

Moriond-EW, 17 March '14

Neutrino's 2014

An Introduction

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In the last ~15 years we have learnt a lot about ν 's!

- ν 's oscillate (no separate lepton number conservation)
- ν 's are massive (at least two of them)
- their masses are very small
- Δm^2_{ij} and mixing angles are measured with fair precision
- Theory: probably ν 's are Majorana particles [can explain small masses and large mixings (see-saw, O_5)]
- an appealing picture: ν 's as probes of GUT's, baryogenesis thru leptogenesis....
- still many open questions: absolute scale of m^2 ? inverse or normal hierarchy? CP viol? flavour symmetry? sterile ν 's? $0\nu\beta\beta$, DM?..



Neutrino Masses



ν oscillations measure Δm^2 . What is m^2 ?

$\Delta m^2_{\text{atm}} \sim 2.5 \cdot 10^{-3} \text{ eV}^2; \quad \Delta m^2_{\text{sun}} \sim 8 \cdot 10^{-5} \text{ eV}^2$

- Direct limits

$m_{\nu e} < 2.2 \text{ eV}$

$m_{\nu \mu} < 170 \text{ KeV}$

$m_{\nu \tau} < 18.2 \text{ MeV}$

End-point tritium β decay (Mainz, Troitsk, future: Katrin)

- $0\nu\beta\beta$

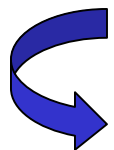
$m_{ee} < 0.2 - 0.7 - ? \text{ eV}$ (nucl. matrix elements)

$m_{ee} = |\sum U_{ei}^2 m_i|$

- Cosmology

$\Omega_\nu h^2 \sim \sum_i m_i / 94 \text{ eV}$ ($h^2 \sim 1/2$)

$\sum_i m_i < 0.23 - 0.8 \text{ eV}$ 95% Planck +BAO+WMAPPol+HighL



Any ν mass $< 0.08 - 0.27 \text{ eV}$

depends on cosmology priors



KATRIN

Expects start of tritium data-taking in 2016

In 3-4 years $\sigma_{\text{syst}} \sim \sigma_{\text{stat}}$

Weinheimer

sensitivity:

$$m_\nu < 0.2\text{eV (90\%CL)}$$

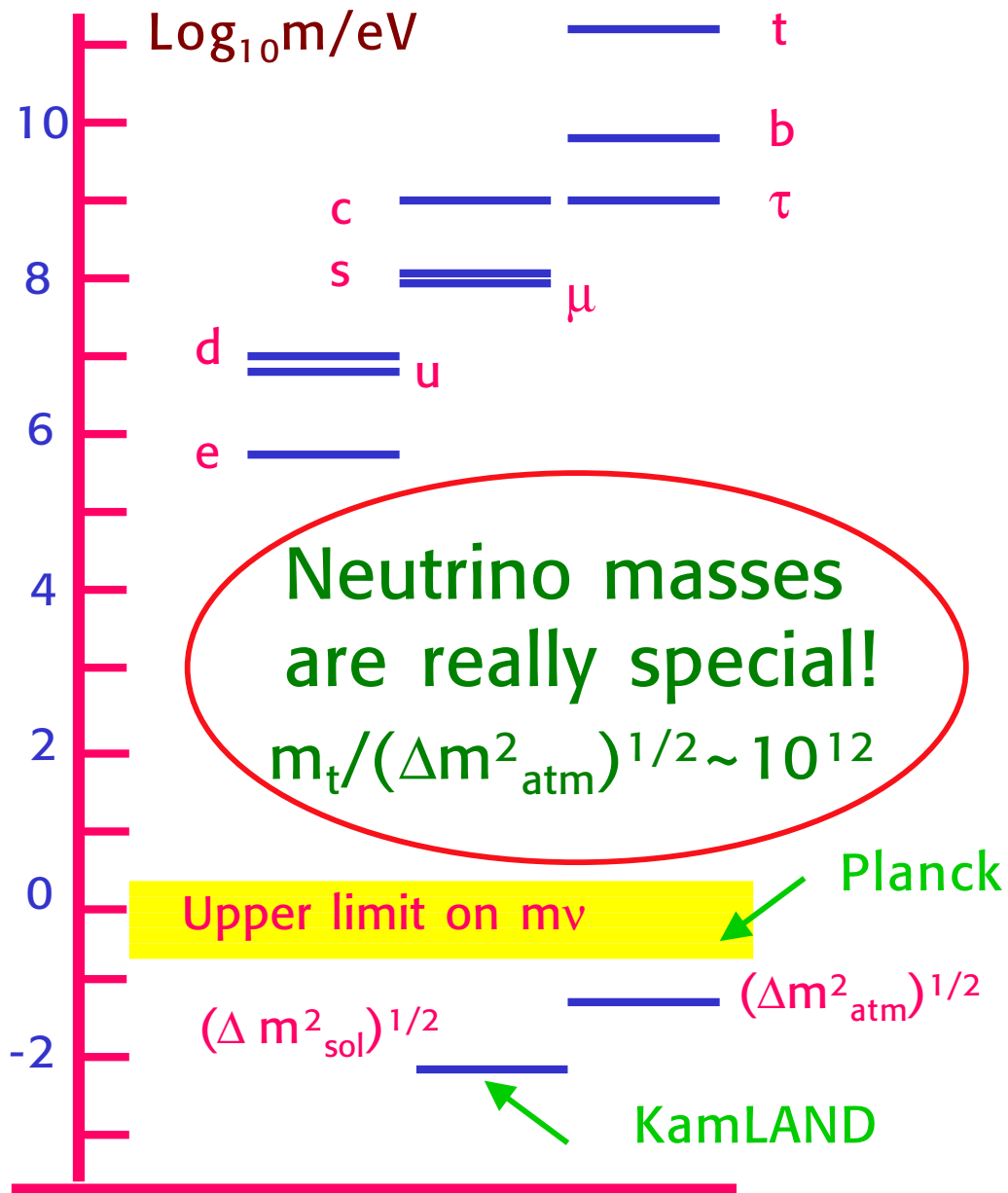
discovery potential:

$$m_\nu = 0.3\text{eV} \quad (3\sigma)$$

$$m_\nu = 0.35\text{eV} \quad (5\sigma)$$

Further in the future
MARE (^{187}Re),
ECHO (^{163}Ho), ...





Massless ν 's?

- no ν_R
- L conserved

But ν_R can well exist and we really have no reason to expect that B and L are exactly conserved

Small ν masses?

- ν_R very heavy
- L not exactly cons.

The SM can be easily extended to include Majorana ν 's



Completing the SM with ν_R

It is sufficient to introduce 3 RH gauge singlets ν_R
[each completing a 16 of $SO(10)$ for one generation]
and not artificially impose that L is conserved

In the SM, in the absence of ν_R , B and L are “accidental”
symmetries [i.e. no renormalizable gauge invariant
B and/or L non-conserving vertices can be built from
the fields of the theory]

But we know that non perturbative terms (instantons)
break B and L and also non renormalizable operators:

Weinberg
dim-5 operator

$$O_5 = \frac{(Hl)_i^T \lambda_{ij} (Hl)_j}{\Lambda} + h.c.$$

With Majorana ν_R renormalizable mass terms are
allowed by gauge symmetries and break L (and B-L)
Large ν_R Majorana mass \rightarrow see-saw mechanism



See-Saw Mechanism

Minkowski; Glashow; Yanagida;
Gell-Mann, Ramond, Slansky;
Mohapatra, Senjanovic.....

 $M \nu_R^T \nu_R$ allowed by $SU(2) \times U(1)$
Large Majorana mass M (as large as the cut-off)

$$m_D \bar{\nu}_L \nu_R$$

Dirac mass m_D from
Higgs doublet(s)

$$\begin{array}{c} \nu_L \\ \nu_R \end{array} \begin{array}{cc} \nu_L & \nu_R \\ \left[\begin{array}{cc} 0 & m_D \\ m_D & M \end{array} \right] \end{array} \quad M \gg m_D$$

Eigenvalues

$$|m_{\text{light}}| = \frac{m_D^2}{M}, \quad m_{\text{heavy}} = M$$



A very natural and appealing explanation:

ν 's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{\text{GUT}}$

$$m_\nu \sim \frac{m^2}{M}$$

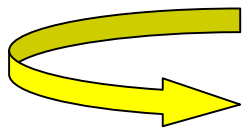
$$m: \leq m_t \sim v \sim 200 \text{ GeV}$$

M: scale of L non cons.

Note:

$$m_\nu \sim (\Delta m_{\text{atm}}^2)^{1/2} \sim 0.05 \text{ eV}$$

$$m \sim v \sim 200 \text{ GeV}$$



$$M \sim 10^{14} - 10^{15} \text{ GeV}$$

This is so impressive that, in my opinion, models with ν_R at the EW scale or around are strongly disfavoured



A great extra bonus of see-saw with heavy Majorana ν_R 's

BG via Leptogenesis near the GUT scale  (after inflation)

Buchmuller, Yanagida,
Plumacher, Ellis, Lola,
Giudice et al, Fujii et al
.....

Only survives if $\Delta(B-L)$ is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{11-12}$ GeV)

L non conserv. & CP violat.'n in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from
 ν oscill's is compatible with BG via (thermal) LG

Buchmuller, Di Bari, Plumacher;
Giudice et al; Pilaftsis et al;
Hambye et al



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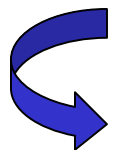
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$\rightarrow m_{ee} = |\sum U_{ei}^2 m_i|$

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$\Omega_\nu h^2 \sim \sum_i m_i / 94 \text{ eV} \quad (h^2 \sim 1/2)$

$\sum_i m_i < 0.23 - 0.8 \text{ eV} \quad 95\% \quad \text{Planck +BAO+WMAPPol+HighL}$

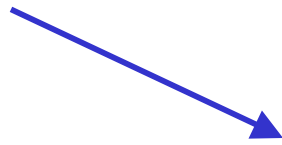


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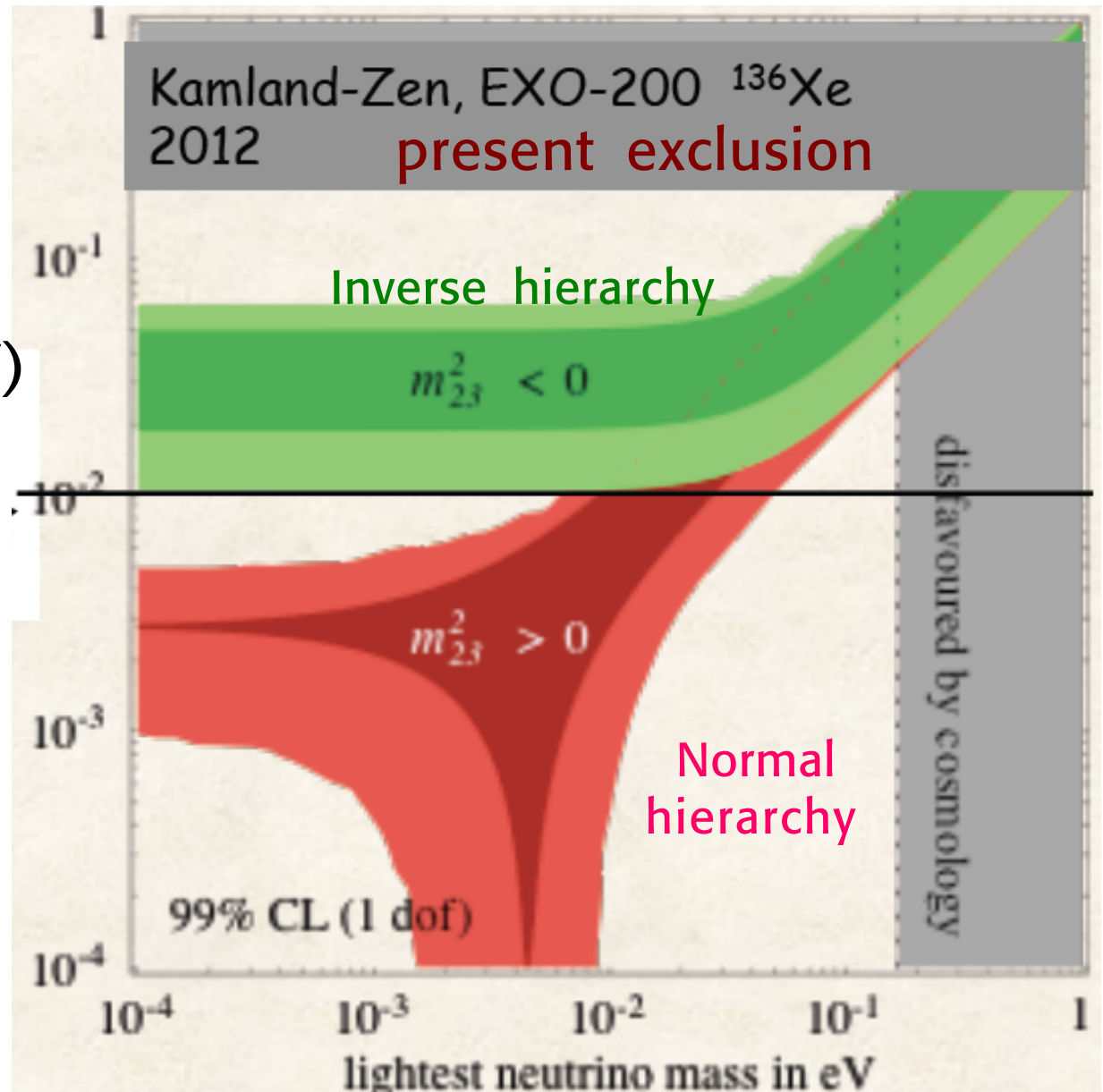
$$m_{ee} = \left| \sum U_{ej}^2 m_j e^{i\alpha_j} \right|$$



$|m_{ee}|(\text{eV})$

next generation
10 meV →

$0\nu\beta\beta$ signal
would establish
Majorana ν 's,
measure m_{ee} and
indicate hierarchy



Determining the type of spectrum is still an open problem

⊕ Better outlook now that θ_{13} has been measured and is large

Present results on neutrinoless DBD

Fiorini

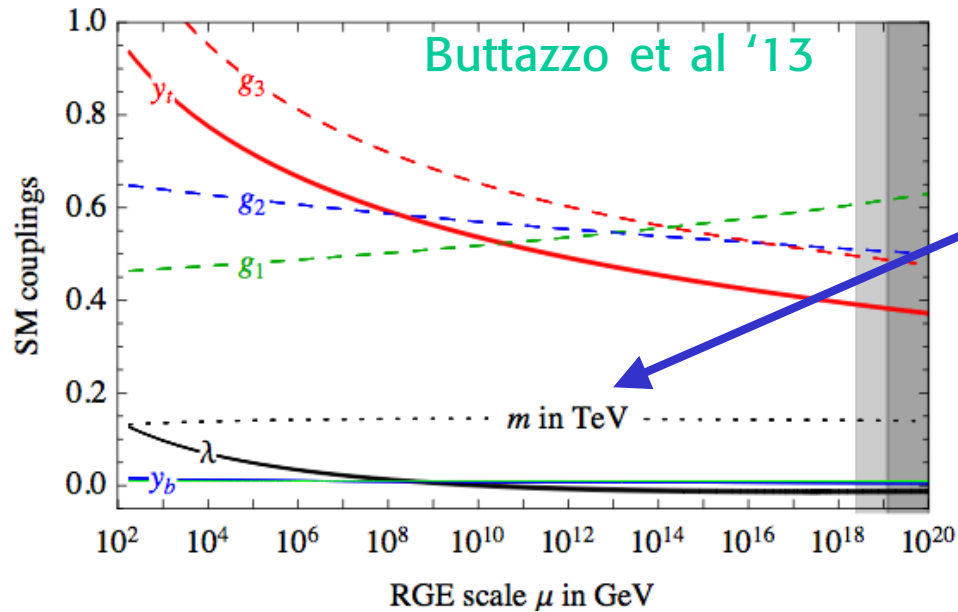
Isotope	Technique	$\tau_{1/2}^{0\nu}$ (y)	$\langle m_{\beta\beta} \rangle$ eV
^{48}Ca	CaF ₂ scint	$>1.4 \times 10^{22}$	<7-45
^{76}Ge (HM)	Ge diode	$>1.9 \times 10^{25}$	<(0.3-1.27)
^{76}Ge (IGEX)	Ge diode	$>1.6 \times 10^{25}$	<(0.33-1.35)
^{76}Ge (Klapdor 2004)	Ge diode	1.2×10^{25}	.38
^{76}Ge (Klapdor 2006)	Ge diode	2.2×10^{25}	.28
^{76}Ge (GERDA I)	Ge diode	$>2.1 \times 10^{25}$	<(0.29-1.1)
^{76}Ge (GERDA+HM+IGEX)	Ge diode	$>3 \times 10^{25}$	<(0.25-.98)
^{82}Se	Foil&track	$>.6 \times 10^{23}$	<(0.89-2.)
^{96}Zr	Foil&track	$>9.2 \times 10^{21}$	<(7.2-19.5)
^{100}Mo	Foil&track	$>1.1 \times 10^{24}$	<(0.31-.79)
^{116}Cd	Scintillator	$>1.7 \times 10^{23}$	<1.7
^{128}Te	Geochem	$>7.7 \times 10^{24}$	<(1.1-1.35)
^{130}Te	Bolometer	$>2.8 \times 10^{24}$	<(0.3-.7)
^{136}Xe	EXO	$>1.6 \times 10^{25}$	<140-380
^{136}Xe	Kamland Zen	$>1.9 \times 10^{25}$	<128-349
^{136}Xe	EXO+Kamzen		<120-250
^{150}Nd	Foil TPC	$>1.8 \times 10^{22}$	

here Ettore forgot the dot: 0.140 etc

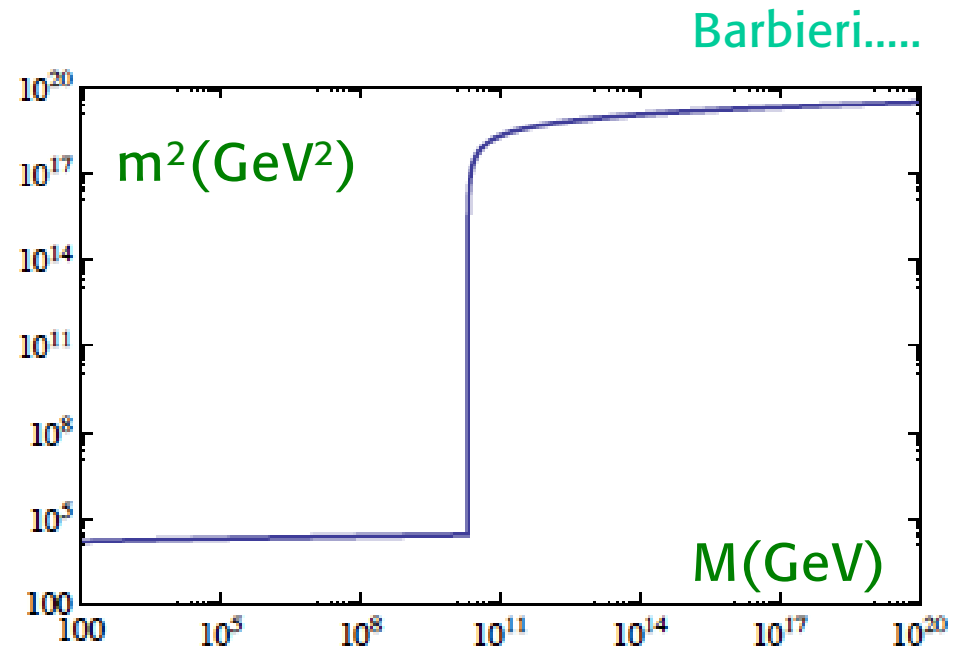
Heavy $\nu_{R'}$
Naturalness
and
Vacuum Stability



Naturalness in a more physical language



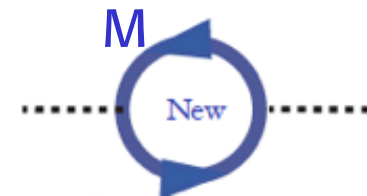
In the renormalized theory the running Higgs mass slowly evolves logarithmically



But in the presence of a threshold at M for a heavy particle coupled to the Higgs, the quadratic sensitivity produces a jump in the running mass

$$M \sim 10^{10} \text{ GeV}, \lambda_H \sim 1, \text{ jump: } m^2 \sim (\lambda_H M)^2 / (16\pi^2)$$

Fine tuning is then needed to explain the small value of m at low energy

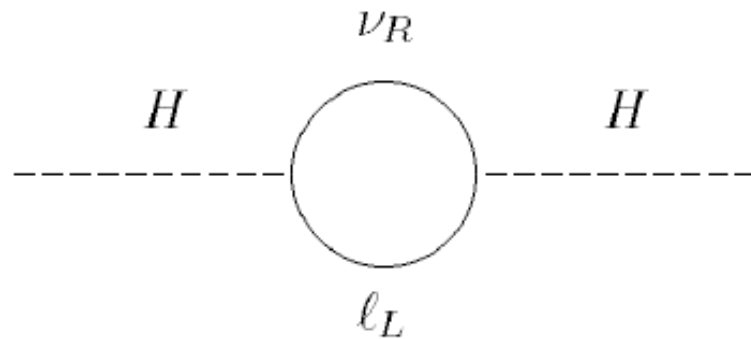


Heavy ν_R well match with GUT's [recall the 16 of SO(10)!]

(if for naturalness SUSY is invoked, one also has the bonus that coupling unification and proton decay are OK, ...)

But so far, no SUSY or any New Physics

If only the SM + Majorana ν 's, then heavy ν_R are unnatural and require fine tuning:



$$\begin{aligned} & \text{for } q \gg M_R \\ \delta\mu^2 & \approx \frac{y_\nu^2}{(2\pi)^2} M_R^2 \log(q/M_R) \\ & \approx \frac{m_\nu M_R^3}{(2\pi v)^2} \log(q/M_R) \end{aligned}$$

$$\mu < 1 \text{ TeV} \longrightarrow M_R < 10^7\text{-}10^8 \text{ GeV}$$

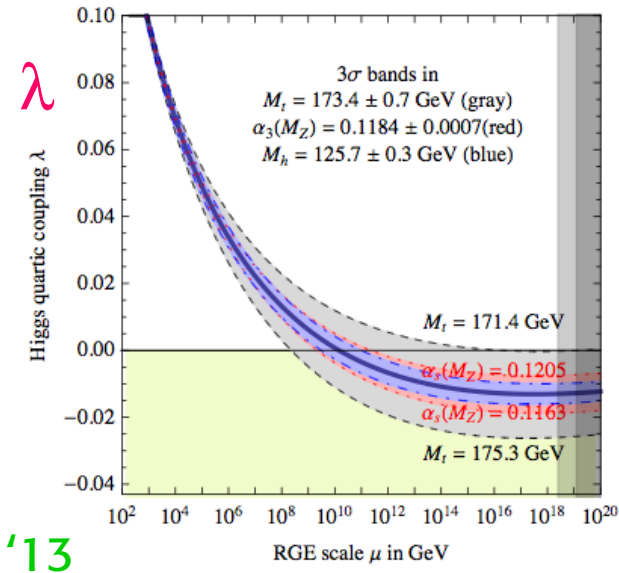
But if no SUSY or any NP there is FT anyway!

Vissani '97
Elias-Miro '11



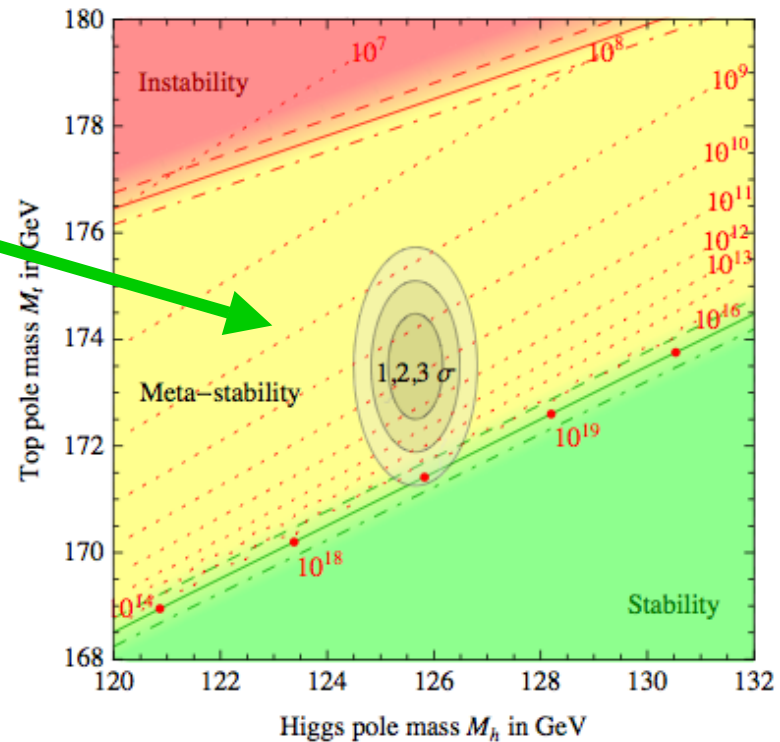
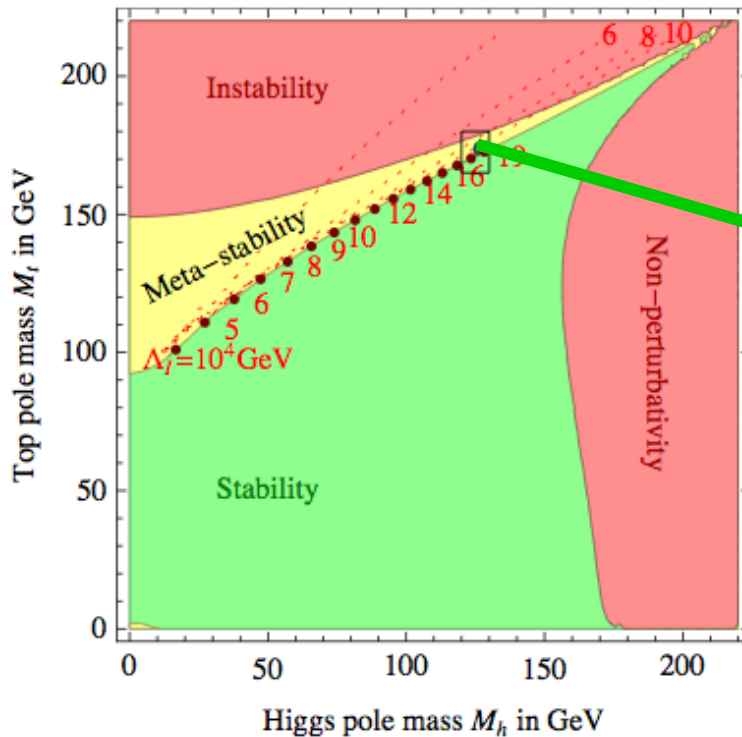
The pure SM evolution of couplings leads to a metastable Universe

The SM evolution up to M_{pl} leads to a narrow critical wedge: a hidden message?



$$\lambda\phi^4$$

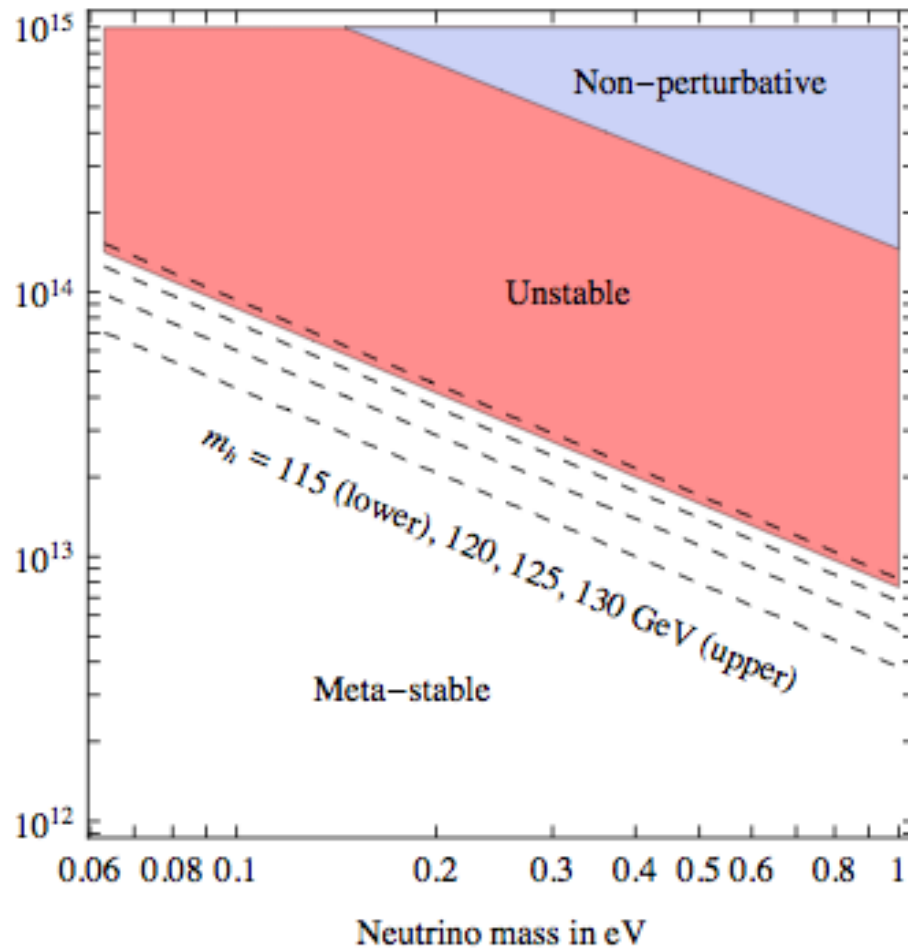
Buttazzo et al '13



Heavy ν_R 's further de-stabilize the vacuum

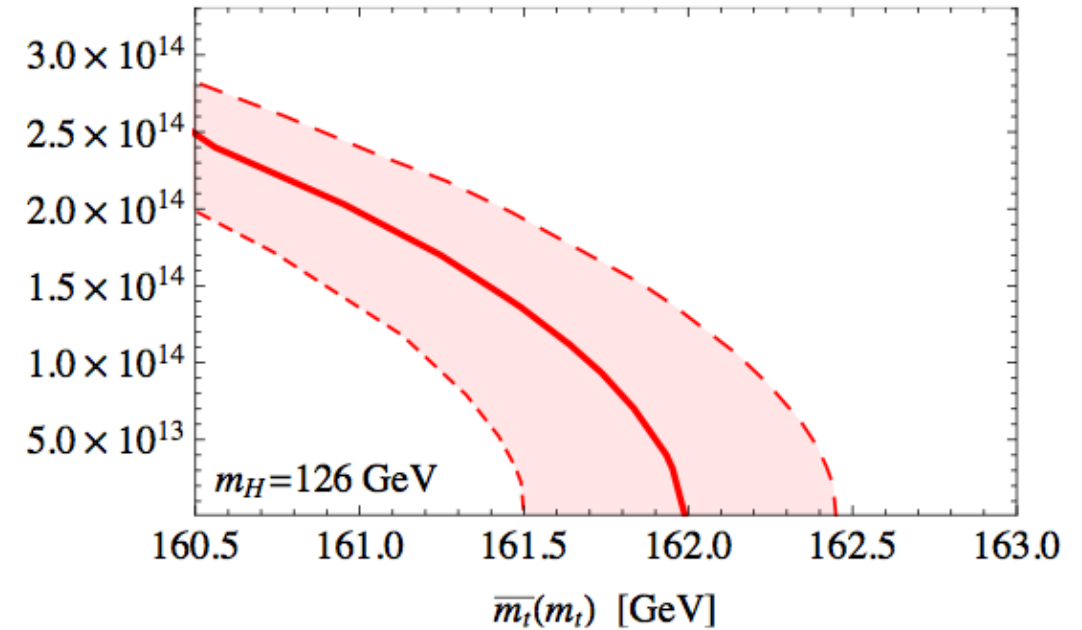
But, for $M < 10^{14}$ GeV, ν_R 's do not make the vacuum unstable

J. Elias-Miro' et al '11



m_{ν_R} [GeV]

Masina'12



(

A drastic conjecture

No new thresholds between m_W and M_{Pl} ?

Shaposhnikov '07--->

And hope that gravity will somehow fix the problem of fine tuning related to the M_{Pl} threshold (with many thresholds it would be more difficult for gravity to arrange the fine tuning)

Giudice EPS'13

For this, one would need to solve all problems like Dark Matter, neutrino masses, baryogenesis.... at the EW scale

In particular no GUT's below M_{Pl}



The ν MSM

Shaposhnikov et al

There are 3 RH ν 's: N_1, N_2, N_3 and the see-saw mechanism

But the N_i masses are all below the EW scale

Actually $N_1 \sim \mathcal{O}(1-10)$ keV, and $N_{2,3} \sim$ GeV with eV splitting

Very small Yukawa couplings are assumed to explain the

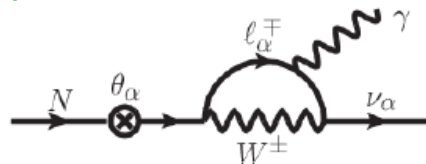
small active ν masses

$$m_\nu = \frac{y_\nu^2 v^2}{M_N}$$

The phenomenology of ν oscillations can be reproduced

N_1 can explain (warm) DM

$N_{2,3}$ can explain the Baryon Asymmetry in the Universe



N_1 decay produces a distinct X-ray line

$$N_1 \rightarrow \nu + \gamma \quad (E_\gamma = m_{N_1}/2)$$

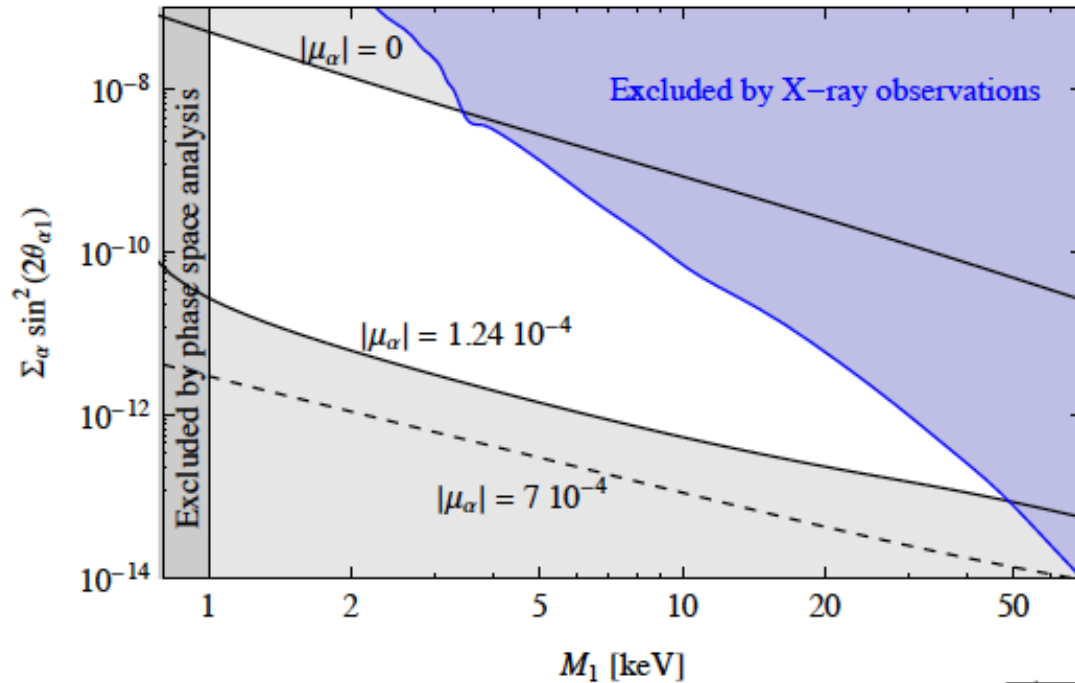
$$\Gamma_\gamma(m_s, \theta) = 1.38 \times 10^{-29} \text{ s}^{-1} \left(\frac{\sin^2 2\theta}{10^{-7}} \right) \left(\frac{m_s}{1 \text{ keV}} \right)^5$$

$N_{2,3}$ could be detected by dedicated accelerator experiments (eg in B decays, $\text{Br} \sim 10^{-10}$)

A LOI for the CERN SPS has been presented



Bonivento et al, ArXiv:1310.1762



Canetti et al '12

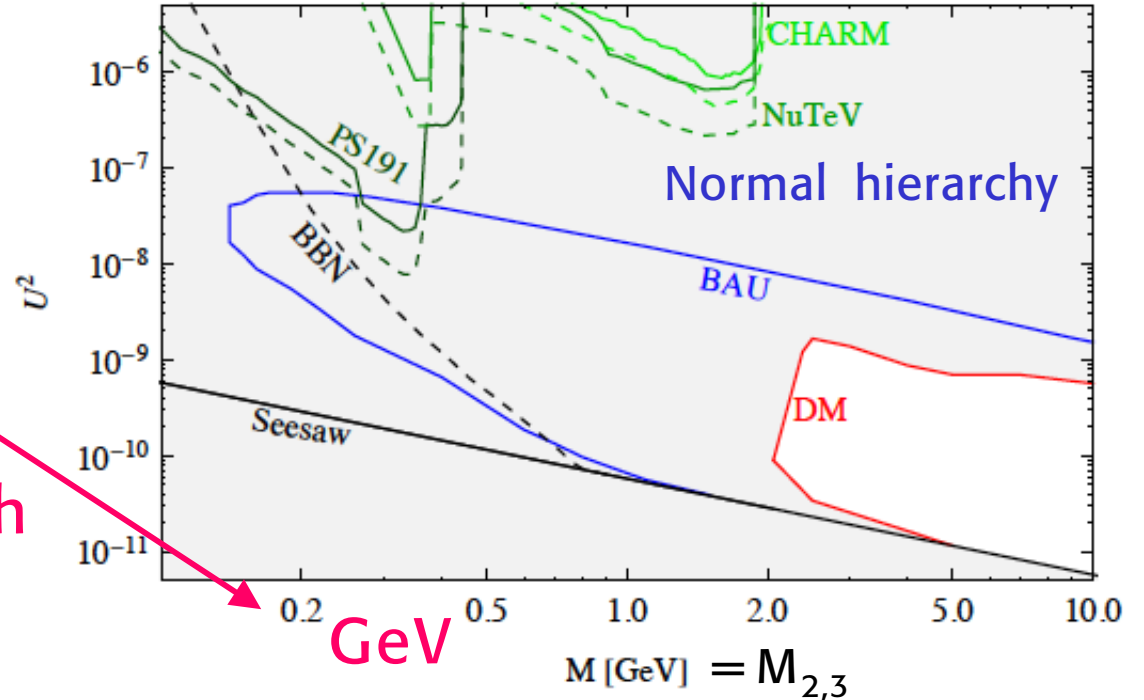
The claim is that all constraints can be satisfied

For DM one needs $1 < M_1 < \sim 100$ keV

keV

No explanation of the mass splitting

Also $N_{1,2}$ are GeV heavy with eV splitting!!!



GeV



A ~ 7 keV sterile N_1 ?

ArXiv:1402.2301

DETECTION OF AN UNIDENTIFIED EMISSION LINE IN THE STACKED X-RAY SPECTRUM OF GALAXY CLUSTERS

ESRA BULBUL^{1,2}, MAXIM MARKEVITCH², ADAM FOSTER¹, RANDALL K. SMITH¹, MICHAEL LOEWENSTEIN², AND SCOTT W. RANDALL¹

¹ Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138.

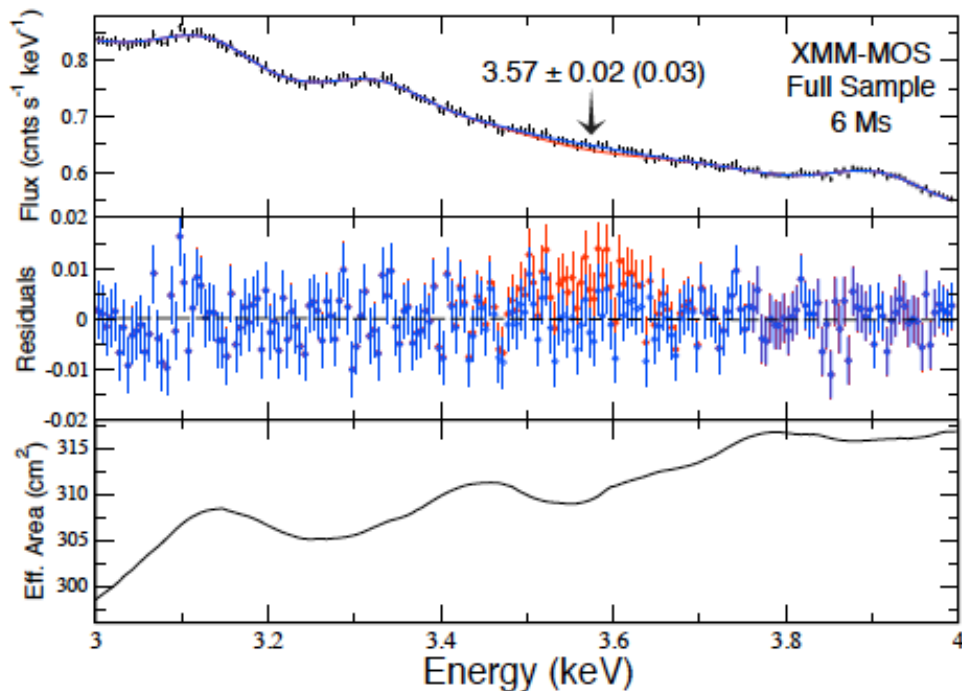
² NASA Goddard Space Flight Center, Greenbelt, MD, USA.

Submitted to *ApJ*, 2014 February 10

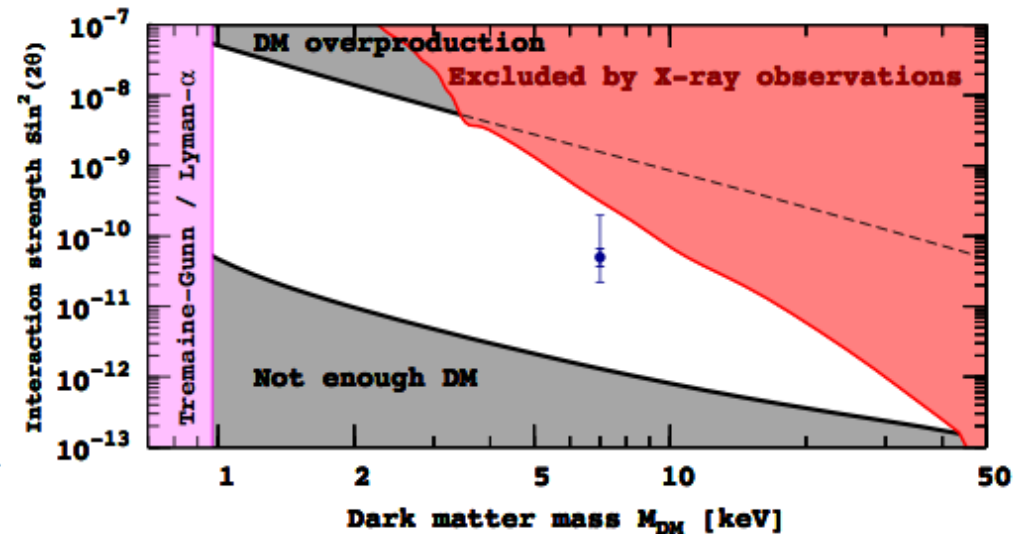
XMM-Newton X-ray observatory

ABSTRACT

We detect a weak unidentified emission line at $E = (3.55 - 3.57) \pm 0.03$ keV in a stacked XMM spectrum of 73 galaxy clusters spanning a redshift range 0.01 - 0.35. MOS and PN observations



Independent analysis by Boyarski et al
ArXiv:1402.4119



(Confirmation from Chandra, Suzaku and eventually, Astro-H needed

$m \sim \text{eV}$ Sterile Neutrinos

A White Paper: K.N. Abazajian et al, ArXiv:1204.5789



Sterile ν 's? A number of "hints" with some "tensions"

(they do not make an evidence but pose an important experimental problem that needs clarification)

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e \quad \nu_\mu \rightarrow \nu_e \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

- LSND and MiniBoone (appearance)
- Reactor anomaly ($\bar{\nu}_e$ disappearance)
- Gallium (ν_e disappearance)

These data hint at sterile neutrinos at ~ 1 eV which would represent a major discovery in particle physics

Important information also from

- $\nu_\mu/\bar{\nu}_\mu$ disappearance expts (MINOS, CDHSW, CCFR...)

-  Neutrino counting from cosmology



Cosmology is fully compatible with $N_{\text{eff}} \sim 3$ but could accept **one** sterile neutrino

The bound from nucleosynthesis is the most stringent (assuming thermal properties at decoupling)

- ▶ BBN: $N_s = 0.22 \pm 0.59$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]
- ▶ BBN: $N_s = 0.64^{+0.40}_{-0.35}$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]
- ▶ BBN: $N_s < 1.2$ (95% CL) Mangano, Serpico, 1103.1261
- ▶ BBN: $N_s < 1.54$ (95% CL) [M. Pettini, et al, arXiv:0805.0594]



A “simple” cosmology emerges from Planck

No evidence for sterile neutrinos

$$N_{\text{eff}} = 3.36 \pm 0.34$$

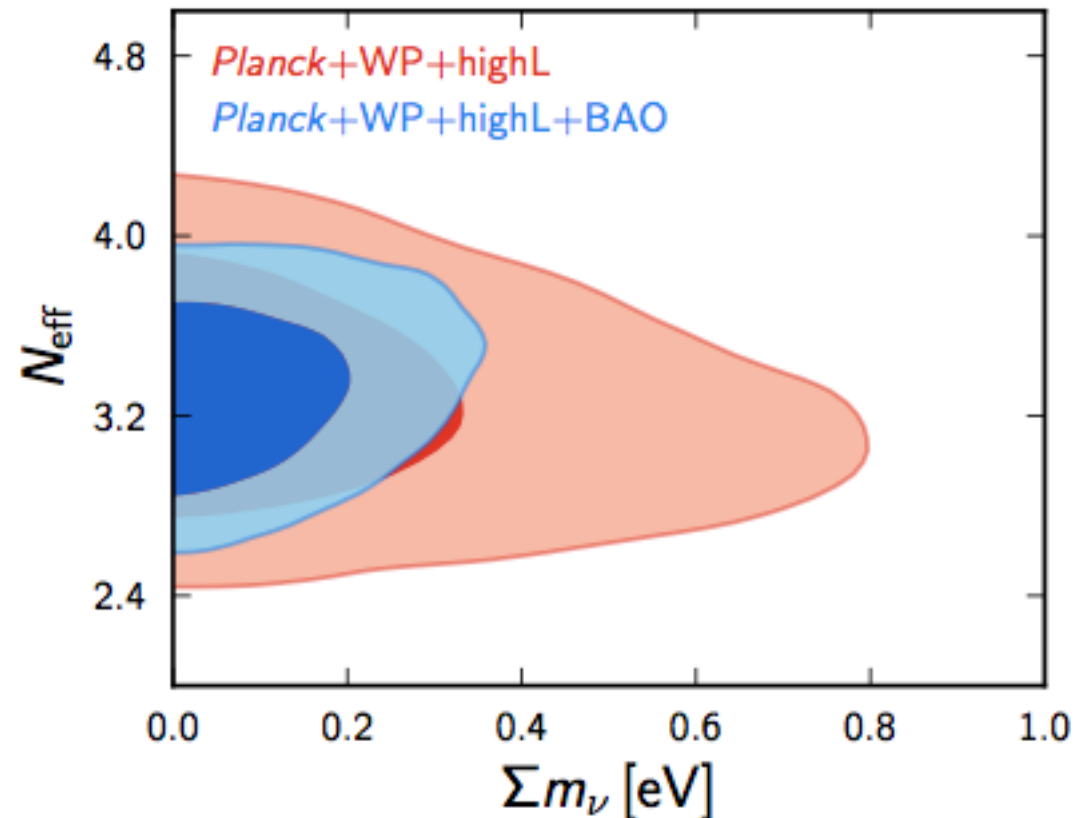
More precise values of cosmological parameters

$$\Omega_{\Lambda} = 0.686 \pm 0.020$$

$$\Omega_m = 0.314 \pm 0.020$$

$$\Omega_b h^2 = 0.02207 \pm 0.00033$$

$$h = 0.674 \pm 0.014$$



$$\Sigma m_{\nu} < 0.23 - 0.80 \text{ eV}$$

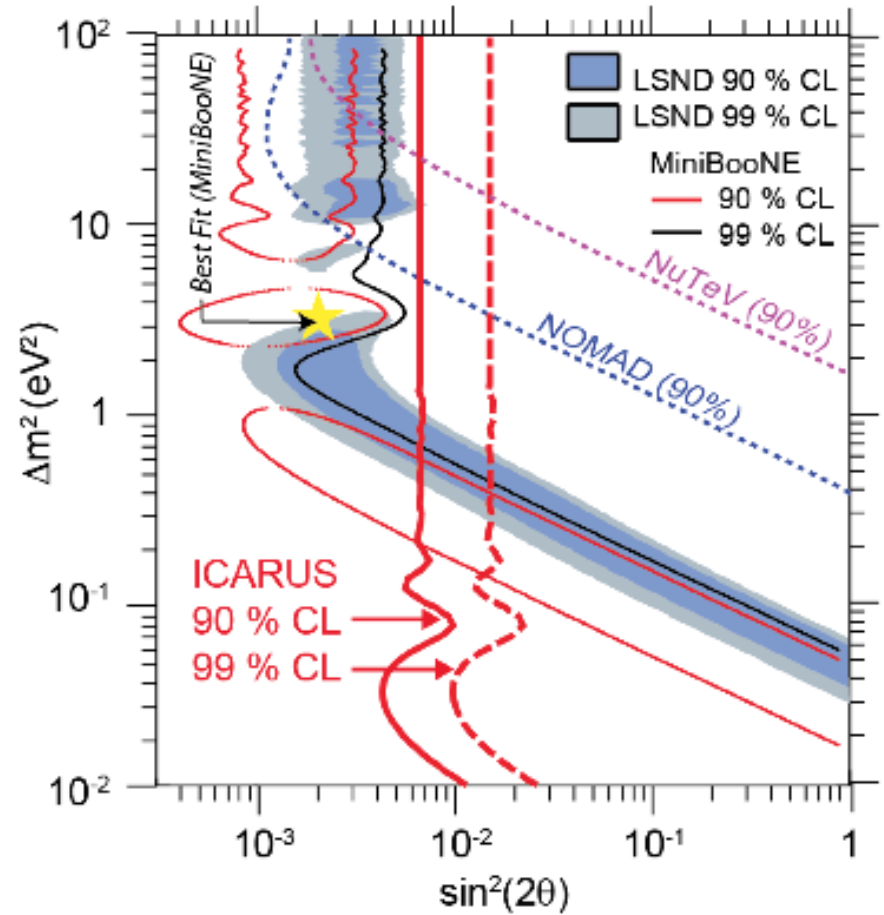
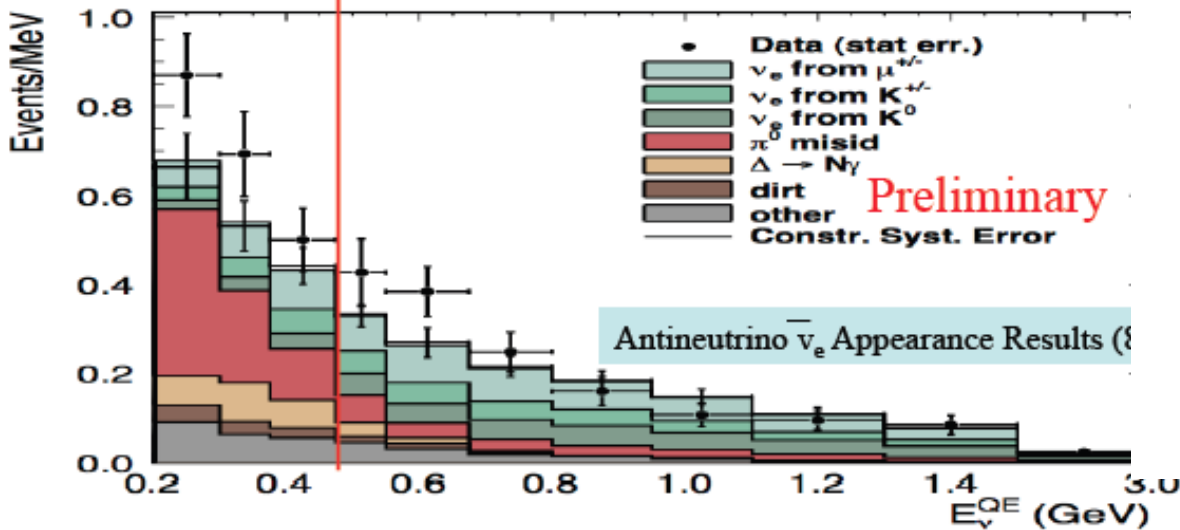
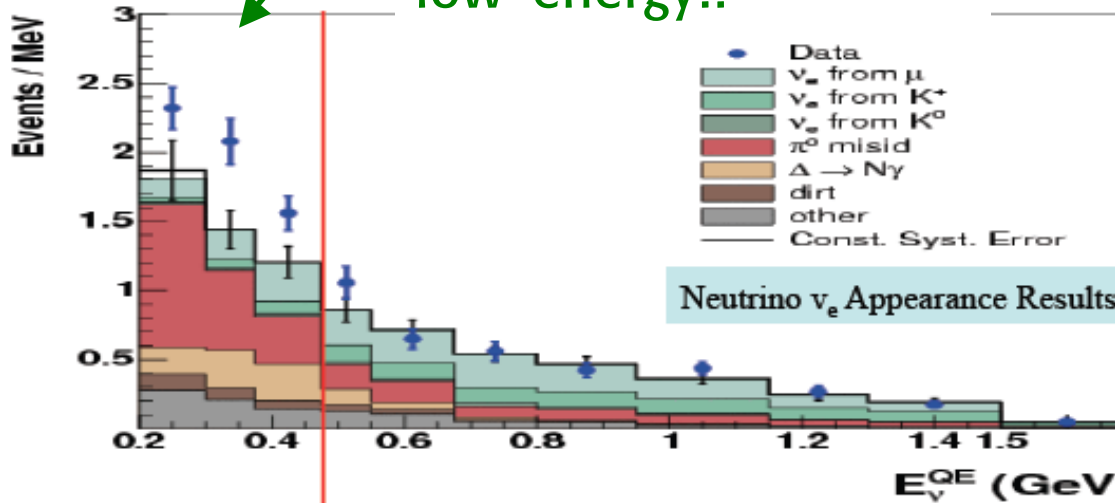


Appearance accelerator experiments

LSND, KARMEN, ICARUS
MiniBoone

MiniBoone supports LSND in $\bar{\nu}_\mu$
but not in ν_μ (or CP viol.?)

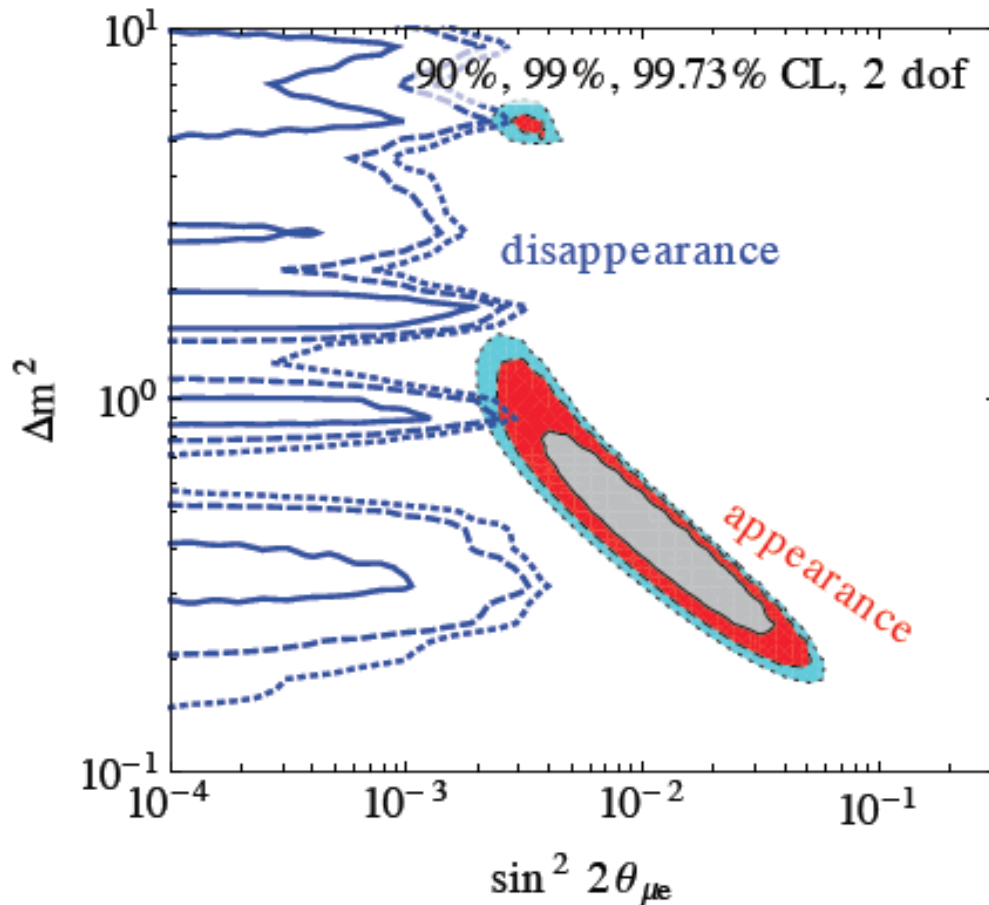
Unidentified excess at low energy!!



ICARUS Coll,1307.4699

No signal in ν_μ disappearance experiments
 (CDHSW, MINOS, CCFR, MiniBooNE-SciBooNE)
 creates a tension with LSND (if no CP viol.)

Kopp et al '13



For example, in 3+1 models
 here is the clash
 between appearance
 (LSND, MiniBoone.....) and
 disappearance (MINOS...)

$$\sin^2 2\theta_{\mu e} = 4|U_{e4}|^2|U_{\mu 4}|^2$$

app. wants
 this large

disapp. wants
 this small

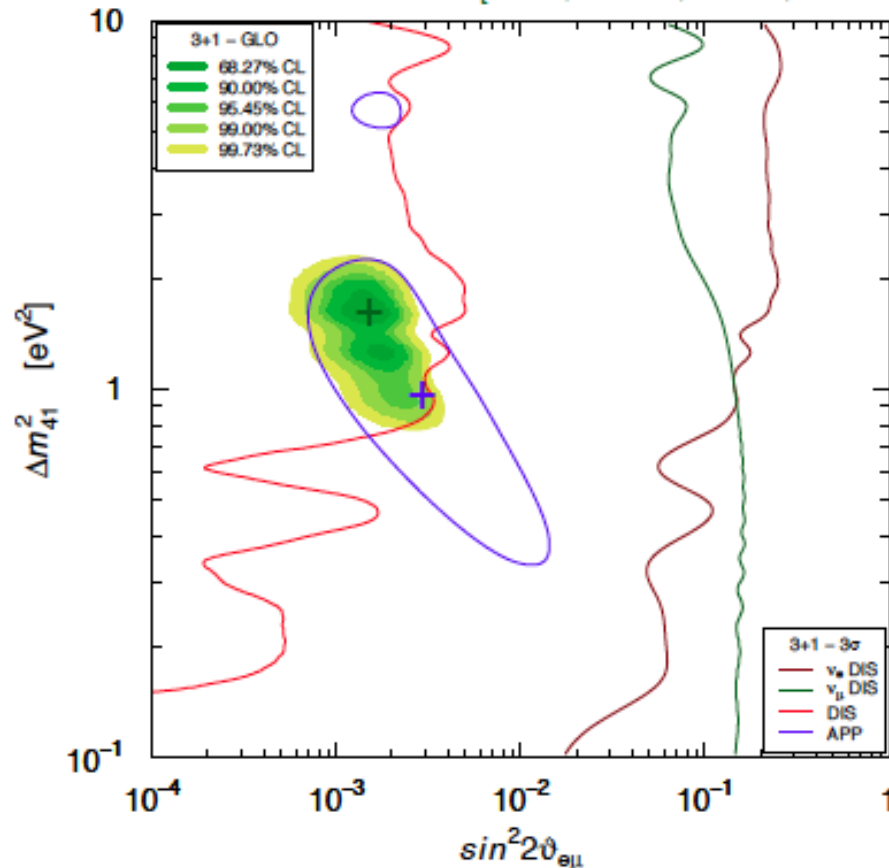


Giunti et al are more positive on the 3+1 fit

The difference comes from the low energy MiniBooNe data (not included here)

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, arXiv:1308.5288]



- ▶ APP $\nu_\mu \rightarrow \nu_e$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$:
 LSND (Y), MiniBooNE (?),
 OPERA (N), ICARUS (N),
 KARMEN (N), NOMAD (N),
 BNL-E776 (N)
- ▶ DIS ν_e & $\bar{\nu}_e$: Reactors (Y),
 Gallium (Y), $\nu_e C$ (N),
 Solar (N)
- ▶ DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N),
 MINOS (N),
 Atmospheric (N),
 MiniBooNE/SciBooNE (N)

MiniBooNE $E > 475$ MeV
 GoF = 29% PGoF = 9%

No Osc. excluded at 6.2σ
 $\Delta\chi^2/\text{NDF} = 46.2/3$



The reactor anomaly (below 100m baseline: SBL) came after a revision of the theoretical flux and of cross-sections

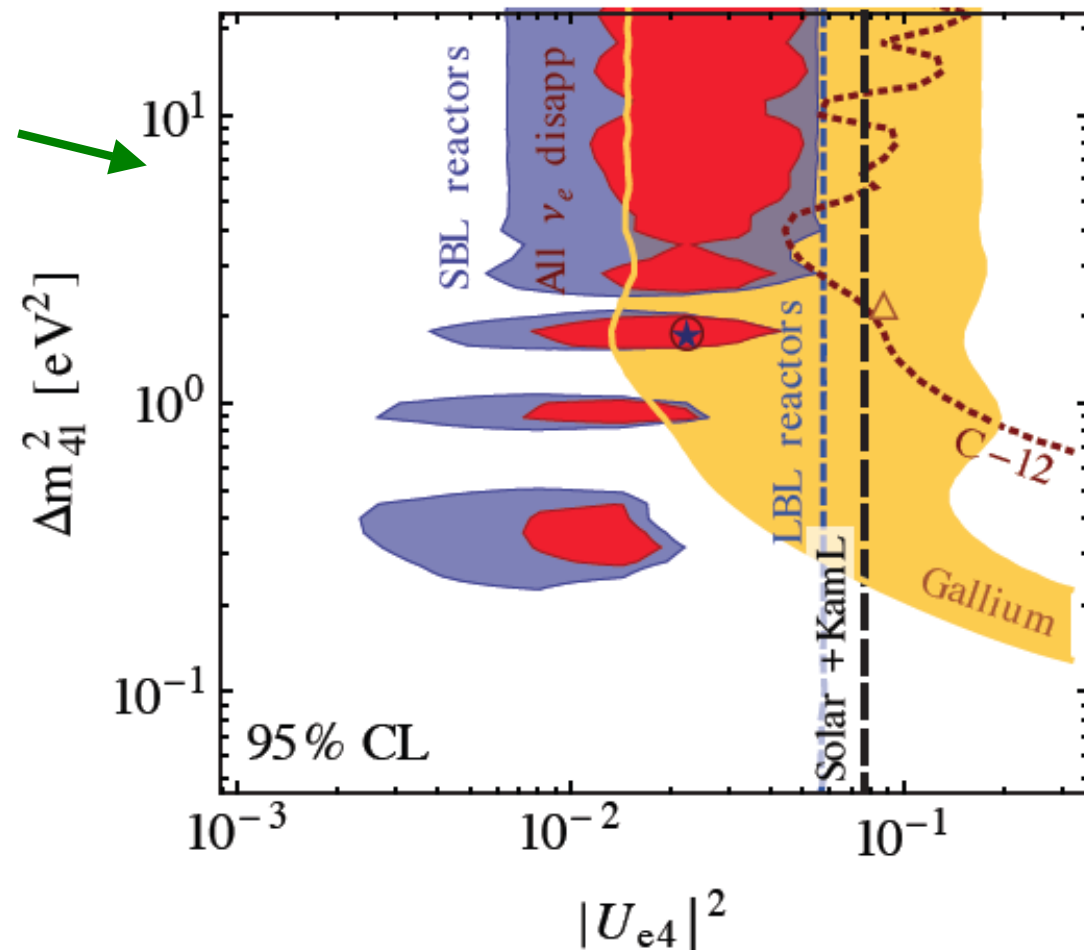
Mueller et al '11; Huber '11

Similarly the Ga anomaly depends on assumed cross section and errors Kaether et al '10; Abdurashitov et al '09

Kopp et al '13

SBL reactors and gallium in 3+1 models

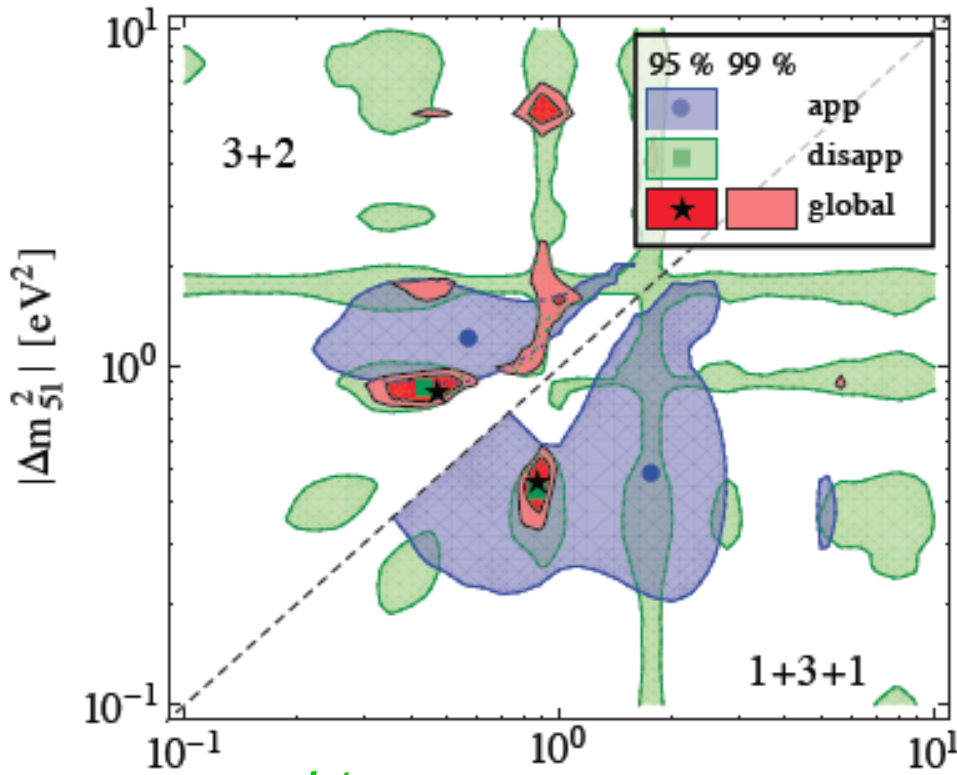
SBL reactors and gallium are not in tension with other measurements



Global fits to all data (1 or 2 sterile neutrinos)

3+1 not very good but acceptable

No great advantage from 3+2 or 1+3+1 (this second is better)



Kopp et al '13

Conrad et al '12

	Δm_{41}^2 [eV ²]	$ U_{e4} $	$ U_{\mu 4} $	Δm_{51}^2 [eV ²]	$ U_{e5} $	$ U_{\mu 5} $
3+1	0.93	0.15	0.17			
3+2	0.47	0.13	0.15	0.87	0.14	0.13
1+3+1	-0.87	0.15	0.13	0.47	0.13	0.17

In all fits (3+1 or 2, 1+3+1) the Δm^2 values are in tension with the cosmology mass bound:

$$\Sigma m_\nu < 0.23 - 0.8 \text{ eV}$$

(partial thermalization?)

The issue of sterile ν 's is very important \longrightarrow experiment

\oplus e.g. Icarus at FNAL, Antonello et al, ArXiv:1312.7252

Models of Neutrino Mixing



3- ν models of masses and mixings

still the main framework

The large ν mixing versus the small q mixing is probably due to the Majorana nature of ν 's

An interplay of different matrices:

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

neutrino diagonalisat'n

charged lepton diagonalisat'n

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \rightarrow \nu_L^T m_\nu \nu_L$$

$$m_\ell \rightarrow \bar{R} m_\ell L$$

$$m_\ell' = V_\ell^\dagger m_\ell U_\ell$$

See-saw

$$m_\nu = m_D^T M^{-1} m_D$$

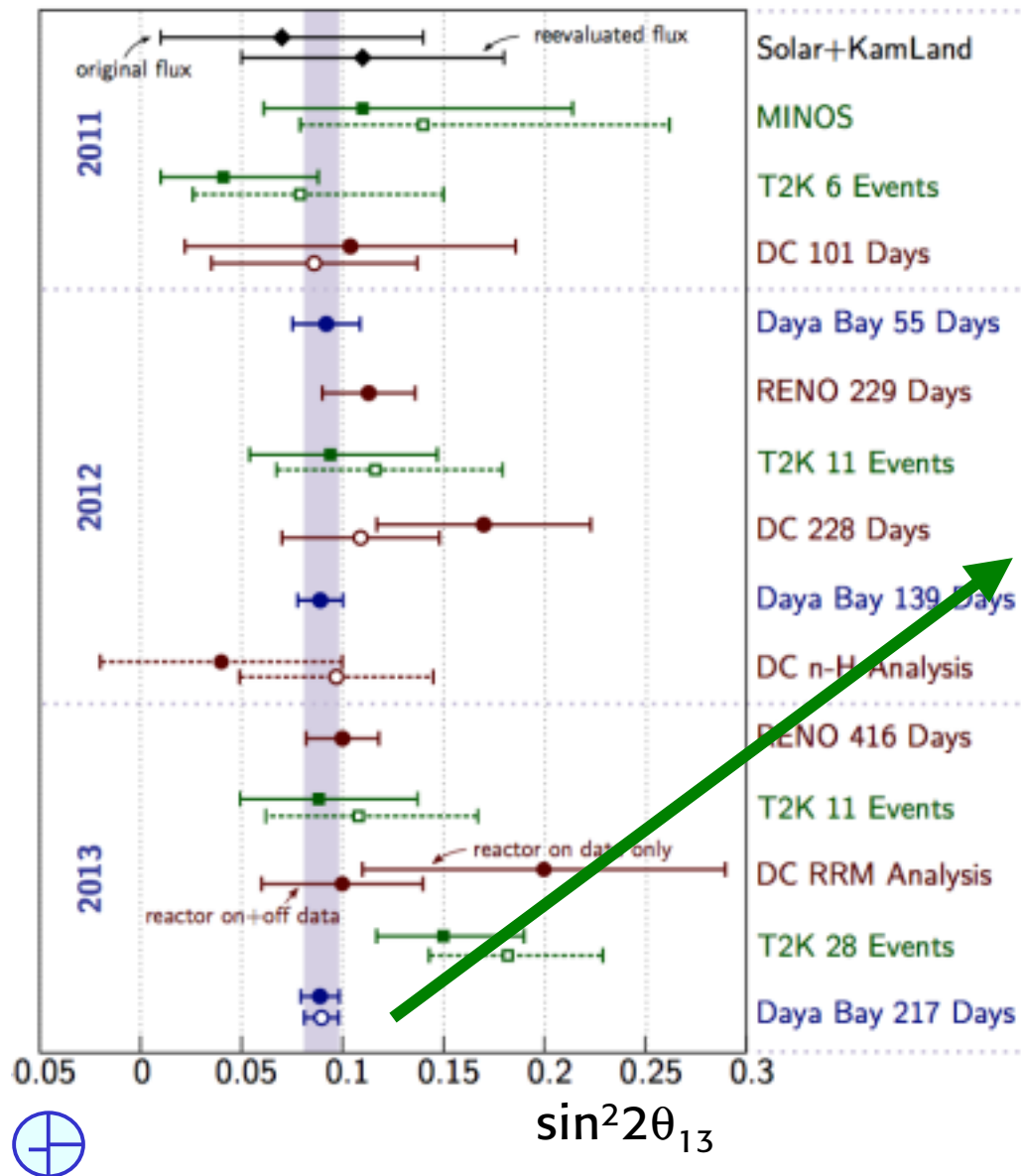
$$m_\ell^{\dagger'} m_\ell' = U_\ell^\dagger m_\ell^\dagger m_\ell U_\ell$$

$$m_\nu' = U_\nu^T m_\nu U_\nu$$

neutrino Dirac mass

neutrino Majorana mass

Now we have a good measurement of θ_{13} !!



$\sim 10 \sigma$ from zero

Daya Bay

$$\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$$

A large impact on model building and on designing new experiments! (hierarchy, δ_{CP} ...)

Empirically

$$\sin^2 \theta_{13} \approx \frac{1}{2} \sin^2 \theta_C$$

or

$$\theta_{13} \approx \theta_C / \sqrt{2}$$

← Capozzi, Fogli et al '13
(free reactor fluxes)

θ_{23} non maximal

(free reactor fluxes)

$\sin^2 \theta_{12}$	$0.302^{+0.013}_{-0.012}$
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$
$\sin^2 \theta_{23}$	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.3}_{-1.3}$
$\sin^2 \theta_{13}$	$0.0227^{+0.0023}_{-0.0024}$
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$
$\delta_{CP}/^\circ$	300^{+66}_{-138}
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.18}_{-0.19}$
$\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}$ (N)	$+2.473^{+0.070}_{-0.067}$
$\frac{\Delta m_{32}^2}{10^{-3} \text{ eV}^2}$ (I)	$-2.427^{+0.042}_{-0.065}$

Parameter	Best fit	1σ range
$\delta m^2/10^{-5} \text{ eV}^2$ (NH or IH)	7.54	7.32 – 7.80
$\sin^2 \theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25
$\Delta m^2/10^{-3} \text{ eV}^2$ (NH)	2.44	2.38 – 2.52
$\Delta m^2/10^{-3} \text{ eV}^2$ (IH)	2.40	2.33 – 2.47
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.34	2.16 – 2.56
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.39	2.18 – 2.60
$\sin^2 \theta_{23}/10^{-1}$ (NH)	4.25	3.98 – 4.54
$\sin^2 \theta_{23}/10^{-1}$ (IH)	4.37	$4.08 - 4.96 \oplus 5.31 - 6.10$
δ/π (NH)	1.39	1.12 – 1.72
δ/π (IH)	1.35	0.96 – 1.59

a start on $\cos \delta$?

Gonzalez-Garcia et al '12 →

By now all mixing angles are fairly well known!



In spite of this progress viable models still span a wide range that goes from very little structure to a lot of symmetry

At one extreme are models dominated by chance

Some examples:

Anarchy
 $U(1)_{FN}$ charges

.....



Froggatt-Nielsen '79

On the other hand the range for each mixing angle has narrowed and precise special patterns can be tentatively identified as starting approximations that, if significant, would lead to specified discrete symmetries:

TriBimaximal (TB), BiMaximal (BM),.....

Discrete non abelian flavour groups A_4 , S_4 , T' , $\Delta(96)$



θ_{13} near the previous bound and θ_{23} non maximal both go in the direction of Anarchy (a great success for Anarchy!)

Anarchy: no order for neutrino mixing

In the neutrino sector no symmetry, no dynamics is needed; only chance Hall, Murayama, Weiner '00.....
de Gouvea, Murayama '12

$\theta_{12}, \theta_{13}, \theta_{23}$ are just 3 random angles, the value of $r = \Delta m_{\text{sun}}^2 / \Delta m_{\text{atm}}^2 \sim 1/30$ is also determined by chance

See-Saw: $m_{\nu} \sim m^T M^{-1} m$ produces some hierarchy (r small) from random m, M . But θ_{13} and r are still too small

In models based on $SU(5) \times U(1)_{\text{FN}}$ one gets more success with the same n. of parameters by charge assignments that mitigate anarchy

GA, Feruglio, Masina '02,'06

GA, Feruglio, Masina, Merlo '12



SU(5)xU(1)

One can try different charge assignments

Recall: $m_u \sim 10 \ 10$

$m_d = m_e^T \sim 5^{\text{bar}} \ 10$

$m_{\nu D} \sim 5^{\text{bar}} \ 1; M_{RR} \sim 1 \ 1$

No structure for leptons



No automatic $\det 23 = 0$



Automatic $\det 23 = 0$



With suitable charge assignments many relevant patterns can be obtained

1st fam. 2nd 3rd

$$\left\{ \begin{array}{l} \Psi_{10}: (5, 3, 0) \\ \Psi_5: (2, 0, 0) \\ \Psi_1: (1, -1, 0) \end{array} \right.$$

Equal 2,3 ch. for lopsided

Model	Ψ_{10}	Ψ_5	Ψ_1
Anarchy (<i>A</i>)	(3,2,0)	(0,0,0)	(0,0,0)
Semianarchy $\mu\tau$ -Anarchy (<i>A</i> $_{\mu\tau}$)	(3,2,0)	(1,0,0)	(2,1,0)
Pseudo $\mu\tau$ -Anarchy (<i>PA</i> $_{\mu\tau}$)	(5,3,0)	(2,0,0)	(1,-1,0)
Hierarchy (<i>H</i>) new	(5,3,0)	(2,1,0)	(2,1,0)

all charges non negative

charges of both signs


here r, θ_{13} are suppressed

From Anarchy and $U(1)_{FN}$ to more symmetry

Larger than $U(1)$ continuous symmetries:

$$\text{e.g. } U(3)_l \times U(3)_e \text{ ----> } U(2)_l \times U(2)_e$$

Blankenburg, Isidori, Jones-Perez '12
Alonso, Gavela, isidori, Maiani'13

At the other extreme from Anarchy 
models with a maximum of order:
based on non abelian discrete flavour groups

(reviews: G.A., Feruglio, Rev.Mod.Phys. 82 (2010) 2701; Kobayashi et al'10;
Grimus, Ludl'11; G.A., Feruglio, Merlo'12 ;
Morisi, Valle'12; King, Luhn'13)

A number of “coincidences” could be hints
pointing to the underlying dynamics

[A particular implementation: sequential dominance - not pursued here
e.g. the recent paper by King, ArXiv:1311.3295]



King.....

TB Mixing

TB mixing is close to the data; less now,
but still: θ_{12}, θ_{23} agree at $< \sim 2\sigma$

$$U = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

and θ_{13} is the smallest angle

At 1σ :

Fogli et al '13

$$\sin^2\theta_{12} = 1/3 : 0.291 - 0.325$$

$$\sin^2\theta_{23} = 1/2 : 0.40 - 0.45$$

$$\sin\theta_{13} = 0 : 0.15 - 0.16$$

A coincidence or a hint?

Called:

Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

θ_{13} largish and θ_{23} non maximal move away from exact TB



(still remains a good first approximation)

LQC: Lepton Quark Complementarity

$$\theta_{12} + \theta_c = (46.4 \pm 0.8)^\circ \sim \pi/4 \quad \leftarrow \text{Gonzalez-Garcia et al '12}$$

Suggests Bimaximal Mixing (BM) corrected by diagonalisation of charged leptons
(in GUT charged leptons may know θ_c)

BM: Group S_4 GA, Feruglio, Merlo '09....
A coincidence or a hint?

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio

GR: Golden Ratio - Group A_5

Feruglio, Paris '11; G. J. Jing et al '11

Cooper et al '12, de Madeiros Verzielas et al '13....

$$\sin^2 \theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5 + \sqrt{5}} \approx 0.276$$

A coincidence or a hint?

$$U_{GR} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ \frac{\sin \theta_{12}}{\sqrt{2}} & -\frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}}{\sqrt{2}} & -\frac{\cos \theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



Cannot all be true hints, perhaps none

TB Mixing naturally leads to discrete flavour groups
(similarly for GR, BM....)

TB Mixing: $U = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

This is a particular rotation matrix with specified fixed angles

TB: Group A4, S4, T'..... A vast literature (Ma, Rajasekaran '01.....)

Some recent works: **A4** Ferreira et al '13; Morisi et al '13; Gonzalez-Felipe et al '13
Holthausen et al '12; Ben Tov et al '12; King et al '12 ...

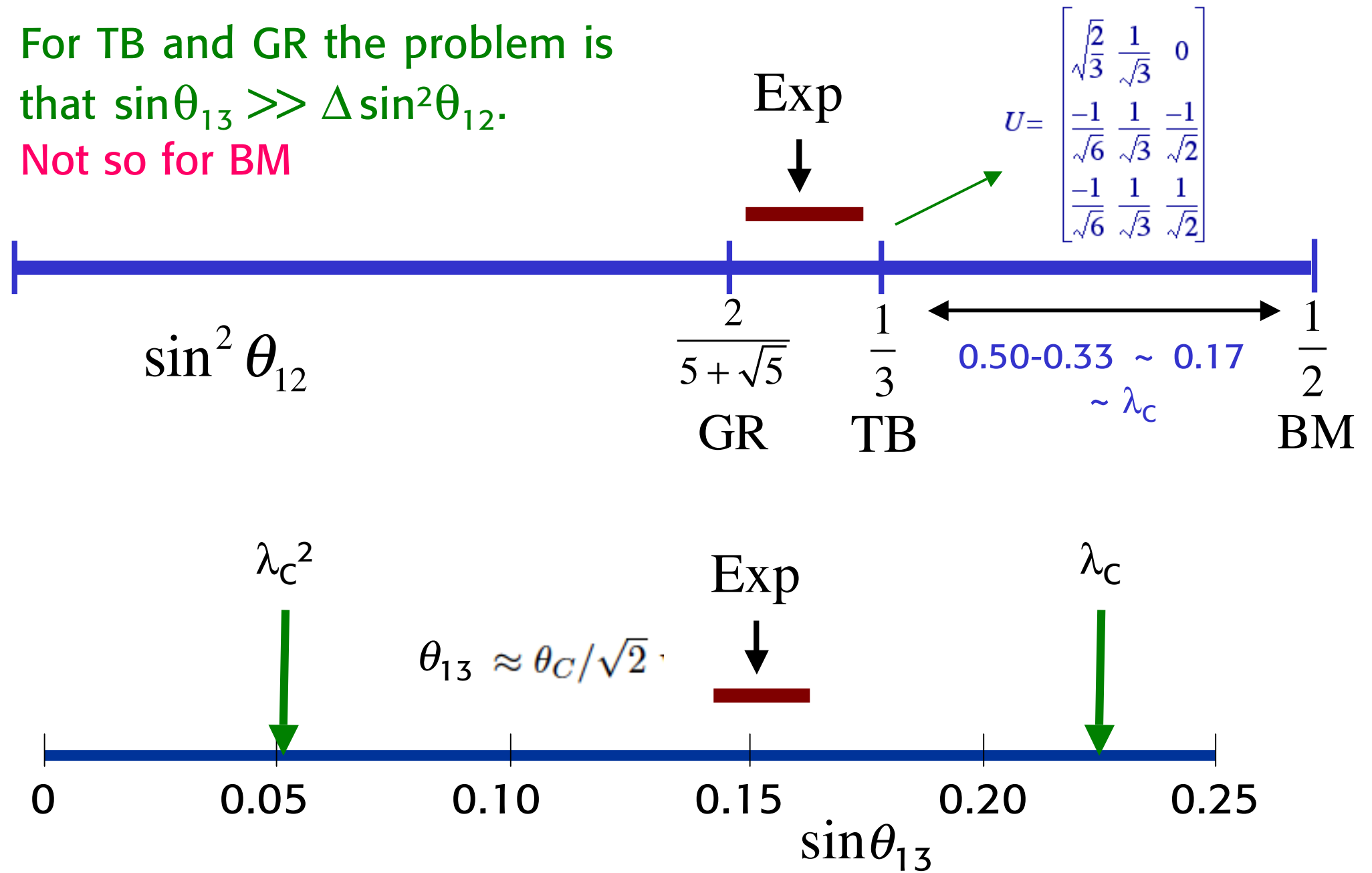
S4 Bazzocchi et al '12; Hagedorn et al '12; Zhao '11.....

T' Chen et al '13; Meroni et al '12; Merlo et al '11.....



For TB and GR the problem is that $\sin\theta_{13} \gg \Delta \sin^2\theta_{12}$.

Not so for BM



With θ_{13} largish TB models need some additional ingredient

Some selected versions are still perfectly viable

GA, Feruglio, Merlo, Stamou '12

e.g. Lin '09 discussed a natural A4 model where $\theta_{13} \sim o(\lambda_C)$, $\theta_{12} \sim o(\lambda_C^2)$
[ch. lepton and ν sectors are kept separate also at NLO]

Yin Lin: ArXiv:0905.3534

Alternatively:

Symmetry requirements have been relaxed

Hernandez, Smirnov '12

eg
 $G_\nu = Z_2$

He, Zee '07 and '11; Grimus, Lavoura '08; Grimus, Lavoura, Singraber '09;
Albright, Rodejohann '09; Antusch, King, Luhn, Spinrath '11; King, Luhn '11
Hall, Ross '13...

Larger groups have been studied

de A. Toorop, Feruglio, Hagedorn '11;
Lam '12 - '13; de Madeiros Verzielas,
Ross '12; Holthausen, Lim, Lindner '12;
Neder, King, Stuart '13....

eg $\Delta(600)$ (!!)

CP violation has been included in the symmetry breaking pattern

$$\oplus G_\nu = Z_2 \times CP$$

Feruglio, Hagedorn, Ziegler '12 - '13;
Ding, King, Luhn, Stuart '13;
Girardi, Meroni, Petcov, Spinrath '13;
Chen et al '14.....

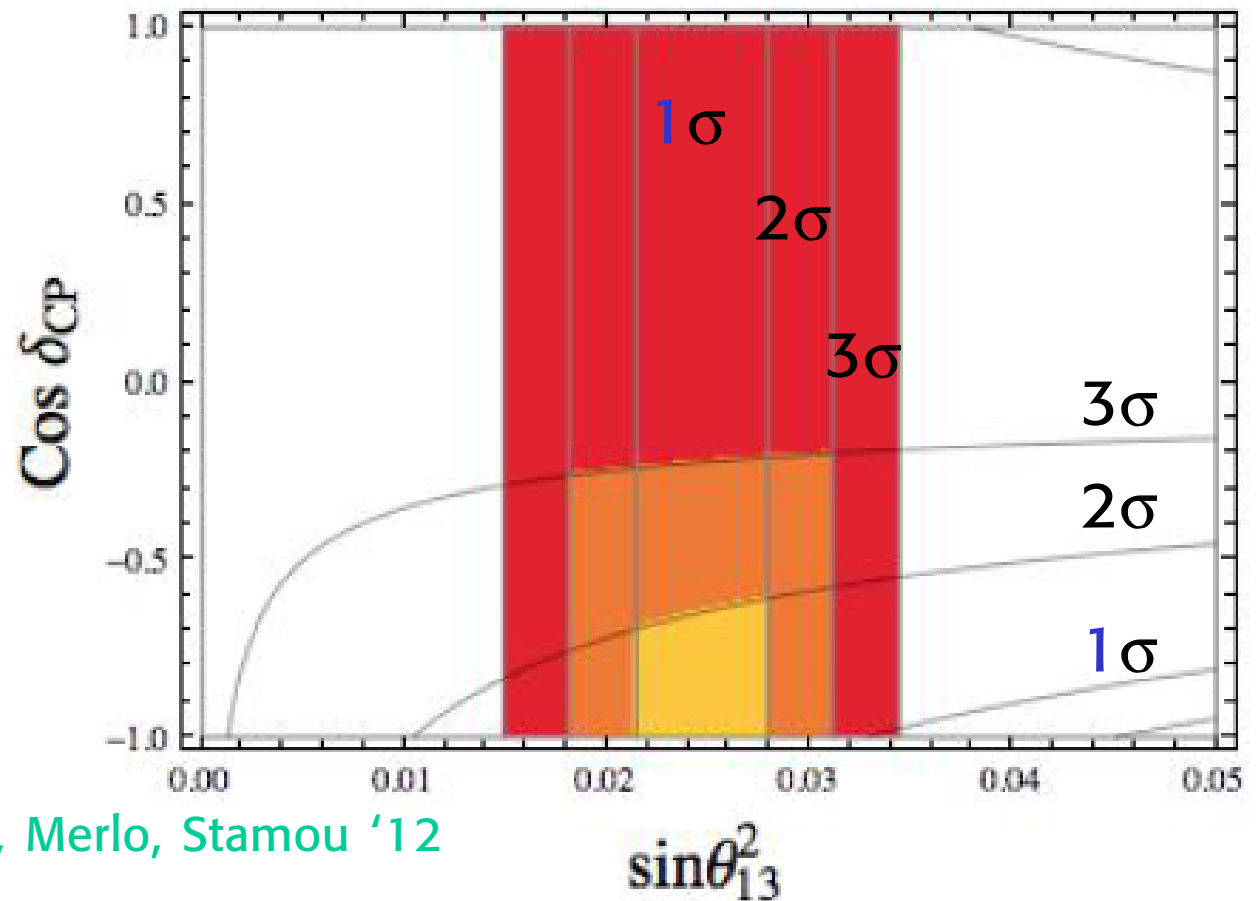
Testable sum rules arise in several models

In Lin model by neglecting small corrections one predicts:

Yin Lin: ArXiv:0905.3534

$$\sin^2 \theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \theta_{13} \cos \delta_{CP}$$

which requires
 $\cos \delta_{CP} < 0$



GA, Feruglio, Merlo, Stamou '12



Bimaximal Mixing

Inspired by the “complementarity” relation:

$$\theta_{12} + \theta_C = (46.4 \pm 0.8)^\circ \sim \pi/4$$

Raidal'04;
Minakata, Smirnov '04

one is led to consider models that give $\theta_{12} = \pi/4$ before corrections from the diag'tion of charged leptons

θ_{13} large is not problematic in this case! GA, Feruglio, Masina '04
....

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

e.g. in GUT's a connection between ch. leptons and quark mixing and θ_C

Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ “weak compl.”
rather than $\theta_{12} + \theta_C \sim \pi/4$



$$\delta_{CP} = \pi + \arg(c_{12}^e - c_{13}^e)$$

$$\sin \theta_{13} = \frac{1}{\sqrt{2}} |c_{12}^e - c_{13}^e| \xi$$

$$\sin^2 \theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}} \text{Re}(c_{12}^e + c_{13}^e) \xi$$

$$\sin^2 \theta_{23} = \frac{1}{2}.$$

For dominance of a single c^e ,
e.g. $c_{13}^e=0$ we have a sum rule

c_{ij}^e rotation angles for ch. leptons

$$\sin^2 \theta_{12} = \frac{1}{2} + \sin \theta_{13} \cos \delta_{CP}$$

equivalent to

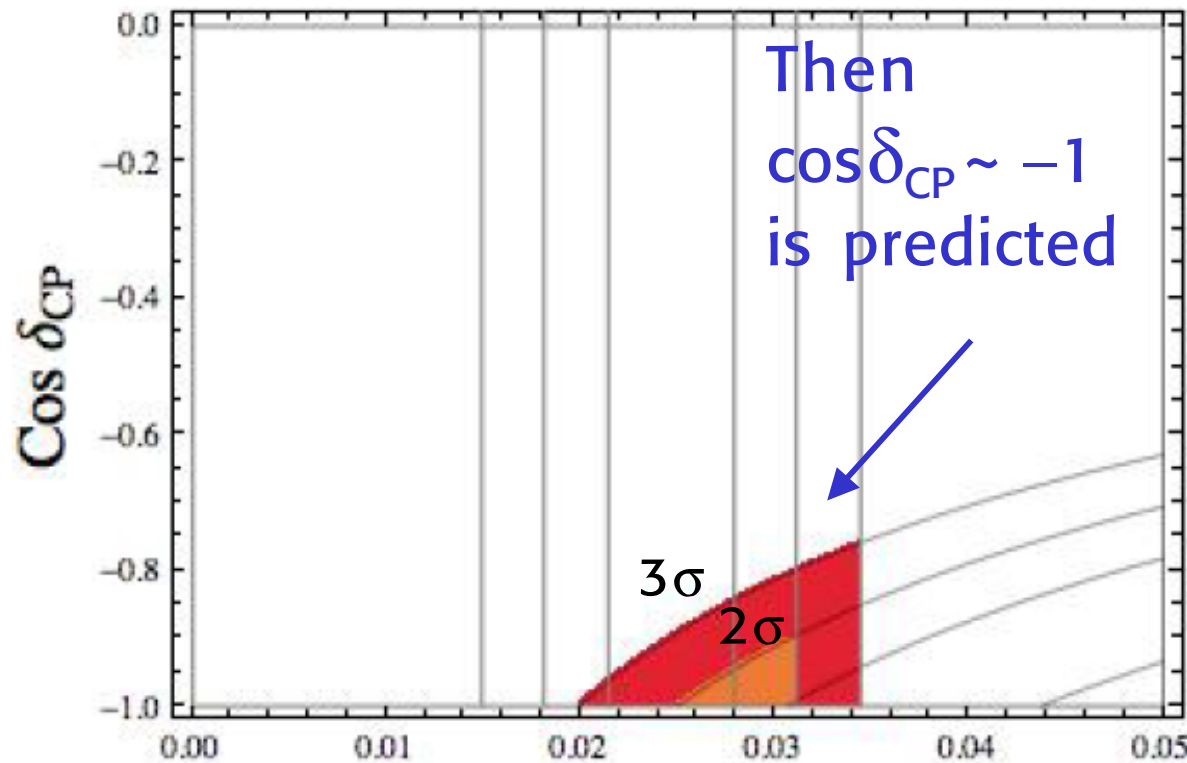
$$\theta_{12} \sim \pi/4 + \sin \theta_{13} \cos \delta_{CP}$$

Masina '05

If the same dominance
of ch. lepton corr.s
is assumed for TB mixing

$$\theta_{12} \sim \theta_{12}^{\text{TB}} + \sin \theta_{13} \cos \delta_{CP}$$

then $\cos \delta_{CP} \sim 0$



GA, Feruglio, Merlo, Stamou '12 $\sin^2 \theta_{13}$

Conclusion

Neutrino physics deals with fundamental issues, is being vigorously studied and our knowledge has much increased in the last 15 years

But many crucial problems remain open: Dirac/Majorana, $|m^2_{ij}|$, hierarchy (normal or inverse), CP viol., sterile ν 's,

Data on mixing angles are much better now but models of neutrino mixing still span a wide range from anarchy to discrete flavour groups

In the near future it will not be easy to decide from the data which ideas are right

The main problem of discrete flavour groups is not so much that θ_{13} is large but that there is no hint from quarks for them

So far no real illumination came from leptons to be combined \oplus with the quark sector for a more complete theory of flavour

Backup



Anarchy and its variants can be embedded in a simple GUT context based on

SU(5)xU(1)_{flavour}



U(1): Froggatt Nielsen '79

Offers a simple description of hierarchies for quarks and leptons, but only orders of magnitude are predicted (large number of undetermined $o(1)$ parameters c_{ab})

$\lambda_C = \sin\theta_C$

The typical order parameter is $o(\lambda_C)$ and the entries of mass matrices are suppressed by $m_{ab} \sim c_{ab} (\lambda_C)^{n_{ab}}$

The exponents n_{ab} are fixed by the charge imbalance



Anarchy can be realised in SU(5) by putting all the flavour structure in $T \sim 10$ and not in $F^{\text{bar}} \sim 5^{\text{bar}}$ and $V_R \sim 1$

$$\begin{array}{ll}
 m_u \sim 10 \cdot 10 & \text{strong hierarchy } m_u : m_c : m_t \\
 m_d \sim 5^{\text{bar}} \cdot 10 \sim m_e^T & \text{milder hierarchy } m_d : m_s : m_b \\
 & \text{or } m_e : m_\mu : m_\tau
 \end{array}$$

Experiment supports that down quark & charged lepton hierarchy is roughly the square root of up quark hierarchy

$$m_\nu \sim V_L^T m_\nu V_L \sim 5^{\text{bar}T} \cdot 5^{\text{bar}} \quad \text{or for see saw } (5^{\text{bar}} \cdot 1)^T (1 \cdot 1) (1 \cdot 5^{\text{bar}})$$

For example, for the simplest flavour group, $U(1)_F$

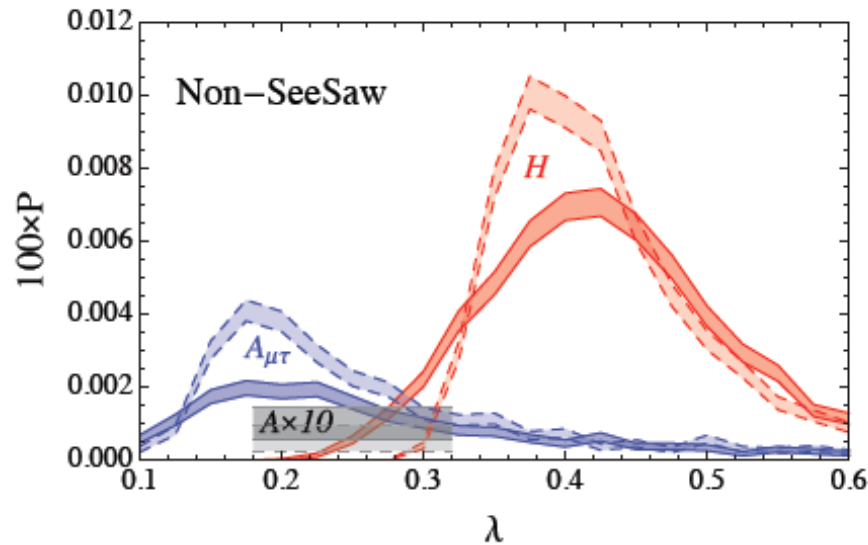
Anarchy

$$\left\{ \begin{array}{l}
 T : (3, 2, 0) \\
 F^{\text{bar}} : (0, 0, 0) \\
 1 : (0, 0, 0)
 \end{array} \right.$$

1st fam. 2nd 3rd



If we embed anarchy in GUT's and explain quark hierarchies in terms of FN charges, then more effective variants of anarchy can be built, where chance is somewhat mitigated



GA, Feruglio, Masina '02,'06
GA, Feruglio, Masina, Merlo '12

Optimal values of $\lambda \sim 0(\lambda_c)$

$A_{\mu\tau}$: $\lambda \sim 0.2$ (non SS), 0.3 (SS)

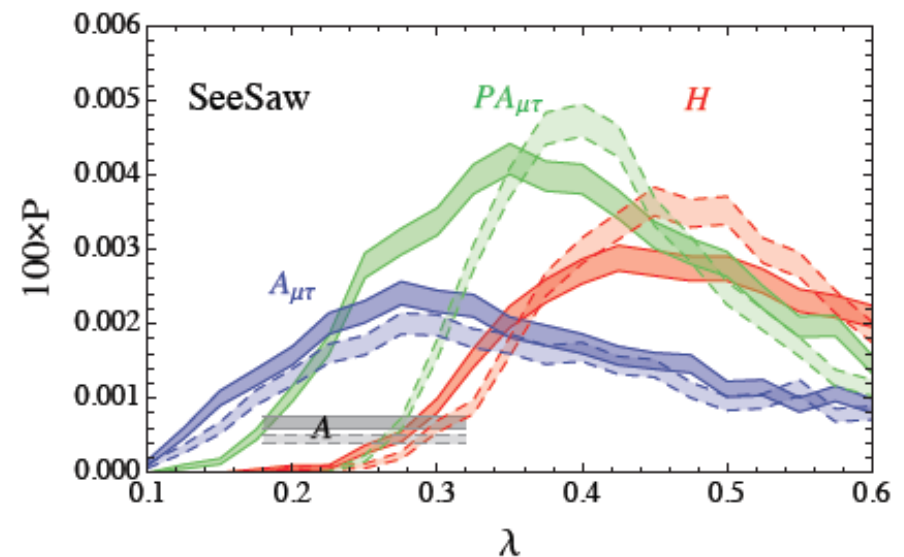
$PA_{\mu\tau}$: $\lambda \sim 0.35-0.4$

H: $\lambda \sim 0.4$ (non SS), 0.45 (SS)



Anarchy (A): both r and θ_{13} small by accident
 $\mu\tau$ -anarchy ($A_{\mu\tau}$): only r small by accident
H, $PA_{\mu\tau}$: no accidents

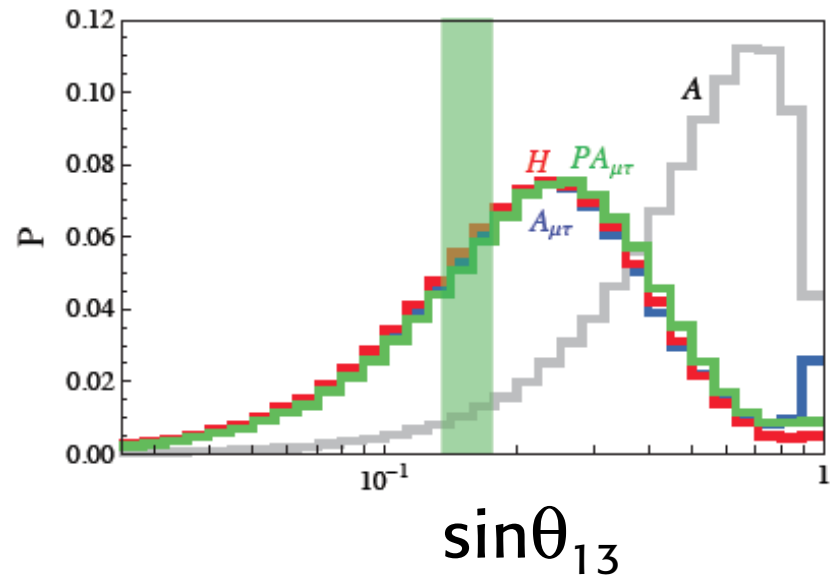
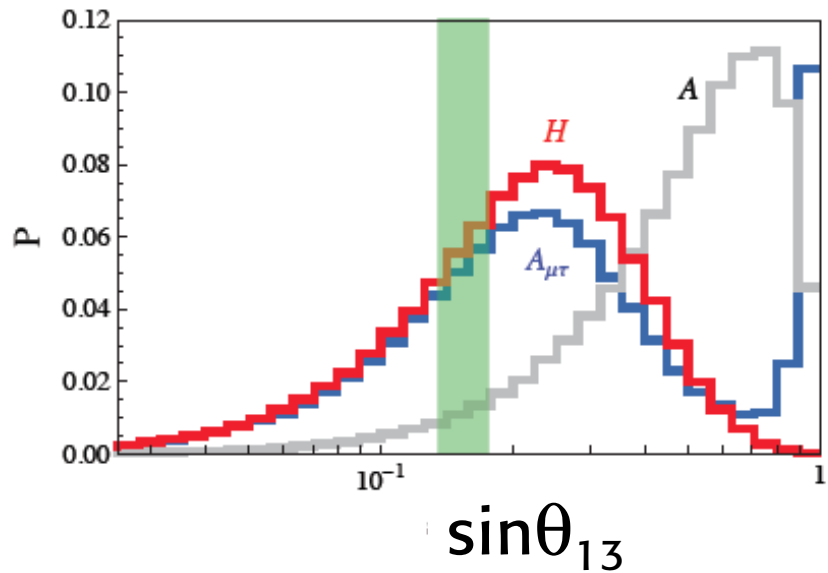
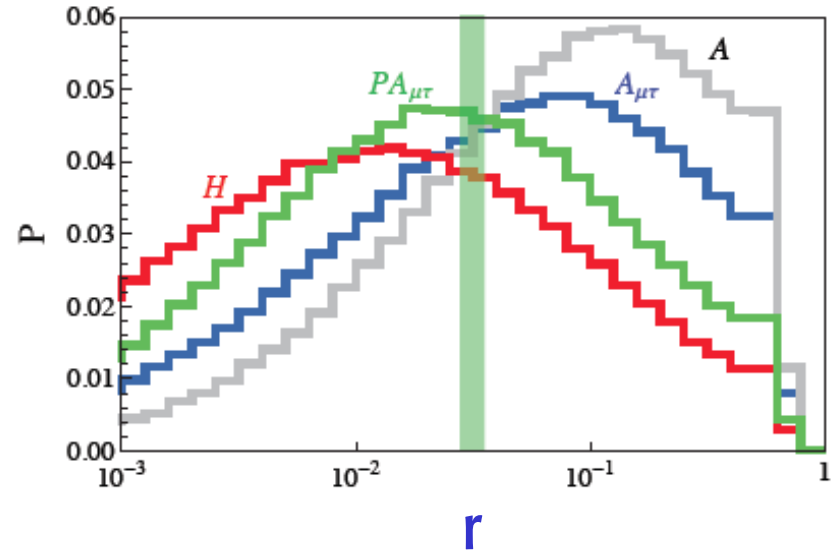
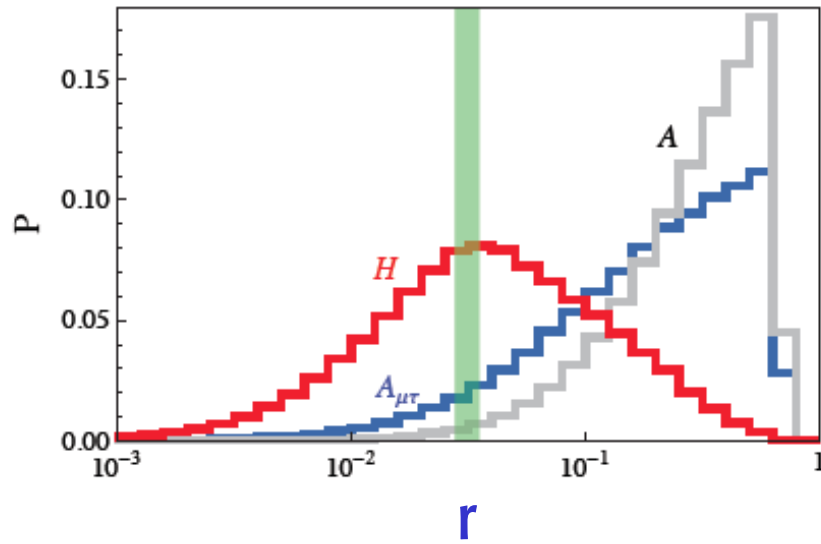
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no see-saw

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell H H \rightarrow \nu_L^T m_\nu \nu_L$$

when all charges are positive
see-saw only affects r



The Lagrangian density is invariant under the discrete flavour group G_f

3 generations $\rightarrow G_f$ with triplet representations

e.g. A4: ch. lepton doublets $l \sim 3$, singlets $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

At LO, G_f spont. breaks down to G_e in the charged lepton sector and to G_ν in the ν sector

$G_f: S4, A4, T' \dots$

G_ν : most general group
leaving $\nu^T m_\nu \nu$ invariant
with generic $m_{\nu i}$ is $G_\nu = Z_2 \times Z_2$
(Majorana ν imply G_ν discrete)

G_e : typical discrete groups
leaving $m_e^\dagger m_e$ invariant
with generic m_{ei} are
 $G_e = Z_n$ ($n > 2$)

⊕ This alignment is crucial and must be natural in a good model

At LO in A4 models TB mixing is exact

When NLO corrections are included from operators of higher dimension in the superpotential, **generically** each mixing angle receives corrections of the same order $\delta\theta_{ij} \sim o(\text{VEV}/\Lambda) \sim o(\xi)$

Typical
predicted
pattern

$$\sin^2 \theta_{12} = \frac{1}{3} + o(\xi) \quad \longleftarrow \sim -0.03$$

$$\sin^2 \theta_{23} = \frac{1}{2} + o(\xi) \quad \longleftarrow \sim -0.07$$

$$\sin \theta_{13} = o(\xi) \quad \longleftarrow \sim 0.15$$

exp
values
of "o(ξ)"

As the needed corrections to θ_{12} and θ_{23} are numerically $o(\lambda_c^2)$, one typically expected $\theta_{13} \sim o(\lambda_c^2)$

⊕ This generic prediction can be altered in special versions