Recent Lattice QCD results relevant for flavor physics phenomenology

Rencontres de Moriond, 15-22 February 2014

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Summary

Introduction

1. Why lattice QCD?
2. Overview of a typical lattice QCD calculation & systematic errors
3. Lattice community: main goal & state of the art

Recent progresses in Heavy flavor physics

1. Verification of unitarity of second row of CKM matrix
2. Flavor puzzle from B neutral meson mixings
3. B decays: $B_s \to \mu^+\mu^-$, $B \to D/K\ell\nu$
4. Radiative decays of charmonium
5. Perspectives

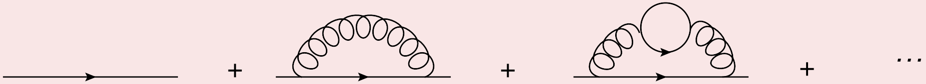
Why lattice QCD

Lattice QCD = QCD

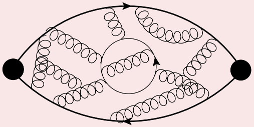
- Only method to solve non-perturbative QCD from the first theory principles
- No need to add parameters apart from those originally present in QCD lagrangian
- Precision, in principle, only limited by available computational power
- All sources of systematic errors can be eliminated

Perturbative vs. nonperturbative

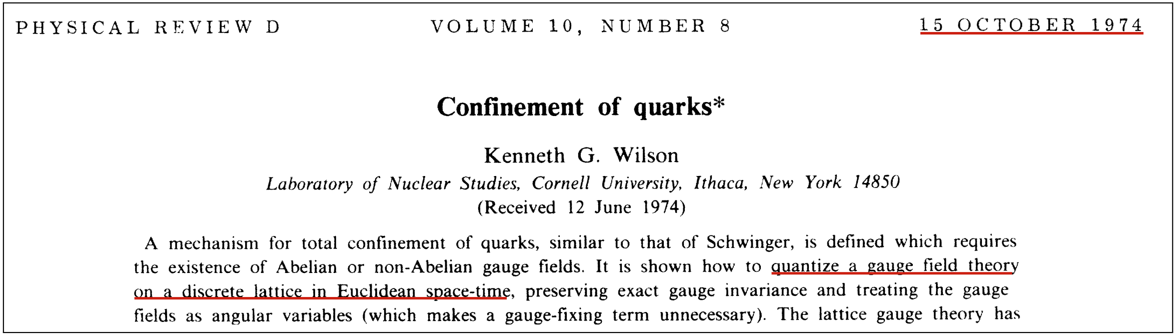
- Perturbative: compute order by order and then summing



- Non-perturbative: take into account directly all the contribution (up to cut-off scales)



Lattice QCD forty years after its introduction



PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has

Wilson prophecy

> "*Fifty years will be necessary for computational resources and algorithms to reach proper maturity*"

Forty years passed since Lattice methods invention...

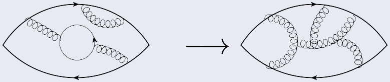
Where do we stand?

Why Lattice QCD is so computationally demanding?

Quark masses dependency

Simulation cost: rapidly grows as quark masses are lowered

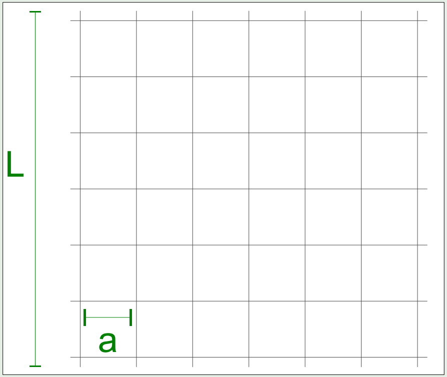
Early solution: quenching = drop virtual pair contributions from partition function



Intermediate solution: consider unphysical light quarks ($M_\pi \sim 300 \div 500$ MeV)

Nowadays: many collaborations (CP-PACS, BMW, RBC/UKQCD ...) use M_π^{phys}

Lattice size dependence



- **Simulation cost:** $[\#points]^{k>1} = \left[(L/a)^4\right]^k$ (scales: $a \ll 1/M_H$, $L \gg 1/M_\pi$)
- **Early solution:** $\#points = 4^4$
- **Nowadays:** $\#points = 48^3 \times 96 \div 64^3 \times 128$
  - D physics: $M_D/M_\pi \sim 15$, $M_{J/\psi}/M_\pi \sim 22$
  - B physics: $M_B/M_\pi \sim 40$, $M_\Upsilon/M_\pi \sim 70$

Main computational problem & State of the art

State of the art

1. Physical light quarks and large volumes ($\gtrsim (6\,\mathrm{fm})^3$)
2. Simulations performed at several lattice spacings
3. Isospin and electromagnetic corrections start to be accounted for

What helped these improvements?

Increase in computing power

Conceptual developments

- Improved regularizations of LQCD
- Better understanding of behavior of Monte Carlo w.r.t (m_0, g_0)

Algorithm breakthroughs

- Multiple timescale Molecular Dynamic integrators
- Deflation, Multigrid, Domain Decomposition solvers, etc.

Growth of community (Lattice 2013 attended by more than 500 people)

Approaching B mesons

Effective theories

- Heavy Quark Effective Theory (continuum expansion in Λ_{QCD}/m_b)
- Nonrelativistic QCD (expansion in quark velocity and in $1/am_b$)
- Propagating Heavy Quarks (reinterpretation in therms of $1/m_b$ expansion)

Extrapolate results from the charm to the bottom region

- Scaling laws often known in effective theories
- Use numeric (or sometimes exact) results in the static limit
- Results become more reliable as lattice spacings get smaller
- Special actions have been designed to deal with b quarks (HISQ, ...)
- Use of step scaling function to separate various scales (a, m_b, L)

CKM unitarity

LQCD helped to check Unitarity of the first row of CKM matrix

$$\underbrace{|V_{ud}|}_{\text{well known}}{}^2 + |V_{us}|^2 + \underbrace{\cancel{|V_{ub}|}}_{\text{negligible}}{}^2 = 1$$

- V_{us}: obtained from $K_{\ell 2}$ and $K_{\ell 3}$
- Needed f_K and $f_+^{K\to\pi}(0)$ from lattice

Similar check of the second row (work in progress)

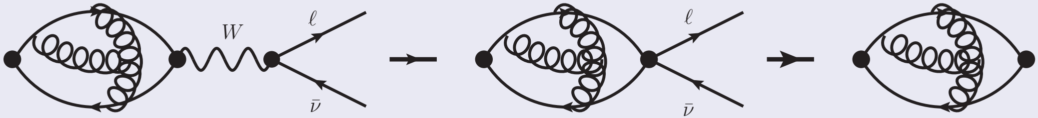
- Need to extract all three CKM entries:

$$\underbrace{|V_{cd}|^2}_{\substack{D\to e\nu \\ \text{or} \\ D\to\pi e\nu}} + \underbrace{|V_{cs}|^2}_{\substack{D_s\to\ell\nu \\ \text{or} \\ D_s\to K\ell\nu}} + \underbrace{|V_{cb}|^2}_{\substack{B_{(s)}\to D_{(s)}\ell\nu \\ \text{or} \\ B^*_{(s)}\to D^*_{(s)}\ell\nu}} = 1$$

- Hadronic quantities entering theoretical expression:
  - leptonic decay constants f_D, f_{D_s} for $D\to e\nu$, $D_s\to\ell\nu$
  - form factors $f_+^{D\to\pi}(q^2)$, $f_+^{B\to D}(q^2)$ ... for $D\to\pi e\nu$, $B\to D\ell\nu$...

Leptonic decays of mesons

Full process — Eff. weak hamiltonian — QCD side



Two point correlation functions

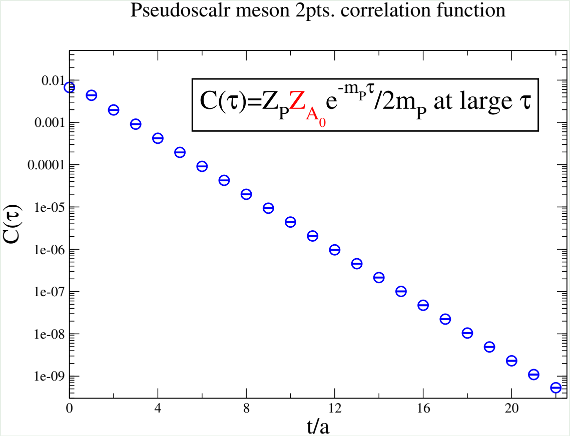
$$\mathcal{B}_{P\to\ell\bar{\nu}} = \underbrace{\left|V_{xy}\right|^2}_{\text{CKM}} \underbrace{\mathcal{K}\left(m_\ell,\, m_M\right)}_{\text{kinematics}} \left| \underbrace{f_P}_{\text{dec. constant}} \right|^2$$

$$f_P = \frac{Z_A}{m_P} = \frac{\langle 0|A_0|P\rangle}{m_P}$$

Z: coupling of current inducing decay

From lattice, 2 point correlation functions:

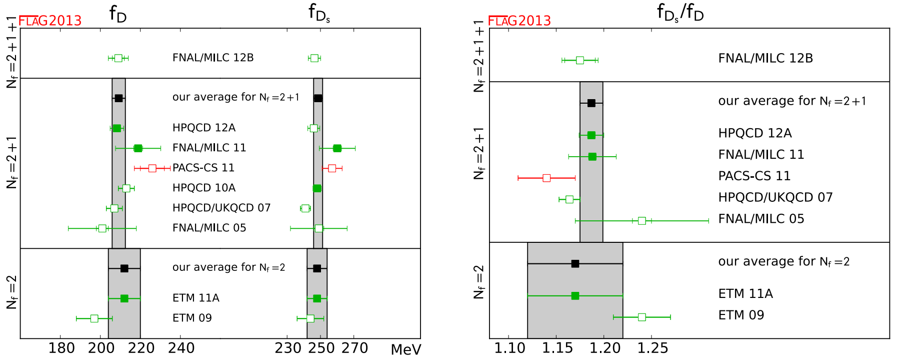
$$C(\tau) = \langle O^\dagger_{A_0}(\tau)\, O_P(0)\rangle, \;\; O = \bar{\psi}\Gamma\psi$$


Pseudoscalr meson 2pts. correlation function

D meson decay constants

Pseudsocalar decay constants, f_D f_{D_s}

Use to compute V_{cd}, V_{cs} and use to check unitarity of 2nd row of CKM



$$|V_{cd}| = 0.2218(35)(95)\,, \quad |V_{cs}| = 1.018(11)(21)\,, \qquad \text{(leptonic decays}, N_f = 2+1)$$
$$|V_{cd}| = 0.2189(83)(94)\,, \quad |V_{cs}| = 1.021(25)(21)\,, \qquad \text{(leptonic decays}, N_f = 2)$$

Flavour Lattice Averaging Group (FLAG) second review published in Oct. '13

- arXiv:1310.8555, 255 pages, 29 Authors from all main lattice collaborations
- Emerging consensus as reference for averages of lattice results
- Good starting point to answer the question: "Which values do I have to take?"

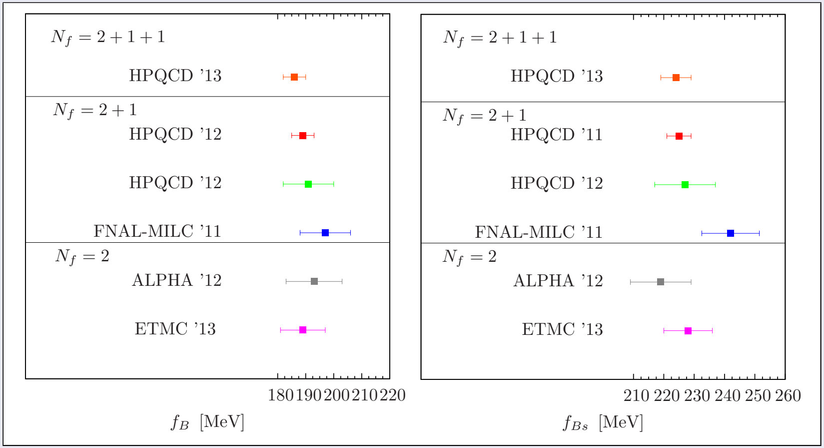
B meson decay constants

Physical motivation for a big question to lattice community: "Can you provide $f_{B_{(s)}}$ at % accuracy?"

- $|V_{ub}|$ from $B \to \tau\nu$, compare with $|V_{ub}|$ from $B \to \pi\ell\nu$ and $B \to X_u$
- f_{B_s} essential $B_s \to \ell^+\ell^-$

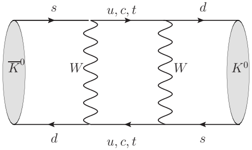
Employed strategies and current situation

FNAL-MILC	Fermilab method	ETMC	Ratios with known static limit
HPQCD	Non Relativistic QCD, or HISQ	ALPHA	HQET + Step Scaling



[Comparison taken from arXiv:1308.1851]

Neutral meson mixing

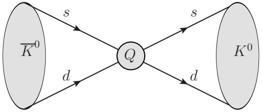


Integrating out the

heavy degrees of freedom

OPE

$$Q_1 = [\bar{s}\gamma^\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d]$$



Beyond the Standard Model

$$\underbrace{\langle \overline{B}^0 | H_{eff}^{\Delta F=2} | B^0 \rangle}_{\text{experiments}} = \frac{G_F^2 M_W^2}{16\pi^2} \sum_{i=1}^{5} \underbrace{C_i(\mu)}_{\text{short distance}} \underbrace{\langle \overline{B}^0 | Q_i(\mu) | B^0 \rangle \left(\equiv \boldsymbol{B_i} \langle \overline{B}^0 | Q_i(\mu) | B^0 \rangle_{VIA} \right)}_{\text{long distance}}$$

From the experimental parametrization of meson oscillation

- and the knowledge of hadronic matrix element computed on the lattice
- we check Standard Model (where $C_{2,..,5} = 0$)
- and gain insight in BSM physics (C_i depend on model details)

See: *UTfit Collaboration and CKM fitter*

B_i from lattice QCD

- Technology pioneered for $\bar{K}^0 - K^0$ system
- Compute from three point functions: $C_{3;i}(\tau) = \langle O^\dagger_{\bar{B}^0}(t_{sep})\, Q_i(\tau)\, O_{B^0}(0) \rangle$
- Mixing pattern among operators complicated on the lattice regularization scheme

Flavor puzzle

Relation to the scale of New Physics

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2} = \begin{cases} \text{couplings \& loop effect of New Physics} \sim 1 \text{ in generic FCNC} \\ \text{scale of New Physics} \end{cases}$$

Computed first time 12 years ago - D.Becirevic et al., JHEP 0204 (2002)

Quenched, no continuum extrapolation, mixing-lattice artifacts, HQET driven...

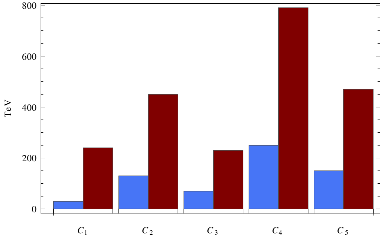
$$\boxed{B_i^{(d)}(m_b)^{\overline{\mathrm{MS}}} = \{0.87(4)(5),\ 0.82(3)(4),\ 1.02(6)(9),\ 1.16(3)(^{+5}_{-7}),\ 1.91(4)(^{+22}_{-7})\}}$$

New results from ETM - N. Carrasco et al., JHEP 1403 (2014) 016

- $N_f = 2$ & extrapolated to m_π^{phys}, 4 lattice spacings, NPR and mixing of continuum

$$\boxed{B_i^{(d)}(m_b)^{\overline{\mathrm{MS}}} = \{0.85(3)(2),\ 0.72(3)(1),\ 0.88(12)(6),\ 0.95(4)(3),\ 1.47(8)(9)\}}$$

- Ongoing joint analysis from Fermilab and MILC Collaborations
- RBC/UKQCD computed $SU(3)$ breaking ratio $\xi^{stat} = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.13(12)$, PRD82,'10



FLAVOR PUZZLE

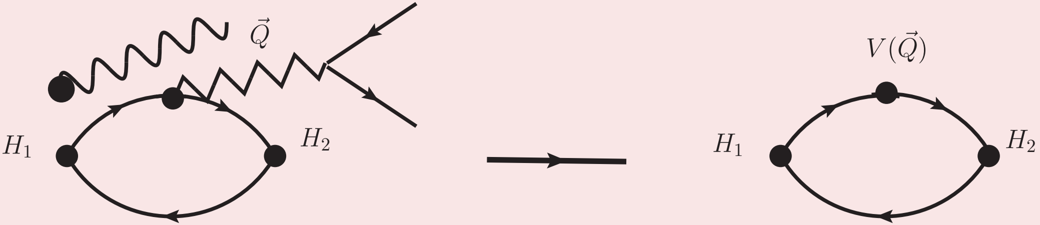
Assuming arbitrary flavor structure: $\Lambda_{NP} > 10^5$ TeV

For New Physics at 1 TeV one needs to:

- forbid FCNC or
- have non trivial flavor structure

Semileptonic & radiative meson decays

Accessible with three point functions



Prototype: $K\ell_3$ decays, now computable at the physical point

Semileptonic decays

- $D \to K\ell\nu$, $\pi\ell\nu$: V_{us}, V_{ud}
- $B \to D^{(*)}\ell\nu$: V_{cb}
- $B \to K\ell\nu$, $\pi\ell\nu$: V_{ub},
  limited to region of large Q^2

Useful for model-independent studies

Radiative decays

Independents from CKM matrix

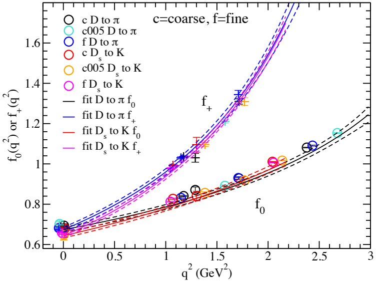
- J/ψ, $h_c \to \eta_c\gamma$
- $\eta' \to J/\psi\gamma$
- $\Upsilon \to \eta_b\gamma$

$D \to P\ell\nu$ partial width in terms of Form Factors

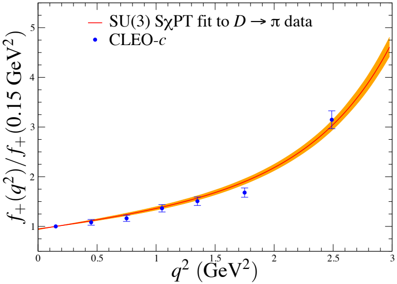
$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = |V_{cx}|^2 \left[K_+\left(q^2\right)\left|f_+^{D\to P}\left(q^2\right)\right|^2 + K_0\left(q^2\right)\left|f_0^{D\to P}\left(q^2\right)\right|^2\right]$$

Preliminary results

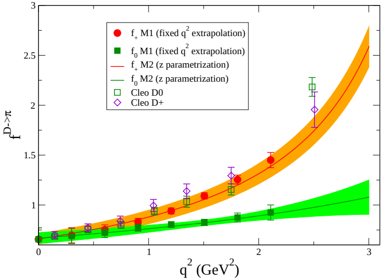
HPQCD; Charm'12 conference



Fermilab + MILC; Latt. '12



D.Becirevic, F.S, V.Lubicz; Latt.'13



Remarks:

- Preliminary results
- More computations devoted just to $f_{+/0}\left(q^2=0\right)$
- f_+ not described by single pole in VMD model

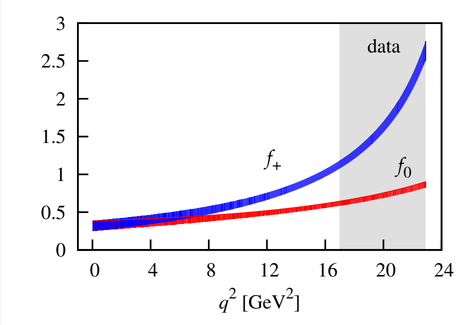
Form factors for $B \to K^{(*)}\ell^+\ell^-$ decays – cf. talks by Descotes-Génon and Altmannshofer

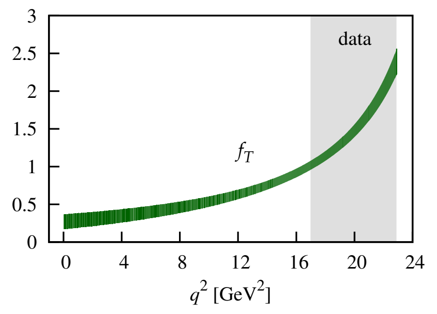
$B \to K\ell^+\ell^-$ - three form factors

$$\langle K(k)|\bar{b}\gamma_\mu s|B(p)\rangle \propto f_+(q^2), f_0(q^2) \qquad \langle K(k)|\bar{b}\sigma_{\mu\nu}q^\nu s|B(p)\rangle \propto f_T(q^2)$$

HPQCD, C. Bouchard et al, PRD88(2013)054509, PRL111(2013)162002

- Staggered light quarks and the non-relativistic expansion of the b-quark on the lattice





- Final results = bands of fitted values fit result, lattice data available only in the shaded area

Remarks

- Major improvement over the quenched results [cf. PRD86(2012)034034]
- New and old values for $f_{0,T}(q^2)$ consistent, new value for $f_+(q^2)$ lower then before
- The new $f_+(m^2_{J/\psi})/f_0(m^2_{\eta_c})$ suggests a sizable violation of the factorization approximation in $\mathcal{B}(B \to \eta_c K)/\mathcal{B}(B \to J/\psi K)$ [cf. arXiv:1312.2858]

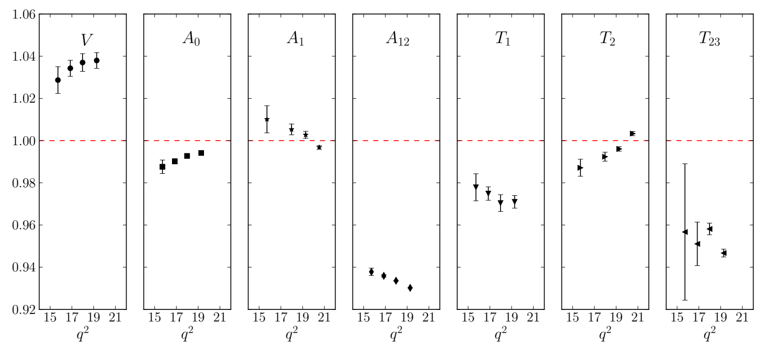
Form factors for $B \to K^{(*)}\ell^+\ell^-$ decays – cf. talks by Descotes-Génon and Altmannshofer

$B \to K^*\ell^+\ell^-$ - seven form factors

$$\langle K^*(k,\varepsilon)|\bar{b}\gamma_\mu s|B(p)\rangle \propto V(q^2) \qquad \langle K^*(k,\varepsilon)|\bar{b}\gamma_\mu\gamma_5 s|B(p)\rangle \propto A_1(q^2), A_2(q^2), A_0(q^2)$$

$$\langle K^*(k,\varepsilon)|\bar{b}\sigma_{\mu\nu}q^\nu s|B(p)\rangle \propto T_1(q^2),\, T_2(q^2),\, T_3(q^2)$$

The case of $B_s \to \phi\ell^+\ell^-$, R.R.Horgat et al, arXiv:1310.3722



- Where $A_{12} = f[A_1(q^2), A_2(q^2)]$ and $T_{23} = f[T_2(q^2), T_3(q^2)]$
- Need results with different approach to heavy quark and light quarks other than staggered

$B \to D\ell\nu$ decays

Popular test of New Physics

$$R(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}, \qquad R(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu)}, \ (\ell = e, \mu)$$

Ratios useful to cancel/reduce theoretical uncertainties in V_{cb}/f.f

BaBar ('12)

$$\boxed{\begin{aligned} R(D) &= 0.440 \pm 0.058 \pm 0.042, & \color{red}{R(D)^{SM}} &\color{red}{= 0.31 \pm 0.02} \\ R(D^*) &= 0.332 \pm 0.024 \pm 0.018, & R(D^*)^{SM} &= 0.252 \pm 0.003 \end{aligned}}$$

- Larger than the SM expectations! New Physics?
- $B \to D\ell\nu$ needs form factors $f_{+,0,T}^{B\to D}$ to check SM and constraint the NP contribution

Form factors for $B_{(s)} \to D_{(s)}$

- Convenient parametrization (HQET motivated) in terms of $\mathcal{G}(w)$

$$\frac{1}{\sqrt{m_{B_{(s)}} m_{D_{(s)}}}} \langle D_{(s)}(k) | V_\mu | B_s(p) \rangle \propto \mathcal{G}(w) + corr.$$

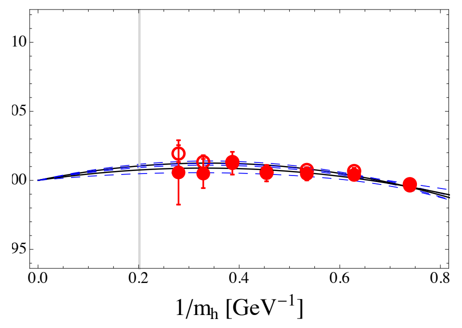
- $\mathcal{G}(1) = 1$ up to radiative and $1/m_h$ correction
- Compute the true $\mathcal{G}(1)$ on the lattice

$B \to D\ell\nu$ decays

New results, M.Atoui et al., arXiv:1310.5238

- Define: $\mathcal{G}(1,\, m_b,\, m_c) = \sigma_n\sigma_{n-1}...\sigma_1\sigma_0 \cancelto{1}{\mathcal{G}\left(1,\, m_c,\, m_c\right)}$, $\quad \sigma_i = \frac{\mathcal{G}(1, \lambda m_h, m_c)}{\mathcal{G}(1, m_h, m_c)}$
- In the elastic case $D_{(s)} \to D_{(s)}$ by definition $\mathcal{G}(1) = 1$
- Towards the static h-quark: $\lim_{m_h \to \infty} \sigma(m_h) = 1$
- Extrapolate constrained σ from c to b, reconstructing $\mathcal{G}\left(1\right)$ from the chain of products

Results & comparison with previous studies



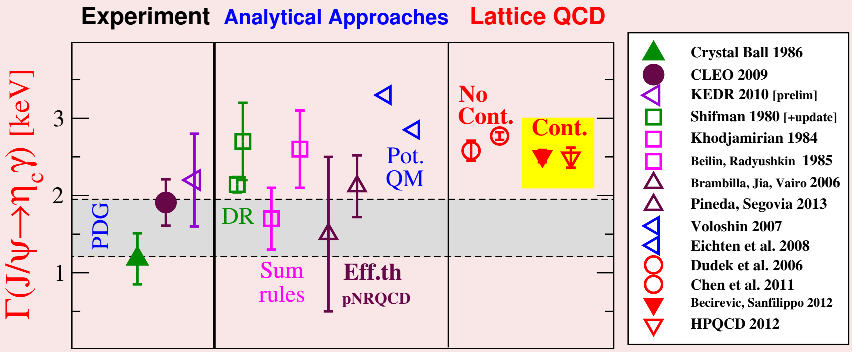
- Final Results: $\mathcal{G}^{B_s \to D_s}\left(1\right) = 1.052(46)$ Unquenched, full QCD heavy quark
- De Divitiis et al. (Phys.Lett.B '07): 1.026(17)
  - ✓ Step scaling method
  - ✗ Quenched
- MILC+Fermilab: 1.074(24), Lattice '04

The case of $B \to D^*\ell\nu$

Very recently Fermilab + MILC reported: $\mathcal{F}\left(1\right) = 0.906(4)(12)$, arXiv:1403.0635

Radiative decays of charmonia

$J/\psi \to \eta_c\gamma$ puzzle solved?



Tension in $h_c \to \eta_c\gamma$?

- D.Becirevic, F.S (2012): $\Gamma_{h_c} = \frac{\Gamma^{LAT}(h_c\to\eta_c\gamma)}{\mathrm{Br}^{\mathrm{Bes\ III}}(h_c\to\eta_c\gamma)} = 1.37(23)\,\mathrm{MeV}$, JHEP 1301 (2013)
- BES III: $\Gamma_{h_c}^{incl} = 0.73(45)(28)\,\mathrm{MeV}$, $\Gamma_{h_c}^{excl} = 0.70(28)(22)\,\mathrm{MeV}$, X Confinement 2012

Future direction of research

- Charmonium excited states decay (e.g. $\eta(2S) \to J/\psi$)
- Bottomonium system, useful for light CP-odd Higgs research

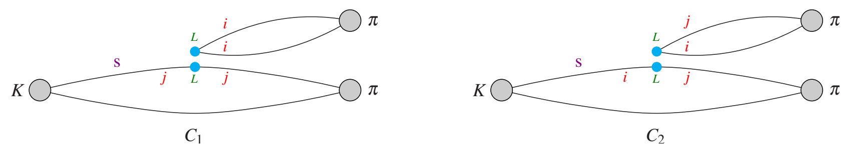
A look into the future

Can we understand CP-violation in difference of $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$?

[LHCb, PRL 108 '12]

"Emerging understanding of the $\Delta I = 1/2$ rule from Lattice QCD"

RBC-UKQCD Collaboration, arXiv:1212.1474



Observed cancellation between diagrams in one isospin channel, and not in the other

Decays beyond inelastic threshold

- Method used for $K \to \pi\pi$ works only for a single final state containing two particles This is evidently not the case for D decays, admitting $\to \pi\pi$, $\to KK$, $\to \pi\pi\pi\pi$...
- First step to include multiple more channels proposed by Hansen and Sharpe, Phys.Rev. D86 (2012): Inclusion of channels with multiple two-particle states $\to \pi\pi$, $\to KK$)
- Still unknown how to treat decay channels admitting many-particle final states with the same quantum number (that is, $D \to \pi\pi\pi\pi$, etc)
- Other side of the coin: resonances above inelastic threshold remains non treatable

Future perspectives

Current status forty years after the formulation of LQCD

Lattice Calculations in the precision era

1. Many simulations include $N_f = 2 + 1 + 1$ physical quarks
2. Continuum limit extrapolation under control for charm physics
3. Many different methods to study b-physics allow for crosschecks
4. Emerging consensus on Lattice averages

Next steps

1. Include Isospin Breaking/Electromagnetic effects
2. Simulate at lattice spacing small enough to treat directly the physical b quark
3. $g - 2$, rare kaon decay amplitudes
4. Blind analysis

Long time perspectives

1. Hadronic decays above inelastic thresholds & full understanding of resonance spectrum
2. Calculate $K_L - K_S$ mass difference and ε_K to sub-percent accuracy
3. Cross-analysis among collaborations?