High intensity photon beams for CERN

Mieczyslaw Witold Krasny LPNHE, Paris, 5 Dec, 2013

Present and future photon beams





nuclear size ~ 10 -15 - 10 -14 m

Relevant photon energies: ~ 2 - 200 MeV

...how to produce monochromatic beams in this energy range?

LCS γ -ray sources





FIG. 2. Energy distribution comparison between a bremsstrahlung spectrum produced by a 15-MeV electron collision with a tungsten target and a back-scattered Compton spectrum produced by a 2-eV photon collision with a 700-MeV electron.

2013 – The Duke University HI_YS facility has the highest γ -beam intensity in the world ~10⁸ γ /s



Parameters of the gamma facilities around the world

Project name	LADON ^a	LEGS	ROKK-1M ^b	GRAAL	LEPS	HIγS ^c
Location	Frascati	Brookhaven	Novosibirsk	Grenoble	Harima	Durham
	Italy	US	Russia	France	Japan	US
Storage ring	Adone	NSLS	VEPP-4M	ESRF	SPring-8	Duke-SR
Electron energy (GeV)	1.5	2.5-2.8	1.4-6.0	6	8	0.24-1.2
Laser energy (eV)	2.45	2.41-4.68	1.17-4.68	2.41-3.53	2.41-4.68	1.17-6.53
γ-beam energy (MeV)	5-80	110-450	100-1600	550-1500	1500-2400	1-100 (158) ^d
Energy selection	Internal	External	(Int or Ext?)	Internal	Internal	Collimation
_	tagging	tagging	tagging	tagging	tagging	
γ-energy resolution (FWHM)						
ΔE (MeV)	2-4	5	10-20	16	30	0.008-8.5
$\frac{\Delta E}{E}$ (%)	5	1.1	1-3	1.1	1.25	0.8-10
E-beam current (A)	0.1	0.2	0.1	0.2	0.1-0.2	0.01-0.1
Max on-target flux (γ/s)	5×10^{5}	5×10^{6}	10 ⁶	3×10^{6}	5×10^{6}	$10^{4}-5 \times 10^{8}$
Max total flux (γ/s)						10^{6} -3 × 10^{9e}
Years of operation	1978-1993	1987-2006	1993-	1995-	1998-	1996-

How to increase the fluxes by several orders of magnitude?

The ThomX Project

ABORATOIRE

DE L'ACCÉLÉRATEUR

INÉAIRE

SYNCHROTRON

Injector			Ring	
Charge		1 nC	Energy	50 MeV (70 MeV possible)
Laser wavelength and p	ulse power	266 nm, 100 μJ	Circumference	16.8 m
Gun Q and Rs		14400, 49 MW/m	Crossing-Angle (full)	2 degrees
Gun accelerating gradie	int	100 MV/m @ 9.4 MW	B _{x,y} @ IP	0.2 m
Normalized r.m.s emitte	ance	8π mm mrad	Emittance x,y (without IBS and Compton)	3 10 ⁻⁸ m
Energy spread		0.36%	Bunch length (@ 20 ms)	30 ps
Bunch length		3.7 ps	Beam current	17.84 mA
Laser and FP cavity			RF frequency	500 MHz
Laser wavelength	10	030 nm	Transverse / longitudinal damping time	1 s /0.5 s
Laser and FP cavity Fre	эр 30	6 MHz	RF Voltage	300 kV
Laser Power	50	0 - 100 W	Revolution frequency	17.8 MHz
FP cavity finesse / gain	3	0000 / 10000	$\sigma_x \otimes IP$ (injection)	78 mm
FP waist	70	0 μm	Tune x / y	3.4 / 1.74
Source			Momentum compaction factor $\boldsymbol{\alpha}_{c}$	0.013
Photon energy cut off	46 keV (@50 MeV), 90	keV (@ 70 MeV)	Final Energy spread	0.6 %
Total Flux	10 ¹¹ -10 ¹³ ph/sec			
Bandwidth	1 % - 10%			
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institut

ESRF

Institut national de la santé et de la recherche médicale

Extreme Light Infrastructure (ELI) - EU project

GUIDING LIGHTS

The three laser-research sites of the Extreme Light Infrastructure (ELI) rely on European infrastructure funds typically meant for civic projects.





Main limitation of the electron-beam generated gamma beams:

Compton scattering cross section is small ~O(10⁻²⁵) cm²

...technological brick walls in:

•ERLs (e-bunches recycled to accelerate subsequent ones)

•High power FELs (to increase the energy of the initial light quanta)

•High Power lasers

•Cavities (to stack laser pulses)

•High energy, large current and small emittance electron beams

Alternative idea discussed in this talk:

Use partially stripped ion beams as the light frequency converter to bypass the technological brick-walls specific to electron beams:

 $\nu \rightarrow 4\gamma^2 \nu$

 γ =E/M - Lorentz factor for the ion beam

A proposal of an "unconventional" use of the LHC and its detectors for the ep(eA) collision programme



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Electron beam for LHC

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Abstract

A method of delivering a small energy spread electron beam to the LHC interaction points is proposed. In this

...can one use the LHC to produce intense photon beams?

Partially stripped ions as electron carriers



 average distance of the electron to the large Z nucleus d ~ 600 fm (sizably higher than the range of strong interactions)

•partially stripped ion beams can be considered as <u>independent electron and</u> <u>nuclear beams</u> as long as the incoming proton scatters with the momentum transfer q >> 300 KeV

•both beams have <u>identical bunch structure</u> (timing and bunch densities), <u>the same β^* , <u>the same beam emittance</u> – the choice of collision type can be done exclusively by the trigger system (no read-out and event reconstruction adjustments necessary)</u>

Quantum optics of ultrarelativistic atoms



Note: $1Ry= 13.6 \text{ eV}; \gamma_L = E/M \sim 2740$ for Pb^{81+} at the LHC

<u>The proposal:</u> LHC as a frequency converter of O(1-10 eV) photons into O(1-400 MeV) γ-rays



Fine tuning of $E_{\gamma-ray}$

The energy of the gamma beam can be tuned by selecting the ion (Z), its storage energy (γ -factor), the atomic level (n), and the laser light wavelength (E_{laser})

Example1:

Pb⁸¹⁺ ion at the top LHC energy , n=2, E_{FEL} =12.2 eV, $E_{\gamma-ray}$ (max) = 366 MeV

Example2:

Argon laser E_{laser} =3.53 eV, Ag⁴⁵⁺ ion, γ =2925, n=2, $E_{\gamma-ray}$ (max) = 121 MeV

Example3:

ThomX laser λ =1030 nm, Ca²⁰⁺ ion, γ =2460, n=2, $E_{\gamma-ray}$ (max) = 20 MeV

The comparison of the "LHC-frequency converter" and LCS sources

Fluxes:

The Rayleigh resonant cross section for partially stripped ions is higher by a factor $(\lambda_{res}/r_e)^2$ than the Thompson cross-section for electrons ($r_e = 3 \times 10^{-15}$ m)

gain in the γ -flux of the order of ~ 10⁷ for the same intensity of the laser light (even if one assumes that only a small fraction of the laser light of 10⁻³ can be absorbed resonantly (beam divergence, momentum spread, Γ_{atomic})

Beam rigidity:

lons bunches are "undisturbed" by the light emission. Electron bunches are.

... partial remedy: e-beam is recycled to accelerate succeeding beam (ERL)

The comparison of the

"LHC-frequency converter" and LCS sources

Energy tunability:

Four dimensional flexibility of the "LHC-converter" ($E_{laser(FEL)}$, γ , Z_{ion} ,n.). Easy to optimize for a required narrow band of the γ -beam energy over a large E γ -ray domain. For the ERL sources two parameter tuning.

Beam divergence:

Excellent: Below 0.3 mrad for both sources

Polarizability

Flexible setting. Reflect, in both cases the polarization of the laser light

Technological challenges

For maximal energies "LHC frequency converter" must be driven by a <100 nm FEL photons. For lower energies standard 100-1000 nm lasers (e.g. CAN lasers) + FP cavities are sufficient Basic question:

Can one produce, accelerate and store partially striped ions at the LHC?

(my studies presented in: M.W. Krasny, CERN seminar 22.06.2004, and M.W. Krasny, Nuclear Instruments and Methods A540 (2005) 222.)

Lead acceleration at CERN present stripping sequence



Survival of partially stripped ions: the LHC lattice



The electric field, E, "seen" by the ion is larger than the field holding an electron in a hydrogen atom!

Survival of partially stripped ions: the LHC lattice



low-Z ions cannot survive!

Survival of bunches of partially stripped ions: temperatures

Longitudinal temperature:

 $kT_{\parallel} = m_{ion}c^2\beta^2 (\sigma(p)/p)^2$

Transverse temperature:

 $kT_{h,v} = m_{ion}c^2\beta^2(\epsilon_N/\langle\sigma_{h,v}\rangle)^2$

 ϵ_N is the normalized emittance

 $<\sigma_{h,v}>$ is the average horizontal (vertical) beam size: $<\sigma_{h,v}^{2}>= \varepsilon_{h,v} R/Q$,

Q – horizontal and vertical tune and R – radius of the machine

Temperatures of the LHC bunches: k $T_{\parallel} \sim 2$ keV, k $T_{h,v} \sim 1$ MeV (at the LHC

injection energy and growing with $\sim \gamma$)

Kinetic energy of the transverse thermal motion larger than binding energy of the electron on the K-shell, even for the highest Z ions !!!

... Can ions survive intra-beam scattering???

Survival of bunches of partially stripped ions: collision frequency

Mean free path of a partially stripped ion in the bunch (approximation of non-interacting gas):

 $c\tau = c\gamma/(\sigma\rho\Delta v)$

For : $\sigma = \pi (10 r_{ion})^2$, $\Delta v - dispersion of Maxwell velocity of ions,$ $\rho - ion bunch density (bunch rest frame) averaged$ over the ring circumference (< β >)

 $\tau \sim 43$ sec ... at the LHC injection energy

lons will be eventually lost due to intra-beam scattering ... unless one finds a method of stabilizing bunch temperatures below the equivalent ionization temperature...

Intra-beam collisions Intra-beam scattering at temperatures: $kT < E_1 - E_2$ 0 Elastic scattering Intra-beam scattering at temperatures: $E_1 - E_{n max} < kT < E_1 - E_2$ photon 0 **Atomic** followed by excitation $hv_k = E_1 - E_k$ Intra-beam scattering at temperatures: $kT > E_1$ Stripped ion **Atomic** followed by a beam particle loss! electron ionization 0

Recipes for accelerating partially stripped ions in high density bunches



Two recipes:

1. External cooling of overheated bunches : $\tau >> t(\gamma_2) - t(\gamma_1)$ 2. Isothermal heat evaporation: $\tau << t(\gamma_2) - t(\gamma_1)$

Isothermal heat evaporation



Numerical example:

<u>A model:</u>

accelerating of bunches of partially stripped ions in an accelerator specified in terms of the following parameters: $N_{ions/bunch} = 10^7$; $\varepsilon_N = 1.5 \times 10^{-6} m$, $\sigma(p)/p = 10^{-4}$; $<\beta>_{circumference} = 67 m$; $\sigma_{excit} = 10^{-18} \text{ cm}^2$ at T = 1 MeV; bunch-length = 7.5 cm (the standard parameters for fully stripped lead ions at the LHC – note a simplification of the full acceleration chain and an "educated guess" of the atomic excitation cross section)

Temperatures at insertions to SPS and LHC:

 $kT_{h,v}(\gamma = \gamma_{PS}=6) = 36 \text{ KeV}$, $kT_{h,v}(\gamma = \gamma_{SPS}=166) = 1000 \text{ KeV}$

Critical acceleration phase for isothermal heat evaporation:

At $\gamma_1 \sim 10$ the bunch temperature approaches the value of $(E_1 - E_2)/k = 69 \text{ keV}$

Acceleration speed at the critical acceleration phase allowing for isothermal heat evaporation :

 $d \gamma/dt \sim 0.8/minute$

Can all that be experimentally verified? Ion striping sequence BNL



Partially stripped gold ions Au ⁷⁷⁺ (two electrons left) were already accelerated and stored at Brookhaven National Laboratory at the energy of 11.6 GeV/nucleon)

Can all that be experimentally verified?

Acceleration of Au (77+) in the AGS at BNL

If the above model is applied to acceleration of gold ions carrying two electrons the onset of self-cooling should be observed already at the acceleration phase of AGS

<u>Note</u>: larger ε_N and smaller $\langle \beta \rangle_{circumference}$ and larger $N_{ions/bunch}$...

... is there any sign temperature stabilization (drop of ε_N)?

Longstanding puzzles at BNL:

- Contrary to expectations, emittance of gold beam is smaller than that for protons
- The emittance of **Au (77+)** beam ejected from AGS is independent of the injection scheme.
- A drop of the normalized emittance is directly observed (non-conclusive due to associated beam-losses).

Survival of partially stripped ions: Ionization losses

 A dominant process leading to losses of partially stripped ions is the ionization process in beam-beam and beam-gas collisions (note a quantum jump in magnetic rigidity of the beam particles)

Ionization cross-sections

Anholt and Becker, Phys.Rev.A36(1987)

Coulomb contribution: (7, 7) (7, 7)

 $\sigma_{\text{Coul}} = s(Z_t, Z_p) (Z_t/Z_c)^2 10^4 \text{ [barn/electron]}$

Transverse contribution: $\sigma_{Tran} = t(Z_t, Z_p) (Z_t/Z_c)^2 10^4 \ln(\gamma^2) \text{ [barn/electron]}$

Where: $s(Z_t, Z_p)$, $t(Z_t, Z_p)$ are slowly (logarithmically) varying functions of the electron carrier Z_c and target Z_t , and γ is the Lorenz factor

Note:

- spin-flip contribution is neglected

- coherent bunch contribution is neglected



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Survival of partially stripped ions: beam-gas collisions

Collisions of Pb⁸¹⁺(1s) ions with the residual gas in the LHC beam pipe – how long can they survive?

 Calculate maximal allowed concentration of molecules to achieve the 10 hour lifetime of the beam

 $\tau^{-1} = \sigma_i \times \rho_i \times c$

Compare with the estimated densities for the gas molecules in the interaction regions by Rossi and Hilleret: (H2 – 1.3x10¹²; CH4 – 1.9x10¹¹; ... CO2 – 2.8x10¹¹ mol/m³)

Result: The safety factor varies between 30 (for the H2 molecules) and 2 (for the CO2 molecules). Better vacuum in arcs.

Survival of partially stripped ions: summary

- Bunch temperature T_b << 1 Ry × Z² at all the acceleration stages -(radiative evaporation cooling, back-up: laser cooling)
- "Stark effect" in the LHC superconducting dipoles (E= 7.3 10¹⁰ V/m) only high Z ions allowed to be the electron carriers at the LHC
- Ionization process

-stringent requirement on the LHC vacuum (concentration of CO₂ is critical - must be kept below ~6x10¹¹ mol/m³ to achieve the **Pb⁸¹⁺(1s)** beam life-time larger that 10 Hours)

 stringent requirements on the allowed collision schemes (partially stripped high Z ions can collide only with the lightest fully stripped ions: p, He, O…)

Disclaimer

All the results presented below are based on the order of magnitude, preliminary estimates, often based on the extrapolations from the present (future) light sources using electron beams. The goal of such an exercise is: (1) to find out if this work is worth to continue and (2) to invite those interested in making a sound quantitative studies to join the camp at this preliminary stage.

In the following I shall often assume the scaling allowing to increase the present (future) sources by a factor $\sim 10^7$ (a factor $\sim 10^{10}$ coming from the ratio of cross sections and a factor 0.001 representing the efficiency of selecting, from the flux of the laser photons, those corresponding to the width of the excited atoms, beam divergence and the beam compaction factor.

I assume that the beam parameters of the partially stripped ion beams and the collision geometry with the laser beam will be set to be those for the electron beams (e.g. for ThomX project) - what cannot be achieved is the same current of both beams and large crossing angle (I shall come back to these questions later on)

For HEP (beam collision mode of the LHC)



γγ Higgs factory in the LHC tunnel: 63 GeV "LEP" + γ beams from the LHC)

Top level parameters		
Collision energy (center of mass)	GeV	126
Luminosity (per IP)	10 ³⁴ cm ⁻² s ⁻¹	0.5
definition of luminosity		γγ > 125 GeV
Luminosity for e-e-	10 ³⁴ cm ⁻² s ⁻¹	3.2
No. of IP		1
No. of Higgs per year (per IP)		10,000
Circumference	km	6.28
P(wall)	MW	80
Polarization e-		80%
Polarization γ		90% (lum. peak)

Table 2: HFiTT accelerator and beam parameters.

63 GeV at "LEP" with respect to the 80 GeV beam for HFiTT (higher frequency photons) both to be compared to ~130 GeV beam for TLEP-Significant reduction of the synchrotron radiation ~ E^4

For HEP (single beam mode of the LHC)



COULD OUTPERFORM THE PRESENT PROJECTS BY SEVERAL ORDES OF MAGNITUDE

Applications beyond HEP (single beam mode of the LHC)



- 1. Medical applications
- 2. Tomography
- 3. Nondestructive assay and segregation of nuclear wastes
- 4. Photo transmutation of nuclear waste using resonant (γ,n) transitions
- 5. γ-ray laser?
- 6. Nuclear fusion and fission



7. Wakefield for plasma acceleration



γ-ray surgery of nuclear waste



Example: (γn) transmutation of a nuclear waste 126Sn with a high life-time of 100 00 years into 125Sn with a life-time 9.64 days

... γ -transmutation not taken (so far) seriously because of lack of high-intensity mono-energetic γ -sources in the range 5-20 MeV...

...no longer the case for the LHC gamma beams! Worth dedicated feasibility studies...

γA and nA cross-sections



Giant Dipole Resonance (GDR) cross section is large – comparable to the neutron cross sections in the MeV region

Facts – nuclear waste

With 145 operating reactors (2001) with a total power of 125 GW, the resulting electrical energy generation in Europe is of about 850 TWh per year and represents ~35% of the total electricity consumption of the European Union.

Most of the hazard from the spent fuel stems from only a few chemical elements - *plutonium, neptunium, americium, curium,* and some *long-lived fission products such as e.g. iodine and technetium* at concentration levels of grams per ton.

Approximately 2500 tons of spent fuel are produced annually in the EU, containing about 25 tons of plutonium and 3.5 tons of the "minor actinides" neptunium, americium, and curium and 3 tons of long-lived fission products (the long term > 100 years radiotoxicity is dominated by the actinides).

Feasibility estimate

FIGURE2. PRINCIPLE ACTINIDES RADIOTOXICITY

Nuclide	Amount (g/ton)
Np236	5.3E-04
Np237	6.5E+02
Pu238	2.3E+02
Pu239	5.9E+03
Pu240	2.6E+03
Pu241	6.8E+02
Pu242	6.0E+02
Pu244	4.2E-02
Am241	7.7E+02
Am242m	2.5E+00
Am243	1.4E+02
Cm242	5.9E-03
Cm243	4.3E-01
Cm244	3.1E+01
Cm245	2.3E+00
Cm246	3.2E-01
Cm247	3.7E-03
Cm248	2.4E-04

Transuranics in spent fuel after 15 years decay.



For $N_{\gamma} \sim 10^{20} \text{ 1/s}$, 100% transmutation efficiency ~ 1 ton of radioactive nuclides can be transmuted over one year (of the order 0.2 of the yearly EU toxic waste production – worth studies).

Transmutation efficiency



Dazhi LI , Kazuo IMASAKI , Ken HORIKAWA , Shuji MIYAMOTO , Sho AMANO & Takayasu MOCHIZUKI(2009) SUBARU facility, Journal of Nuclear Science and Technology, 46:8, 831-835, DOI 10.1080/18811248.2007.9711592.

Issues which must be investigated:

1.Possibility of a significant increase of the transmutation efficiency by using neutrons produced in the GDR process

2. Number of light-ion collision points (each ion needs to be used ~2000 times over one LHC turn to convert 10^{20} eV photons to gammas – for very large Z can be done over ~10 cm long collision zone – larger for smaller Z ions)

3. The power consumption of the LHC gamma-beam production operation mode to produce the flux of 10^{20} photons at 10 MeV is ~ 170 MW (to be compared e.g. with the TLEP proposal which needs 300 MW). Note, each ion would loose of the order of 0.3% of its energy in one LHC ring turn.

Issues which must be investigated:

4. The GDR process competes with the conversion process – only ~10% of photons undergo nuclear absorption, remaining ones mainly convert into electron-positron pair

5. Need an energy recovery scheme to use the 17 MW of dissipated heat to produce electrical power for the LHC cavities and a cost recovery scheme e.g. positron beams for CLIC.

6. How to build the transmutation station(dissipated power ~17 MW)



High intensity electron and positron beams – cost recovery

Conclusions:

1. There are no obvious showstoppers to produce, accelerate and store high energy beams of partially stripped ions in the existing HEP laboratories equipped with ion accelerators.

2. Such beams can be used as, by far, more effective photon frequency boosters than the electron beams (the possible efficiency gain of ~ a factor of up to 10^7)

3. For very high energy beams, (e.g. LHC beams) the high intensity gamma beam can be generated using conventional lasers and cavities.

4. At lower energy similar energy range can be attained using the X-ray FELs.

5. Production of intense gamma beams could open new research and application domains.

6. Using such beams for the transmutation of nuclear waste looks sufficiently promising to continue detailed quantitative studies.

extra transparencies_

PIE*@LHC:

Pb⁸¹⁺(1s)-p example

- <u>CM energy (ep collisions)</u> = 205 GeV
- <u>β at IP</u> = 0.5 m
- <u>Transverse normalized emittance</u> = $1.5 \mu m$
- Number of ions/bunch = 10^8
- Number of protons/bunch = 4×10^9
- <u>Number of bunches</u> = 608
- Luminosity = $0.4 \times 10^{29} \text{ cm}^{-2} \text{ s}^{-1}$

Technological challenges

Need optical cavities for (100 nm - 400 nm) wavelength. Multilayer mirrors using high refraction index materials (AL2O3, HFO2, ZRO2) and low refraction index material (SiO2) deposited on silicium or sapphire. The roughness must be controlled to better than 1 angstrom. Very recent technological progress: Mackowski- Lyon, Jena (Germany)*





Fig.3. Coupling of γ ray to nuclear giant resonance of ¹²⁹I. Crosssections of gamma ray photon for the typical interactions target is indicated. Curve a shows pair and creation, b corresponds to Compton by target scatter atom electron and c corresponds to giant resonance and d corresponds phototo electron effect. Curve e γ ray photon by denotes Compton scattering.



Figure 1: Layout of the CLIC positron source. Red box show the part which concerns the positron production and capture (zoomed in Figure 2).

Before collisionAfter collisionElectron $p^i = (\mathcal{E}_e/c, \vec{p})$ $p'^i = (\mathcal{E}'_e/c, \vec{p'})$ Photon $\hbar k^i = (\hbar \omega/c, \hbar \vec{k})$ $\hbar k'^i = (\hbar \omega'/c, \hbar \vec{k'})$

where \mathcal{E}_e and \vec{p} denote the energy and momentum of the electron and \vec{k} is the wave number of the photon. Using conservation of four-momentum and for low-intensity electromagnetic waves, we can express the energy of the outgoing photon as:

$$E_{\gamma} \equiv \hbar \omega' = \frac{\hbar \omega (1 - \beta \cos \theta_{\rm i})}{1 - \beta \cos \theta_{\rm f} + \frac{\hbar \omega}{g_{\rm e}} (1 - \cos \theta_{\rm ph})},\tag{1}$$

where \hbar is Planck's constant, $\hbar\omega$ is the energy of the incoming photon, E_{γ} ($\hbar\omega'$) is the energy of the outgoing photon, θ_i and θ_f are the angles between the momentum of the incoming and outgoing photons and that of the incident electron, ($\cos \theta_i = \hat{p} \cdot \hat{k}$) and $\cos \theta_f = \hat{p} \cdot \hat{k}'$), and θ_{ph} is the angle between two photons ($\cos \theta_{ph} = \hat{k} \cdot \hat{k}'$).

For a collision between relativistic electrons and low-energy photons, the energy of the scattered photons is peaked along the direction of the incident electrons. The scattered photon energy has a maximum value in a head-on collision with $\theta_i = \pi$ and $\theta_f = 0$,

$$\hbar\omega' = \frac{\gamma^2 (1+\beta)^2}{1+R_0} \hbar\omega,\tag{2}$$

where γ is the Lorentz boost factor, $\beta = \frac{v}{c}$, and $R_0 = 2\gamma^2(1+\beta)\hbar\omega/\mathcal{E}_e$ is the recoil term. When the recoil is small $(R_0 \ll 1)$, the maximum scattered photon energy is approximately, $\hbar\omega'_{max} \approx \gamma^2(1+\beta)^2\hbar\omega \approx 4\gamma^2\hbar\omega$, where the second approximation holds for ultra-relativistic electrons with $\gamma \gg 1$. It is clear that the energy boost factor comes from two consequent Lorentz transformations between the laboratory frame and the electron-rest frame; each gives a boost of $\gamma(1+\beta)$.

Medium lived elements

$_{55}^{137}Cs (\tau = 30.1 y) + n \Rightarrow$	$_{55}^{138}Cs \xrightarrow{\beta^{-}(33.2 m)} \sim _{56}^{138}Ba (stable)$
$^{134}_{55}Cs~(\tau=2.06~y)~+~n~\Rightarrow$	$^{135}_{55}Cs\ (\tau = 2.6 \times 10^6\ y)$
$^{90}_{38}Sr(\tau = 29.1 \text{ y}) + n \Rightarrow$	$\frac{91}{38}$ Sr $\stackrel{\beta^{-}(6.63\ h)}{\sim}$ $\frac{91}{39}$ Y $\stackrel{\beta^{-}(58.51\ d)}{\sim}$ $\frac{91}{40}$ Zr (stable)
$^{90}_{39}$ Y ($\tau = 64.1$ h) + n \Rightarrow	$\frac{91}{39}Y \xrightarrow{\beta^{*}(58.51 d)} \frac{91}{40} Zr (stable)$

Long lived elements

$$\begin{array}{l} {}^{135}_{55}Cs\,(\tau=2.6\times10^{6}\,y)\ +\ n\ \Rightarrow\ {}^{136}_{55}Cs\, {}^{\beta^{*}(13.16\,d)}\ {}^{136}_{56}Ba\,(stable) \\ {}^{129}_{53}I\,(\tau=6.6\times10^{7}\,y)\ +\ n\ \Rightarrow\ {}^{130}_{53}I\ {}^{\beta^{*}(12.36\,h)}\ {}^{130}_{54}Xe\,(stable) \\ {}^{126}_{50}Sn\,(\tau=1.0\times10^{5}\,y)\ +\ n\ \Rightarrow\ {}^{127}_{50}Sn\, {}^{\beta^{*}(12.4\,m)}\ {}^{127}_{51}Sb\ {}^{\beta^{*}(3.85\,d)}\ {}^{127}_{52}Te\ {}^{\beta^{*}(9.35\,h)}\ {}^{127}_{53}I\,(si) \\ {}^{99}_{43}Tc\,(\tau=2.1\times10^{5}\,y)\ +\ n\ \Rightarrow\ {}^{100}_{43}Tc\ {}^{\beta^{*}(15.8\,s)}\ {}^{100}_{44}Ru\,(stable) \\ {}^{93}_{40}Zr\,(\tau=1.5\times10^{6}\,y)\ +\ n\ \Rightarrow\ {}^{94}_{40}Zr\,(stable) \\ {}^{79}_{34}Se\,(\tau=6.5\times10^{4}\,y)\ +\ n\ \Rightarrow\ {}^{80}_{34}Se\,(stable) \\ \end{array}$$