CURVED
EXTRA-DIMENSIONS

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Work in progress with **Giacomo Cacciapaglia** and **Aldo Deandrea**

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Outline

Introduction: Why 2UED is attractive

Survey of Positively Curved Geometries

Constructing a Reasonable Model

Conclusion: Where We Are and Where We Go

Why is 2UED attractive ?

Dark Matter!

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A stable excitation of a neutral SM field could be Dark Matter!

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Two limiting factors: Isometries and fermions

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Survey of Positively Curved Geometries

Positively curved geometries

Uniformization theorem

All positively curved 2D surfaces can be described as S^2/G with G a discrete subgroup of $O(3)$

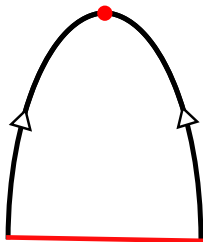# Positively curved geometries

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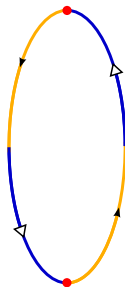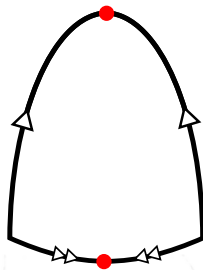First question: Which of these have non-trivial isometries ?

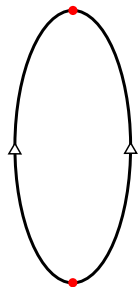# Orbifolds with symmetries
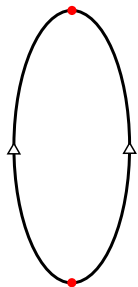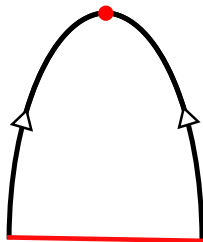

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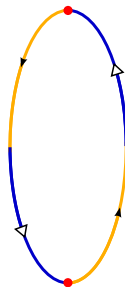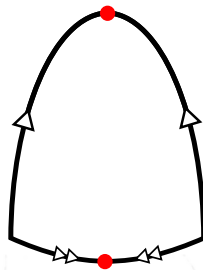# Orbifolds with symmetries

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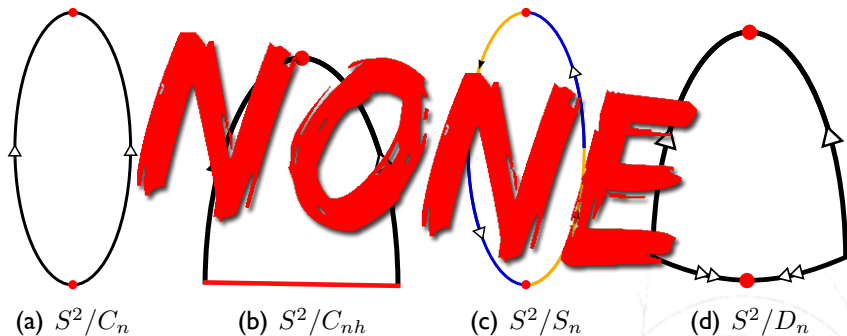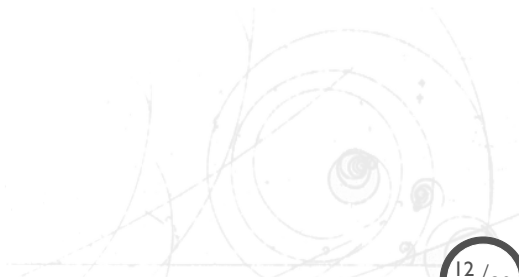
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Method from Randjbar-Daemi, Salam and Strathdee

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If X cancels $\pm\Omega$ one of the chiralities has a zero-mode.

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Start by writing the Standard Model Lagrangian in 6D with the new gauge field and an additional Higgs field :

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + |D_M H|^2 + \mu^2 |H|^2 - \frac{\lambda}{2}|H|^4$$

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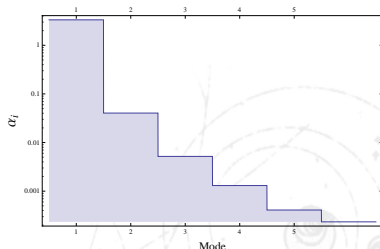
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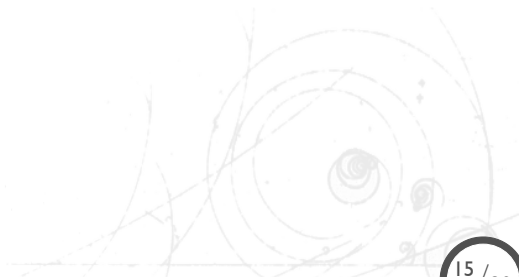
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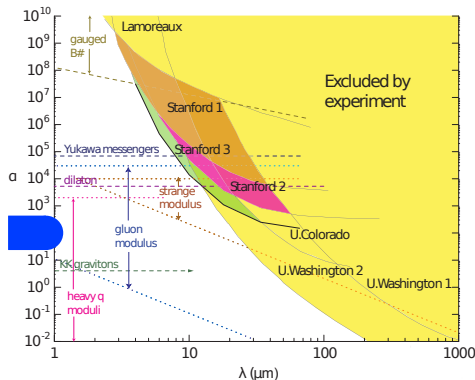
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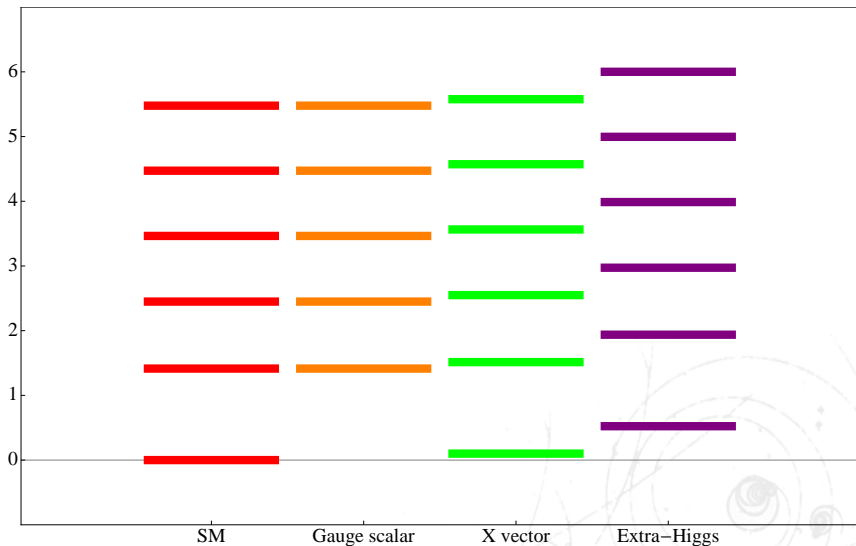Compatible with short-range gravity tests

Spectrum of the model

Some Experimental Remarks

X boson properties

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Summing up the model

Spherical Extra-Dimensions are hard to construct.

- Chiral fermions do not come easily
- Need to add two extra-fields: X and H'
- Unsatisfying because
  - X and H' are rather untestable
  - They are ugly
  - Stability seems to be an issue

- Works for now but needs corrected masses

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- Quantum corrections could play a significant role as M_{pl} goes down

Thank you for your attention