SCATTERING RATES AND SPECTATOR EFFECTS IN LEPTOGENESIS

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In recent years tremendous progress was made towards a more precise understanding of the Leptogenesis mechanism. Some of the recent developments are reviewed, like new results for scattering rates and effects from partially equilibrated spectator processes, and their impact on the final baryon asymmetry is discussed.

1 Introduction

Baryogenesis, i.e. the generation of a baryon anti-baryon asymmetry in the early universe, is one of the most important open problems in particle physics, and a strong motivation for physics beyond the standard model. One popular solution is provided by the Leptogenesis [1] mechanism, where a lepton asymmetry is generated from the CP-violating out of equilibrium decays of heavy right-handed neutrinos, and partially converted into a baryon asymmetry through electroweak sphaleron processes.

An attractive feature of Leptogenesis is that it requires only a very small extension of the standard model (SM): adding two right-handed neutrinos with Majorana masses and Yukawa couplings to the lepton doublets is sufficient, although three right-handed neutrinos are usually considered. At the same time these right-handed neutrinos provide a natural explanation for the smallness of the neutrino masses through the see-saw mechanism. After electroweak symmetry breaking, a mass of order $m_\nu = \frac{Y^2 v^2}{M}$ is generated for the left-handed SM neutrinos, where $M \gg v$ is the mass scale of the right-handed neutrinos and $v = 174$ GeV is the vacuum expectation value of the SM Higgs field. Sub-electronvolt (eV) neutrino masses can therefore be explained with natural values for the Yukawa couplings Y provided that $M \sim 10^{7-14}$ GeV.

2 Basics of Leptogenesis

The interactions relevant for Leptogenesis are described by the following Lagrangian,

$$\mathcal{L} = -\tfrac{1}{2} M_i \bar{N}_i N_i - Y_{i\alpha} \bar{N}_i \tilde{H}^\dagger \ell_\alpha - h_{\alpha\beta} \bar{e}_{\mathrm{R}\alpha} H^\dagger \ell_\beta + \mathrm{h.c.} \qquad (1)$$

where M_i are the Majorana masses of the right-handed neutrinos N_i, $Y_{i\alpha}$ and $h_{\alpha\beta}$ are the neutral and charged Yukawa couplings, respectively, and H, ℓ_α and $e_{R\beta}$ are the Higgs and lepton doublets and the charged lepton singlets of the SM, with lepton flavour index α.

Lepton number (and thus also $B-L$) is broken by the Majorana masses, and CP violation can be provided by complex phases in the neutral Yukawa matrix Y, such that two of the three Sakharov conditions are already satisfied. The third condition, departure from thermal equilibrium, can be realised if the right-handed neutrinos depart from thermal equilibrium when the temperature of the universe, T, drops below the their masses. For this the decay rate $\Gamma(N_1 \to \ell H)$ should not be much faster than the expansion rate of the universe $H(T=M_1)$.

It is intriguing that Γ/H depends on the same parameter combination Y^2/M as the light neutrino masses, and that the condition $\Gamma/H \sim 1$ is compatible with m_ν of order meV, in agreement with the range of neutrino masses required to explain neutrino oscillation data.

Obtaining a prediction for the final baryon asymmetry in terms of the Lagrangian parameters introduced above is non-trivial. In the simplest case, when finite temperature effects can be neglected and the masses of the right-handed neutrinos are hierarchical, one still has to solve Boltzmann equations for the abundance Y_{N_1} of the lightest right-handed neutrino and for the lepton asymmetries Y_ℓ:

$$\frac{d}{dz}Y_{N1} = -\bar{\mathcal{C}}_{N1}\left(Y_{N1} - Y_{N1}^{\rm eq}\right), \tag{2a}$$

$$\frac{d}{dz}Y_\ell = \frac{1}{2}\bar{S}\left(Y_{N1} - Y_{N1}^{\rm eq}\right) - \bar{W}\left(Y_\ell + \frac{1}{2}Y_H\right). \tag{2b}$$

Here $\bar{S}$ is the source term for the CP asymmetry coming from loop corrections to the decay $N_1 \to \ell H$, $\bar{\mathcal{C}}_{N_1}$ is the tree level decay rate and $\bar{W}$, the so called washout term, corresponds to the inverse decay $\ell H \to N_1$. The time variable is conveniently replaced by $z = M_1/T$, with larger z corresponding to lower temperatures, i.e. later times.

Washout is important when the decay parameter $K_1 = \Gamma_1/H \equiv \bar{\mathcal{C}}_{N_1}(z=1)$ is larger than one. The advantage of this so called *strong washout regime* is that the final asymmetry is determined at a late time and becomes independent from both initial asymmetries and from the initial N_1 distribution.

Two things complicate the solution of Eqns. (2). First, since Y_ℓ is in general a matrix in lepton flavour space, interactions mediated by the charged lepton Yukawas $h_{\alpha\beta}$ will have to be included. Their main effect is to project the asymmetry onto charged lepton mass eigenstates with different effective washout rates, such that the final asymmetry is modified. Since the tau and muon Yukawa interactions reach thermal equilibrium at temperatures where the Leptogenesis mechanism might still be active, a precise knowledge of these rates is necessary to properly incorporate this effect.

The second complication arises from the presence of a Higgs asymmetry Y_H in (2). This asymmetry has to be related to Y_ℓ in order for the equation to be solvable, however the exact relation depends on the temperature range since "Higgs-number" is also violated by other interactions like the top and bottom Yukawas. Furthermore once electroweak sphalerons reach thermal equilibrium Y_ℓ should be replaced by Y_{B-L}. The net effect in both cases is that the washout and thus the final asymmetry is modified. Effects on the final asymmetry due to $B-L$ preserving interactions have been named *spectator effects* in the literature [2,3].

In the following sections some recent results in the calculation of flavour equilibration rates and on the treatment of spectator effects will be presented.

3 Flavour Equilibration Rates

In the early universe, all SM particles can be considered approximately massless. Therefore a process of the form $H \leftrightarrow \ell e_R$, which would be leading order in the charged lepton Yukawa h, is kinematically forbidden.

Figure 1 – Examples for the types of diagrams that contribute to the lepton flavour equilibration rate. Double lines denote resummed propagators.

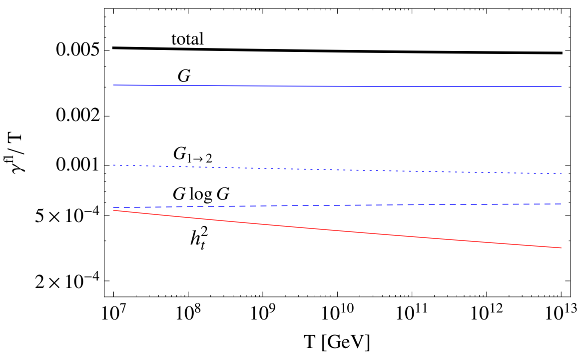

Figure 2 – Flavour equilibration rate γ^{fl}/T as a function of the temperature. Shown are the individual contributions as well as the full result, which is dominated by the $2 \to 2$ scattering involving gauge bosons.

If one allows the absorption or emission of an additional gauge boson from the thermal plasma, the resulting $2 \to 2$ scattering processes are kinematically accessible, such that processes sensitive to the coupling h can appear at order $g^2|h|^2$. At the same time the tree level process $H \leftrightarrow \ell e_R$ also become allowed if thermal masses are included in the calculation, again leading to a contribution of order $g^2|h|^2$. Furthermore it was recently pointed out[4] that the resummation of multiple emissions and absorptions of soft gauge bosons in the thermal plasma also leads to a contribution which is parametrically of order $g^2|h|^2$.

In [5] we have presented the first calculation of the flavour equilibration rate γ^{fl} for charged lepton Yukawa mediated interactions, including all effects at order $g^2|h|^2$. For performing the calculation, we have made use of the 2PI formalism, i.e. only self energy diagrams which are two particle irreducible are included in the calculation.

Contributions that would normally arise from bubble insertions in the one loop diagrams are instead included by calculating the one loop self energy with resummed propagators for the lepton and Higgs fields. This has the advantage that both the $2 \to 2$ scatterings and the effects of thermal masses and thermal widths are included in the same diagram. Moreover the divergencies in the $2 \to 2$ scatterings, which appear when a massless lepton with vanishing momentum is exchanged in the t-channel, are automatically regulated by the thermal masses included in the propagators, without the need for an ad-hoc prescription.

There are also proper 2PI two loop diagrams, which correspond to interference effects of different scattering processes. These were calculated using tree level finite temperature propagators, since corrections to the propagators would lead to contributions of order $g^4|h|^2$ which are of higher order. Examples for the two types of diagrams calculated in this work are presented in Fig. 1. The regulated divergence leads to a logarithmic enhanced contribution which is formally of order $g^2 \log g$. We have explicitly expanded the resummed lepton propagator to isolate the divergence and determine its coefficient.

Our final result for the charged lepton flavour equilibration rate is shown in Fig. 2. The rate is dominated by the $2 \to 2$ processes involving gauge bosons, while the non-perturbative $1 \to 2$ processes as well as the log enhanced contributions and those from scatterings involving top

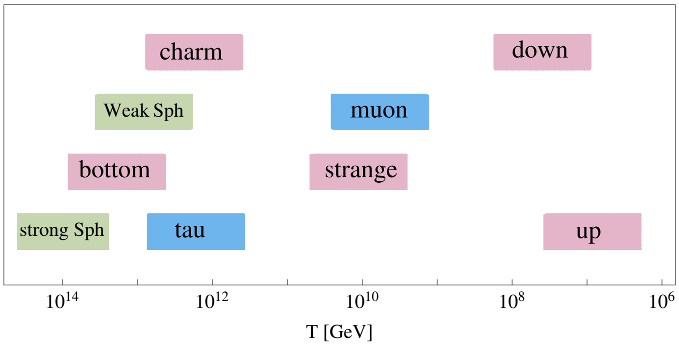

Figure 3 – Equilibration temperatures of different SM interactions. Bands range from T_X to $20 \times T_X$, where T_X is the equilibration temperature of interaction X, to indicate the regions where partial equilibration of spectators can be important for leptogenesis predictions.

quarks are sub-dominant. The temperature dependence arises since the gauge couplings have to be renormalised at a scale $\mu = 2\pi T$, but is negligible in practice. Our result $\gamma^{\rm fl} = 5 \times 10^{-3}|h|^2 T$ agrees with previous estimates e.g. in [6]. Since not all contributions to this rate were included in those estimates, this agreement is probably accidental.

The above calculation can also be used to obtain the right-handed neutrino production rate, simply by changing the hyper charge factors in the intermediate results and redoing the momentum integration with a different statistical weight. A calculation of the right-handed neutrino production rate in the relativistic regime was previously reported in [7]. Our results for the top Yukawa induced contributions as well as for the log enhanced term agree with those results, however there is some disagreement on the dominant contribution from gauge boson scatterings. Whether this due to the different methods for regulating the t-channel divergence remains to be seen.

4 Spectator effects

In the literature, spectator effects have so far been treated only in the limiting cases where a specific spectator is either fully equilibrated or completely decoupled. It is however difficult to imagine a temperature range where this treatment is adequate, since different spectator effects reach thermal equilibrium throughout most of the temperature range where Leptogenesis is possible.

To see this more clearly, in Fig. 3 we show the estimated equilibration temperatures for standard model interactions in the temperature range of 10^6 GeV to 10^{14} GeV. For the quark Yukawa interactions the results of [5] were used to obtain new estimates for the equilibration rates [8]. Most notably it can be seen that charm and tau Yukawa interactions as well as muon and strange Yukawas equilibrate at the same time, so quark spectator effects should be included when the corresponding lepton flavour effects are evaluated.

Now it is clear that when a spectator interaction comes into thermal equilibrium when the Leptogenesis mechanism is still active, neither of the limiting cases can provide a correct description. Instead it is straightforward to include the spectator as dynamical degree of freedom into the Boltzmann equations. The final asymmetry can then be compared to those obtained using the approximations of either fully equilibrated or negligible spectator interactions, to estimate the numerical importance of the effect.

The final asymmetry in Leptogenesis is usually determined when the washout process freezes out. In the strong washout regime, $K_1 \gg 1$, this usually happens around $z_f \approx \mathcal{O}(10)$, i.e. at a temperature $T_f = M_1/z_f$. Now consider a spectator with equilibration temperature T_X. By varying M_1 we can go from a regime where $T_f \gg T_X$ to a regime with $T_f \ll T_X$. Keeping the washout strength K_1 fixed, we expect the final asymmetry to smoothly interpolate between the

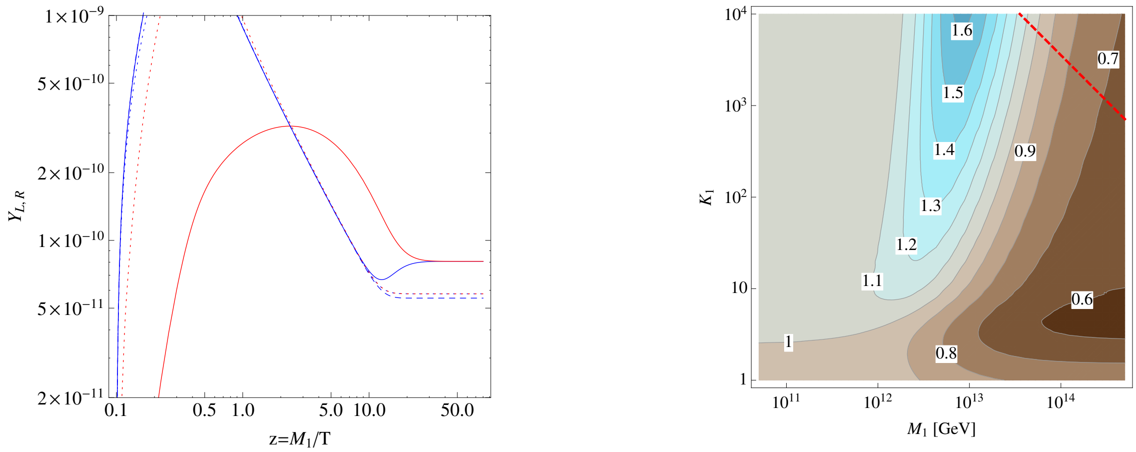

Figure 4 – Left: Solution of Eqns. (3) (solid lines) compared with the limiting cases of fully equilibrated (dotted) or fully negligible (dashed) tau Yukawa interactions. Right: Total asymmetry as function of washout K_1 and Leptogenesis scale M_1, divided by total asymmetry obtained in the limiting case of fully equilibrated flavours.

limiting cases, which we will confirm later.

However it turns out that in the intermediate regime, the final asymmetry can be larger than that in both limiting cases, and effect that is more prominent the stronger the washout is. This can be understood as follows: First, in the strong washout regime, the asymmetry at earlier times $z > z_f$ can be significantly larger than the final freeze-out asymmetry. The spectator processes can transfer part of this asymmetry into spectator degrees of freedom, where they are protected from washout. If the spectator is not fully equilibrated, a fraction of the larger early time asymmetry can be hidden from washout until it ends, thus resulting in a larger final asymmetry.

To see this more clearly it is instructive to consider a simple example where the tau Yukawa interaction reaches thermal equilibrium near the freeze-out temperature. The spectator degree of freedom here is the right-handed tau lepton, in which no lepton asymmetry is generated directly, and which is not washed out. For simplicity we assume that the lepton asymmetry is generated purely in the left-handed tau lepton doublet, such that no flavour off-diagonal asymmetries are generated. Then the system is described by the following Boltzmann equations [9,8]:

$$\frac{dY_\ell}{dz} = \frac{\bar{S}}{2}\left(Y_{N1} - Y_{N1}^{\rm eq}\right) - \bar{W} Y_\ell - h_\tau^2 \gamma^{{\rm fl}\delta\ell} \frac{T_{\rm com}}{T M_1}(Y_\ell - Y_{\rm R})\,, \tag{3a}$$

$$\frac{dY_{\rm R}}{dz} = -2 h_\tau^2 \gamma^{{\rm fl}\delta\ell} \frac{T_{\rm com}}{T M_1}(Y_{\rm R} - Y_\ell)\,, \tag{3b}$$

where $\gamma^{\rm fl}$ is the rate calculated in the previous section, $T_{\rm com}$ is the co-moving time and Y_ℓ and Y_R are the entropy normalised asymmetries in left-handed and right-handed tau leptons.

The left plot in Fig. 4 shows the solution of (3) for a choice of parameters where the effect is prominent. The asymmetry in the left-handed doublets (blue line) rises quickly until around $z \sim 0.5$ the washout takes over. Parts of that large asymmetry are transferred to the right-handed leptons (red line) and remains hidden there until the washout ends around $z \sim 12$. Only afterwards the spectators reach full thermal equilibrium (corresponding to $Y_{\rm R} - Y_\ell = 0$).

Compared with the case of fully equilibrated tau Yukawa interactions (dotted lines), where the asymmetry in the right-handed fields closely tracks that in the left-handed fields, the final asymmetry is enhanced by about 50%. When neglecting tau Yukawas the final asymmetry is even lower, since now it is only stored in the two left-handed degrees of freedom, while no asymmetry in the right-handed fields is present.

In the right plot of Fig. 4 we show how this effect behaves as function of the washout strength K_1 and of the Leptogenesis scale M_1. Shown is the total asymmetry obtained from

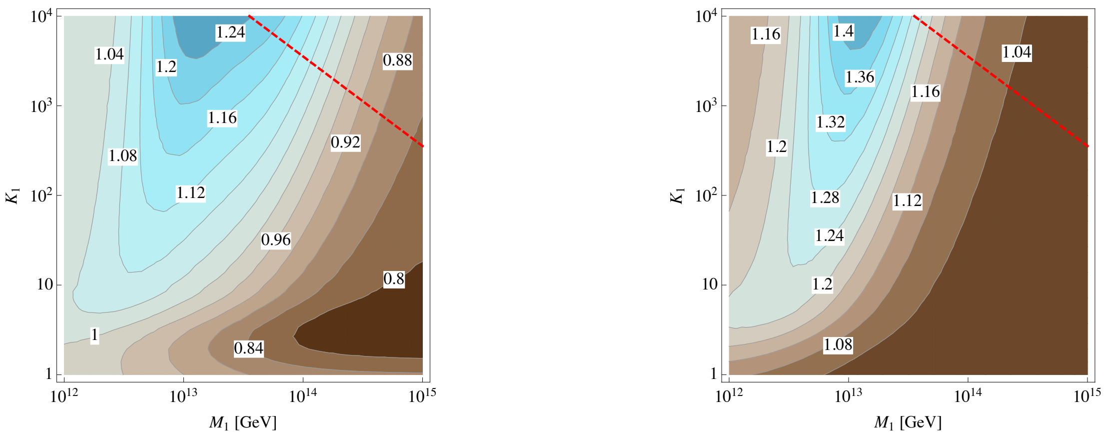

Figure 5 – Left: Final asymmetry with bottom Yukawa and weak sphaleron effects, divided by the limiting case where both interactions are fully equilibrated. Right: Same result, but divided by limiting case with fully equilibrated bottom Yukawas and neglected weak sphaleron effects.

solving Eqns. (3) divided by the solution of the same equations in the $\gamma^{\rm fl} \to \infty$ limit. The enhancement is larger for stronger washout, as expected. Furthermore we see that the full solution, as a function of the scale M_1, interpolates smoothly between the two limiting cases: the ratio goes to unity for low M_1, where the tau Yukawa is fully equilibrated, and to $2/3$ for very large scales, where the asymmetry is only stored in the two left-handed degrees of freedom.

Full alignment of flavours is difficult to realise in practice, and other interactions are relevant in this temperature range, so the above result can only serve as a toy example to illustrate this effect. Instead from Fig. 3 we see that a consistent treatment of the bottom Yukawa and weak sphaleron spectator effects is possible, in a regime where charm and tau Yukawas are still negligible, while the strong sphaleron can be treated as fully equilibrated. The results are shown in Fig. 5. Compared to several limiting cases that were explored in the literature, we find that the results of a full treatment of spectator effects can deviate by up to 40%, leading to an enhanced asymmetry in most cases.

5 Conclusions

We have discussed new results on scattering rates [5] and on the treatment of spectator effects [8] in Leptogenesis. This is part of an ongoing effort to achieve precise predictions for the lepton asymmetry produced by this mechanism. Besides corrections that can reach up to 50%, partially equilibrated spectators also re-introduce some dependence on initial conditions in the strong washout regime of Leptogenesis. Further studies are necessary to understand the full impact of these effects.

References

1. M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
2. W. Buchmüller and M. Plümacher, Phys. Lett. B **511** (2001) 74.
3. E. Nardi, Y. Nir, J. Racker and E. Roulet, JHEP **0601** (2006) 068.
4. A. Anisimov, D. Besak and D. Bodeker, JCAP **1103** (2011) 042.
5. B. Garbrecht, F. Glowna and P. Schwaller, Nucl. Phys. B **877** (2013) 1.
6. S. Davidson, E. Nardi and Y. Nir, Phys. Rept. **466** (2008) 105.
7. D. Besak and D. Bodeker, JCAP **1203** (2012) 029.
8. B. Garbrecht and P. Schwaller, arXiv:1404.2915 [hep-ph].
9. M. Beneke, B. Garbrecht, C. Fidler, M. Herranen and P. Schwaller, Nucl. Phys. B **843** (2011) 177.