$B_s \to \mu^+ \mu^-$ and Electroweak Interactions

"49th Rencontres de Moriond 2014
Electroweak interactions and Unified Theories"
La Thuile
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Based on works with
Christoph Bobeth, Emanuel Stamou [PRD 89, 034023 (2014)]
Christoph Bobeth, Thomas Hermann, Mikolaj Misiak, Emanuel Stamou and Matthias Steinhauser [PRL 112, 101801 (2014)]

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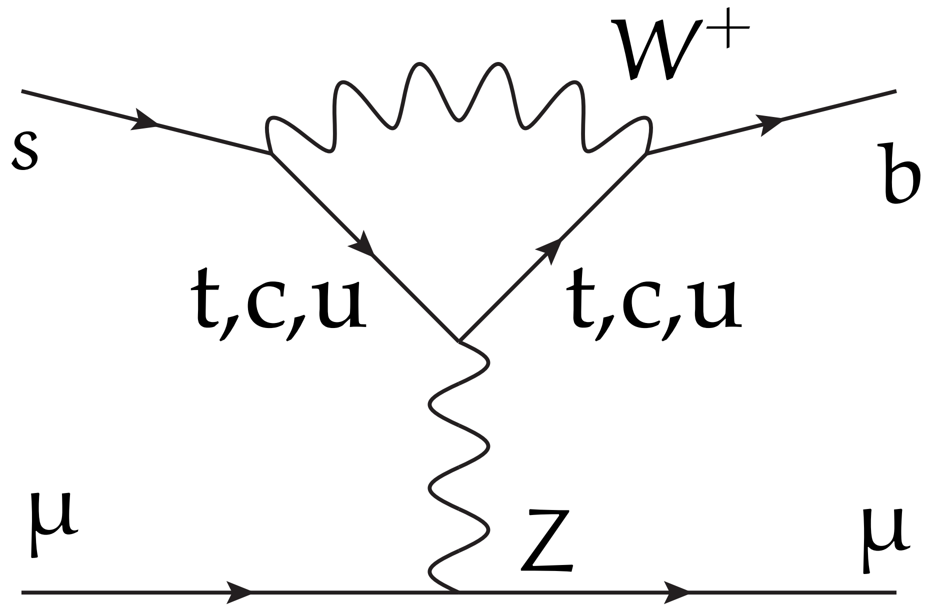
$B_s \rightarrow \mu^+ \mu^-$ in the Standard Model



+ Box diagrams

B_s is pseudoscalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l} \gamma_\mu \gamma_5 l)$$

Dominant operator in the SM

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

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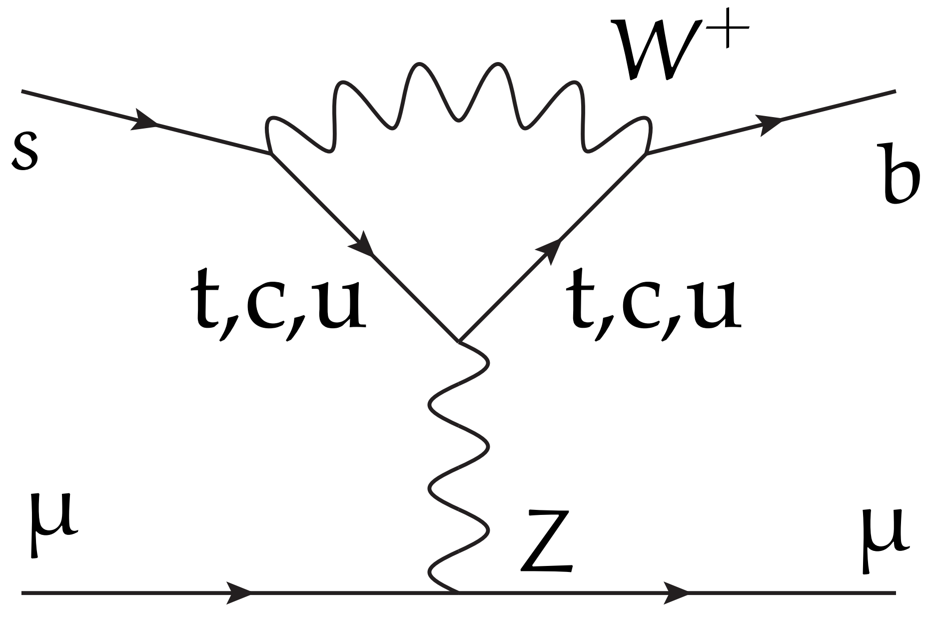

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Effective Lagrangian in the SM:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \left(C_A Q_A + C_S Q_S + C_P Q_P \right) + \text{h.c.}$$

Scalar operators: $Q_S = (\bar{b}_R q_L)(\bar{l} l)$ $\quad Q_P = (\bar{b}_R q_L)(\bar{l} \gamma_5 l)$

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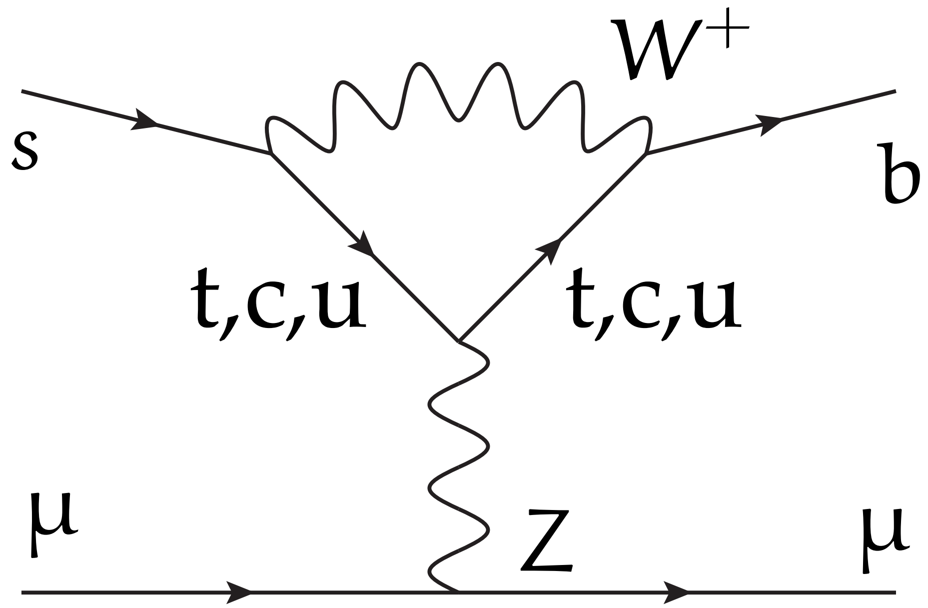

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Old normalisation [Buras ...]:

$$\mathcal{L}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} \left(C_S Q_S + C_P Q_P + C_A Q_A\right)$$

Theory Status at NLO

Standard Model: C_S & C_P are highly suppressed

C_A is known at NLO in QCD [Buras, Buchalla; Misiak, Urban `99]

$C_A(m_t\ /\ M_W)^{NLO} = 1.0113\ C_A(m_t\ /\ M_W)^{LO}$

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For pure QCD determine $< \mu^- \mu^+ | Q_A | B_s >$ from
$< 0\ | \bar{b}\, \gamma^\mu \gamma_5\, s | B_s > = i\, p^\mu f_{Bs}$ (f_{Bs}= 227.7(4.5)MeV [FLAG])

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QED & Electroweak are considered only at LO

QED corrections

B_s decay into a 2 lepton final state always helicity suppressed

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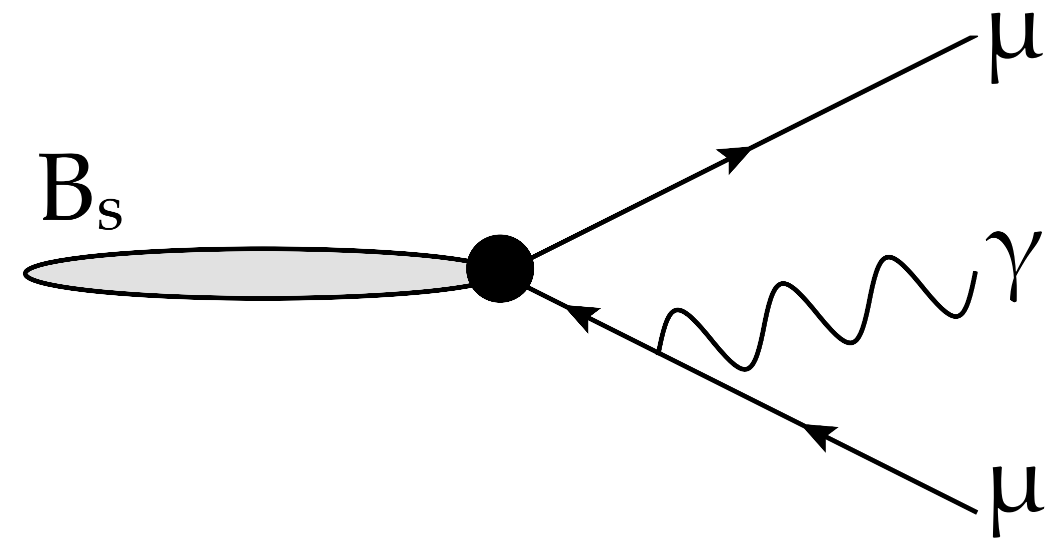
Soft photon radiation from muons: Theoretical branching ratio is fully inclusive of bremsstrahlung. There would be sizeable corrections otherwise [Buras, Girrbach, Guadagnoli, Isidori] arXiv:1208.0934.



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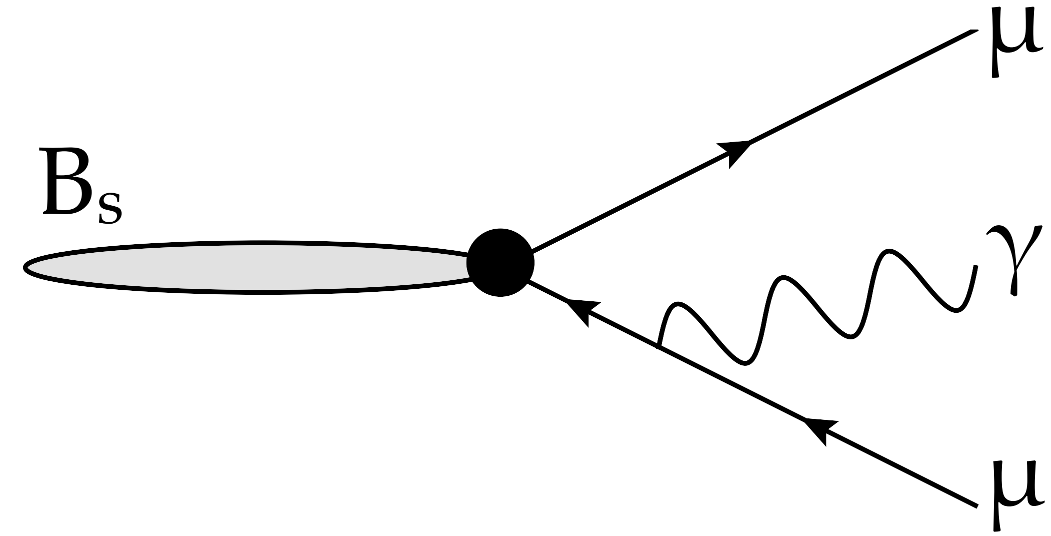
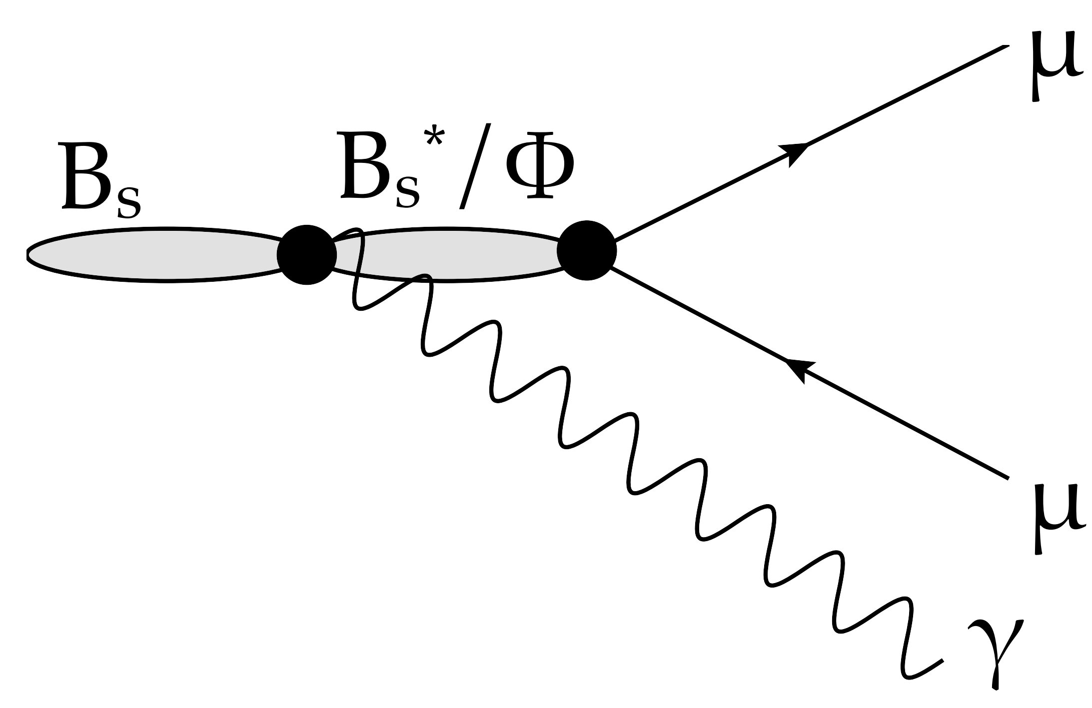
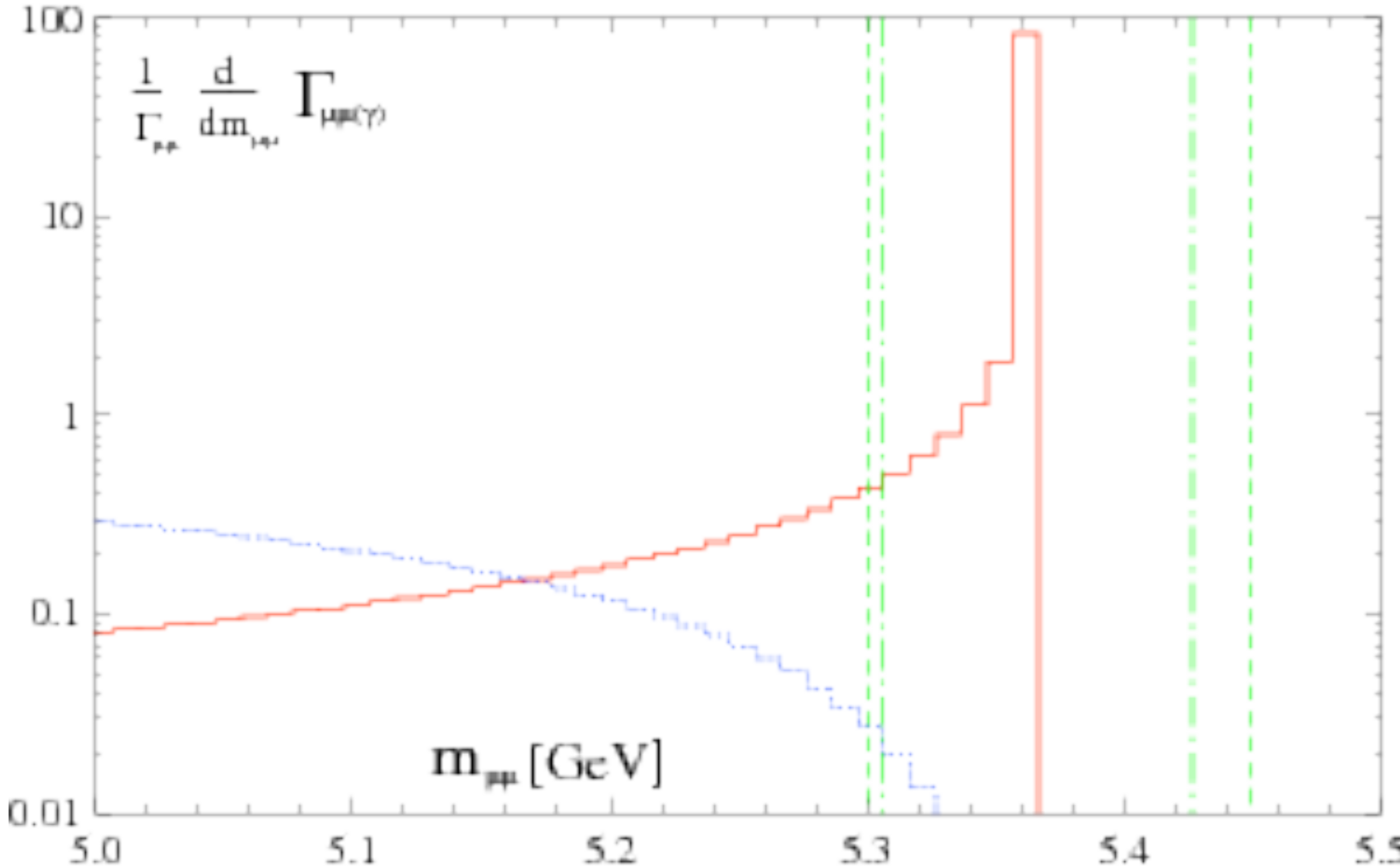


Direct emission is IR safe (B_s is neutral) and phase space suppressed for invariant mass $m_{\mu\mu}$ close to M_{Bs}. [Aditya, Healey, Petrov] arXiv: 1212.4166



Illustration

Consider an experimental signal window for the invariant mass of the muon pair $m_{\mu\mu}$



Simulate signal fully inclusive of bremsstrahlung (PHOTOS)

Direct emission is a background in the signal window

Comparing Theory and Experiment

Bremsstrahlung taken into account by the experiment and direct emission treated as background.

The B_s system has a non-zero decay width difference:
$\rightarrow$ instantaneous $\neq$ time integrated branching ratio
[de Bruyn, Fleischer et. al. `12] This correction is precisely known.

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→ Only electroweak corrections and QED to $C_A(\mu_b)$ are potentially large – with m_{top}/M_W, $1/s_W$, $\alpha_e \log^2(M_W/m_b)$

Electroweak Corrections

Consider $$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A Q_A + \text{h.c.}$$

$G_F \alpha / \sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F \alpha / \sin^2\theta_W \, C_A(m_t/M_W)$ invariant under electroweak scheme change

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This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

Electroweak Scheme Uncertainties

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^{*} V_{ts}}{\sin^2 \theta_W} C_A\left(\frac{m_t}{M_W}\right) Q_A + \text{h.c.}$$

	MS-bar	OS	unct. $B_s\, \mu^+ \mu^-$
$\sin \theta_W$	0,231	0,223	4 %
m_t(QCD-MS-bar)	163,5 GeV	164,8 GeV	1 %

Electroweak scheme shift larger then pure theory uncertainty quoted in previous literature. There an hybrid of MS-bar (coupings) and on-shell (masses) renormalisation was used.

To find out what is the best scheme we have to explicitly calculate 2-loop electroweak corrections!

Renormalisation Schemes

1. On-shell scheme: Determine M_W including loop corrections from input: results in sin θ_W, m_t and M_W counterterms to $C_A^{(EW)}$.

2. MS-bar scheme: Fit g_1, g_2, v, λ, m_t from data i.e. from G_F, α, M_Z, M_t, M_H

3. Hybrid scheme: Masses on-shell couplings MS-bar

4. OS2: Use $G_F^2 M_W^2$ normalisation and on-shell scheme

Note: QCD is MS-bar renormalised for all schemes
i.e. we use a QCD MS-bar top mass at a fixed scale

Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence

if we go from 1-loop to 2-loop

Note: α(nq=6) used for plot

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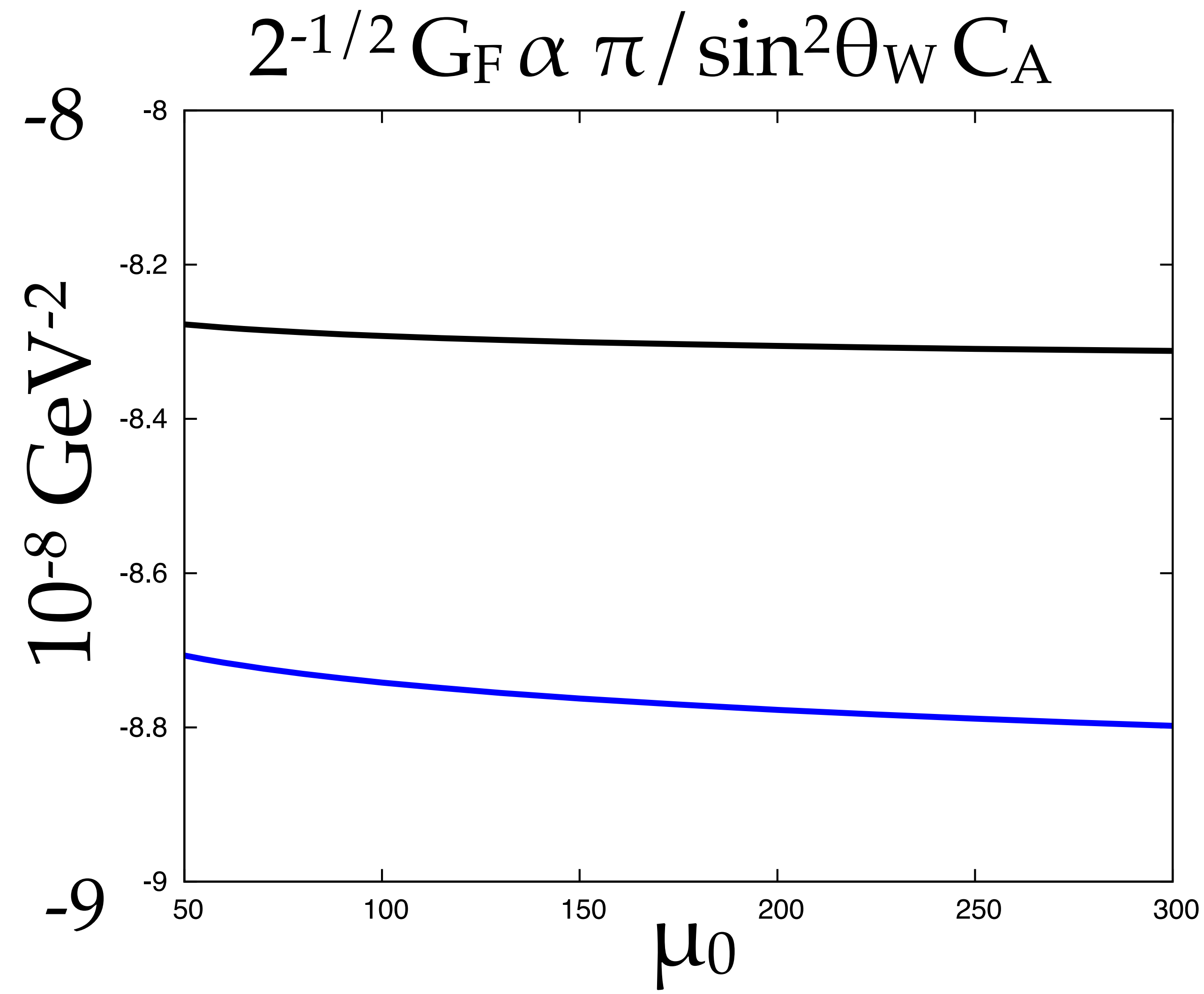

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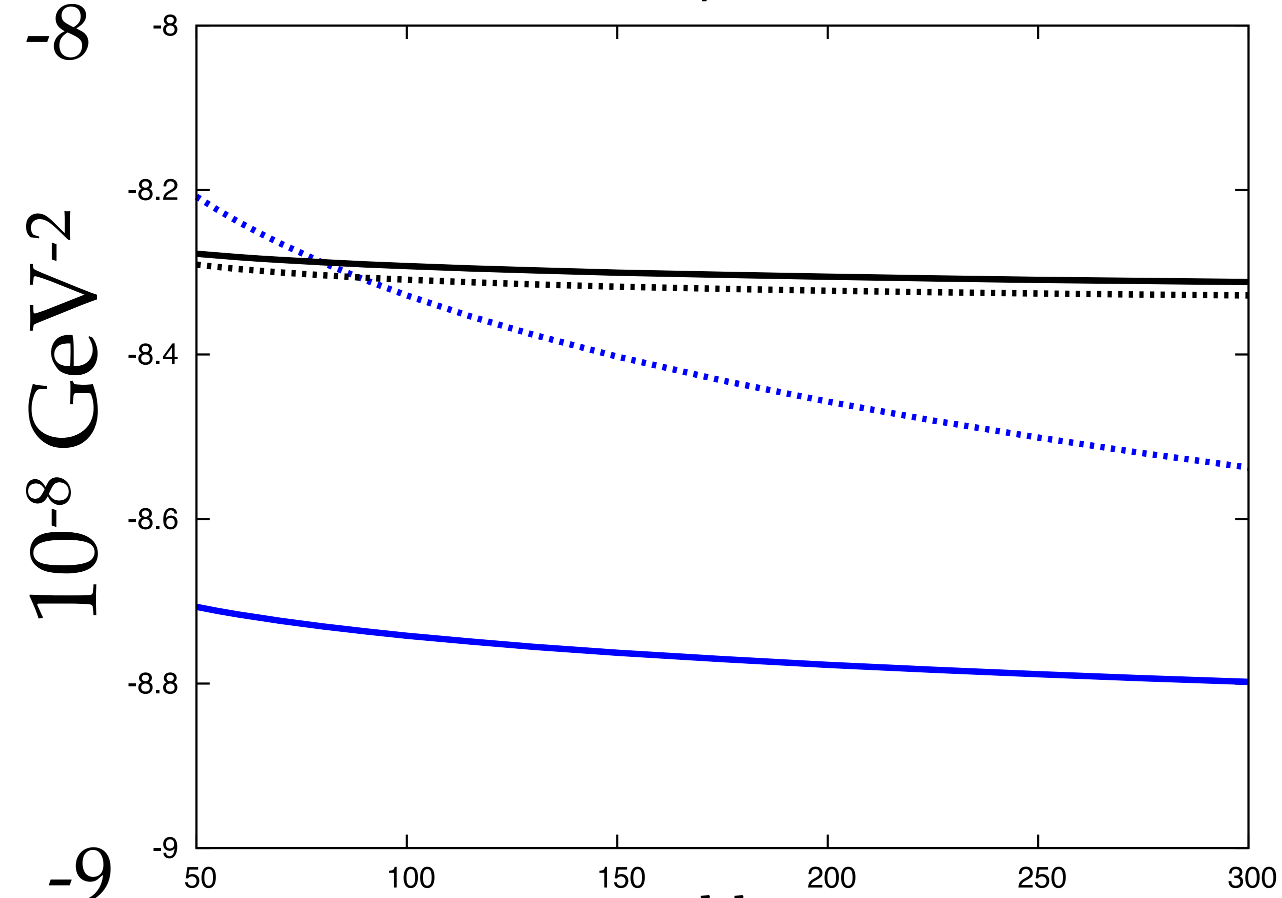
2. large scale dependence for the MS-bar scheme

$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



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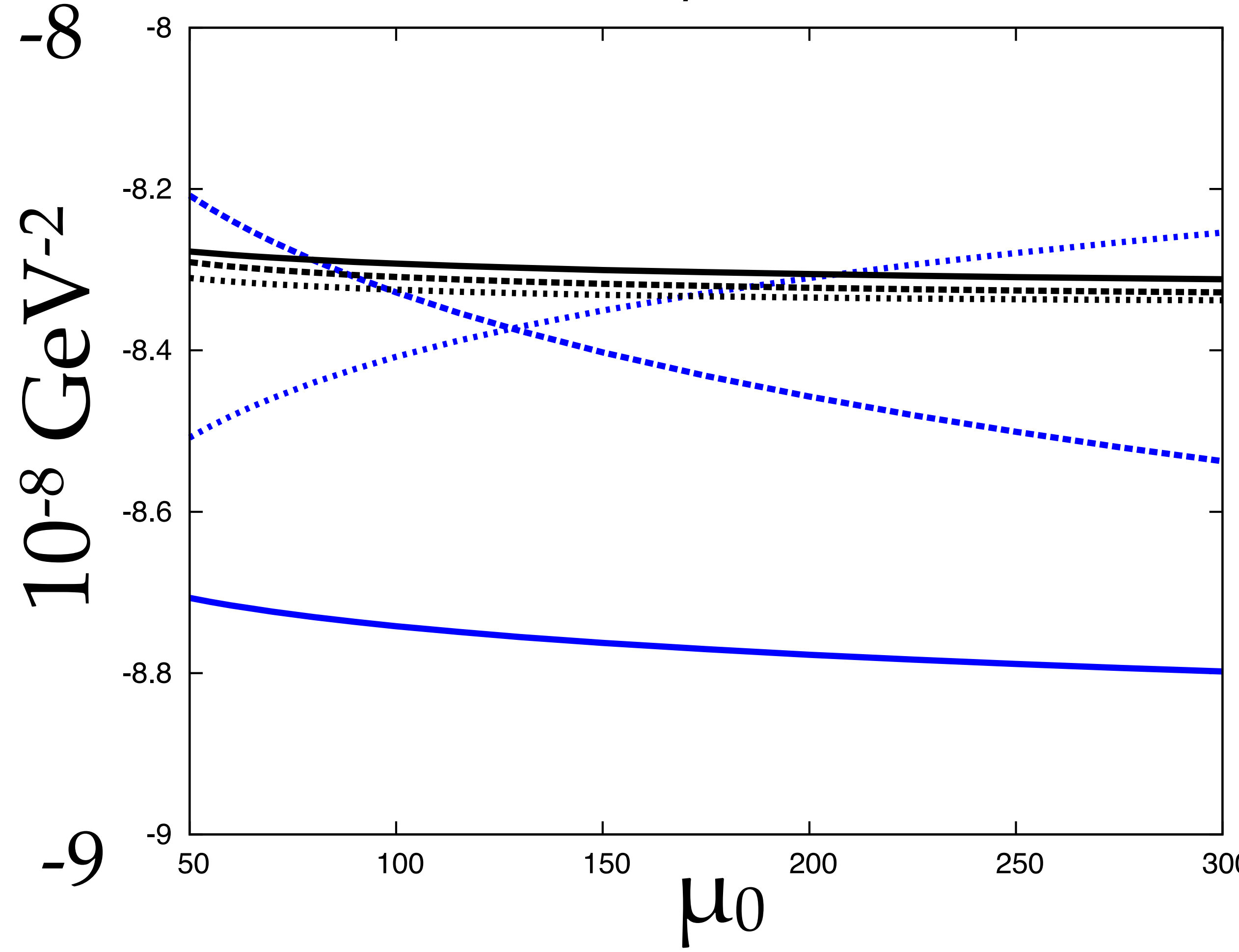

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4. $G_F^2 M_W^2$ normalisation removes `artificial´ scale and parameter dependence

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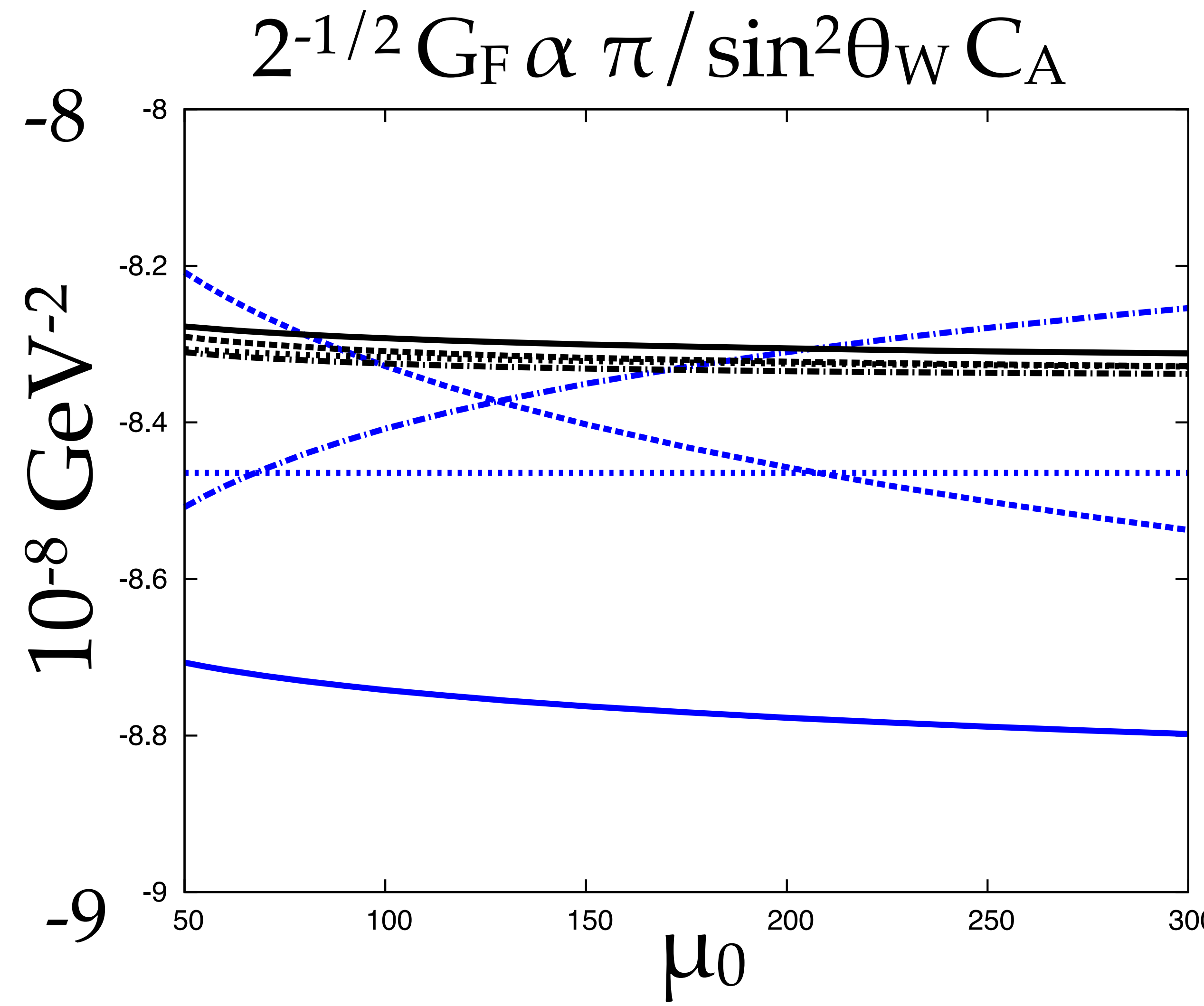
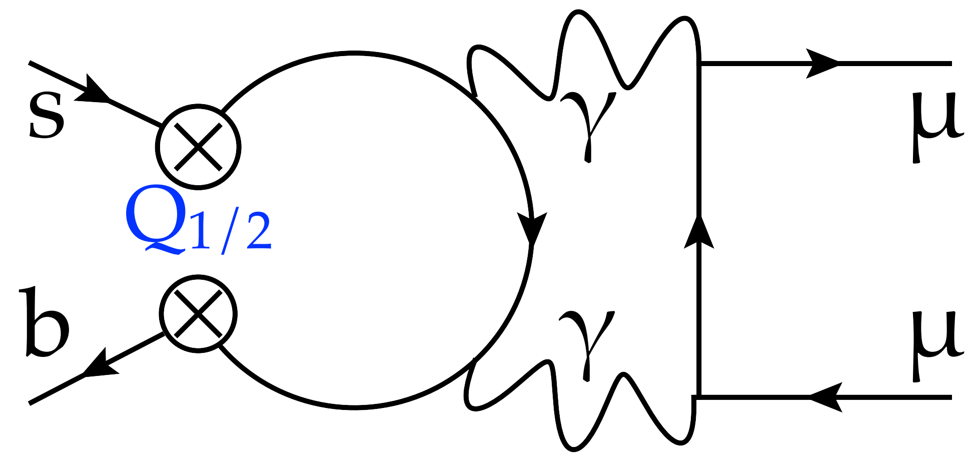


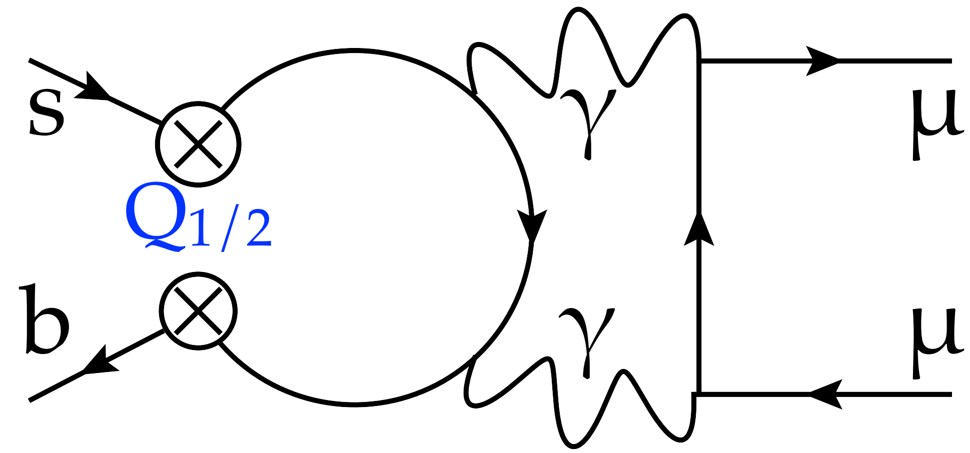
EW corrections reduce modulus of Wilson Coefficient and remove 7 % scale uncertainty in the BR

Renormalisation Group Equation



Renormalisation Group Equation

Log enhanced QED corrections known [Bobeth, Gambino, MG, Haisch `03; Huber et. al. `05, Misiak `11]



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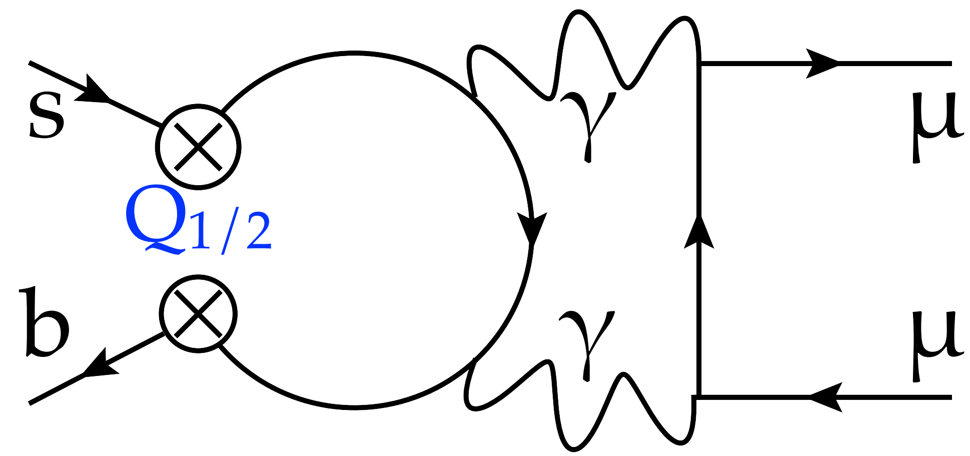
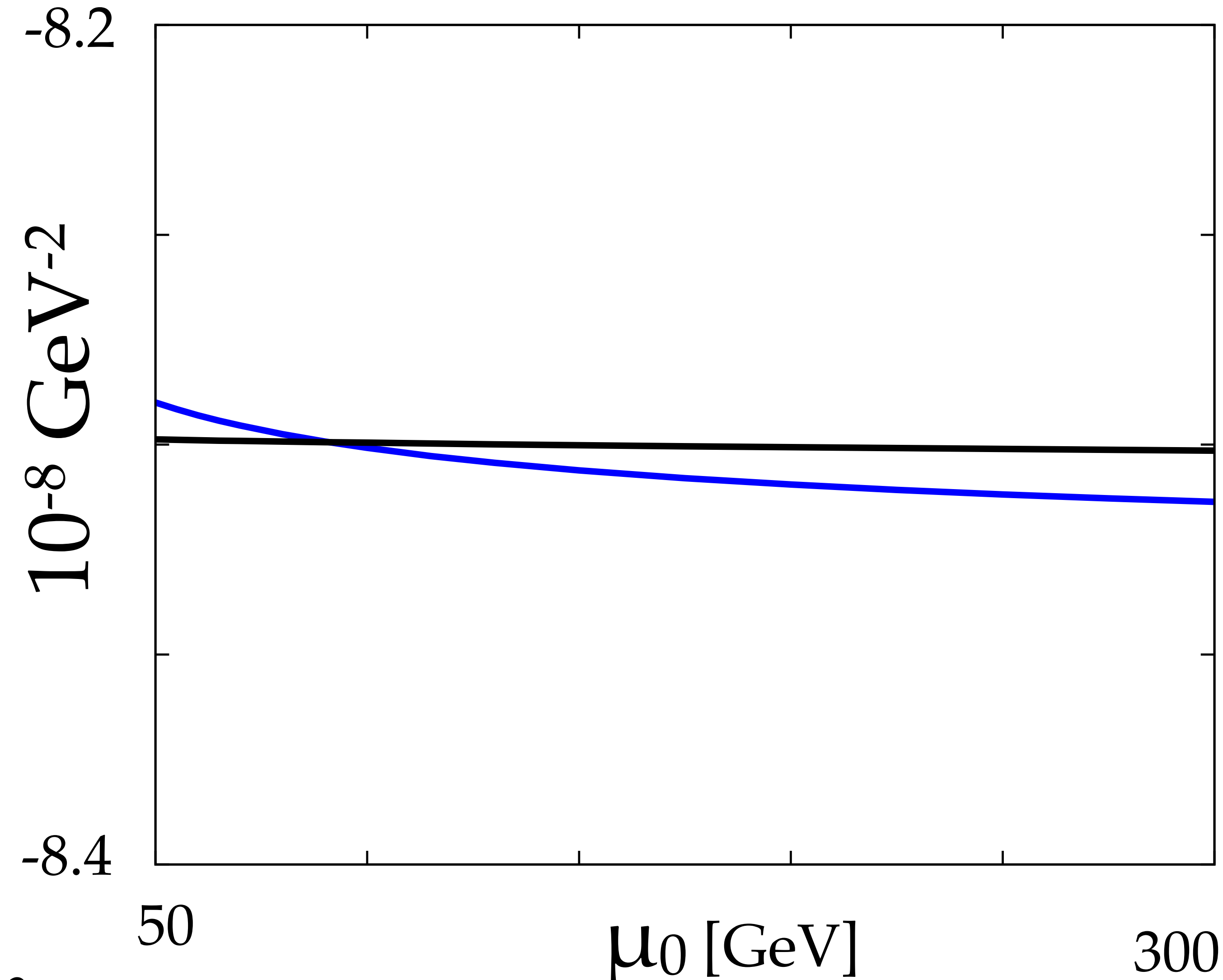
Study residual scale dependence for the G_F^2 M_W^2 normalised results

$G_F^2 M_W^2 C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2 M_W^2 C(\mu_0)$ is only residually scale dependent.



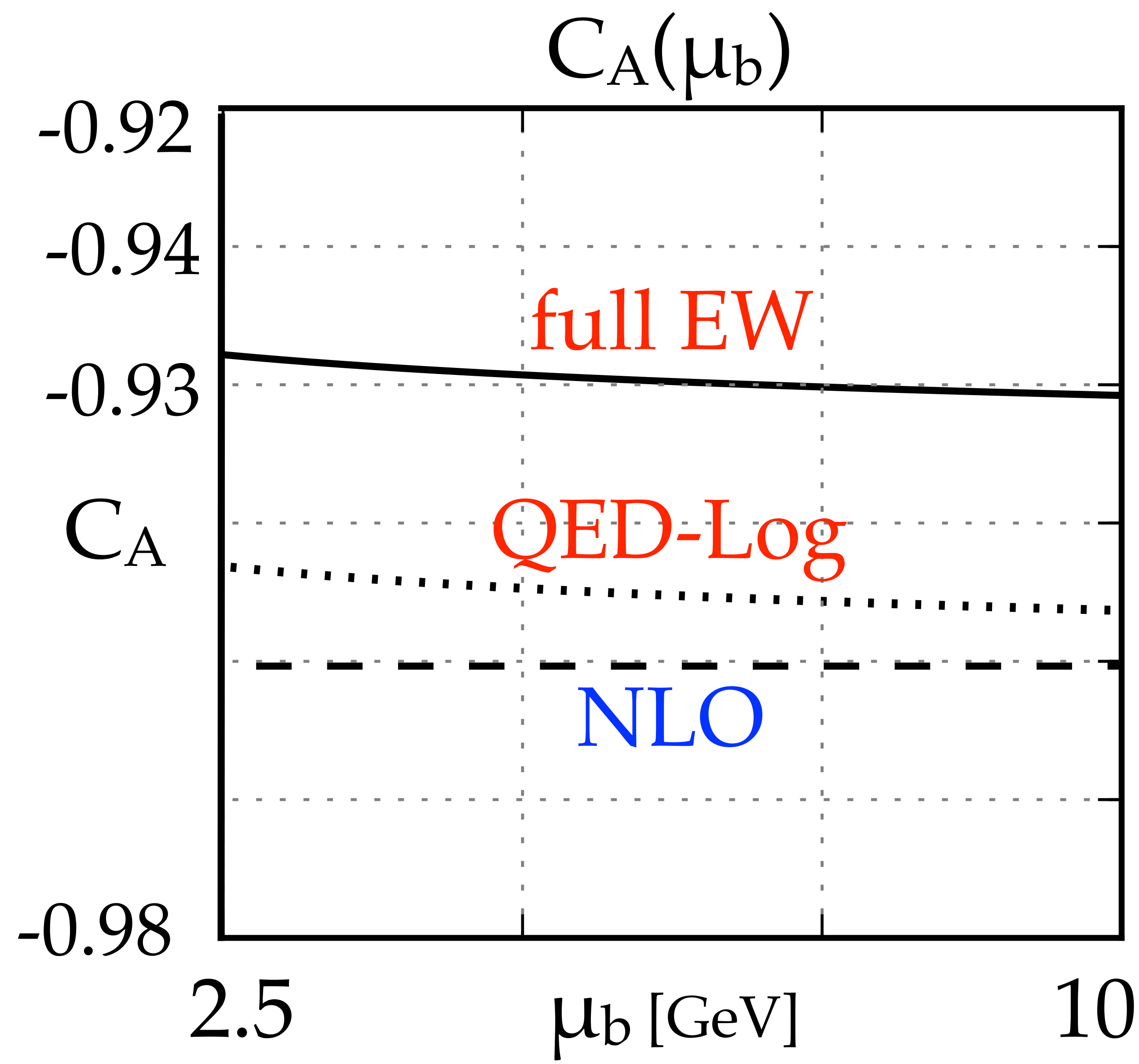
$G^2_F M^2_W C_A(M_Z)$



Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient further.

Varying μ_b in $U(\mu_b, m_t)\, G_F^2\, M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.



The 0.3% scale dependence is not canceled at the scale μ_b

Remaining scale uncertainty

The remaining 0.3% μ_b scale dependence will only be removed after non-perturbative QED corrections are included.

I.e. QED⊗QCD Matrix elements of

$$
\begin{aligned}
Q_1 &= (\bar{b}\gamma_\mu T^a q_L)(\bar{q}\gamma_\mu T^a s_L) \\
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Q_V &= (\bar{b}\gamma_\mu s_L)(\bar{l}\gamma_\mu l)
\end{aligned}
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could be considered, but they are $O(\alpha/\pi) \lesssim 0.3\%$ – our error estimate

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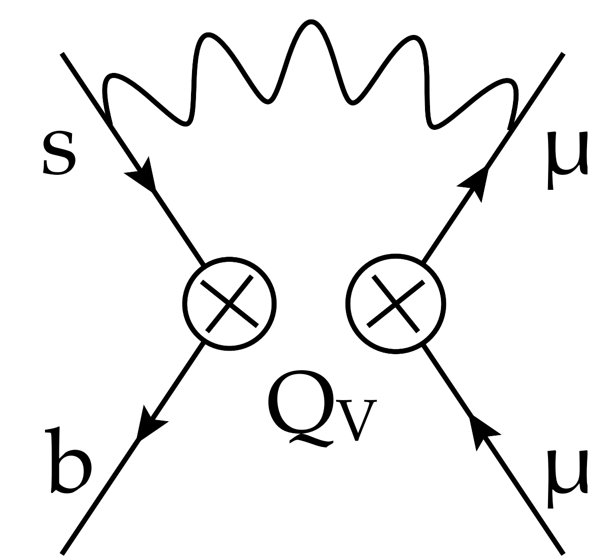

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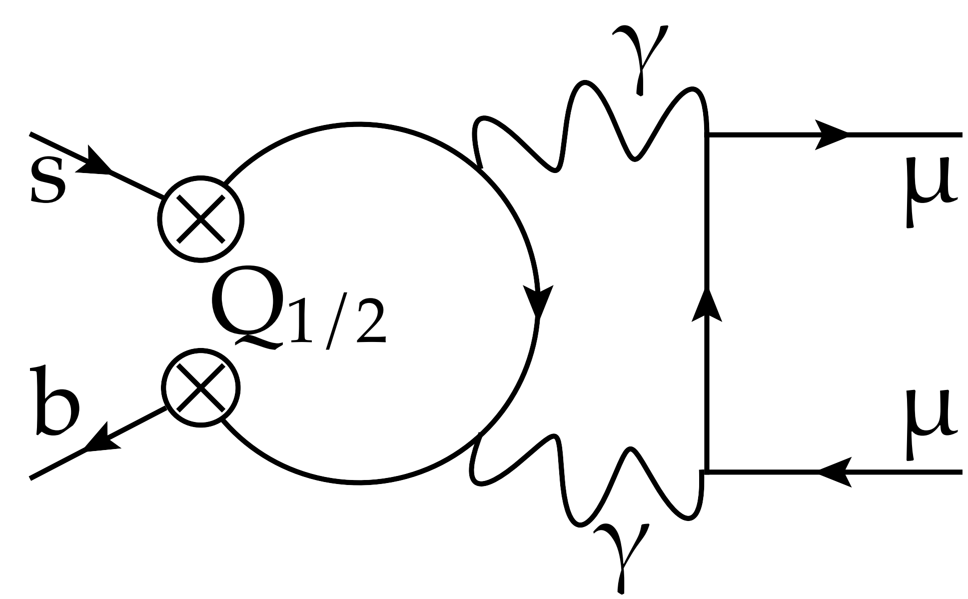

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Conclusions

7% electroweak scheme ambiguity in $B_s \to \mu^+ \mu^-$ is removed

3-5% Shift is relevant if compared with the experimental accuracy and significant for the theory uncertainty

$B_s \to \mu^+ \mu^-$ can provide a precision probe of scalar, but also axial vector coupling interactions