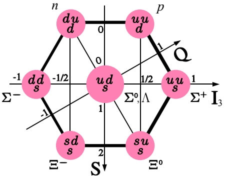

Isospin breaking effects from lattice QCD and QED

Antonin Portelli (University of Southampton)

21st of January 2014 – RPP 2014

Motivations

Isospin symmetry breaking

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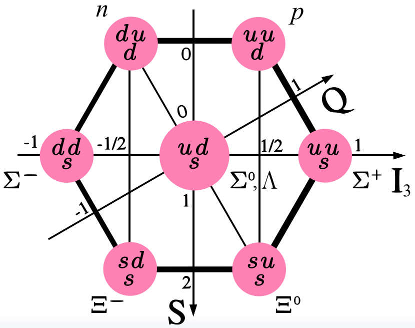
Isospin breaking effects

Sum of two little effects of the same order ($\sim 1\%$), eventually competing.

Octet baryon mass splittings

There are 3 stable baryon multiplets formed with u, d and s quarks :



Mass splittings are experimentally known [PDG 2012] :

$$M_p - M_n = -1.29333214(43)\ \mathrm{MeV}$$
$$M_{\Sigma^+} - M_{\Sigma^-} = -8.08(08)\ \mathrm{MeV}$$
$$M_{\Xi^0} - M_{\Xi^-} = -6.85(21)\ \mathrm{MeV}$$

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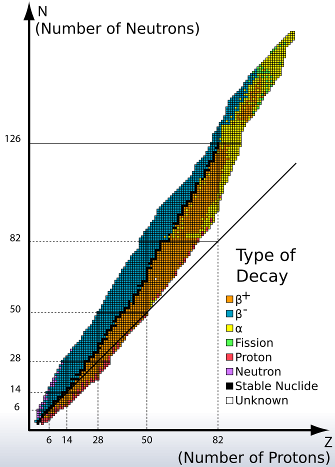

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It determines through β decay **the stable nuclides chart**.

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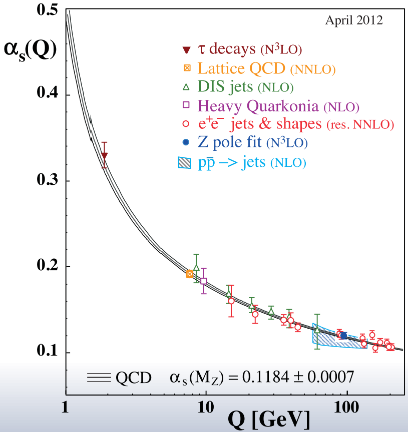
Lattice QCD could give a way to solve numerically this problem.

Predicting a $1‰$ effect through lattice simulation is a **considerable computational challenge**.

Lattice QCD+QED

QCD at low energies



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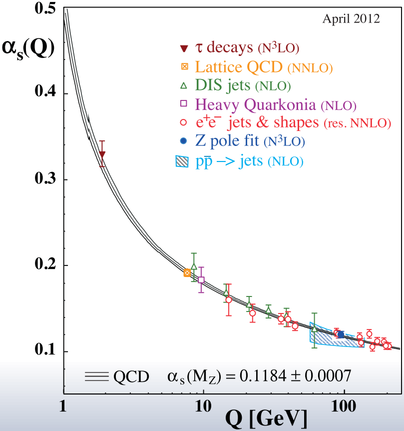

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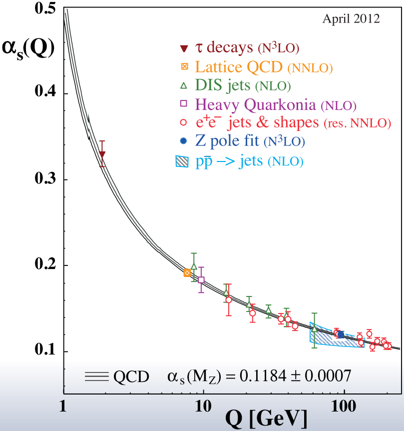

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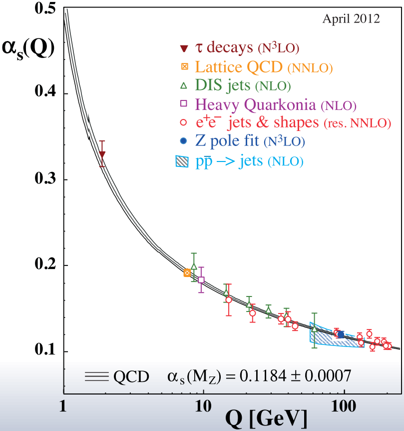

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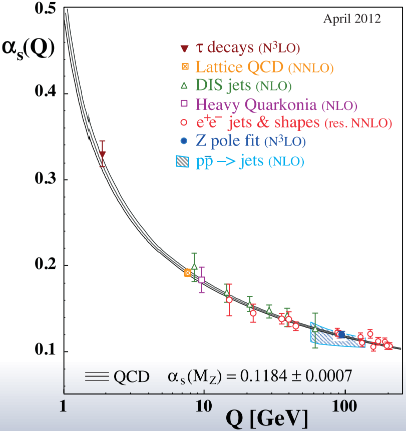

- At nuclear energies (~ 1 GeV) the strong coupling constant becomes large.
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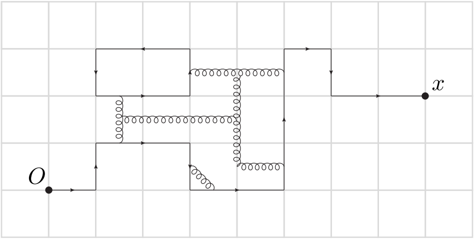
- **Non-perturbative framework needed** for hadronic QCD.

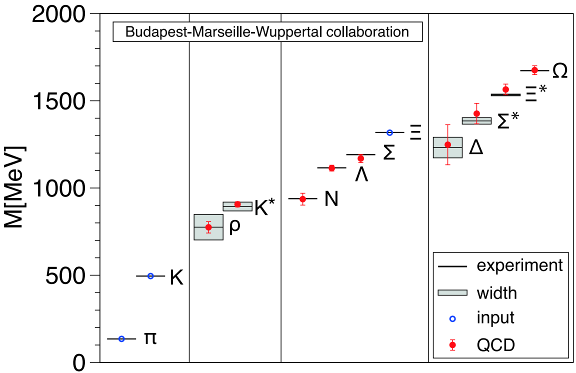
Lattice QCD



Numerical Monte-Carlo evaluation of QCD path integral :

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{D}\, U_\mu \, O_{\mathrm{Wick}}[D^{-1}] \, \det(D) \exp(-S_{\mathrm{gauge}})$$

Light QCD isospin spectrum solved



[BMWc 2008, Science, hep-lat/0906.3599]

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Infinite volume limit is correct.
Power-like finite size effects expected.

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We assume 10% **of relative quenching error** on EM effects.

Isospin breaking effects on hadron masses

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- 16 volumes from $(2\ \mathrm{fm})^3$ to $(6\ \mathrm{fm})^3$ with $M_\pi L > 4$ (negligible QCD finite volume effects).

Valence masses

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- 3 additional points: unphysical values for α.

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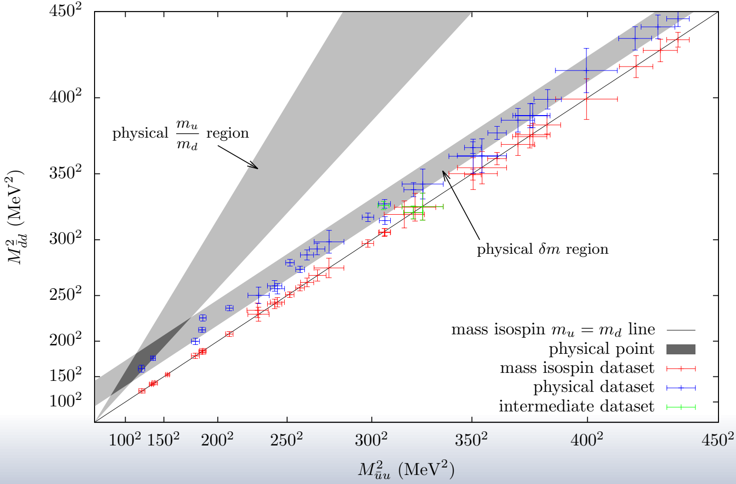

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Additionally there is $\mathrm{O}(10\%)$ of quenching error on EM splittings.

Preliminary results: quark masses

Kaon splitting is strongly related to δm:

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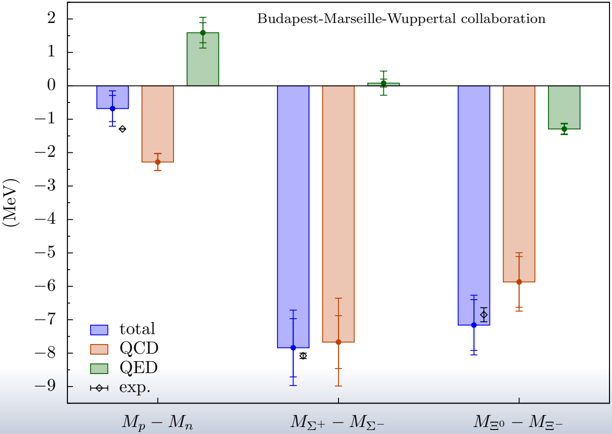
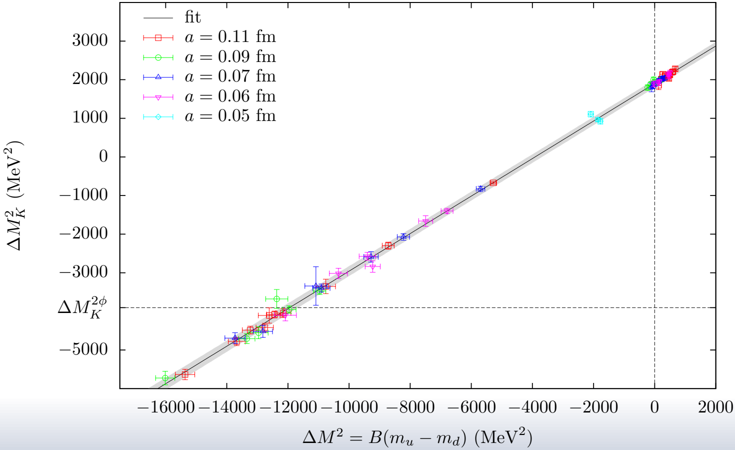
Improvement of the PDG precision by a factor ~ 8

Baryon octet splittings

[BMWc, PRL 111(25), p. 252001, hep-lat/1306.2287]



ΔM_K^2 fit example



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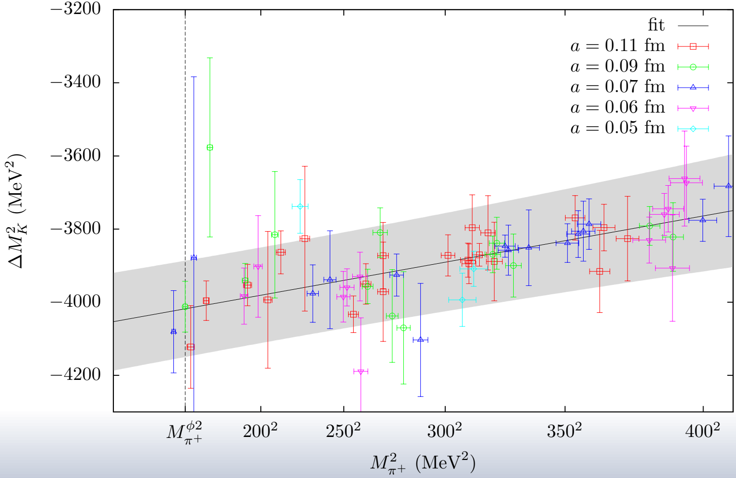
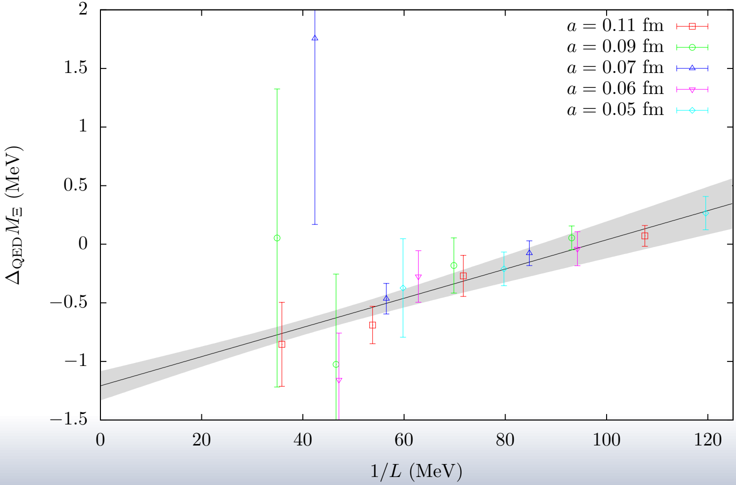


ΔM_{Ξ} FV effects



Epilogue

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- m_u is ~ 10 sigmas away from 0!
- Large power-like FV effects;
- For some important quantities, electro-quenching may already be the dominant source of uncertainty.

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- Our result for the β-decay existence ($M_n - M_p - m_e > 0$) has only a 1.3σ significance. Reaching the 5σ: ***ab-initio* proof of the stability of nuclear matter**.
- More generally: lattice QCD+QED is an important step toward **complete simulations of the Standard Model** at low energies.

Thank you.

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