

Baryon-to-meson transition distribution amplitudes:

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- 1 Introduction: DAs, GPDs, TDAs
- 2 Forward and backward kinematical regimes
- 3 πN TDAs: definition, properties, support, spectral representation, chiral constraints
- 4 Factorized Ansatz for quadruple distributions.
- 5 $N\bar{N} \rightarrow \pi\gamma^* \rightarrow \pi\ell^+\ell^-$ and $N\bar{N} \rightarrow \pi J/\psi$ cross section estimates
- 6 Summary and Outlook

B. Pire, K. S., L. Szymanowski Phys. Rev. D **82**, 094030 (2010)

B. Pire, K. S., L. Szymanowski, Phys. Rev. D **84**, 074014 (2011)

J.P. Lansberg, B. Pire, K. S., L. Szymanowski, Phys. Rev. D **85**, 054021 (2012)

J.P. Lansberg, B. Pire, K. S., L. Szymanowski, Phys. Rev. D **86**, 114033 (2012)

B. Pire, K. S., L. Szymanowski, Phys. Lett. B **724**, 99 (2013)

- Factorization theorems for hard reactions: amplitude as convolution of perturbative and non-perturbative parts.
- Main objects: matrix elements of QCD light-cone ($z^2 = 0$) operators.

- Quark bilinear light-cone operator:

$$\langle A | \bar{\Psi}(0)[0; z] \Psi(z) | B \rangle$$

⇒ PDFs, meson DAs, GPDs, transition GPDs, etc.

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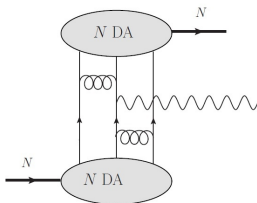
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- $\langle A | = \langle 0 |$; B - baryon ⇒ baryon DA. QCD description of nucleon e.m. FF.

Nucleon e.m. FF

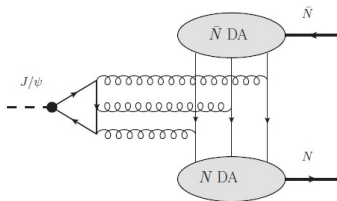
Brodsky & Lepage'81 Efremov & Radyushkin'80



Charmonium decay

$$J/\psi \rightarrow \bar{N} + N$$

Brodsky & Lepage'81 Chernyak, Ogloblin, and Zhitnitsky'89



$$\langle A | \Psi(z_1)[z_1; z_2] \Psi(z_2)[z_2; z_3] \Psi(z_3)[z_3; z_1] | B \rangle$$

- Let $\langle A |$ be a light meson state ($\pi, \eta, \rho, \omega, \dots$) B - a baryon \Rightarrow baryon-to-meson TDAs.

Common features with

- baryon DAs: same operator;
- GPDs: $\langle B |$ and $|A\rangle$ are not of the same momentum \Rightarrow skewness:

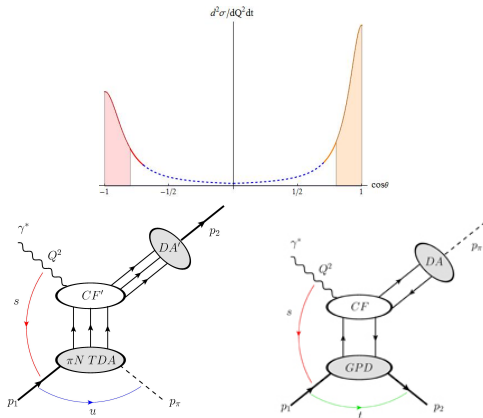
$$\xi = -\frac{(p_A - p_B) \cdot n}{(p_A + p_B) \cdot n}.$$

Factorization regimes for hard meson production

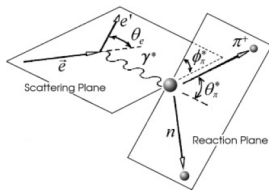
Generalized Bjorken limit ($-q^2 = Q^2$, W^2 - large; $x_B = \frac{Q^2}{2p \cdot q}$ - fixed)

Two complementary regimes:

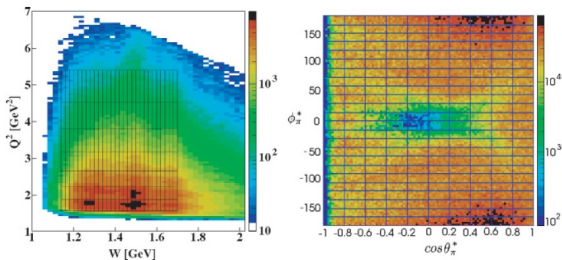
- $t \sim 0$ (forward peak) factorized description in terms of GPDs J. Collins, L. Frankfurt, M. Strikman'97;
- $u \sim 0$ (backward peak) factorized description in terms of TDAs L. Frankfurt, M. V. Polyakov, M. Strikman et al.'02;



Backward meson electroproduction @ CLAS I



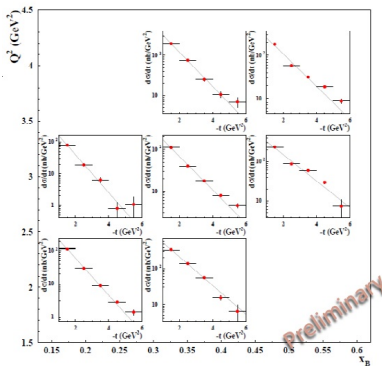
- Data from JLab @ 6 GeV exist for the backward $\gamma^* p \rightarrow \pi^+ n$. Analysis on-going [Kijun Park](#).



- Kinematical coverage for π^+ of the CLAS experiment [K. Park et al., PRC77:015208, 2008](#).

Backward meson electroproduction @ CLAS II

- Analysis of backward $\gamma^* p \rightarrow \pi^0 p$. V. Kubarovsky, CIPANP 2012.



$$\frac{d\sigma}{dt} = A \cdot e^{Bt} \quad (\text{away from the forward peak})$$

- G. Huber clear signal from backward ω production at Jlab Hall C.

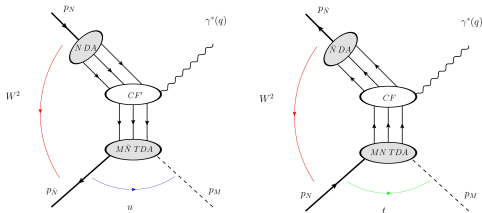
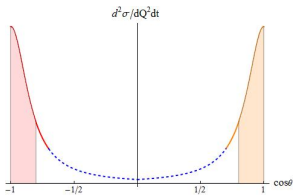
Baryon to meson TDAs at $\bar{P}ANDA$ I

- Factorized description of

$$\bar{N} + N \rightarrow \gamma^*(q) + \pi \rightarrow \ell^+ + \ell^- + \pi$$

in terms of MN TDAs.

- Two regimes (forward and backward). C invariance \Rightarrow perfect symmetry. (Lansberg et al.'12)



$\bar{P}ANDA$ @ GSI-FAIR

- $E_p \leq 15$ GeV; $W^2 \leq 30$ GeV²

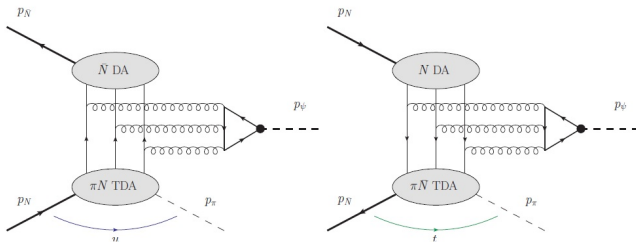
- Planned to be done with the proton FF studies in the timelike region.
- M.C. Mora Espí, F.Maas, M. Zambrana (in preparation): detailed feasibility studies of $\bar{p}p \rightarrow e^+e^-\pi^0$ @ $\bar{P}ANDA$.

Baryon to meson TDAs at \bar{P} ANDA II

- Charmonium production in association with a pion **Pire et al.'13**

$$\bar{N} + N \rightarrow J/\psi + \pi.$$

- Same TDAs \Rightarrow test of universality.
- Forward and backward regimes.



J.P.Lansberg, B.Pire & L.Szymanowski'07:

$$\begin{aligned}
 & 4(P \cdot n)^3 \int \left[\prod_{i=1}^3 \frac{dz_i}{2\pi} e^{ix_i z_i (P \cdot n)} \right] \langle \pi(p_\pi) | \varepsilon_{c_1 c_2 c_3} \Psi_\rho^{c_1}(z_1 n) \Psi_\tau^{c_2}(z_2 n) \Psi_\chi^{c_3}(z_3 n) | N(p_1, s_1) \rangle \\
 &= \delta(2\xi - x_1 - x_2 - x_3) i \frac{f_N}{f_\pi M} \\
 & \times \left[V_1^{\pi N} (\hat{P}C)_{\rho\tau} (\hat{P}U)_\chi + A_1^{\pi N} (\hat{P}\gamma^5 C)_{\rho\tau} (\gamma^5 \hat{P}U)_\chi + T_1^{\pi N} (\sigma_{P\mu} C)_{\rho\tau} (\gamma^\mu \hat{P}U)_\chi \right. \\
 & + V_2^{\pi N} (\hat{P}C)_{\rho\tau} (\hat{\Delta}U)_\chi + A_2^{\pi N} (\hat{P}\gamma^5 C)_{\rho\tau} (\gamma^5 \hat{\Delta}U)_\chi + T_2^{\pi N} (\sigma_{P\mu} C)_{\rho\tau} (\gamma^\mu \hat{\Delta}U)_\chi \\
 & \left. + \frac{1}{M} T_3^{\pi N} (\sigma_{P\Delta} C)_{\rho\tau} (\hat{P}U)_\chi + \frac{1}{M} T_4^{\pi N} (\sigma_{P\Delta} C)_{\rho\tau} (\hat{\Delta}U)_\chi \right]
 \end{aligned}$$

- $P = \frac{1}{2}(p_1 + p_\pi)$; $\Delta = (p_\pi - p_1)$; $n^2 = p^2 = 0$; $2p \cdot n = 1$; $\sigma_{P\mu} \equiv P^\nu \sigma_{\nu\mu}$;
- C : charge conjugation matrix;
- $f_N = 5.2 \cdot 10^{-3} \text{ GeV}^2$ (V. Chernyak and A. Zhitnitsky'84);
- $\xi = -\frac{\Delta \cdot n}{2P \cdot n}$
- 8 TDAs: $H(x_1, x_2, x_3, \xi, \Delta^2, \mu^2) \equiv \{V_i, A_i, T_i\}(x_1, x_2, x_3, \xi, \Delta^2, \mu^2)$
- c.f. 3 leading twist nucleon DAs: V^P, A^P, T^P

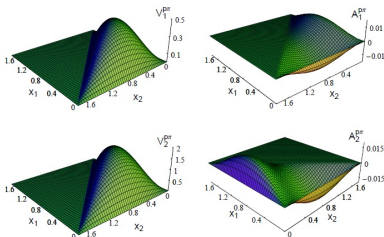
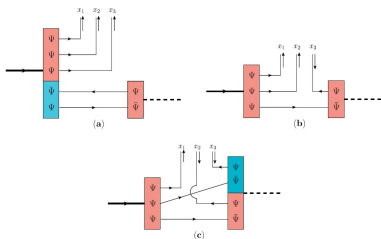
Interpretation and modelling of πN TDAs I

- Mellin moments in $x_i \Rightarrow \pi N$ matrix elements of local operators

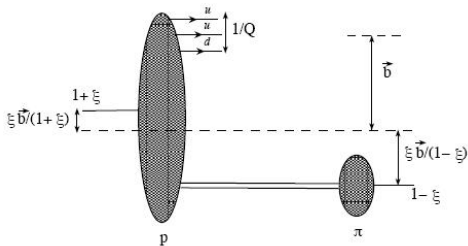
$$\left[i\vec{D}^{\mu_1} \dots i\vec{D}^{\mu_{n_1}} \Psi_\rho(0) \right] \left[i\vec{D}^{\nu_1} \dots i\vec{D}^{\nu_{n_2}} \Psi_\tau(0) \right] \left[i\vec{D}^{\lambda_1} \dots i\vec{D}^{\lambda_{n_3}} \Psi_\chi(0) \right].$$

Can be studied on the lattice [Y. Aoki et al.](#).

- πN TDAs provides information on the next to minimal Fock state. Light-cone quark model interpretation [B. Pasquini et al. 2009](#):



- Impact parameter space interpretation: the Fourier transform $\Delta_T \rightarrow b_T$ of TDAs \Rightarrow transverse picture of pion cloud in the proton



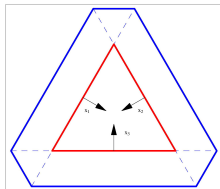
Fundamental theoretical requirements for πN TDAs:

B. Pire, L.Szymanowski, KS'10,11:

- 1 restricted support in x_1, x_2, x_3 : intersection of three stripes $-1 + \xi \leq x_i \leq 1 + \xi$ ($\sum_i x_i = 2\xi$)
 - 2 polynomiality in ξ of the Mellin moments in x_i
 - 3 isospin + permutation symmetry
 - 4 crossing: πN TDA \leftrightarrow πN GDA
 - 5 chiral properties: soft pion theorem **P. Pobylitsa, M. Polyakov and M. Strikman'01** constrains πN GDA at the threshold $\xi = 1, \Delta^2 = M^2$ in terms of nucleon DAs
 - 6 QCD evolution
- Spectral representation **A. Radyushkin'97** for πN TDAs: polynomiality and support:

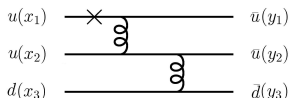
$$\begin{aligned} & H(x_1, x_2, x_3 = 2\xi - x_1 - x_2, \xi) \\ &= \left[\prod_{i=1}^3 \int_{\Omega_i} d\beta_i d\alpha_i \right] \delta(x_1 - \xi - \beta_1 - \alpha_1 \xi) \delta(x_2 - \xi - \beta_2 - \alpha_2 \xi) \\ & \times \delta(\beta_1 + \beta_2 + \beta_3) \delta(\alpha_1 + \alpha_2 + \alpha_3 + 1) F(\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3); \end{aligned}$$

- Ω_i : $\{|\beta_i| \leq 1, |\alpha_i| \leq 1 - |\beta_i|\}$ are copies of the usual DD square ;
- $F(\dots)$: six variables that are subject to two constraints \Rightarrow **quadruple distributions**



Calculation of the amplitude

- LO amplitude for $\bar{p}p \rightarrow \gamma^* \pi^0$ can be computed as in J.P. Lansberg, B. Pire and L. Szymanowski'07
- 21 diagrams contribute



$$\mathcal{I} \sim \int_{-1+\xi}^{1+\xi} d^3x \delta(x_1 + x_2 + x_3 - 2\xi) \int_{-1}^1 d^3y \delta(1 - y_1 - y_2 - y_3) \left(\sum_{\alpha=1}^{21} R_{\alpha} \right)$$

Each R_{α} , has the structure:

$$R_{\alpha} \sim K_{\alpha}(x_1, x_2, x_3) \times Q_{\alpha}(y_1, y_2, y_3) \times$$

[combination of πN TDAs] \times [combination of nucleon DAs]

$$R_1 = \frac{q^{\mu} (2\xi)^2 [(V_1^{p\pi^0} - A_1^{p\pi^0})(V^p - A^p) + 4T_1^{p\pi^0} T^p + 2\frac{\Delta^2}{M^2} T_4^{p\pi^0} T^p]}{(2\xi - x_1 + i\epsilon)^2 (x_3 + i\epsilon) (1 - y_1)^2 y_3}$$

c.f. $\int_{-1}^1 dx \frac{H(x, \xi)}{x \pm \xi \mp i\epsilon} \int_0^1 dy \frac{\phi_M(y)}{y}$ for HMP

$p\bar{p} \rightarrow \pi\gamma^*$ amplitude and $\bar{p}p \rightarrow \gamma^*\pi \rightarrow \ell^+\ell^-\pi$ cross section

$$\mathcal{M}_{s_p s_{\bar{p}}}^\lambda = -i \frac{(4\pi\alpha_s)^2 \sqrt{4\pi\alpha_{em}} f_N^2}{54f_\pi} \frac{1}{Q^4} \left[S_{s_p s_{\bar{p}}}^\lambda \mathcal{I}(\xi, \Delta^2) - S'_{s_p s_{\bar{p}}}{}^\lambda \mathcal{I}'(\xi, \Delta^2) \right],$$

where

$$S_{s_p s_{\bar{p}}}^\lambda \equiv \bar{V}(p_{\bar{p}}, s_{\bar{p}}) \hat{\epsilon}^*(\lambda) \gamma_5 U(p_p, s_p);$$

$$S'_{s_p s_{\bar{p}}}{}^\lambda \equiv \frac{1}{M} \bar{V}(p_{\bar{p}}, s_{\bar{p}}) \hat{\epsilon}^*(\lambda) \hat{\Delta}_T \gamma_5 U(p_p, s_p),$$

$\bar{p}p \rightarrow \gamma^*\pi \rightarrow \ell^+\ell^-\pi$ cross section

$$\frac{d\sigma}{dtdQ^2 d\cos\theta_\ell} = \int d\varphi_\ell \frac{2\pi e^2 (1 + \cos^2\theta_\ell)}{Q^2} \frac{|\overline{\mathcal{M}_T}|^2}{64W^2(W^2 - 4M^2)(2\pi)^4}.$$

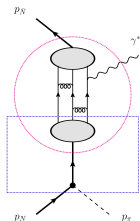
- Off-shell photon is transversally polarized at leading twist \Rightarrow characteristic behavior in lepton azimuthal angle: $1 + \cos^2 \theta_l$
- $1/Q^8$ scaling behavior of the $p\bar{p} \rightarrow \gamma^* \pi$ cross section
- Non-zero imaginary part of the amplitude.

How to model quadruple distributions?

- No enlightening $\xi = 0$ limit as for GPDs
- In the limit $\xi \rightarrow 1$ πN TDAs are fixed due to soft pion theorems in terms of nucleon DAs
- Start from $\xi = 1$ limit rather than the forward limit $\xi = 0$ to fix the overall magnitude of quadruple distributions
- Phenomenological solutions for nucleon DA (COZ, KS, GS, BLW, BK, Heterotic Ansatz etc.) can be taken as numerical input

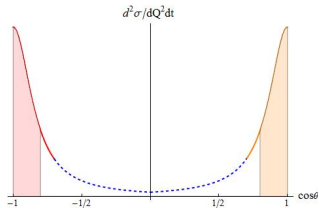
Two component model

- u -channel nucleon exchange is complementary to the spectral representation: D -term
- non-zero in the ERBL-like region
 $0 \leq x_i \leq 2\xi$



$$\frac{d\sigma}{dt dQ^2 d \cos \theta_\ell} = \int d\varphi_\ell \frac{2\pi e^2 (1 + \cos^2 \theta_\ell)}{Q^2} \frac{|\overline{\mathcal{M}}_T|^2}{64 W^2 (W^2 - 4M^2) (2\pi)^4}.$$

- Useful cut: $|\Delta_T^2|$ -cut \Leftrightarrow cut in θ_{CMS} .
- This helps to focus on forward (backward) regime.

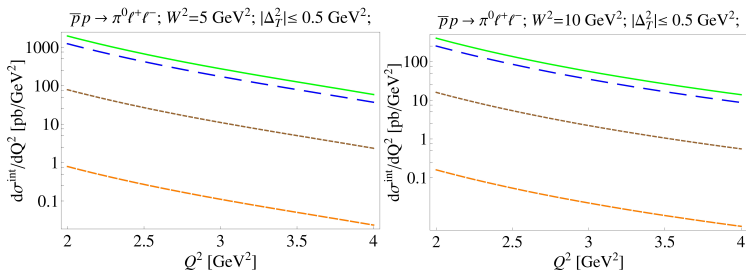


Integrated cross section

$$\frac{d\sigma^{\text{int}}}{dQ^2} (|\Delta_T^2|_{\text{max}}) \equiv \int_{t_{\text{min}}}^{t_{\text{max}}} dt \int d\theta_\ell \frac{d\sigma}{dt dQ^2 d \cos \theta_\ell}$$

$\bar{p}p \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ cross section

- Nucleon pole dominates over quadruple distribution part for PANDA conditions
- Numerical input: COZ, KS, BLW NLO, BLW NNLO phenomenological solutions for nucleon DAs

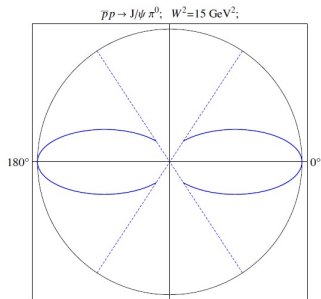
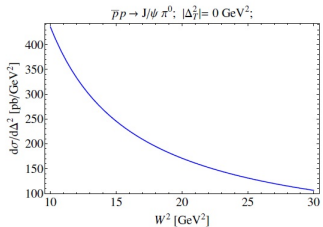
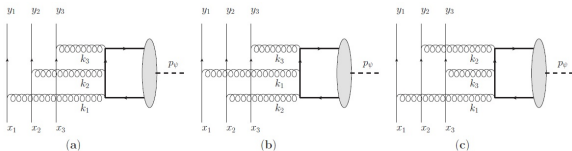


- Cross section of $\bar{p}n \rightarrow \pi^- \gamma^* \rightarrow \pi^- \ell^+ \ell^-$ is larger by factor 2. But requires neutron target.

$N \bar{N} \rightarrow J/\psi \pi$ at PANDA I

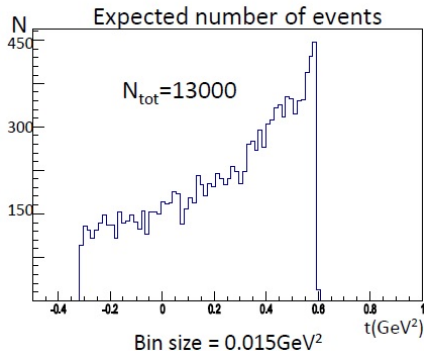
Amplitude calculation and cross section estimates B. Pire, KS, L. Szymanowski'13

Unpolarized cross section and angular distribution



First feasibility studies

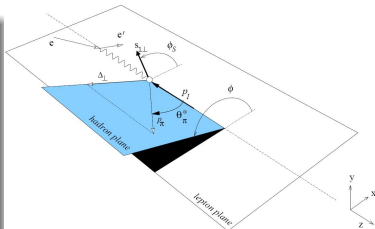
- **B. Ma & B. Ramstein**, PANDA Collaboration meeting at Bochum, September 2013: a new EvtGen Model in PandaRoot
- First study for $W^2 = 12.25 \text{ GeV}^2$, near forward regime
- Assumed integrated luminosity: 2 fb^{-1} (4 months of beamtime at full luminosity); 100% efficiency: this number will be reduced by efficiency and analysis cuts, but it is very promising!



- 1 Nucleon to meson TDAs provide new information about correlation of partons inside hadrons
- 2 We strongly encourage to try to detect near forward and backward signals for various mesons (π , η , ω , ρ): there could be interesting physics around!
- 3 Theoretical understanding is growing up: spectral representation for πN TDA based on quadruple distributions; factorized Ansatz for quadruple distributions with input at $\xi = 1$ is proposed
- 4 Some experimental success achieved for backward $\gamma^* N \rightarrow N' \pi$, $\gamma^* N \rightarrow N' \eta$ (q^2 - spacelike) @ Jlab already at 6 GeV (and more is expected at 12 GeV)
- 5 $\bar{p} N \rightarrow \pi \ell^+ \ell^-$ (q^2 - timelike) and $\bar{p} N \rightarrow \pi J/\psi$ @ PANDA would allow to check universality of TDAs
- 6 Open questions: proof of factorization theorems, interpretation in the impact parameter space, analytic properties of the amplitude

Transverse Target Single Spin Asymmetry $\gamma^* N \rightarrow \pi N$

- TSA = $\sigma^\uparrow - \sigma^\downarrow \sim \text{Im part of the amplitude}$
- it probes the contribution of the DGLAP-like regions
- One expects a TSA vanishing with Q^2 and W^2 for (simple) baryon-exchange approaches
- Non vanishing and Q^2 -independent TSA within TDA approach



$$\mathcal{A} = \frac{1}{|\vec{s}_1|} \left(\int_0^\pi d\tilde{\phi} |\mathcal{M}_T^{s_1}|^2 - \int_\pi^{2\pi} d\tilde{\phi} |\mathcal{M}_T^{s_1}|^2 \right) \left(\int_0^{2\pi} d\tilde{\phi} |\mathcal{M}_T^{s_1}|^2 \right)^{-1}$$

