Fermion Masses and Mixing from a Minimum Principle

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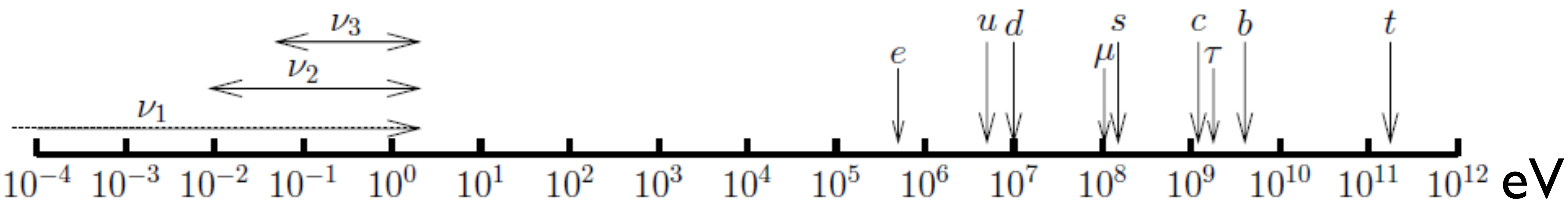
Work in collaboration with M.B. Gavela, L. Merlo, D. Hernandez, G. Isidori, L. Maiani, S. Rigolin

Flavour Puzzle

why are masses so separated from one another?



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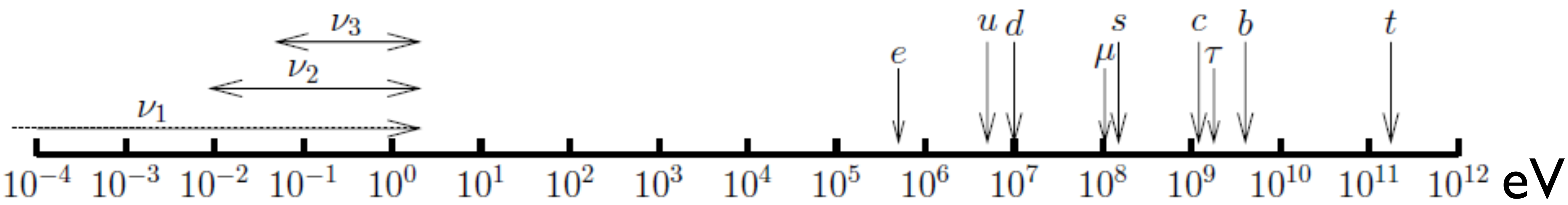

lightness of neutrinos
due to a Majorana character?

Flavour Puzzle

why is the mixing pattern so different?

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

Leptons

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

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Maybe because of the
Majorana Character?

The Standard Theory

Gauge Group

Fermions

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	$\frac{1}{6}$
U_R	3	1	$\frac{2}{3}$
D_R	3	1	$-\frac{1}{3}$
ℓ_L	1	2	$-\frac{1}{2}$
E_R	1	1	-1

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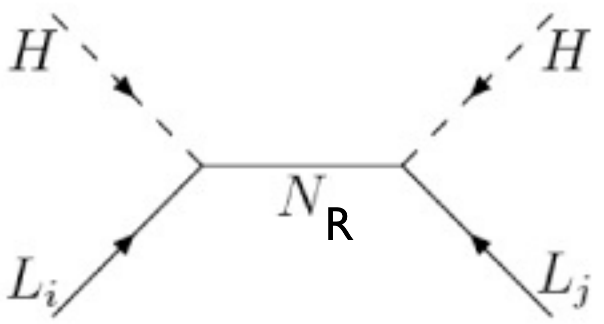
E_R	1	1	-1
N_R	1	1	0

Neutrino Masses: type I seesaw

Addition of a gauge singlet

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
N_R	1	1	0



with the only "bare" and Majorana mass allowed

$$\mathscr{L}_{seesaw} = -\bar{\ell}_L Y_\nu \tilde{H} N_R - \overline{N}_R^c \frac{M}{2} N_R + h.c.$$

after EWSB and
for vY<<M

$$m_\nu = Y_\nu \frac{v^2}{2M} Y_\nu^T$$

$Y_\nu \sim 1 \; M \sim 10^{15}\text{GeV}$

$Y_\nu \sim 10^{-6} \; M \sim 10^{3}\text{GeV}$

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi$$

that is:

$$U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

$\mathcal{G}^q_{\mathcal{F}}$ Quarks

[Georgi, Chivukula, 1987
D'Ambrosio, Giudice, Isidori, Strumia, 2002]

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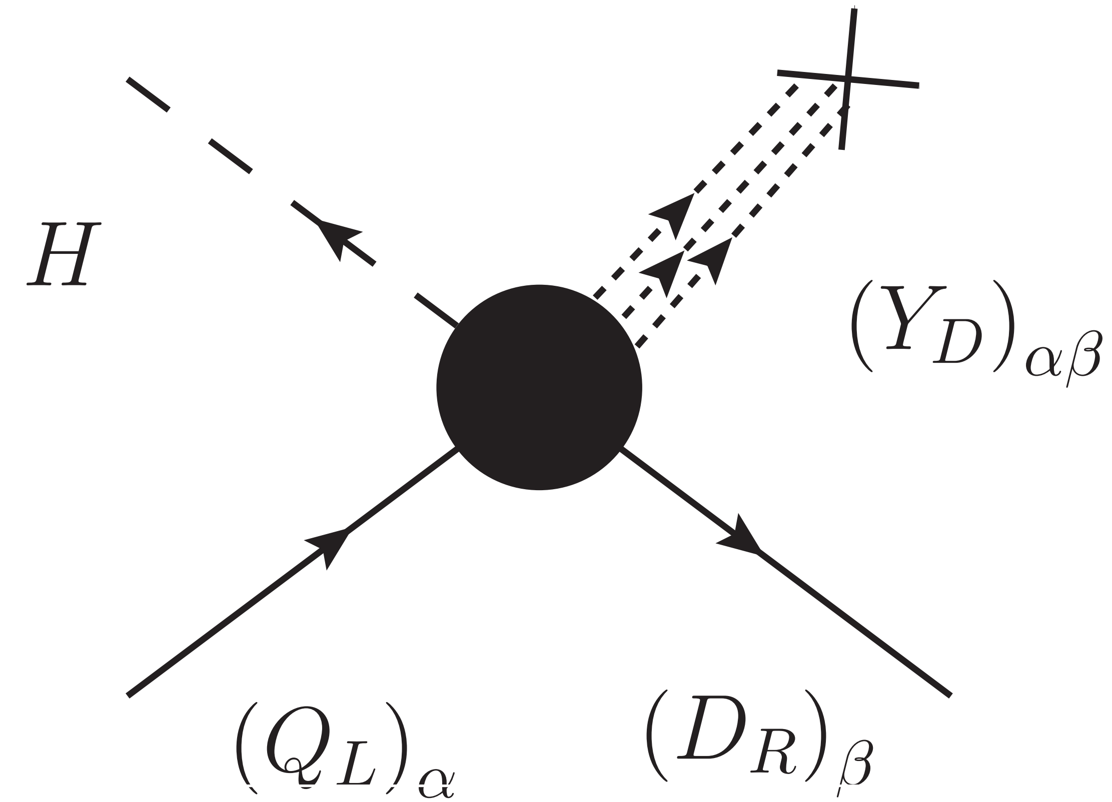
that is:

$$U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \times U(3)_{\ell_L} \times U(3)_{E_R} \times O(3)_{N_R}$$

$\mathcal{G}^q_{\mathcal{F}}$ Quarks	$\mathcal{G}^l_{\mathcal{F}}$ Leptons

Flavour Symmetry Breaking

...which we assume spontaneously broken to generate the Yukawa couplings



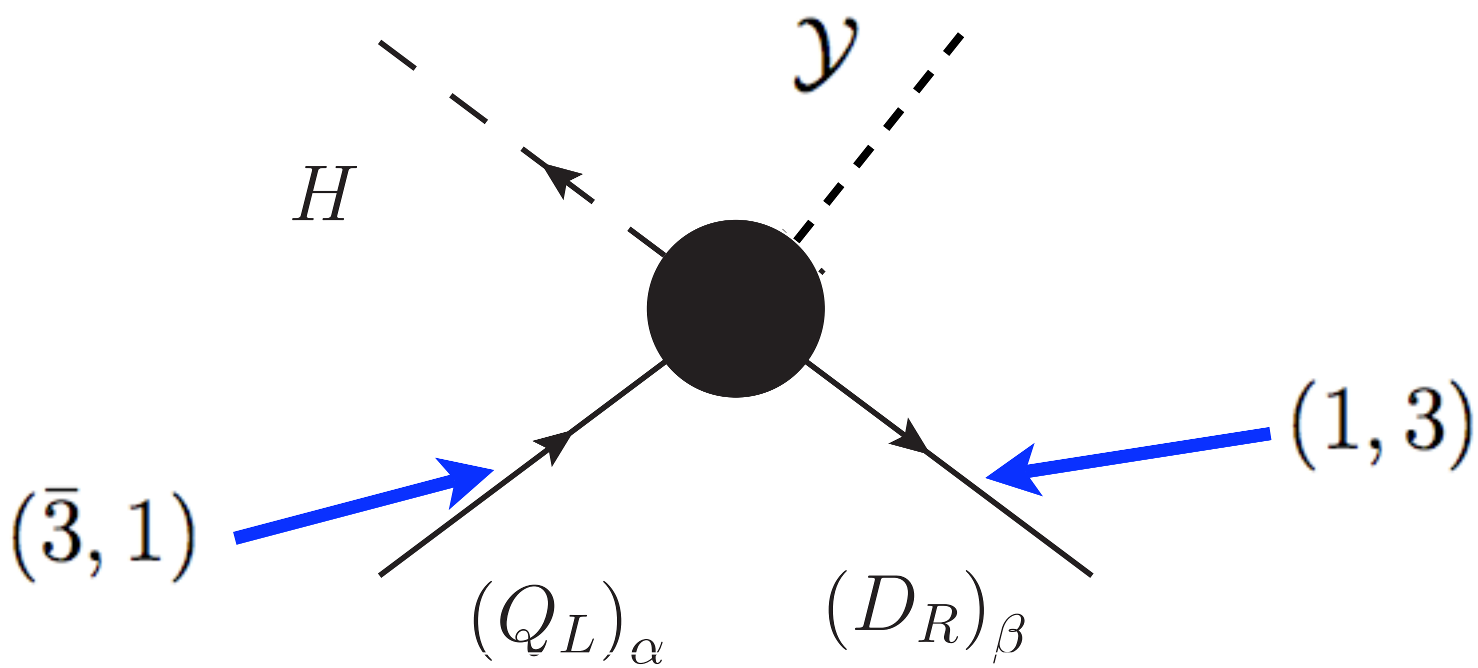
[C. D. Froggat, H. B. Nielsen
Berezhiani, Rosi,
Cabibbo, Maiani
Michel, Radicati ...]

$$Y = \frac{\langle \Phi^n \rangle}{\Lambda_f^n}$$

Flavour Fields

...which we assume spontaneously broken to generate the Yukawa couplings

$$\overline{Q}_L \frac{\mathcal{Y}}{\Lambda_f} U_R \tilde{H}$$



$n = 1$ (d=5)

A single and therefore "bi-fundamental" field

$\mathcal{Y} \sim (3, \bar{3})$

Flavour Field's Scalar Potential

The potential shall respect

- *Gauge invariance* $\mathcal{G}$
- *Flavour invariance* $\mathcal{G}_{\mathcal{F}}$

$$V(\Phi) = V(I(\Phi))$$

This means that the potential depends on invariant combinations of the fields: "*I*"

Minimization

a variational principle fixes the vevs of the Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i}\frac{\partial V}{\partial I_j} \equiv J_{ij}\frac{\partial V}{\partial I_j} = 0\,,$$

This is an homogenous linear equation;

study rank of the Jacobian $J_{ij} = \partial I_j / \partial y_i,$

Natural Breaking Patterns

$$\det(J) = 0$$ [Cabibbo, Maiani, 1969]

identifies especial solutions
with unbroken symmetry $\mathcal{H}$

$$\mathcal{G} \to \mathcal{H}$$

The "most" natural ones are the maximal subgroups

guaranteed a extremum point in any function [Michel, Radicati, 1969]

e.g. $SU(5) \to SU(3) \times SU(2) \times U(1)$ [Glashow, Georgi, 1974]

Quarks

Bi-fundamental Flavour Fields

eigenvalues only

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right],$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right].$$

eigenvalues and mixing

[Feldmann, Jung, Mannel; Jenkins, Manohar]

Jacobian Analysis: Mixing

$$\det(J_{UD}) = \left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)$$
$$\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)$$
$$\times |V_{ud}||V_{us}||V_{cd}||V_{cs}|$$

the rank is reduced the most for:

V_{CKM}= PERMUTATION

no mixing: reordering of states

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

$$\not{\mathcal{G}}_{\mathcal{F}}^{q} \quad : \quad U(3)^3 \to U(2)^3 \times U(1)$$

a hierarchical mass spectrum without mixing

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

[RA, Gavela, Merlo, Rigolin]

Leptons

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

$$\#\,\text{d.o.f. in } \mathcal{Y}_{E,\nu} \text{ - } \dim(\mathcal{G}^{\ell}_{\mathcal{F}}) = 15$$
$$2\times 18 \qquad\qquad 2\times 9+3$$

The better suited parametrization is the bi-unitary

$$\mathcal{Y}_\nu = \Lambda_f\, \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathcal{Y}_E = \Lambda_f \mathbf{y}_E\,;$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

the connection with neutrino masses being

$$U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu \mathcal{U}_L^T\,,$$

Leptons

Eigenvalues only

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right],$$

$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right], \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

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$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

New Invariants w.r.t. Quarks

Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for V_{CKM}

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times\,|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}|$$

the rank is reduced the most for $\mathcal{U}_R\mathcal{U}_R^T$ *being a permutation*

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...but **maximal** mixing

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}, \qquad m_{\nu_2} = \frac{v^2}{M} y_{\nu_2}y_{\nu_3}\,, \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2\,.$$

+degeneracy and maximal Majorana phase

[RA, Gavela, Hernandez, Merlo, Rigolin
RA, Gavela Isidori, Maiani]

Lepton Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern:

$$\not{\mathcal{G}}^l_{\mathcal{F}} \quad : \quad U(3)^2 \times O(3) \to U(2) \times U(1)$$

brings along hierarchical charged leptons

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1} & 0 & 0 \\ 0 & y_{\nu_2}/\sqrt{2} & -iy_{\nu_2}/\sqrt{2} \\ 0 & y_{\nu_3}/\sqrt{2} & iy_{\nu_3}/\sqrt{2} \end{pmatrix},$$

and (at least) two degenerate neutrinos and maximal angle and Majorana phase

$\theta_{23} = 45°$;

Majorana Phase Pattern (1,1,i)

& Mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix}U^T_{PMNS} = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

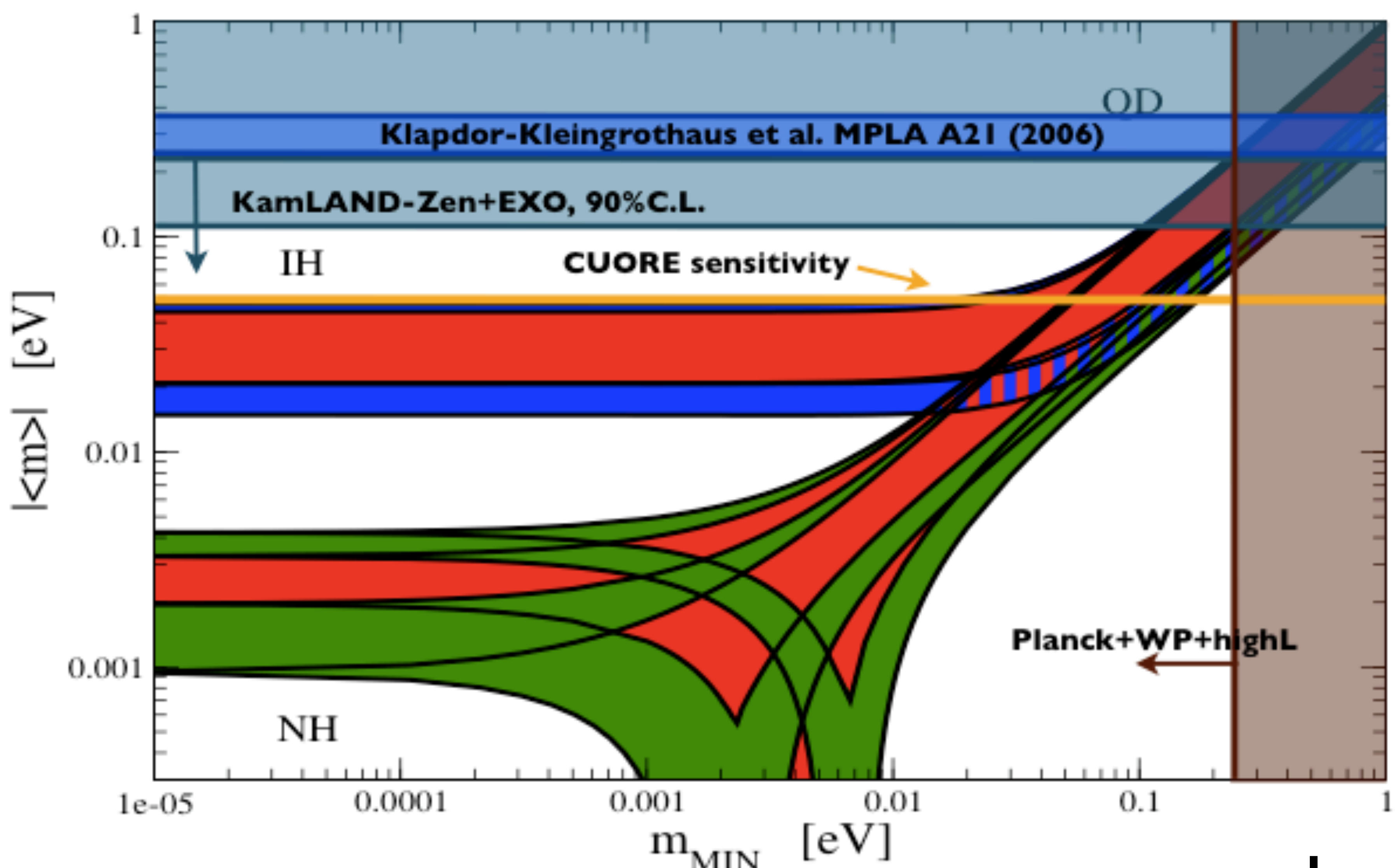
$\theta_{23} \simeq \pi/4$, θ_{12} large , $\theta_{13} \simeq \epsilon$

Majorana Phases $(e^{i\alpha}, 1, i)$

degenerate spetrum

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!



rough estimate

~0.1eV

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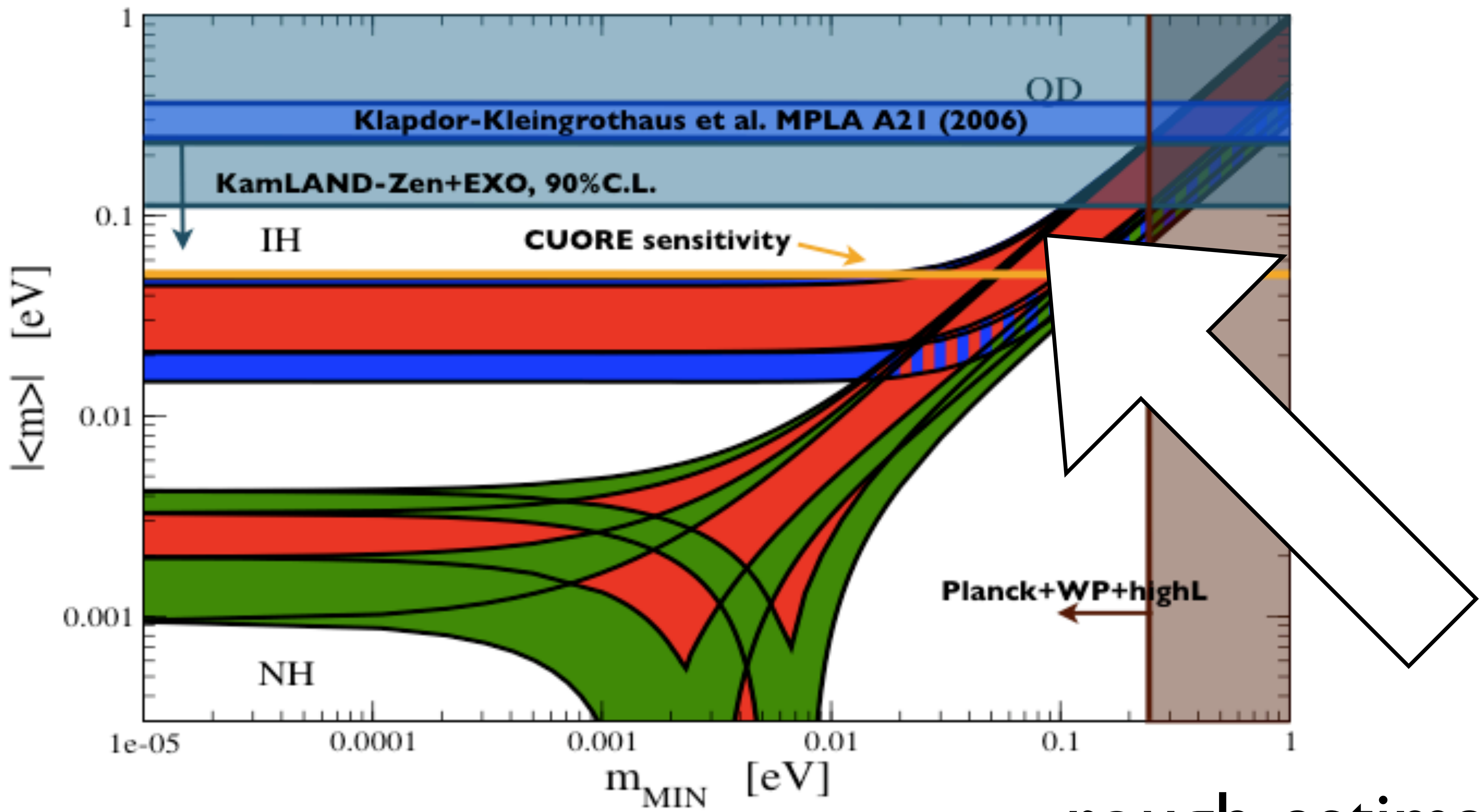

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Where do the differences in Mixing originate?

From the

MAJORANA vs DIRAC nature of fermions

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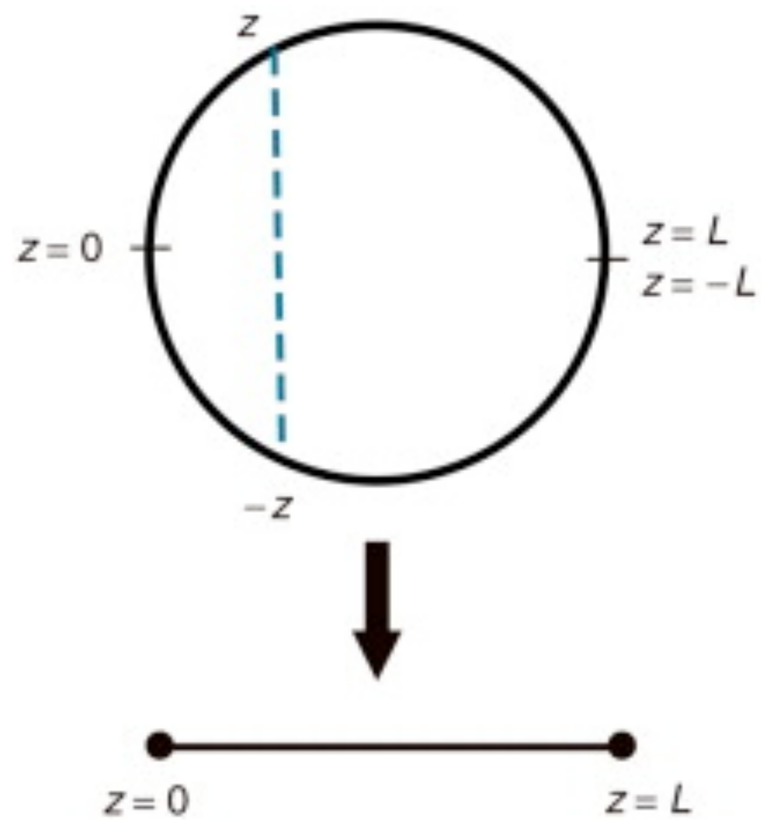
THANK YOU

Backup

Boundaries Exhibit Unbroken Symmetry

Extra-Dimensions Example



<u>The smallest boundaries are extremal points of any function</u>

[Michel, Radicati, 1969]

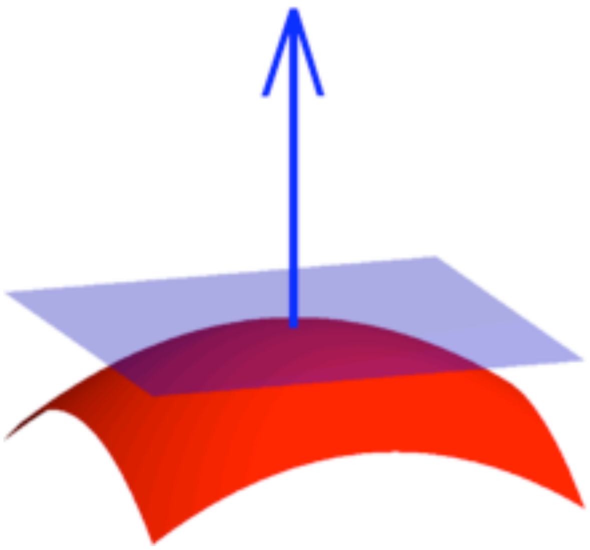
Boundaries

for a reduced rank of the Jacobian,

$$\det(J) = 0$$

there exists (at least) a direction δy_i in which a variation of the field variables does not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



that is a Boundary of the *I-manifold*

[Cabibbo, Maiani, 1969]