Recent Lattice QCD results relevant for heavy quark flavor phenomenology

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Forty years after its introduction lattice Quantum Chromodynamics (LQCD) is nowadays a reliable method to study the nonperturbative regime of strong interactions, which is of vital interest for flavor physics phenomenology of the Standard Model (SM) and beyond. After long and painstaking efforts, thanks to a striking hardware and algorithmical evolution, the LQCD methods entered an era in which all the main sources of systematic uncertainties affecting a typical LQCD simulation can be kept under control. The increasing reliability of LQCD results opens the possibilities for new precision tests in the flavor sector of SM. In that respect, I illustrate in this talk the impact of recent improvement of LQCD on the phenomenology. A particular attention is given to leptonic and semileptonic decays of D (and B) mesons, and to decays of charmonia. We also discuss the impact of LQCD results on the New Physics searches.

1 Introduction

In the last several years we witnessed a great advancement of LQCD techniques. On one side the introduction of massively parallelized computing facilities allowed to perform simulations on very large lattice volumes (e.g. $48^3 \times 96$ or $64^3 \times 128$ points, corresponding to $\mathcal{O}(10^9 \div 10^{10})$ degrees of freedom), by splitting computations across thousands of different processors, efficiently communicating with each other. On the other side the introduction of improved regularizations of LQCD (such as HISQ, Stout techniques or Domain Wall fermions) allowed to lower the light dynamical quark mass in the simulations and to formally reproduce the physical pion mass case, thus overcoming the known limitations of the simulations with the simple Wilson or Staggered regularizations. Finally, the implementation of modern algorithmic features such as Multiple time-scale integrators in the Hybrid Monte Carlo, or the Deflation and Domain Decomposition preconditioning of the linear problem related to solving the Dirac equation on the lattice, in a globally improved understanding of Monte Carlo simulation behavior as a function of physical parameters, allowed us to keep the computational cost under control when considering large volumes simulations at physical pion mass. As a result, several collaborations (such as PACS-CS, RBC-UKQCD, MILC-Fermilab) have recently presented first results of LQCD simulations in which the effects of pairs of light quarks corresponding to physical mass u and d quarks are taken into account in the sea, on large lattices ensuring that the finite volume effects are kept under the percent level. Having already become used to taking the continuum limit of simulation results since many years, it goes by itself to say that the three major sources of systematic uncertainties affecting a typical lattice computation (physical light quark mass, continuum and infinite volume limit extrapolations) are now well kept under control.

A significant progress has also been made at taking into account the isospin breaking and electromagnetic effects on the lattice, i.e. without having to rely on effective theories. The adoption of sophisticated methods to treat heavy quarks on the lattice allowed a better control of the discretization errors and provided a way to make solid predictions for the heavy quark physics phenomenology.

In the present talk I report on a selection of results concerning hadronic properties for which, thanks to all of the aforementioned progress, has been possible to perform the computations in such a way that all the main systematic uncertainties relative to QCD are kept under control.

2 Charm physics

By comparing the experimentally measured rate of the leptonic decays, $\Gamma\left(D_{(s)} \to \ell\nu\right)$, and of the semileptonic ones, $\Gamma\left(D \to \pi\ell\nu\right)$ and $\Gamma\left(D_{(s)} \to K\ell\nu\right)$, with their theoretical predictions one can determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cs}|$ and $|V_{cd}|$. Using one of the two estimates for the matrix element $|V_{cb}|^{incl}$ or $|V_{cb}|^{excl}$, one can check the unitarity of the second row of the CKM matrix in a way similar to what has already been done for the first row [1,2,3]. To this end, a sufficiently accurate estimate of the leptonic decay constants $f_{D_{(s)}}$ and of the vector form factors $f_+^{D\to\pi}(q^2)$, $f_+^{D_{(s)}\to K}(q^2)$ are needed to get a reliable control over the theoretical uncertainties. During the past several years the improvement of LQCD methods allowed many LQCD collaborations to carry out a number of unquenched computations of these quantities, by using various approaches and techniques. Besides one should emphasize an important progress in studying the radiative decays of charmonia, as that could be used as a proof of validity of LQCD in studying the non-perturbative properties of QCD related to flavor physics, independently from the CKM matrix.

2.1 $D_{(s)}$ leptonic decays

The Standard Model expression for the decay width of the process $PS \to \ell\bar{\nu}_\ell$, in which a pseudoscalar meson PS decays into a lepton-neutrino pair reads:

$$\Gamma\left(PS \to \ell\bar{\nu}_\ell\,(\gamma)\right) = \frac{G_F^2\,|V_{PS}|}{8\pi} f_{PS}^2 M_{PS}^2 M_\ell^2 \left(1 - \frac{M_\ell^2}{M_{PS}^2}\right)\left[1 + \frac{\alpha}{\pi} C_{PS}\right], \tag{1}$$

where G_F is the Fermi constant, M_ℓ and M_{PS} are the lepton and the pseudo-scalar meson masses, C_{PS} is a coefficient parameterizing the electromagnetic radiative corrections, with α being the QED coupling. V_{PS} is the CKM matrix element, i.e. the coupling between the up (U) and down (D) type quark of the Weak current, and f_{PS} is the meson decay constant which encodes the non-perturbative QCD effects and it is defined via the hadronic matrix element $\langle 0|\,A_\mu - V_\mu\,|PS\rangle = \langle 0|\,A_\mu\,|PS\rangle = f_{PS}P_\mu$ where $A_\mu = \bar{U}\gamma_\mu\gamma_5 D$. The experimental measurement of the decay width, together with the knowledge of the decay constant f_{PS} allow one to extract the corresponding CKM matrix element. In Fig. 2.1 (taken from last FLAG II collaboration report [4], where a full list of references can be found) we show the comparison of the values for f_D and f_{D_s} as obtained by various group using various QCD discretization schemes. The overall agreement of results is very good, so that one can take their average and compare it with the experimental measurements to get:

$$|V_{cd}| = 0.222(10)\,, \quad |V_{cs}| = 1.018(24)\,, \quad [N_f = 2+1] \tag{2}$$

$$|V_{cd}| = 0.219(13)\,, \quad |V_{cs}| = 1.021(33)\,, \quad [N_f = 2] \tag{3}$$

where we separately show the results based on $N_f = 2+1$ or $N_f = 2$ dynamical quark simulations.

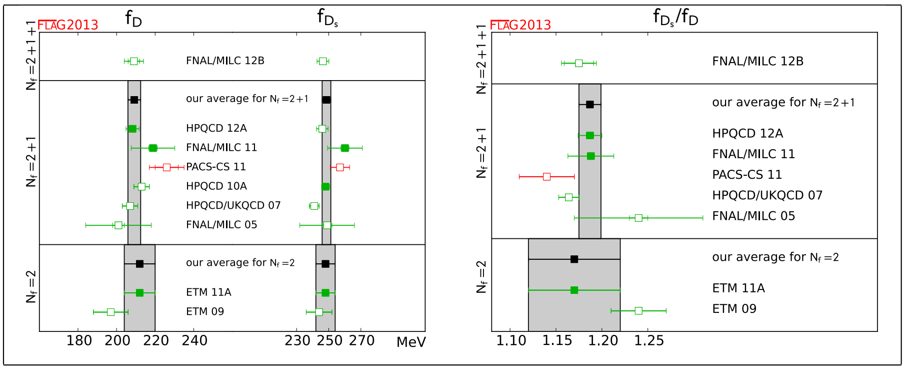

Figure 1 – Left panel: comparison of the decay constants f_D and f_{D_s} as obtained by different lattice collaborations. Right panel: the $SU(3)$ flavor breaking ratio f_{D_s}/f_D. Plots taken from [4].

2.2 $D \to \pi\ell\nu$ and $D_{(s)} \to K\ell\nu$ semileptonic decays

In the Standard Model the theoretical expression for the differential decay rate of the D_s-meson semileptonic decay into a lighter pseudoscalar meson P and a lepton pair $\ell\nu$, reads $\frac{d}{dq^2}\Gamma(D_{(s)} \to P\ell\nu) = G_F^2|V_{CKM}|^2\left[\mathcal{K}_+ f_+(q^2) + \mathcal{K}_0 f_0(q^2)\right]^2$, where G_F is the Fermi constant, V_{CKM} is the appropriate CKM matrix element, $\mathcal{K}_+$ and $\mathcal{K}_0$ are the kinematical factors depending on the particle masses and their momenta, and $f_{+,(0)}(q^2)$ is the vector (scalar) form factor parameterizing the hadronic matrix elements. For the experimentally accessible processes involving an electron or a muon in the final state, the coefficient $\mathcal{K}_0$ (proportional to m_ℓ^2) is very small and therefore the scalar form factor contribution to the process is negligible. The $D \to K$ decay form factor $f_+(q^2)$ can be obtained from studying the three point correlation functions from which we extract the hadronic matrix element of the vector current between the D and K mesons, which decomposes as:

$$\langle K|\, V_\mu \,|D\rangle = p_\mu f_+(q^2) + q_\mu \frac{m_D^2 - m_K^2}{q^2}\left[f_0(q^2) - f_+(q^2)\right], \quad p = p_D + p_K,\; q = p_D - p_K\,, \tag{4}$$

and similarly for the $D_s \to K$ and $D \to \pi$ case. In the last several years three lattice collaborations (FNAL/MILC, ETM and HPQCD) approached computed the $D \to K/\pi$ form factors by using modern sophisticated LQCD techniques, adopting different methods to non-perturbatively renormalize the matrix element, and performing the continuum and chiral extrapolations. In Fig. 2 we collect the plots depicting the form factors as functions of q^2. HPQCD produced results for $f_{+,0}^{D\to K}(q^2)$ and $f_0^{D\to\pi}(q^2)$ covering the full kinematical range, and managed to successfully compare the shape of the $f_+^{D\to K}$ form factor with that measured in experiments. By taking advantage of the equality $f_+(0) = f_0(0)$, they also determined the form factor at $q^2 = 0$ and extracted the CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$. In this way they were able to verify the unitarity of the second row of the CKM matrix. The same test was repeated by the FLAG collaboration [4], including the results for matrix elements obtained by other lattice groups. In that way they were able to estimate the difference from unity of the sum of squares of $|V_{cd}|$, $|V_{cs}|$ and $|V_{cb}|$ which they found to be:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.04(6)\,, \tag{5}$$

thus compatible with zero. In their preliminary analysis, the ETM collaboration compared the $D \to \pi$ form factor $f_+(q^2)$ computed on the lattice with the Vector Meson Dominance (VMD) model, but in such a way that they also computed the value of the residuum of the form factor at its first pole on the same set of lattice configuration. They showed that the inclusion of the first state is not sufficient to saturate the form factor and that the VMD does not work at all. In the analysis they considered the full set of form factors that enter the theoretical predictions

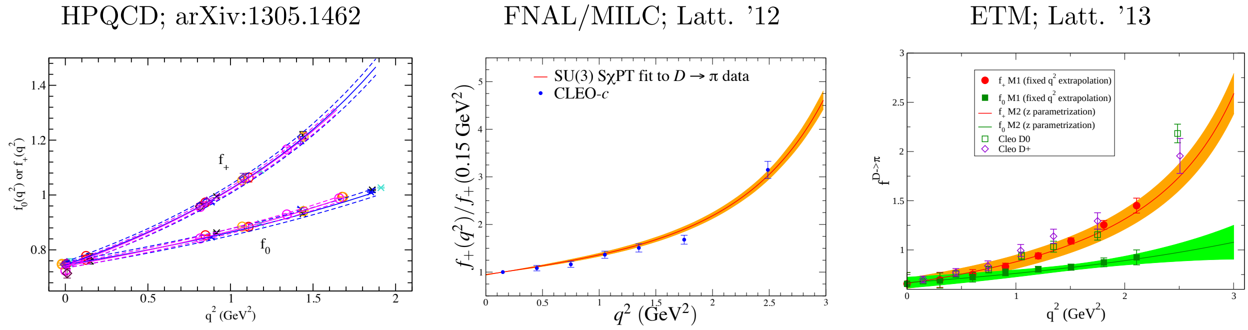

Figure 2 – Comparison of three different determinations of the $D \to K$ and $D \to \pi$ form factors. Left panel: results of HPQCD collaboration for $f_{+,0}^{D\to K}(a^2)$. Center panel: preliminary results presented by Fermilab/MILC collaboration at Lattice 2012 conference [6] for $D \to \pi$ vector form factor $f_+(q^2)$. Right panel: author's preliminary results presented at Lattice 2013 conference [7].

of $D_{(s)} \to K/\pi\ell\nu$ in the most generic extension of the Standard Model, that required the computation of the form factor f_T, in addition to the already mentioned f_+ and f_0, extracting it from the matrix element of tensor current between D and π states, namely:

$$\langle P(p')|\bar{q}\sigma_{\mu\nu}c|\overline{D}(p)\rangle = -i(p_\mu p'_\nu - p'_\mu p_\nu)\frac{2f_T(q^2,\mu)}{m_D + m_P}. \tag{6}$$

Using the accurate experimental values for the decay widths, together with the established values of V_{CKM}, one can use the three form factors computed in LQCD to set bounds on the coupling to additional operator (apart from the Standard Model one, namely $\bar{U}\gamma_\mu^L D$) thus constraining the effects of New Physics Beyond the Standard Model. Similarly, one can use leptonic and semileptonic decays to test the models predicting the existence of a heavy (sterile) neutrino (c.f. [5]).

In the future the increase in precision of the computation of the $D_{(s)}$ semileptonic form factors will allow to increase the accuracy of the determination of the CKM matrix elements, and improve the bounds on physics beyond the SM. The expertise acquired in studying D meson form factors set the grounds for more challenging determination of the B meson properties.

2.3 Radiative decays of charmonia

The radiative decays of charmonia offer a possibility of performing valuable tests. One can either test the LQCD approach to high accuracy or test the SM independently from the CKM matrix. The decay $J/\psi \to \eta_c\gamma$ has been a subject of extensive theoretical and experimental studies since several decades. The current experimental value quoted by PDG [8] is,

$$\Gamma(J/\psi \to \eta_c\gamma) = 1.58(37)\ \text{keV}. \tag{7}$$

It has been obtained after averaging two experimental results, namely $\Gamma(J/\psi \to \eta_c\gamma) = 1.18(33)$ keV by Crystal Ball [9], and the more recent one obtained by CLEOc, 1.91(28)(3) keV [10]. The currently running KEDR experiment [11], instead, suggests a larger value, 2.2(6) keV. It is fair to say that the current experimental situation is unclear and a dedicated charm physics experiment at BESIII is expected to clarify the situation in the future.

Prior to 2012 the theoretical situation concerning prediction of $\Gamma(J/\psi \to \eta_c\gamma)$ was not better. The predictions obtained by means of various approaches (Dispersive analysis [12], QCD sum rules [13,14], effective theory of non-relativistic QCD [15], and potential quark models [16,17]) as well as LQCD performed in quenched approximation [18] or at single lattice spacing [19] presented a globally confusing picture, nevertheless pointing to values comparable with those measured by the latest experiments (see Fig. 2.3).

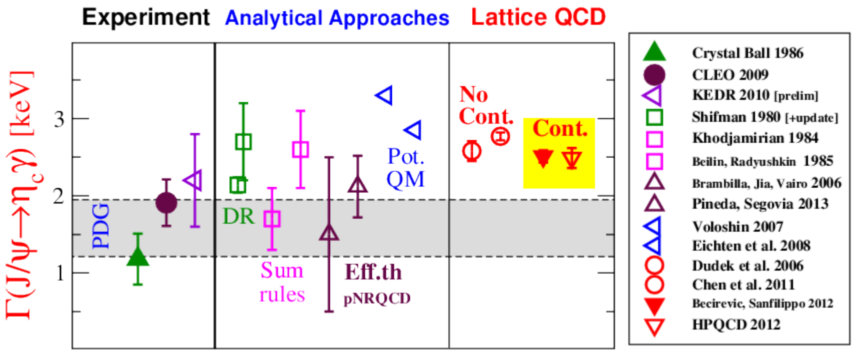

Figure 3 – Comparison of various experimental results(left panel), theoretical predictions based on analytical approaches (central panel) and lattice numerical calculation for the $J \to \psi\gamma$ decay rate. The horizontal grey band is PDG average of the two leftermost experiments.

In 2012 two different lattice groups [20,21] computed the decay width of this process by taking into account for the first time the effects of light sea quark loops (more specifically, up and down quarks in the case of ref. [20], and also the strange quark in [21]), and by performing the extrapolation to both the chiral and continuum limits. By relying on the so-called Twisted Boundary condition [22] the two computations provided the form factor value relevant to the physical photon case, i.e. $q^2 = 0$, thereby avoiding the extrapolations from large, unphysical momenta, to $q^2 = 0$. The two analyses were based on two very different lattice regularizations: the Twisted Mass (Wilson-like) at maximal twist in the case of ref. [20], resulting in $\Gamma(J/\psi \to \eta_c\gamma) = 2.64(11)$ keV, and the Highly Improved Staggered Quark (Rooted Staggered-like) in the case of ref. [21], with the result 2.49(19) keV. A strikingly good agreement between the two so precise determinations based on two very different approaches stresses the power reached by the LQCD method. The theory prediction for this decay width is therefore clarified and a call for an improved experimental determination is in order. If the experimental precision could be increased, the $J/\psi \to \eta_c\gamma$ mode could become a precision test of nonperturbative QCD. Otherwise, the results obtained for charmonium can be used for an equally interesting study, namely to test the presence of a light CP-odd Higgs state that is predicted in various two-higgs doublet models (2HDM). Furthermore, a study of the $\Upsilon(2S) \to \eta_b\gamma$ process could help us to improve the determination of η_b mass which would in turn improve the comparison of the measured and predicted hyperfine splitting $\Delta = m_{\Upsilon(1S)} - m_{\eta_b}$. In this way one could also check for the presence of a CP-odd Higgs state in bottomium systems as well.

3 B physics

The physics of the b quark offers a rich set of processes to test of the Standard Model and search for the effects of physics beyond. For example, the value of the two entries $|V_{ub}|$ and $|V_{cb}|$ of CKM matrix as extracted by means of different phenomenological inputs (inclusive and exclusive B decays) are in disagreement among themselves. On the other side, a comparison of the recently measured $Br(B_s \to \mu^+\mu^-)$ with theory offers the possibility to check for the presence of New Physics. Lastly, the $B \to K^{(*)}\ell^+\ell^-$ decays raised a great interest recently due to significant tension between the experimental measurements and theory predictions. For all of these processes LQCD is crucial for providing the reliable method to compute the hadronic matrix elements entering the theory prediction. Motivated by the importance of the phenomenological implications, the lattice community dedicated great effort to the b-physics. The major issue for a lattice computation of b-hadronic matrix elements is that of treating the large range of energies between the hadronic scale $[\mathcal{O}(\Lambda_{QCD})]$ and the meson masses containing a b quark(s) $[\mathcal{O}5 \div 10$ GeV$]$. To accommodate in the same box with finite volume and finite lattice spacing box these two scales, one would need considering huge lattices, of $10^2 \div 10^3$ points, which are clearly beyond the reach of currently available computing resources. Nonetheless thanks to a number of innovative approaches (improved regularizations, using judicious ratios to suppress the cut-off effects, and make use of different effective field theory approaches to separate the scales involved in the problem) made it possible to make reliable predictions accessible with currently available computer facilities and algorithms.

3.1 $B_{(s)}$ leptonic decay

The B and B_s leptonic decay constants f_B and f_{B_s} are of central importance in flavor physics, for different reasons. The value of the first, in conjunction with the experimental measurement of $\mathcal{B}(B \to \tau\nu)$ allows us to extract the value of $|V_{ub}|$, CKM matrix element the value of which is by itself longly debated: when determined through the inclusive $B \to X_u \ell\nu$ decay it amounts [23] to $|V_{ub}| = 4.40(15)(20) \times 10^{-3}$, while exclusive $B \to \pi\ell\nu$ decay at Belle and BaBar resulted in $3.47(22) \times 10^{-3}$, and $3.37(21) \times 10^{-3}$, respectively. An independent determination of $|V_{ub}|$ from the leptonic decay would help solving the tension. Since the lattice spacings adopted in typical lattice simulations are still not larger than the inverse b quark mass, a special care must be devoted to discretization effects. Various lattice collaborations have applied different strategies: FNAL-MILC adopted the Fermilab method based on a non-relativistic interpretation of the cut-off effects; HPQCD employed the Non Relativistic QCD and more recently the HISQ regularization to extrapolate from masses very close to the physical b-quark mass; the Alpha collaboration instead used Heavy Quark Effective Theory, together with the step scaling to separate the light and heavy scales; finally, ETM defined ratios of decay constants at heavy quark masses that differ by a known exact factor. The benefit of that approach is that the value of those ratios in the static limit is fixed by the heavy quark symmetry. The results of these studies are reported in the left panel of Fig. 3.1, where a good level of precision and a remarkably good agreement among different determinations has been reached. Unfortunately, the limited experimental precision by Belle and BaBar does not allow for a reliable extraction of $|V_{ub}|$. As of now, by using the Belle measurement for $\mathcal{B}(B \to \tau\nu)$ one obtains $|V_{ub}| = 3.8(5)(2)$, while by using the BaBar experimental input one gets $|V_{ub}| = 5.2(7)(2)$. A better precision on the experimental side is therefore mandatory to solve the above-mentioned problem.

Although the B_s cannot decay leptonically (in the Standard Model), the value of its leptonic decay constant is of major importance for the prediction of the penguin/box induced decay $B_s \to \mu^+\mu^-$. In the right panel of Fig. 3.1 we show a comparison of the results obtained from the same lattice analyses discussed above in the case of f_B. A very good agreement among various determinations is found. The great precision reached by them individually shows the robustness of the employed methods, and stresses the progress made in the past several years in the computation this quantity on the lattice. The error in the determination of f_{B_s} will soon approach the level at which the inclusion of electromagnetic corrections, beyond the factorization approximation, will be necessary in order to match the increasing precision of the experimentally measured $B_s \to \mu^+\mu^-$ decay rate (see talk by M. Gorbahn at this conference).

3.2 $B_{(s)} \to D^{(*)}_{(s)}\ell\nu$ decays

A popular test of the Standard Model is based on verification of the agreement between theoretical predictions and experimental measurements of ratios of branching fractions of semileptonic $B_{(s)} \to D^{(*)}_{(s)}\ell\nu$ decays differing by the lepton flavor, considering the quantity:

$$R(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}, \qquad R(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu)} \ (\ell = e,\, \mu) \ . \tag{8}$$

In the ratio many of the experimental and theoretical uncertainties (in particular, $B \to D$ form factor normalization and CKM matrix elements) cancel out. In 2012 the BaBar collabo-

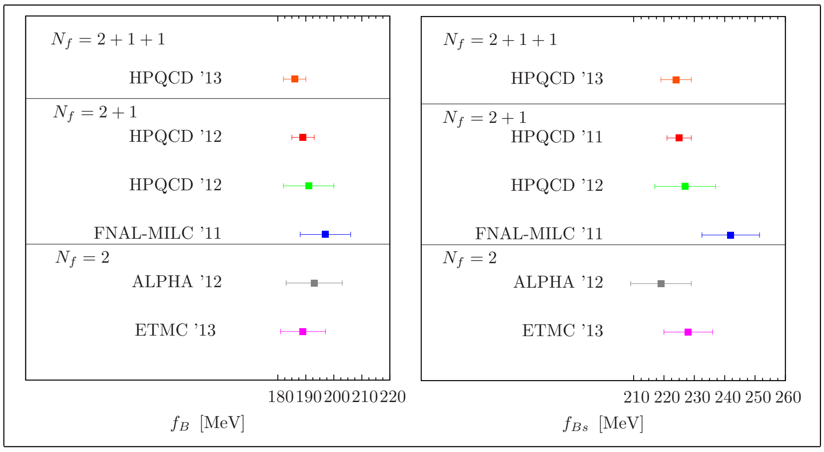

Figure 4 – Comparison of the result for the B and B_s decay constants as obtained by various lattice collaborations. Plot taken from [24].

ration reported a deviation of their measurement of $R(D)$ w.r.t the Standard Model prediction, in particular:

$$R(D)^{BaBar} = 0.440 \pm 0.071, \qquad R(D)^{SM} = 0.31 \pm 0.02 \tag{9}$$

It has been suggested that the observed discrepancy could be attributed to a presence of Physics Beyond the Standard Model. To put this suggestion on a more solid basis, an accurate theoretical calculation of the $B \to D$ form factor by means of modern LQCD methods, is required. In particular the experiments fit their data to the shape of the form factors parameterized according to the expressions proposed and derived in ref. [25], reporting the combination of $|V_{cb} \cdot \mathcal{G}(1)|$, where $\mathcal{G}(1)$ is the relevant form factor extracted at zero recoil, $w = 1$ corresponds to $q^2_{max} = (m_B - m_D)^2$ as the two are related via $q^2 = m_B^2 + m_D^2 - 2m_Bm_Dw$. So far a few LQCD calculations have been dedicated to the computation of the form factor $\mathcal{G}$ needed to normalize the experimental result and determine V_{cb}. Recently, the authors of ref. [26] studied the case of the slightly simpler $B_s \to D_s$ process, in view of exploring the more demanding and interesting case of $B \to D$ case.

This determination relies on the fact that the elastic $D_s \to D_s$ form factor $\mathcal{G}(1, m_c, m_c)$ is equal to 1, thanks to conservation of the electric charge. Also in the limit $\lim_{m_h \to \infty}$ the form factor $\mathcal{G}(1, m_h, m_c)$ is known to be 1 up to radiative and $1/m_h$ [27]. These two bounds constrain the form factor behavior as a function of the heavy quark mass, and therefore studying the form factor $\mathcal{G}(1, m_h)$ for various values of the heavy quark mass m_h between the charm and bottom quark masses, it becomes possible to extrapolate the lattice data computed for the b quark mass lower than the physical one. Computing the form factor $\mathcal{G}$ at two different values of the heavy quark mass m_h and $\lambda \cdot m_h$, one can build the ratio:

$$\sigma = \frac{\mathcal{G}(1, \lambda m_h, m_c)}{\mathcal{G}(1, m_h, m_c)}\,, \tag{10}$$

so that starting with $m_h = m_c$ one can reconstruct $\mathcal{G}(1, m_b, m_c)$ from the chain equation:

$$\mathcal{G}(1, m_b, m_c) = \sigma_n \sigma_{n-1} ... \sigma_1 \sigma_0 \underbrace{\mathcal{G}(1, m_c, m_c)}_{1}\,, \tag{11}$$

being $\sigma(i) = \frac{\mathcal{G}(1,\lambda^{i+1} m_c, m_c)}{\mathcal{G}(1,\lambda^i m_b, m_c)}$, $\lambda = \left(\frac{m_b}{m_c}\right)^{1/n}$. The value of σ_i close to the physical b quark mass, where lattice data is not accurate and affected by significant cut-off effects can be reconstructed by fitting the value of σ_i close to the physical charm, constrained by the fact that σ is 1 in the static limit by construction, as depicted in Fig. 5. The advantage of studying ratios of quantities

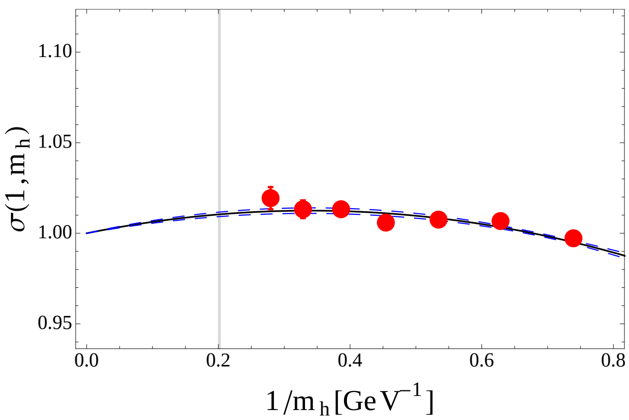

Figure 5 – Reconstruction of $\sigma(m_h)$ (defined in the text) using lattice data close to the charm quark. The two set of points show the result of two different possible extrapolation to the continuum limit (plot taken from [26]).

at different values of the heavy quark mass comes from the large cancellation of statistical and systematic errors between numerator and denominator, and has been suggested for the fist time in ref. [28].

The final result is therefore obtained by applying eq. 11, using the fitted form of $\sigma(m_h)$ to evaluate σ_i, leading to: $\mathcal{G}(1) = 1.052(46)$. This number can be compared with the one obtained in ref. [29] for the case of $B \to D$, namely 1.026(17) in which the step scaling function has been used to separate the high and low energy scales involved in the process, but in the quenched approximation. The result is also comparable with the one obtained in ref. [30], $\mathcal{G}(1) = 1.074(24)$, an unquenched computation, at a single lattice spacing. In the future, increase in statistics will help providing more accurate results for such form factors and settle the issue of $R(D)$ on a more quantitative basis. Note also that in ref. [26] the authors reported on the first lattice computation of the scalar and tensor form factors for the $B_{(s)} \to D_{(s)}\ell\nu$ decays, which are essential for a study of the current discrepancy between theory and experiment for $R(D)$ and in view of various extensions of the Standard Model.

This method has not been employed in computing the hadronic form factor entering the theoretical description of the $B \to D^*\ell e\nu$ decay. A key quantity for that decay (equivalent to $\mathcal{G}(1)$ discussed above) is the form factor $\mathcal{F}(1)$. Very recently that quantity has been computed to an impressive accuracy in ref. [31] by using the Fermilab approach to the heavy quark and with the staggered light quarks, $\mathcal{F}(1) = 0.906(4)(12)$.

3.3 $B \to K^{(*)}\ell^+\ell^-$ decays

The $B \to K^{(*)}\ell^+\ell^-$ decay has been subject to theoretical debate in reference to a tension manifested in the global fit of $b \to s$ transition (see talk by S.Descotes-Génon and W.Altmannshofer at this conference). The form factors relevant to these processes have been only recently computed in an unquenched ($N_f \neq 0$) environment. The HPQCD collaboration has reported [32] the results of their computation of the $B \to K$ form factors $f_{+,0,T}$ by means of Non-Relativistic QCD (NRQCD), and for a range of q^2's close to the maximal $q^2_{max} = m_B^2 - m_D^2$, and then extrapolating to low q^2's as shown in fig. 6. In this new study the values for $f_{0,T}(q^2)$ results consistent with the previous one [33], but $f_+(q^2)$ results is found to be smaller. It is interesting to note that [34] the new $f_+(m^2_{J/\psi})/f_0(m^2_{\eta_c})$ suggests a sizable violation of the factorization approximation in $Br(B \to \eta_c K)/Br(B \to J/\psi K)$. Notice also that FNAL/MILC collaboration presented their preliminary results for the form factors in ref. [35]. The authors of ref. [36], instead, studied the cases of $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays, where seven different form factors contribute. Again they used NRQCD and considered transferred momenta close to q^2_{max}.

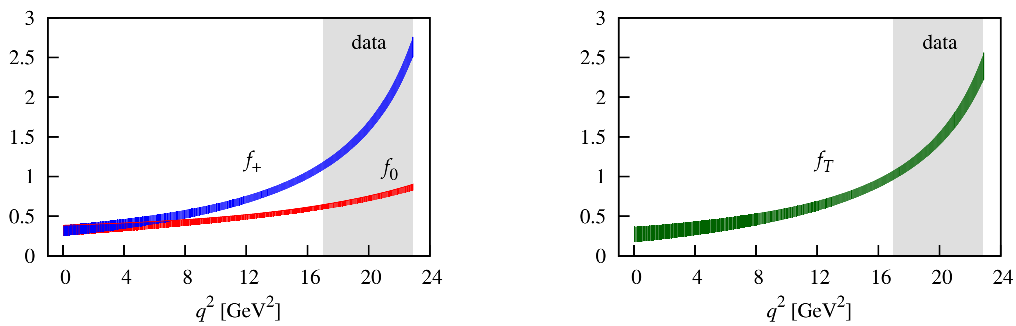

Figure 6 – Calculation of the form factors relevant for the $B \to K\ell^+\ell^-$ decay performed by HPQCD collaboration [32]. Colored bands are the results of fit to the lattice data computed on the range of q^2 covered by the grey area.

4 Conclusions

In this talk we showed in what way the recent progress in LQCD can help the Standard Model phenomenology and provide a way to look for the effects of physics beyond the Standard Model through a comparison between the experimental data and the theoretical estimates of a large number of flavor physics processes. The improvement of computing power and of the used algorithms, allow to eliminate the major sources of systematic errors affecting LQCD computations. The ongoing progress is allowing to consider heavy quark ever closer to the physical b-quark mass. Thanks to many theoretical developments the effects of electromagnetism and mass difference between u and d quark masses are starting to being accounted directly from the first principle of the theory [37,38], allowing in the future to perform even more precise tests of the Standard Model. LQCD is nowadays able to provide reliable results for a number of matrix elements, mainly those needed for semileptonic and leptonic decays. LQCD helped checking the unitarity of the 1st row of the CKM matrix and today is narrowing the precision of the test of the 2nd row. Beside these decays, vital for the flavor physics phenomenology, a steady progress in understanding the QCD dynamics of non-leptonic decays has been made as well. A particularly interesting results in the direction of solving the $\Delta I = 1/2$ rule, has been presented in ref. [39,40].

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