Updated SM predictions of $B_q \to \ell^+\ell^-$ at NLO EW and NNLO QCD

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in collaboration with Martin Gorbahn, Thomas Hermann
Mikolaj Misiak, Emmanuel Stamou and Matthias Steinhauser

– arXiv:1311.0903 –

Outline

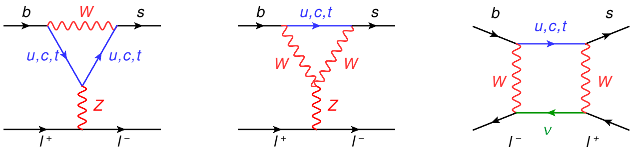
I) NNLO QCD corrections to to C_A

II) Updated $Br(B_q \to \ell^+\ell^-)$ @ NNLO QCD + NLO EW

!!! Throughout only Standard Model (=SM) is considered

Motivation to study $B_q \to \ell^+\ell^-$

- test of the SM at loop-level (FCNC decay) $\Rightarrow$ loop-suppressed



$\Rightarrow$ @ LO in QED, hadronic uncertainty only from $B_{d,s}$ decay constant

- in addition helicity suppressed

  $\Rightarrow$ sensitive to non-SM (pseudo-) scalar interactions

- important B-decay @ LHCb, CMS & ATLAS $\Rightarrow$ first measurements from 2013:

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$
$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}$$

LHCb (3/fb) + CMS (25/fb)

[LHCb arXiv:1307.5024]
[CMS arXiv:1307.5025]

- exp. prospects for $B_s \to \mu^+\mu^-$:

  @ LHCb with 50 fb^{-1} : $\sim 0.15 \times 10^{-9} = 5\%$ error of SM (only stat. err) [LHCb arXiv:1208.3355]

  @ CMS with 100 fb^{-1} : 15% error of SM [Kai-Feng Chen, KEK Flavor Factory WS, 2014]

Higher order QCD and EW corrections to $B_q \to \ell^+\ell^-$

B-Meson decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

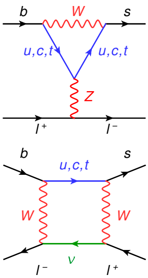
electroweak IA	$\gg$	ext. mom'a in B restframe	$\gg$	QCD-bound state effects
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV		$\Lambda_{\text{QCD}} \approx 0.5$ GeV

B-Meson decays are a Multi-scale problem . . .

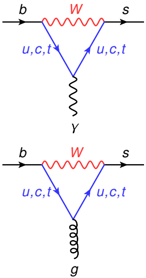
. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe	OPE = expansion in
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV	$M_B^2/M_W^2 \sim 0.004$

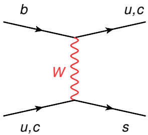
$b \to s\,\ell^+\ell^-$



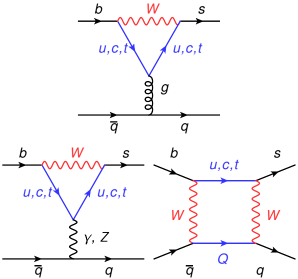
$b \to s\gamma$ & $b \to sg$



$b \to s\,(u\bar{u},\, c\bar{c})$



$b \to s\,q\bar{q}\,(q = u, d, s, c, b)$

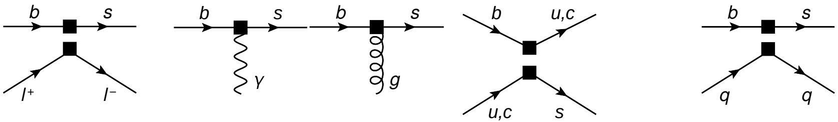


B-Meson decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in *B* restframe	The extension of Fermi's theory of weak interactions to *B* decays
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV		$M_B \approx 5$ GeV	

semi-leptonic | electro- & chromo-mgn | charged current | QCD & QED -penguin



C_i = Wilson coefficients: contains short-dist. pmr's (heavy masses $M_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to next-to-next-to-leading order in QCD

$\mathcal{O}_i$ = higher-dim. operators: flavour-changing coupling of light quarks (+ gluons and photons)

$$\mathcal{L}_{\text{eff}}^{\Delta B=1} \propto V_{CKM} \left[\sum_{V,A} C_i \, \mathcal{O}_i^{\ell\bar{\ell}} + \sum_{7\gamma,\, 8g} C_i \, \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \right] + \mathcal{O}\left(\frac{M_B^2}{M_W^2} \right)$$

“Matching” = Determination of Wilson coefficients

⇒ C_i are determined by requiring equality of full theory (=SM) and effective theory (=EFT) amplitudes order by order in expansion in couplings G_F, α_s (QCD) and α_e (QED)

relevant for $B_s \to \ell^+\ell^-$

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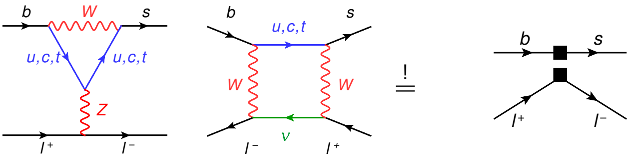
relevant for $B_s \to \ell^+ \ell^-$

$G_F \frac{\alpha_e}{s_W^2}$

LO



$\to$ $C_A^{(00)}$

$\sim$ $Y^{(0)}$

[Inami/Lim Prog.Theor.Phys. 65 (1981) 297]

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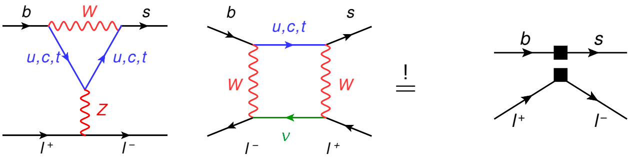


$\to C_A^{(00)}$

$\sim Y^{(0)}$

[Inami/Lim Prog.Theor.Phys. 65 (1981) 297]

EFT is “simple” at LO EW (& all orders in QCD), because only one operator

$$\mathcal{O}_A^{\ell\bar\ell} = [\bar s \gamma_\mu \gamma_5 b][\bar\ell \gamma^\mu \gamma_5 \ell] \qquad \left(\text{usually known as } \mathcal{O}_{10}^{\ell\bar\ell} : C_A = -\frac{C_{10}}{2}\right)$$

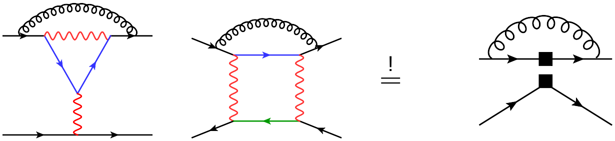
contributes to $B_q \to \ell^+\ell^-$ ⇒ $\mathcal{O}_A^{\ell\bar\ell}$ is a conserved current under QCD (→ no “Running”)

!!! (pseudo-) scalar $\mathcal{O}_{S(P)}^{\ell\bar\ell} \propto [\bar s \gamma_5 b][\bar\ell 1 (\gamma_5)\ell]$ from Higgs-pinguins are suppressed by $M_{B_s}^2/M_W^2$

Adding QCD corrections . . .

$$G_F \frac{\alpha_e}{s_W^2} \alpha_s$$

NLO QCD

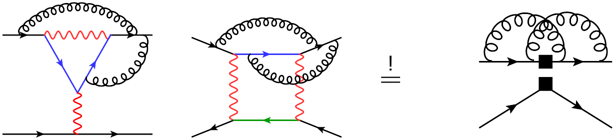


$$\to C_A^{(10)}$$

[Buchalla/Buras NPB 398 (1993) 285, NPB 400 (1993) 225, Misiak/Urban hep-ph/9901278]

$$G_F \frac{\alpha_e}{s_W^2} \alpha_s^2$$

NNLO QCD



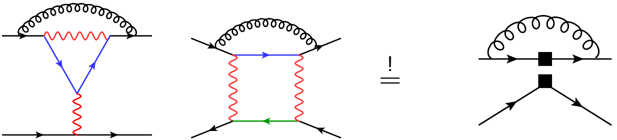
$$\to C_A^{(20)}$$

[Hermann/Misiak/Steinhauser arXiv:1311.1347]

Adding QCD corrections . . .

$G_F \frac{\alpha_e}{s_W^2} \alpha_s$

NLO QCD



$\rightarrow C_A^{(10)}$

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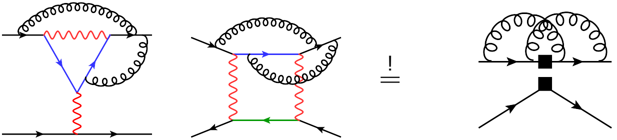
$G_F \frac{\alpha_e}{s_W^2} \alpha_s^2$

NNLO QCD



$\rightarrow C_A^{(20)}$

[Hermann/Misiak/Steinhauser arXiv:1311.1347]

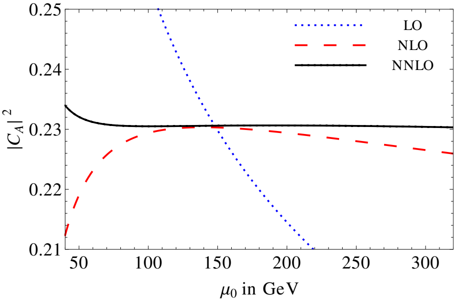
$$C_A = C_A^{(00)} + \frac{\alpha_s}{4\pi} C_A^{(10)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(20)}$$

implicite – via $m_t(\mu_0)$ (= $\overline{\text{MS}}$ QCD) – and explicite μ_0 dependence

$m_t(m_t) = 163.5$ GeV

$m_t(\mu_0 = 50 \text{ GeV}) = 180.8$ GeV

$m_t(\mu_0 = 300 \text{ GeV}) = 156.2$ GeV



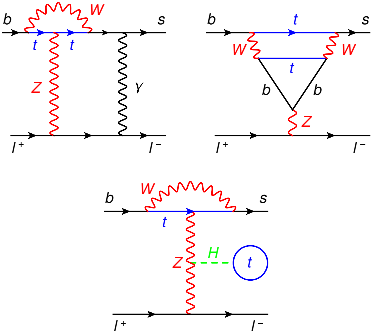
$\Rightarrow$ NNLO QCD crrs. reduce μ_0-dep. from

1.8% at NLO → 0.2% at NNLO on $Br \sim |C_A|^2$

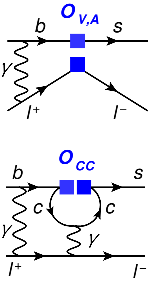
Adding NLO EW corrections . . .

$$G_F \frac{\alpha_e}{s_W^2} \alpha_e$$

NLO EW



$$\stackrel{!}{=}$$



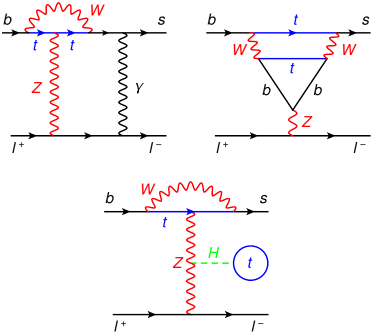
$$\rightarrow C_A^{(01)}$$

[Bobeth/Gorbahn/Stamou arXiv:1311.1348]

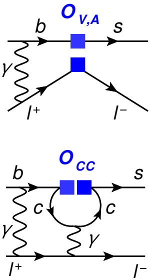
Adding NLO EW corrections . . .

$$G_F \frac{\alpha_e}{s_W^2} \alpha_e$$

NLO EW



$\stackrel{!}{=}$



$\rightarrow \quad C_A^{(01)}$

[Bobeth/Gorbahn/Stamou arXiv:1311.1348]

See talk by Martin Gorbahn

Including QED-Log's + combination with NNLO QCD

⇒ Log-enhanced QED corrections known

[Bobeth/Gambino/Gorbahn/Haisch hep-ph/0312090, Huber/Lunghi/Misiak/Wyler hep-ph/0512066]

Choose OS-2 as default scheme

Solution of RGE

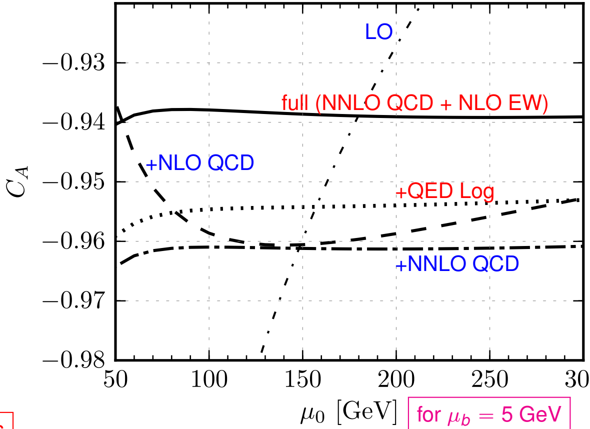
$$C_A(\mu_b) = \sum_i \left[U(\mu_b, \mu_0)\right]_{A,i} C_i(\mu_0)$$

→ $U(\mu_b, \mu_0)$ = evolution operator

→ $C_i(\mu_0)$ = Wilson coefficients at high scale

!!! $C_A(\mu_b)$ μ_0–independent

⇒ full EW corr's reduce *Br* by 4% compared to NNLO QCD



for $\mu_b = 5$ GeV

Estimate of higher order uncertainties

1) μ_0 variation between $[m_t/2, 2m_t]$ → about 0.2% (QCD) + 0.2% (EW)

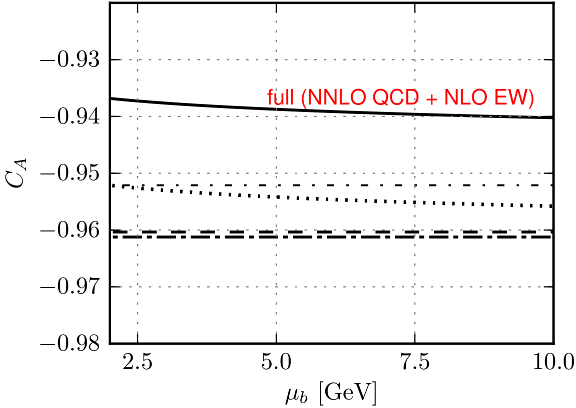
2) additional EW scheme dep. from diff. of OS-2 and HY scheme → about 0.2%

on $|C_A(\mu_b)|^2$

Residual μ_b dependence of C_A ...

... gives measure of uncertainties due to lacking virtual QED corrections to matrix elements of operators: $\mathcal{O}_{A,V,cc}$

0.3% residual μ_b-dependence on $|C_A(\mu_b)|^2$



Updated $Br(B_q \to \ell^+\ell^-)$ at NNLO QCD + NLO EW

Branching ratio $B_q \to \ell^+\ell^-$

Up to now discussed (NNLO QCD + NLO EW) corrections to $C_A(\mu_b)$
$\Rightarrow$ need to calculate (matrix elements)^2 to obtain Br including NLO QED

$$Br \;\sim\; \left|\langle \ell\bar{\ell}|\mathcal{H}_{\rm eff}|\bar{B}\rangle\right|^2 \;\sim\; \left|C_A(\mu_b)\right|^2 \langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}\rangle^2 + \mathcal{O}(\alpha_e)$$

1) Hadronic matrix element (to all orders in QCD and LO in QED):

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\overline{B}_q(p_B)\rangle \equiv i\, f_{B_q}\, p_B^\mu$$

$$f_{B_d} = (190.5 \pm 4.2)\,\text{MeV}$$
$$f_{B_s} = (227.7 \pm 4.5)\,\text{MeV}$$

[FLAG (lattice average) 2013]

2) Account for B_s-mixing: fully time-integrated CP-averaged Br [De Bruyn et al. arXiv:1204.1737]

Branching ratio $B_q \to \ell^+\ell^-$

Up to now discussed (NNLO QCD + NLO EW) corrections to $C_A(\mu_b)$
$\Rightarrow$ need to calculate (matrix elements)^2 to obtain *Br* including NLO QED

$$Br \sim \left|\langle \ell\bar{\ell}|\mathcal{H}_{\rm eff}|\bar{B}\rangle\right|^2 \sim \left|C_A(\mu_b)\right|^2 \langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}\rangle^2 + \mathcal{O}(\alpha_e)$$

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!!! for consistency, should include $\mathcal{O}(\alpha_e)$ corrections:

A) bremsstrahlung

can be sizeable depending on photon energy cuts in experiment

B) virtual corrections

expected to be small $\alpha_e/\pi \sim 1/400 \sim 0.25\%$, but non-perturbative $\rightarrow$ no theoretical framework

Standard Model predictions @ (NLO EW + NNLO QCD)

in OS-2 Scheme

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}$$
$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}$$

$\overline{\mathcal{B}}(B_s \to e^+e^-) = (8.54 \pm 0.55) \times 10^{-14}$, $\overline{\mathcal{B}}(B_s \to \tau^+\tau^-) = (7.73 \pm 0.49) \times 10^{-7}$,

$\overline{\mathcal{B}}(B_d \to e^+e^-) = (2.48 \pm 0.21) \times 10^{-15}$, $\overline{\mathcal{B}}(B_d \to \tau^+\tau^-) = (2.22 \pm 0.19) \times 10^{-8}$

[CB/Gorbahn/Hermann/Misiak/Stamou/Steinhauser arXiv:1311.0903]

Error budget	f_{B_q}	CKM	τ_H^q	M_t	α_s	non-param.	other param.	$\sum$
$\overline{\mathcal{B}}_{s\mu}$	4.0%	4.3%	1.3%	1.6%	0.1%	1.5%	$< 0.1\%$	6.4%
$\overline{\mathcal{B}}_{d\mu}$	4.5%	6.9%	0.5%	1.6%	0.1%	1.5%	$< 0.1\%$	8.5%

⇒ CKM: use here $|V_{cb}|_{\text{incl}} = 0.0424 \pm 0.0009$ [Gambino/Schwanda arXiv:1307.4551]

rather than $|V_{cb}|_{\text{excl}} = 0.0396 \pm 0.0009$ as used as input by [UTfit post-EPS13]

⇒ non-parametric uncertainties

- 0.3% from $\mathcal{O}(\alpha_{em})$ corrections from $\mu_b \in [m_b/2, 2m_b]$
- $2 \times 0.2\%$ from $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s\alpha_{em})$ matching corrections from $\mu_0 \in [m_t/2, 2m_t]$
- 0.3% from top-mass conversion from on-shell to $\overline{\text{MS}}$ scheme
- 0.5% further uncertainties (power corrections $\mathcal{O}(m_b^2/M_W^2), \ldots$)

Another precise SM prediction @ (NLO EW + NNLO QCD)

Ratio of $\overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$ and B_q-mass difference ΔM_{B_q}

$$\kappa_{q\ell} \equiv \frac{\overline{\mathcal{B}}(B_q \to \ell^+\ell^-)}{\Delta M_{B_q}} \frac{\Gamma_H^q}{(G_F M_W m_\ell)^2 \beta_{q\ell}} \overset{\text{(SM)}}{=} \frac{3|C_A(\mu_b)|^2}{\pi^3 C_{LL}(\mu_b) B_{B_q}(\mu_b)}$$

NLO EW crr. to ΔM_q [Gambino/Kwiatkowski/Pott hep-ph/9810400]

- decay constant f_{B_q} and CKM cancel in ratio
- BUT depends on

  $\Rightarrow B_{B_q}(\mu_b)$ bag factor of $\Delta B = 2$ hadronic matrix element
- depends on perturbatively calculable

  $\Rightarrow C_{LL}(\mu_b)$ Wilson coefficient of $\Delta B = 2$ operator

SM prediciton with comparable theory uncertainty to $\overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$

$\Rightarrow$ dominated by uncertainties of lattice predictions of $B_{B_q}(\mu_b)$

$$\kappa_{s\ell} = (1.26 \pm 0.07)\%$$

$$\kappa_{d\ell} = (1.32 \pm 0.12)\%$$

Conclusion

SM prediction of $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ has $\lesssim$ 7% theory uncertainty

$\Rightarrow$ mainly f_{B_s} (4.0%), V_{cb} (4.3%), non-parametric (1.5%)

included are

- NNLO QCD crrs. reduce μ_0-dep. of $m_t(\mu_0)$ — from 1.8% at NLO $\to$ 0.2% at NNLO
- NLO EW crrs. enhanced by: $\dfrac{1}{\sin^2\theta_W}, \quad \dfrac{m_t^2}{M_W^2}, \quad \ln^2 \dfrac{m_b^2}{M_W^2}$

  reduce LO-scheme dependence — from $\gtrsim$ 8% $\to$ 0.6% $\lesssim$ at NLO
- the size of NLO EW crrs. is about — $\sim$ (3 . . . 5)% (depending on μ_0)
- photon bremsstrahlung corrections due to PHOTOS

Lacking: Virtual QED corrections to matrix elements $\sim \alpha_e/\pi$

$\Rightarrow$ expected to be small from μ_b-variation

$\Rightarrow$ HOWEVER, nonperturbative physics involved below scales $\mu_b \sim m_b$

Backup Slides

Impact of NLO EW crrs. at μ_0 . . . neglecting operator mixing

$\Rightarrow$ α_e always $\overline{\text{MS}}$ and m_t always $\overline{\text{MS}}$ w.r.t QCD

$\Rightarrow$ 3 different EW renormalization schemes:

- OS) masses and ($s_W^2 = 1 - M_W^2/M_Z^2$) on-shell renormalized
- $\overline{\text{MS}}$) masses and s_W^2 in minimal subtraction
- HY) masses on-shell but s_W^2 in minimal subtraction

$\Rightarrow$ 2 normalizations of $\mathcal{L}_{\text{eff}}$ [Misiak arXiv:1112.5978]

1) $$\frac{4G_F}{\sqrt{2}}\frac{\alpha_e}{s_W^2}\left[C_A^{(00)} + \frac{\alpha_e(\mu_0)}{4\pi}C_A^{(01)}(\mu_0)\right]$$

2) $$\frac{G_F^2 M_W^2}{\pi^2}\left[C_A^{(00)} + \frac{\alpha_e(\mu_0)}{4\pi}C_A^{(01)}(\mu_0)\right]$$

Impact of NLO EW crrs. at μ_0 ... neglecting operator mixing

$\Rightarrow$ α_e always $\overline{\text{MS}}$ and m_t always $\overline{\text{MS}}$ w.r.t QCD

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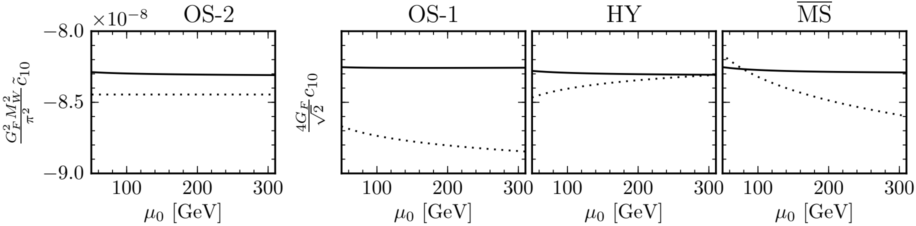
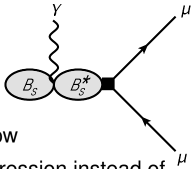


scheme dependence @ LO: $[-8.9\ , -8.2\ ] \to \pm 8\%$ at *Br*

reduces @ NLO: $[-8.31, -8.25] \to \pm 0.8\%$ at *Br*

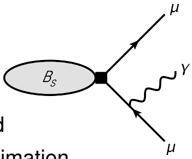
LO = dotted
NLO = solid

Bremsstrahlung: initial and final state



Initial State Radiation

- tiny in signal window
- phase-space suppression instead of helicity suppression
- can be avoided with cuts



Final State Radiation

- helicity suppressed
- soft-photon approximation
- extrapolated from signal window over all $m^2_{\mu^+\mu^-}$ via PHOTOS by LHCb and CMS

ISR

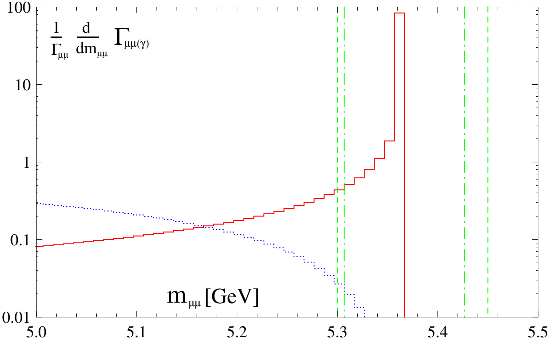
[Aditya/Healey/Petrov arXiv:1212.4166]

FSR

[Buras et al. arXiv:1208.0934]

experimental signal windows (LHCb, CMS)

[LHCb arXiv:1307.5024, CMS arXiv:1307.5025]



What should theorists include in predictions of *Br*?

- no need to include ISR → currently removed by cuts

  !!! it should be counted as background in MC

  ??? should not be included in PHOTOS in experimental analysis

  (especially when experimental accuracy increases)

- FSR should be included due to cuts in experimental analysis

  BUT already accounted for by LHCb and CMS using PHOTOS

  (extrapolation along red curve to zero on previous slide)

  ⇒ this corresponds to the limit where:

  photon-inclusive *Br* = non-radiative *Br*

  [Buras/Girrbach/Guadagnoli/Isidori arXiv:1208.0934]

- virtual corrections → requires nonperturbative method

  ⇒ NOT included → assign 0.3% uncertainty from μ_b-variation