

Dipole and quadrupole amplitudes and semi-inclusive observables at the LHC

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QCD at high density

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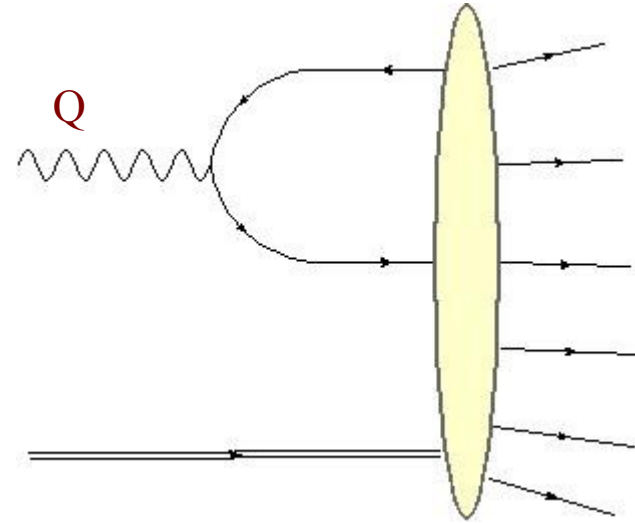
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This regime is very interesting theoretically. Parton saturation may also have important phenomenological consequences at the LHC.

QCD at high density: How to test it?

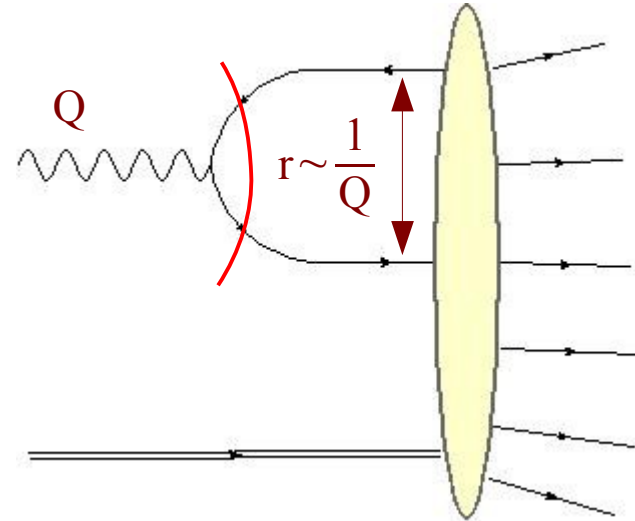
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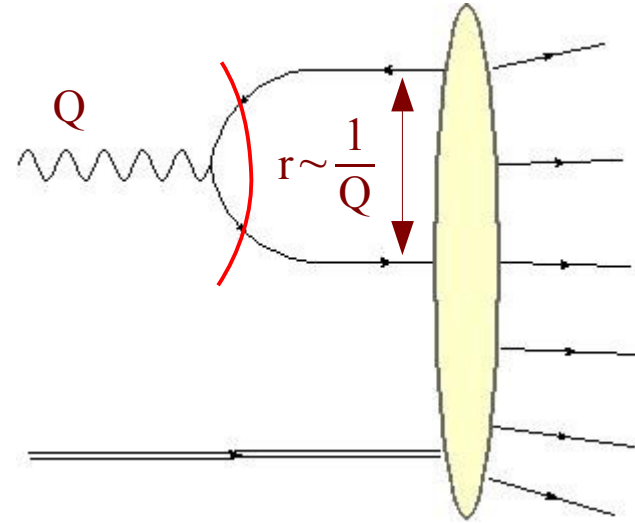
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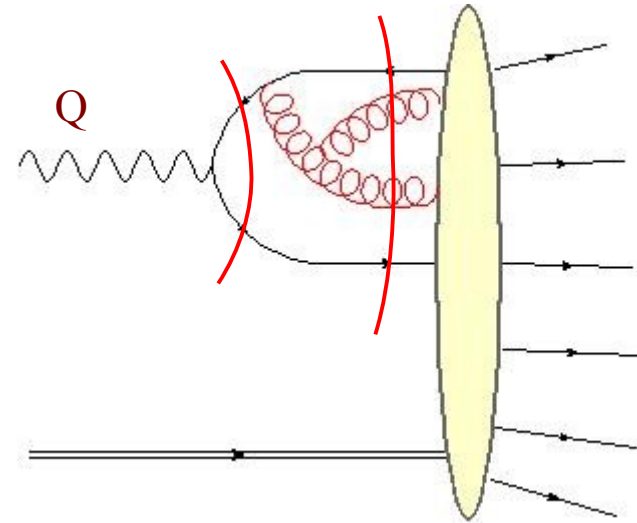


A lot of understanding of the dipole scattering amplitude was gained at HERA, at the border of the dense/saturation regime of QCD!

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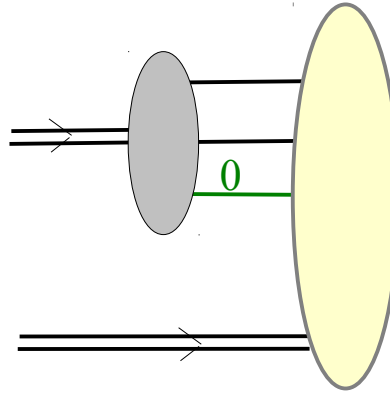
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*On the theoretical size, it is “easy” to formulate the QCD evolution of the dipole amplitude with the energy as **radiative corrections to the dipole wave function**.*

BFKL (at low density), BK, JIMWLK equations (accounting for high-density effects)

QCD at high density: How to test it?

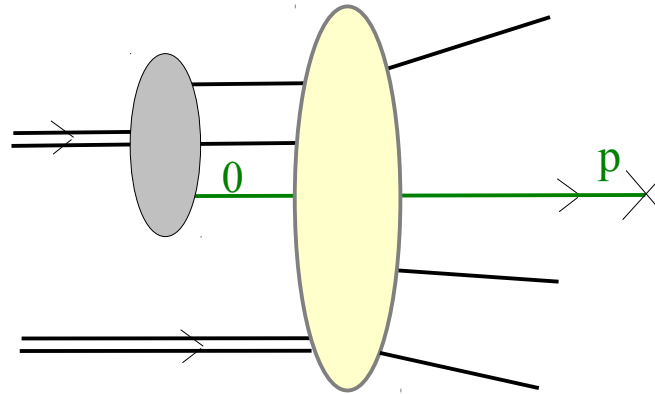
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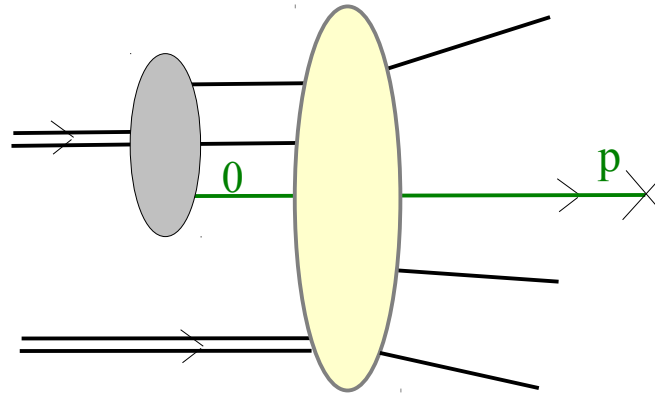


*Observe a jet of
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 $p \sim Q_s$*

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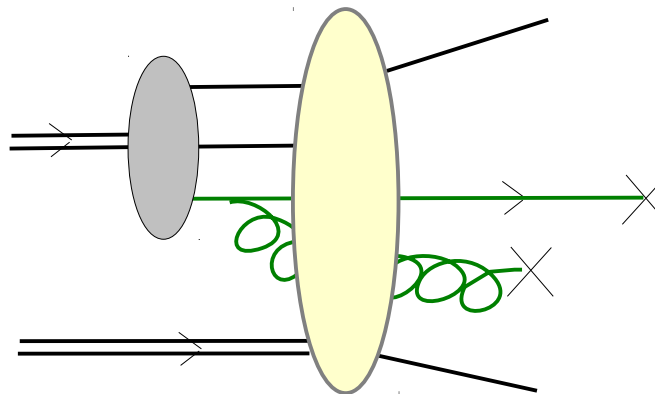
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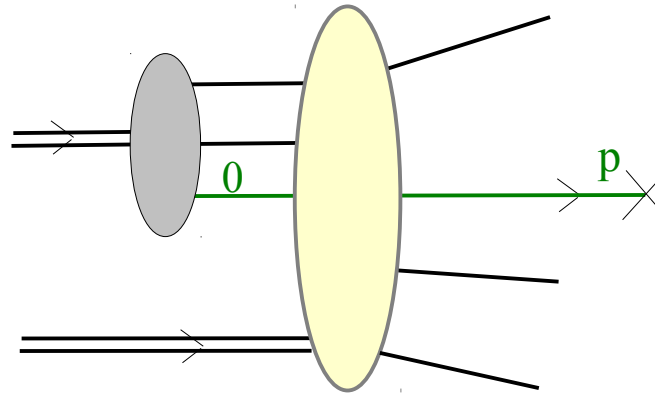


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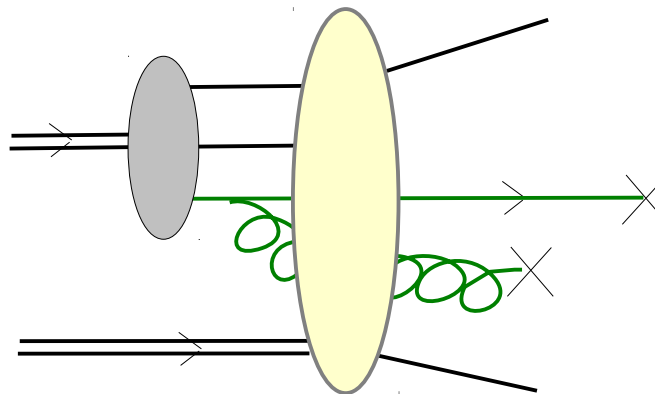
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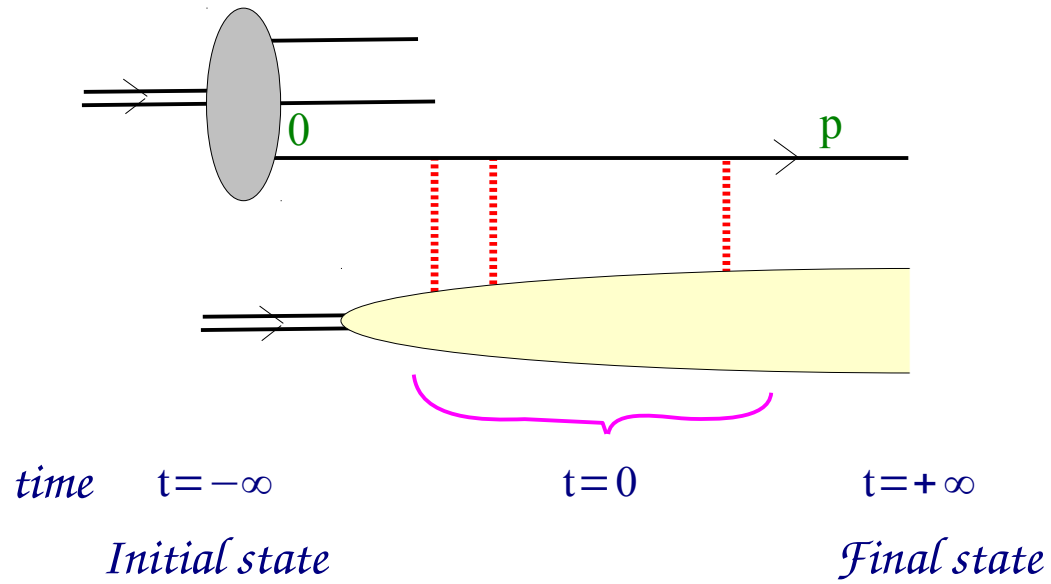
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These observables are more tricky to formulate in QCD!

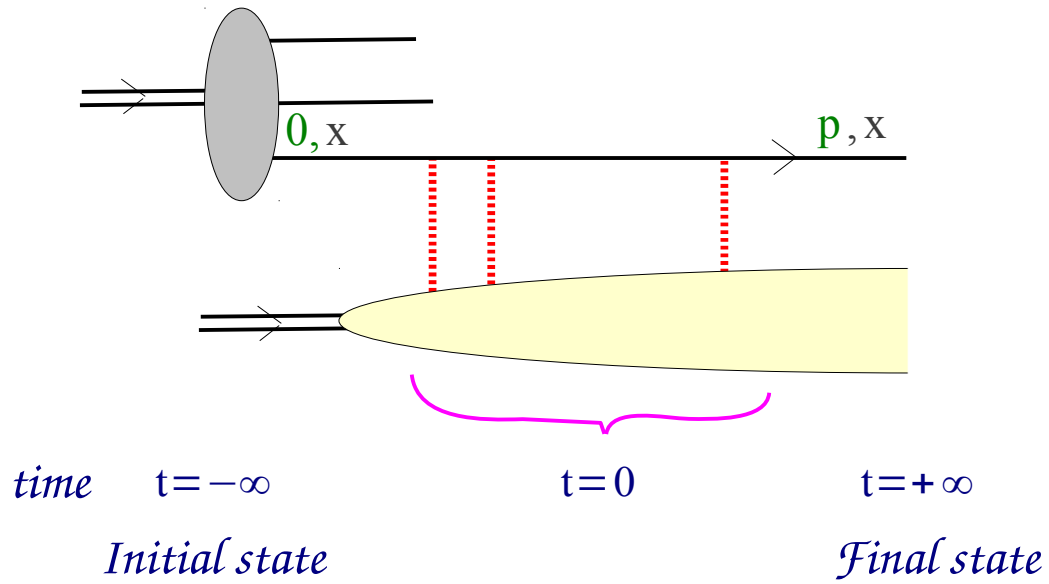
Outline

- ★ Production processes in $p\mathcal{A}$ in terms of dipole/quadrupole amplitudes*
- ★ Robustness under quantum evolution*

Formulation of p_T -broadening

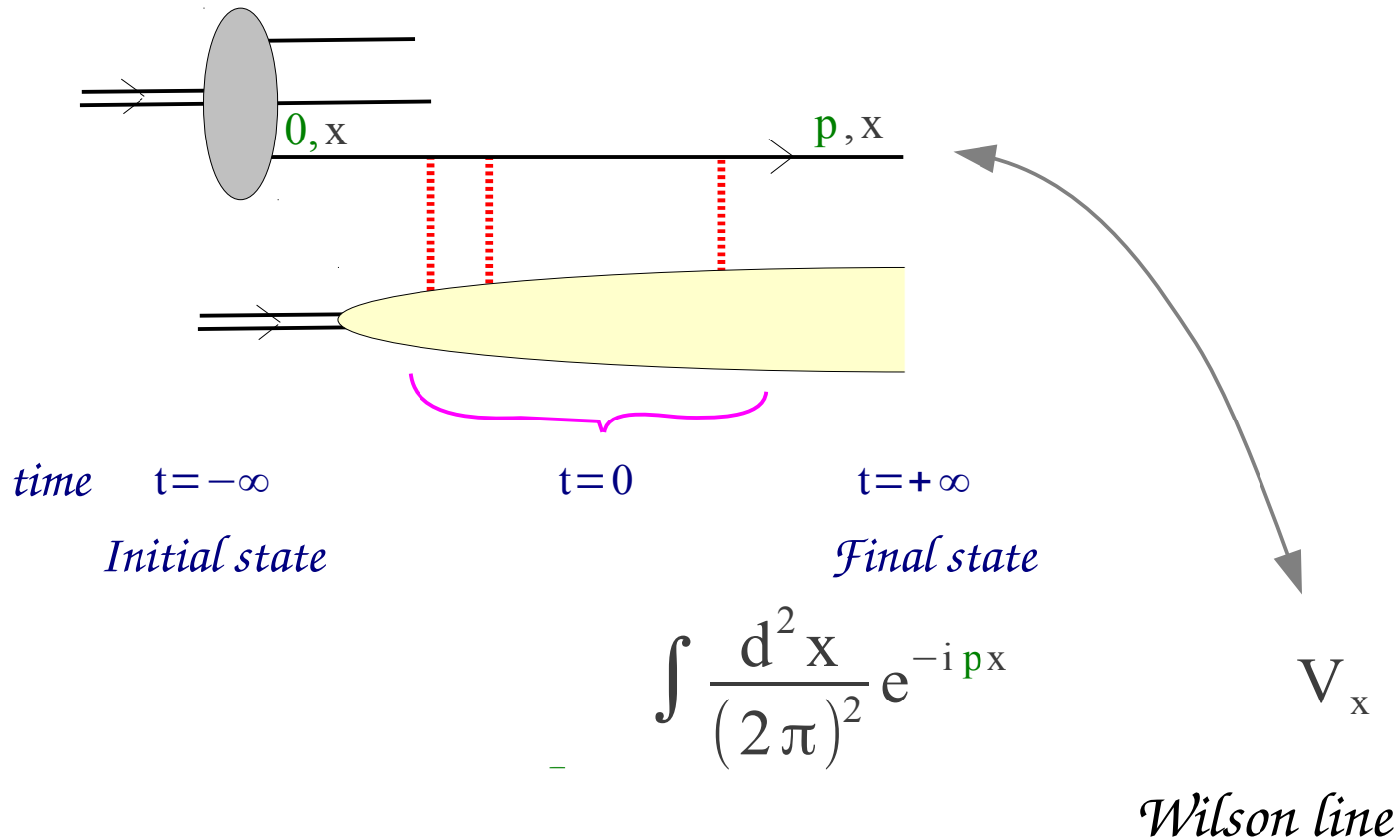


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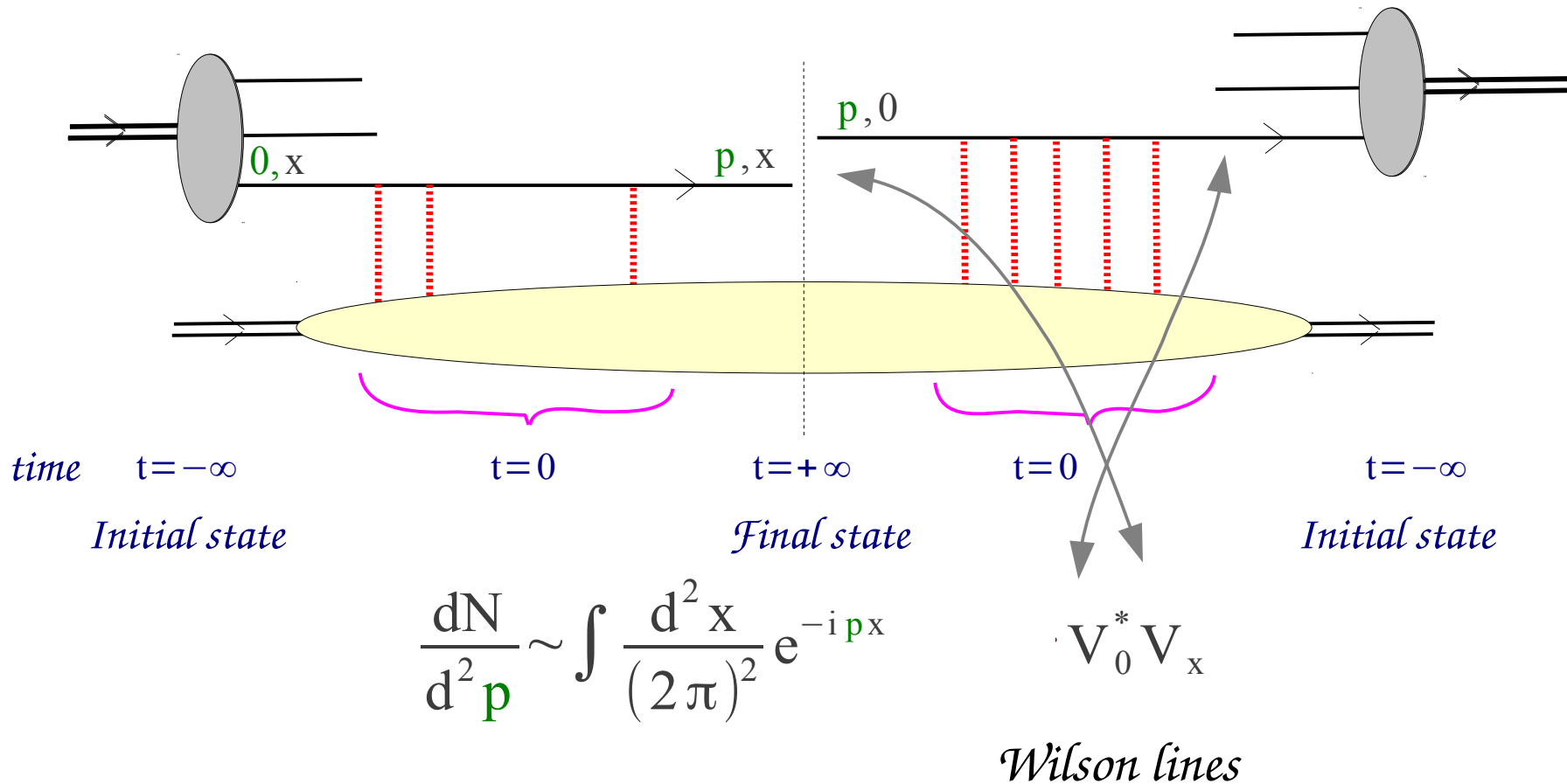


$$\int \frac{d^2 \mathbf{x}}{(2\pi)^2} e^{-i \mathbf{p} \cdot \mathbf{x}}$$

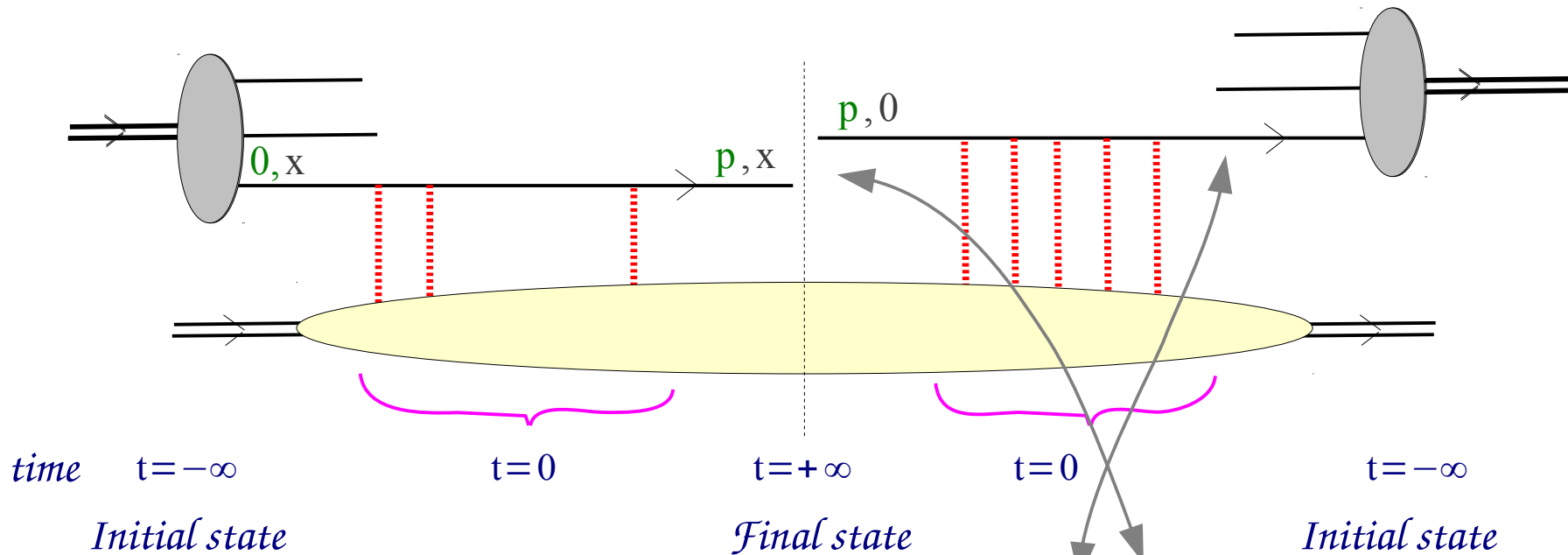
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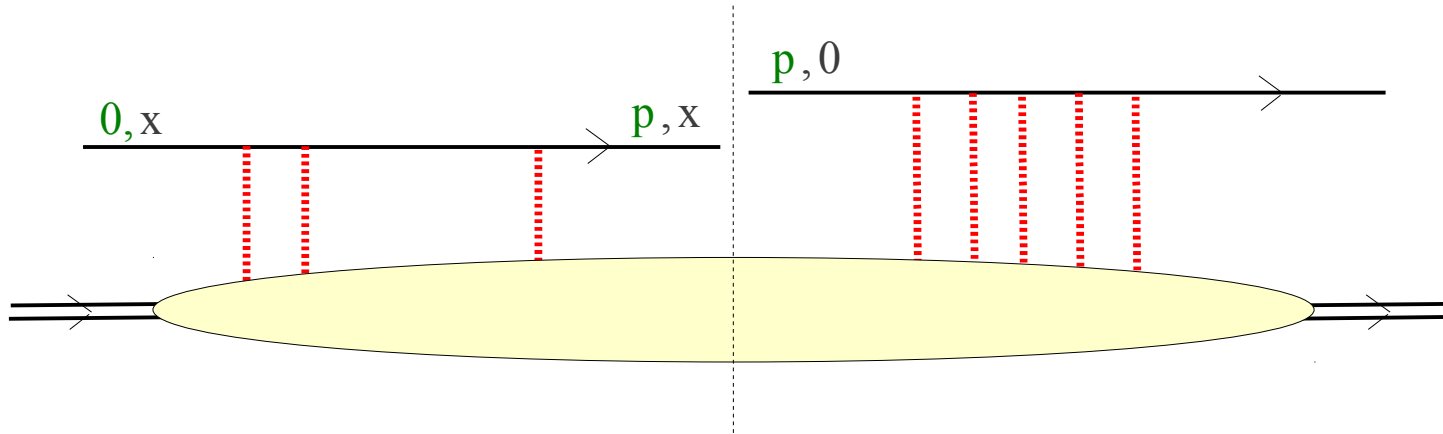
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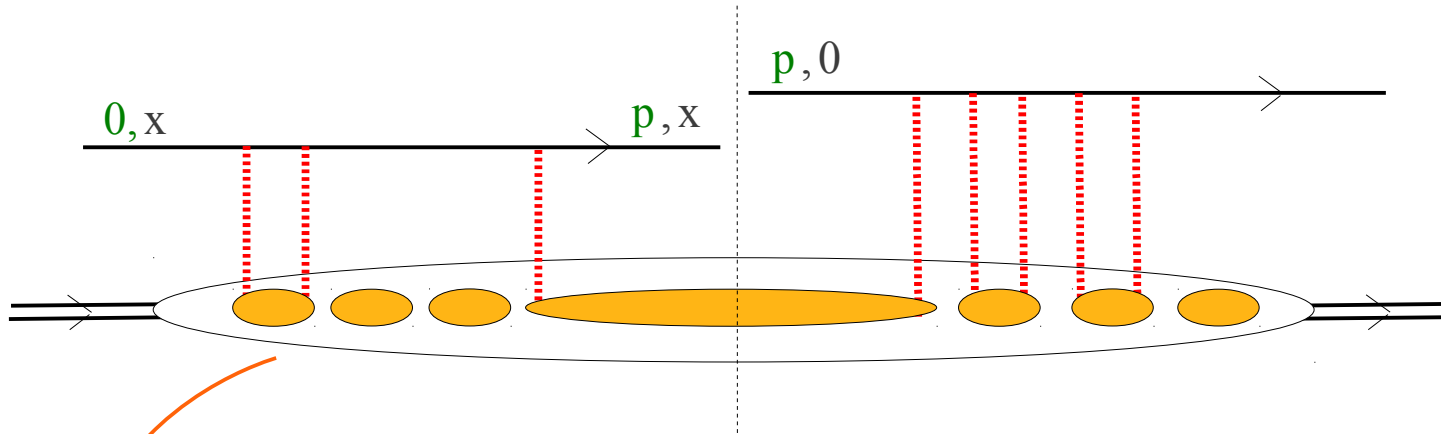
Wilson lines

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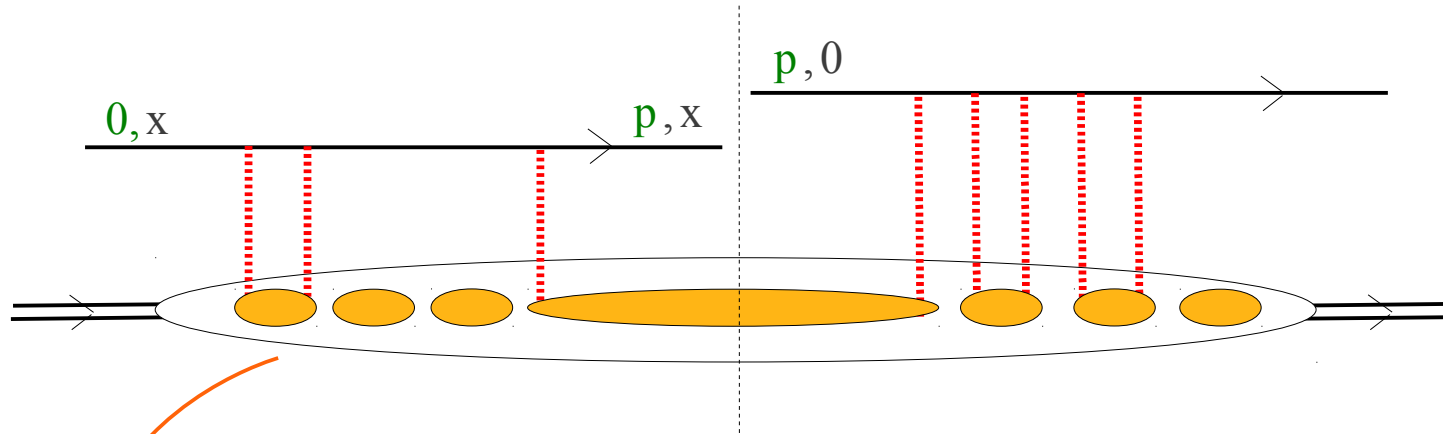
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McLerran-Venugopalan model
(assumes 2-gluon exchanges at most)

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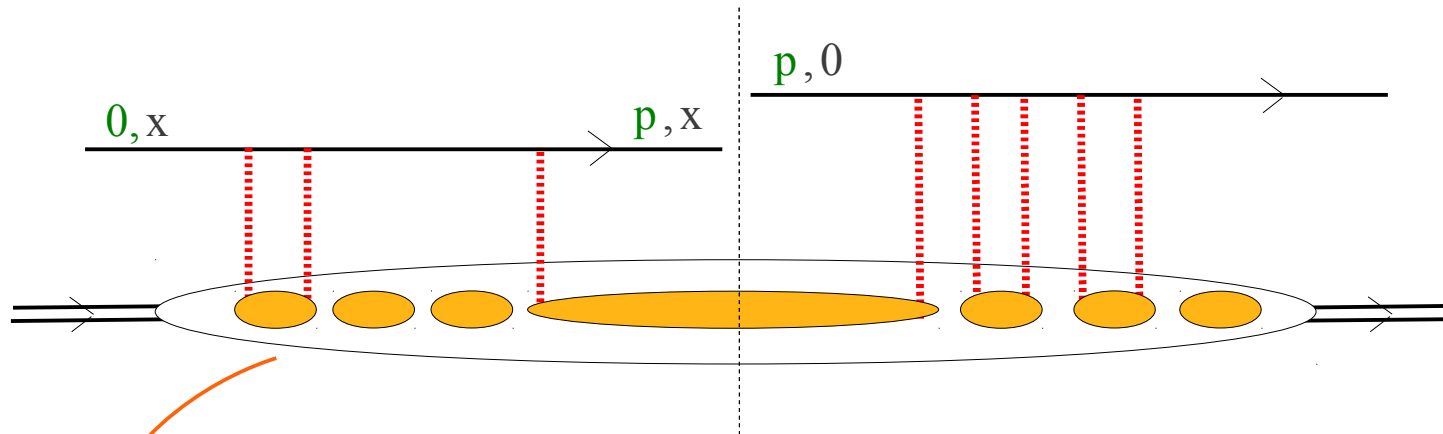


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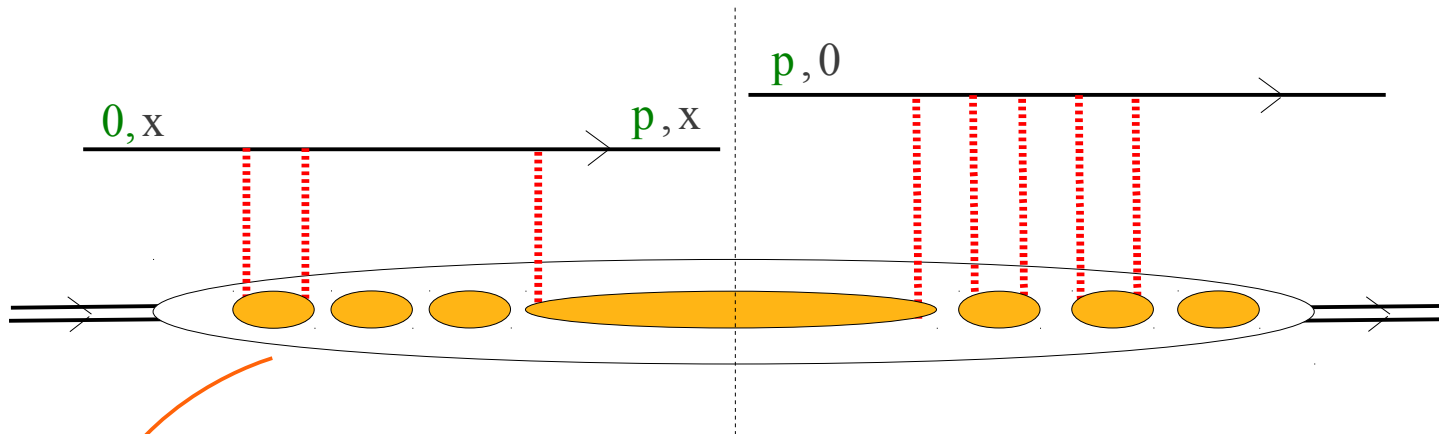


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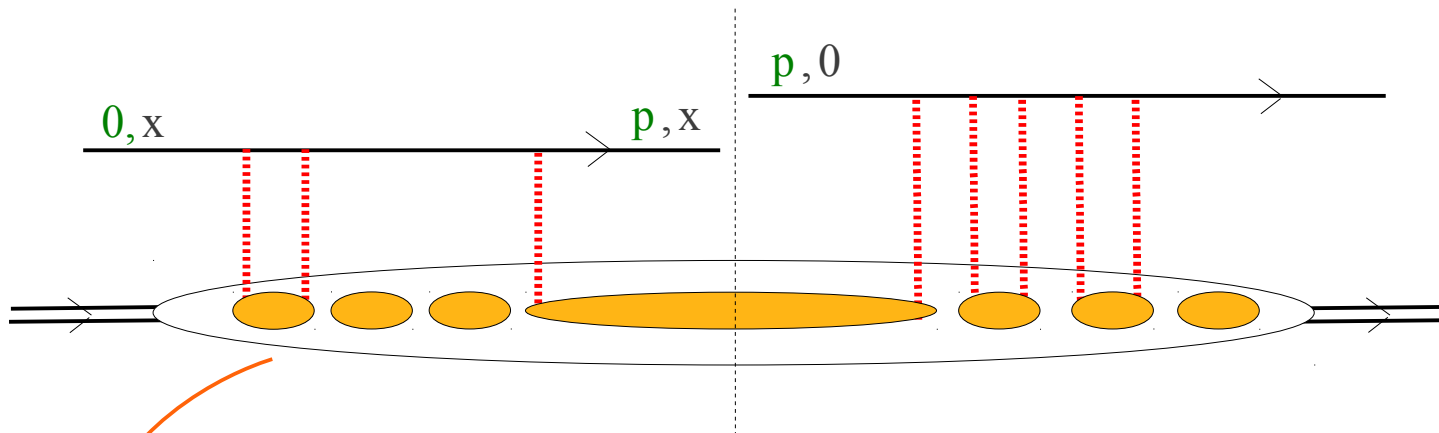
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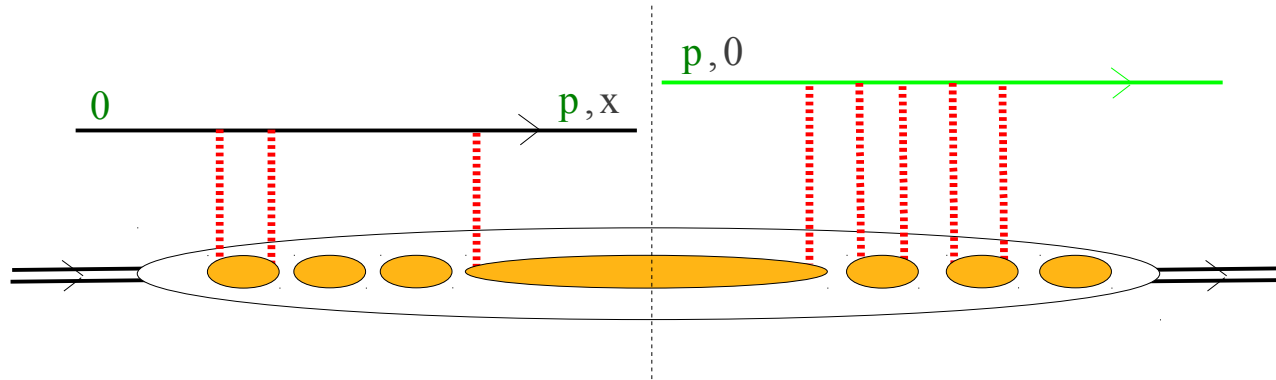
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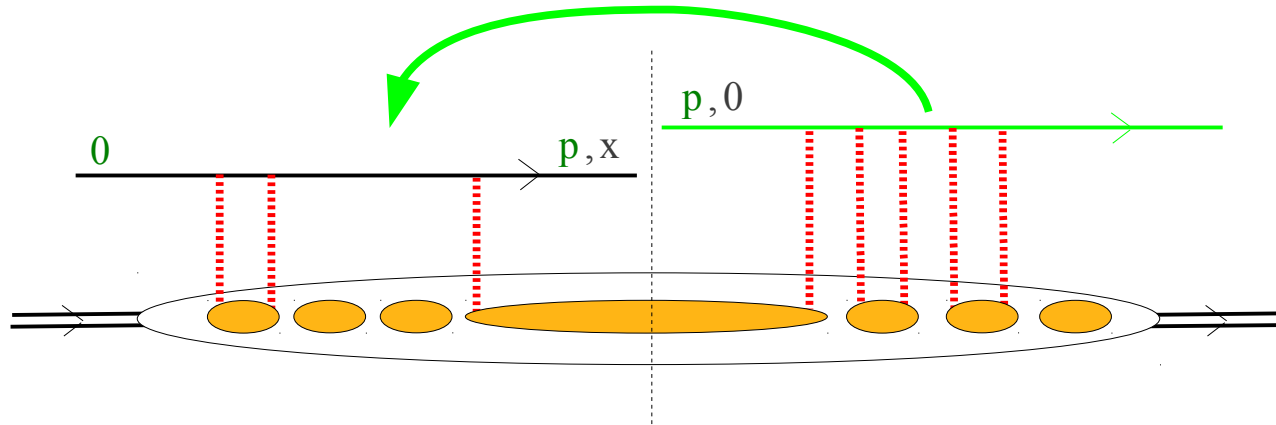
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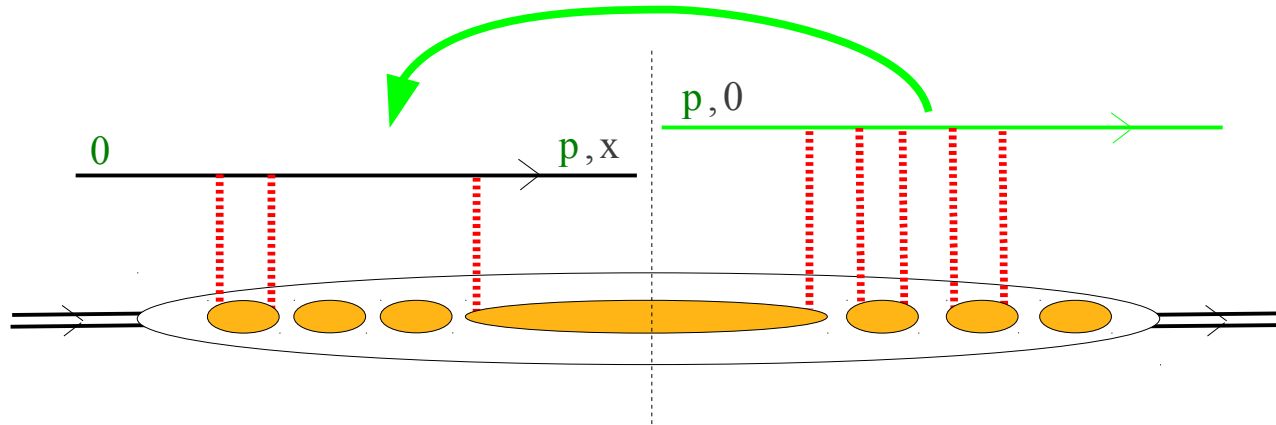
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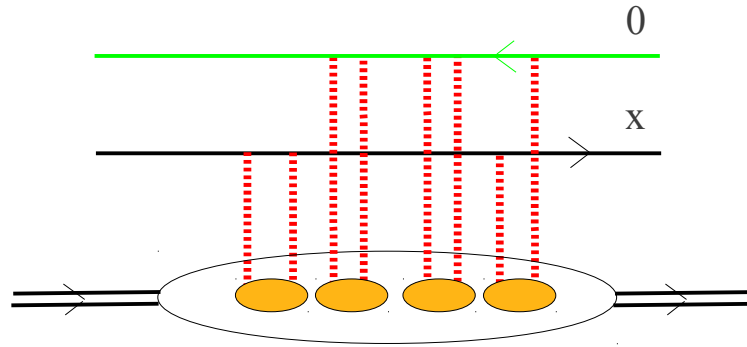
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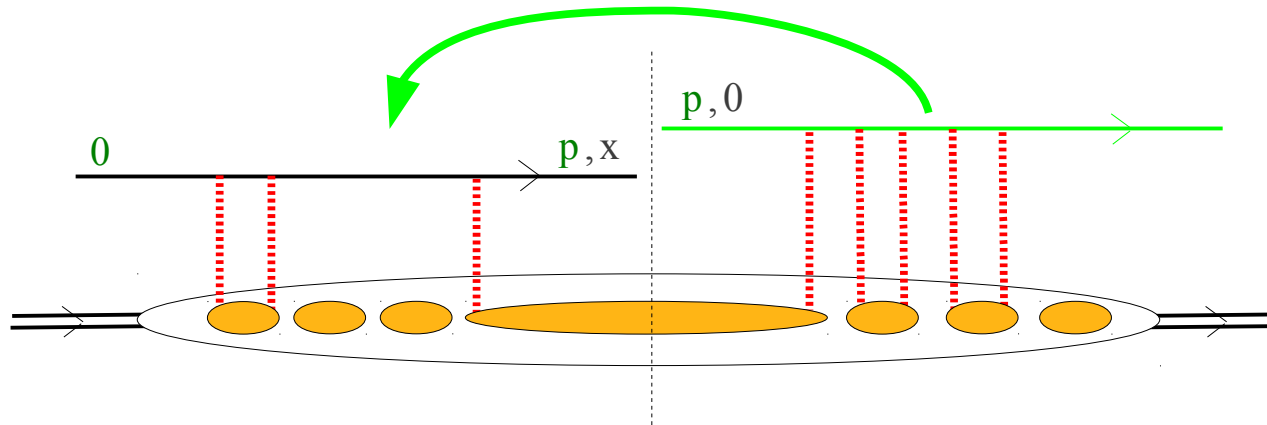
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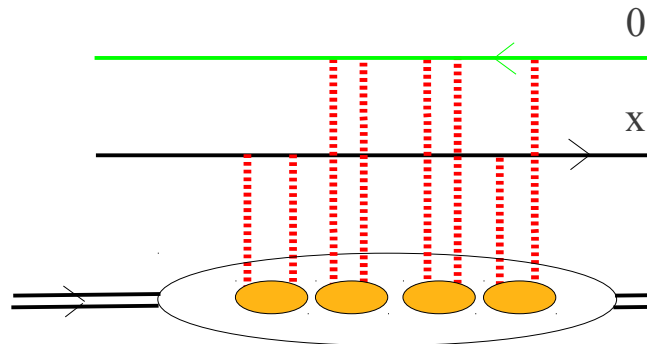
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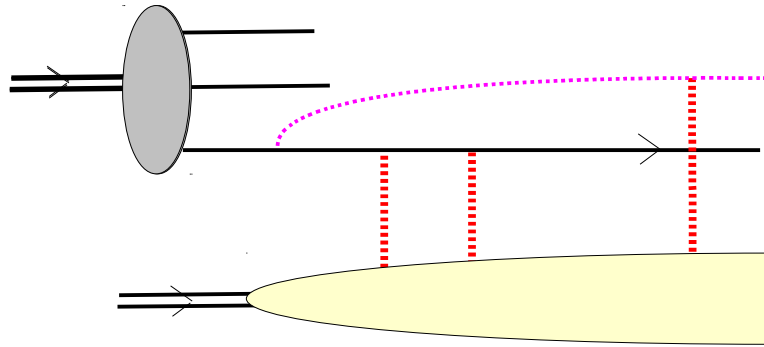
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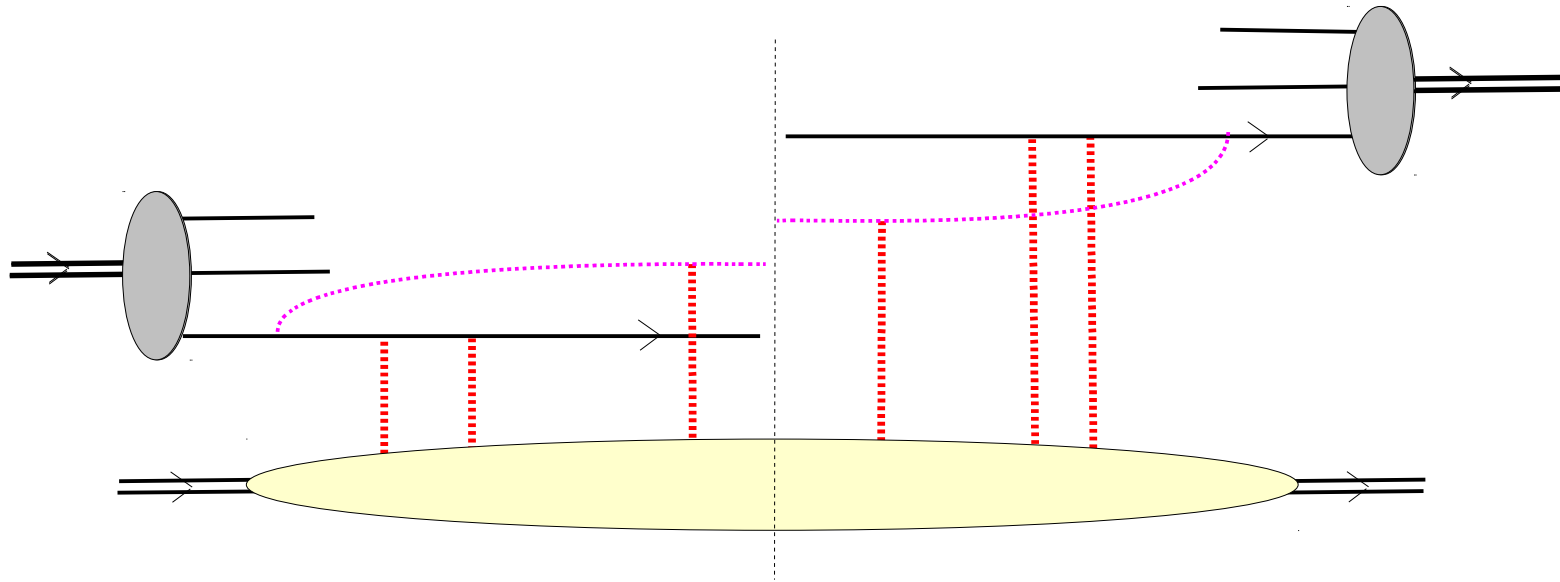
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Intuitively: just bend the quark line in the complex conjugate amplitude to an antiquark line to transform it to a dipole amplitude!

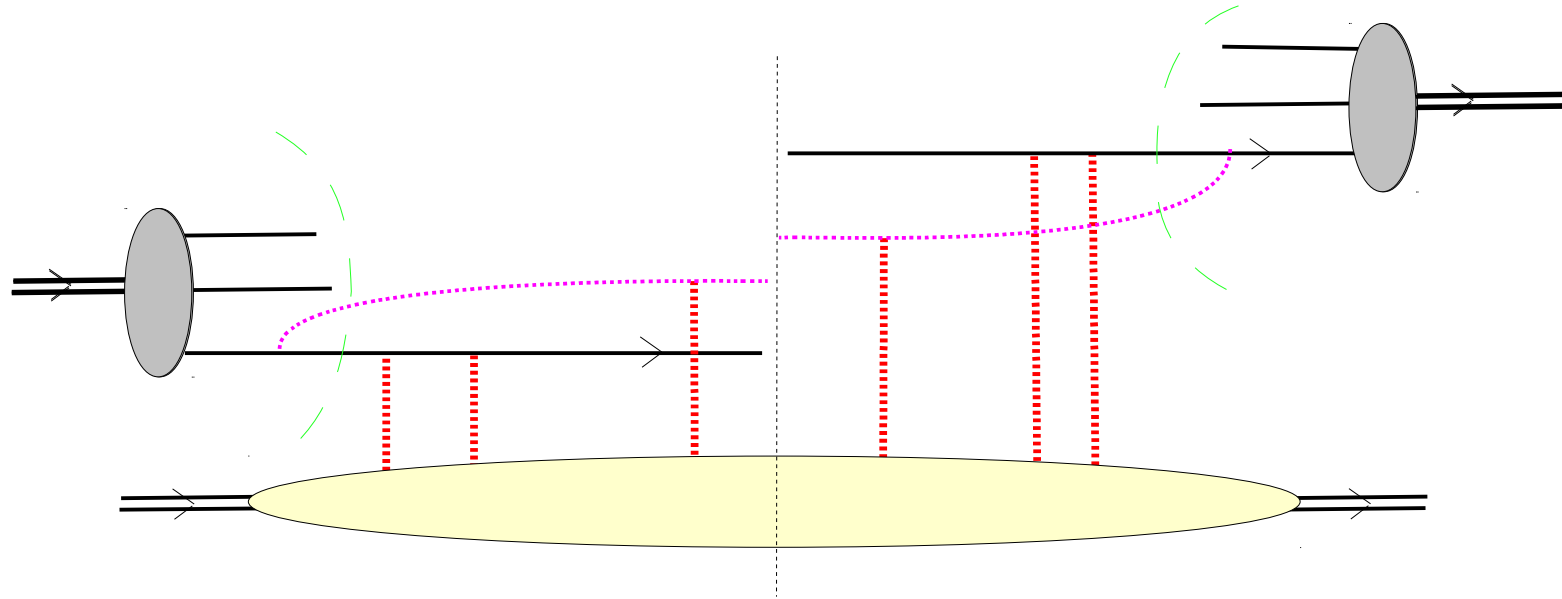
Formulation of dijet correlations



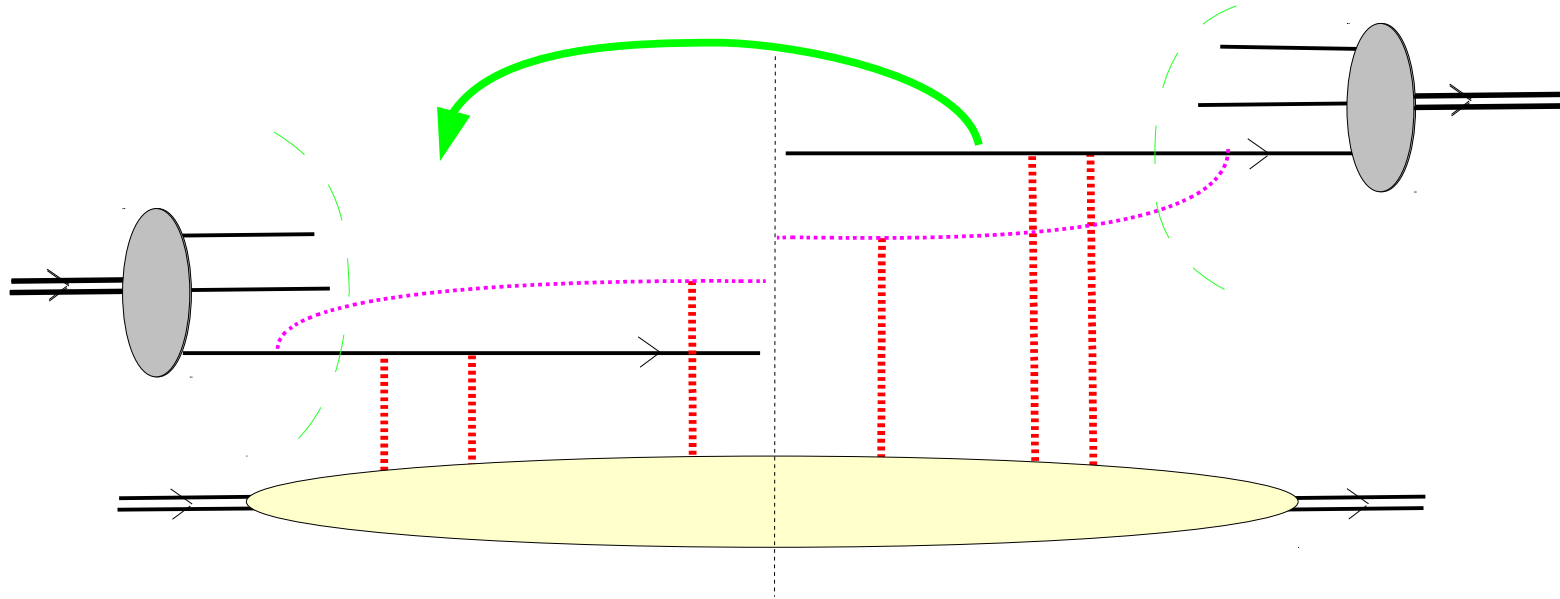
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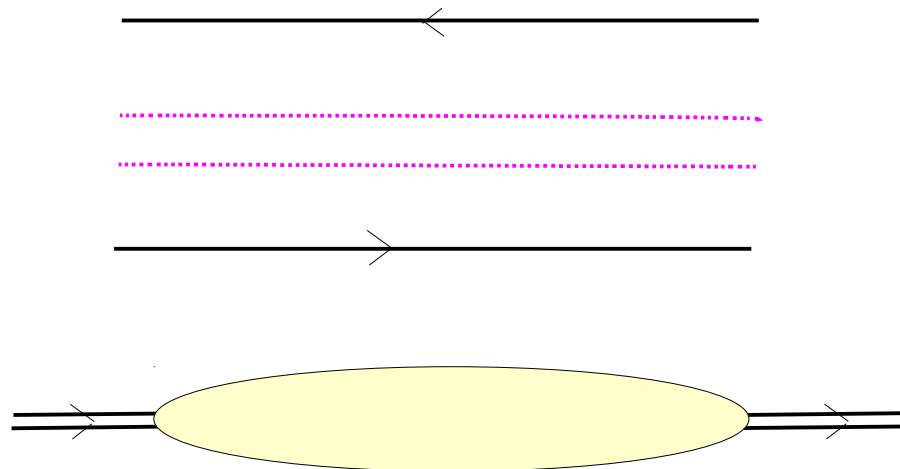
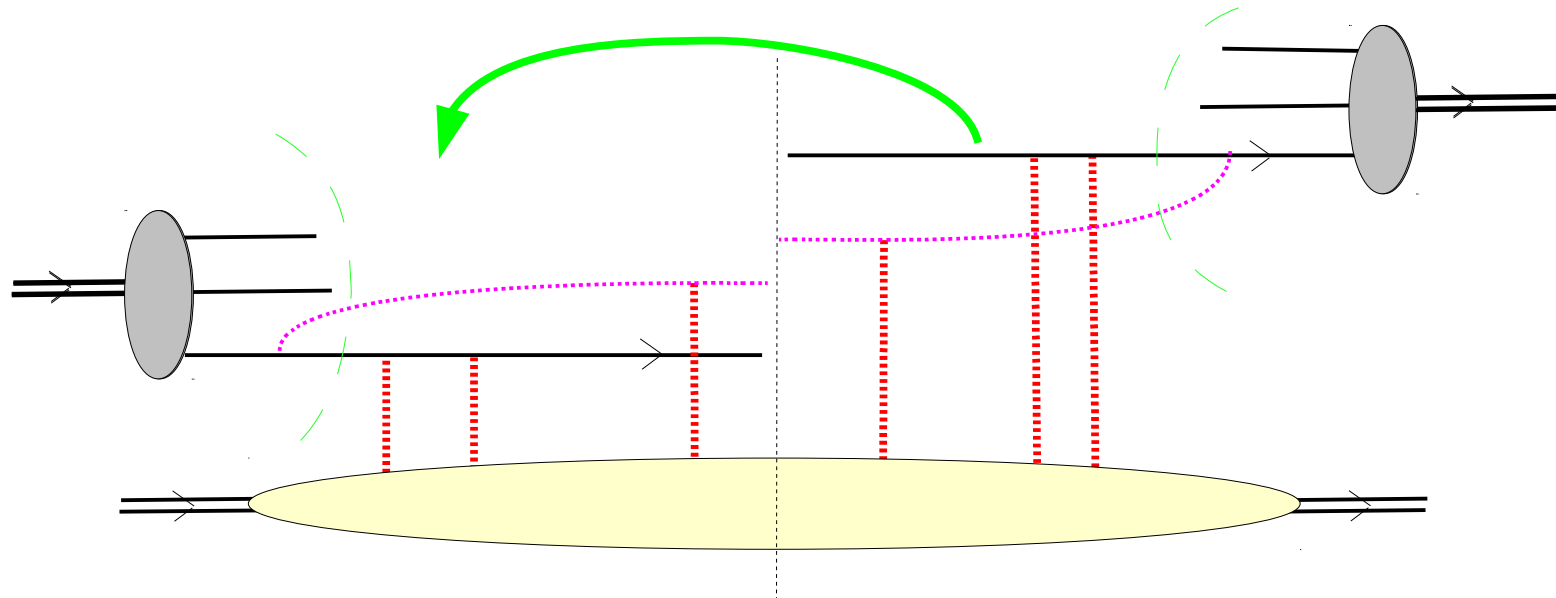
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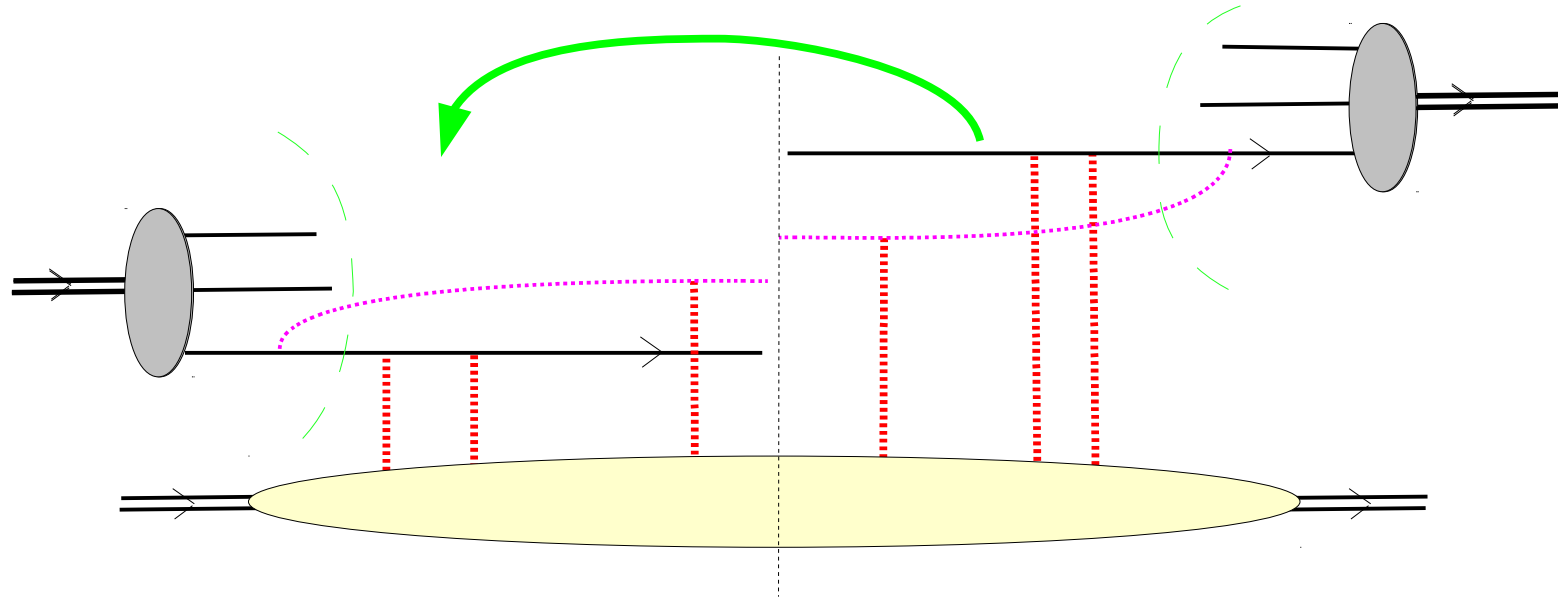
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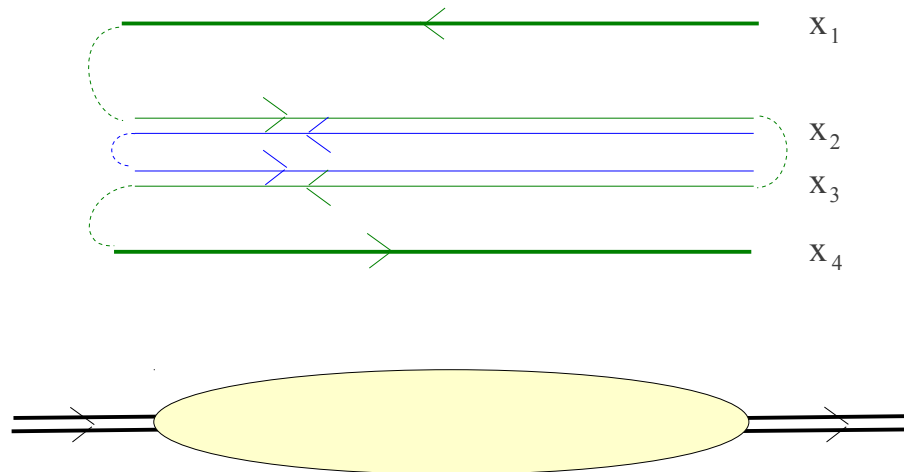
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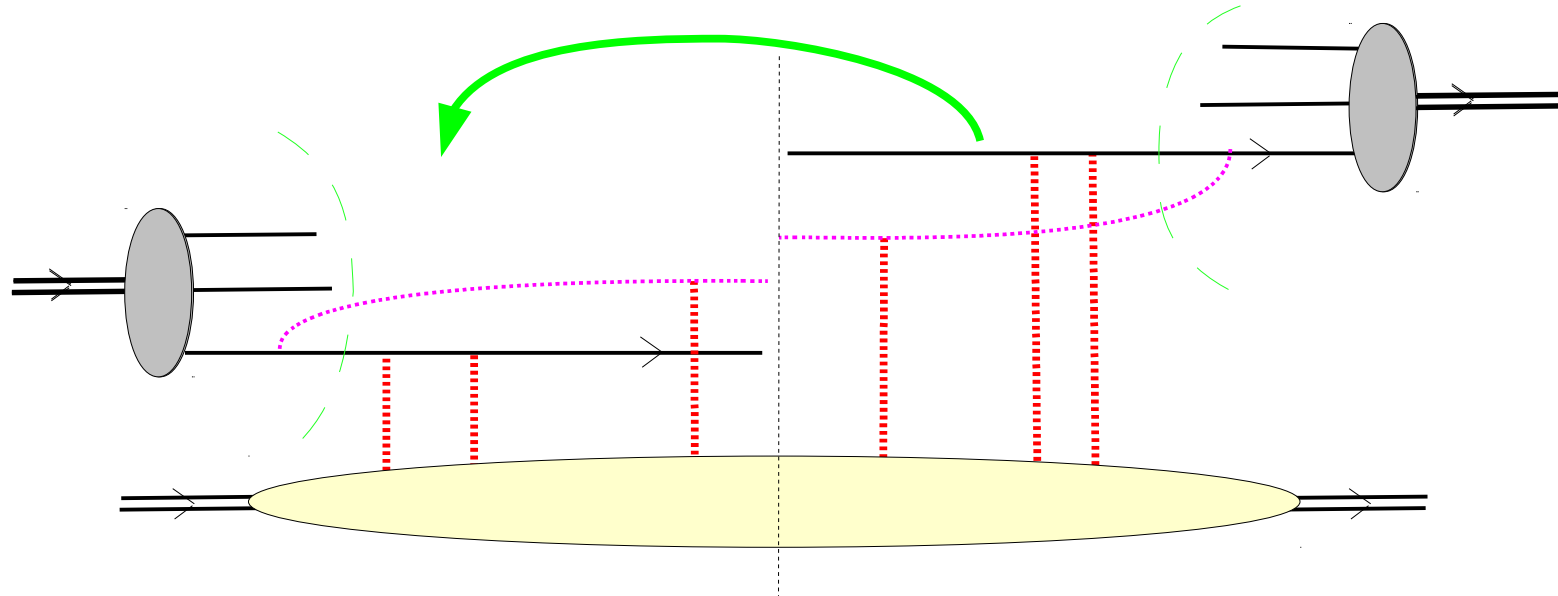
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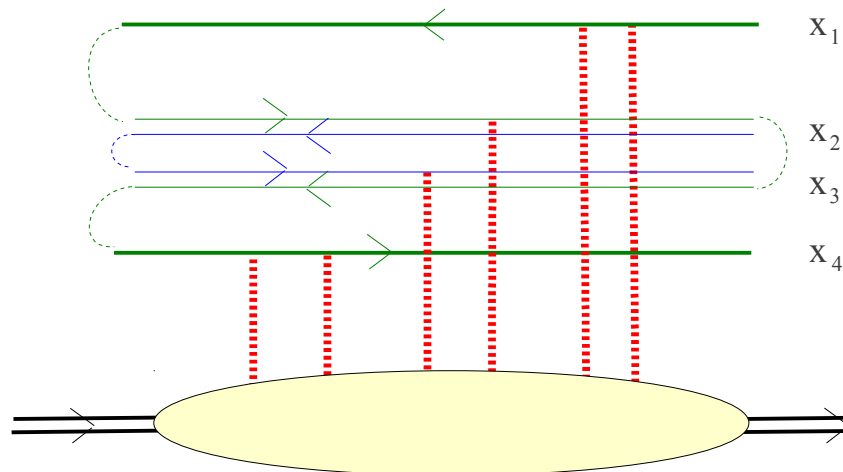
*Large
number-of-color
limit*



Formulation of dijet correlations



*Large
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$$S \propto \left\langle \text{Tr} \left(V_{x_2}^* V_{x_3} \right) \right\rangle$$

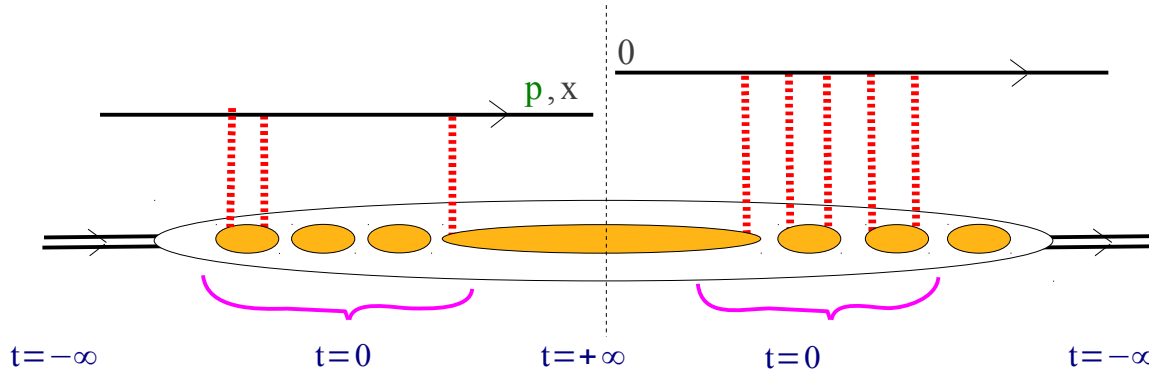
$$Q \propto \left\langle \text{Tr} \left(V_{x_1}^* V_{x_2} V_{x_3}^* V_{x_4} \right) \right\rangle$$

Outline

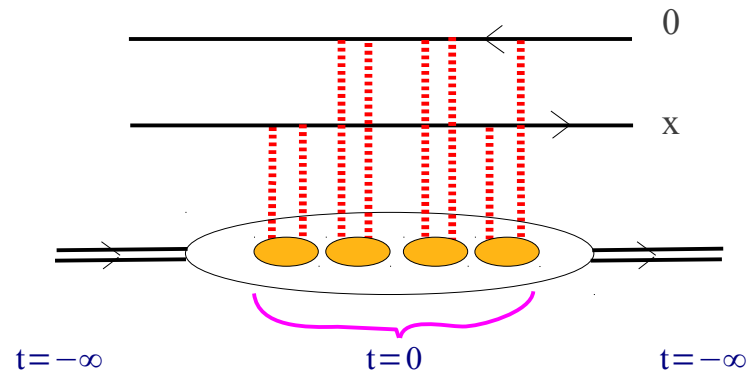
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Quantum corrections

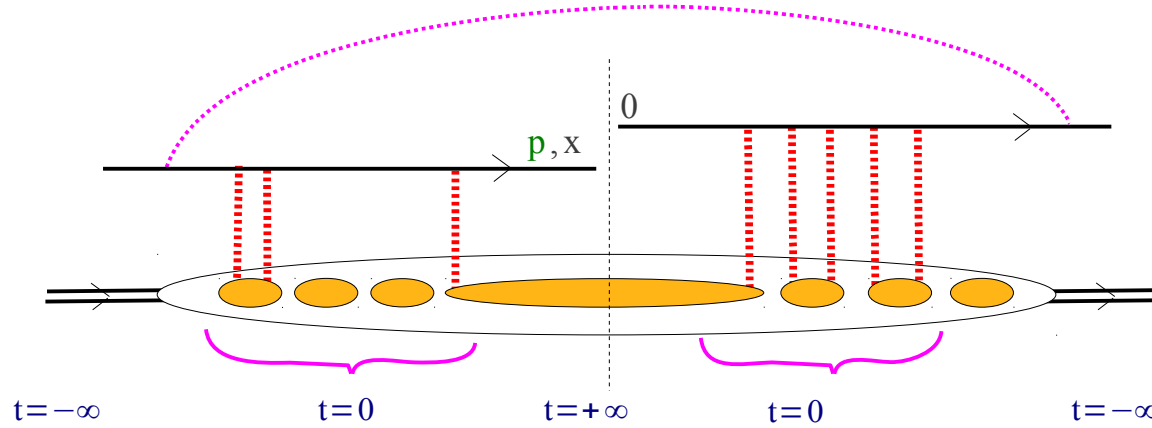


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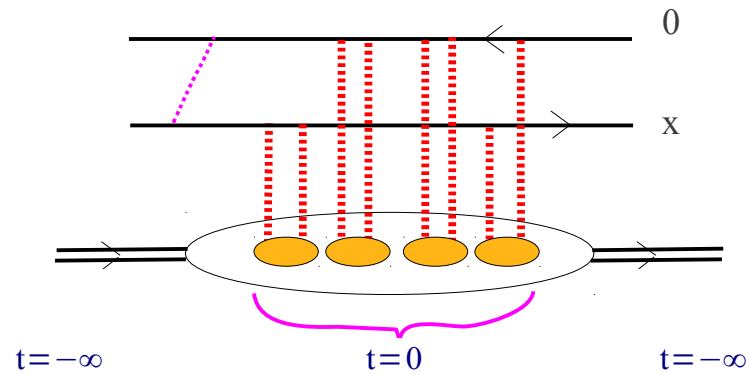


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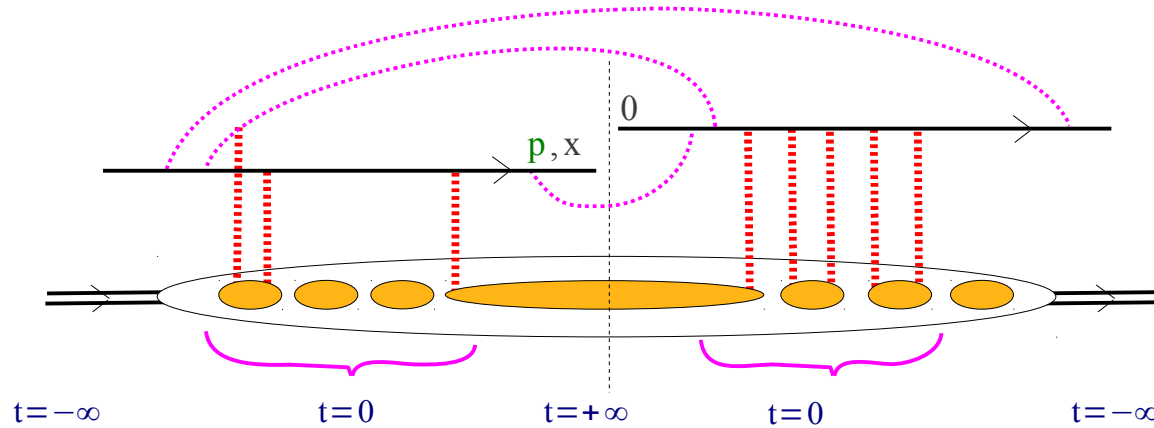


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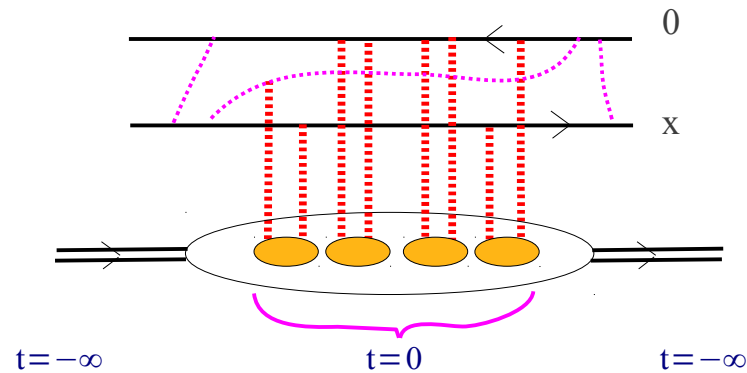


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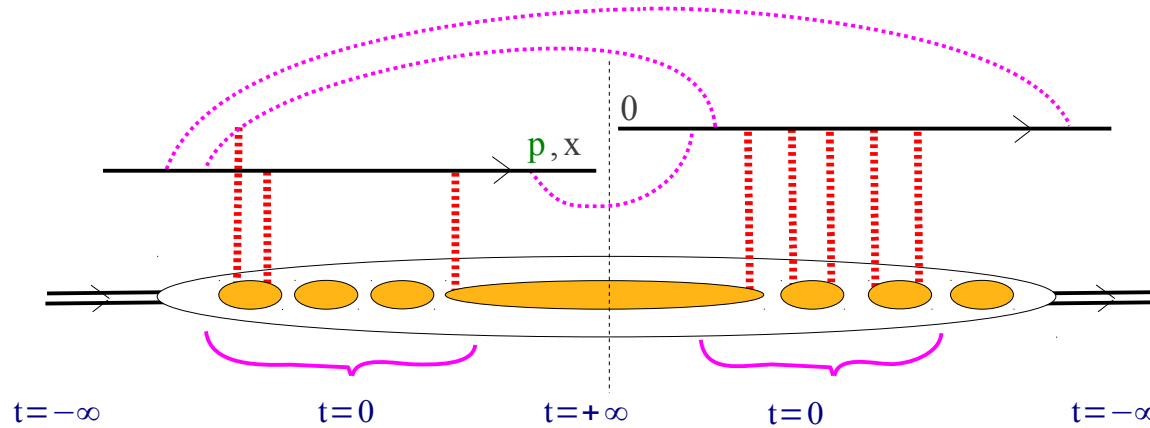


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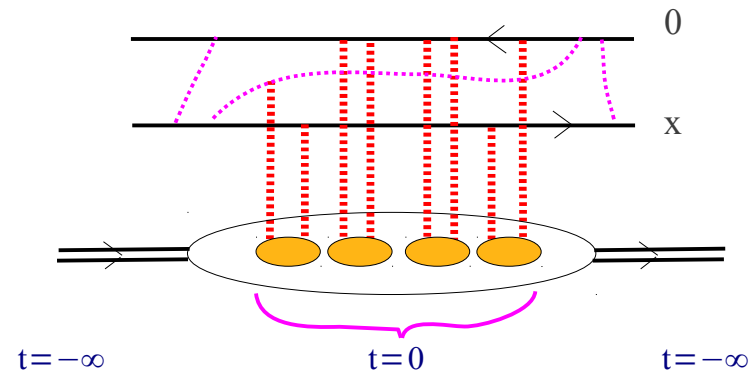


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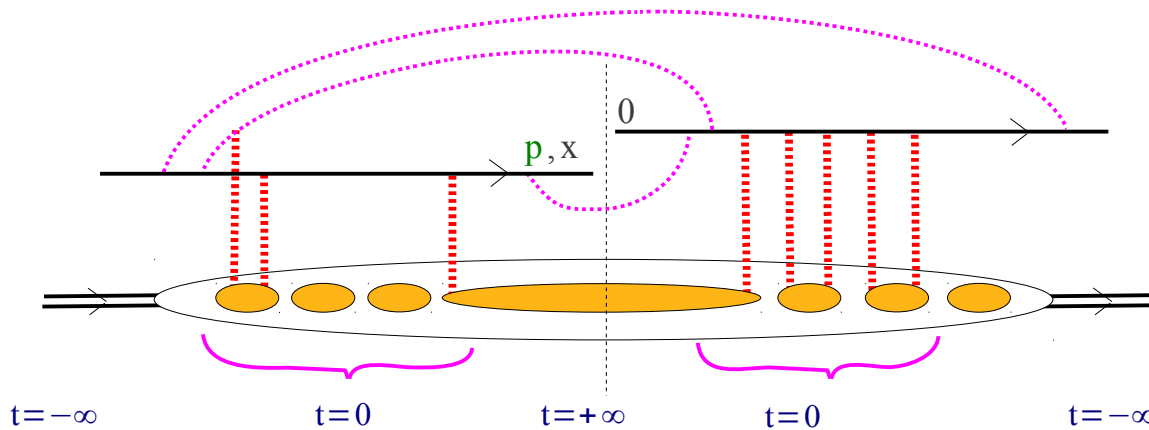


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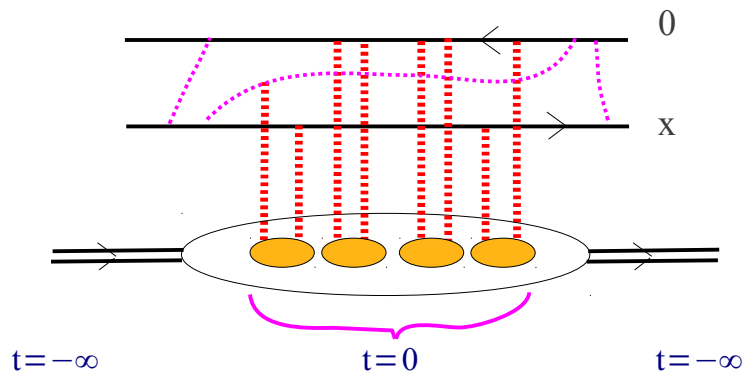


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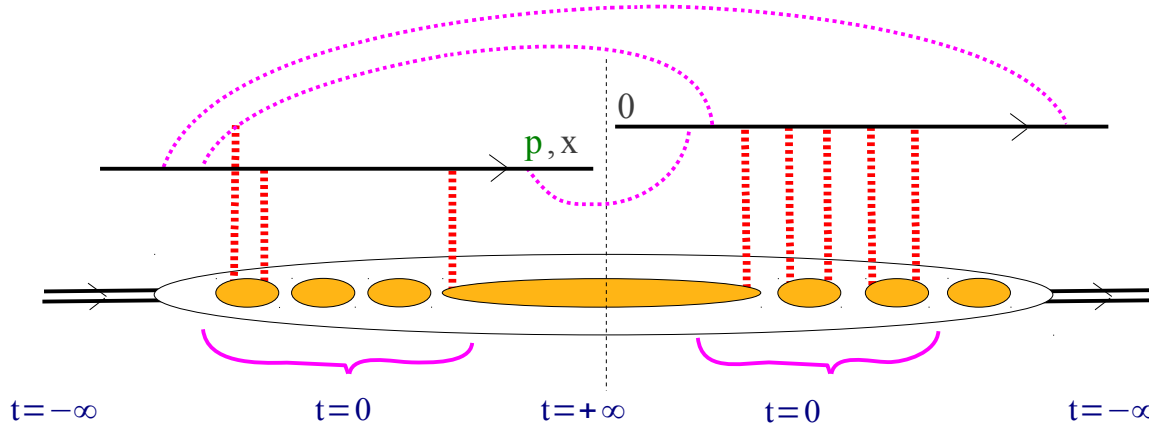


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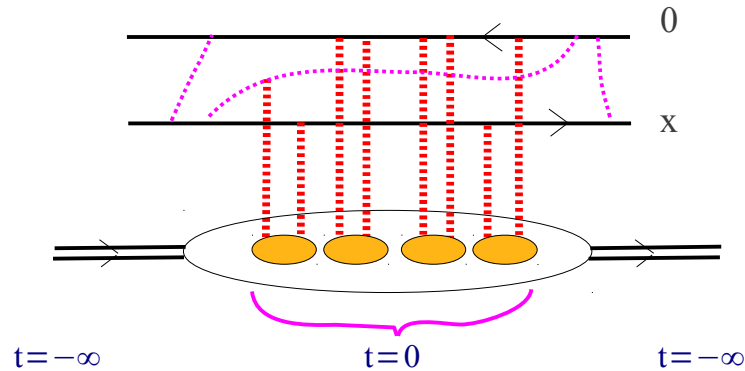


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*Identity
holds at
leading order
(1 gluon, iterated)*

Kovchegov et al

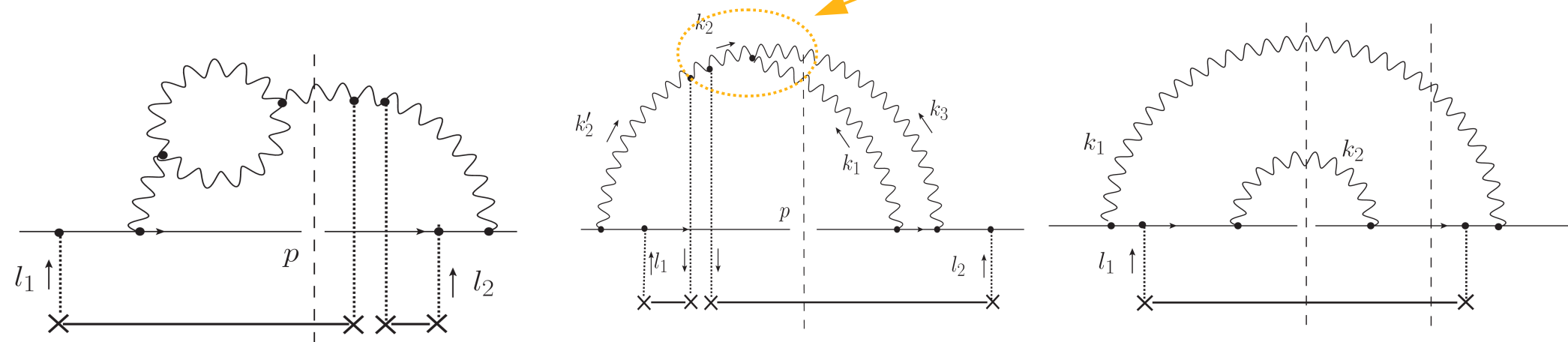
Quantum corrections: check at next-to-leading order

Hundreds of graphs on both (broadening and dipole) sides!

Mueller, Munier (2012)

May be grouped in 3 classes:

3-gluon vertex to compute exactly!



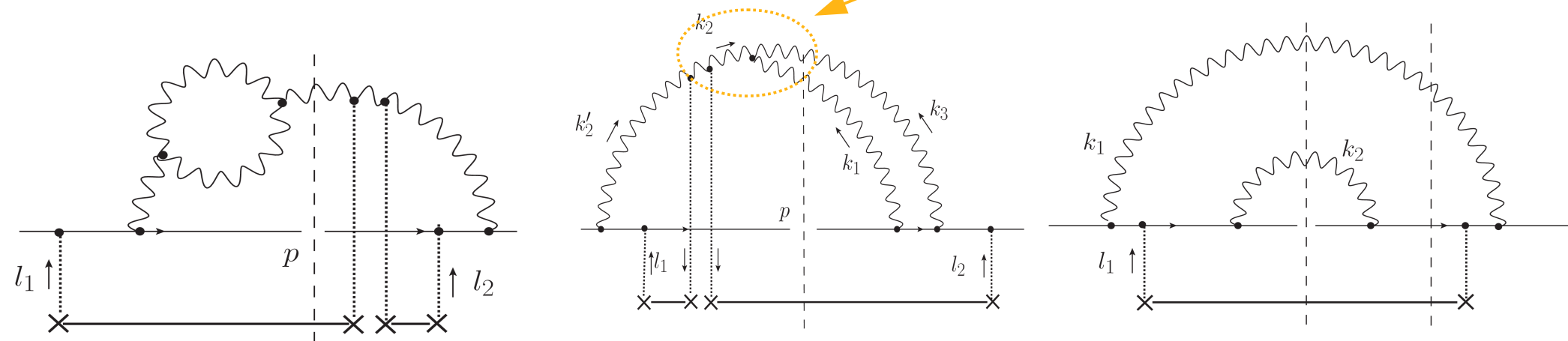
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The correspondence between broadening and dipole scattering is preserved at $\mathcal{N}LO$!

This statement is also true for the dijet/quadrupole correspondence.

Summary and outlook

- ★ *One and two-particle inclusive observables at the LHC are related to **dipole** and **quadrupole** amplitudes respectively, at semi-classical level, but also when quantum corrections are included at least **to next-to-leading order**.*

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- ★ Can one understand more features of the quadrupole amplitude?

Dominguez, Mueller, Munier, Xiao (2011)

- ★ Can one constrain this object experimentally?