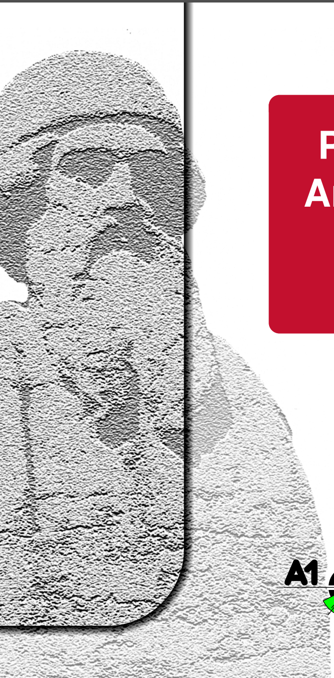

Proton Radius Puzzle: Global Analysis of Electron Scattering Data and New Experiments from MAMI

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Outline

1. Introduction:
   Form factor $\leftrightarrow$ charge distribution
2. The Mainz measurements
3. Analysis of the world data
4. New Experiments
5. Summary

Proton / Neutron

mass

$m_p = 938.272046(21)\,\mathrm{MeV/c^2}$

Discovered by E. Rutherford (1919)

$m_n = 939.565379(21)\,\mathrm{MeV/c^2}$

Discovered by J. Chadwick (1939)

size

moments of electric charge and magnetization distribution derived from **form factor** measurements

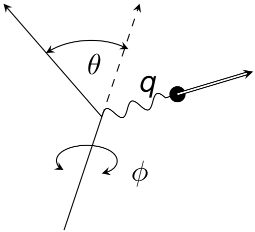
shape

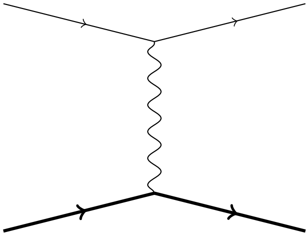
departure from spherical symmetry determined in $N \longrightarrow \Delta$ measurements

stiffness

electric and magnetic polarizabilities extracted from Virtual Compton scattering (VCS)

Cross section and form factors for elastic e-p scattering





The cross section:

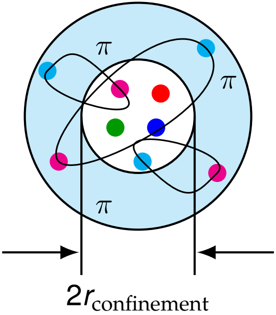
$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\left(1+\tau\right)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Low-Q form factors

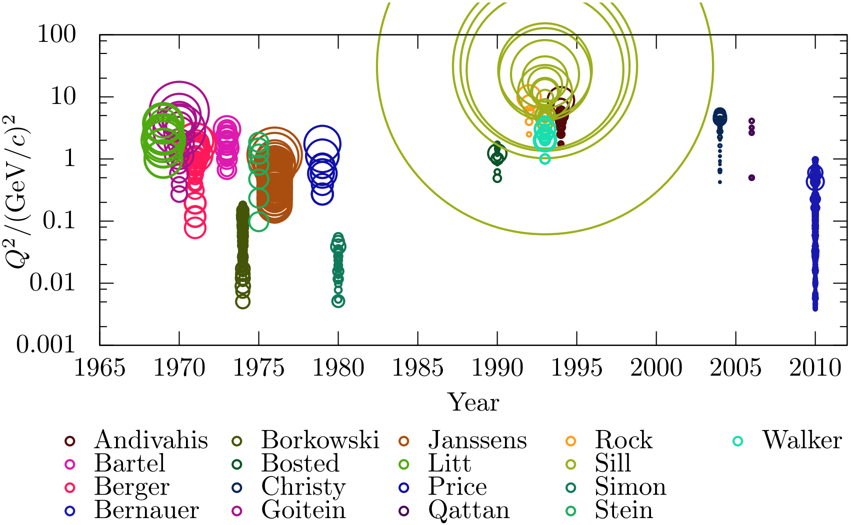
What more can we learn from the form factors?



- Low-Q $\Longleftrightarrow$ Long range structure
- How big is the proton?
- Is there evidence for a pion cloud?

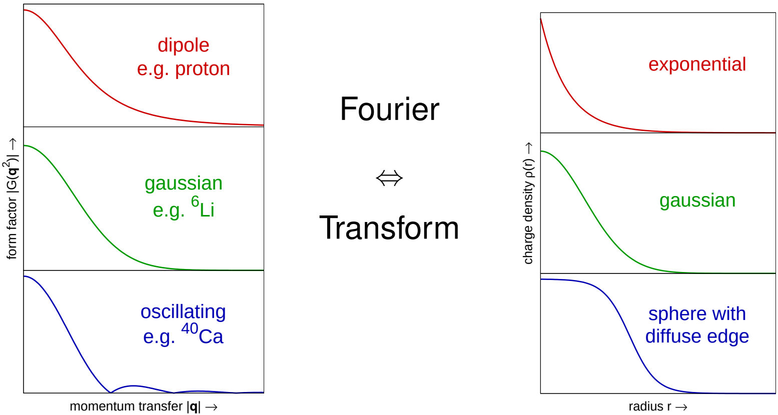
$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left. \frac{d\left(G_M/\mu_p\right)}{dQ^2} \right|_{Q^2=0}$$

Timeline of proton cross section data



Classical picture

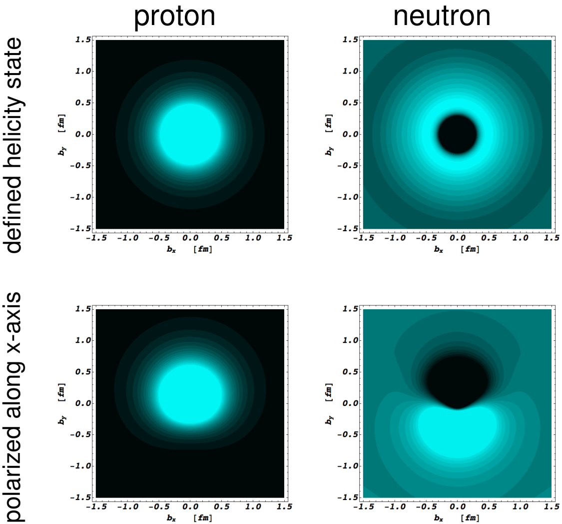
form factor: $G(q^2) = \frac{1}{e} \int_0^\infty \rho(r) \, \frac{\sin qr}{qr} \, 4\pi r^2 \, dr$



charge distribution: $\rho(r) = \frac{e}{(2\pi)^3} \int_0^\infty G(q^2) \, \frac{\sin qr}{qr} \, 4\pi q^2 \, dq$

Light-front picture

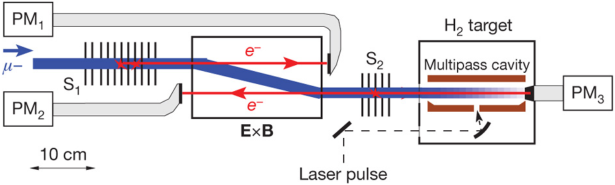
Infinite Momentum Frame: 3D distribution gets ”squashed”.

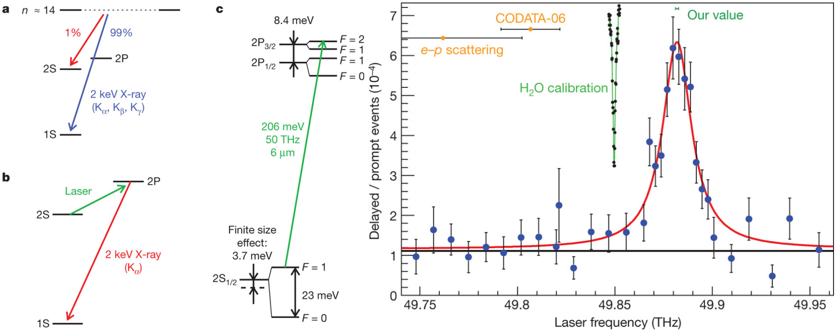


From: M. Vanderhaeghen, Th. Walcher, arXiv:1008.4225v1

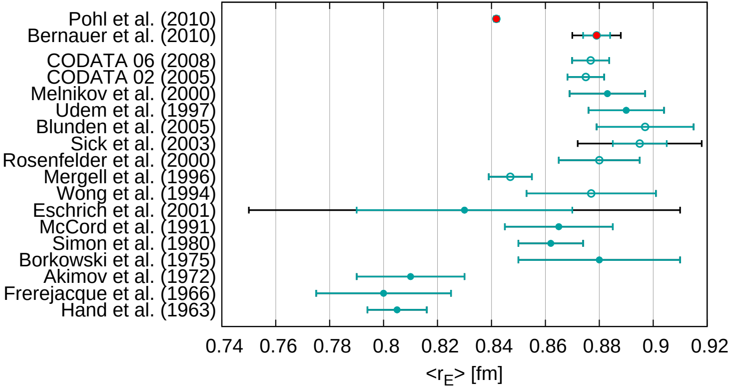
The radius puzzle – Lamb shift in μH

Nature **466**, 213-216 (8 July 2010)





The radius puzzle – Lamb shift in μH



Filled dots: Results from new measurements.
Hollow dots: Reanalysis of existing data.

Discussion of the Lamb shift / electron scattering discrepancy

- **Muonic hydrogen (Lamb Shift)**

  $r_p = 0.84184(67)\,\mathrm{fm}$

  R. Pohl *et al.*, Nature **466**, 213-216 (2010)
- **Mainz form factor measurement**

  $r_p = 0.879(8)\,\mathrm{fm}$

  J.C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).
- **Analysis of previous ep scattering data**

  $r_p = 0.895(18)\,\mathrm{fm}$

  I. Sick, Phys. Lett. **B576** 62-67 (2003).
- **Electronic hydrogen - (CODATA)**
  (Hyperfine structure and Lamb shift)

  $r_p = 0.8768(69)\,\mathrm{fm}$

  P.J. Mohr *et al.*, Rev. Mod. Phys. **80** 633-730 (2008).

Discrepancy is between
muonic **and** *electronic* **measurements**

Early attempts to resolve the discrepancy

3^{rd} Zemach-Moment

De Rújula's toy model

- A. De Rújula, "QED is not endangered by the proton's size", Phys. Lett. **B693**, 555 (2010).
- **Sum of "single pole" and "dipole"**

$$\rho_{\text{Proton}}(r) = \frac{1}{D}\left[\frac{M^4 e^{-Mr}\cos^2(\theta)}{4\pi r} + \frac{m^5 e^{-mr}\sin^2(\theta)}{8\pi}\right]$$
$$D \equiv M^2\cos^2(\theta) + m^2\sin^2(\theta)$$

using $M = 0.750\,\mathrm{GeV/c^2}$, $m = 0.020\,\mathrm{GeV/c^2}$, and $sin^2(\theta) = 0.3$ and

$$\rho_{(2)}(r) = \int d^3r_2\,\rho_{\mathrm{charge}}(|\vec{r} - \vec{r}_2|)\,\rho_{\mathrm{charge}}(r_2)$$

we get the **third Zemach moment**:

$$\langle r^3\rangle_{(2)} = \int d^3r\,r^3\rho_{(2)}(r) = 36.2\,\mathrm{fm}^3$$

We put $\langle r^3 \rangle_{(2)} = 36.2\,\mathrm{fm}^3$ in the Lamb shift formular:

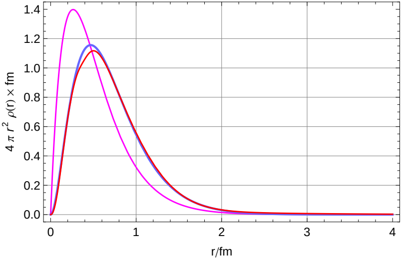
$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \left(209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\mathrm{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\mathrm{fm}^3} \right) \mathrm{meV}$$

and get $r_p = 0.878\,\mathrm{fm}$

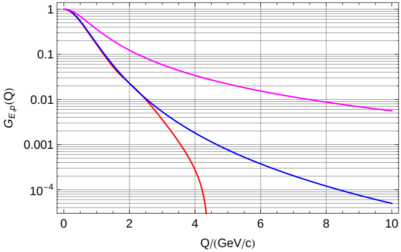
problem solved

De Rújula's toy model – is excluded by experiment

charge distribution



form factor



- De Rújula's toy model
- standard dipole
- Bernauer-Arrington fit assembly

M.O.D., J.C. Bernauer, Th. Walcher, Phys. Lett. **B696**, 343 (2011)

Another attempt to resolve the discrepancy

Structures (dips, thorns) at very low Q^2

Nuclear Finite Size Effects In Light Muonic Atoms

J. L. Friar, Annals Phys. **122** (1979) 151.

Finite-size shift in the energy of the *n*th s-state through order $(Z\alpha)^6$:

$$
\begin{aligned}
\Delta E_n &= \frac{2\pi}{3}\left|\phi_n(0)\right|^2 Z\alpha \times \\
&\quad \left(\left\langle r^2\right\rangle - \frac{Z\alpha\mu}{2}\left\langle r^3\right\rangle_{(2)} + (Z\alpha)^2 F_{\text{REL}} + (Z\alpha\mu)^2 F_{\text{NR}}\right) \\
&= 5200\,\frac{\mu\text{eV}}{\text{fm}^2}\left\langle r^2\right\rangle - 9.1\,\frac{\mu\text{eV}}{\text{fm}^3}\left\langle r^3\right\rangle_{(2)} \\
&\quad +0.28\,\frac{\mu\text{eV}}{\text{fm}^2}F_{\text{REL}} + 0.064\,\frac{\mu\text{eV}}{\text{fm}^4}F_{\text{NR}} \\
&\simeq 5200\,\frac{\mu\text{eV}}{\text{fm}^2}\left\langle r^2\right\rangle - 9.1\,\frac{\mu\text{eV}}{\text{fm}^3}\left\langle r^3\right\rangle_{(2)} \\
&\simeq 5200\,\frac{\mu\text{eV}}{\text{fm}^2}r_p^2 - 35\,\frac{\mu\text{eV}}{\text{fm}^3}r_p^3
\end{aligned}
$$

$$
\text{with}\quad \frac{\left\langle r^3\right\rangle_{(2)}}{\left\langle r^2\right\rangle^{3/2}} = \frac{35\sqrt{3}}{16}
$$

(valid for Dipole FF, only)

Nuclear Finite Size Effects In Light Muonic Atoms

How small are F_{REL} and F_{NR}?
Dipole form factor

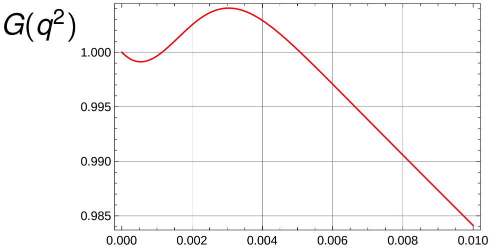
$$
\begin{aligned}
0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times F_{\text{REL}} &= 0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times 4.7\,\mathrm{fm}^2 = 1.3\,\mu\mathrm{eV} \\
0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times F_{\text{NR}} &= 0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times 0.54\,\mathrm{fm}^4 = 0.035\,\mu\mathrm{eV}
\end{aligned}
$$

Now we construct a FF that **"resolves"** the discrepancy:

$$
G(q^2) = (1-x)\left(1+\frac{q^2}{m^2}\right)^{-2} + x\,\exp\left(-\frac{q^4}{2\sigma^2}\right)
$$

$$
\text{with} \quad x = -1.7\%,\ m = 0.776,\ \sigma = 0.04^2
$$

"Thorn" form factor



$$0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times F_{\mathrm{REL}} = 0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times -58\,\mathrm{fm}^2 = -16\,\mu\mathrm{eV}$$

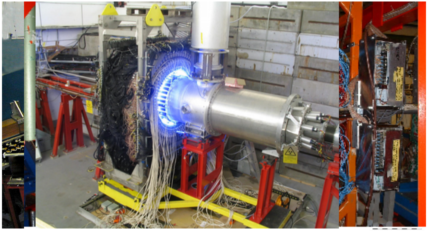
$$0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times F_{\mathrm{NR}} = 0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times 7100\,\mathrm{fm}^4 = 450\,\mu\mathrm{eV}$$

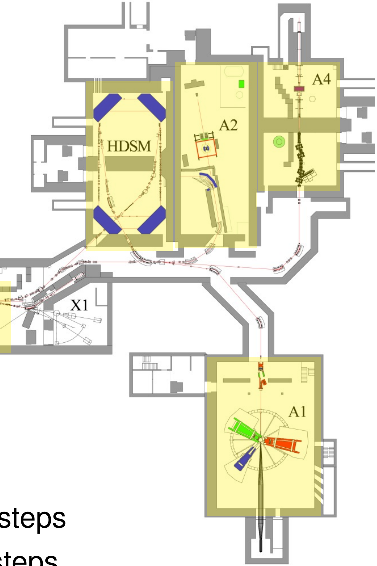
The "Dipole" approximation completely breaks down

Similar conclusion: J. D. Carroll, A. W. Thomas, J. Rafelski and G. A. Miller, "Proton form-factor dependence of the finite-size correction to the Lamb shift in muonic hydrogen," arXiv:1108.2541

The Mainz measurements

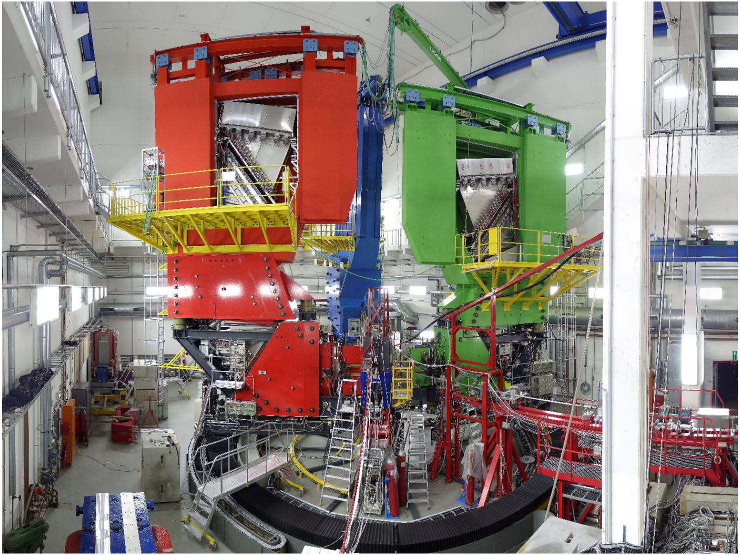
The Mainz Microtron MAMI





- MAMI-A: 180 MeV fixed
- MAMI-B: 855 MeV, 15 MeV steps
- MAMI-C: 1.6 GeV, 15 MeV steps

The Mainz high-precision p(e,e')p measurement: Three spectrometer facility of the A1 collaboration

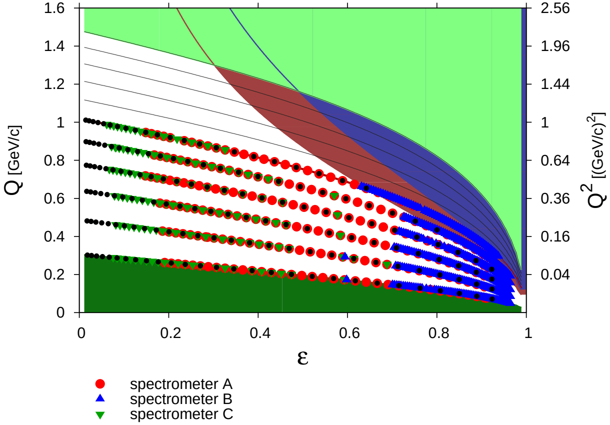


Design goal: High precision through redundancy

- Statistical precision: 20 min beam time for <0.1%
- Control of luminosity and systematic errors:
  Measure all quantities in as many ways as possible:
  - Beam current:
    Foerster probe (usual way) $\Longleftrightarrow$ pA-meter
    $\longrightarrow$ measures down to extremely low currents for small θ
  - Luminosity:
    current $\times$ density $\times$ target length
    $\Longleftrightarrow$ third magnetic spectrometer as monitor
  - Overlapping acceptance
  - Where possible: Measure at the same scattering angle with two spectrometers

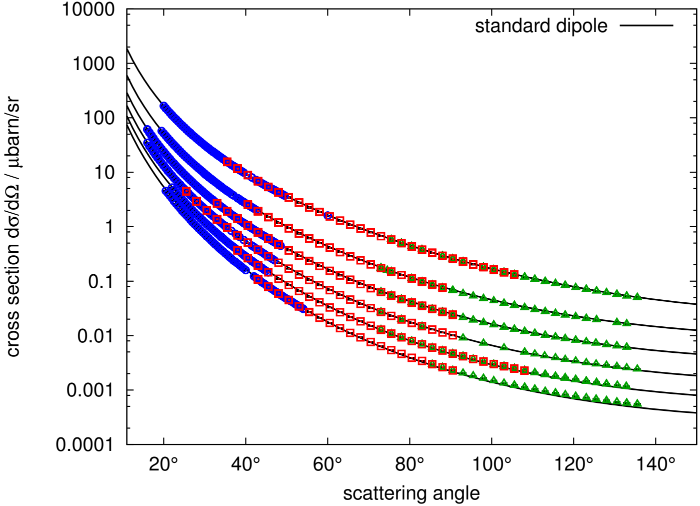
Measured settings and future (high Q^2) expansion

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{\varepsilon(1+\tau)} \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

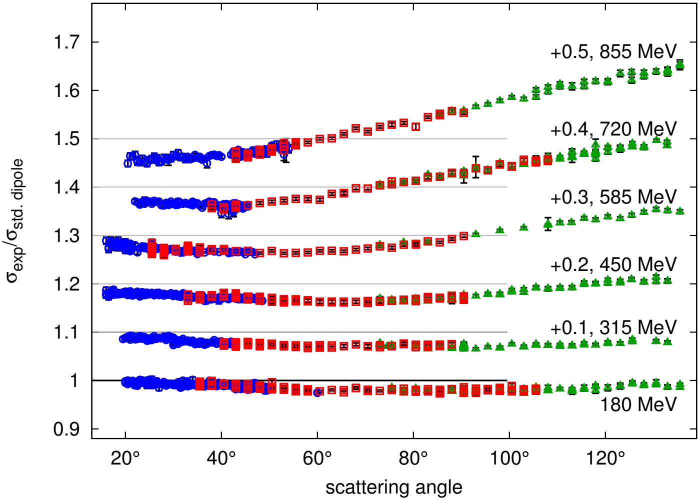


- ∼90 GByte on disc
- ∼1400 Settings
- $> 10^9$ events.

Cross sections

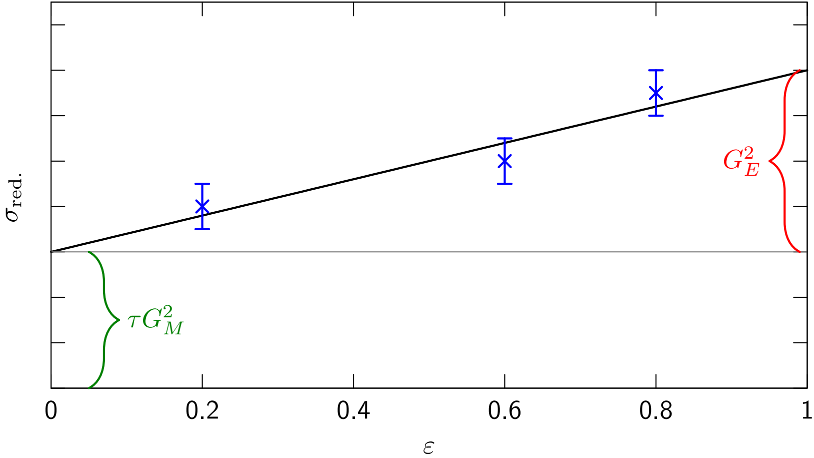


Cross sections / standard dipole



Rosenbluth method

$$\sigma_{red} = \varepsilon\,(1+\tau)\,\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$



Rosenbluth with a twist!

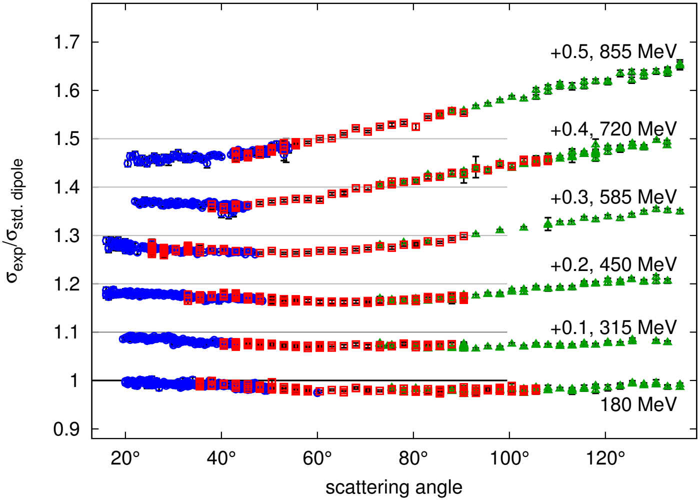
Instead of independent separation at discrete Q^2:

Super-Rosenbluth separation

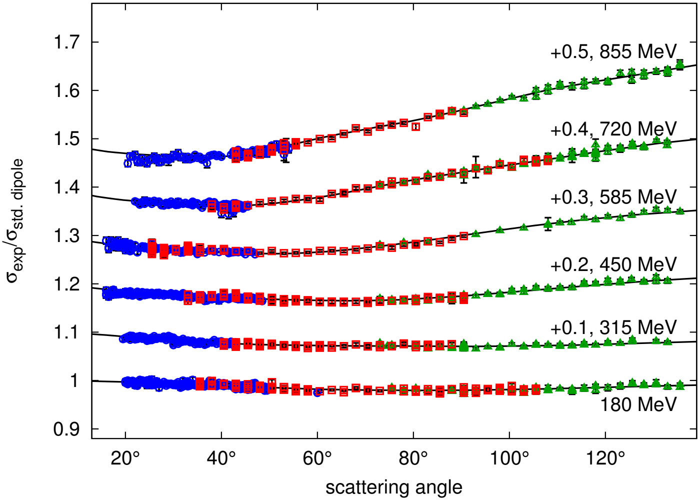
Take (many, flexible) models for form factors, plug them into cross section formula. Fit to all cross section data in one go!

- Feasible due to fast computers.
- All data at all Q^2 and ε values contribute to the fit, i.e. full kinematical region used, no projection (to specific Q^2) needed.
- Global normalization fixed to static limits, $G_E(0) = 1$, $G_M(0) = \mu_p$.

Cross sections / standard dipole

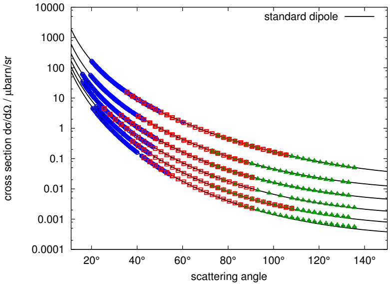


Cross sections + spline fit



Advantage of the Super-Rosenbluth Separation

Rosenbluth formular gives additional constraints

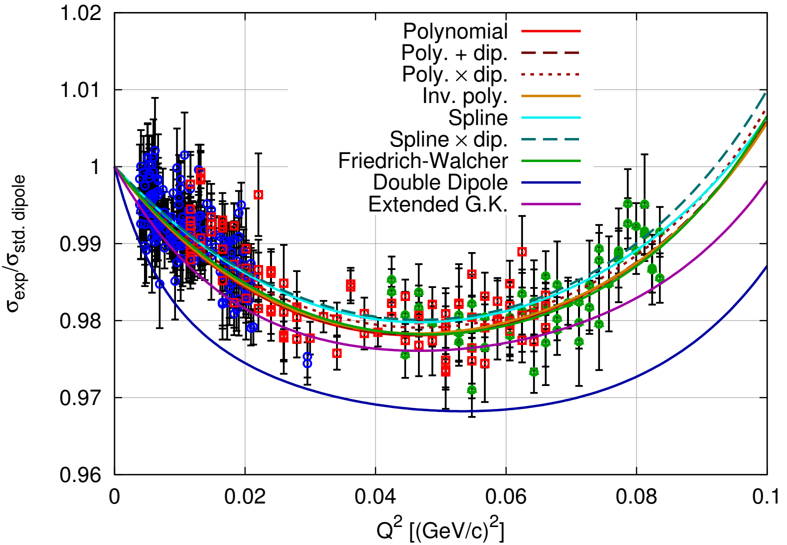


31 normalization parameters allow a statistical analysis.

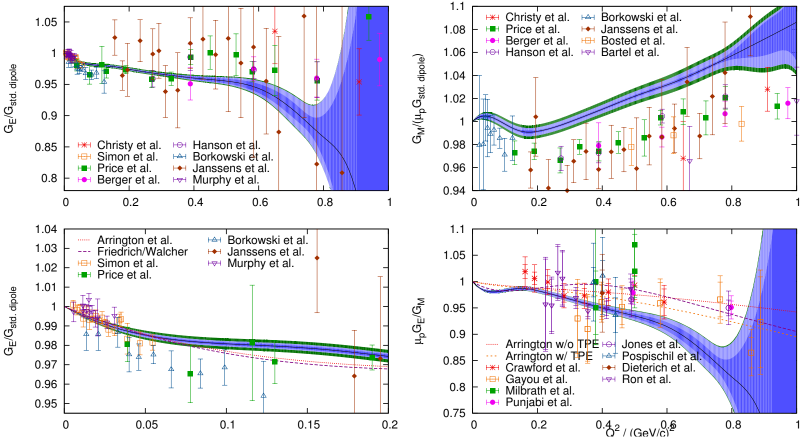
The luminosity is a major source of the systematic error

$\longrightarrow$ **Here, it becomes a statistical error**

Result: Cross section fits (180 MeV)



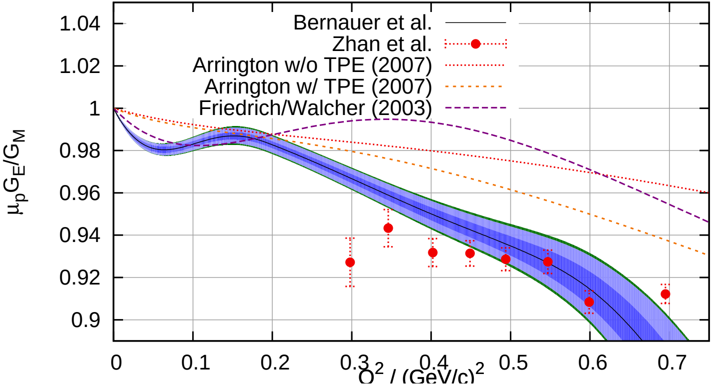
Form factor results



Jan C. Bernauer *et al.*, “High-precision determination of the electric and magnetic form factors of the proton”,
PRL 105, 242001 (2010), arXiv:1007.5076

Form factor results: G_E / G_M ratio

Recoil Polarimetry

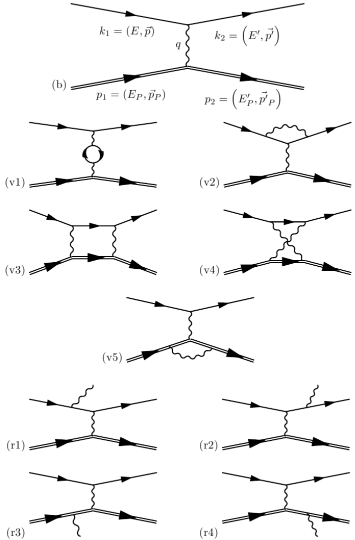


Jan C. Bernauer *et al.*, PRL 105, 242001 (2010), arXiv:1007.5076

X. Zhan *et al.*, Phys.Lett. B705 (2011) 59-64, arXiv:1102.0318

J. Arrington *et al.*, Phys. Rev. C76 (2007) 035205, arXiv:0707.1861

Feynman graphs of leading and next to leading order for elastic scattering



All graphs are taken into account:

- vacuum polarization (v1):
  e, (μ, τ)
  Maximon/Tjon (2000) and Vanderhaeghen et al. (2000)
- electron vertex correction
- Coulomb distortion
  (two photon exchange)
- real photon emission

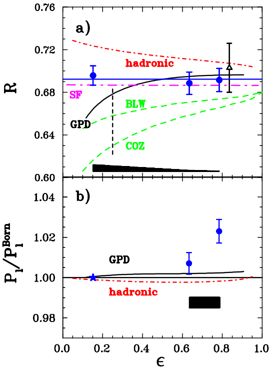
Comments on Coulomb distortion and TPE

- Coulomb distortion:
  Exchange of one hard and multiple soft photons
  Feshbach (1948), Mo and Tsai (1969).

- Two-photon exchange (TPE) with and w/o excited intermediate states:
  Exchange of two hard photons

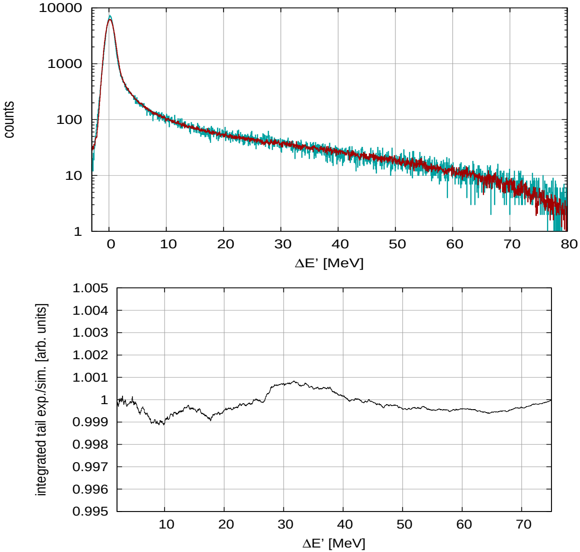
  Still not reliable and highly debated

  Figure shows a recent experimental result from JLab.



Meziane, M. et al.: *Search for effects beyond the Born approximation in polarization transfer observables in $\vec{e}p$ elastic scattering*, PRL 106, 132501 (2011), arXiv:1012.0339

Description of the radiative tail



Inclusion of world data

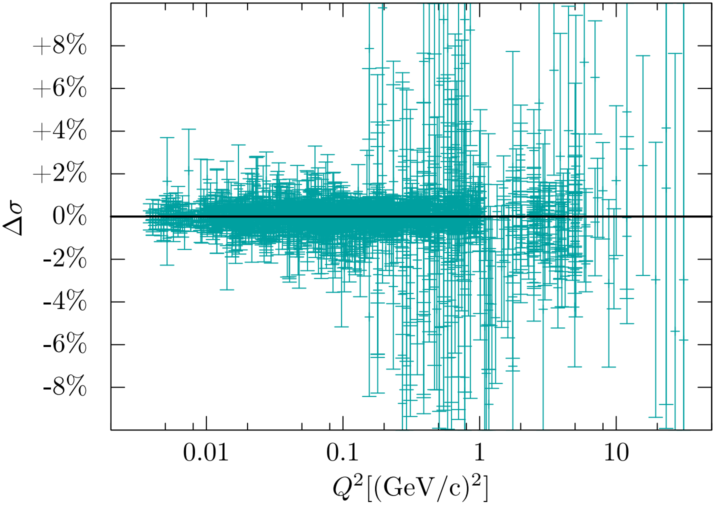
J.C. Bernauer *et al.*: The electric and magnetic form factors of the proton, arXiv:1307.6227

Inclusion of world data

- Extend data base with world data
  $\Longrightarrow$ Cross check, extend Q^2 reach
- Take **cross sections** from Rosenbluth exp's
- Sidestep unknown error correlation
  - Update / standardize radiative corrections
  - One normalization parameter per source (Andivahis: 2)
- Two models:
  - Splines with **variable** knot spacing
    $\Longrightarrow$ Adapt knot density to data density
  - Padé-Expansion
    $\Longrightarrow$ Low(er) flexibility, for comparison

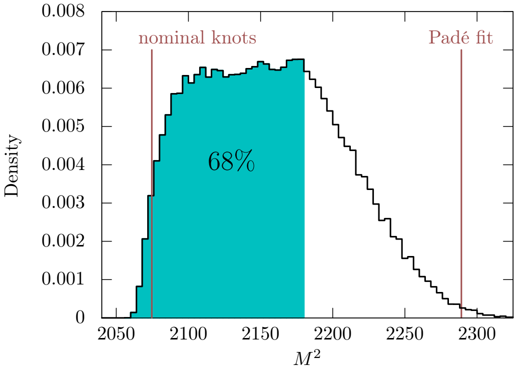
L. Andivahis *et al.*,
Phys. Rev. D50, 5491 (1994).
F. Borkowski *et al.*,
Nucl. Phys. B93, 461 (1975).
F. Borkowski *et al.*,
Nucl.Phys. A222, 269 (1974).
P. E. Bosted *et al.*,
Phys. Rev. C 42, 38 (1990).
M. E. Christy *et al.*,
Phys. Rev. C70, 015206 (2004)
M. Goitein *et al.*,
Phys. Rev. D 1, 2449 (1970).
T. Janssens *et al.*,
Phys. Rev. 142, 922 (1966).
J. Litt *et al.*,
Phys. Lett. B31, 40 (1970).
L. E. Price *et al.*,
Phys. Rev. D4, 45 (1971).
I. A. Qattan *et al.*,
Phys. Rev. Lett. 94, 142301 (2005).
S. Rock *et al.*,
Phys. Rev. D 46, 24 (1992).
A. F. Sill *et al.*,
Phys. Rev. D 48, 29 (1993).
G. G. Simon *et al.*,
Nucl. Phys. A 333, 381 (1980).
S. Stein *et al.*,
Phys. Rev. D 12, 1884 (1975).
R. C. Walker *et al.*,
Phys. Rev. D 49, 5671 (1994).

It works!



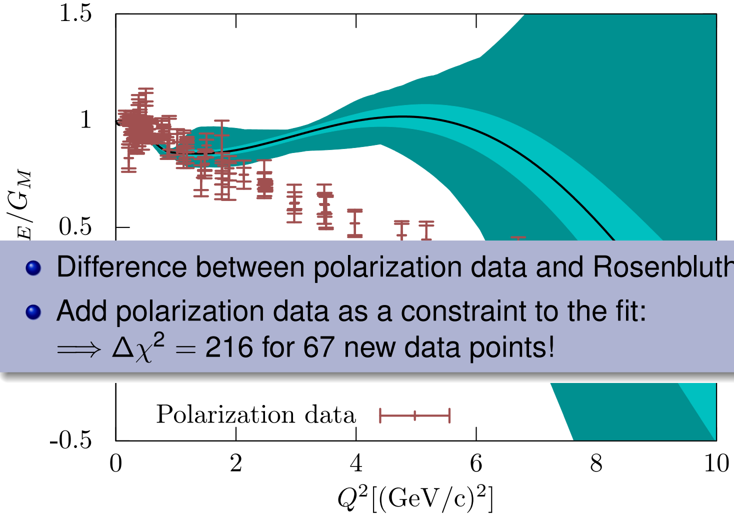
Model dependence

- Spline model has variable knot spacing
- Vary knots, refit, record χ^2.
- Select the 68% best tries.
- Construct envelope of models.



Band will cover at least 68% of all model variations!

Form factor ratio G_E/G_M



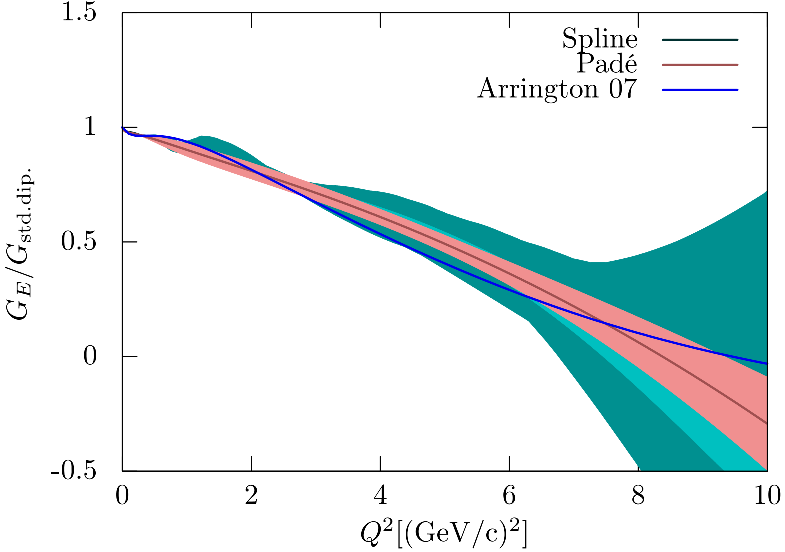
- Difference between polarization data and Rosenbluth data
- Add polarization data as a constraint to the fit:
  $\Longrightarrow \Delta\chi^2 = 216$ for 67 new data points!

Two Photon Exchange - A parametrisation

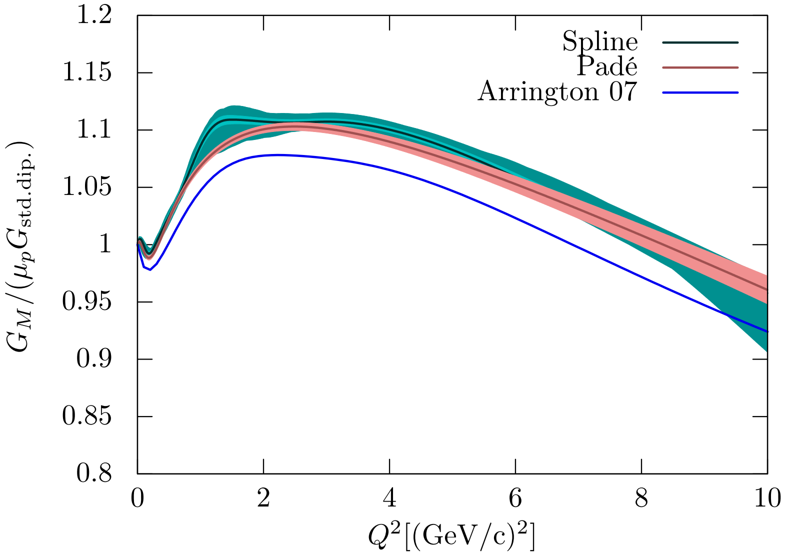
- Available data is sparse
- Mostly Q^2 dependence
- Few data on ε dependence
- Only possible to fit simple model
- In addition to Feshbach Coulomb-correction!

$$\delta = a \cdot (1 - \varepsilon) \cdot \log\left(1 + b \cdot Q^2\right)$$

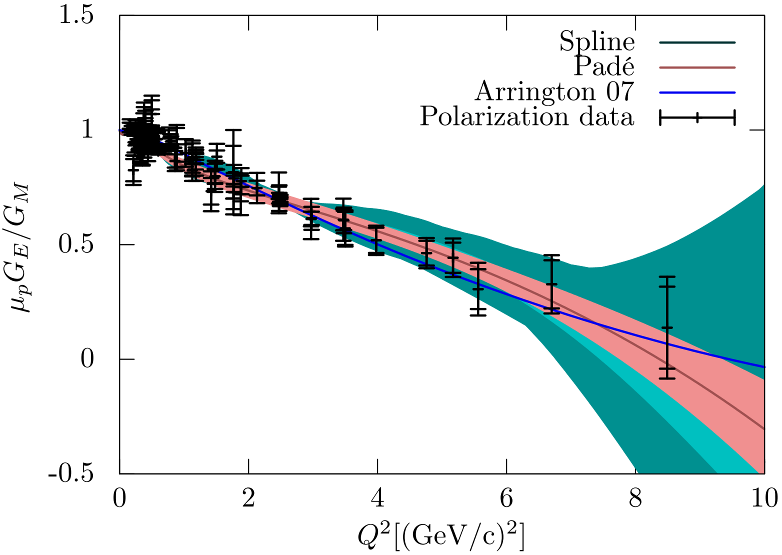
G_E fit incl. polarized data



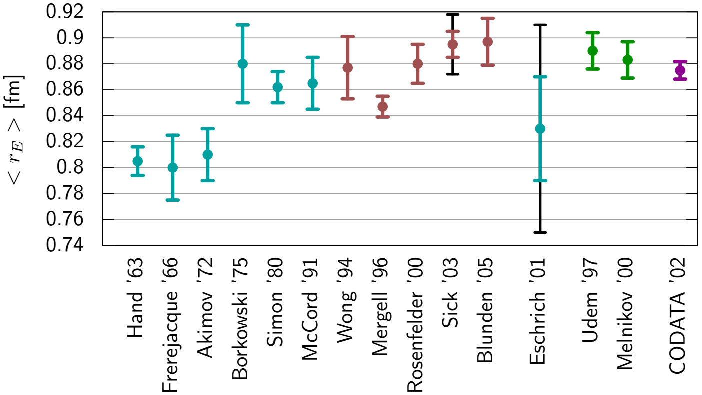
G_M fit incl. polarized data



G_E / G_M fit incl. polarized data



Back to low Q^2 – Charge Radius



Electric and magnetic radius

Final result from flexible models

$$\left\langle r_E^2 \right\rangle^{\frac{1}{2}} = 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{model}} \pm 0.004_{\text{group}} \text{ fm},$$

$$\left\langle r_M^2 \right\rangle^{\frac{1}{2}} = 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{model}} \pm 0.002_{\text{group}} \text{ fm}.$$

Results with world data

	$\left\langle r_E^2 \right\rangle^{\frac{1}{2}}$	$\left\langle r_M^2 \right\rangle^{\frac{1}{2}}$
+ Rosenbluth data	0.878	0.772
+Rosenbluth and Polarization data	0.878	0.769

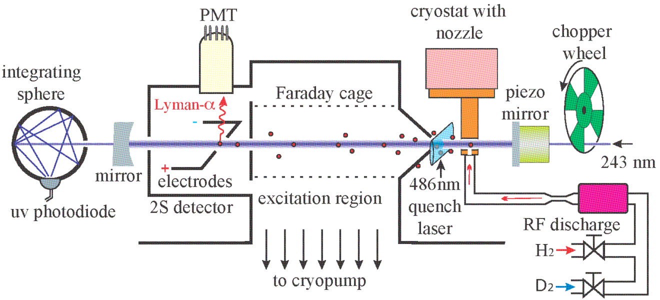
(Eur.Phys.J. D33 (2005) 23-27: Zemach and magnetic radius of the proton from the hyperfine splitting in hydrogen: 0.778(29) fm)

New Experiments

- JLab
  - Very low Q^2 experiment, near 0°
  - Form factor ratio at very low Q^2
- PSI
  - MUSE: elastic μp scattering **Guy Ron**
  - Lamb shift measurements on muonic helium
- MAMI
  - Initial state radiation
  - Measurement of the elastic $A(Q^2)$ form factor of the deuteron at very low momentum transfer

Precision Measurement of the Hydrogen-Deuterium 1S–2S Isotope Shift

Parthey, Christian G. and Matveev, Arthur and Alnis, Janis and Pohl, Randolf and Udem, Thomas and Jentschura, Ulrich D. and Kolachevsky, Nikolai and Hänsch, Theodor W.: *Precision Measurement of the Hydrogen-Deuterium 1S–2S Isotope Shift*, Phys. Rev. Lett. **104**, 233001 (2010).



Beam apparatus for two-photon spectroscopy on the hydrogen / deuterium atomic beam.

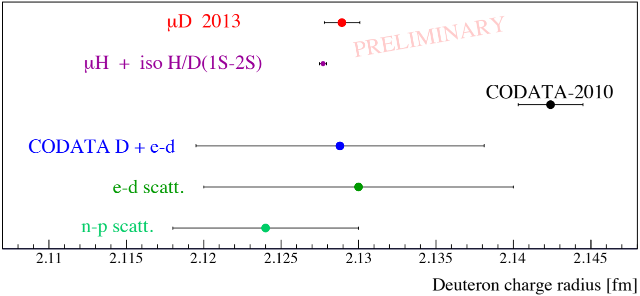
Deuteron radius from the H-D isotope shift and muonic hydrogen

Proton radius: The challenge continues

Combining H-D isotope shift and μH:

$$\left.\begin{array}{rcl} r_d^2 - r_p^2 &=& 3.82007(65)\,\mathrm{fm}^2 \\ r_p &=& 0.84087(39)\,\mathrm{fm} \end{array}\right\} \Rightarrow r_d = 2.12771(22)\,\mathrm{fm}$$

A. Antognini *et al.*, Science 339 (2013) 417-420



Paul Indelicato, Mainz, 2013

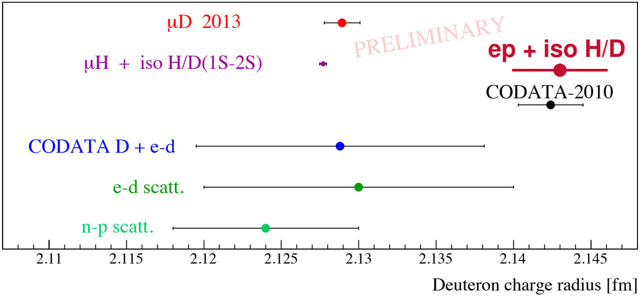
Deuteron radius from the H-D isotope shift and muonic hydrogen

Proton radius: The challenge continues

Combining H-D isotope shift and e-p elastic scattering:

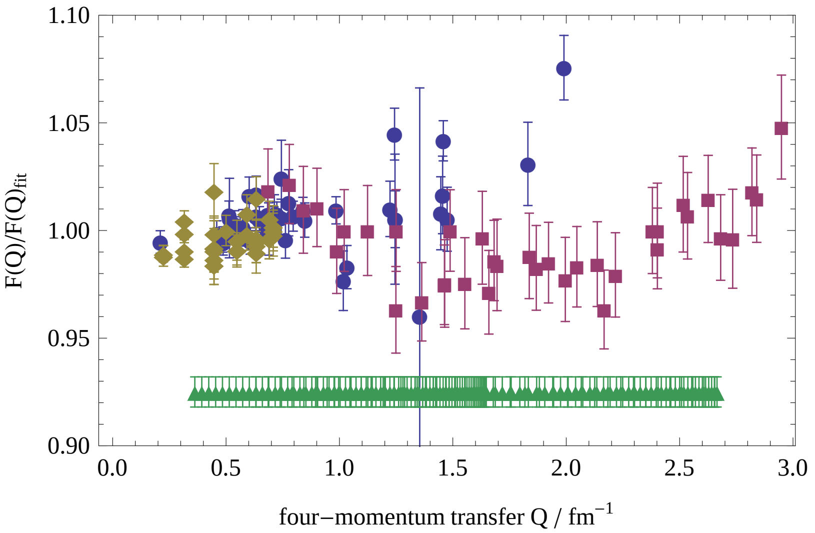
$$\left.\begin{aligned} r_d^2 - r_p^2 &= 3.82007(65)\,\mathrm{fm}^2 \\ r_p &= 0.879(8)\,\mathrm{fm} \end{aligned}\right\} \Rightarrow r_d = 2.143(3)\,\mathrm{fm}$$

J.C. Bernauer *et al.*, Phys.Rev.Lett. 105 (2010) 242001



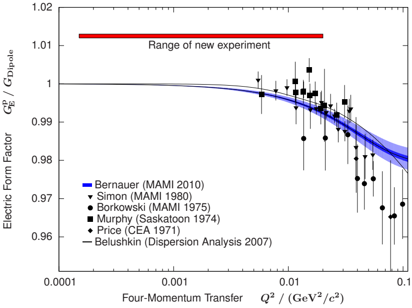
Paul Indelicato, Mainz, 2013

World low Q^2 data and predicted errors



♦ Berard (1973), • Simon (1981), ■ Platchkov (1990), ▲ MAMI

Possible Improvements on Proton Radius

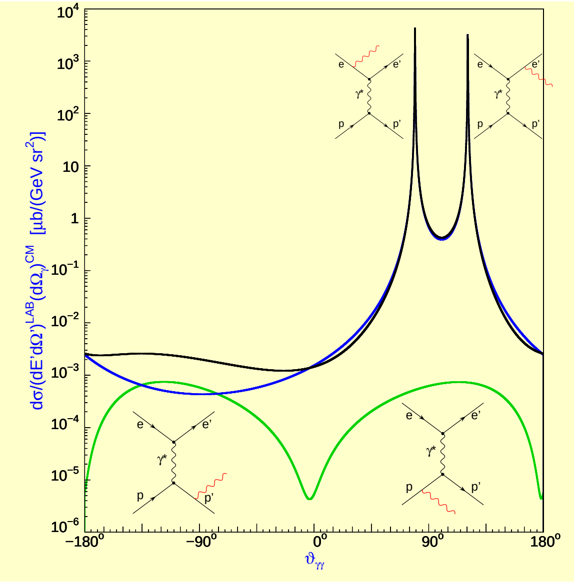


Where should we determine the root-mean-square-radius?

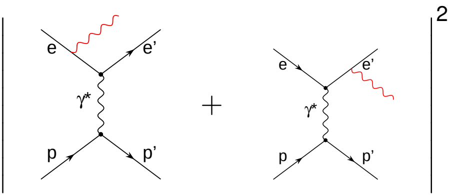
$$\langle r_E^2 \rangle = -6 \, \frac{d}{dQ^2} \, G_E(Q^2) \Big|_{Q^2=0}$$

- Extrapolation to $Q^2 = 0$, no absolute cross section!
- Sufficient “Lever arm” for radius determination
  $\rightarrow Q^2 \approx 0.2\mathrm{GeV}^2/c^2$
- Reduction of higher orders $\rightarrow$ linear fit

Virtual Compton Cross Section $H(e, e')p\gamma$

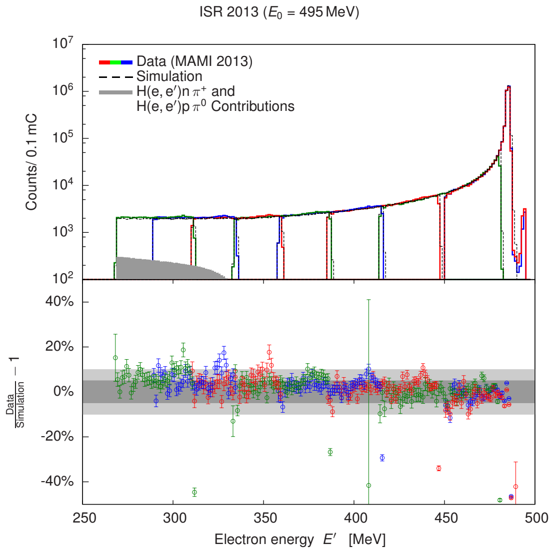


Initial state radiation

$d\sigma \approx$ 

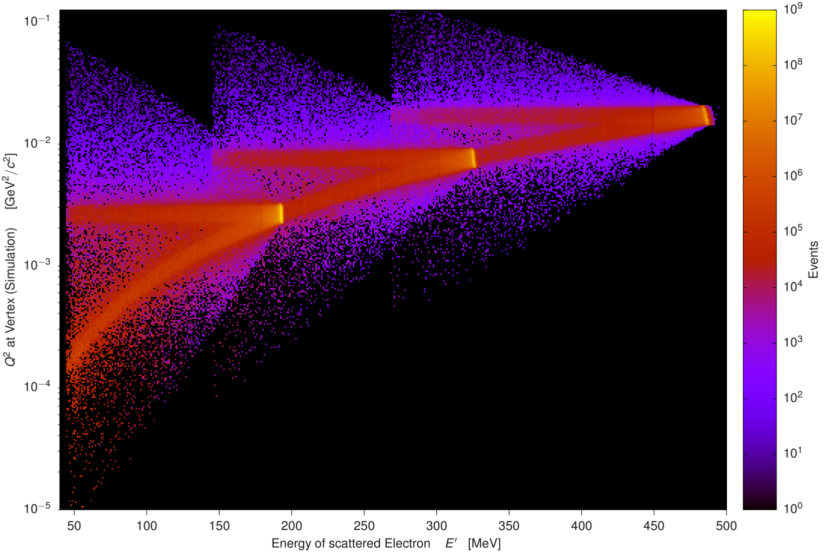
- $H(e,e')p\gamma$ Cross section dominated by Initial/Final State Radiation
- Final State Radiation: Four-Momentum-Transfer Q^2 constant
- Initial State Radiation: Continuous Q^2 range
- Experiment
  - First spectrometer for “Normalization” at elastic peak
  - Start with second spectrometer setup at elastic peak
  - Measurement of the radiative tail
    by change of the magnetic field of second spectrometer
  - Keep *everything* else constant!

Initial State Radiation



M. Mihovilovič *et al.*, Data taking August/September 2013

Initial State Radiation



M. Mihovilovič *et al.*

Summary

1. Form factor $\leftrightarrow$ charge distribution
2. The Mainz measurements
   - High-precision p(e,e')p cross sections
   - Inclusion of the world data
3. Outlook
   - New Experiments at PSI, JLab, and MAMI

Final result from flexible models

$$\left\langle r_E^2 \right\rangle^{\frac{1}{2}} = 0.879 \pm 0.005_{\mathrm{stat.}} \pm 0.004_{\mathrm{syst.}} \pm 0.002_{\mathrm{model}} \pm 0.004_{\mathrm{group}}\ \mathrm{fm},$$

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