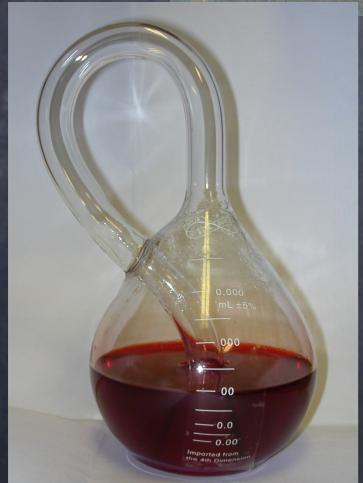
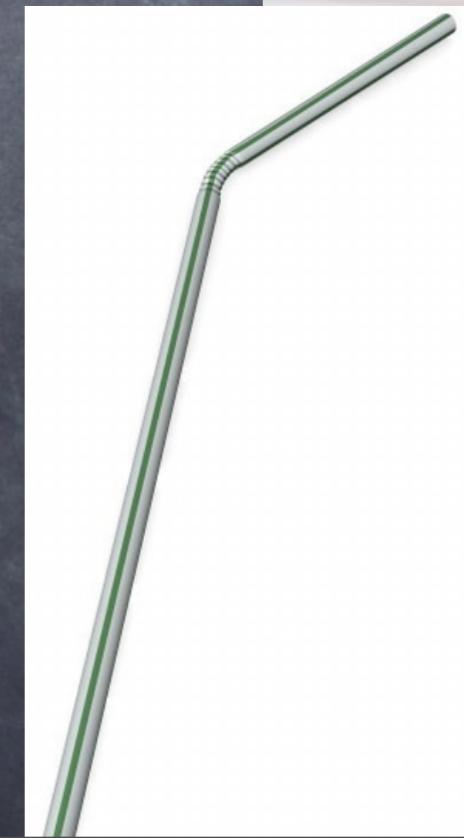


# Dark Matter from Extra Dimensions

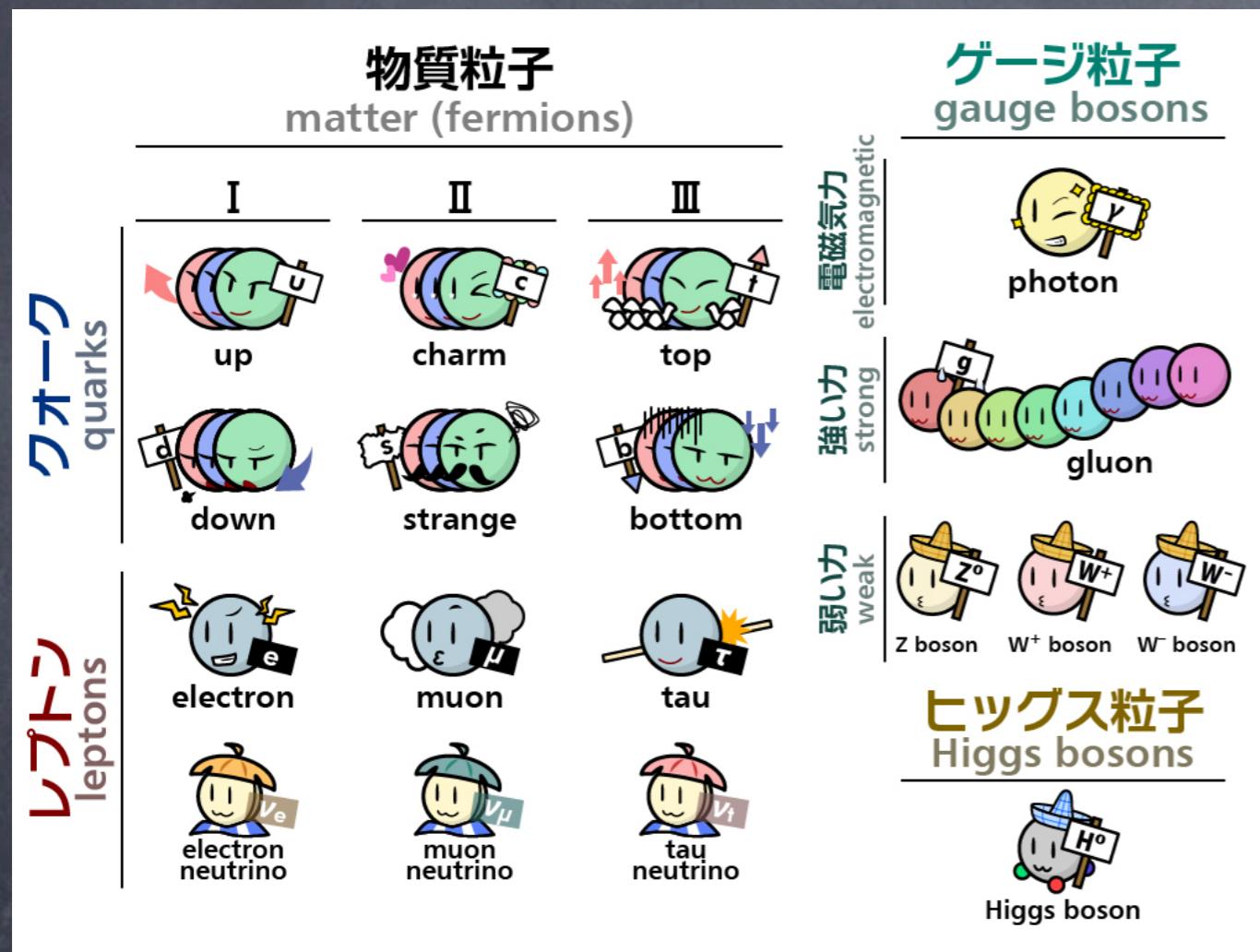
Giacomo Cacciapaglia  
IPN Lyon (France)



News from the dark  
Montpellier, 04-12-2013



# Why do we need BSM?



The Higgs boson has been discovered.

The Standard Model is now complete!



Ian MacNicol / AFP - Getty Images

- The discovery of the Higgs boson has brought the Naturalness problem to reality! New Physics at the TeV scale needed more than before!
- There are other unresolved puzzles: what is Dark Matter made of?

# WIMP miracles in BSM theories

- ⦿ Supersymmetry:

extension of the Poincaré algebra  
including boson/fermion symmetry

$$s=1/2 \Leftrightarrow [s=0]$$

+

R-parity

- ⦿ Extra space dimensions:

more trivial extension of  
the Poincaré algebra!

Nordstöm (1914), Kaluza (1921) and Klein (1926)  
realised that extra dimensions have interesting features:

$$g^{MN} = \begin{pmatrix} & g^{\mu\nu} & \\ & a^\nu & \\ a^\mu & & \phi \end{pmatrix}$$

Can this be the photon?

Later rediscovered in string theory:  
they can be larger than  
the string (Planck) scale  
(Antoniadis, 2001)

# Extra Dimension primer

Action for a massless scalar in D-dimensions

$$S = \int d^D x \left\{ \partial_\mu \phi^\dagger \partial^\mu \phi - \sum_{j=5}^{D-4} \partial_j \phi^\dagger \partial_j \phi \right\}$$

Expansion in 4-dim fields on compact extra space:

$$\phi(x_\mu, x_j) = \int \frac{d^4 p}{(2\pi)^4} e^{ip_\mu x^\mu} \sum_{\vec{k}} \varphi_{\vec{k}}(p_\mu) f_{\vec{k}}(x_j)$$

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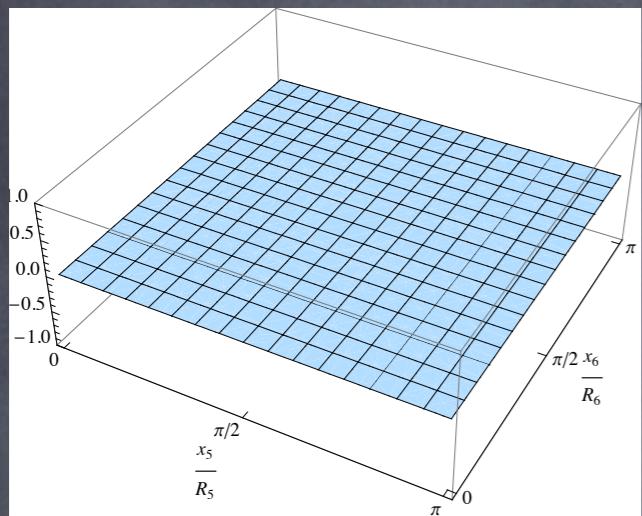
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- D-dim fields correspond to tower of massive 4-dim fields



# Extra Dimension primer

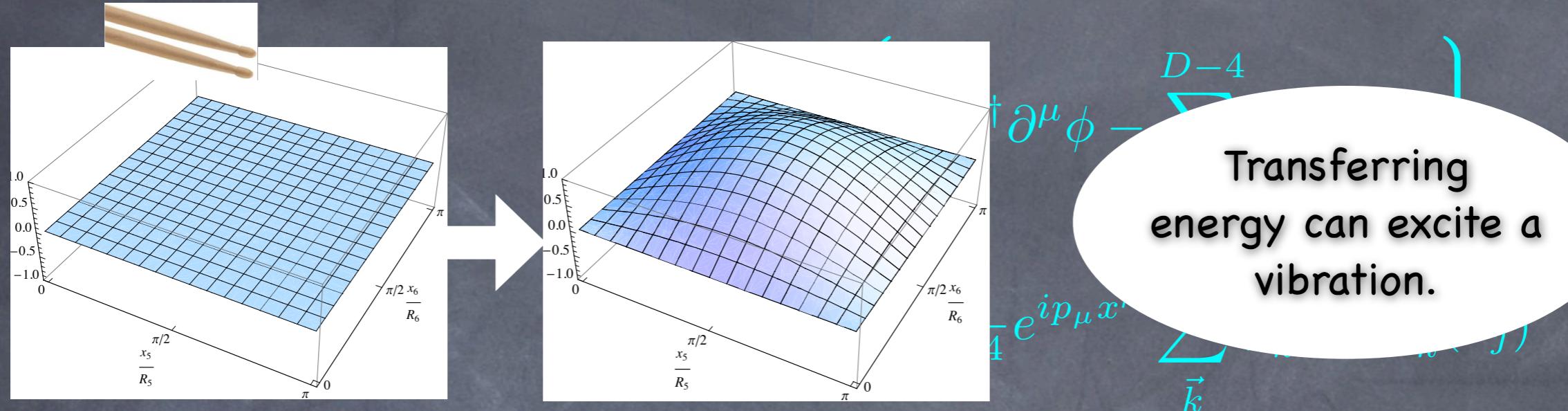


$$S = \int \left( \frac{1}{2} \partial_\mu \phi(x_\mu, x_j) \partial^\mu \phi(x_\mu, x_j) + \sum_k \vec{k}(x_j) \cdot \vec{\nabla} \phi(x_\mu, x_j) \right)$$

The extra space is like a vibrating membrane, a drum!

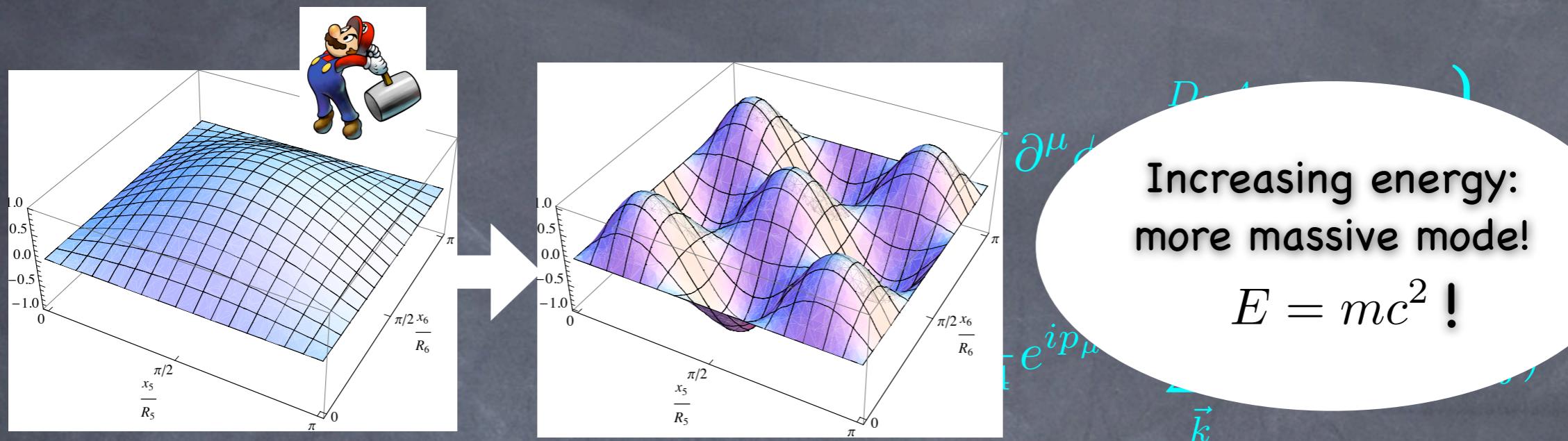
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# Extra Dimension primer



- D-dim fields correspond to tower of massive 4-dim fields
- k's are like frequencies of vibrating membrane!

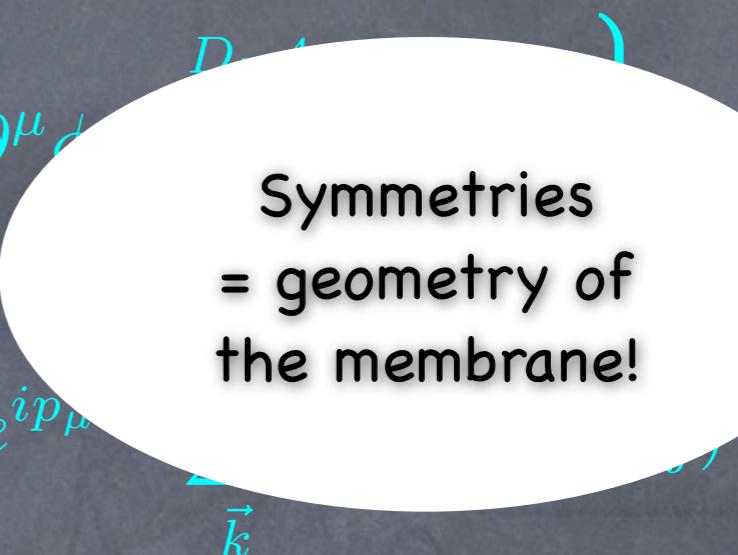
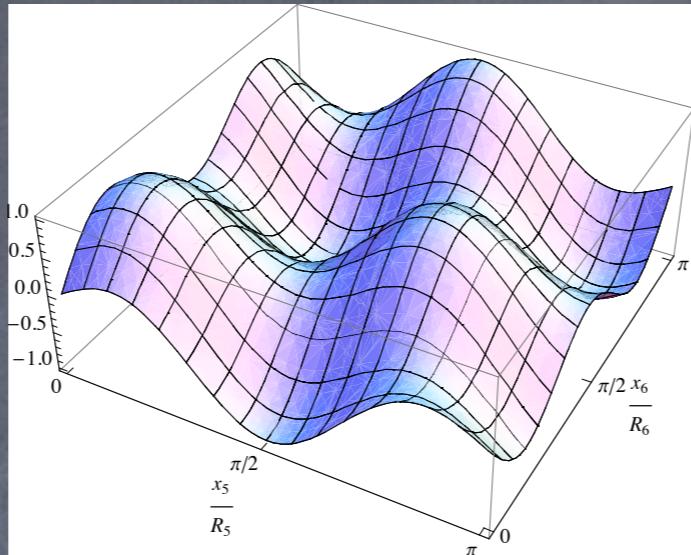
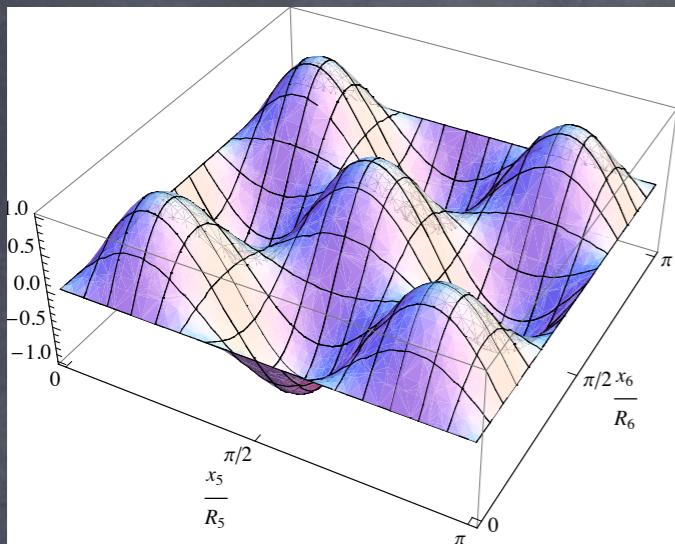
# Extra Dimension primer



- D-dim fields correspond to tower of massive 4-dim fields
- k's are like frequencies of vibrating membrane!
- Masses and interactions determined by the wave functions  $f_{\vec{k}}(x_i)$  !



# Extra Dimension primer

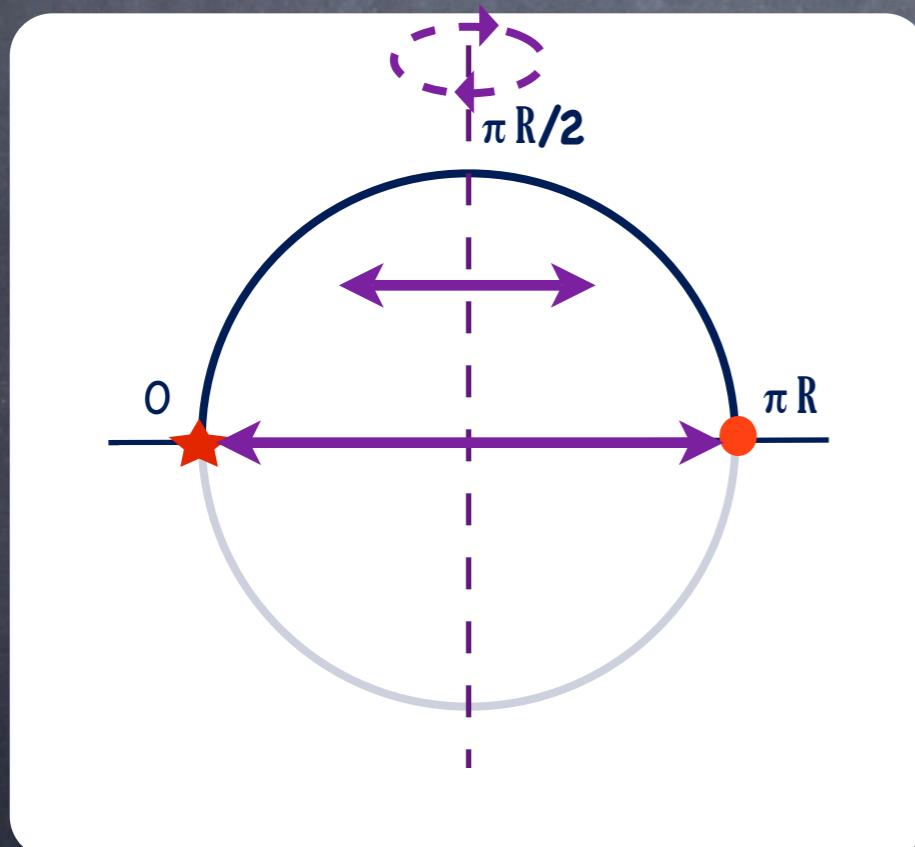


- D-dim fields correspond to tower of massive 4-dim fields
- k's are like frequencies of vibrating membrane!
- Masses and interactions determined by the wave functions  $f_{\vec{k}}(x_i)$  !
- Symmetries of the compact space = global symmetries of 4-dim fields: transformation properties of the wave functions!
- Can such symmetry stabilise the Dark Matter?

# Stability of the Dark Matter “requires” a symmetry!

Can it arise “naturally” from extra dimensions?

- Symmetries of the compact space ARE parities for the Kaluza-Klein modes!
- The physics is in the wave functions: for instance



Orbifold  $S^1/\mathbb{Z}_2$

$$x_5 \rightarrow \pi R - x_5$$

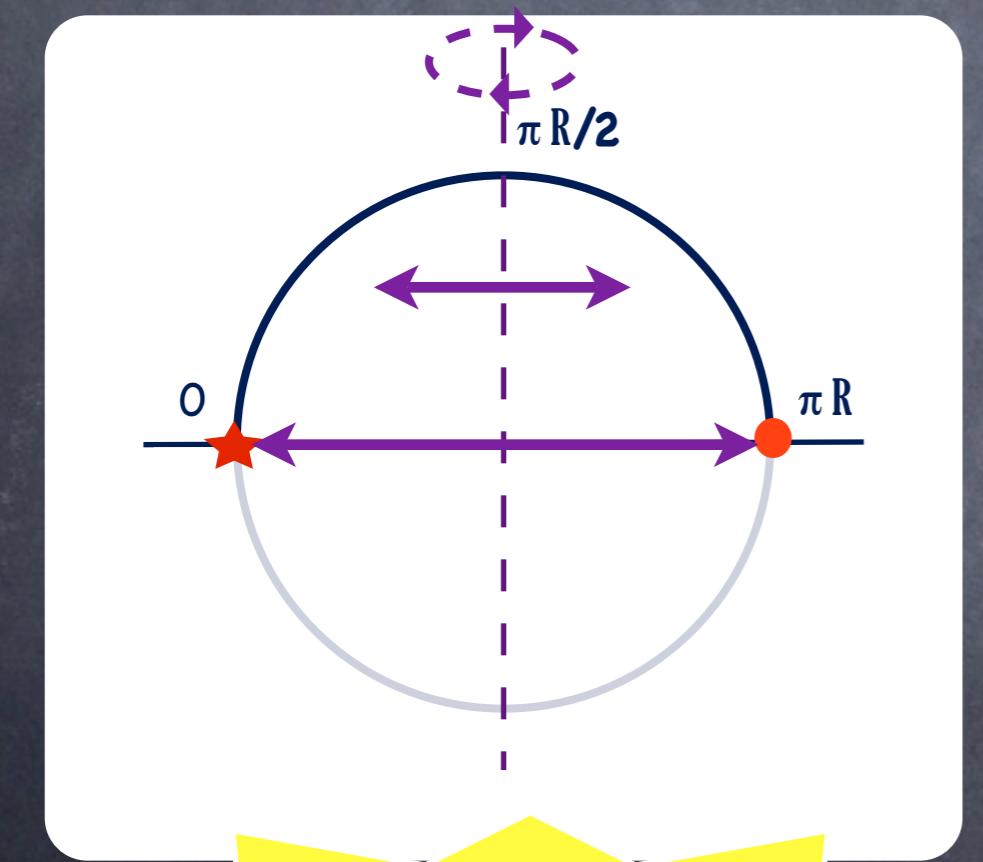
$$\cos\left(k \frac{x_5}{R}\right) \rightarrow (-1)^k \cos\left(k \frac{x_5}{R}\right).$$

However, fixed points (in red)  
are NOT invariant!

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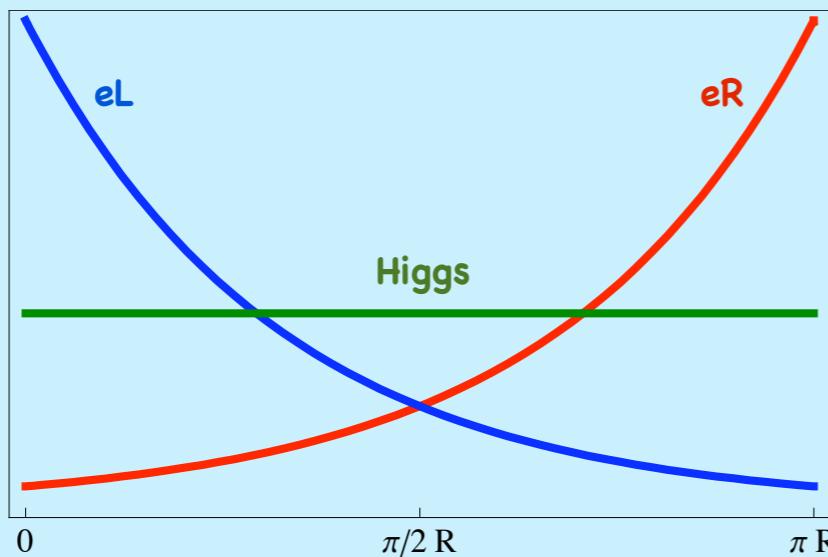
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However, fixed points (in red)  
are NOT invariant!

# Stability of the Dark Matter “requires” a symmetry!

Can it arise “naturally” from extra dimensions?

In Gauge-Higgs Unification models, or models of flavour,  
fermion localisation is essential!



Bulk fermion masses break  
the KK parity!

Already pointed out by  
Barbieri, Contino, Creminelli, Rattazzi, Scrucca  
[hep-th/0203039](https://arxiv.org/abs/hep-th/0203039)

KK-parity is ad-hoc  
symmetry!

KK-parity absent  
in interesting models!

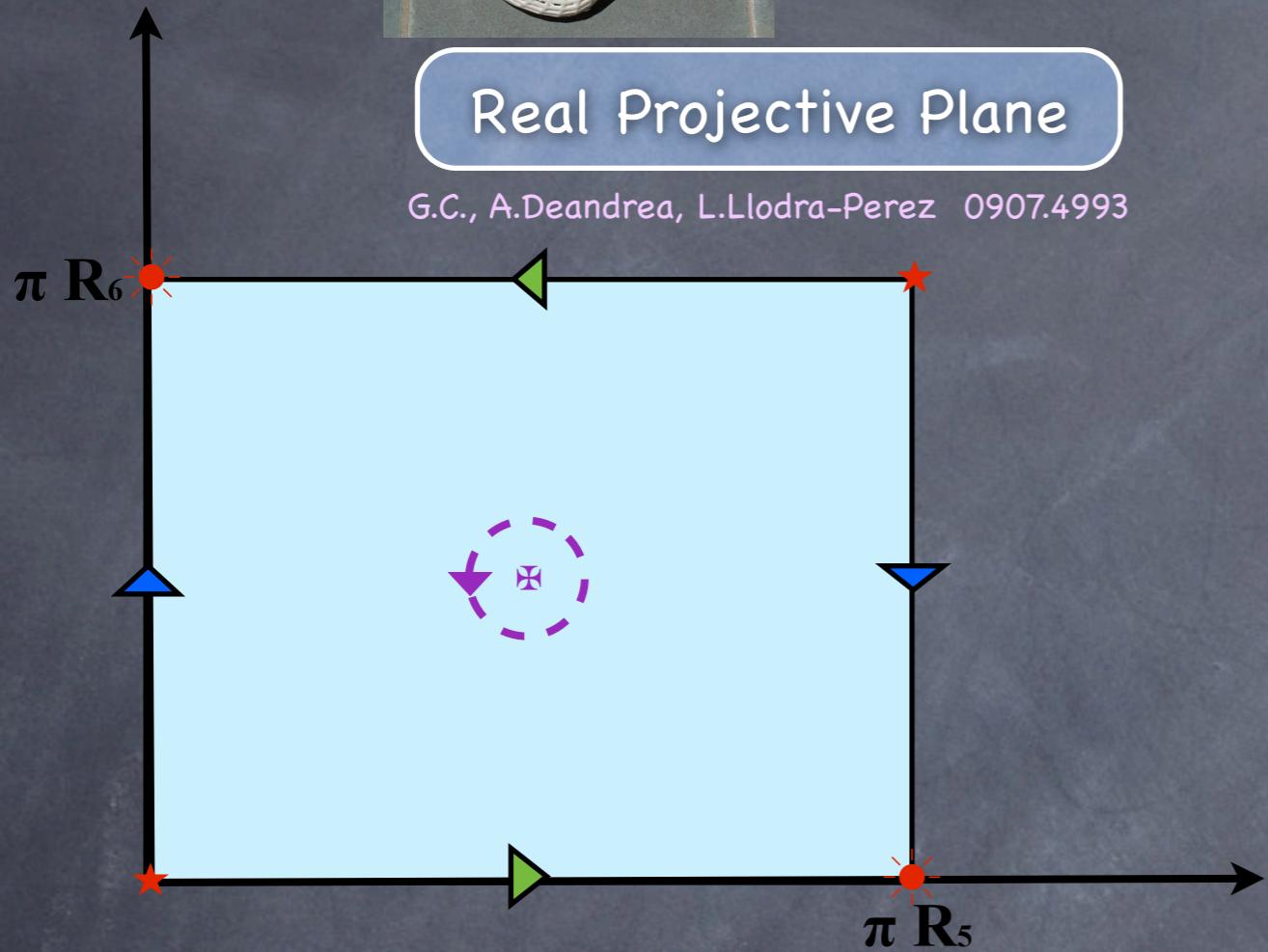


# RPP vs Chi2



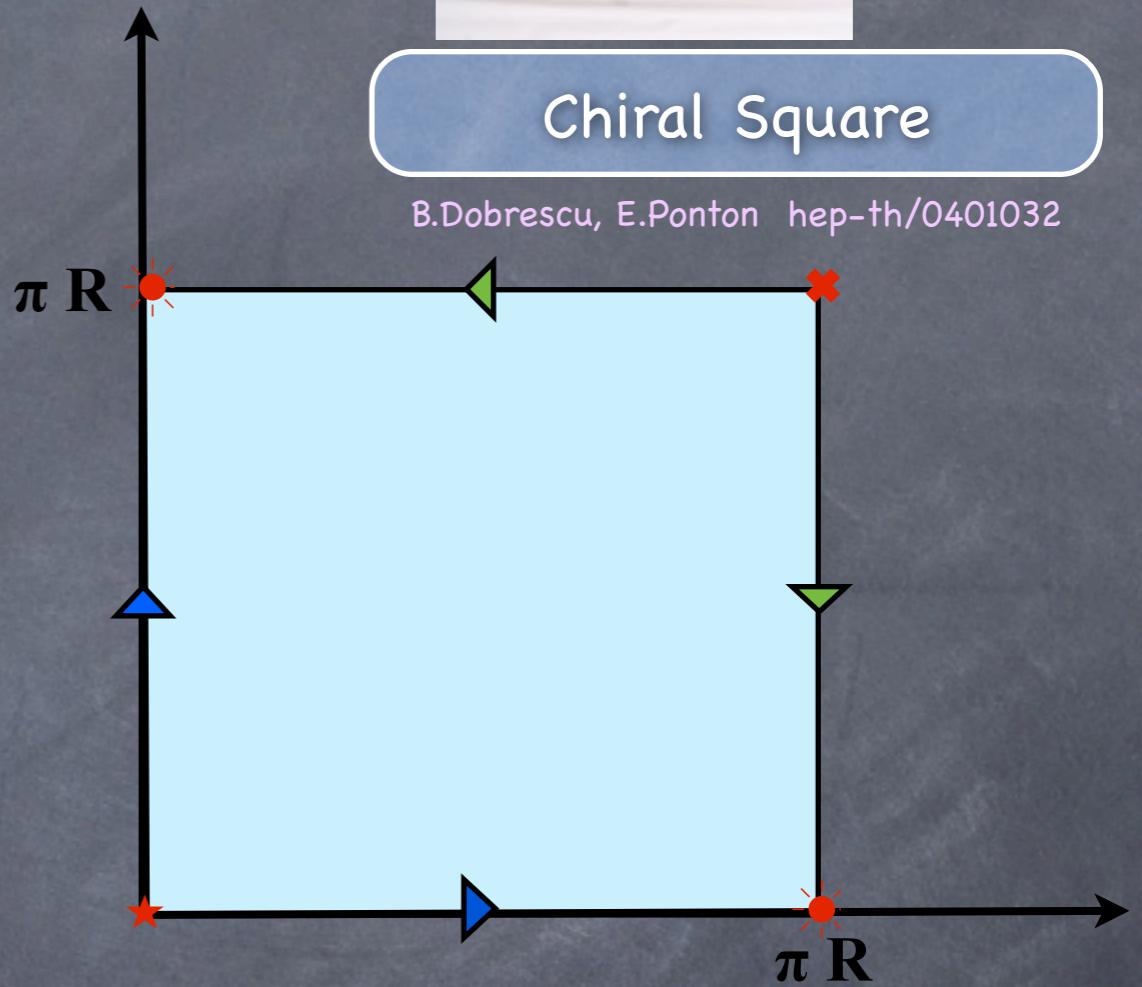
Real Projective Plane

G.C., A.Deandrea, L.Llodra-Perez 0907.4993



Chiral Square

B.Dobrescu, E.Ponton hep-th/0401032



👉 Natural KK parity

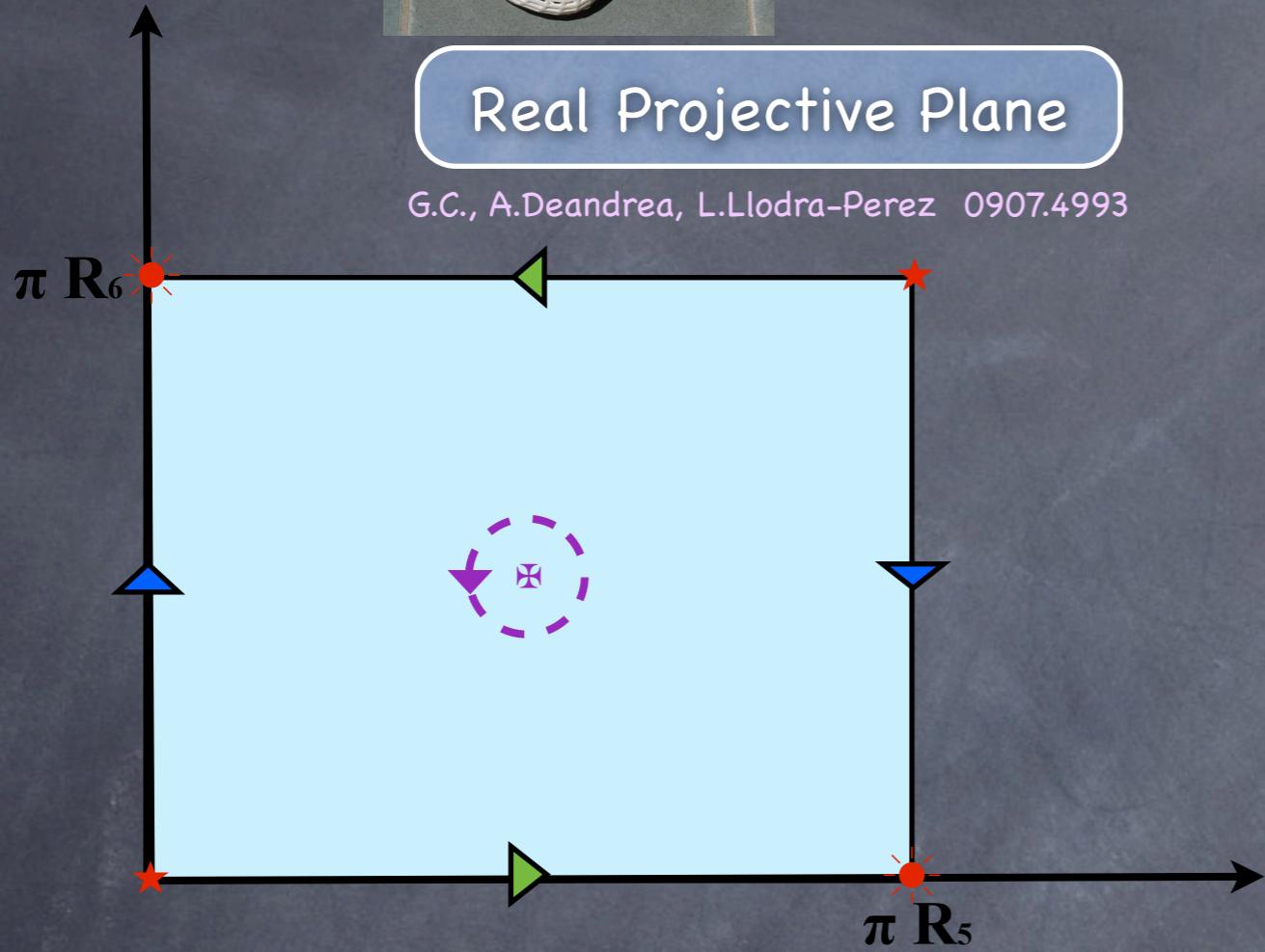


# RPP vs Chi2



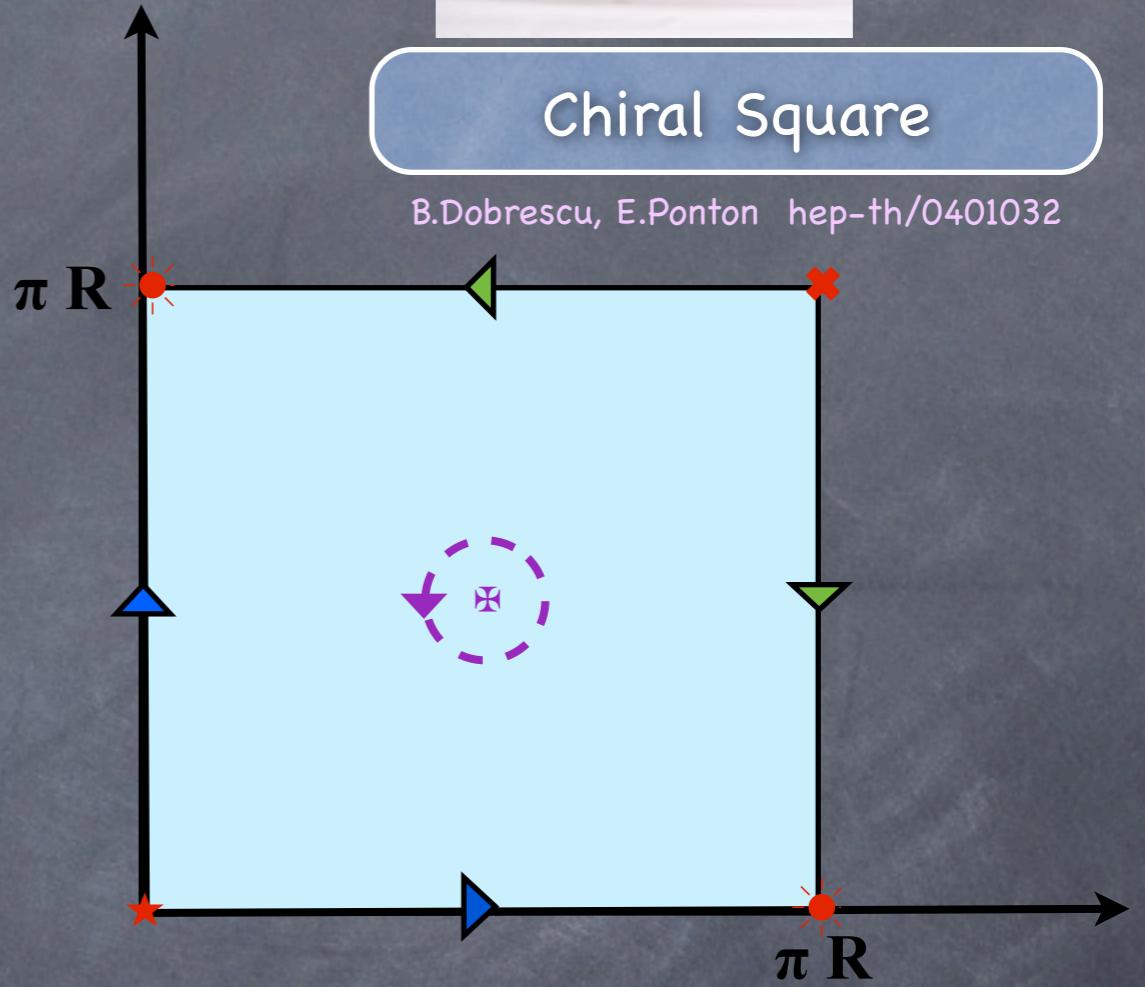
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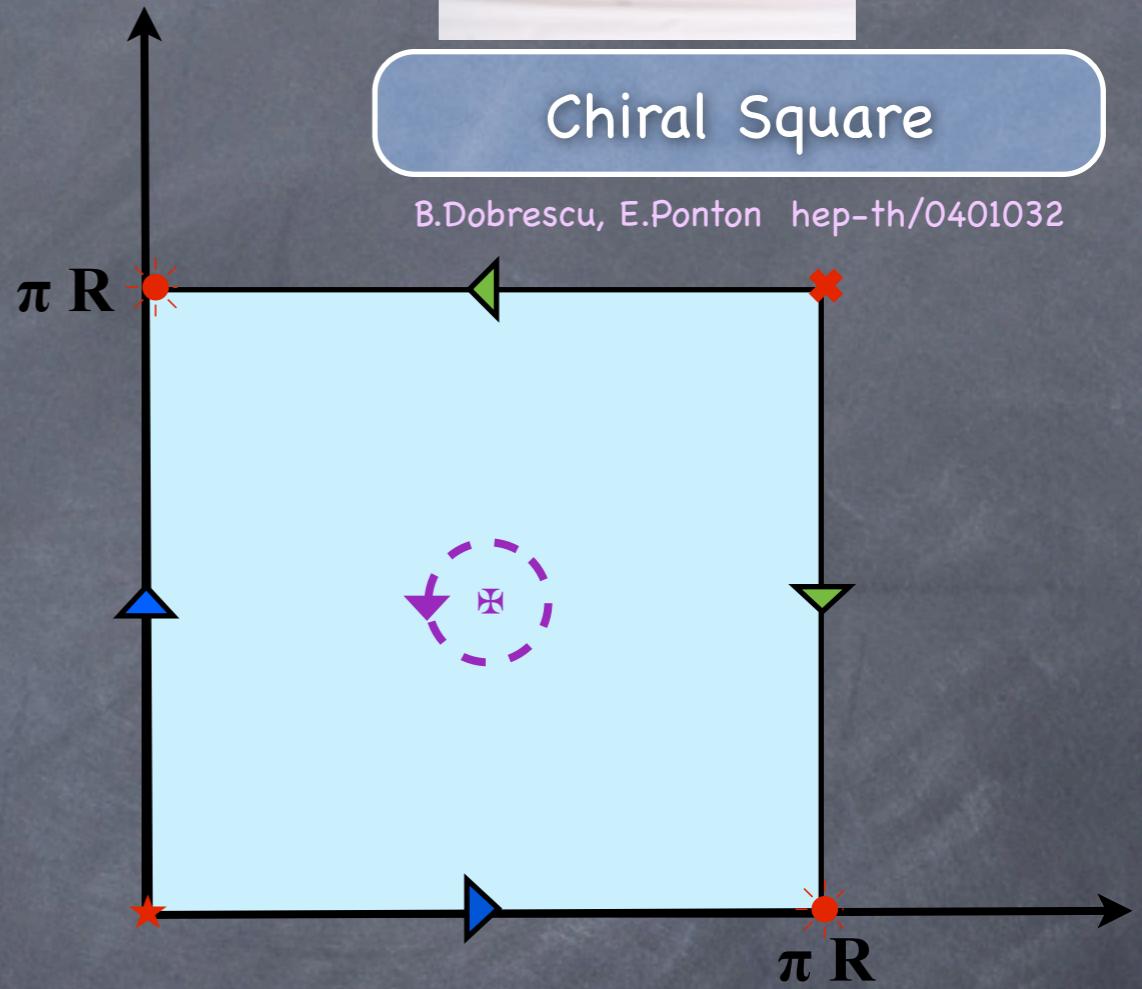
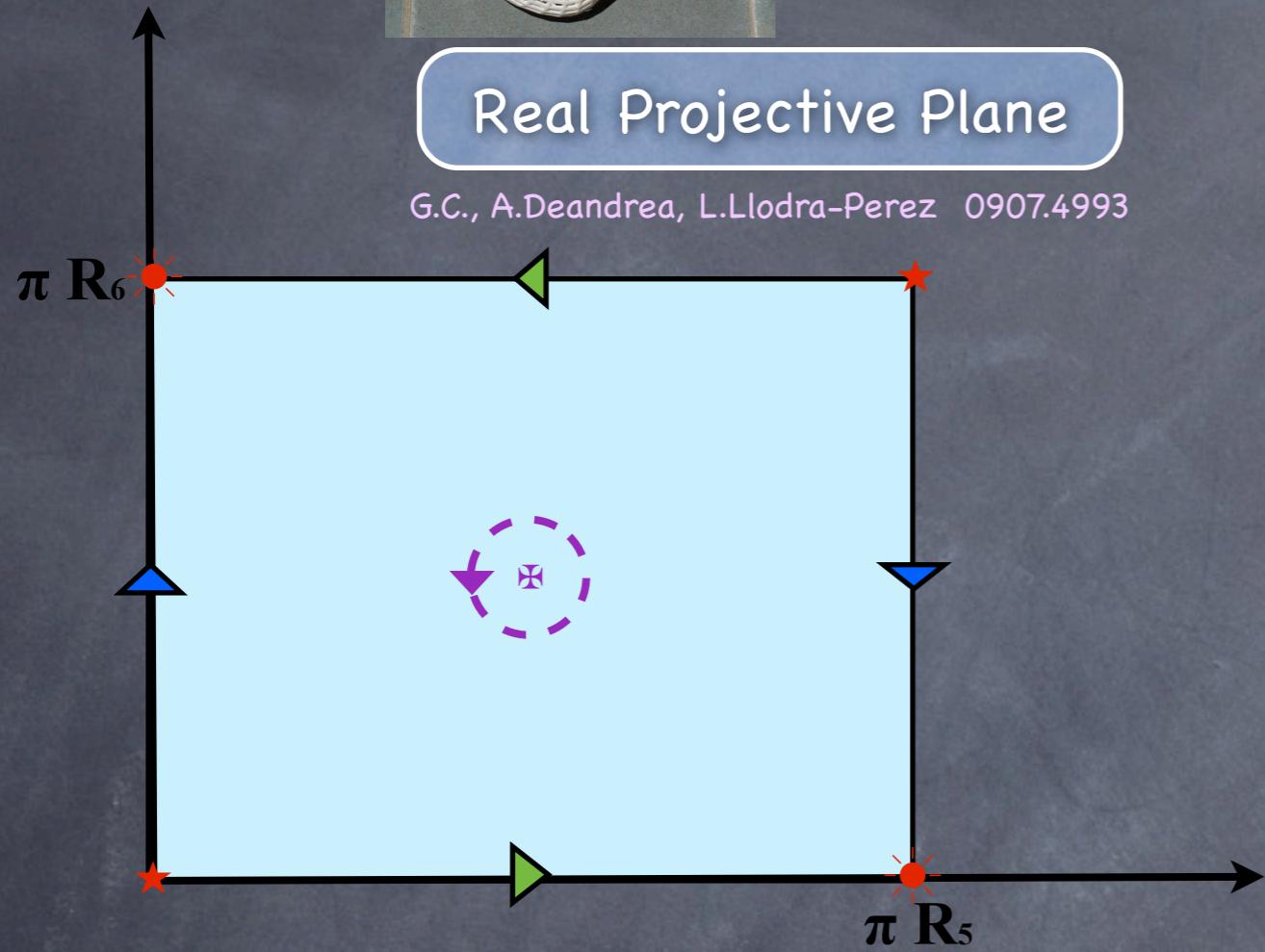


☞ Natural KK parity (no assumptions!)

☞ KK parity only if  $\star = \times$



# RPP vs Chi2



- ☛ Natural KK parity (no assumptions!)
- ☛ 2 independent radii

- ☛ KK parity only if  $\star = \times$
- ☛ 1 radius

KK modes labelled by two integers!  
KK parity defined as  $(-1)^{k+l}$   
Spectra are NOT the same!!!

# Tree level spectra

	+	-	-	+	+	+
$p_{KK} = (-1)^{k+l}$	$(0,0) - (0)$ $m = 0$	$(1,0) - (1)$ $m = 1$	$(0,1)$ $m = 1$	$(1,1)$ $m = 1.41$	$(2,0) - (2)$ $m = 2$	$(0,2)$ $m = 2$
Gauge bosons $G, A, Z, W$	<span style="color: cyan;">RPP</span> <span style="color: cyan;">✓</span> <span style="color: magenta;">Chi2</span> <span style="color: yellow;">5D</span> <span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>		<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: cyan;">✓</span>
Gauge scalars $G, A, Z, W$		<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span>	<span style="color: cyan;">✓</span>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span>	<span style="color: magenta;">✓</span>	
Higgs boson(s)	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>		<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: cyan;">✓</span>
Fermions	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span> <small>* Chiral</small>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: cyan;">✓</span>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: cyan;">✓</span> <span style="color: magenta;">✓</span> <span style="color: yellow;">✓</span>	<span style="color: cyan;">✓</span>

Mass splitting given by loops and...  
VERY DIFFERENT in the two cases!

# Loop-corrected spectra

## Tier (1,0)

	RPP	Chi2
$G_s$	678	516
$G_\mu$	-	717
$Z_s, W_s$	615	475
$Z_\mu, W_\mu$	-	547
$DM \rightarrow A_s$	600	444
$A_\mu$	-	500
H	-	542

	RPP	Chi2
mKK	600	516
★ Q	645	645
U	642	630
D	639	625
L	607	537
E	602	537

Mass splittings are  
much larger for the Chi2!!

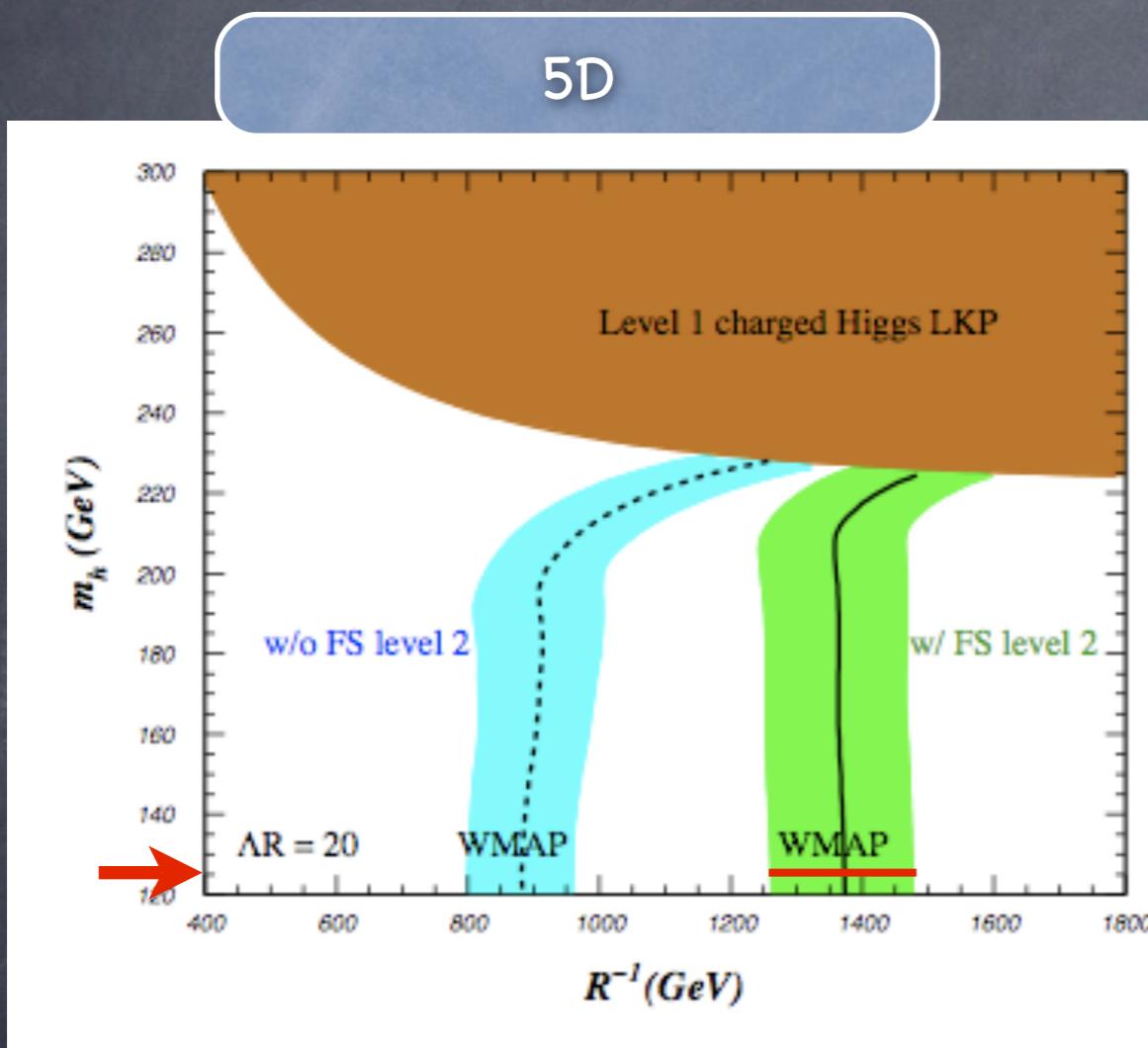
# Loop-corrected spectra

## Tier (1,0)

	RPP	Chi2		RPP	Chi2
$G_s$	178	516	many	600	516
$G_\mu$					645
$Z_s, W$			The lightest odd state (DM) is typically the least interacting one: the partner of the hypercharge GB		630
$Z_\mu, W$					625
$A_s$			Spin = 1 in 5D Spin = 0 in 6D		537
$A_\mu$					537
H	-	542			Mass splittings are much larger for the Chi2!!

$\text{DM} \rightarrow$

# Preferred mass ranges: WMAP (Fermi) data



DM is a  $U(1)_Y$  gauge boson,  
thus smallish annihilation xsec!

The process:

$$A_{(1)} l_{(1)} \rightarrow A_{(2)} l \rightarrow \bar{q} q l$$

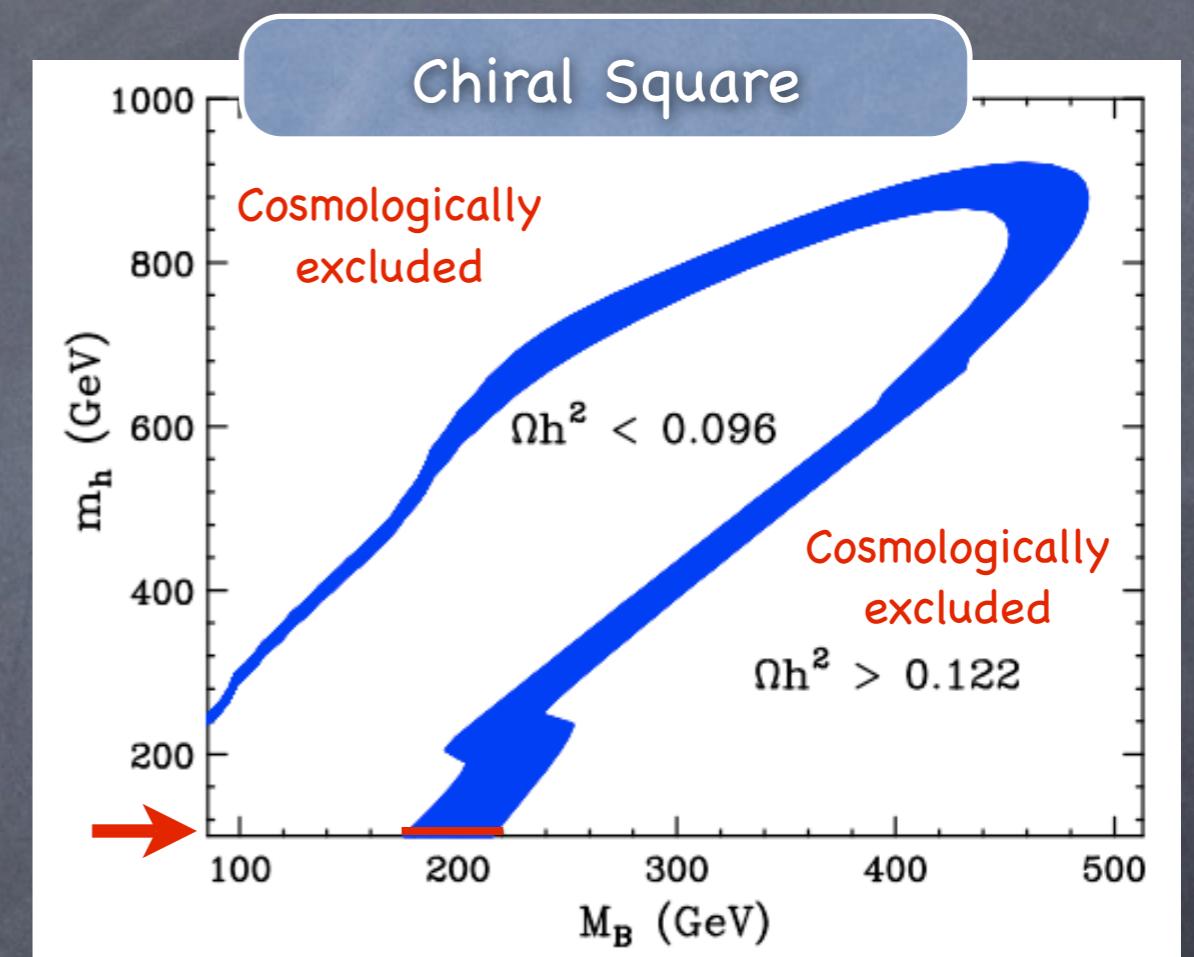
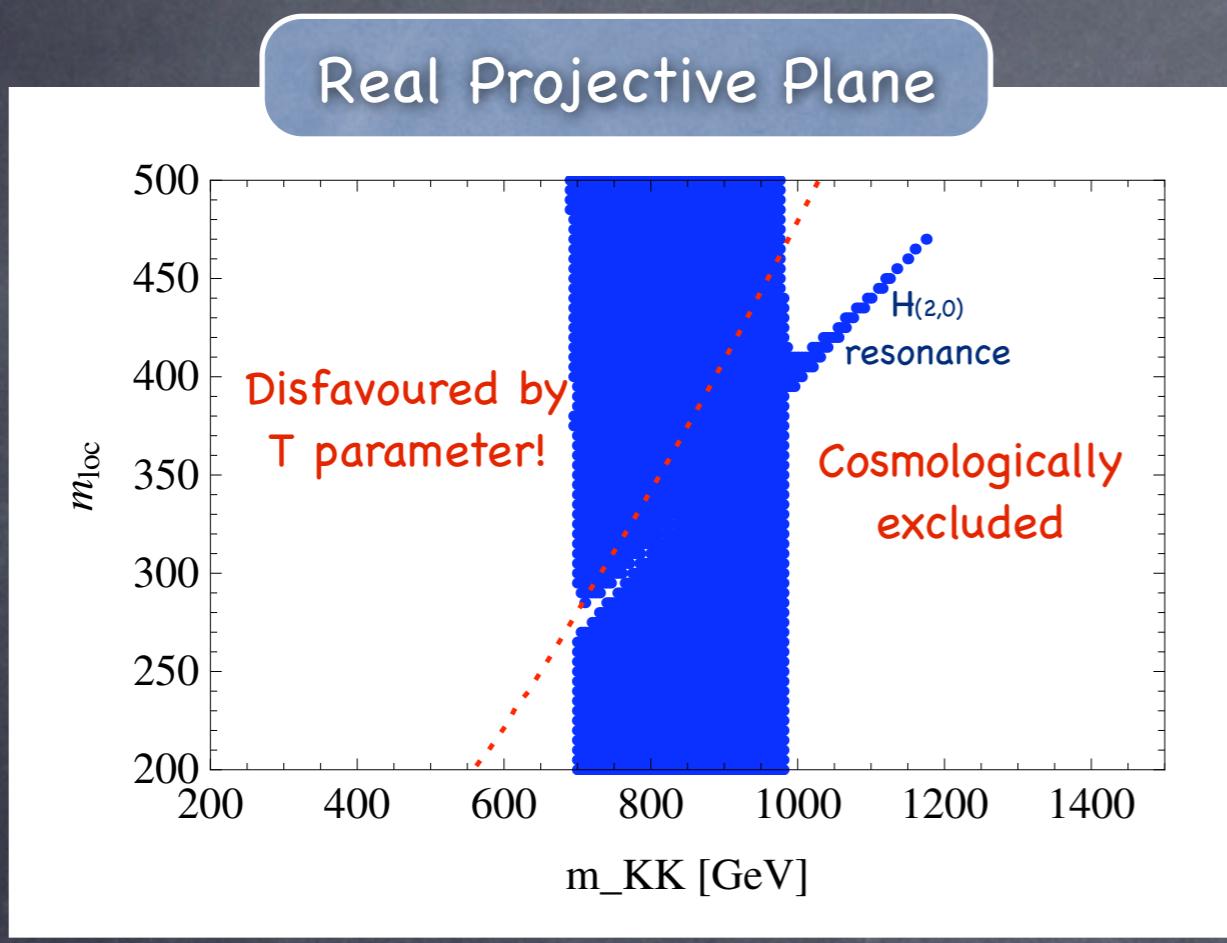
contributes about 50% of the  
total effective xsec!

Co-annihilation of the leptons  
does the rest.

G.Belanger, M.Kakizaki, A.Pukov 1012.2577

$1250 < m_{KK} < 1450$

# Preferred mass ranges: WMAP (Fermi) data



A.Arbey, G.C., A.Deandrea, B.Kubik 1210.0384

$$700 < m_A < 1000$$

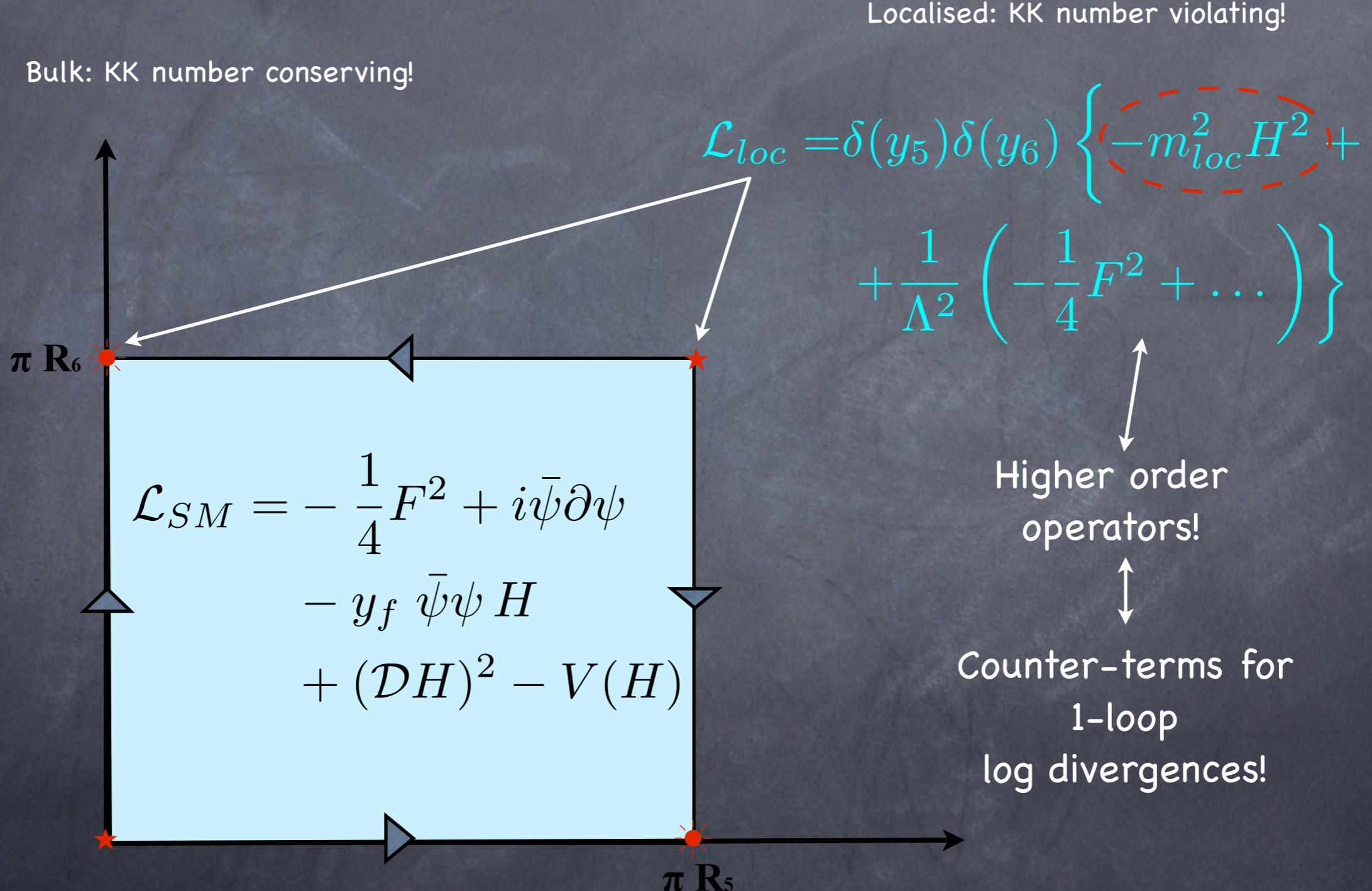
$$700 < m_{\text{KK}} < 1000$$

B.Dobrescu, D.Hooper, K.Kong, R.Mahbubani 0706.3409

$$180 < m_A < 220$$

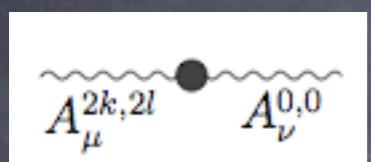
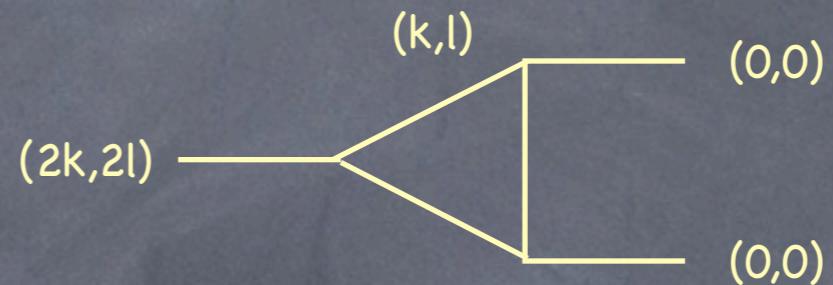
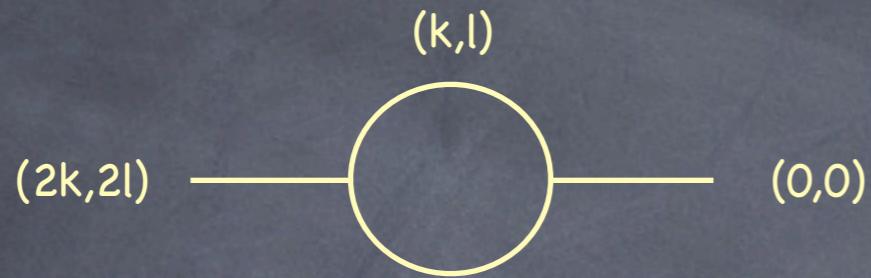
$$210 < m_{\text{KK}} < 255$$

# Spectrum of the SM



# 6D loops from 4D

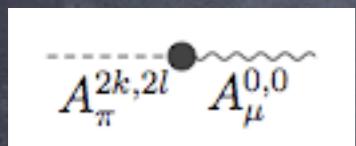
G.C., A.Deandrea, J.Llodra-Perez 1104.3800



$$i\Pi^{\mu\nu} = \frac{g^2 C_2(G) \delta^{ab}}{16\pi^4} \frac{i\pi^2}{2\epsilon} [2M^2(\xi + 3)g^{\mu\nu} - (\xi - 5)(q^2 g^{\mu\nu} - q^\mu q^\nu)]$$

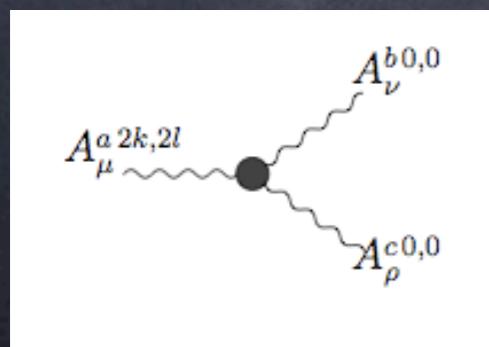
$$= \boxed{\frac{g^2 C_2(G) \delta^{ab}}{16\pi^4} \frac{4i\pi^2}{\epsilon} [q^2 g^{\mu\nu} - q^\mu q^\nu]}$$

$$\frac{1}{\epsilon} = \log \Lambda R$$



$$i\Pi^\mu = \frac{g^2 C_2(G) \delta^{ab}}{16\pi^4} \frac{i\pi^2}{2\epsilon} [iM(\xi + 3)q^\mu] = \boxed{0}$$

In red  
 $\xi = -3$   
 gauge

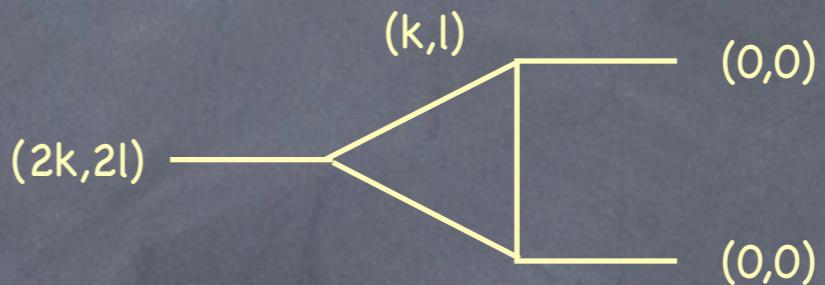
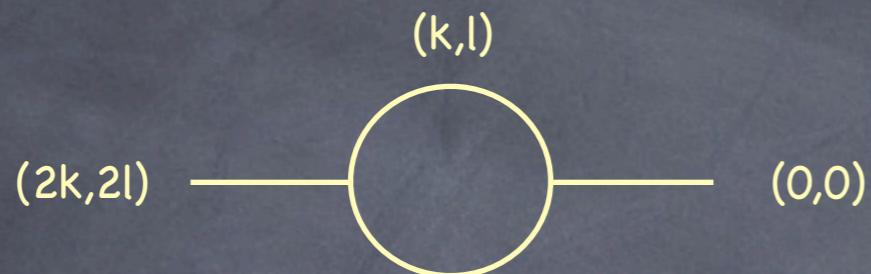


$$iV^{\mu\nu\rho} = \frac{ig^3 f^{abc} C_2(G)}{16\pi^4} \frac{i\pi^2}{4\epsilon} (-3\xi + 7) \mathcal{O}^{\mu\nu\rho}$$

$$= \boxed{\frac{ig^3 f^{abc} C_2(G)}{16\pi^4} \frac{4i\pi^2}{\epsilon} \mathcal{O}^{\mu\nu\rho}}$$

# 6D loops from 4D

G.C., A.Deandrea, J.Llodra-Perez 1104.3800



$$i\Pi^{\mu\nu} \quad A_\mu^{2k,2l} \bullet A_\nu^{0,0}$$

$$A_\pi^{2k,2l} \bullet A_\mu^{0,0}$$

In  $\xi = -3$  gauge the divergences match with the gauge-invariant counterterms!

$$[\nu - q^\mu q^\nu)]$$

$$\frac{1}{\epsilon} = \log \Lambda R$$

$$\frac{r_{10}}{4\pi^2 \Lambda^2 R^2} = \frac{r_{1\pi}}{4\pi^2 \Lambda^2 R^2} = \frac{g^2 C(G)}{16\pi^2} \log \Lambda R$$

In red  
 $\xi = -3$   
 gauge

$$iV^{\mu\nu\rho} = \frac{ig^3 f^{abc} C_2(G)}{16\pi^4} \frac{i\pi^2}{4\epsilon} (-3\xi + 7) \mathcal{O}^{\mu\nu\rho}$$

$$= \boxed{\frac{ig^3 f^{abc} C_2(G)}{16\pi^4} \frac{4i\pi^2}{\epsilon} \mathcal{O}^{\mu\nu\rho}}$$

# Phenomenology

5D

$m_{KK} \sim 1.4 \text{ TeV}$

Chiral Square

$m_{KK} \sim 200 \text{ GeV}$

Real Projective Plane

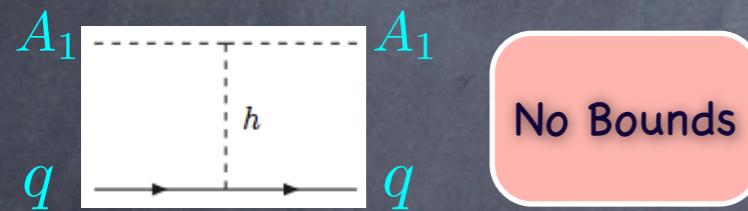
$m_{KK} \sim 900 \text{ GeV}$

# Phenomenology

5D

$$m_{KK} \sim 1.4 \text{ TeV}$$

Small DD xsec!  
(Higgs exchange)



Chiral Square

$$m_{KK} \sim 200 \text{ GeV}$$

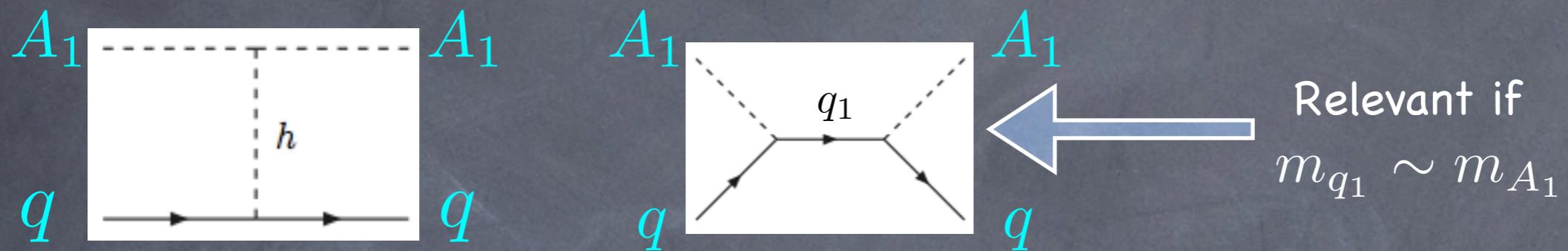
Larger DD xsec!  
(spin-0, but  
much lighter masses)

No Bounds

Real Projective Plane

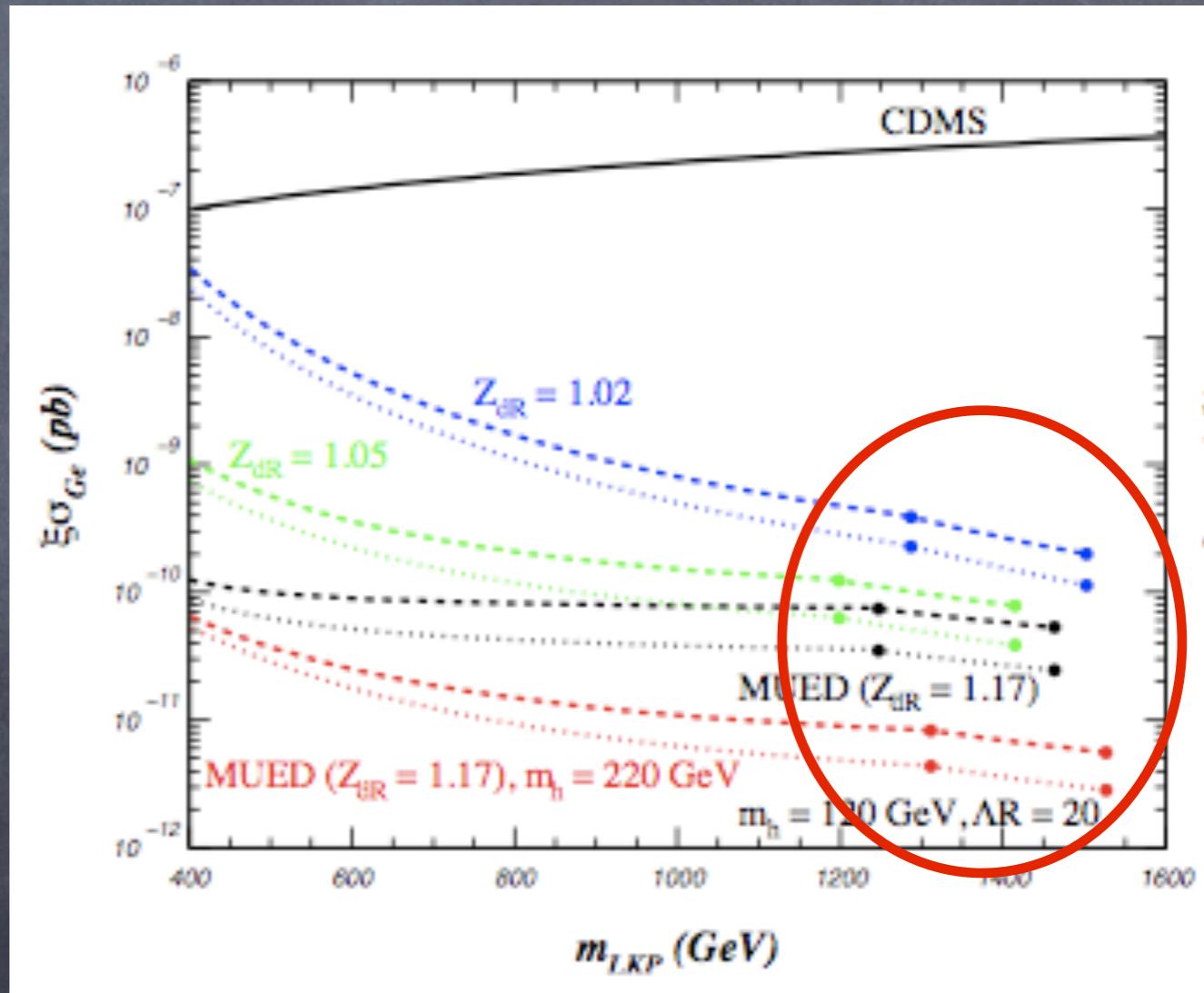
$$m_{KK} \sim 900 \text{ GeV}$$

# Direct detection: 5D



G.Belanger, M.Kakizaki, A.Pukov 1012.2577

- Small scattering cross sections (Higgs mediated)
- Very far from experimental bounds!

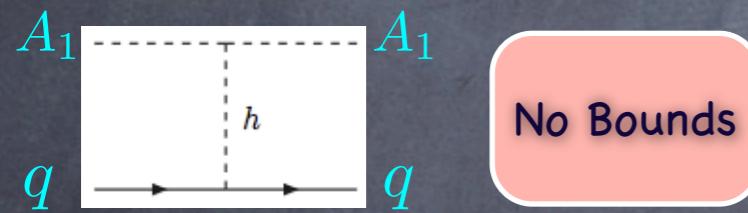


# Phenomenology

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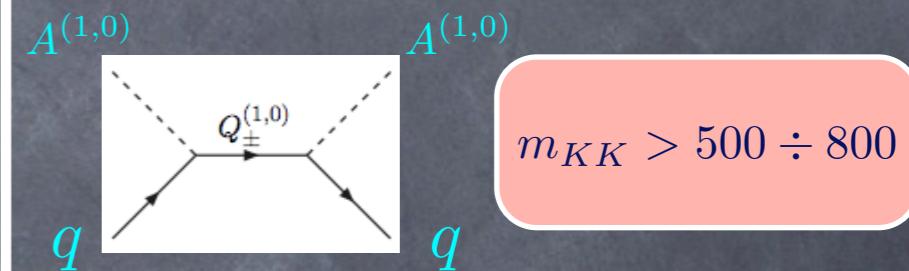
Larger DD xsec!  
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No Bounds

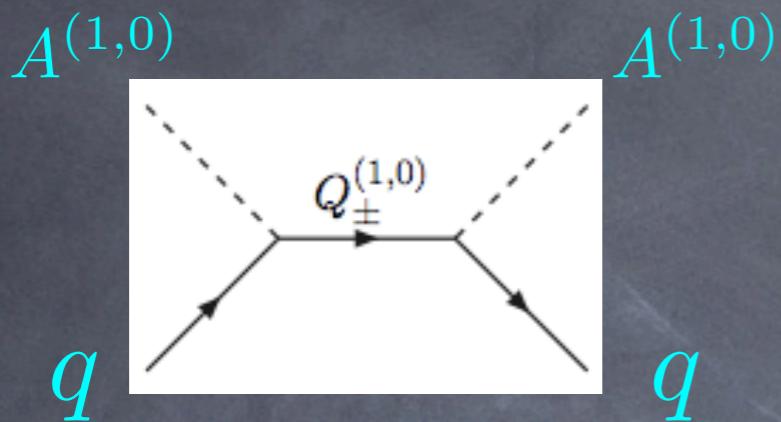
Real Projective Plane

$$m_{KK} \sim 900 \text{ GeV}$$

Enhanced DD xsec!  
(s-channel KK quark  
near resonance)

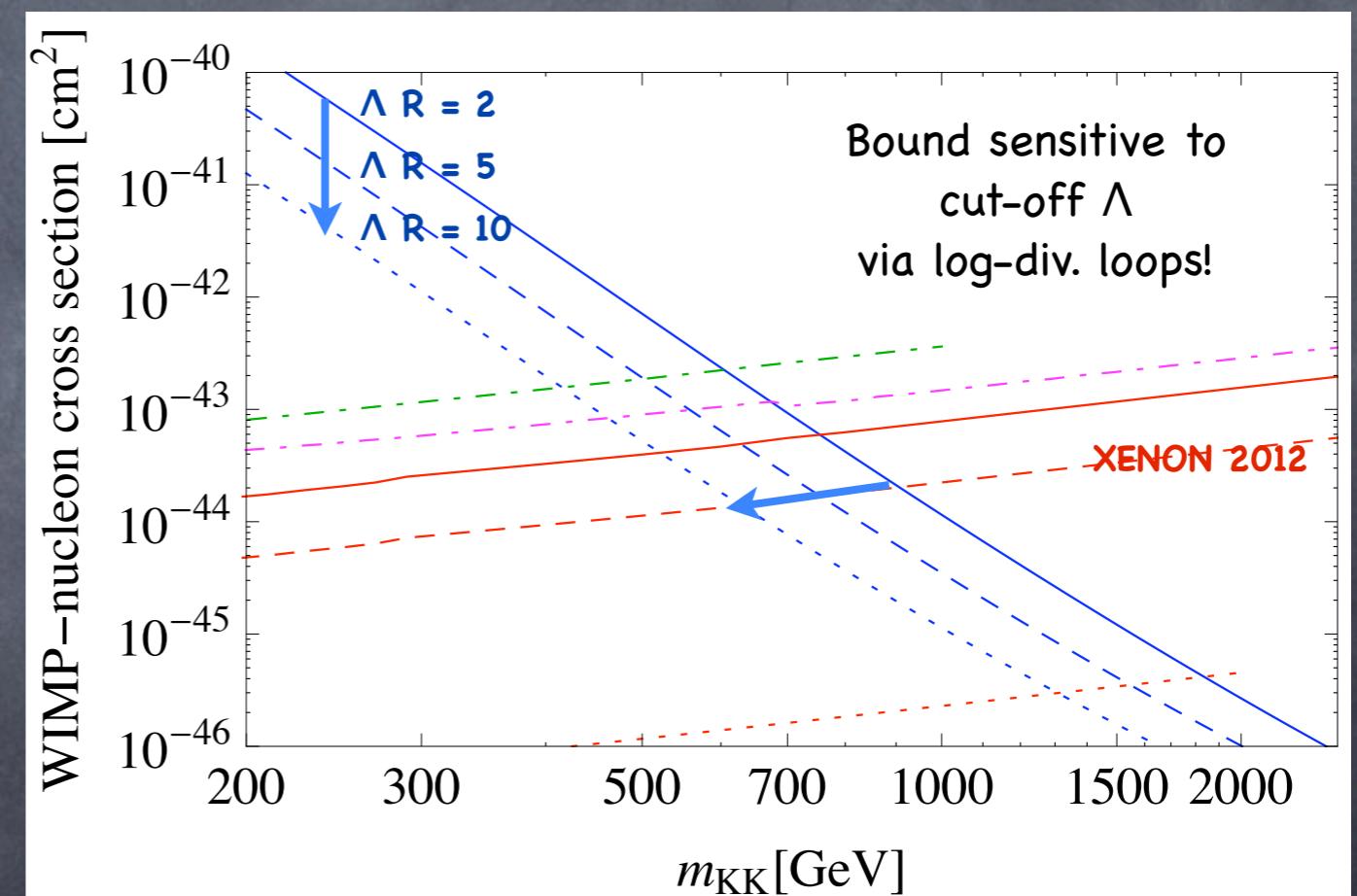
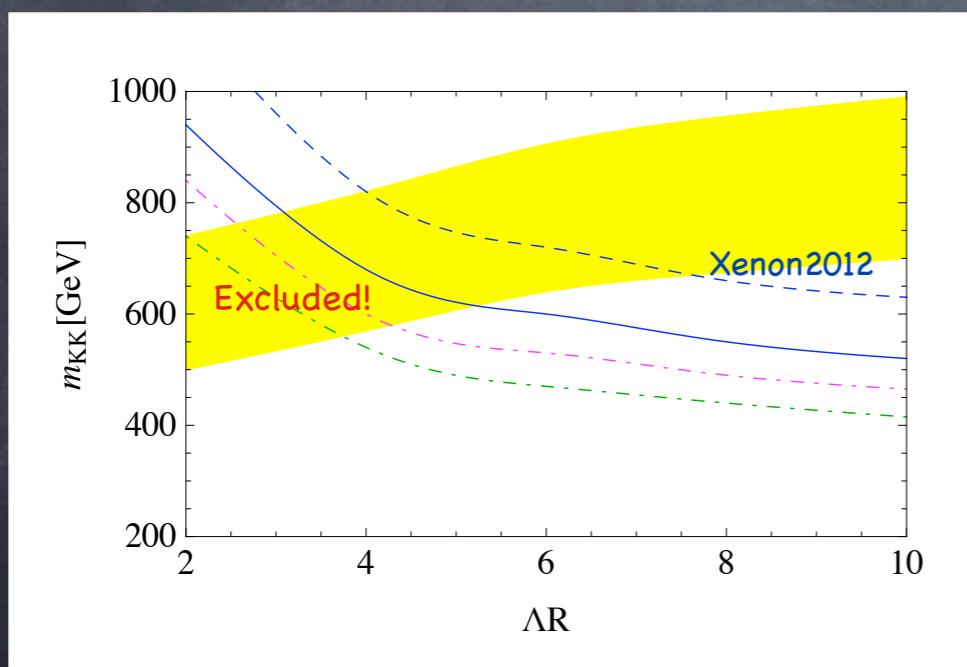


# Direct detection: RPP



Dominant as splitting is very small!  
Value of loop corrections is crucial!

A.Arbe, G.C., A.Deandrea, B.Kubik 1210.0384

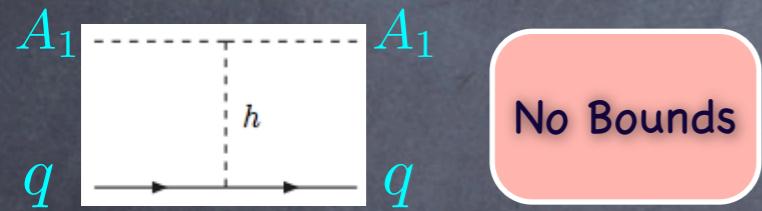


# Phenomenology

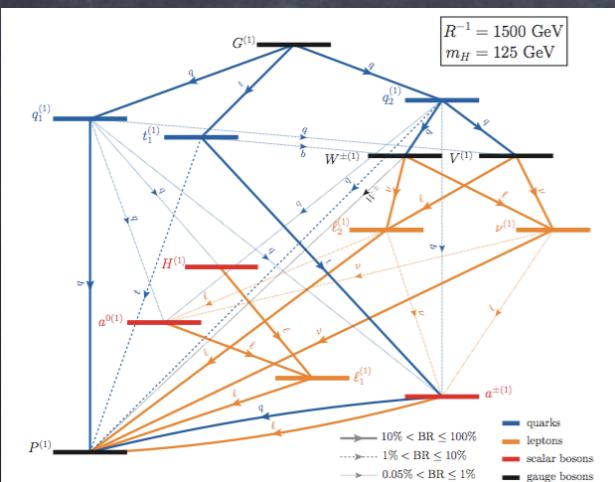
5D

$$m_{KK} \sim 1.4 \text{ TeV}$$

Small DD xsec!  
(Higgs exchange)



Susy-like signatures  
(High mass - hard to probe)



Chiral Square

$$m_{KK} \sim 200 \text{ GeV}$$

Larger DD xsec!  
(spin-0, but  
much lighter masses)

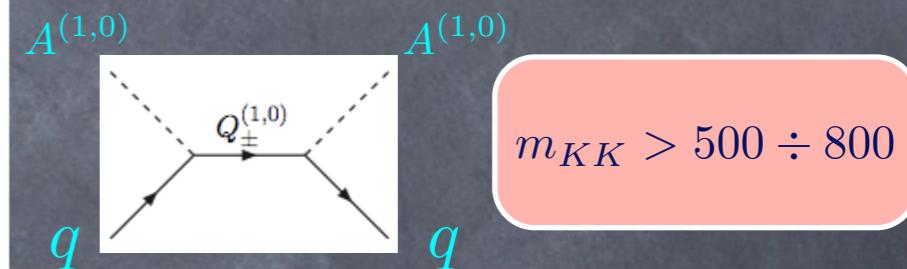
No Bounds



Real Projective Plane

$$m_{KK} \sim 900 \text{ GeV}$$

Enhanced DD xsec!  
(s-channel KK quark  
near resonance)

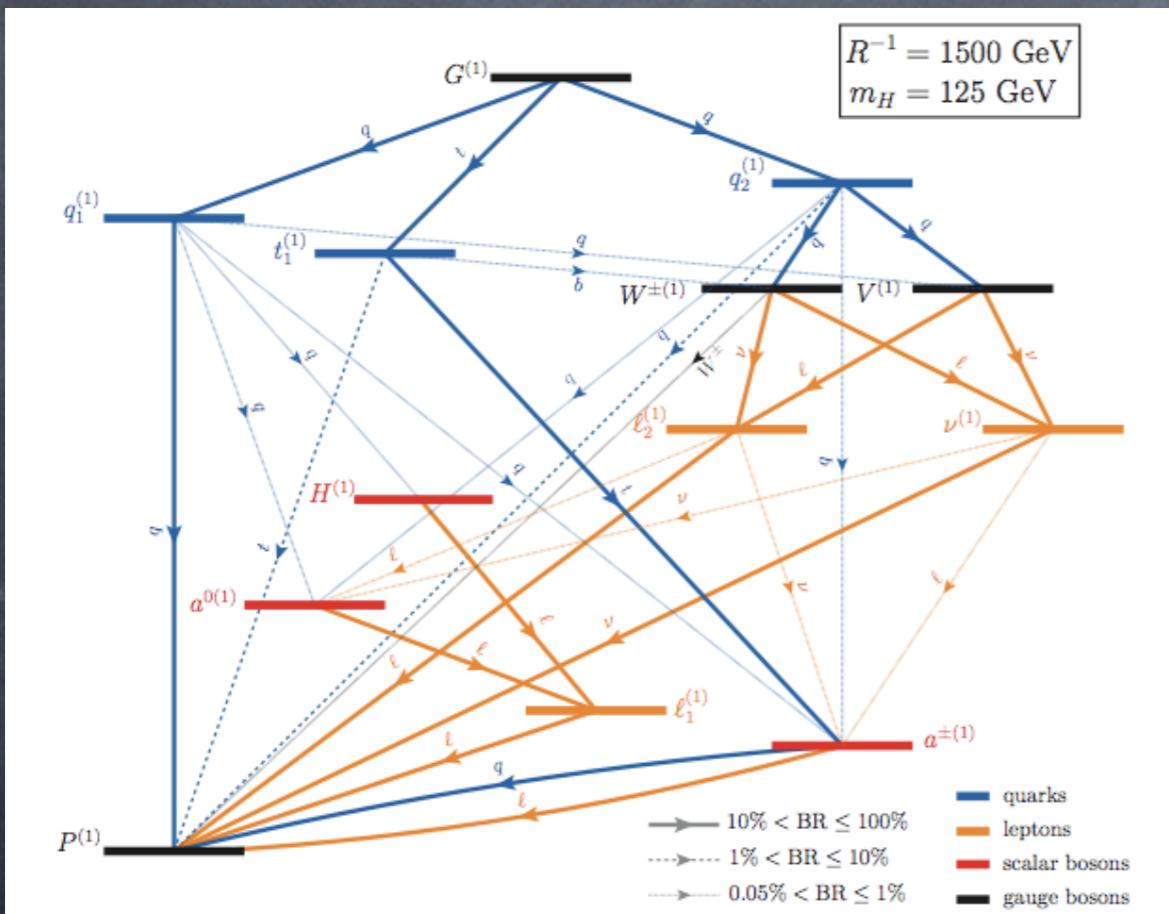


$m_{KK} > 500 \div 800$

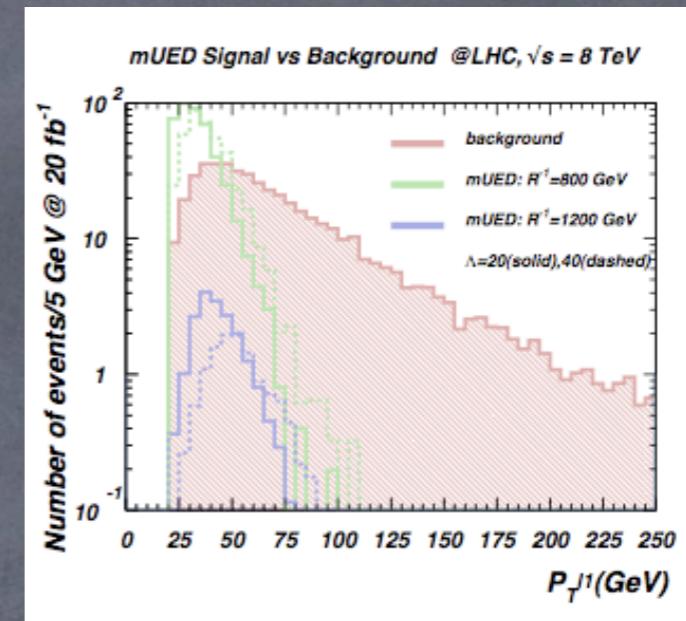
# LHC bounds: 5D

Very complex decay patterns!

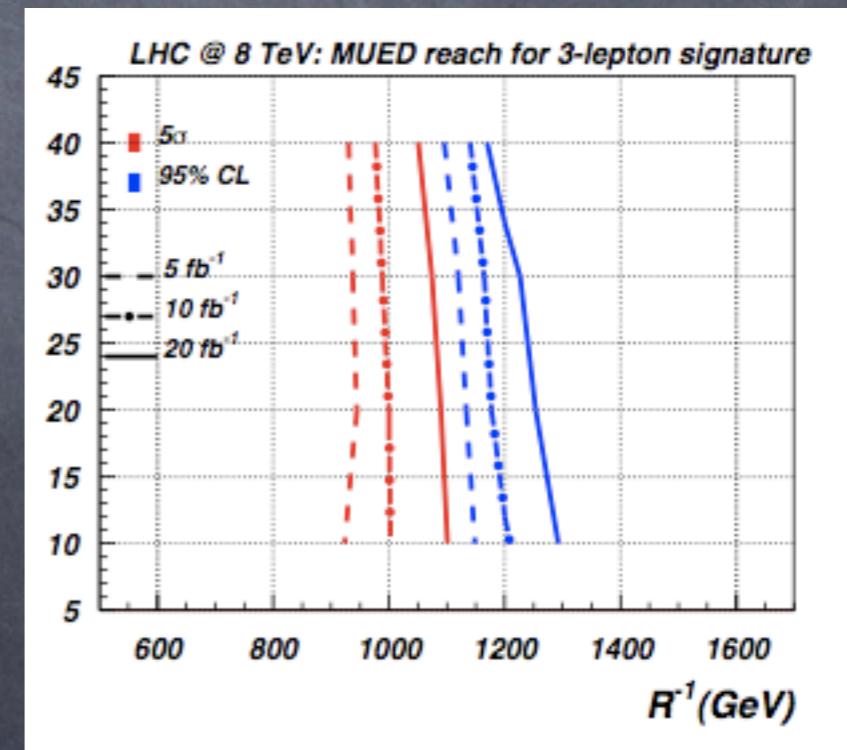
May use the presence of many (soft) leptons coming from chain decays.



For jets + MET signatures, see  
A.Datta, A.Datta, S.Poddar 1111.2912



A.Belyaev, M.Brown, J.Moreno, C.Papineau 1212.4858

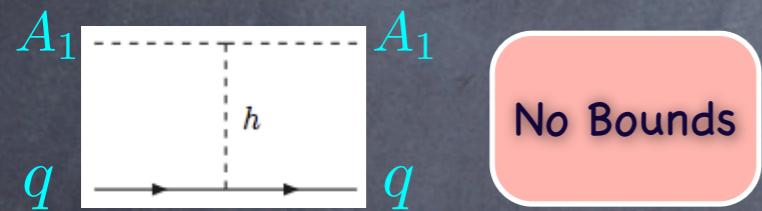


# Phenomenology

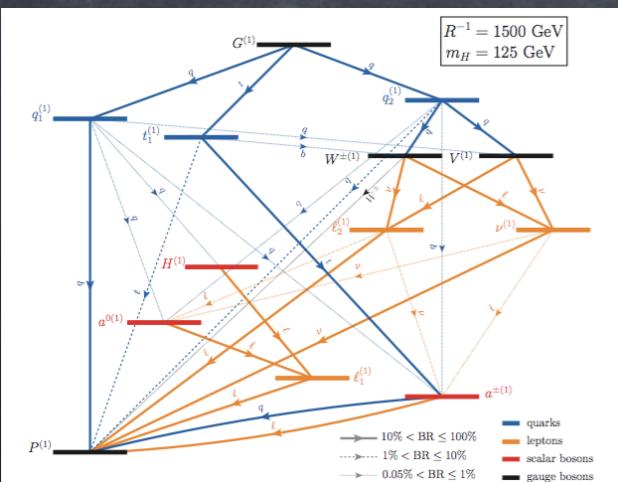
5D

$m_{KK} \sim 1.4 \text{ TeV}$

Small DD xsec!  
(Higgs exchange)



Susy-like signatures  
(High mass - hard to probe)



Chiral Square

$m_{KK} \sim 200 \text{ GeV}$

Larger DD xsec!  
(spin-0, but  
much lighter masses)

No Bounds

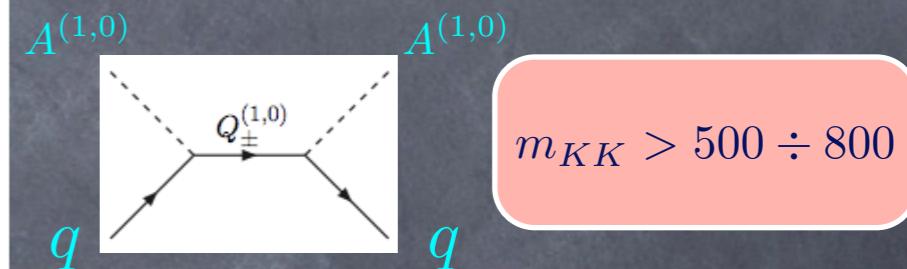
Susy-like signatures  
(small mass likely excluded)



Real Projective Plane

$m_{KK} \sim 900 \text{ GeV}$

Enhanced DD xsec!  
(s-channel KK quark  
near resonance)



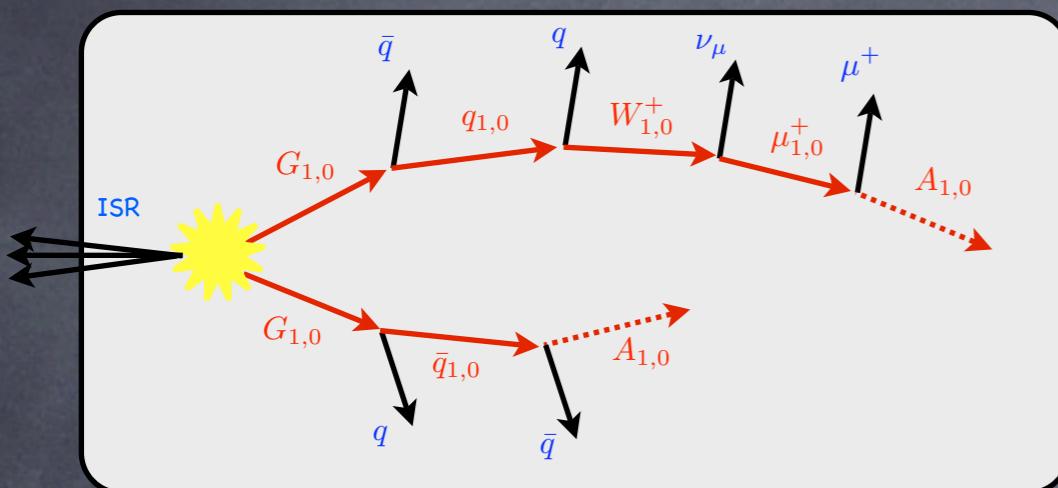
Compressed spectra

2nd KK tier light enough  
to be seen in resonances!

Many handles on the model!

# LHC bounds: RPP

Very complex decay patterns,  
but SMALL mass splittings!



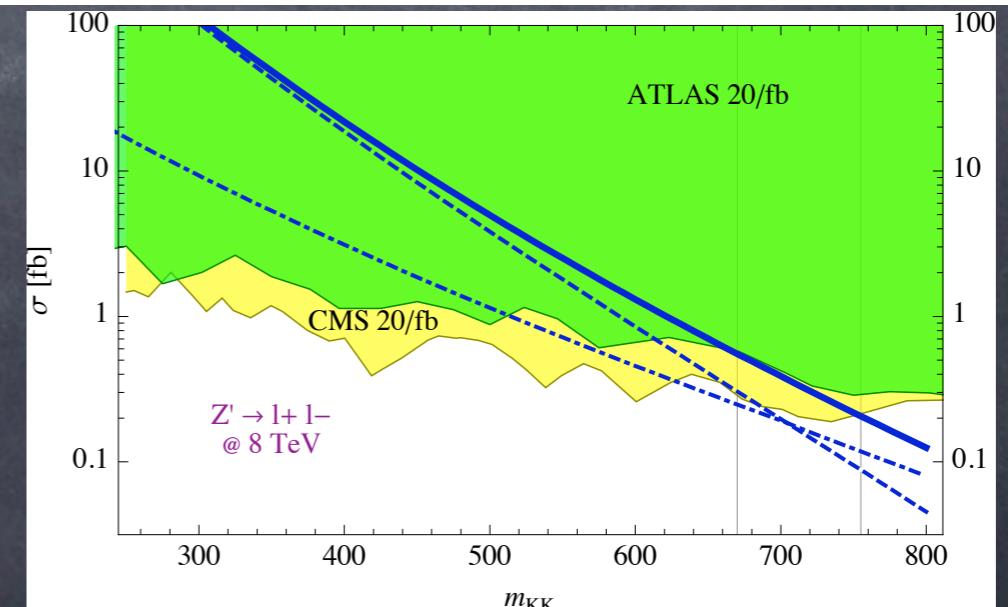
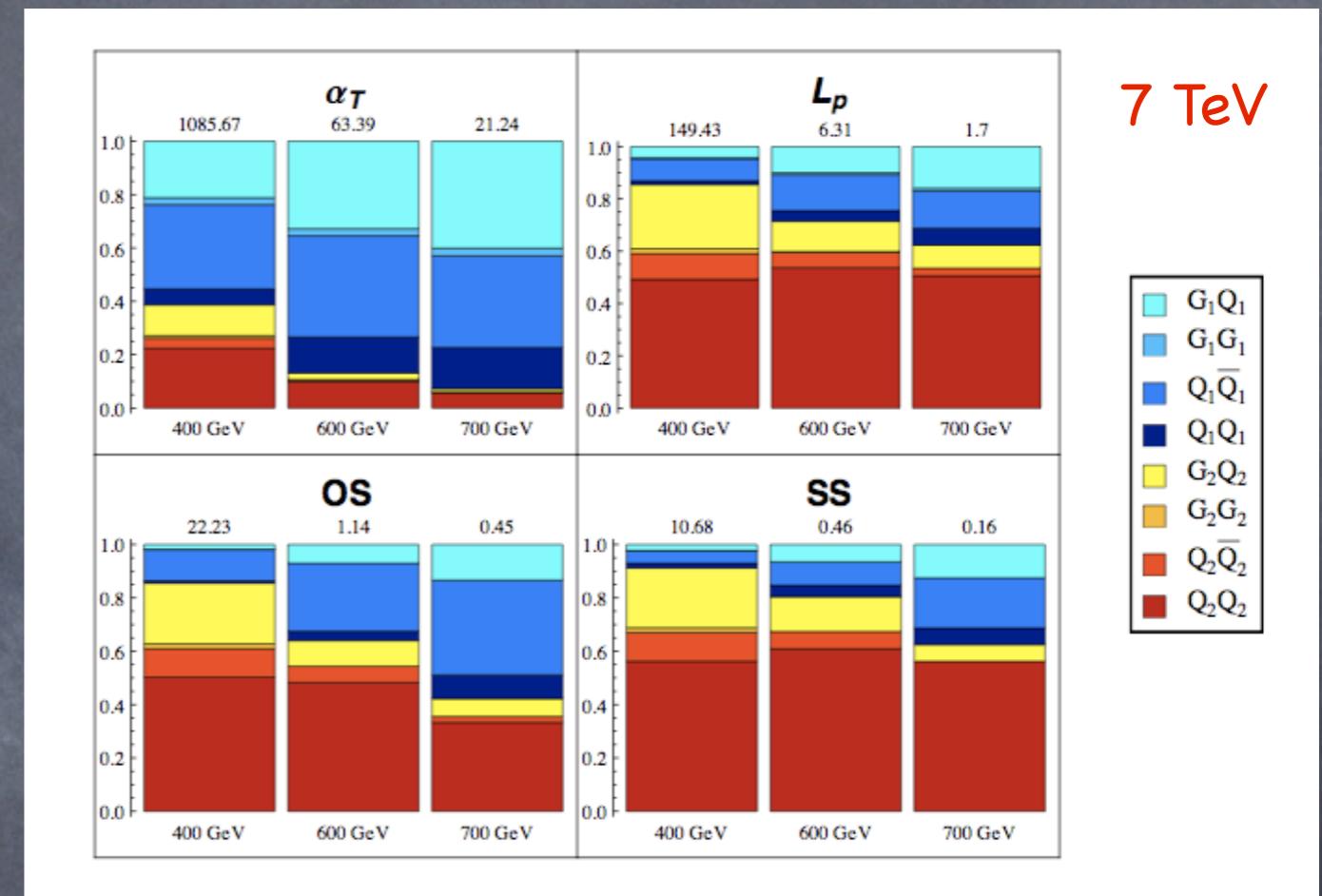
Tool by J.Marrouche and L.Panizzi

Bound between  
600 and 700 GeV!

Tier-2 gauge bosons can decay directly  
into two fermions:  
di-lepton resonances!

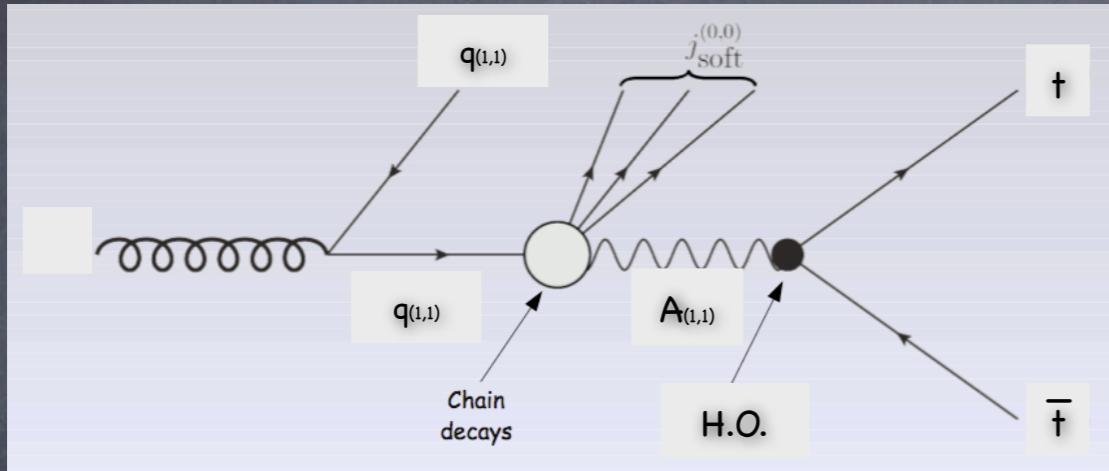
G.C., B.Kubik 1209.6556

G.C., A.Deandrea, J.Ellis, L.Panizzi, J.Marrouche 1302.4750

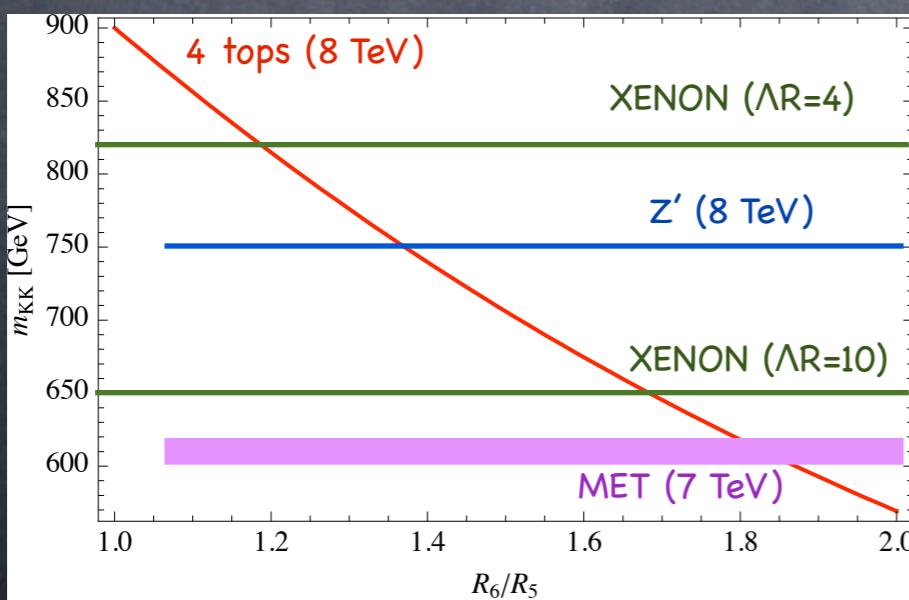
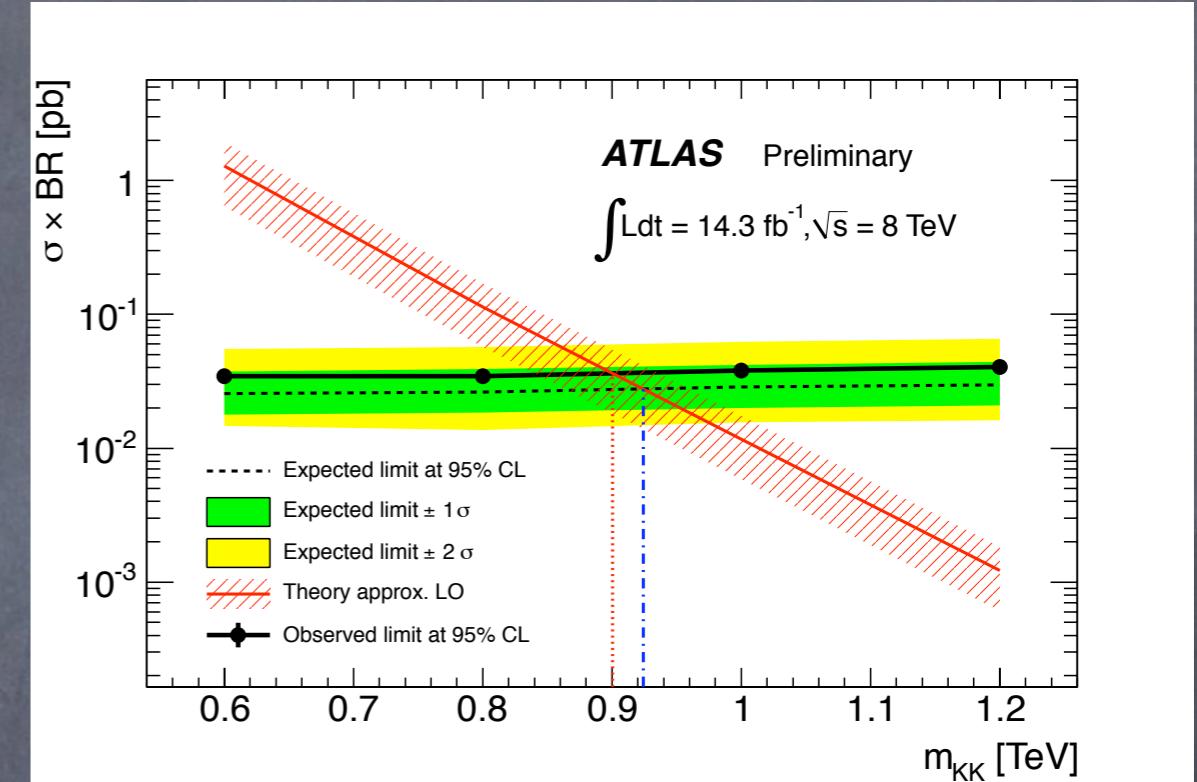


# LHC bounds: RPP

4-top final state: search in same-sign dileptons



G.C., R.Chierici, A.Deandrea, L.Panizzi, S.Perries, S.Tosi  
1107.4616



Dedicated ATLAS search  
in same-sign di-lepton final states.

ATLAS-CONF-2013-051

# Summary:

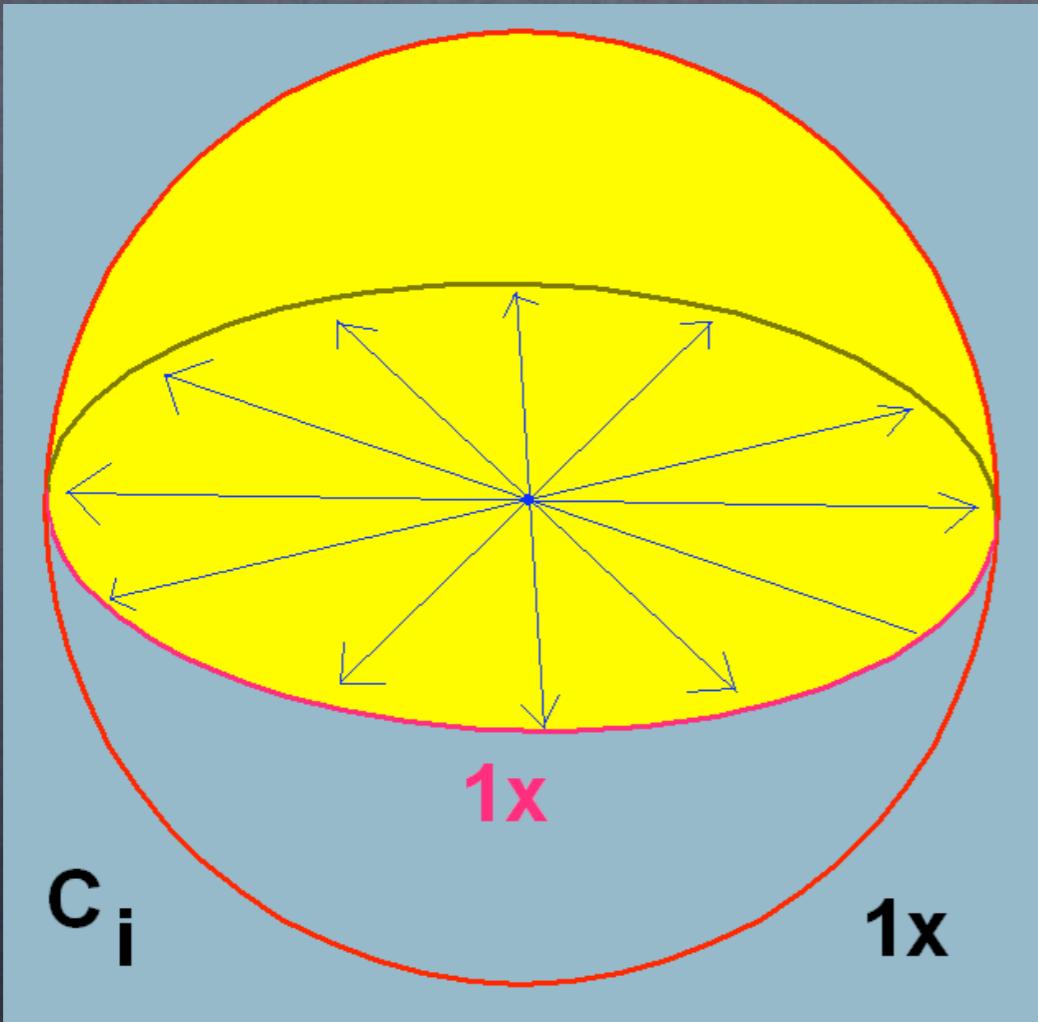
- The phenomenology depends on the geometry of the extra dimensions: 5D, chiral square, Real projective plane...
- The particle content is always complex: co-annihilation is important, no trivial connection between relic abundance and detection!
- Pheno depends crucially on the loop-corrected spectra and coupling: detailed calculations always indispensable!
- Why limit ourselves to flat spaces?

# DM from a sphere

- Spherical RPP

H.Dohi, K-y.Oda, 1004.3722

- Note: due to curvature, fermions have no zero modes.



Wikipedia: spherical symmetry groups

KK modes labelled by  
angular momentum  $(l,m)$

$$m_{(l,n)}^2 = \frac{l(l+1)}{R^2}$$

No fixed points:  
tiny finite loop corrections

Angular momentum is  
conserved on the orbifold!

Each KK tier contains  
a stable DM candidate!