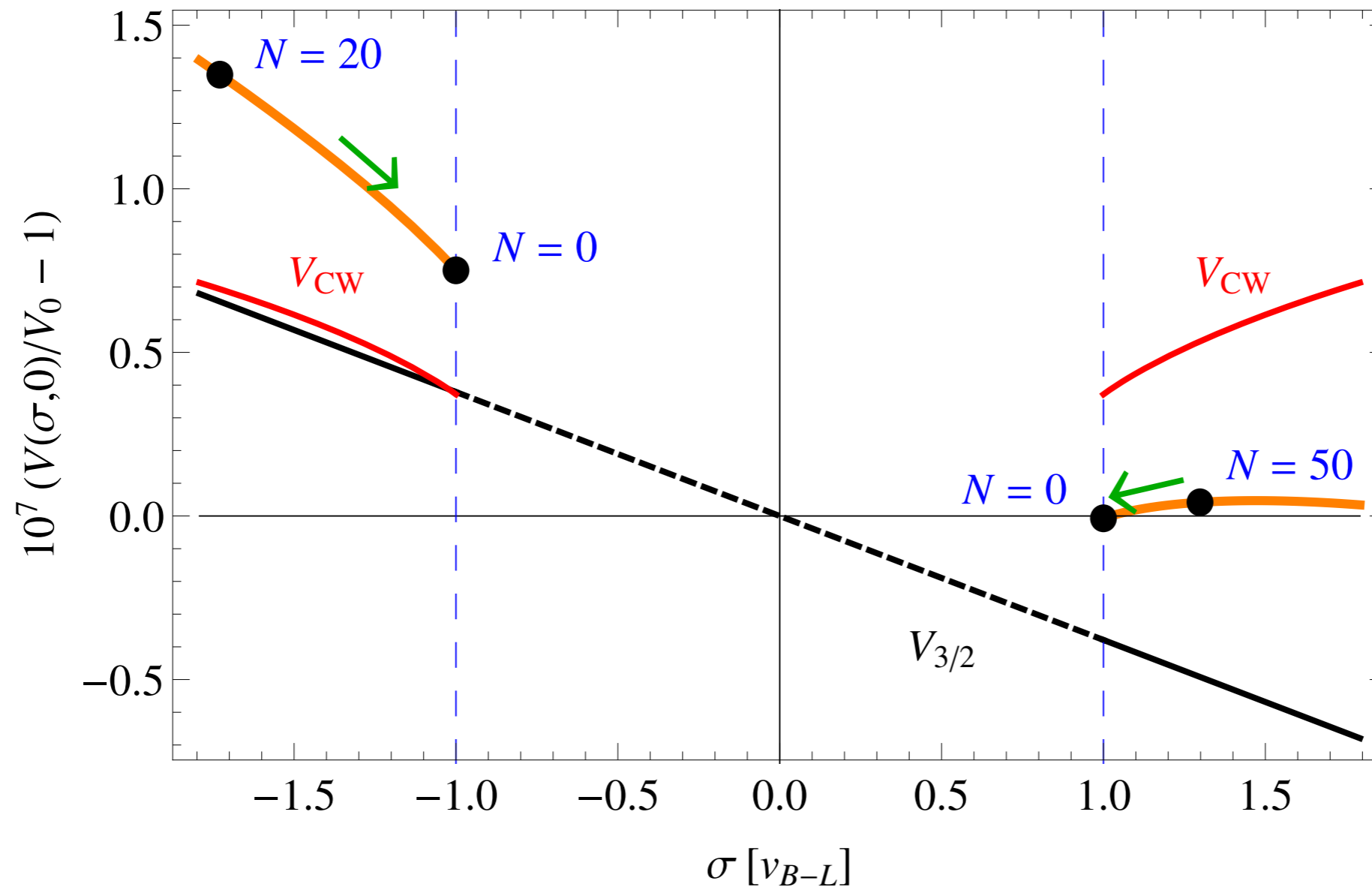
$$V_{\text{CW}}(\phi) = \frac{\lambda^4 v^4}{32\pi^2} \ln \left( \frac{|\phi|}{v/\sqrt{2}} \right) + \dots ,$$

$$V_{\text{SUGRA}}(\phi) = \frac{\lambda^2 v^4}{8M_{\text{P}}^4} |\phi|^4 + \dots ,$$

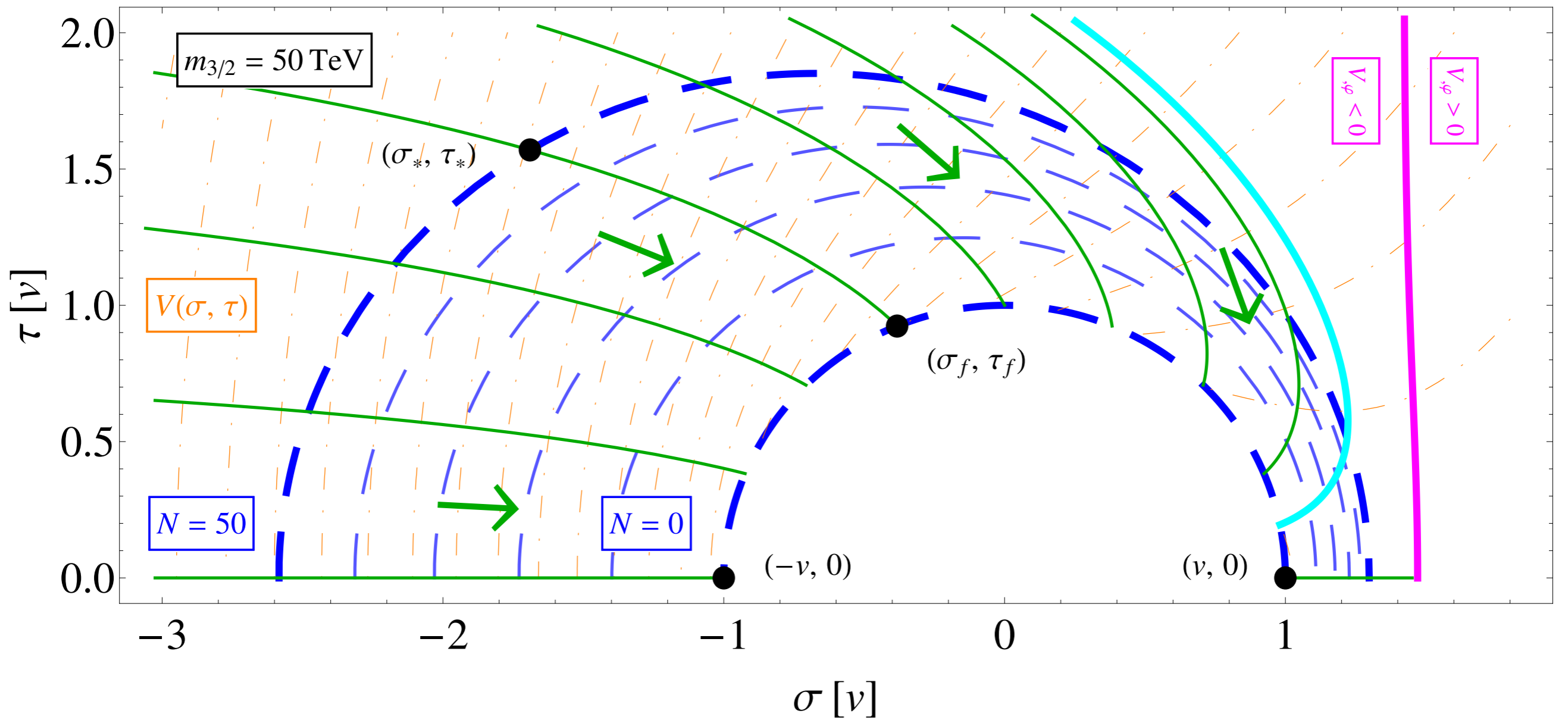
$$V_{3/2}(\phi) = -\lambda v^2 m_{3/2}(\phi + \phi^*) + \dots$$

linear term turns hybrid inflation into two-field model in complex plane;  
strong effect on inflaton observables, now dependent on trajectory, i.e.  
initial conditions! Inflation consistent with Planck data now possible  
(otherwise very difficult)

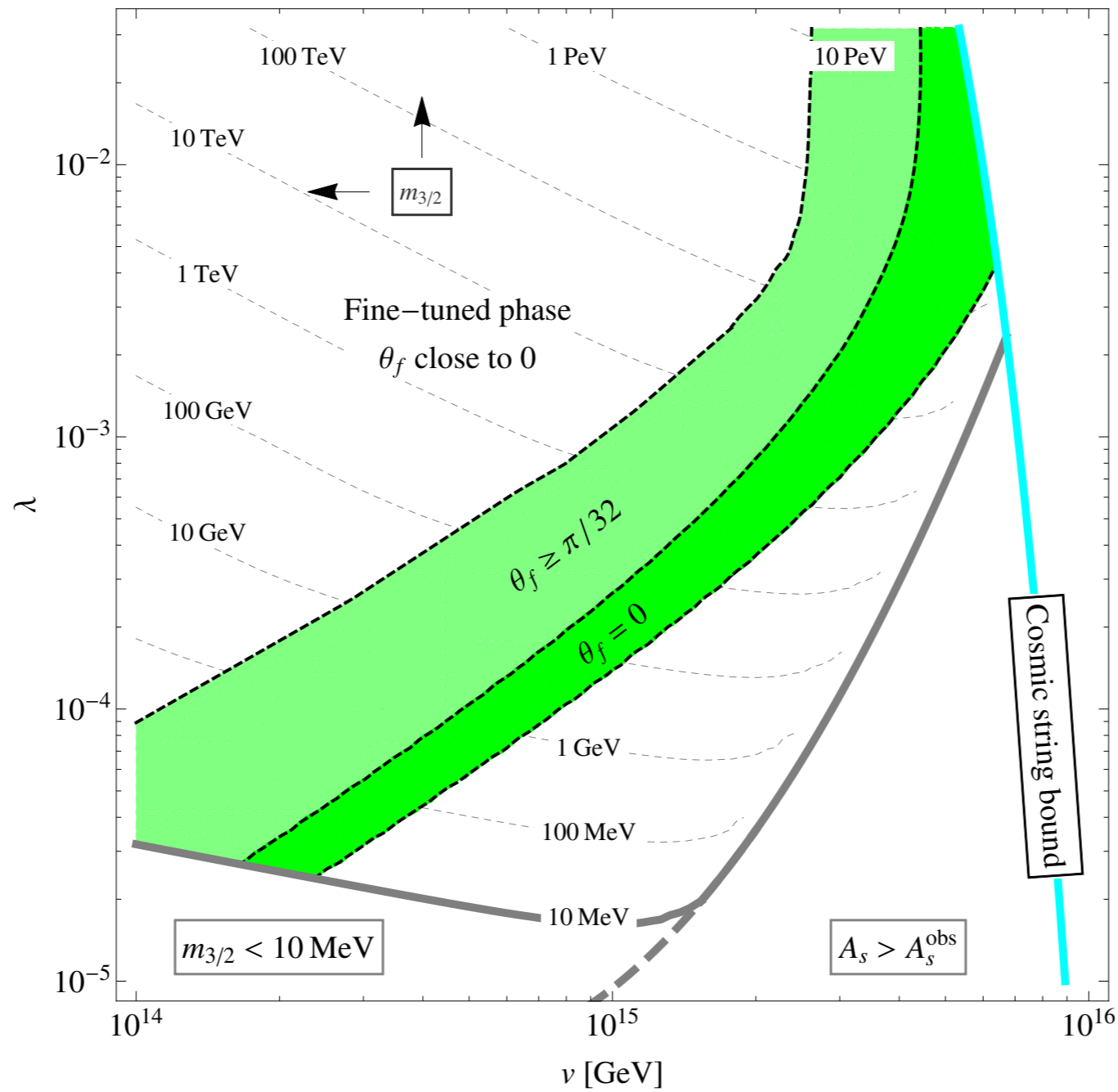
$$v = 3.6 \times 10^{15} \text{ GeV}, \quad \lambda = 2.1 \times 10^{-3}, \quad m_{3/2} = 50 \text{ TeV}$$



Linear term leads to new ‘hill-top’ regime of inflaton potential (Pallis & Shafi '13); allows smaller spectral index than ordinary hybrid inflation



Two-field dynamics of complex inflaton in field space; all trajectories provide enough e-folds of expansion but only one yields the correct spectral index  $n_s \simeq 0.96$



Parameter scan: relations between scale of B-L breaking, gravitino mass and scalar spectral index (no problem!) cosmic string bound automatically fulfilled!

# Comments on BICEP2 results

Prediction of hybrid inflation model for tensor-scalar ratio:

$$r \lesssim 2 \times 10^{-6}$$

Planck data:

$$r < 0.11$$

recent BICEP2 data:

$$r \simeq 0.2$$

Hybrid inflation is inconsistent with BICEP2. Is BICEP2 correct? (recent rumors: more dust than expected ... ?!)

The BICEP2 result is very exciting, it would give evidence for the grand unification energy scale during inflation! This is welcome for leptogenesis. The discussed reheating mechanism can also be combined with 'chaotic' type of inflation (2 versions possible, work in progress).



