**M2-NPAC 2013-2014**
**Particles and Symmetries**
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# TD 4
# Weak interaction

*Unless mentioned otherwise, use the PDG booklet to find the necessary particle properties, experimental measurements and physical constants.*

## Exercise 1

The reaction $^{14}O \to {}^{14}N^{*}\, e^{+}\, \nu_e$ is a nuclear $\beta^{+}$ decay. It is the unique decay mode of the isotope $^{14}O$. The corresponding decay width is:

$$\Gamma_n = \frac{G_n^2 E_0^5}{30\pi^3},$$

where $E_0$ is the energy released during the reaction.

a) Verify the dimensional homogeneity of this expression.

b) We give $E_0$=1.81 MeV and the radioactive half-life period of the $^{14}O$ $T_{1/2}$=70.64 s. Find the numerical value of $G_n$.

c) The width of the $\mu$ decay to an electron and neutrinos ($\mu^{-} \to e^{-}\, \overline{\nu}_e\, \nu_\mu$) is

$$\Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3}.$$

Using $\tau_\mu = 2.2\ 10^{-6}$ s, find the numerical value of $G_\mu$.

d) Compare $G_n$ et $G_\mu$, and comment.

## Exercise 2

We consider the decay of $D^{+}$ ($c\overline{d}$) and $D^{0}$ ($c\overline{u}$) mesons to final states with one positron:

(i). $D^{+} \to e^{+}X$ (ii). $D^{0} \to e^{+}X$

Using the lifetimes of the D mesons and the branching ratios of the two processes[1] compute the value of $\Gamma(D^{+} \to e^{+}X)/\Gamma(D^{0} \to e^{+}X)$. Comment the result.

## Exercise 3 "Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos" : Phys. Rev. Lett., 9, 36 (1962).

In this exercise we are interested the historical experiment with the AGS accelerator in Brookhaven, in which the $\mu$ neutrino was discovered. AGS accelerated protons to an energy of $E_p$=15 GeV. The experimental device is

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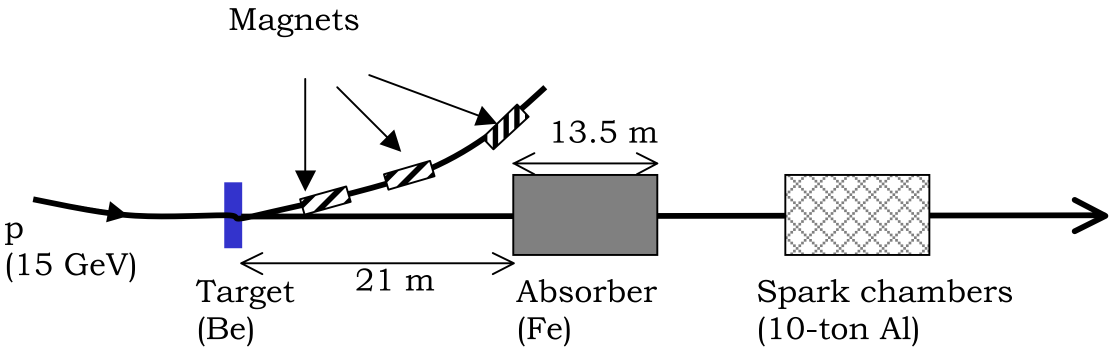
[1] A branching ratio of this kind is called "inclusive", in contrast to an "exclusive" branching ratio, where all the final state particles are specified.

described in the figure below. The protons strike a fixed beryllium target, producing, among other particles, charged pions by inelastic strong-interaction processes of the type $pp \to pp\pi^+\pi^-$. The final-state charged pions decay in flight, essentially according to the weak-interaction processes $\pi^+ \to \mu^+\nu$ and $\pi^- \to \mu^-\bar{\nu}$. We indicate that about 95% of them are $\pi^+$.



a) Knowing that the approximate average total energy of the final-state charged pions is 0.5 GeV, estimate the part of these pions that decay before the absorber. What is the role of the absorber?

b) We now consider the neutrino-nucleon (ν N) interactions $\nu_l\, n \to p\, l^-$ and $\bar{\nu}_l\, p \to n\, l^+$ in the spark chamber. $\ell$ is a lepton (an electron or a muon). We want to obtain an estimation of the cross section of these processes.
   1. For a reaction to be detectable, the lepton must have a sufficient energy to cross a few aluminum layers (slabs) in the spark chamber and leave a track. Show that, in these reactions, the initial- and final-state momenta in the center of mass are of the same order of magnitude ($p_i^* \sim p_f^*$).
   2. Supposing, for simplicity, that $p_i^* = p_f^*$, and knowing that the transition amplitude of the two interactions is given by $T(\nu\, N) \sim 4sG_F$ ($G_F$ is the Fermi constant and $\sqrt{s}$ the center of mass energy), show that the total cross section of the two reactions, $\sigma(\nu_\mu\, N)$, is proportional to $sG_F^2$, and precise the proportionality constant[2].
   3. Find the order of magnitude of $\sigma(\nu_\mu\ N)$ in fb (femto-barn), with $\sqrt{s} \sim 1$ GeV.
   4. What is the role of the spark chambers? Why this experiment used such massive spark chambers (10 tonnes)?

c) The aim of the experiment was to observe the presence of certain processes and the non-presence of other ones. Precise and explain this statement.

d) We will now focus on the spark chambers and the trigger system.
   1. In Fig. 3, the white spaces represent the spark chambers. When a charged particle hits the slabs A, B, C or D, an electrical signal is sent to a logical system, responsible for the triggering of a camera system.

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[2] The computation of the proportionality constant is only suggested here as an exercise. Given the approximations done, and the fact that we are only interested in an order of magnitude, stating that this constant is of order 1 would suffice.

Which are the characteristics of an event in which it is interesting to trigger and take a photograph? Suggest a logic for the trigger system.

2. Figure 4 shows the superposition of a few photographs corresponding to events of the same class. What are these events?
3. Figure 5 shows tracks of three charged particles in a spark chamber. Knowing that the particles involved are an electron, a muon and a pion of the same momentum (~10 GeV), associate a particle to each part of the figure. Explain your arguments.

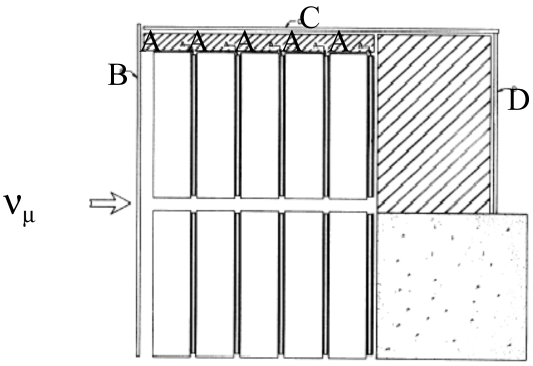

FIG. 3. Spark chamber and counter arrangement. *A* are the triggering slabs; *B*, *C*, and *D* are anticoincidence slabs. This is the front view seen by the four-camera stereo system.

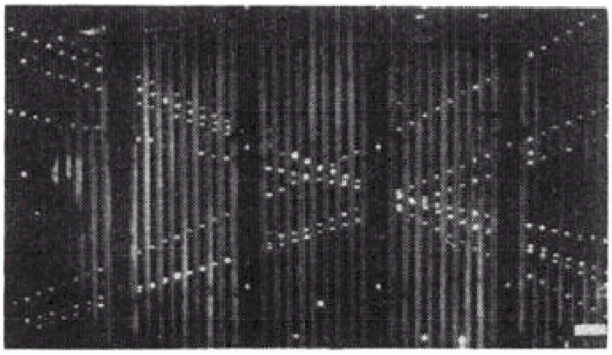

Figure 4

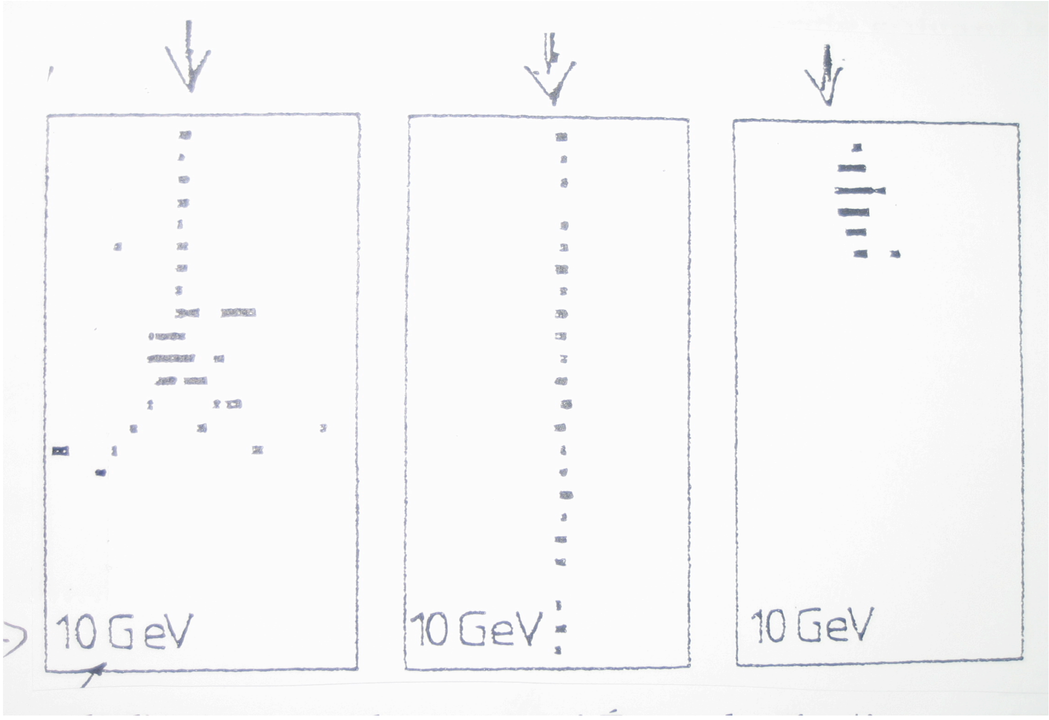

Figure 5

e) Figure 6 shows two among the few dozens of historical events that allowed the discovery of the μ neutrino. Describe and explain these events, knowing that in both of them there is a $\nu_\mu$ entering the spark chamber on the left[3]. Draw the corresponding Feynman diagrams. To answer, recall your arguments from the previous question. Neglect the littriangle on the right hand side of the bottom figure.

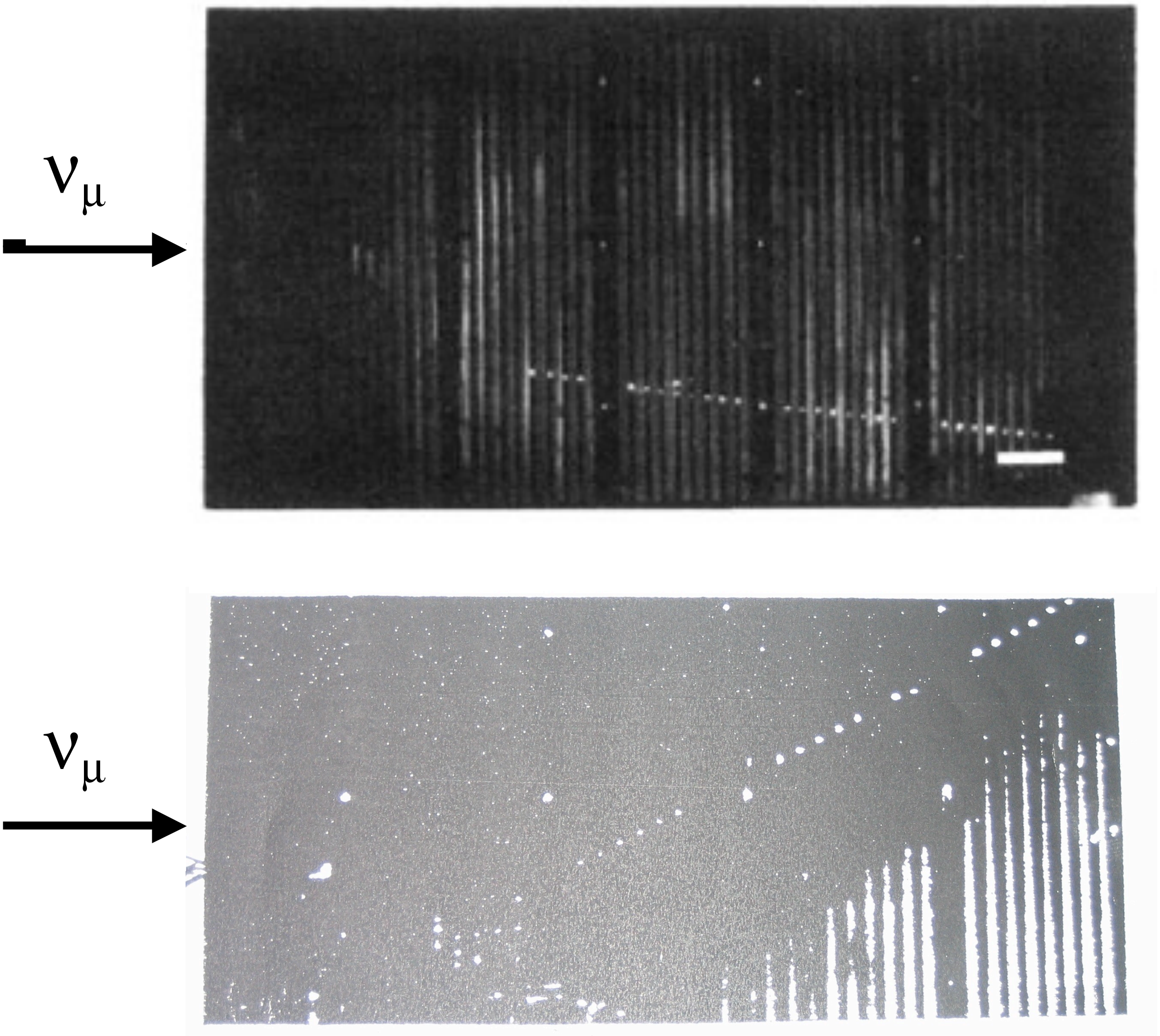

Figure 6

**f)** The signal events in this experiment are rather rare. Most of the observed events are due to background. It is therefore important to well identify this background, and study it thoroughly. Suggest a way to modify the experimental device in order to only take background events for this study.

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[3] The argumentation is the same for $\bar{\nu}_\mu$. However, we remind that they constitute only 5% of the neutrino beam. Moreover, the cross section corresponding to $\bar{\nu}_\mu$ is about 30% of the other.

## Exercise 4

A few hadronic decays of strange particles obey to the old empirical selection rule ΔI=1/2, whose naïve interpretation is the transition of a s-quark (I=0) to a d-quark (I=1/2). To begin, suppose that this rule is valid.

a) Predict the value of the ratio $\frac{\Gamma\left(\Lambda \to n\pi^0\right)}{\Gamma\left(\Lambda \to n\pi^0\right)+\Gamma\left(\Lambda \to p\pi^-\right)}$, using Clebsh-Gordan coefficients. Compare to the ratio of experimentally-measured widths.

b) In the same way, predict $\frac{\Gamma\left(\Omega^- \to \Xi^0\pi^-\right)}{\Gamma\left(\Omega^- \to \Xi^-\pi^0\right)}$ and compare to experimental results. Explain why the empirical rule of ΔI=1/2 is not satisfied in this case.

## Exercise 5

Using Feynman diagrams, explain why the reaction $\bar{\nu}_\mu e^- \to e^- \bar{\nu}_\mu$ gives a unique evidence of the existence of neutral currents (Z boson exchange), unlike the reactions $\bar{\nu}_e e^+ \to \bar{\nu}_e e^+$ et $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$.

## Exercise 6

We remind that the $D^0$ meson is a bound cu state with a mass of 1865 MeV.

a) We are first interested in the decay of the $D^0$ within the spectator-quark model, where only the heavier quark participates in the reaction. Draw the Feynman diagrams of the dominant transitions with respect to the CKM-matrix elements involved.

b) Neglecting other processes, deduce an estimation of the branching ratio of the semileptonic decay $D^0 \to \mu^+\nu_\mu X$, where X represents a (or several) hadrons. To answer, suppose that the phase space is similar for all the processes in a) above (justify this assumption). Compare your estimated branching ratio with the one in the PDG. Comment.

c) Repeat the exercise with the $B^0$ meson (bd) and estimate the semileptonic branching ratio $BR(B^0 \to \mu^+\nu_\mu X)$.

d) Like the muon, the b-quark inside the B meson decays via weak interaction. Use the analogy between these two cases to estimate the lifetime of the $B^0$. For each of these cases, draw the Feynman diagrams of the dominant processes. We remind that, following Fermi's approximation, the width of the muon is of the form $\Gamma_\mu = K m_\mu^5$. We will use the values $m_b$ = 4.5 GeV, $V_{cb}$ = 0.04, $m_\mu$ = 105.7 MeV and $\tau_\mu = 2.2\ 10^{-6}$ s.

e) How can you explain the fact that the $B^0$ has a longer lifetime of the $D^0$ despite the fact that $m_B > m_D$?

## Exercise 7

Write explicitly a few semileptonic decay processes of a top quark in a u-quark (possibly as a "cascade" of a few consecutive decays). Find the three processes whose contribution to the amplitude is maximal. Draw the corresponding Feynman diagrams, associating to each adequate vertex a CKM-matrix element. Give the approximate ratio of the cross-sections of the three dominant processes.

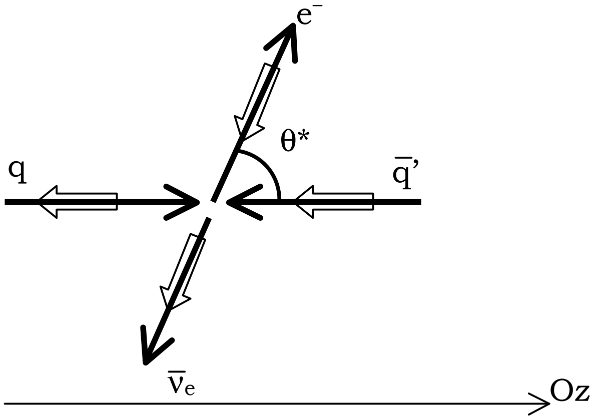
## Exercise 8

The W boson is produced in high-energy $p\bar{p}$ collision following the process $p\bar{p} \to W^- X$. It is detected via the decay $W^- \to e^-\bar{\nu}_e$.

a) The leptonic decay branching fraction BR($W \to \ell\, \nu$)$\approx$10%, independently of the flavor of the lepton $\ell$. If the W had a 0-spin, would the branching fractions of its decay to a muon and an electron be the same? To answer, use arguments related to kinematics and phase space.

b) Draw one of the Feynman diagrams of the process $p\bar{p} \to W^- X$ and another one for $p\bar{p} \to Z^0 X$. These two processes allowed the discovery of the W and Z bosons in CERN in the 80s.

c) Justify the fact that the only allowed helicity configuration for the interaction $qq \to e\nu$, in the limit $m_q = m_{\bar{q}} = 0$ is the following:



d) The angle $\theta^*$ is defined between the direction of the outgoing electron in the center of mass of the W boson and the flight direction of the W in the laboratory frame. The latter also defines the positive sense of the z-axis. We suppose that the W boson has spin 1. Deduce the angular distribution of the electron in the center of mass of the W ($d\sigma/d\cos\theta^*$). Use the expression of the Wigner matrix elements given in the PDG, the additional notes and the figure above. We do not require the full expression of the differential cross section, but only its angular dependence $f(\theta^*)$ ($d\sigma/d\cos\theta^* \propto f(\theta^*)$).

e) Same question, supposing that the spin of the W is 2. Suggest a method to determine which one of the two W-spin hypotheses is correct.

## Exercise 9

We now examine the reaction $e^+e^-\to\mu^+\mu^-$. We specify that the center-of-mass energy is big enough for the muons to be ultrarelativistic, and small enough so that the reaction is mediated by a photon (J=1).

a) When can you say about the helicities of the interacting particles? What are the possible projections of the total angular momenta in the initial and final states on the axes defined by the directions of the negatively-charged particles?

b) Draw the four schemes, describing the possible configurations of momenta and helicities of the incoming and outgoing particles in the center-of-mass frame. The angle $\theta$ is defined between the directions of the incoming $e^-$ and the outgoing $\mu^-$ in this frame.

c) Using the Wigner matrix elements $d^J_{\lambda_1\lambda_2}(\theta)$, give the $\theta$–dependence of the amplitude corresponding to each one of the schemes drawn in a) above.

d) Compare your results to Fig. 1 below. We indicate the the helicity amplitudes are the same for all the four helicity configurations.

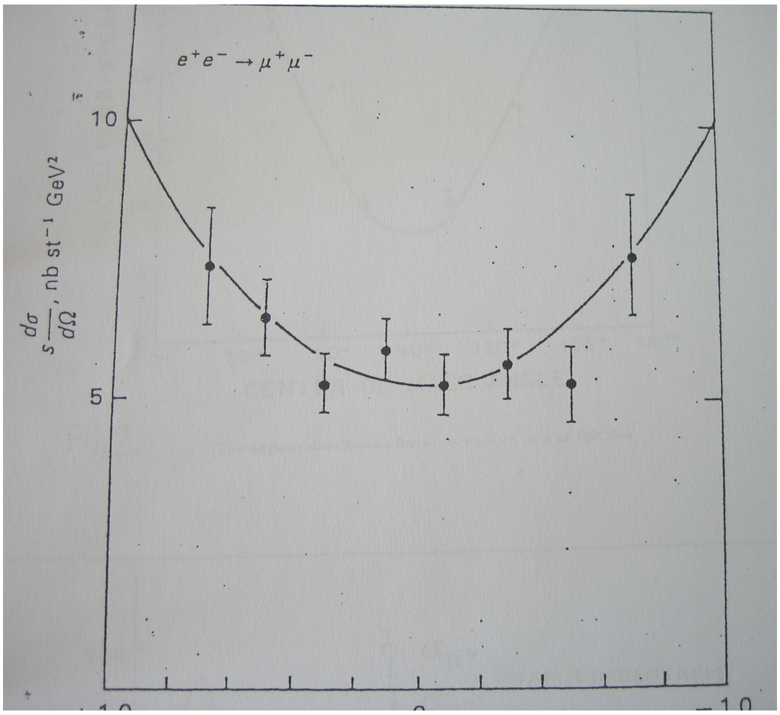


*Figure 1: Measured $e^+e^- \to \mu^+\mu^-$ cross section as a function of cos(θ)*

## Exercise 10

We now examine the reaction $\pi^+ p \underline{\ \Delta^{++}\ } \pi^+ p$

a) Draw the four schemes describing the possible configurations of momenta and helicities of incoming and outgoing particles in the center-of-mass frame. Considering parity transformation, show that there are only two independent helicity amplitudes.

b) We suppose that, for these two amplitudes, the factors that do not contain any angular dependence (helicity amplitudes) are the same. Write the angular dependence of the differential cross section, in terms of

Wigner matrix elements, for a $\Delta^{++}$ resonance of a given spin J. We note as $\theta^*$ the angle between the z-axis and the z'-axis, which are oriented in the direction of the incoming pion and the outgoing proton, respectively, in the center of mass frame.

c) Deduce the angular distribution of the outgoing protons in the centre of mass, in the cases J=1/2 and J=3/2. Compare the result to Fig. 2 below.

d) Does this result allow determining the spin of the $\Delta$ resonance, in case it is a-priori unknown?

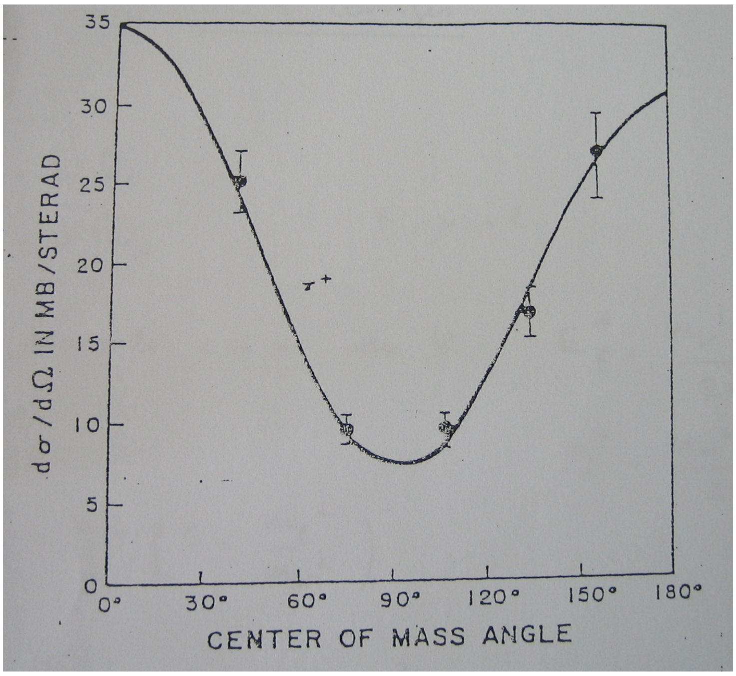


*Figure 2: The $\pi^+p$ cross section as a function of the center of mass angle*

e) Using the additional notes, give the expression of the maximal cross section for the process $\pi^+p \rightarrow \Delta^{++} \rightarrow \pi^+p$. Compute it numerically for different values of J and compare to the maxima of the different distributions in Fig. 3. Show that this result allows extracting the same spin of the $\Delta^{++}$ resonance as above.

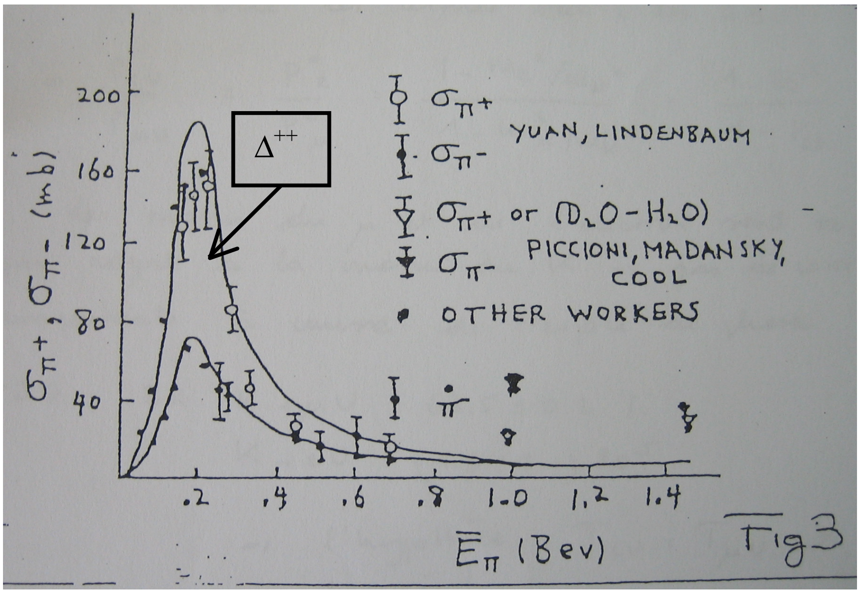


*Figure 3*