

Lepton flavor violation in the scotogenic model

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Based on:

T. Toma, A. Vicente, arXiv:1312.2840
(to appear in JHEP)

Lepton flavor violation and new physics

Neutrino oscillations prove that lepton flavor is **not** conserved: there is **lepton flavor violation (LFV)**

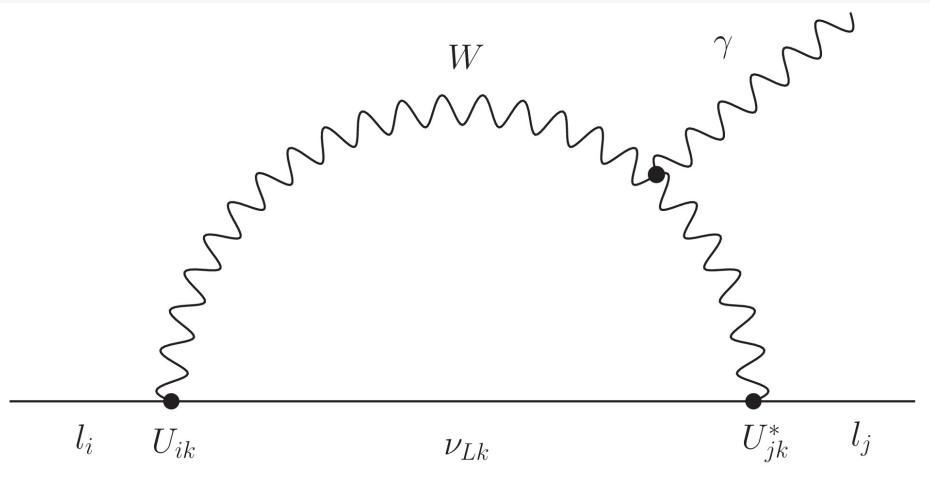
However... what about **charged lepton flavor violation (cLFV)**?

Lepton flavor violation and new physics

Neutrino oscillations prove that lepton flavor is **not** conserved: there is **lepton flavor violation (LFV)**

However... what about **charged lepton flavor violation (cLFV)**?

SM + neutrino masses



$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_k U_{ek} U_{\mu k}^* \frac{m_{\nu k}^2}{m_W^2} \right|^2 \lesssim 10^{-54}$$

Since neutrino masses are the **only source** of LFV, all cLFV amplitudes are strongly suppressed (in fact, GIM suppressed)

Lepton flavor violation and new physics

The observation of **cLFV** would be a clear signal of **physics beyond the Standard Model (BSM)**

- New interactions

Renormalizable: $(Y_\nu)_{ij} L_i H \nu_{Rj}$

Non-renormalizable: $\frac{c_{ij}}{\Lambda^2} H^\dagger H L_i H e_{Rj}$

- New sectors (coupled to the charged leptons)

Example: sleptons in SUSY: $(m_L^2)_{ij}$

Experimental projects

Great experimental perspectives!

LFV Process	Present Bound	Future Sensitivity
$\mu \rightarrow e\gamma$	5.7×10^{-13}	6×10^{-14} (MEG)
$\tau \rightarrow e\gamma$	3.3×10^{-8}	$\sim 10^{-8} - 10^{-9}$ (B factories)
$\tau \rightarrow \mu\gamma$	4.4×10^{-8}	$\sim 10^{-8} - 10^{-9}$ (B factories)
$\mu \rightarrow 3e$	1.0×10^{-12}	$\sim 10^{-16}$ (Mu3e)
$\tau \rightarrow 3e$	2.7×10^{-8}	$\sim 10^{-9} - 10^{-10}$ (B factories)
$\tau \rightarrow 3\mu$	2.1×10^{-8}	$\sim 10^{-9} - 10^{-10}$ (B factories)
$\mu^-, \text{Au} \rightarrow e^-, \text{Au}$	7.0×10^{-13}	—
$\mu^-, \text{SiC} \rightarrow e^-, \text{SiC}$	—	2×10^{-14} (DeeMe)
$\mu^-, \text{Al} \rightarrow e^-, \text{Al}$	—	$10^{-15} - 10^{-17}$ (COMET)
$\mu^-, \text{Ti} \rightarrow e^-, \text{Ti}$	4.3×10^{-12}	$10^{-17} - 10^{-18}$ (Mu2e) $\sim 10^{-18}$ (PRISM/PRIME)

LFV : How to look for?

In order to unravel the **physics behind LFV** (and neutrino masses!) we must:

- **Search for LFV in as many observables as possible:** they might have information about different sectors of the theory
- **Study the relations among different observables** (ratios, correlations, hierarchies...)
- **Understand the origin of such relations:** what is the underlying physics?

This is exactly what we want to do for **the scotogenic model**

The scotogenic model

Also known as...

The inert doublet model

The radiative seesaw

Ma's model

The scotogenic model

E. Ma, PRD 73 (2006) 077301 [hep-ph/0601225]

Field	$SU(2)_L \times U(1)_Y$	Z_2
L_i	(2, -1/2)	+
e_i	(1, 1)	+
ϕ	(2, -1/2)	+
N_i	(1, 0)	-
η	(2, -1/2)	-

σκότος
skotos = darkness



← Inert (or dark) doublet

Dark
Matter!

$$\mathcal{L}_N = \overline{N}_i \not{\partial} N_i - \frac{m_{N_i}}{2} \overline{N}_i^c N_i + y_{i\alpha} \eta \overline{N}_i \ell_\alpha + \text{h.c.}$$

$$\begin{aligned} \mathcal{V} = & m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) \\ & + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\lambda_5}{2} \left[(\phi^\dagger \eta)^2 + (\eta^\dagger \phi)^2 \right] \end{aligned}$$

The scotogenic model

E. Ma, PRD 73 (2006) 077301 [hep-ph/0601225]

$$\begin{aligned}\mathcal{V} = & m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) \\ & + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\lambda_5}{2} \left[(\phi^\dagger \eta)^2 + (\eta^\dagger \phi)^2 \right]\end{aligned}$$

Inert scalar sector: $\eta^\pm \quad \eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$

$$\begin{aligned}m_{\eta^+}^2 &= m_\eta^2 + \lambda_3 \langle \phi^0 \rangle^2 \\ m_R^2 &= m_\eta^2 + (\lambda_3 + \lambda_4 + \lambda_5) \langle \phi^0 \rangle^2 \\ m_I^2 &= m_\eta^2 + (\lambda_3 + \lambda_4 - \lambda_5) \langle \phi^0 \rangle^2\end{aligned} \quad \rightarrow \quad m_R^2 - m_I^2 = 2\lambda_5 \langle \phi^0 \rangle^2$$

Radiative neutrino masses

Tree-level:

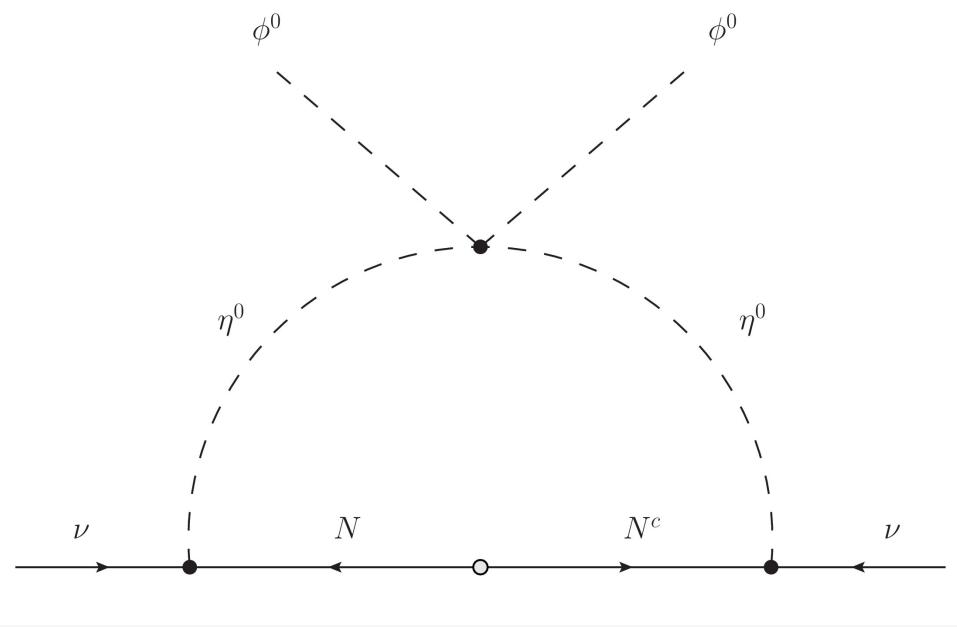
Forbidden by the Z_2 symmetry

Radiative generation of
neutrino masses



Additional
loop suppression

1-loop:



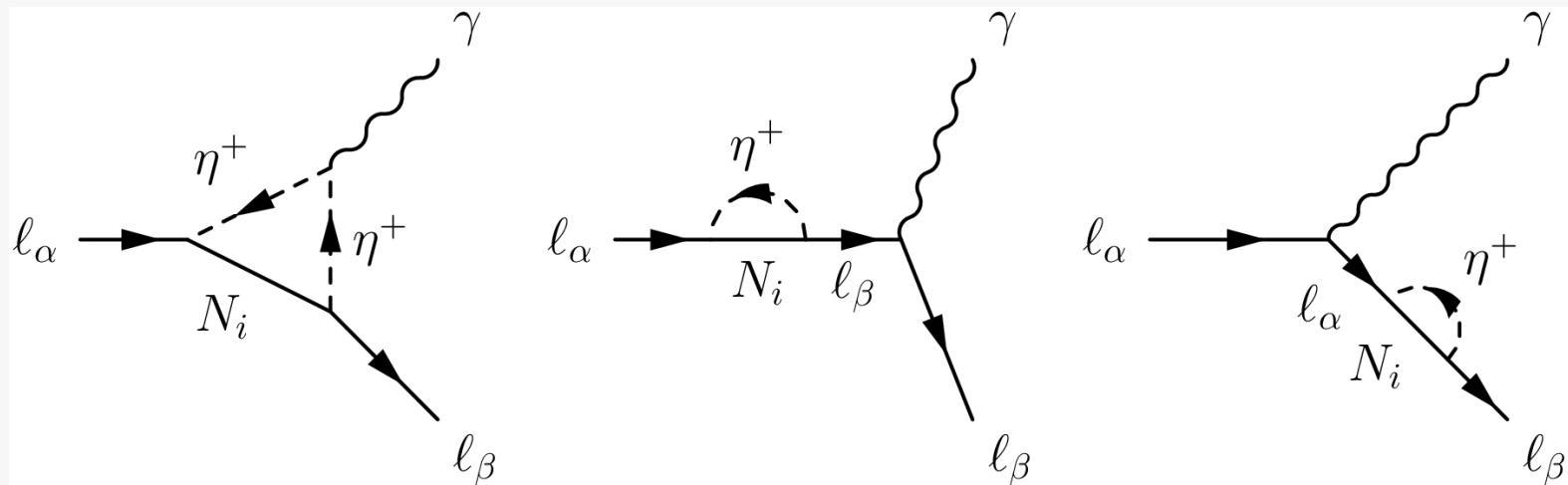
$$\begin{aligned}
 (m_\nu)_{\alpha\beta} &= \sum_{i=1}^3 \frac{y_{i\alpha} y_{i\beta}}{(4\pi)^2} m_{N_i} \left[\frac{m_R^2}{m_R^2 - m_{N_i}^2} \log \left(\frac{m_R^2}{m_{N_i}^2} \right) - \frac{m_I^2}{m_I^2 - m_{N_i}^2} \log \left(\frac{m_I^2}{m_{N_i}^2} \right) \right] \\
 &\simeq \sum_{i=1}^3 \frac{2\lambda_5 y_{i\alpha} y_{i\beta} \langle \phi^0 \rangle^2}{(4\pi)^2 m_{N_i}} \left[\frac{m_{N_i}^2}{m_0^2 - m_{N_i}^2} + \frac{m_{N_i}^4}{(m_0^2 - m_{N_i}^2)^2} \log \left(\frac{m_{N_i}^2}{m_0^2} \right) \right]
 \end{aligned}$$

(with $m_R^2 \simeq m_I^2 \equiv m_0^2$)

LFV in the scotogenic model: Analytical results

$$\ell_\alpha \rightarrow \ell_\beta \gamma$$

J. Kubo et al, PLB 642 (2006) 18
 E. Ma, M. Raidal, PRL 87 (2001) 011802



$$\text{Br}(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{3(4\pi)^3 \alpha_{\text{em}}}{4G_F^2} |A_D|^2 \text{Br}(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)$$

$$A_D = \sum_{i=1}^3 \frac{y_{i\beta}^* y_{i\alpha}}{2(4\pi)^2} \frac{1}{m_{\eta^+}^2} F_2(\xi_i)$$

↳ $(\xi_i \equiv m_{N_i}^2 / m_{\eta^+}^2)$

$$\ell_\alpha \rightarrow 3 \ell_\beta$$

T. Toma, A. Vicente, arXiv:1312.2840

4-fermion process: many more **diagrams** and **operators**

- Photon penguins
- Z-boson penguins
- Higgs penguins
- Boxes

$$\ell_\alpha \rightarrow 3 \ell_\beta$$

T. Toma, A. Vicente, arXiv:1312.2840

4-fermion process: many more diagrams and operators

- Photon penguins
- Z-boson penguins
- ~~Higgs penguins~~
- Boxes



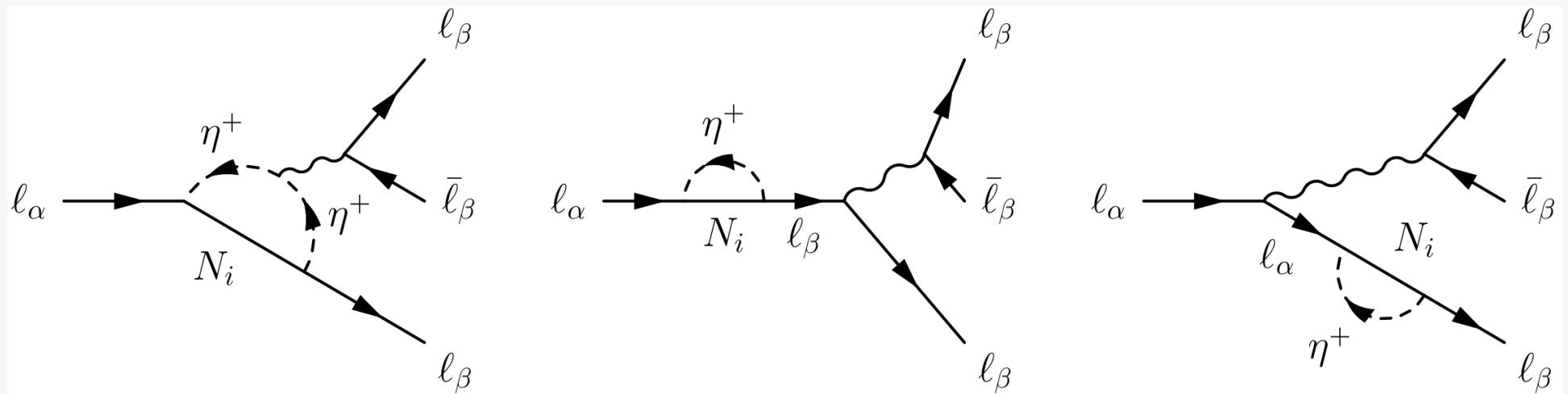
Negligible for the
first two generations
of charged leptons

$[\mu \rightarrow 3e]$

$$\ell_\alpha \rightarrow 3 \ell_\beta$$

$$\ell_\alpha(p) \rightarrow \ell_\beta(k_1)\bar{\ell}_\beta(k_2)\ell_\beta(k_3)$$

T. Toma, A. Vicente, arXiv:1312.2840



Photon penguins

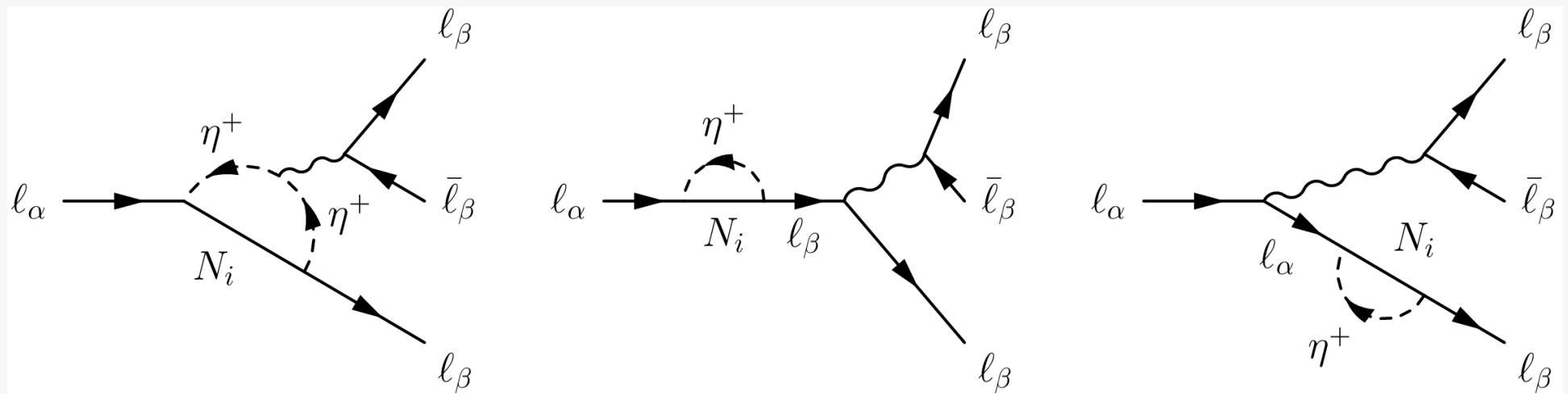
$$i\mathcal{M}_\gamma = ie^2 \bar{u}(k_1) \left[\textcolor{red}{A_{ND}} \gamma^\mu P_L + \frac{m_\alpha}{q^2} \textcolor{red}{A_D} \sigma^{\mu\nu} q_\nu P_R \right] u(p) \bar{u}(k_3) \gamma_\mu v(k_2) - (k_1 \leftrightarrow k_3)$$

$$A_{ND} = \sum_{i=1}^3 \frac{y_{i\beta}^* y_{i\alpha}}{6(4\pi)^2} \frac{1}{m_{\eta^+}^2} G_2(\xi_i) \quad A_D = \sum_{i=1}^3 \frac{y_{i\beta}^* y_{i\alpha}}{2(4\pi)^2} \frac{1}{m_{\eta^+}^2} F_2(\xi_i)$$

$$\ell_\alpha \rightarrow 3 \ell_\beta$$

$$\ell_\alpha(p) \rightarrow \ell_\beta(k_1)\bar{\ell}_\beta(k_2)\ell_\beta(k_3)$$

T. Toma, A. Vicente, arXiv:1312.2840



Z-boson penguins

$$i\mathcal{M}_Z = \frac{iF}{m_Z^2} \bar{u}(k_1)\gamma^\mu P_R u(p)\bar{u}(k_3)\gamma_\mu (g_L^\ell P_L + g_R^\ell P_R) v(k_2) - (k_1 \leftrightarrow k_3)$$

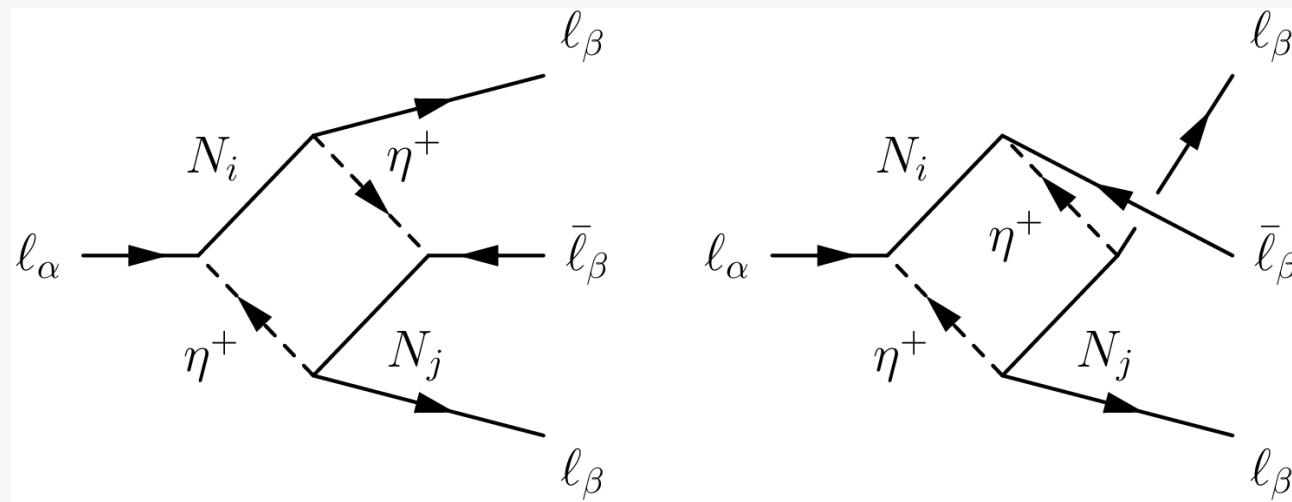
$$F = \sum_{i=1}^3 \frac{y_{i\beta}^* y_{i\alpha}}{2(4\pi)^2} \frac{m_\alpha m_\beta}{m_{\eta^+}^2} \frac{g_2}{\cos \theta_W} F_2(\xi_i)$$

Highly suppressed
by charged lepton
masses

$$\ell_\alpha \rightarrow 3 \ell_\beta$$

$$\ell_\alpha(p) \rightarrow \ell_\beta(k_1)\bar{\ell}_\beta(k_2)\ell_\beta(k_3)$$

T. Toma, A. Vicente, arXiv:1312.2840



Boxes

$$i\mathcal{M}_{\text{box}} = ie^2 \textcolor{blue}{B} [\bar{u}(k_3)\gamma^\mu P_L v(k_2)] [\bar{u}(k_1)\gamma_\mu P_L u(p)]$$

$$e^2 B = \frac{1}{(4\pi)^2 m_{\eta^+}^2} \sum_{i,j=1}^3 \left[\frac{1}{2} D_1(\xi_i, \xi_j) y_{j\beta}^* y_{j\beta} y_{i\beta}^* y_{i\alpha} + \sqrt{\xi_i \xi_j} D_2(\xi_i, \xi_j) y_{j\beta}^* y_{j\beta} y_{i\beta} y_{i\alpha} \right]$$

$$\ell_\alpha \rightarrow 3\ell_\beta$$

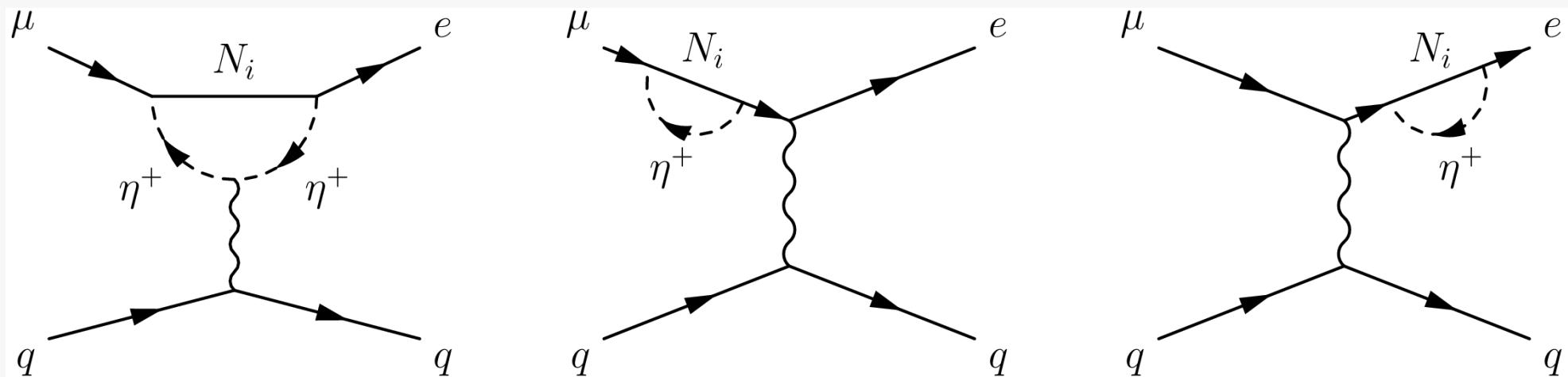
$$\begin{aligned}
\text{Br}(\ell_\alpha \rightarrow 3\ell_\beta) &= \frac{3(4\pi)^2 \alpha_{\text{em}}^2}{8G_F^2} \left[|A_{ND}|^2 + |A_D|^2 \left(\frac{16}{3} \log \left(\frac{m_\alpha}{m_\beta} \right) - \frac{22}{3} \right) \right. \\
&\quad + \frac{1}{6} |B|^2 + \frac{1}{3} (2|F_{RR}|^2 + |F_{RL}|^2) \\
&\quad \left. + \left(-2A_{ND}A_D^* + \frac{1}{3}A_{ND}B^* - \frac{2}{3}A_DB^* + \text{h.c.} \right) \right] \\
&\quad \times \text{Br}(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)
\end{aligned}$$

Photon penguins
Z-boson penguins
Boxes

$$\text{with } F_{RR,RL} = \frac{F g_{R,L}^\ell}{g_2^2 \sin^2 \theta_W m_Z^2}$$

$\mu - e$ conversion in nuclei

T. Toma, A. Vicente, arXiv:1312.2840



- No box contributions from the inert doublet (they do not couple to the quark sector)
- The phenomenology is determined by A_D , A_{ND} and F

↑
negligible

LFV in the scotogenic model: Phenomenological discussion

$$\mu \rightarrow e\gamma \quad \text{vs} \quad \mu \rightarrow 3e$$

Most LFV phenomenological studies focus on the radiative decay $\mu \rightarrow e\gamma$ and simply ignore other LFV observables. Two reasons:

- The great performance of the MEG experiment
- The **dipole dominance** in many models of interest

$\mu \rightarrow e\gamma$ VS $\mu \rightarrow 3e$

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————— [DIPOLE DOMINANCE ASSUMPTION] —————

| $A_D \gg A_{ND}, F, B, \dots$ |

| $\text{Br}(\mu \rightarrow 3e) \simeq \frac{\alpha_{\text{em}}}{3\pi} \left(\log \left(\frac{m_\mu^2}{m_e^2} \right) - \frac{11}{4} \right) \text{Br}(\mu \rightarrow e\gamma)$ |

| $\Rightarrow \quad \text{Br}(\mu \rightarrow 3e) \ll \text{Br}(\mu \rightarrow e\gamma)$ |

—————

$\mu \rightarrow e\gamma$ VS $\mu \rightarrow 3e$

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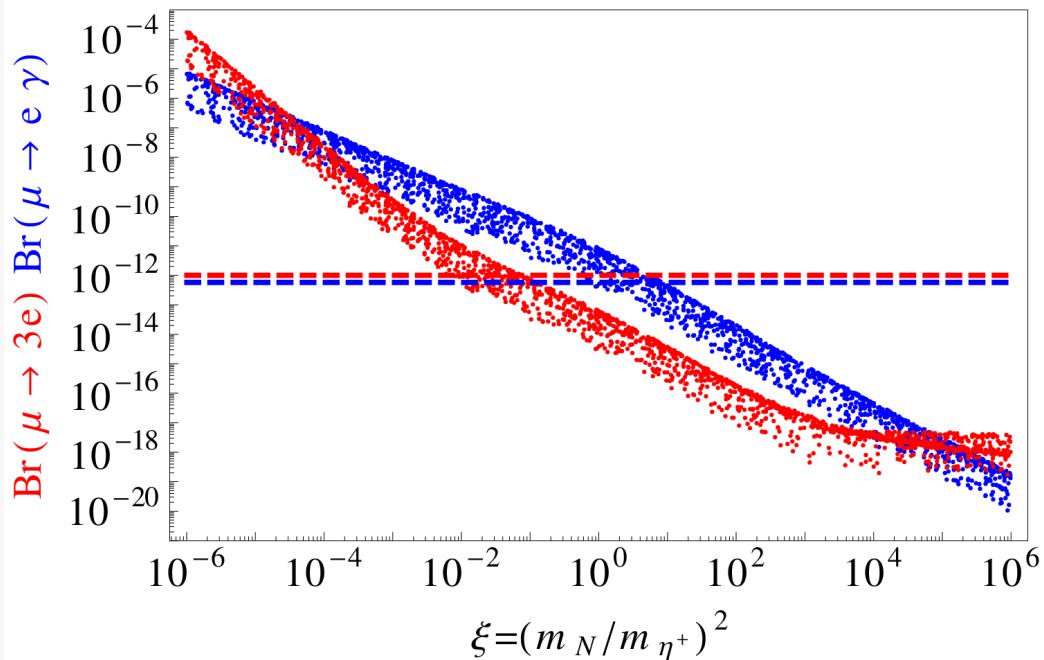
$$\boxed{\begin{array}{c} \text{[DIPOLE DOMINANCE ASSUMPTION]} \\ | \qquad A_D \gg A_{ND}, F, B, \dots \\ | \qquad \text{Br}(\mu \rightarrow 3e) \simeq \frac{\alpha_{\text{em}}}{3\pi} \left(\log \left(\frac{m_\mu^2}{m_e^2} \right) - \frac{11}{4} \right) \text{Br}(\mu \rightarrow e\gamma) \\ | \qquad \Rightarrow \qquad \text{Br}(\mu \rightarrow 3e) \ll \text{Br}(\mu \rightarrow e\gamma) \end{array}}$$

All LFV studies in the scotogenic model **assume dipole dominance**.

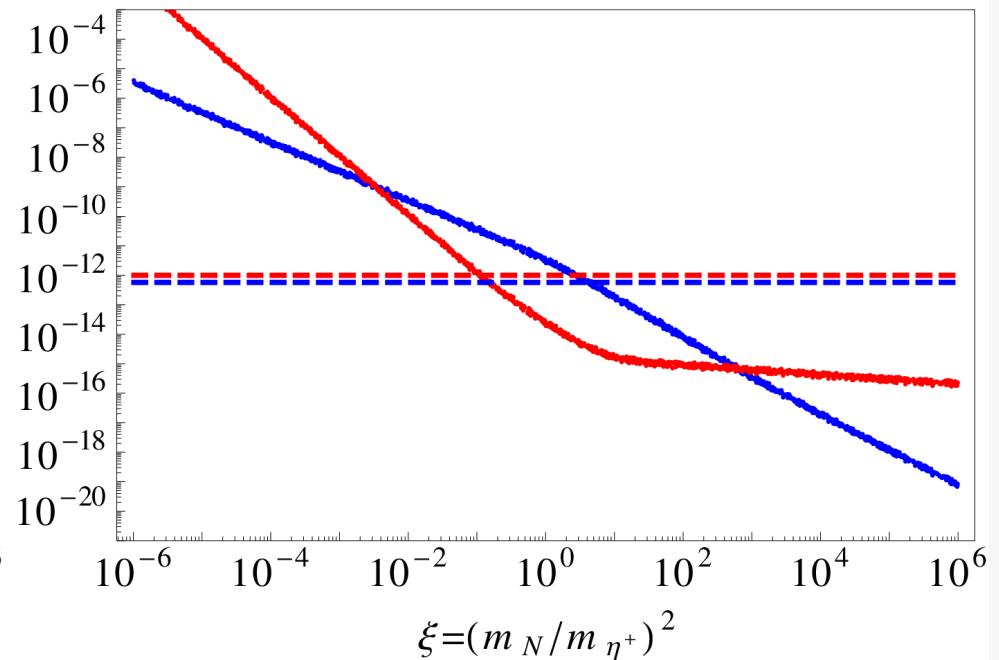
J. Kubo et al, PLB 642 (2006) 18, D. Aristizabal Sierra et al, PRD 79 (2009) 013011, D. Suematsu et al, PRD 79 (2009) 093004, A. Adulpravitchai et al, PRD 80 (2009) 055031

$\mu \rightarrow e\gamma$ VS $\mu \rightarrow 3e$

Normal Hierarchy



Inverted Hierarchy



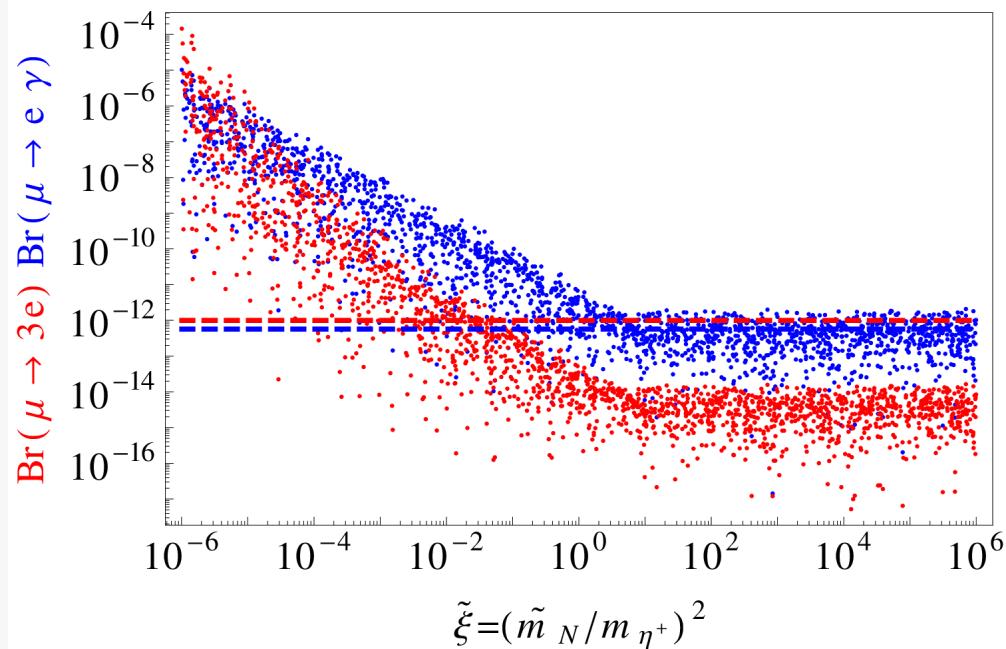
Degenerate right-handed neutrinos. Fixed $m_{\eta^+} = 1$ TeV and $m_{\nu_1} = 10^{-3}$ eV (lightest neutrino mass), $\lambda_5 = 10^{-9}$ and random Dirac phase.

It is possible to have $\text{Br}(\mu \rightarrow 3e) > \text{Br}(\mu \rightarrow e\gamma)$!

[Note that one can globally reduce all LFV rates by increasing λ_5]

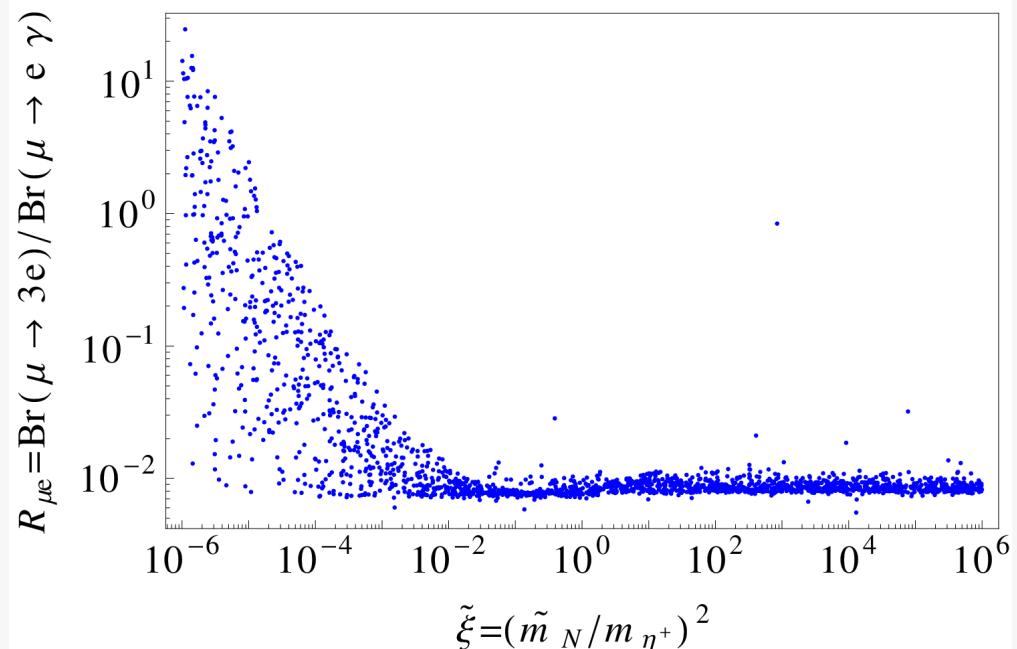
$\mu \rightarrow e\gamma$ VS $\mu \rightarrow 3e$

Also for non-degenerate right-handed neutrinos:



$$m_N = (\tilde{m}_N, \bar{m}_N^{(1)}, \bar{m}_N^{(2)})$$

$$\tilde{\xi} = (\tilde{m}_N / m_{\eta^+})^2$$



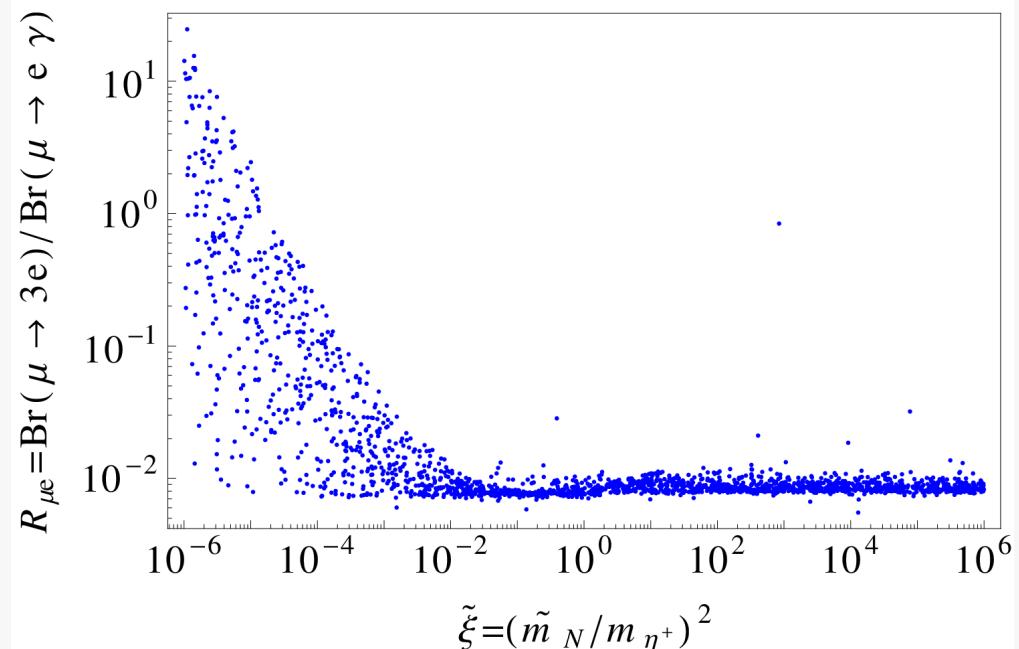
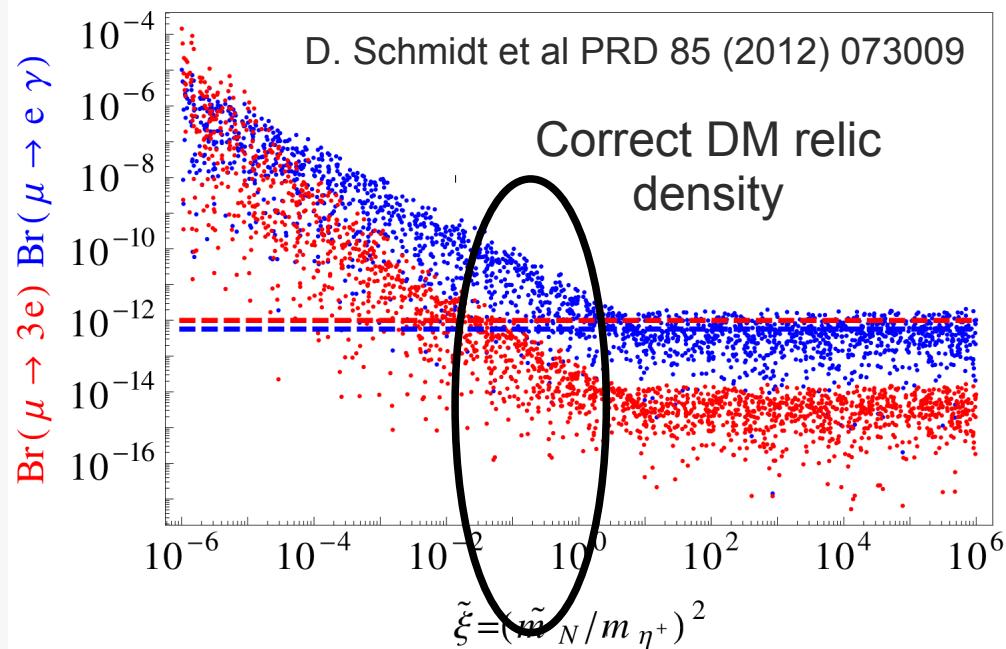
$$\bar{m}_N^{(1)} = 2 \text{ TeV}, \bar{m}_N^{(2)} = 3 \text{ TeV}$$

$$m_{\eta^+} = 1 \text{ TeV}, m_{\nu_1} = 10^{-3} \text{ eV}$$

We do not find any significant qualitative difference with the **degenerate** case

$\mu \rightarrow e\gamma$ VS $\mu \rightarrow 3e$

Also for non-degenerate right-handed neutrinos:



$$m_N = (\tilde{m}_N, \bar{m}_N^{(1)}, \bar{m}_N^{(2)})$$

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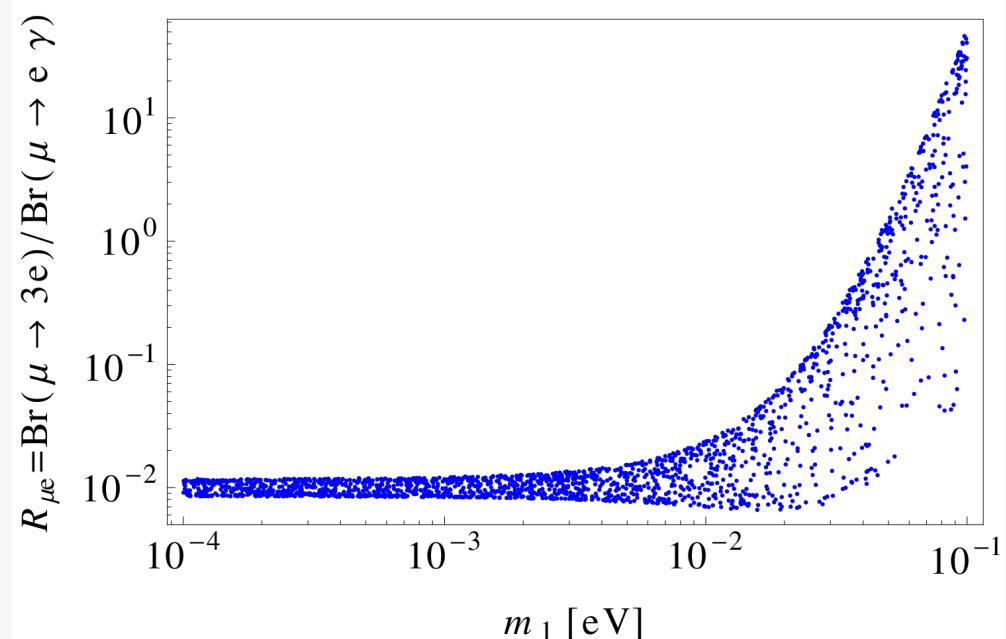
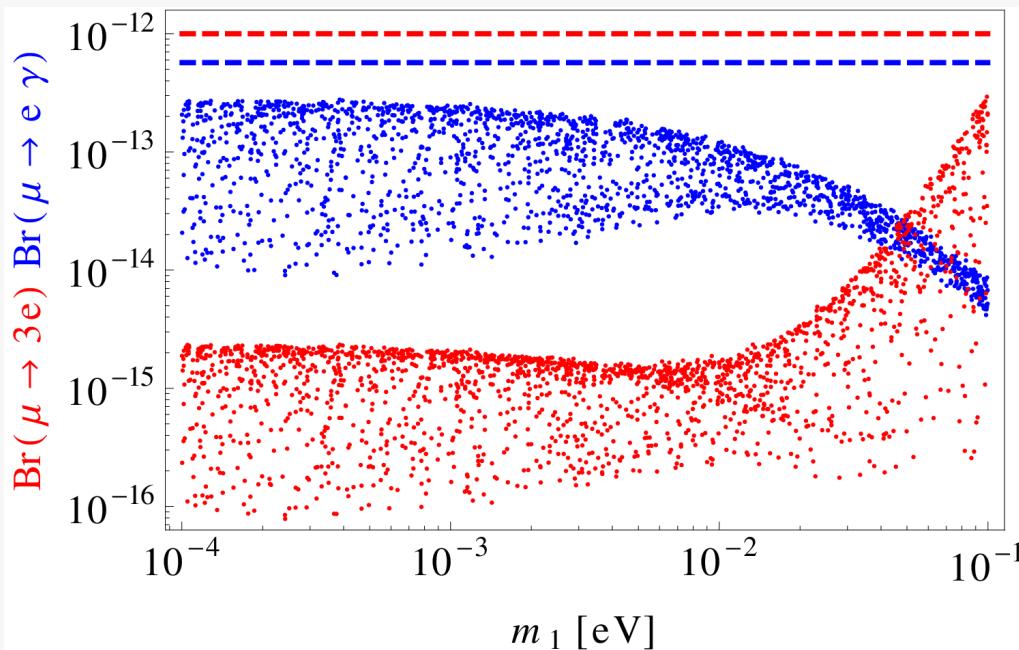
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We do not find any significant qualitative difference with the **degenerate** case

Low energy neutrino parameters

The low energy neutrino parameters may also have a strong impact on the phenomenology



Degenerate right-handed neutrinos. Fixed $m_{\eta^+} = 4$ TeV and $m_N = 1$ TeV and random Dirac phase.

Box contributions get enhanced for degenerate (active) neutrinos

$\mu - e$ conversion in nuclei

In $\mu - e$ conversion in nuclei there are no **box contributions**. Is this process **dipole** dominated?

$$\frac{\text{CR}(\mu - e, \text{Nucleus})}{\text{Br}(\mu \rightarrow e\gamma)} \approx \frac{f(\text{Nucleus})}{428} \quad \text{with } f(\text{Nucleus}) \sim 1.1 - 2.2$$

$\mu - e$ conversion in nuclei

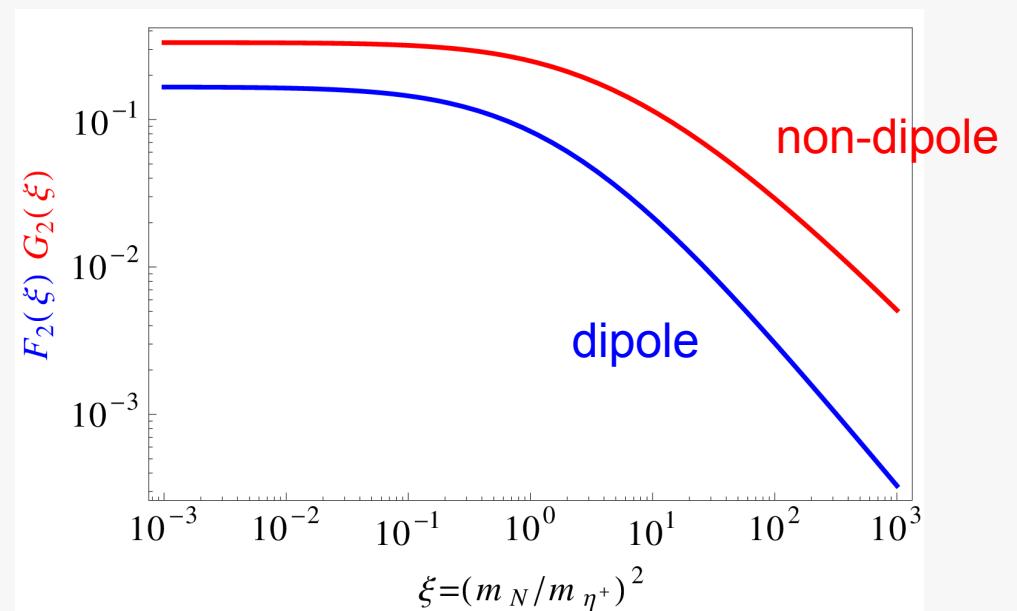
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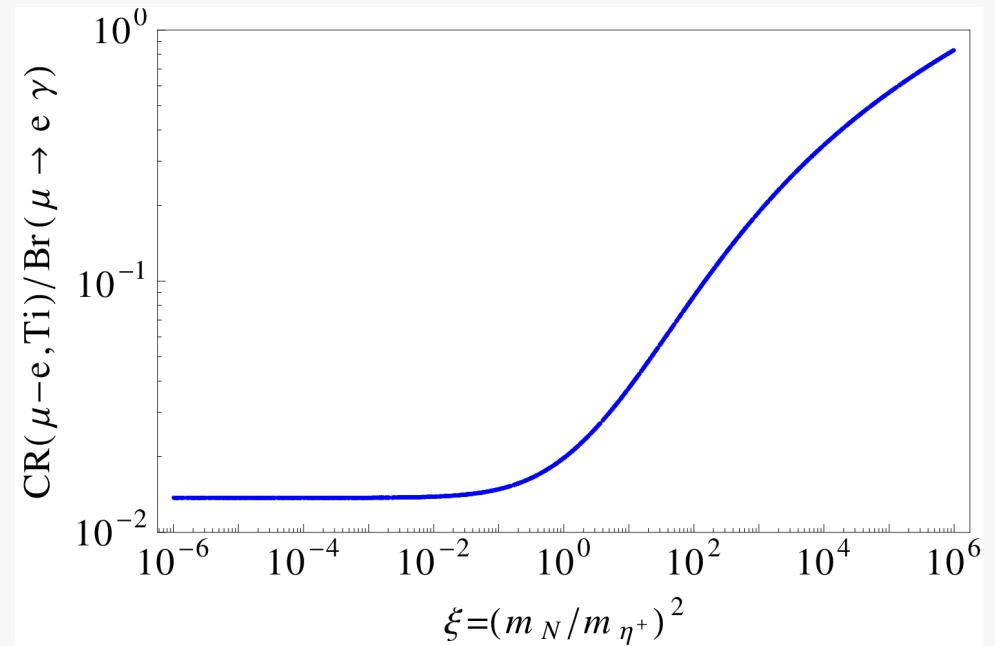
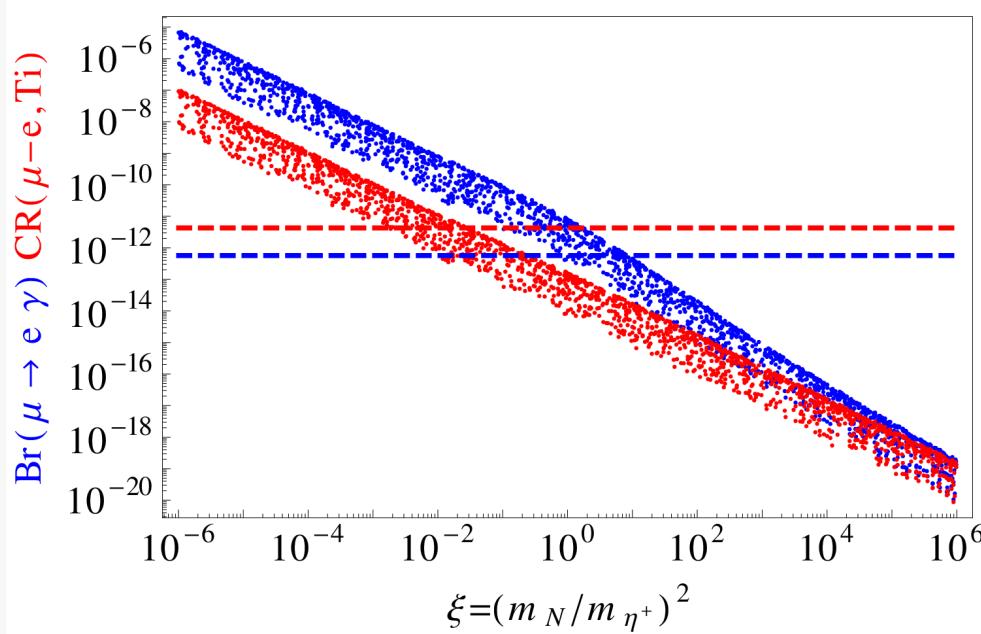
Not necessarily...

Photonic
dipole vs non-dipole

A_D A_{ND}



$\mu - e$ conversion in nuclei



Degenerate right-handed neutrinos. Fixed $m_{\eta^+} = 1$ TeV and $m_{\nu_1} = 10^{-3}$ eV (lightest neutrino mass), $\lambda_5 = 10^{-9}$ and random Dirac phase.

- The **conversion rate in nuclei** can be as large as $\text{Br}(\mu \rightarrow e\gamma)$
- Remember the great sensitivities in **future experiments**

Summary and remarks

Summary and remarks

Box diagrams dominate the LFV amplitudes in some parts of the parameter space. In particular, large Yukawa couplings and large mass hierarchies typically lead to enhanced box diagrams. This makes $\ell_\alpha \rightarrow 3\ell_\beta$ more constraining than $\ell_\alpha \rightarrow \ell_\beta\gamma$

Interestingly, the rate for $\mu - e$ conversion in nuclei can also be enhanced **beyond the dipole contribution** in some regions of the parameter space. The conversion rate can be as large as the branching ratio for $\mu \rightarrow e\gamma$

We have constructed parameter points where all the requirements for **right-handed neutrino dark matter** are met: $m_N < m_\eta$, large Yukawa couplings and m_N in the appropriate range (as found in **dedicated studies**). The scenario is a little fine-tuned but viable

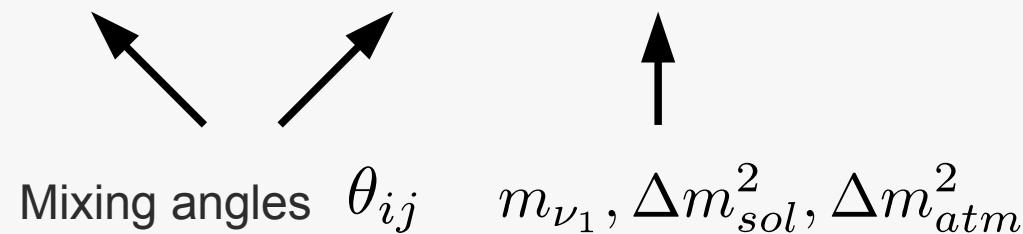


Thank you!

Backup slides

Radiative neutrino masses

$$U_{\text{PMNS}}^T m_\nu U_{\text{PMNS}} = m_\nu^{\text{diag}}$$



T. Toma, AV, arXiv:1312.2840

$$m_\nu = y^T \Lambda y$$

$$\Lambda_{ij} = \frac{m_{N_i}}{(4\pi)^2} \left[\frac{m_R^2}{m_R^2 - m_{N_i}^2} \log \left(\frac{m_R^2}{m_{N_i}^2} \right) - \frac{m_I^2}{m_I^2 - m_{N_i}^2} \log \left(\frac{m_I^2}{m_{N_i}^2} \right) \right] \delta_{ij}$$

$$y = \sqrt{\Lambda}^{-1} R \sqrt{\hat{m}_\nu} U_{\text{PMNS}}^\dagger$$

Modified
Casas-Ibarra parameterization

Dark matter

The lightest particle charged under Z_2 is stable: **dark matter candidate**

Fermion Dark Matter: N_1

- It can only be produced via **Yukawa** interactions
- Potential problems with lepton flavor violation: is it compatible with the current bounds?

J. Kubo, E. Ma, D. Suematsu, PLB 642 (2006) 18, D. Aristizabal Sierra, J. Kubo, D. Restrepo, D. Suematsu, O. Zapata, PRD 79 (2009) 013011, D. Suematsu, T. Toma, T. Yoshida, PRD 79 (2009) 093004, D. Schmidt, T. Schwetz, T. Toma, PRD 85 (2012) 073009, ...

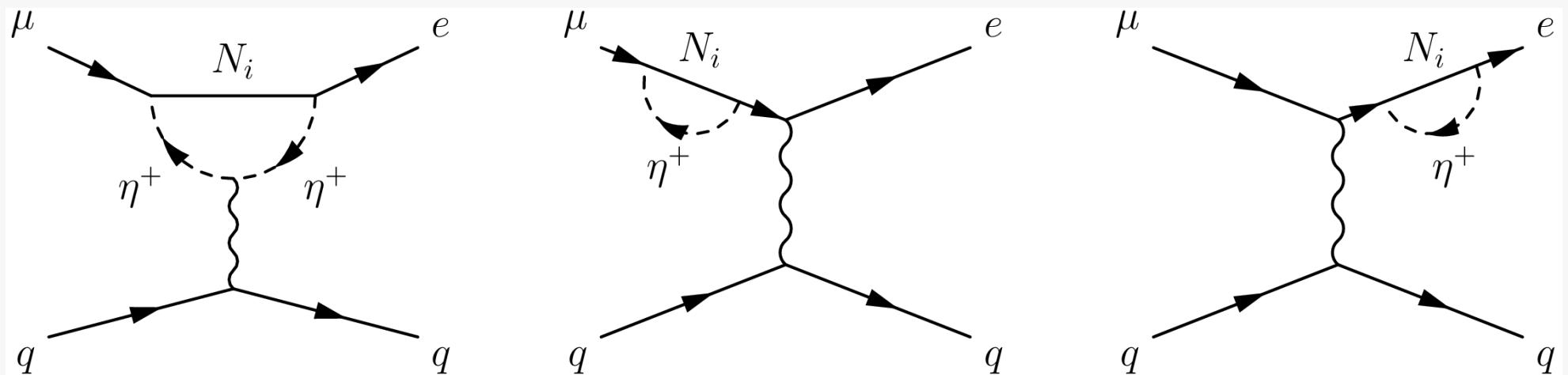
Scalar Dark Matter: the lightest neutral η scalar, η_R or η_I

- It also has **gauge** interactions
- Not correlated to lepton flavor violation

R. Barbieri, L. J. Hall, V. S. Rychkov, PRD 74 (2006) 015007, M. Cirelli, N. Fornengo, A. Strumia, NPB 753 (2006) 178, L. L. Honorez, E. Nezri, J. F. Oliver, M. H. G. Tytgat, JCAP 0702 (2007) 028, Q.-H. Cao, E. Ma, PRD (2007) 095011, S. Andreas, M. H. G. Tytgat, Q. Swillens, JCAP 0904 (2009) 004, E. Nezri, M. H. G. Tytgat, G. Vertongen, JCAP 0904 (2009) 014, T. Hambye, F.-S. Ling, L. L. Honorez, J. Roche, JHEP 07 (2009) 090, L. L. Honorez, C. E. Yaguna, JHEP 1009 (2010) 046 and JCAP 1101 (2011) 002, S. Kashiwase, D. Suematsu, PRD 86 (2012) 053001, ...

$\mu - e$ conversion in nuclei

T. Toma, A. Vicente, arXiv:1312.2840



$$\begin{aligned} \text{CR}(\mu - e, \text{Nucleus}) = & \frac{p_e E_e m_\mu^3 G_F^2 \alpha_{\text{em}}^3 Z_{\text{eff}}^4 F_p^2}{8 \pi^2 Z} \\ & \times \left\{ \left| (Z + N) \left(g_{LV}^{(0)} + g_{LS}^{(0)} \right) + (Z - N) \left(g_{LV}^{(1)} + g_{LS}^{(1)} \right) \right|^2 + \right. \\ & \left. \left| (Z + N) \left(g_{RV}^{(0)} + g_{RS}^{(0)} \right) + (Z - N) \left(g_{RV}^{(1)} + g_{RS}^{(1)} \right) \right|^2 \right\} \frac{1}{\Gamma_{\text{capt}}} \end{aligned}$$

$\mu - e$ conversion in nuclei

T. Toma, A. Vicente, arXiv:1312.2840

$$g_{XK}^{(0)} = \frac{1}{2} \sum_{q=u,d,s} \left(g_{XK(q)} G_K^{(q,p)} + g_{XK(q)} G_K^{(q,n)} \right)$$

$$g_{XK}^{(1)} = \frac{1}{2} \sum_{q=u,d,s} \left(g_{XK(q)} G_K^{(q,p)} - g_{XK(q)} G_K^{(q,n)} \right)$$

$$g_{LV(q)} = g_{LV(q)}^\gamma + g_{LV(q)}^Z$$

$$g_{RV(q)} = g_{LV(q)} \Big|_{L \leftrightarrow R}$$

$$g_{LS(q)} \approx 0$$

$$g_{RS(q)} \approx 0$$

$$g_{LV(q)}^\gamma = \frac{\sqrt{2}}{G_F} e^2 Q_q (\textcolor{red}{A}_{ND} - A_D)$$

$$g_{RV(q)}^Z = -\frac{\sqrt{2}}{G_F} \frac{g_L^q + g_R^q}{2} \frac{F}{m_Z^2}$$

Loop functions

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{6(1-x)^4}$$

$$G_2(x) = \frac{2 - 9x + 18x^2 - 11x^3 + 6x^3 \log x}{6(1-x)^4}$$

$$D_1(x, y) = -\frac{1}{(1-x)(1-y)} - \frac{x^2 \log x}{(1-x)^2(x-y)} - \frac{y^2 \log y}{(1-y)^2(y-x)}$$

$$D_2(x, y) = -\frac{1}{(1-x)(1-y)} - \frac{x \log x}{(1-x)^2(x-y)} - \frac{y \log y}{(1-y)^2(y-x)}$$

$\mu \rightarrow e\gamma$ vs $\mu \rightarrow 3e$

$$R_{\mu e} = \frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e\gamma)}$$

$R_{\mu e} < 1 \Rightarrow \text{Br}(\mu \rightarrow e\gamma)$ is the most relevant observable

Usual **assumption**

In case of pure dipole dominance $R_{\mu e} \sim 0.006$

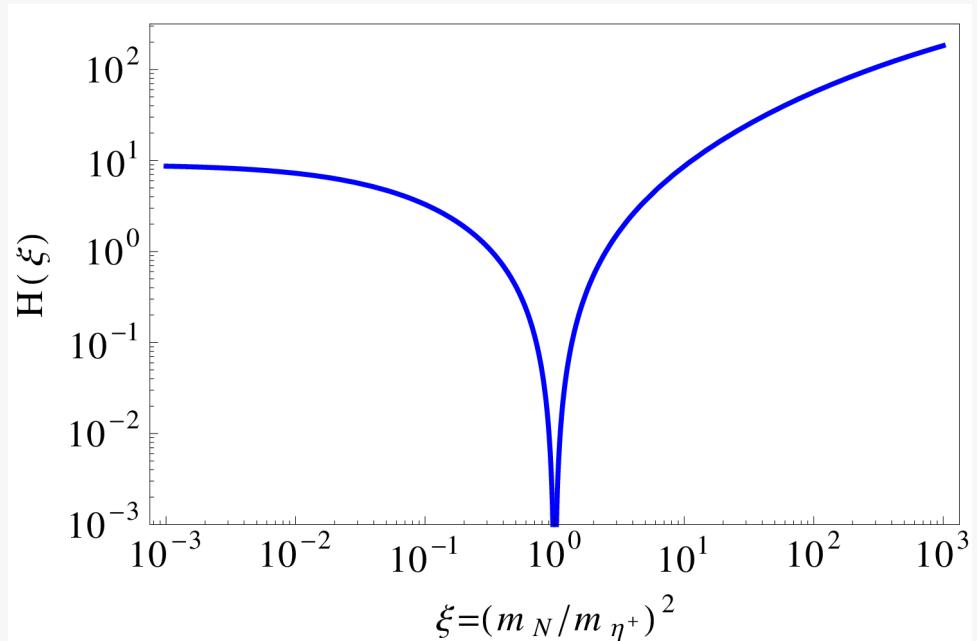
$R_{\mu e} > 1 \Rightarrow \text{Br}(\mu \rightarrow 3e)$ can be more constraining

$\mu \rightarrow e\gamma$ vs $\mu \rightarrow 3e$

Assuming that **boxes** dominate in $\mu \rightarrow 3e$
one can estimate

$$R_{\mu e} \sim \frac{y^4}{48\pi^2 e^2} H(\xi)$$

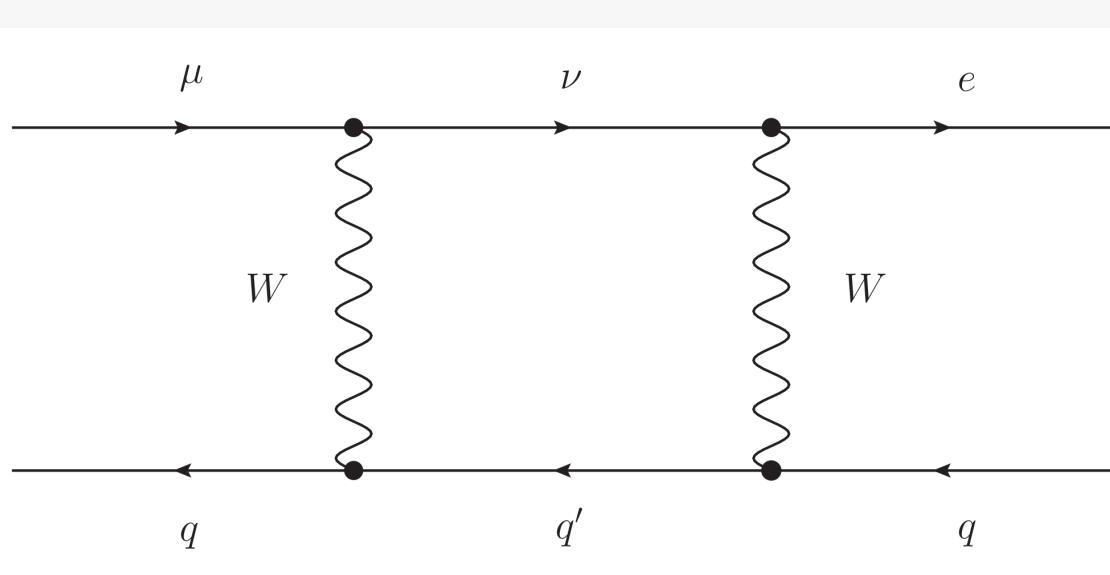
$$H(\xi) = \left(\frac{\frac{1}{2}D_1(\xi, \xi) + \xi D_2(\xi, \xi)}{F_2(\xi)} \right)^2$$



\Rightarrow Large **Yukawa** couplings and large **mass hierarchy**

A side comment...

This is **similar** to what is found for $\mu - e$ conversion in nuclei in models with **light right-handed neutrinos**



A. Ilakovac, A. Pilaftsis
PRD 80 (2009) 091902

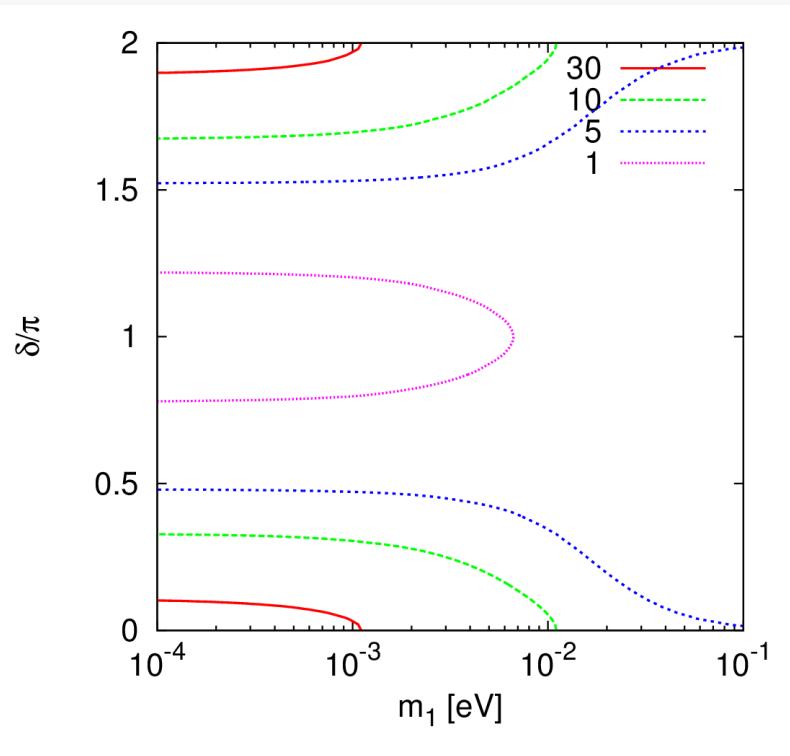
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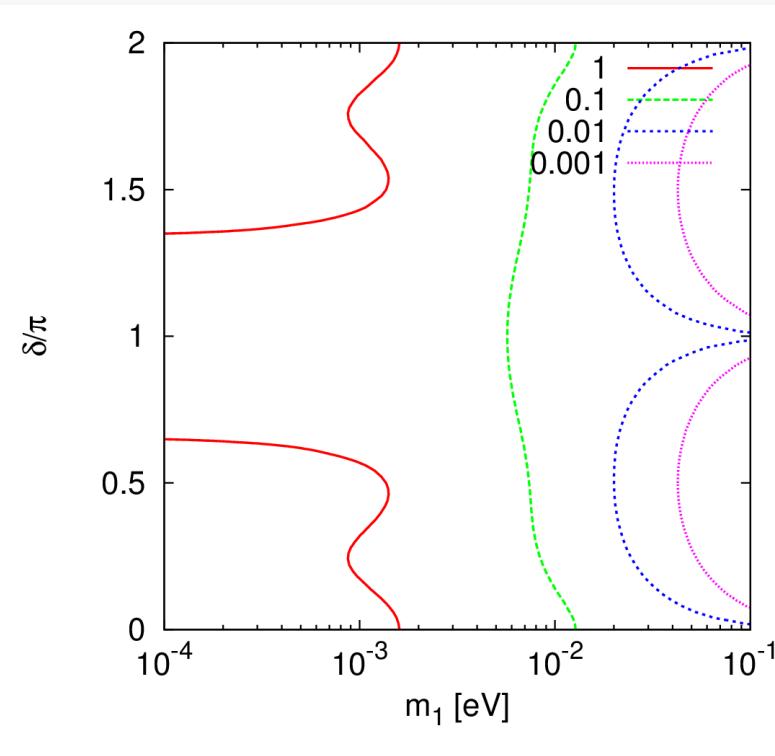
A. Ilakovac, A. Pilaftsis, L. Popov
arXiv:1212.5939

Low energy neutrino parameters

$$\text{Br}(\ell_\alpha \rightarrow 3\ell_\beta)/\text{Br}(\ell_{\alpha'} \rightarrow 3\ell_{\beta'})$$



$$\ell_\alpha = \mu, \ell_{\alpha'} = \tau, \ell_\beta = \ell_{\beta'} = e$$



$$\ell_\alpha = \ell_{\alpha'} = \tau, \ell_\beta = \mu, \ell_{\beta'} = e$$

When the D_2 piece dominates the 3-body flavor ratios are **R matrix independent** (for degenerate RH neutrinos)