The fate of the fine-tuning of the Higgs mass within a finite field theory

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cnrs dépasser les frontières

Motivations

- In a bottom-up approach, one should determine on physical grounds the scale above which a theory is not valid

- To do that, one should be able to extract a typical energy/momentum scale from the calculation of physical observables

- These scales should not be mixed up with (spurious) scales originating from the divergence of (ill-defined) bare amplitudes

- One should look for schemes which lead to completely finite bare amplitudes from the very beginning (without any limit to perform at the end of the day!)

  - The Taylor-Lagrange regularization scheme

P. Grangé and E. Werner, J. Phys. A: Math. Theor. 44 (2011) 385402

Construction of the physical fields

❑ Definition of the physical fields

N. Bogoliubov, 1950's

E. Stueckelberg,
A. Petermann, 1953

- Fields should be considered as **distributions**
- Functional Φ with respect to a **test function** ρ

ex.: scalar field $\phi(x)$ $\quad \Phi(\rho) = \int d^4y \; \phi(y) \; \rho(y)$

- **Physical field** $\varphi(x)$ by means of the translation operator T_x

$$\varphi(x) \equiv T_x \Phi(\rho) = \int d^4y \; \phi(y) \; \rho(x-y)$$

❑ Properties of the test functions

- belongs to the Schwartz space $\mathcal{S}$ of **fast decrease functions**
  - ➥ decrease at **infinity faster than any power of x,** as well as all its derivatives
  - ➥ property **conserved by Fourier transform**

- in momentum space

$$\rho(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip.(x-y)} f(p_0^2, \mathbf{p}^2)$$

- decomposition of the **physical field**

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{f(\epsilon_p^2, \mathbf{p}^2)}{2\epsilon_p} \left[a_p^\dagger e^{i\mathbf{p}.\mathbf{x}} + a_p e^{-i\mathbf{p}.\mathbf{x}}\right]$$

❑ Physical interpretation of the test function

- $\varphi(x)$: **average over the initial field** with a weight ρ
  - ➥ if ρ has a space-time extension a : **average over a volume a^4**

$$\rho_a(x) \longrightarrow \varphi_a(x)$$

- to recover a **"local" field theory**, one should investigate the limit $a \longrightarrow 0$
- **scale invariance** inherent to this limit since also $\dfrac{a}{\eta} \longrightarrow 0$ with $\eta > 1$

so that a priori $\rho_a(x) \longrightarrow \rho_\eta(x)$ and $\varphi_a(x) \longrightarrow \varphi_\eta(x)$

➢ for the **Fourier transform** of ρ_a

$$f_a \xrightarrow{a \to 0} f_\eta \sim cte$$

➢ it is sufficient to consider $f_\eta \sim 1$

➥ **Poincaré group equations invariant** without renormalization of the fields

➢ calculation of any amplitude

$$\mathcal{A}_\eta = \int dX \; T(X) \; f_\eta(X)$$

with a one dimensional variable X for simplicity

ex.: $X = \frac{k_E^2}{\Lambda^2}$, Λ arbitrary scale

$T(X)$: **singular distribution** : $\mathcal{A}_\eta$ **divergent** if no test functions

❑ Explicit construction of the test function

➢ we shall first consider a sequence of test functions f_α with compact support

$$f_\alpha(H) = 0 \quad \text{, with} \quad H \equiv X_{max}$$

so that

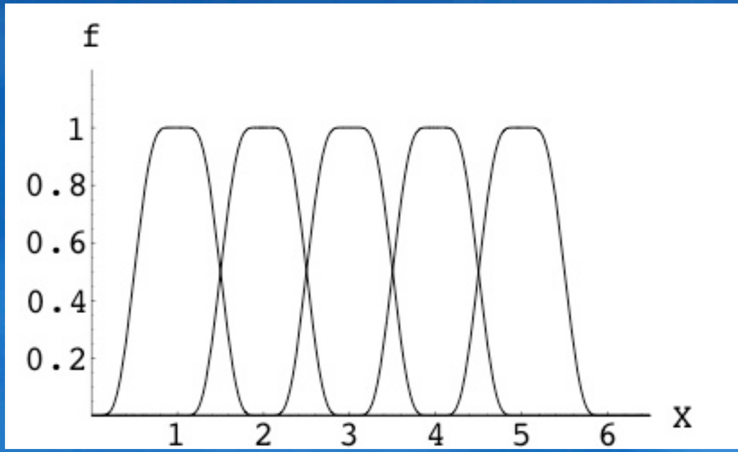
$$\mathcal{A}_\alpha = \int dX \; T(X) \; f_\alpha(X)$$

➢ f_α chosen as a partition of unity (PU)

➥ $\mathcal{A}_a$ independent of the particular choice of a PU

➢ construction of a PU

$$f(x) = \sum_{j=0}^{N-1} u(x - jh)$$



➢ in a given limit $\alpha \to 1^-$ $\quad f_\alpha(x) \to 1$

➢ in this limit, one should recover the original test function

$$\lim_{\alpha \to 1^-} \mathcal{A}_\alpha \equiv \mathcal{A}_\eta$$

➥ This limit should be independent of X_{max}

➢ To do that, one needs a **particular construction** of the test function f_α

➥ **Ultra-soft cut-off** (“dynamical” cut-off)

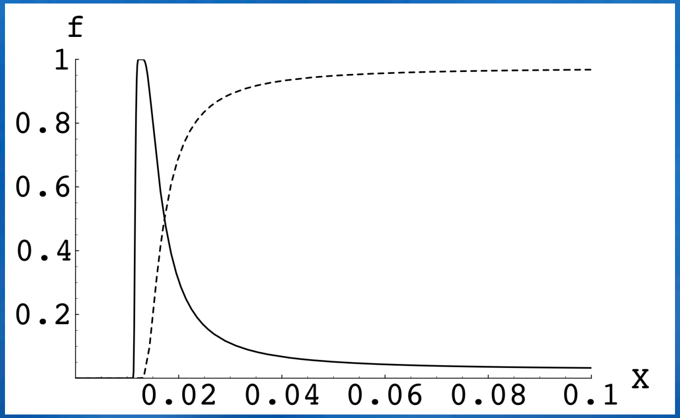
$$H \rightarrow H(X) \equiv \eta^2 \, X^\alpha \; + \; cte \qquad\qquad \eta^2 > 1$$

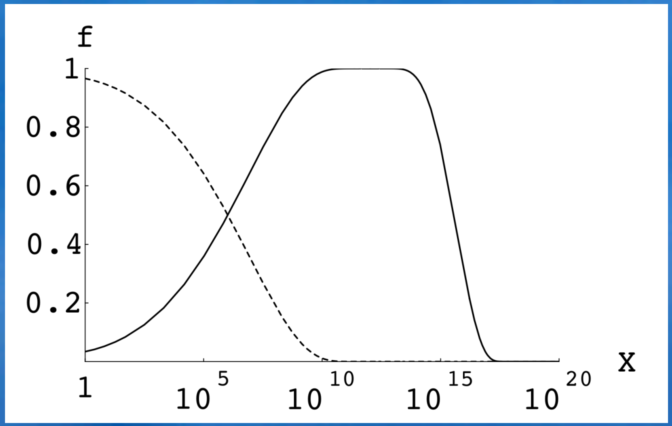
Rem.: not at all unique example

➥ **upper limit** of f_α defined by $X_{max} = H(X_{max})$

$$X_{max} = (\eta^2)^{\frac{1}{1-\alpha}}$$

$$\lim_{\alpha \rightarrow 1^-} X_{max} = \infty$$





➢ **the Taylor-Lagrange regularization scheme**

Construction of (finite) extended bare amplitudes

- **Extension in the ultra-violet domain**

  - **Apply the Lagrange formula for the Taylor remainder of** $f_\alpha = R_k\ f_\alpha$

$$f(\lambda X) = -\frac{X}{\lambda^k k!}\int_\lambda^\infty \frac{dt}{t}(\lambda - t)^k \partial_X^{k+1}\left[X^k f(Xt)\right]$$

λ **intrinsic scale** **ex.:** $T(X) = \dfrac{1}{X+\lambda}$

  - **one should thus calculate** $\mathcal{A}_\alpha = \int_0^\infty dX\ T(X)\ f_\alpha(X)$ $\qquad \alpha \to 1^-$

  - **by integration by part after use of the Lagrange formula**

$$\mathcal{A}_\alpha = \int_0^\infty dX\ \tilde{T}_\alpha^{>}(X)\ f_\alpha(X)$$

In the limit $\alpha \to 1^-$, $\tilde{T}^>_\alpha(X) \to \tilde{T}^>_\eta(X)$ with

$$T^>_\eta(X) = \frac{(-X)^k}{\lambda^k k!}\partial_X^{k+1}\left[XT(X)\right]\int_\lambda^{\eta^2}\frac{dt}{t}(\lambda - t)^k$$

➢ because of the **derivatives** in $\tilde{T}_\eta(X)$, the amplitude is now **completely finite**

$$\mathcal{A}_\alpha \to \mathcal{A}_\eta = \int_0^\infty dX\ \tilde{T}^>_\eta(X)$$

➥ depends on the **arbitrary scale** η^2

➥ if $T(X) = \dfrac{1}{X+\lambda}$ $\qquad \tilde{T}^>_\eta(X) = \mathrm{Ln}\left(\dfrac{\eta^2}{\lambda}\right)$

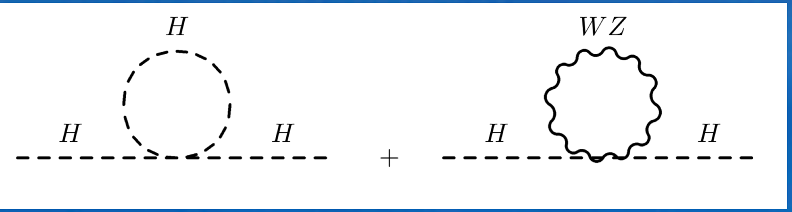
❑ Extension in the infra-red domain

➢ Typical distribution $T^<(X) = \dfrac{1}{X^{k+1}}$ with no intrinsic scale

➢ **extended** distribution

$$\tilde{T}^<(X) = \frac{(-1)^k}{k!}\partial_X^{k+1}\ \mathrm{Ln}(\tilde{\eta}X) \equiv Pf\left[\frac{1}{X^{k+1}}\right]$$

Application to radiative corrections in the Higgs sector



➥ usual interpretation in a cut-off scheme

$$M_H^2 = M_0^2 + b\ \Lambda_C^2$$

➥ For Λ_C very large, fine-tuning between M_0^2 and Λ_C^2 to get $M_H \simeq 125\ \mathrm{GeV}$

➥ Mixing of physical scales with spurious (mathematical) scales from an ill-defined integral

➥ Calculation in the Taylor-Lagrange regularization scheme

$$\Sigma = -\frac{3M_H^4}{32\pi^2 v^2}\ \ln(\eta^2)$$

➥ Equivalent to dimensional regularization (once renormalized) with

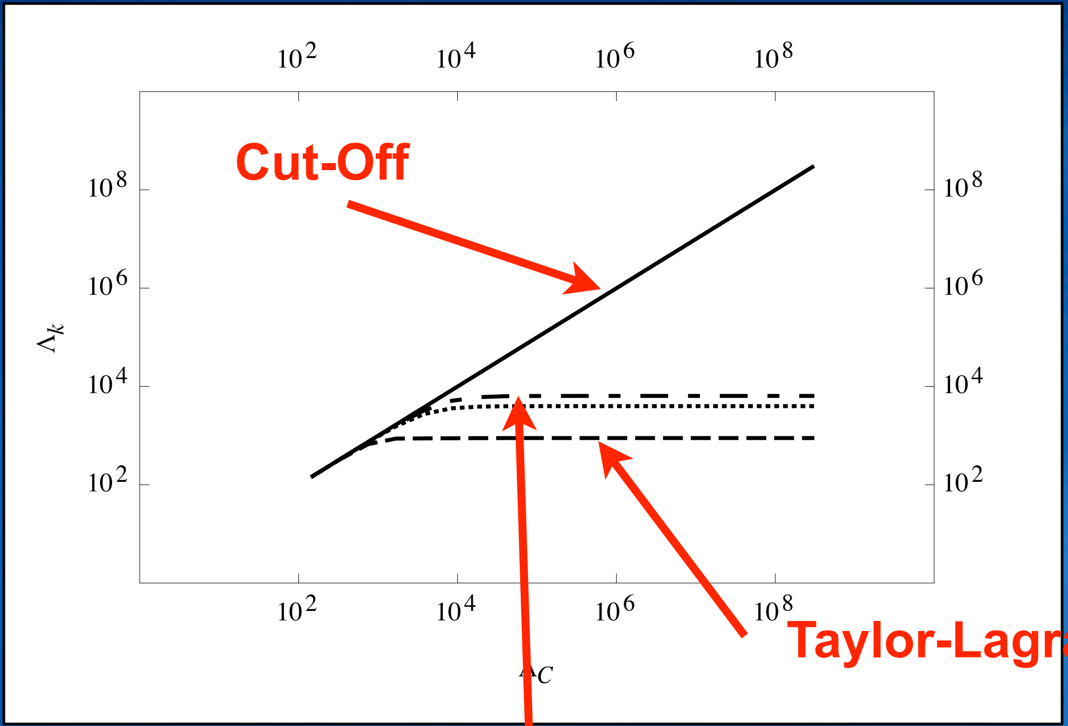
$$\mu^2 = \eta^2\ M_H^2$$

P. Grangé et al., Phys. Rev. D88 (2013) 125015

➢ **Physical interpretation in terms of physical momentum intrinsic scale**

➥ intrinsic scale Λ_k defined by

$$\frac{\bar{\Sigma}(p^2)}{\Sigma(p^2)} = 1 - \epsilon \qquad \epsilon \simeq 1\%$$



with

$$\Sigma(p^2) = \int_0^{\Lambda_C^2} dk_E^2 \; \sigma(k_E^2, p^2)$$

and

$$\bar{\Sigma}(p^2) = \int_0^{\Lambda_k^2} dk_E^2 \; \sigma(k_E^2, p^2)$$

➥ compared to **fully renormalized self-energy** (at two different p^2)

$$\Sigma_R(p^2) = \Sigma(p^2) - \Sigma(M_H^2) - (p^2 - M_H^2) \left. \frac{d\Sigma(p^2)}{dp^2} \right|_{p^2 = M_H^2}$$

➥ **finite typical scale** in Taylor-Lagrange **in the bare amplitude already**, but not in a cut-off scheme

➥ the **same finite scale** on the fully renormalized amplitude

Final remarks

- field strengths, bare masses and coupling constants do depend on the arbitrary scale η^2
- physical observables of course should not, at each order of perturbation theory in terms of physical coupling constants
- mass-dependent renormalization group equations