

## Methods to solve correlated systems : Stochastic approach

Goal : Solve correlated quantum systems such as Nuclei by using a new approach. Firstly, applied to open quantum systems.

Supervisor :

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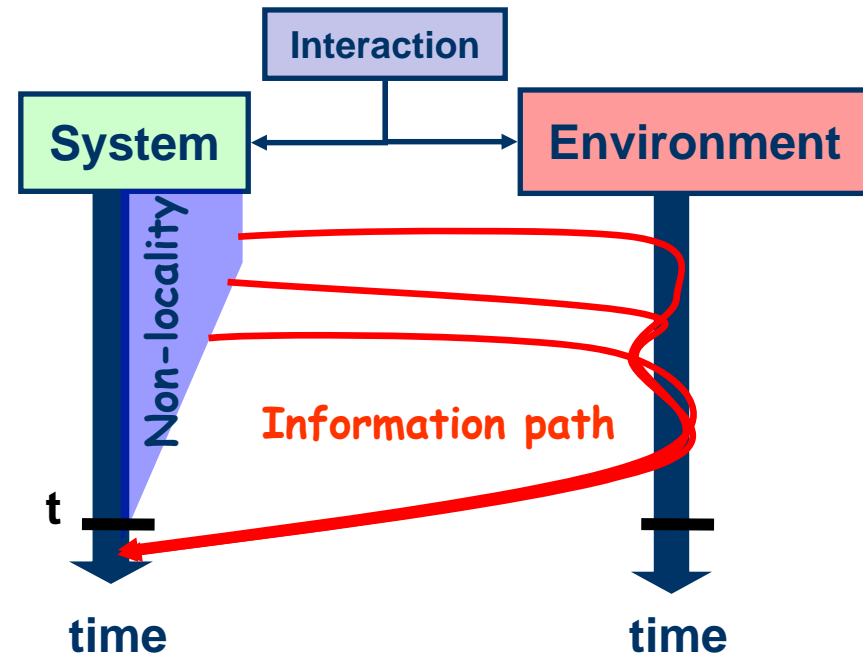
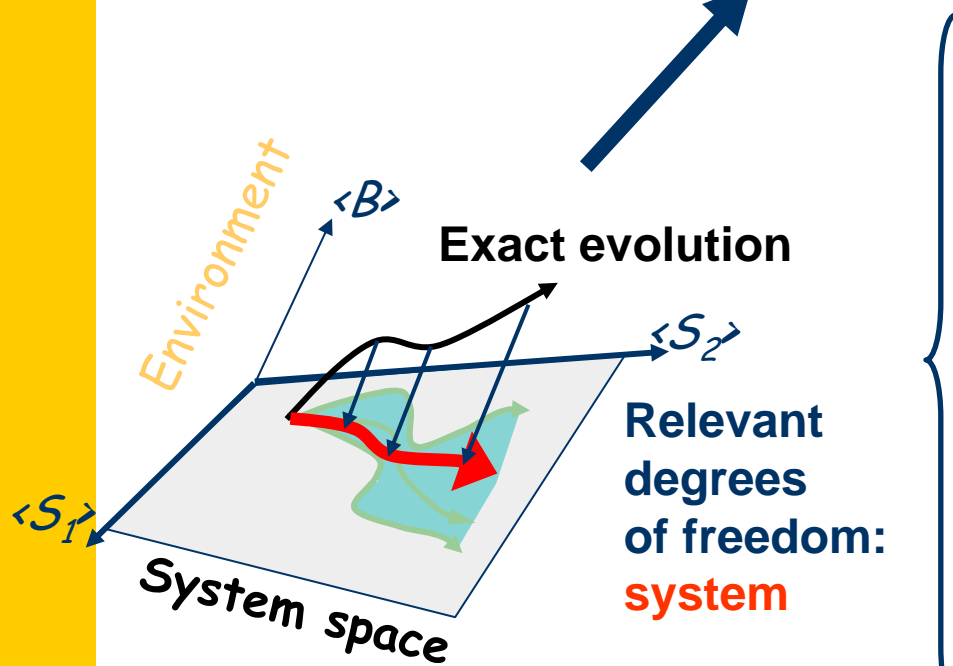
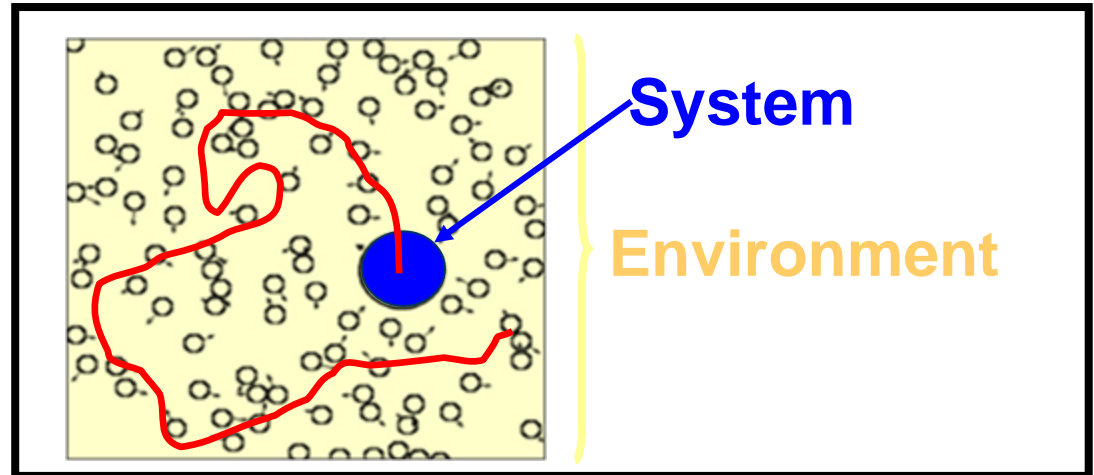
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# Introduction to open quantum systems

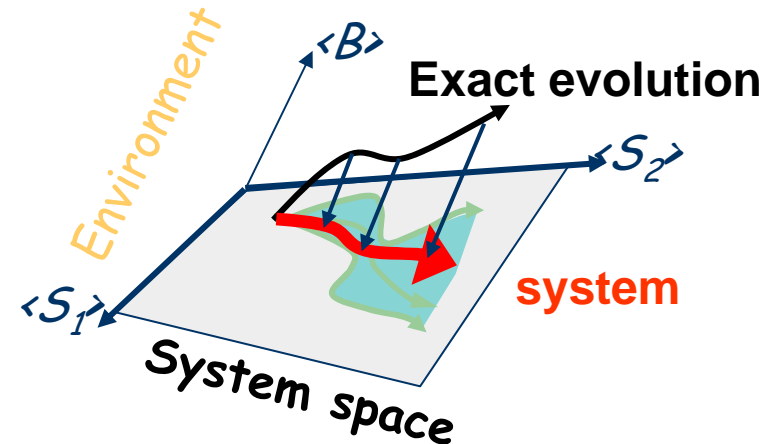
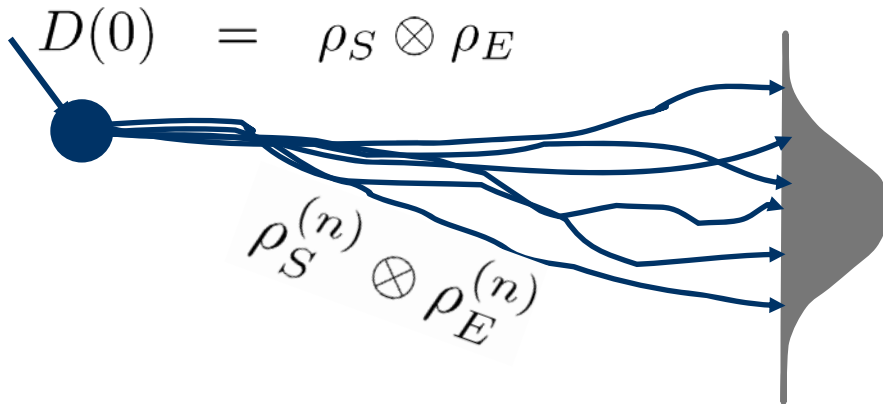
R. Brown, (1827).



# Stochastic formulation for reduced dynamic

The system density can be written as an average :

$$D_{exact}(t) = \sum_n \Lambda_n \rho_S^{(n)} \otimes \rho_E^{(n)}$$



Langevin equation :

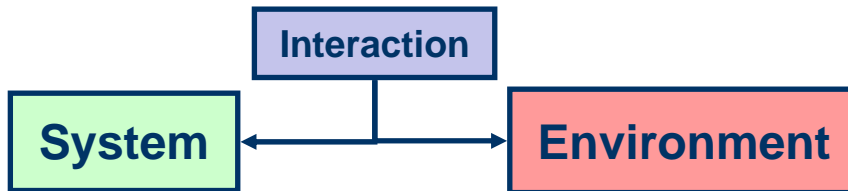
$$d\tilde{X}_i^n = G_i[\vec{X}^{(n)}(t), \dots, \vec{X}(t), t]dt + B_{i,j}(\vec{X}(t))dW_j(t)$$

Deterministic part      Noise part plays a diffusion rôle

# Generalities on open quantum systems

*Denis Lacroix, Physical Review E (2008)*

## Open quantum mechanics



- Liouville von Neumann equation :

$$i\hbar \frac{dD}{dt}(t) = [H, D(t)]$$

- Equivalent Stoch. Density Eq. :

$$\overline{d\xi_S d\xi_E} = \frac{dt}{i\hbar}$$

$$\overline{d\lambda_S d\lambda_E} = -\frac{dt}{i\hbar}$$

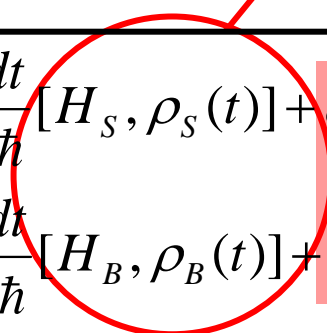
$$d\rho_S(t) = \frac{dt}{i\hbar} [H_S, \rho_S(t)] + d\xi_S Q \rho_S(t) + d\lambda_S \rho_S(t) Q$$

$$d\rho_B(t) = \frac{dt}{i\hbar} [H_B, \rho_B(t)] + d\xi_E B \rho_B(t) + d\lambda_E \rho_B(t) B$$

$$H = H_S + H_B + Q \otimes B$$

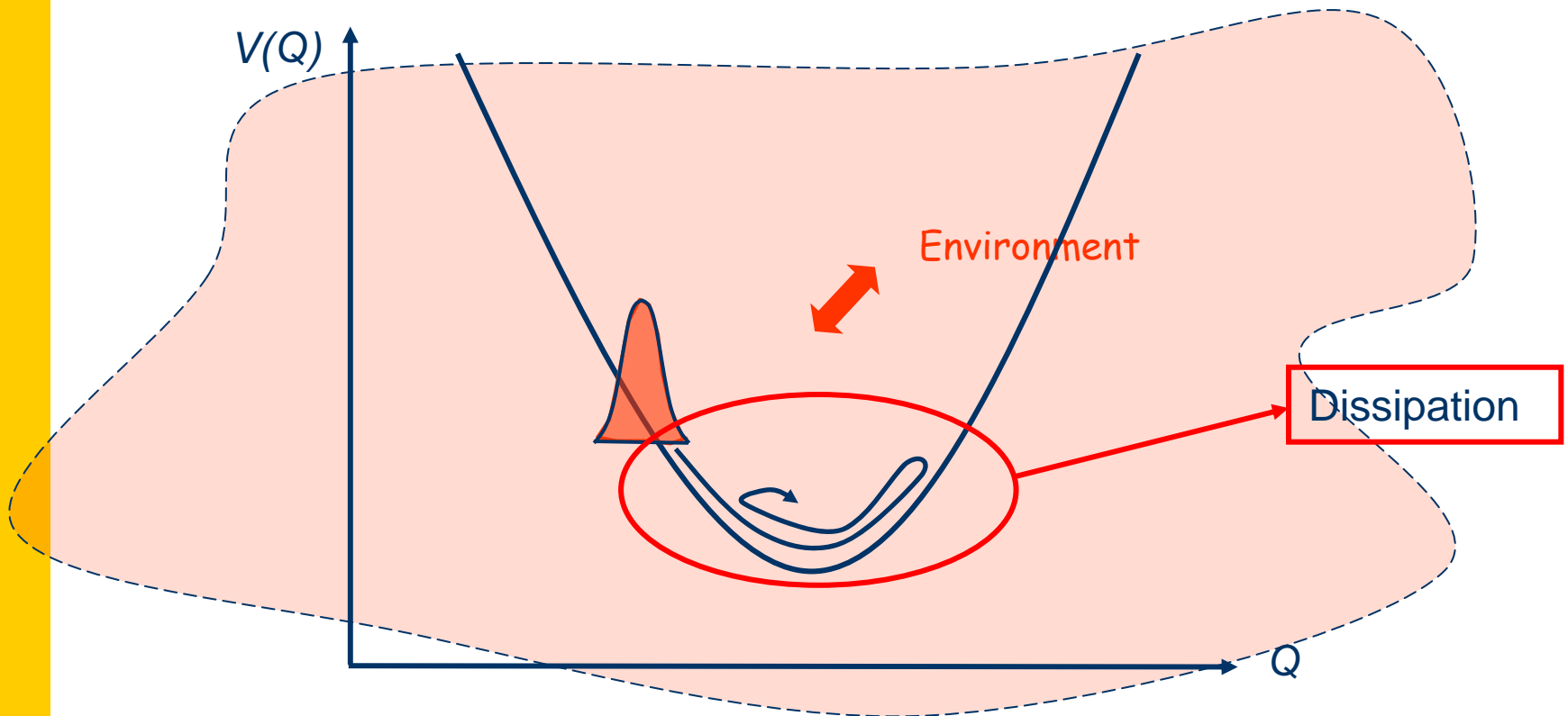
Langevin eq. determinist part

Langevin eq. diffusion part



# Applications to open quantum systems

1. This method has been applied to spin boson model.
2. M2 internship work : Application to the Caldeira Leggett model.



# Application : Caldeira Leggett model

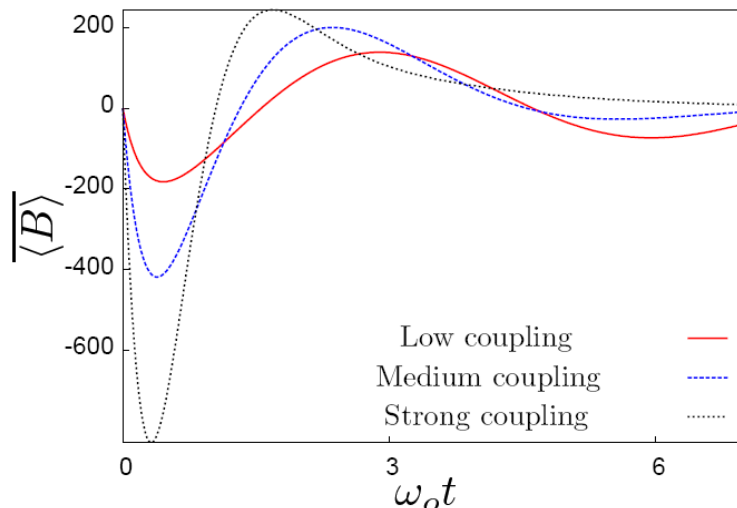
Observable expectation values are given by

$$d\langle P \rangle = -m_0 \omega_0^2 \langle Q \rangle dt - \langle B \rangle \frac{2du_s}{\hbar} dt - \hbar dv_s$$

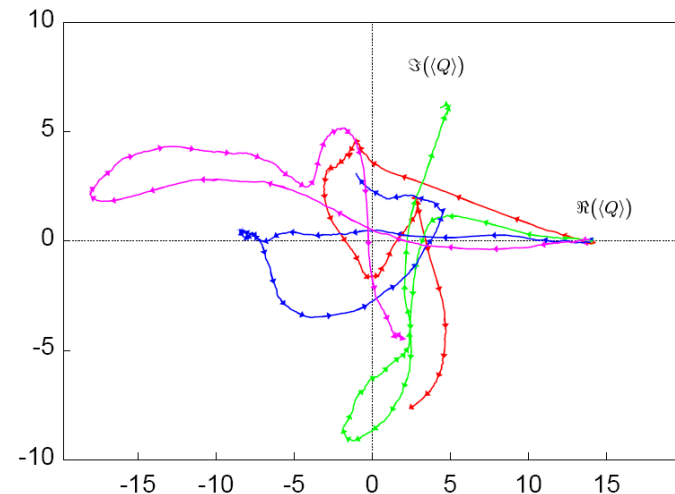
$$d\langle Q \rangle = \frac{\langle P \rangle}{m_0} dt - 2du_s \sigma_{QQ}$$

Random part

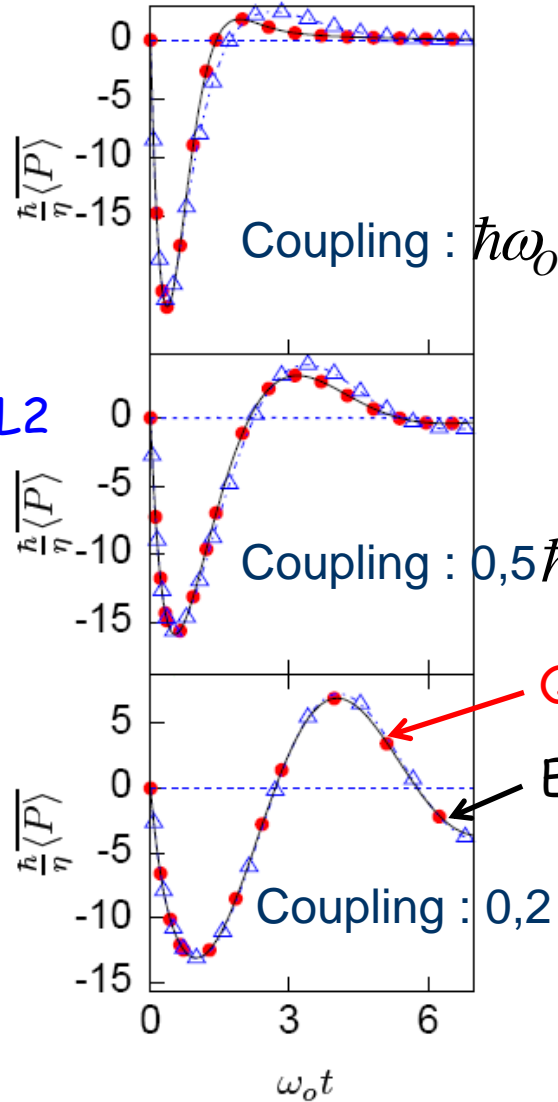
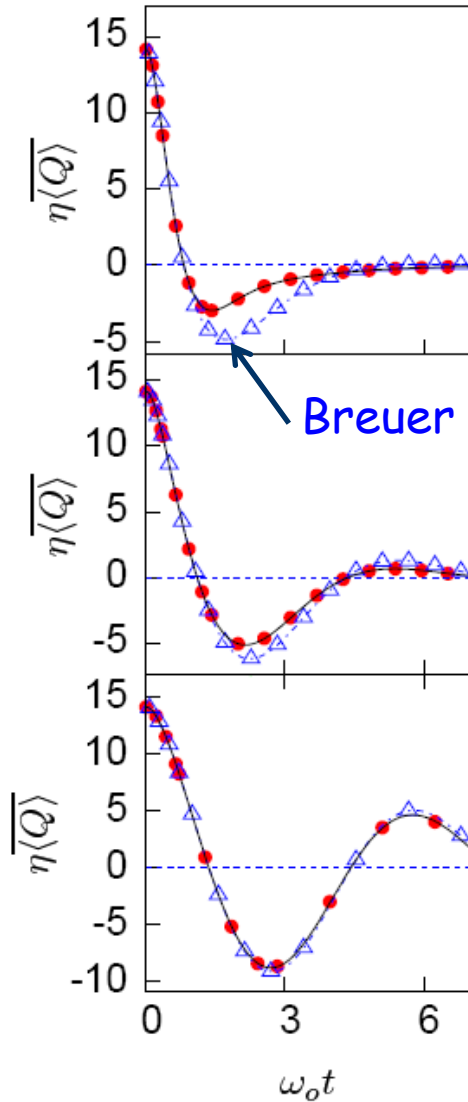
1° Average evolution of the bath observable B



2° Expectation values of Q and P in the complex plane



# Evolution of the first moments



Using a Drude spectral density:

$$J(\omega) = \frac{\eta 2m_0}{\pi} \omega \frac{\Omega^2}{\omega^2 + \Omega^2}$$

$$\Omega = 5\hbar\omega_0$$

$$\hbar\omega_0 = 14\text{MeV}$$

$$T = \hbar\omega_0$$

Quantum+Stat

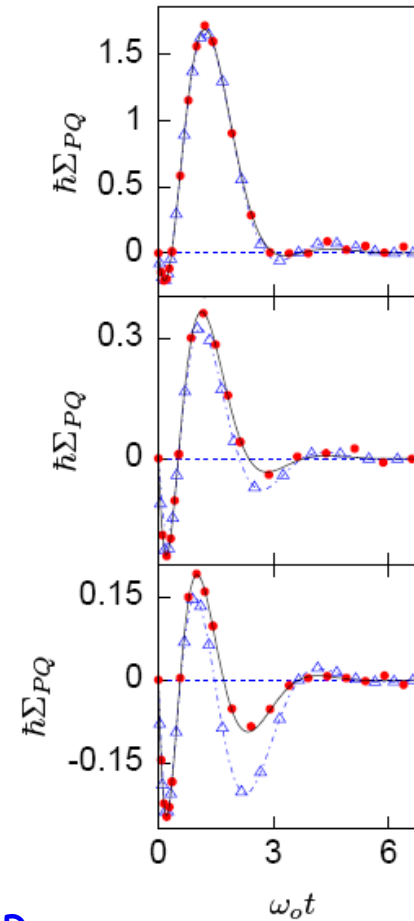
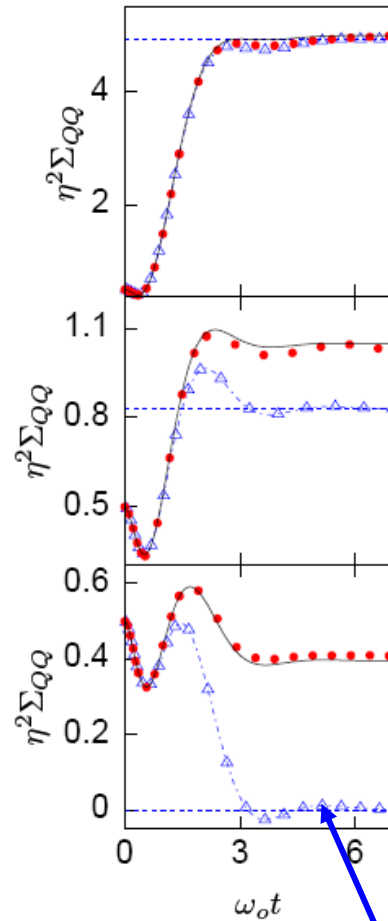
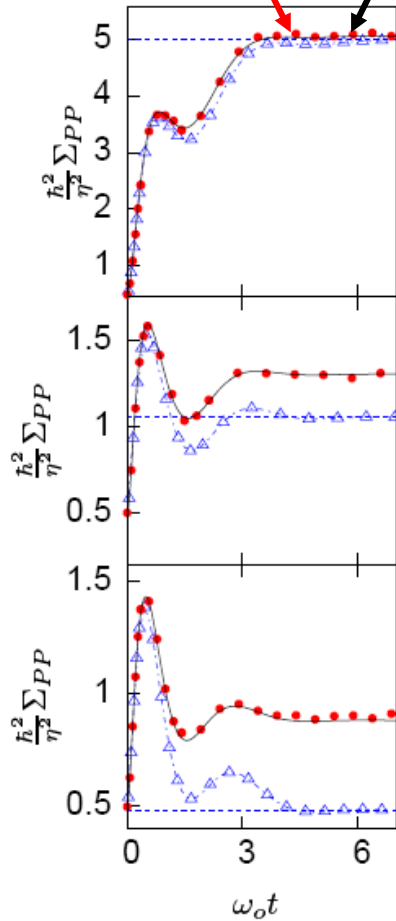
Exact

# 2<sup>nd</sup> moments evolution

Quantum+Stat

Exact

Coupling :  $0,5 \hbar \omega_0$



$T : 5 \hbar \omega_0$

$T = \hbar \omega_0$

$T : 0,1 \hbar \omega_0$

Breuer  
TCL2



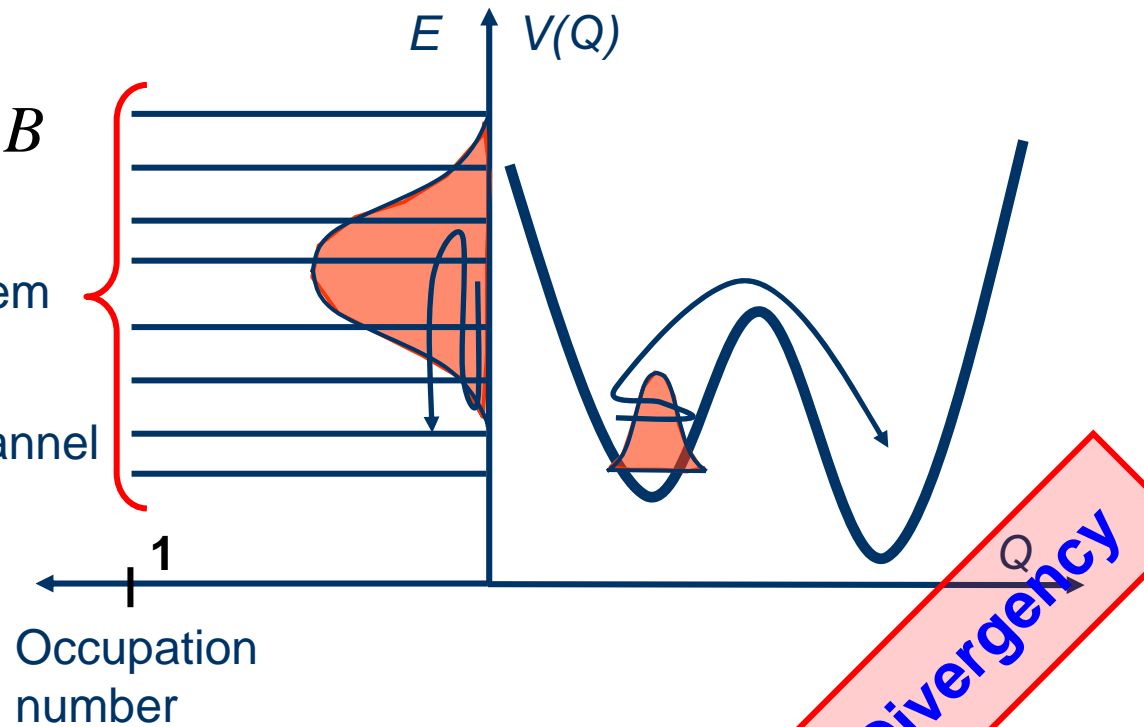
# Applications to open quantum systems

1. This method has been applied to spin boson model.
2. M2 internship work : Application to the Caldeira Leggett model.
3. Last three month : Application to any kind of potential with different methods

$$H = H_S + H_B + Q \otimes B$$

Eigenstates of the system alone

- Coupled channel

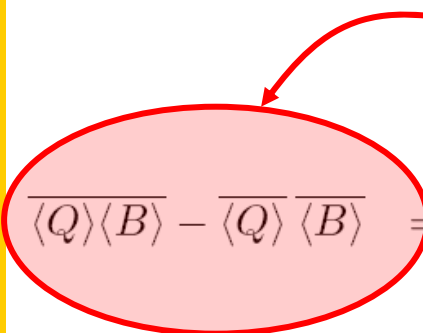


# Applications to open quantum systems

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Quantum+Statistical fluctuations

$$\Sigma_{QQ} \equiv \overline{\langle Q^2 \rangle} - \overline{\langle Q \rangle}^2 = \sigma_{QQ} + \overline{\langle Q \rangle}^2 - \overline{\langle Q \rangle}^2$$

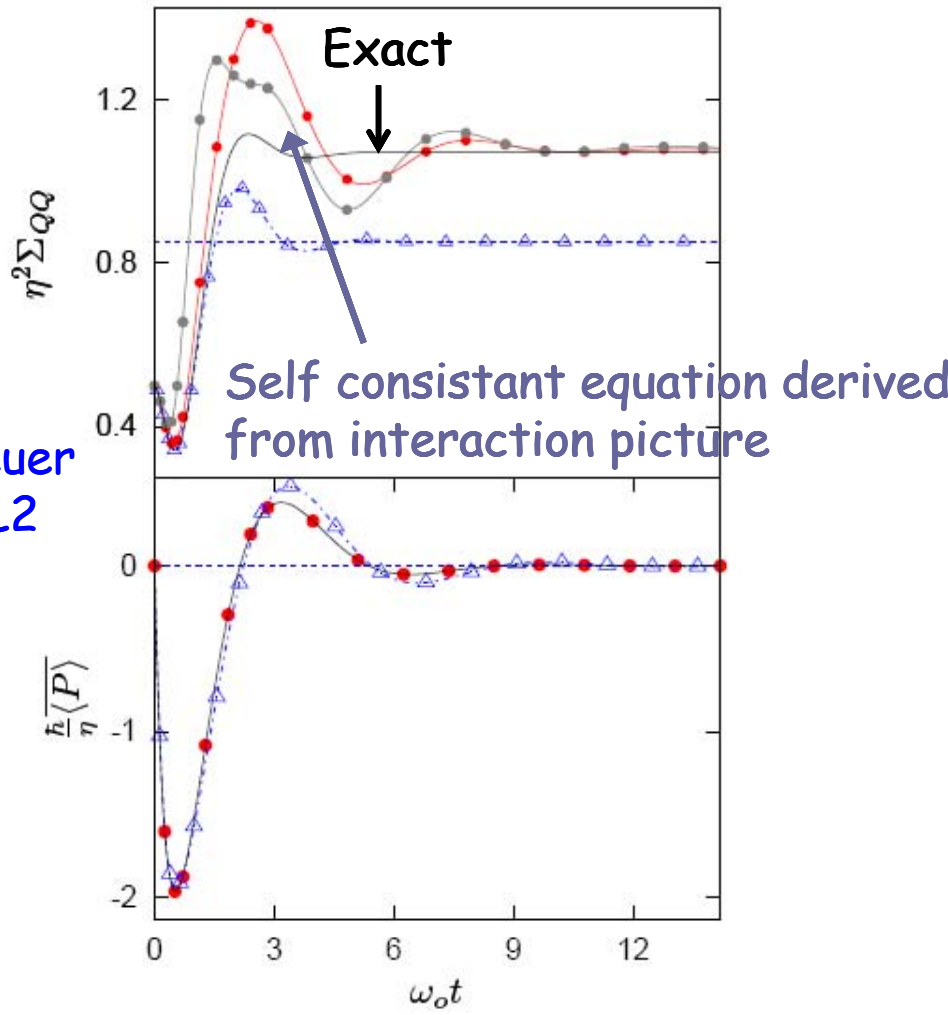
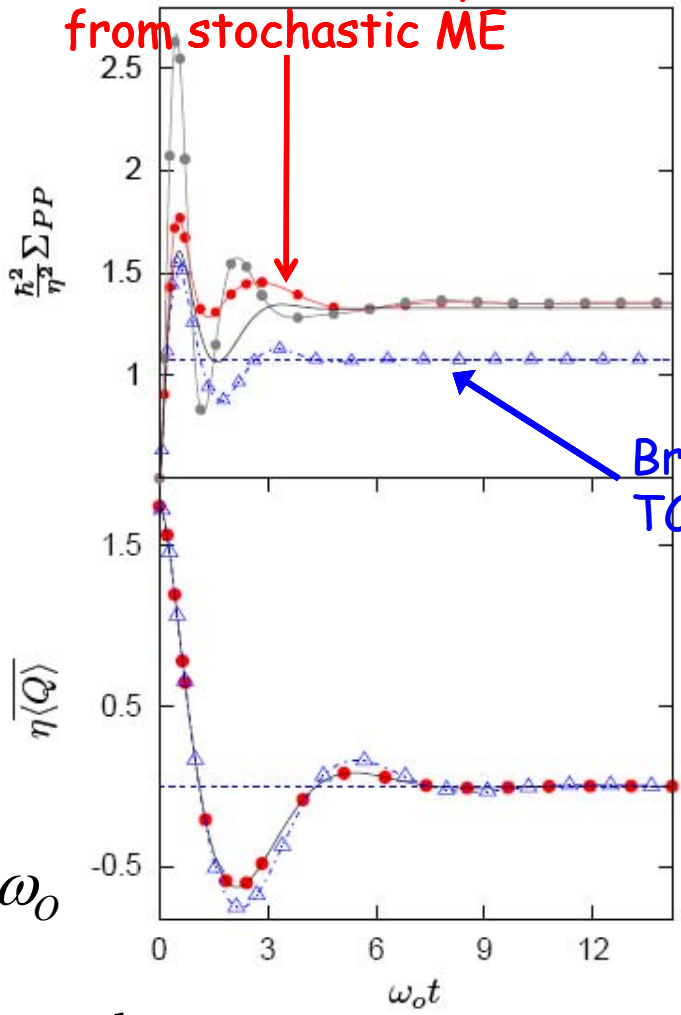

$$\overline{\langle Q \rangle \langle B \rangle} - \overline{\langle Q \rangle} \overline{\langle B \rangle} = -\frac{1}{\hbar} \int_0^t D(t-s) ds \left( \cos(\omega_o(t-s)) S_{QQ}(s) - \sin(\omega_o(t-s)) \frac{S_{PQ}(s)}{m_o \omega_o} \right) - \frac{1}{2} \int_0^t \frac{ds}{m_o \omega_o} D_1(t-s) \sin(\omega_o(t-s))$$

+ higher order

- Self consistent calculation

# Self consistent calculation : for harmonic oscillator

Self consistent equation derived from stochastic ME



$T = \hbar\omega_0$

Coupling :  $0,5\hbar\omega_0$

## Conclusion & perspective

- A new formulation has been obtained with the help of stochastic approach.
- Experience on memory kernel, numerical simulation & computation acquired.
- Introduction on open quantum theory.

- Find a way to get convergence of the stochastic approach?
- Application to the N body problem.