Lattice QCD and baryon physics: recent results

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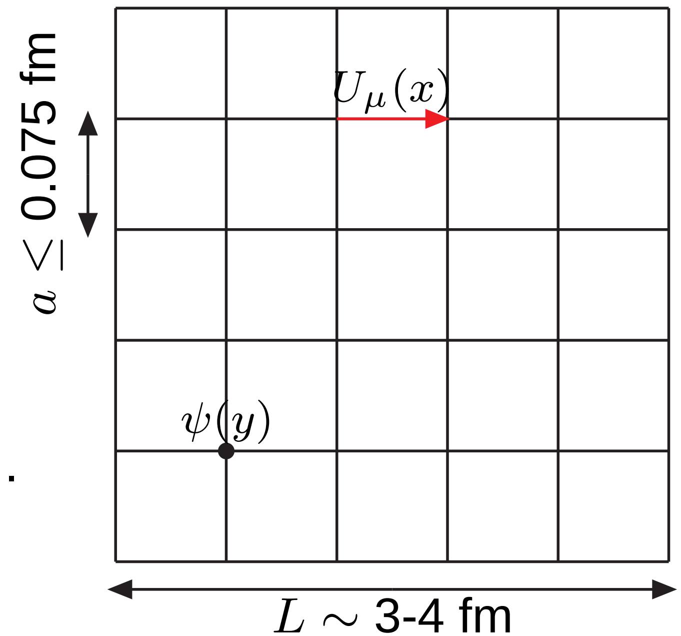
- Prerequisite
- Hadron form factors and structure functions
- Multihadron states and nuclear potential
- Isospin breaking effects
- New Physics in the baryon sector
- Outlook

Prerequisite

Discretisation of QCD in a finite volume of Euclidean space-time.

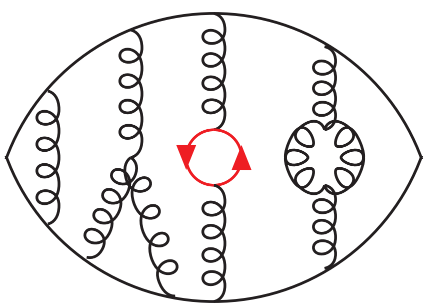
The lattice spacing a is a non perturbative UV cut-off of the theory.



Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0 A_\mu(x+\frac{a\hat{\mu}}{2})}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses m_i.

Computation of Green functions of the theory from first principles:



$$\langle O(U,\psi,\bar{\psi})\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,O(U,\psi,\bar{\psi})e^{-S(U,\psi,\bar{\psi})}$$

$$\mathcal{Z} = \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}e^{-S(U,\psi,\bar{\psi})}$$

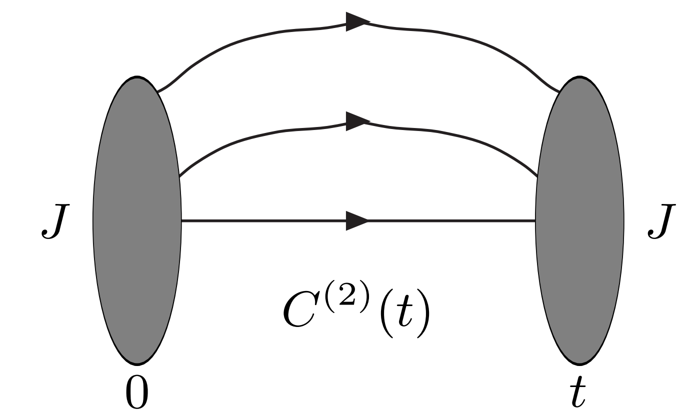
$$S(U,\psi,\bar{\psi}) = S^{\mathrm{YM}}(U) + \bar{\psi}^i_x M^{ij}_{xy}(U)\psi^j_y$$

$$\mathcal{Z} = \int \mathcal{D}U\,\mathrm{Det[M(U)]}e^{-S^{\mathrm{YM}}(U)} \equiv \int \mathcal{D}U e^{-S_{\mathrm{eff}}(U)}$$

Monte Carlo simulation: $\langle O\rangle \sim \frac{1}{\mathrm{N_{conf}}}\sum_i O(\{U\}_i)$: we have to build the statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\mathrm{eff}}}$. Incorporating the quark loop effects hidden in $\mathrm{Det}[M(U)]$ is particularly expensive in computer time.

2pts and 3pts correlators

Extraction of masses and decay constants of bound states and hadronic matrix elements:

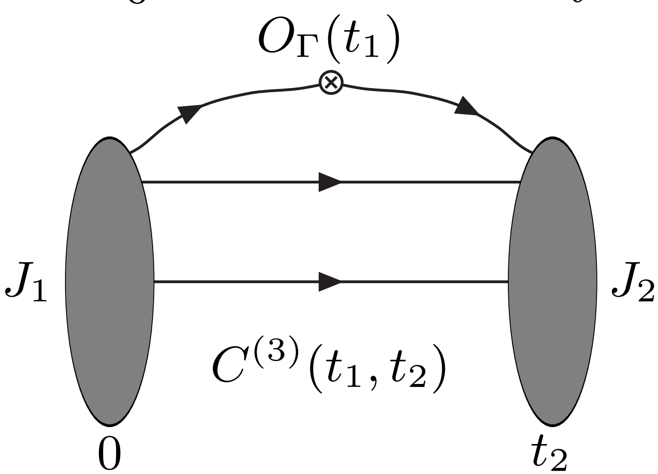


$$C^{(2)}_{JJ}(t) = \mathrm{Tr}\left[\left(\frac{1\pm\gamma_0}{2}\right)\sum_{\vec{x}}\langle\Omega|\mathcal{T}[J(\vec{x},t)J^\dagger(0)]|\Omega\rangle\right]$$

$$= \sum_n \frac{\mathcal{Z}_n^2 e^{-E_n t}}{2E_n}$$

$$\langle\Omega|J|n(0,s)\rangle = \mathcal{Z}_n u(0,s) \quad \langle n(0,s)|m(0,s')\rangle = 2E_n\delta_{mn}\delta_{ss'}$$

$$C^{(2)}_{JJ}(t) \xrightarrow{(E_1-E_0)t\gg 1} \frac{\mathcal{Z}_0^2 e^{-E_0 t}}{2E_0}$$



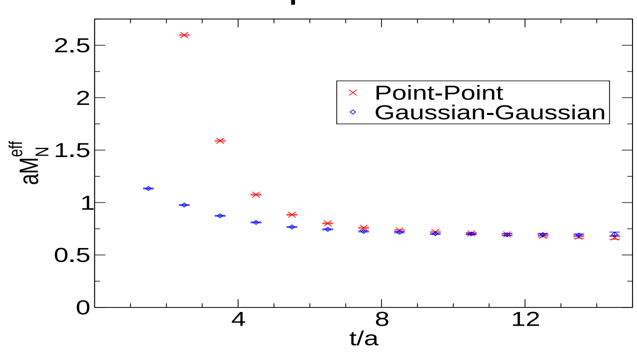
$$C^{(3)}_{J_1,J_2,O_\Gamma}(t_1,t_2) = \mathrm{Tr}[\Gamma_+\Gamma\sum_{\vec{x},\vec{y}}\langle\Omega|\mathcal{T}[J_2(\vec{y},t_2)O_\Gamma(\vec{x},t_1)J_1^\dagger(0)]|\Omega\rangle]$$

$$\xrightarrow{t_1,t_2-t_1\gg 0} \frac{\mathcal{Z}_{0,J_1}\mathcal{Z}_{0,J_2}}{2E_{0,J_1}2E_{0,J_2}} e^{-E_{0,J_1}t_1} e^{-E_{0,J_2}(t_2-t_1)}$$

$$\times\mathrm{Tr}[\Gamma_+\Gamma\langle H_0^{J_2}|O_\Gamma|H_0^{J_1}\rangle]$$

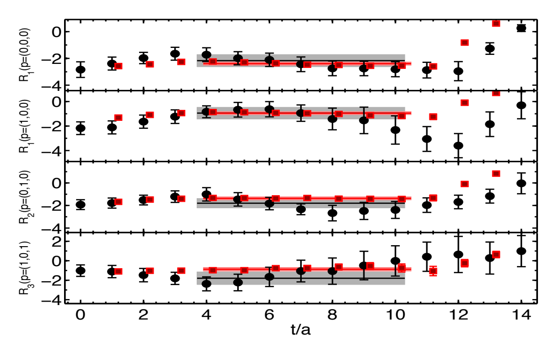
[S. Dürr *et al*, '08]

nucleon 2pts correlator



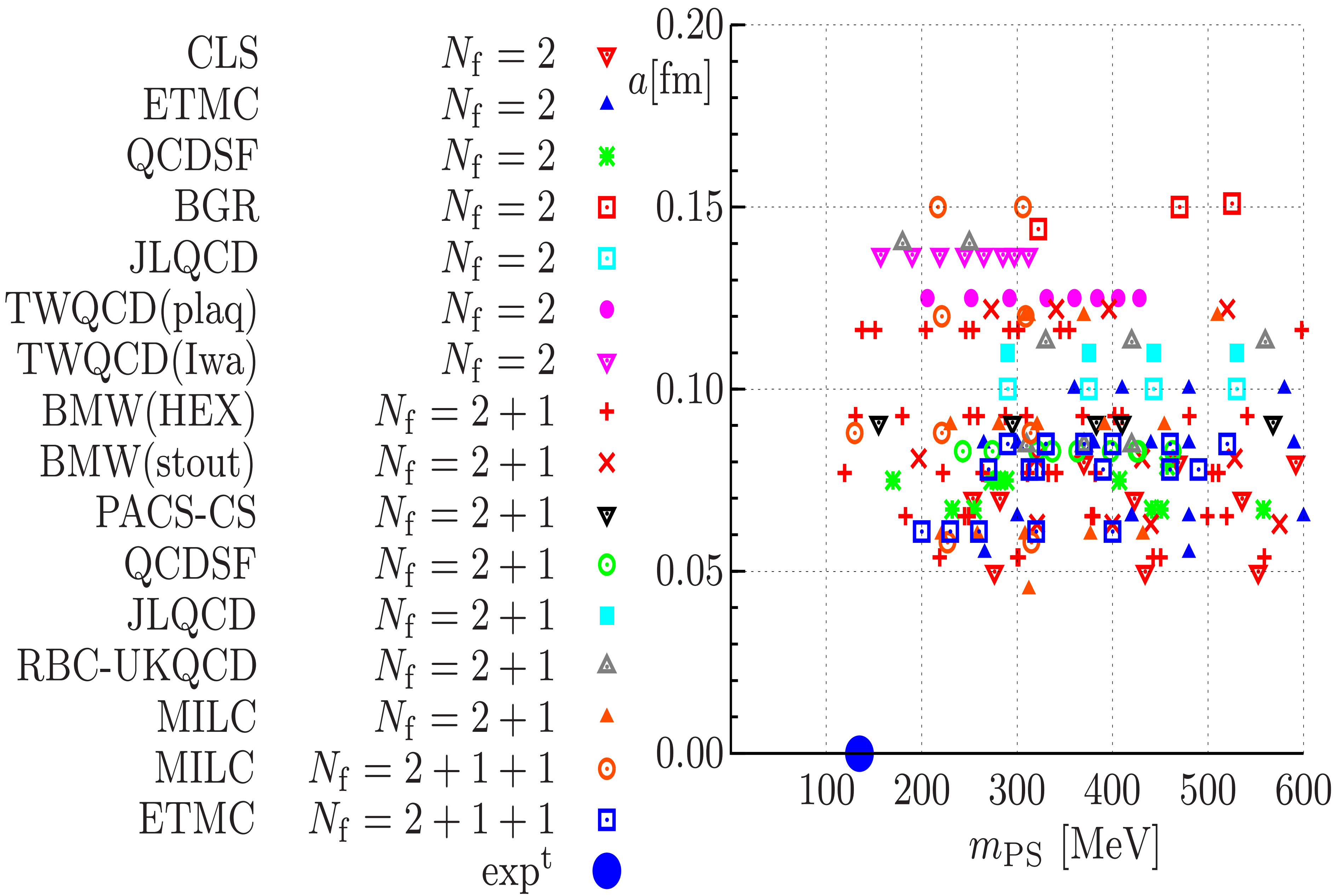
[A. Alexandrou *et al*, '10]

ratios of 3pts and 2pts nucleon correlators



Simulations set up

Nowadays, simulations are quite close to the physical point.



Hadron form factors and structure functions

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda}\langle N(p')|\bar\psi(-\lambda n/2)\Gamma\mathcal{P}[\exp(ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha))]\psi(\lambda/2)|N(p)\rangle$$

x is the momentum fraction of the parton, $\bar P = \frac{p+p'}{2}$, $q = p - p'$, $n^2 = 0$, $\bar P \cdot n = 1$,
$\xi = -n\cdot q/2$

$$\Gamma = \not n:\ F_\Gamma = \frac{1}{2}\bar u_N(p')\left[\not n H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m_N} E(x,\xi,q^2)\right]u_N(p)$$

$$\Gamma = \not n\,\gamma^5:\ F_\Gamma = \frac{1}{2}\bar u_N(p')\left[\not n\gamma^5 \tilde H(x,\xi,q^2) + i\frac{n\cdot q\gamma^5}{2m_N}\tilde E(x,\xi,q^2)\right]u_N(p)$$

Expanding the light cone operator, one obtains a sum over twist-2 operators $O_\Gamma^{\mu\mu_1\cdots\mu_n}$:

$$O_{\not n}^{\mu\mu_1\cdots\mu_n} = \bar\psi\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \qquad O_{\not n\gamma^5}^{\mu\mu_1\cdots\mu_n} = \bar\psi\gamma^5\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1}\cdots\overleftrightarrow{D}^{\mu_n\}}\psi$$

$$\langle N(p)|O_{\not n}^{\mu\mu_1\cdots\mu_n\}}|N(p)\rangle = \langle x^n\rangle_{\mathrm{q}} = \int_0^1 dx\, x^n(q(x) - \bar q(x))$$

$$\langle N(p)|O_{\not n\gamma^5}^{\mu\mu_1\cdots\mu_n\}}|N(p)\rangle = \langle x^n\rangle_{\Delta\mathrm{q}} = \int_0^1 dx\, x^n(\Delta q(x) + (-1)^n\Delta\bar q(x))\ \text{(helicity)}$$

$$\langle N(p)|O_{n_\alpha\sigma^{\alpha\mu}}^{\rho\mu_1\cdots\mu_n\}}|N(p)\rangle = \langle x^n\rangle_{\delta\mathrm{q}} = \int_0^1 dx\, x^n(\delta q(x) - (-1)^n\delta\bar q(x))\ \text{(transversity)}$$

$$\langle N(p',s')|O_{\not n}^{\mu\mu_1\cdots\mu_n\}}|N(p,s)\rangle = \bar u(p',s')\left[\sum_{i=0,\mathrm{even}}^{n}\left(A_{n+1,i}(q^2)\gamma^{\{\mu} + B_{n+1,i}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha}{2m_N}\right)\right.$$

$$\left. q^{\mu_1}\ldots q^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n\}} + \mathrm{mod}(n,2)C_{n+1,0}(q^2)\frac{1}{m_N}q^{\{\mu}q^{\mu_1}\ldots q^{\mu_n\}}\right]u(p,s)$$

$$\langle N(p',s')|O^{\mu\mu_1\cdots\mu_n\}}_{\not{n}\gamma^5}|N(p,s)\rangle = \bar{u}(p',s')\Bigg[\sum_{i=0,\text{even}}^{n}\left(\tilde{A}_{n+1,i}(q^2)\gamma^5\gamma^{\{\mu} + \tilde{B}_{n+1,i}(q^2)\frac{q^\mu}{2m_N}\right)$$

$$q^{\mu_1}\ldots q^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n\}}\Bigg]u(p,s)$$

$n=1$: ordinary nucleon form factors $A_{10}(q^2) = F_1(q^2)$, $\tilde{A}_{10}(q^2) = G_A(q^2)$, $B_{10}(q^2) = F_2(q^2)$, $\tilde{B}_{10}(q^2) = G_P(q^2)$

$n=2$: $\langle x\rangle_q = A_{20}(0)$, $\langle x\rangle_{\Delta q} = \tilde{A}_{20}(0)$; quark spin $J_q = \frac{1}{2}[A_{20}(0) + \tilde{A}_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$; spin sum rule $\frac{1}{2} = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$; $\langle x\rangle_g = 1 - A_{20}(0)$

Axial charge of the nucleon and quark momentum fraction

Since many years lattice results on “gold plated” quantities (g_A, $\langle x\rangle_{u-d}$) were disturbing, in disagreement with their (well under control) experimental measurement.

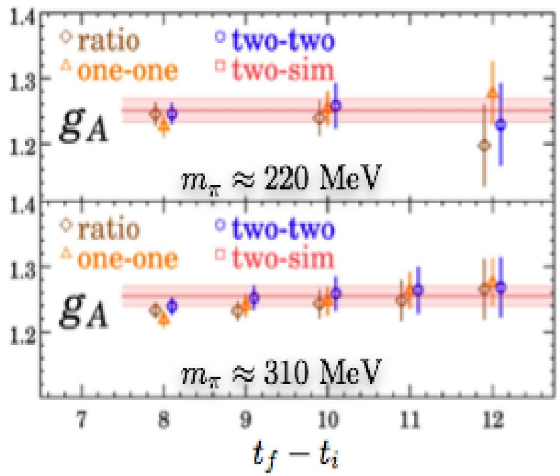
The lattice community has done an important effort in examining in detail the possible sources of systermatics.

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = g_A\bar{u}_p\gamma_\mu\gamma^5 u_p$$

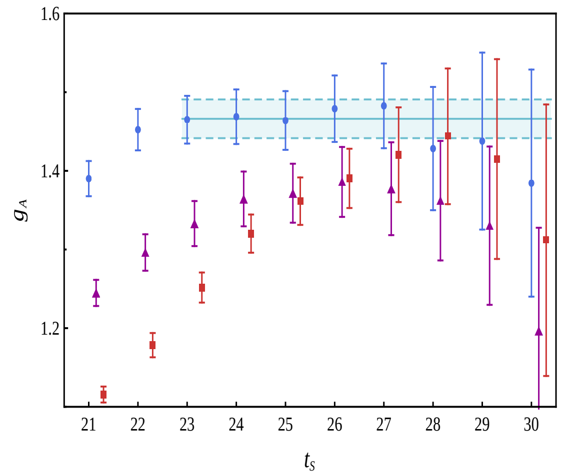
Most of the lattice determinations of g_A are 10 - 15% smaller than what says experiments: $g_A^{\exp} = 1.2701(25)$

It is crucial to remove properly the contributions from excited states.

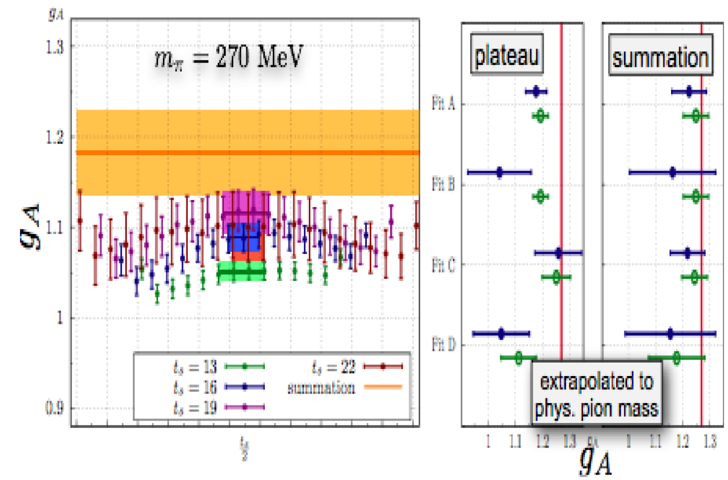
[H. W. Lin *et al*, '13]
2-states exponential fit



[B. Owen *et al*, '12]
variational method



[S. Capitani *et al*, '12]
summation over $t_{\rm inser}$



Variational method: approach to define an operator O_J^n weakly coupled to other states than $|n\rangle$ [C. Michael, '85; M. Lüscher and U. Wolff, '90; B.B. *et al*, '09].

Compute an $N \times N$ matrix of correlators $C^{ij}_{JJ}(t) = \sum_{\vec{x},\vec{y}} \mathrm{Tr}[\Gamma^0 \langle \Omega | \mathcal{T}[O^i_J(\vec{x},t) O^j_J(\vec{y},0)] | \Omega \rangle]$ with $O^i_J(\vec{x},t) = \epsilon^{abc} \sum_{\vec{z}} (\bar{q}^a(\vec{x},t) C\Gamma q^b(\vec{x},t)) \Phi(|\vec{x}-\vec{z}|)^i_J q^c(\vec{z},t)$.

Solve the generalised eigenvalue problem:

$$C^{ij}(t)\, v_n^j(t,t_0) = \lambda_n(t,t_0)\, C^{ij}(t_0)\, v_n^j(t,t_0)$$

$$aE_n^{\rm eff}(t,t_0) = -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right)$$

It is crucial to remove properly the contributions from excited states.

[H. W. Lin *et al*, '13]
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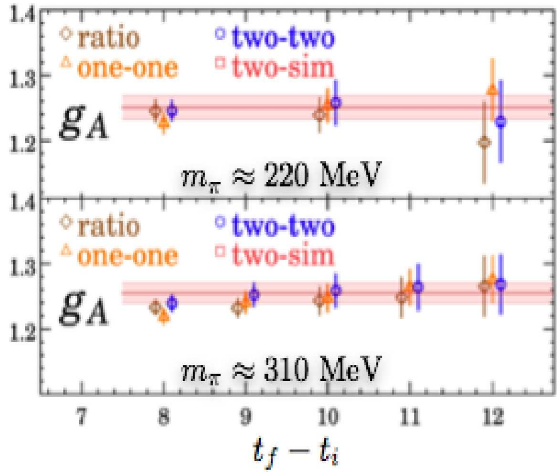

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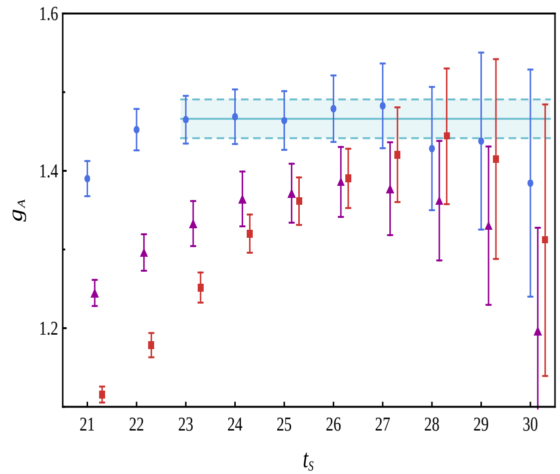

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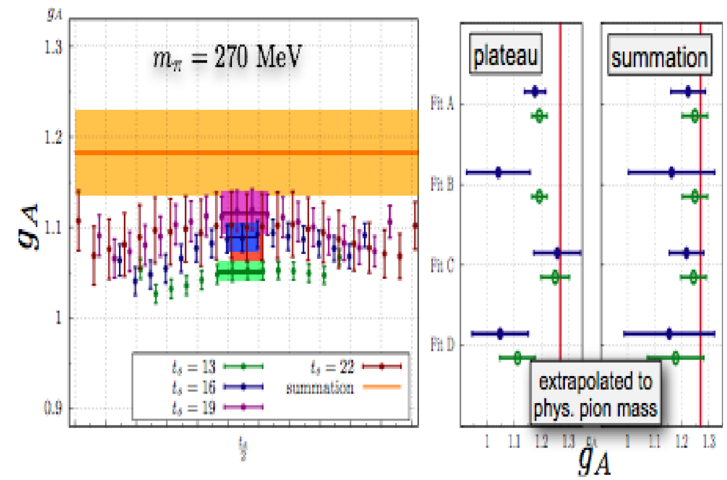

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Compute an $N \times N$ matrix of correlators $C_{JJ}^{ij}(t) = \sum_{\vec{x},\vec{y}} \mathrm{Tr}[\Gamma^0 \langle \Omega | \mathcal{T}[O_J^i(\vec{x},t) O_J^j(\vec{y},0)] | \Omega \rangle]$ with $O_J^i(\vec{x},t) = \epsilon^{abc} \sum_{\vec{z}} (\bar{q}^a(\vec{x},t) C\Gamma q^b(\vec{x},t)) \Phi(|\vec{x}-\vec{z}|)_J^i q^c(\vec{z},t)$.

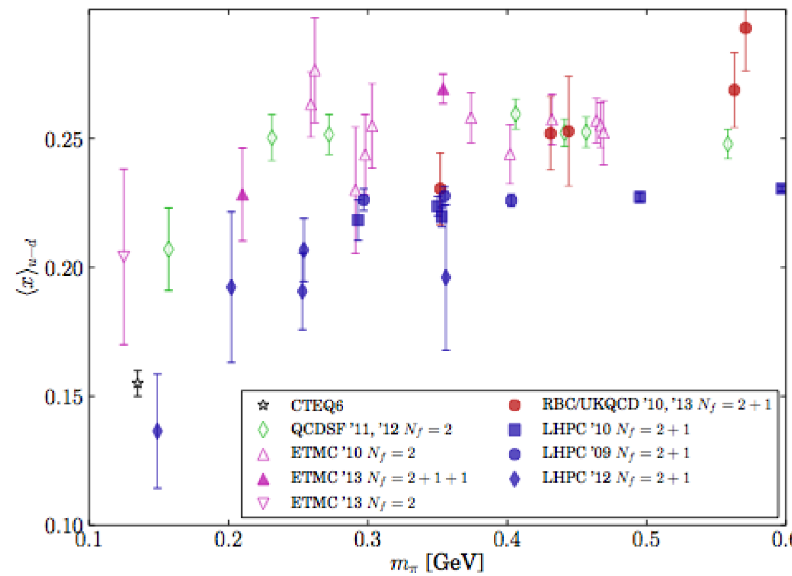
Solve the generalised eigenvalue problem:

$$
\begin{aligned}
C^{ij}(t)\, v_n^j(t,t_0) &= \lambda_n(t,t_0)\, C^{ij}(t_0)\, v_n^j(t,t_0) \\
\langle M_n^{(h)} | O_\Gamma | M_m^{(h')} \rangle &\propto \frac{\sum_{ij} v_n^i(t_s - t, t_0) C^{(3)}_{i\Gamma j}(t,t_s) v_m^j(t,t_0)}{B_n(t_s-t) B'_m(t)} \\
B_n(t) &= \sum_{ij} v_n^i(t,t_0) C^{(2)}_{ij}(t) v_n^j(t,t_0)
\end{aligned}
$$

$$\langle x \rangle_{u-d} = \int dx\, x\, (u(x) + d(x) - \bar{u}(x) - \bar{d}(x))$$

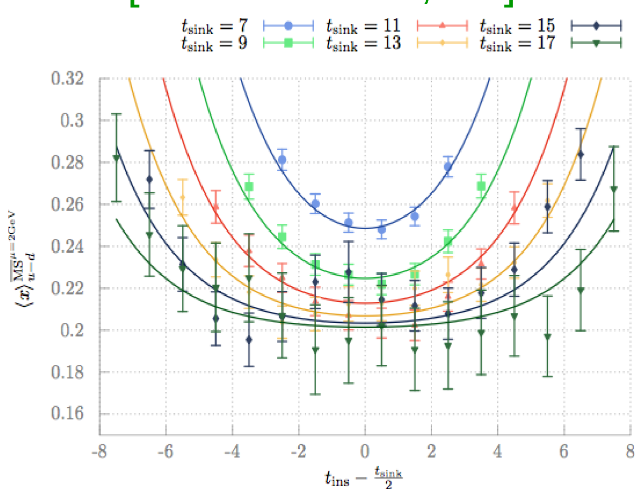
$$\langle x \rangle_{u-d}^{\overline{\mathrm{MS}}\, 2\mathrm{GeV}} = 0.155(5) \quad \langle N(p)|\bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,|N(p)\rangle = \langle x \rangle_q \bar{u}_p \gamma_{\{\mu}p_{\nu\}}u_p$$

[S. Syritsyn, lattice 2013]

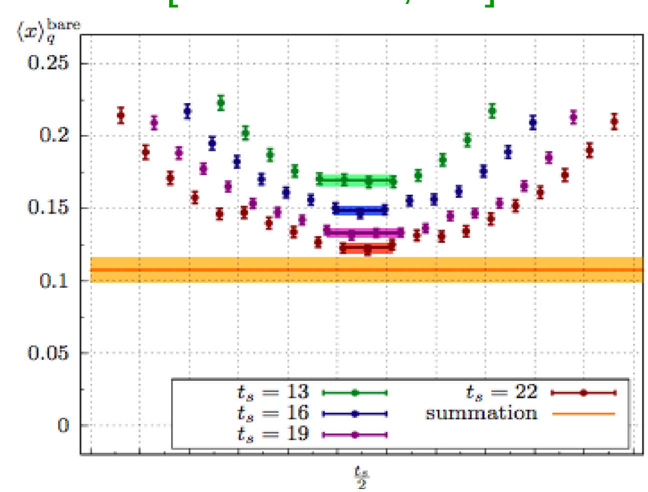


Again, excited states need to be carefully extracted out of the signal.

[S. Collins *et al*, '13]



[T. Rae *et al*, '13]



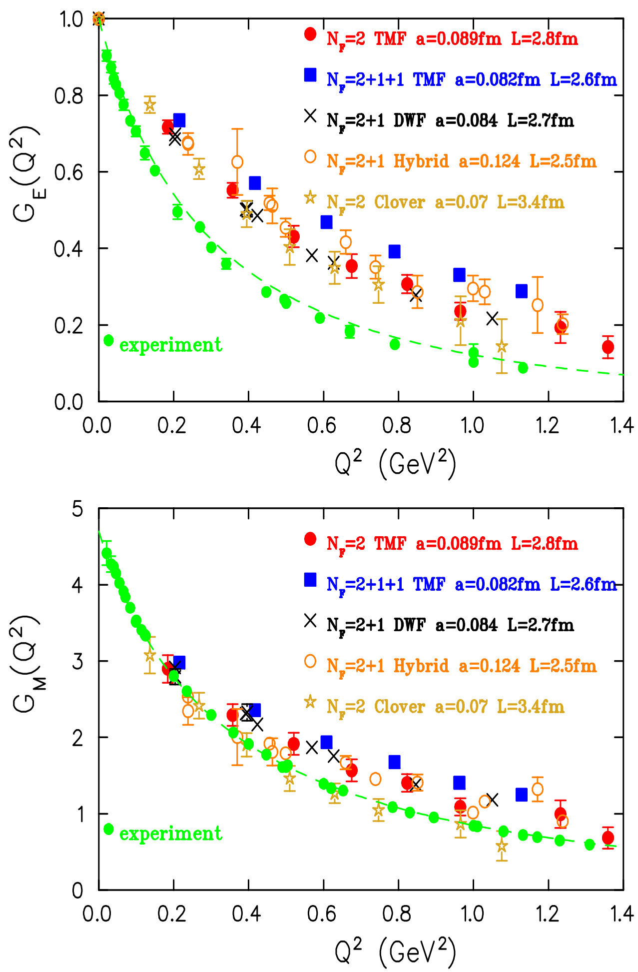
Nucleon form factors

Vector form factors: $\langle N(p+q)|\bar{q}\gamma^\mu q|N(p)\rangle = \bar{u}_{p+q}\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]u_p$

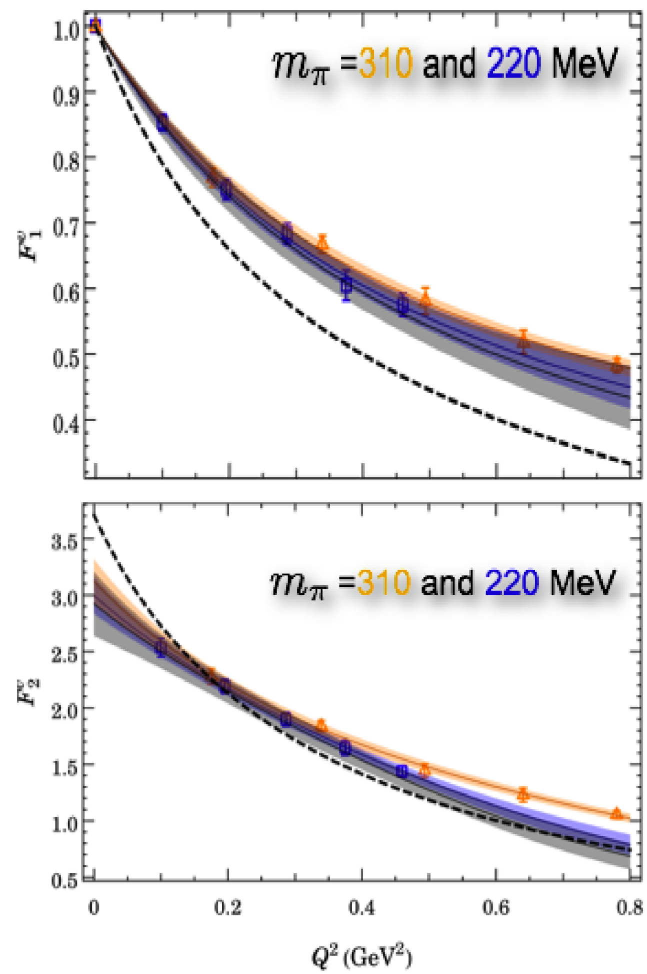
Sachs form factors: $G_E = F_1 - \frac{Q^2}{2M_N^2}F_2$, $G_M = F_1 + F_2$.

Lattice results obtained at $\mathrm{N_f} = 2$, $2+1$ and $2+1+1$; inverse quadratic polynomial in Q^2 describes better the data for F_1 than a dipole expression. For F_2, the best fit is $\frac{F_2(0)}{1+\alpha Q^2+\beta Q^6}$.

[C. Alexandrou *et al*, '13]

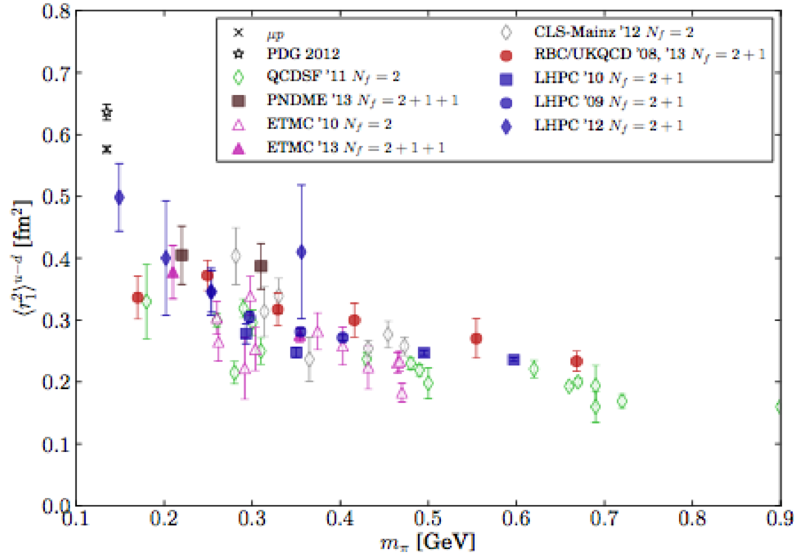


[T. Bhattacharya *et al*, '13]



Dirac radius of the proton: $F_1(Q^2) = F(0)\left[1 - \frac{1}{6}Q^2\langle r_1^2\rangle + \mathcal{O}(Q^4)\right]$.

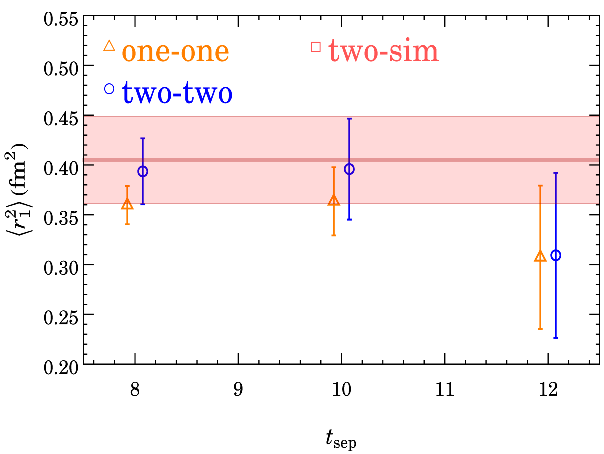
[S. Syritsyn, lattice 2013]



χPT predicts a logarithmic divergence in m_π^2. Finite size effects are questionnable, taking care of excited states is also relevant.

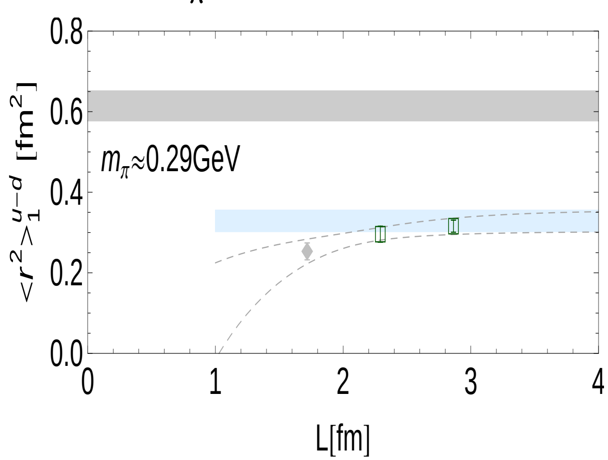
[H. W. Lin *et al*, '13]

$m_\pi = 220$ MeV



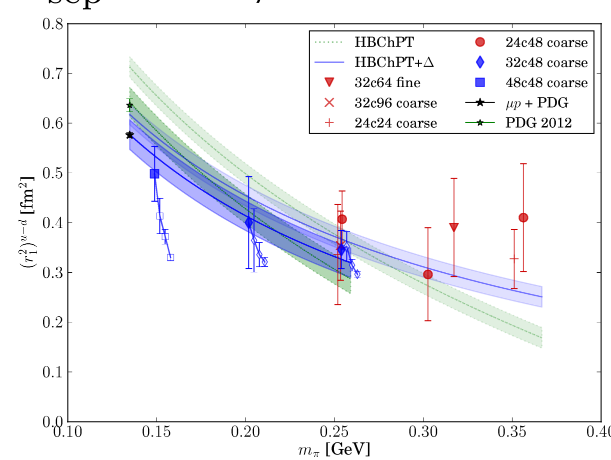
[S. Collins *et al*, '11]

$m_\pi = 290$ MeV



[R. Green *et al*, '12]

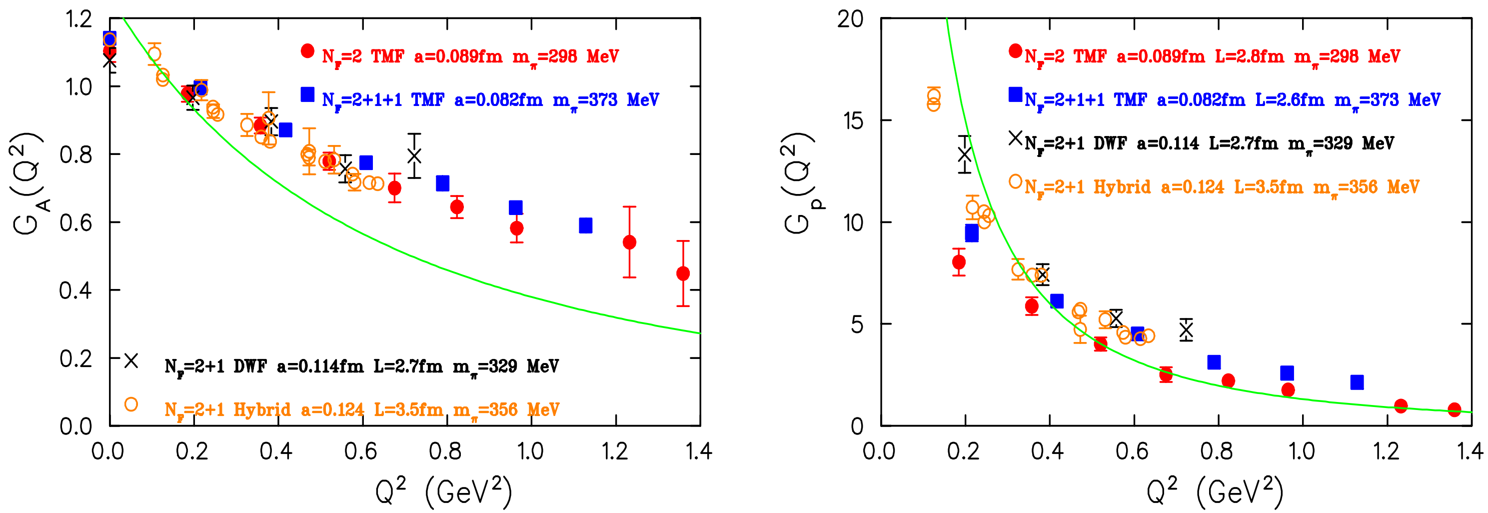
$t_{\rm sep} = 0.93$, 1.16 and 1.39 fm



Axial and pseudoscalar form factors:

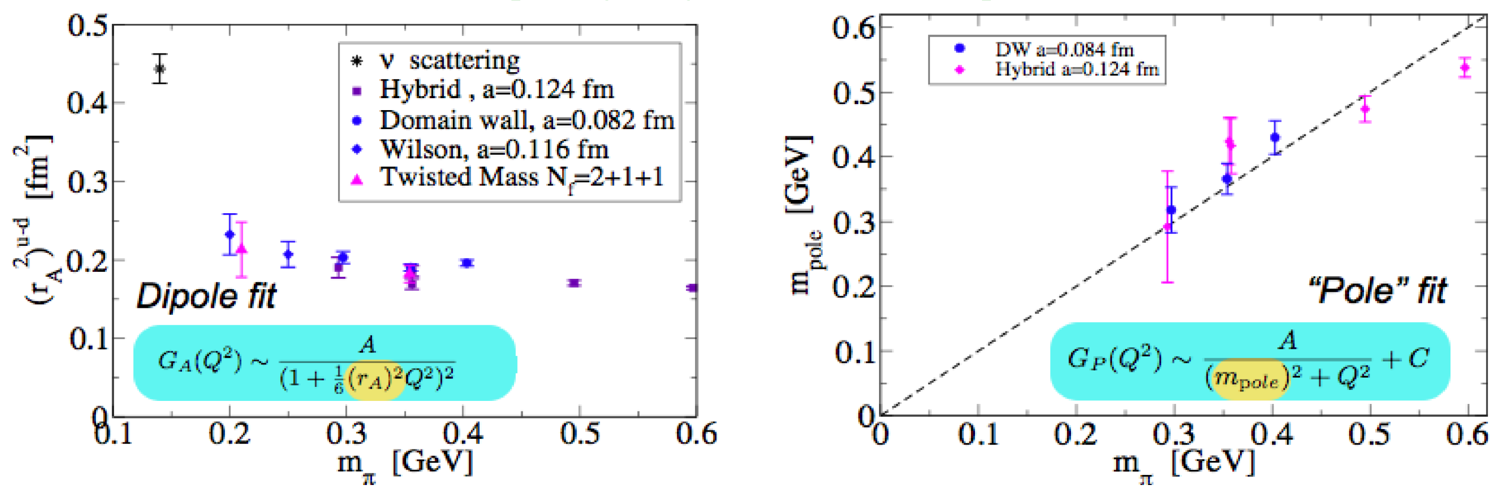
$$\langle N(p+q)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = \bar{u}_{p+q}\left[G_A(Q^2)\gamma^\mu\gamma^5 + G_P(Q^2)\frac{q^\mu}{2M_N}\right]u_p$$

[C. Alexandrou *et al*, '13]



One can extract the radius $\langle r_A^2\rangle$ and the "pole mass" m_P associated to the form factors.

[S. Syritsyn, lattice 2013]



Nucleon s-quark form factors: $G^s_{E,M,A}(Q^2) \lesssim 1\%\ G^{u/d}_{E,M,A}(Q^2)$

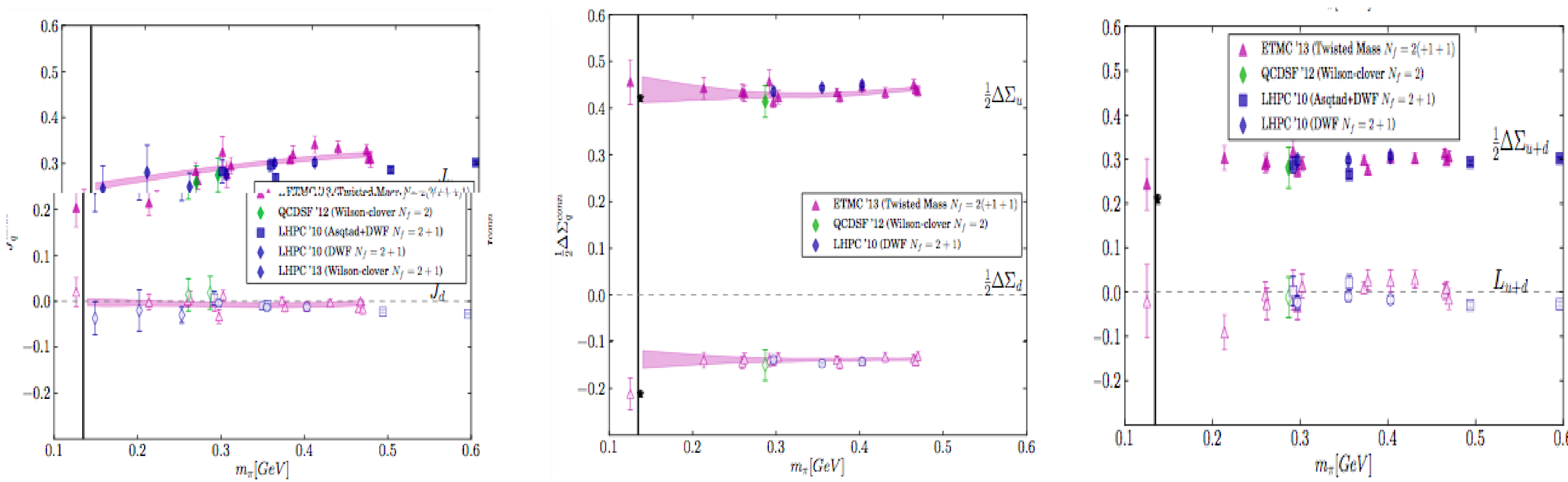
[R. Babich *et al*, '10; T. Doi *et al*, '10]

Spin of the proton

Spin sum rule: $J_{\mathrm{glue}} + \sum_q J_q = \frac{1}{2}$

$J_q = \frac{1}{2}\Delta\Sigma_q + L_q$: $|L_{u+d}| \ll \frac{1}{2}\Delta\Sigma_{u+d}$, $|J_u| \sim 40 - 50\%$, $|J_d| \lesssim 10\%$

[S. Syritsyn, lattice 2013]



The disconnected contributions to $\Delta\Sigma^{u,d,s}$ is small $\Longrightarrow$ the total quark angular momentum is $\sim$ 20 – 30%; no calculation yet of $2J^{\mathrm{unq}}_{\mathrm{glue}}$: $2J^{\mathrm{N_f=0}} = 0.254(76)$ [K. Liu *et al*, ’12].

Distribution Amplitude

$$|N,\uparrow\rangle \quad \propto \quad \int dx_1\, dx_2\, dx_3\, \delta(1-x_1-x_2-x_3)\frac{\varphi(x_i)}{2\sqrt{24x_1x_2x_3}}$$
$$[|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle - |u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle]$$

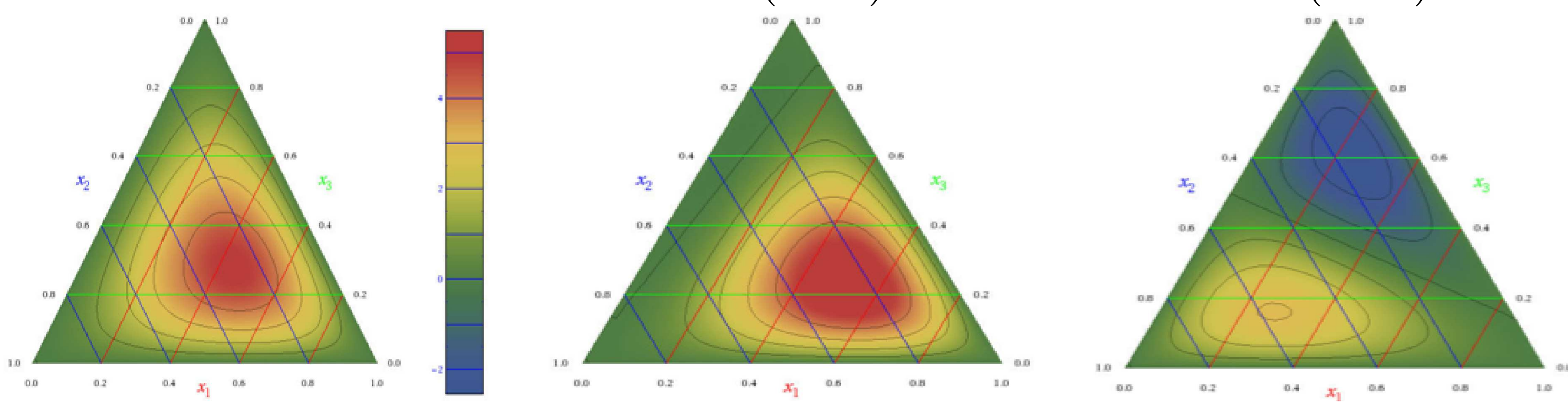
$$\varphi(x_i,\mu^2) \quad = \quad 120x_1x_2x_3\left[1 + c_{10}(x_1-2x_2+x_3)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8}{3\beta_0}}\right.$$
$$\left. + \quad c_{11}(x_1-x_3)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{20}{9\beta_0}} + \ldots\right]$$

On the lattice, one computes the moments of DA $\langle\Omega|O_{\alpha\beta\gamma}(x)|N\rangle$ where O is a local 3-quark operator with at most 2 derivatives:

$$\varphi^{lmn} = \int dx_1\, dx_2\, dx_3\, x_1^l x_2^m x_3^n \varphi(x_1,x_2,x_3) \quad \{c_{1j},c_{2j}\} \leftrightarrow \{\varphi^{lmn}|l+m+n=1,2\}$$

[R.Schiel *et al*, '13]

Nucleon $\quad N^*(1650) \quad N^*(1535)$



Parton distribution functions

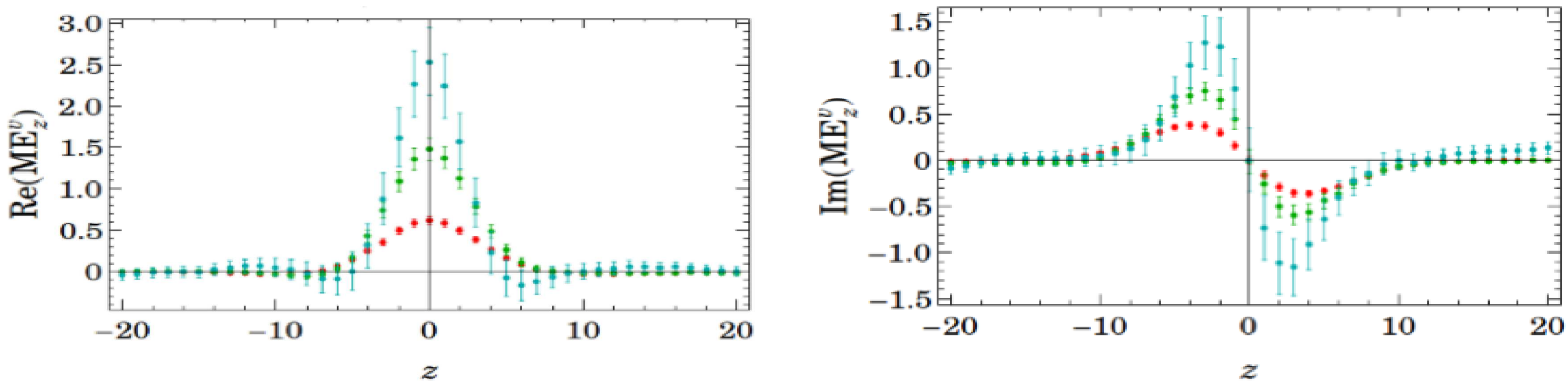
The usual PDF is defined by

$$q(x,\mu) = \int \frac{dx}{4\pi} e^{ix(z_- P_+)} \langle P | \bar{q}(z_-)\gamma^+ \exp[-ig \int_0^{z_-} dt A_+(t)] q(0) | P \rangle$$

Other possibility: boost the hadron, rotate the gauge links along a spatial direction [X. Ji, '13]

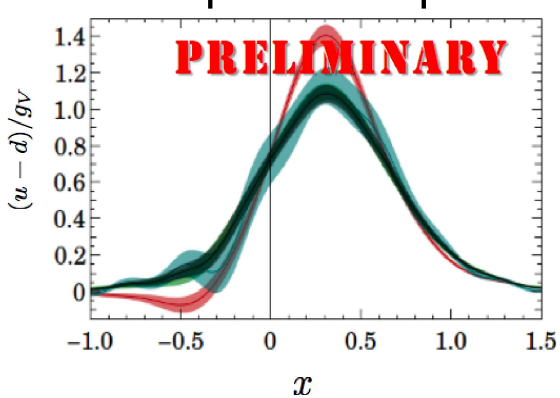
$$\tilde{q}(x,\mu,P_+) = \int \frac{dx}{4\pi} e^{ix(zP_z)} \langle P | \bar{q}(z)\gamma^+ \exp[-ig \int_0^{z} dt A_z(t)] q(0) | P \rangle + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_z^2, M^2/P_z^2)$$

[H. W. Lin *et al*, '13]: $m_\pi = 310$ MeV, $a = 0.12$ fm, $P_z = \frac{2\pi}{L}\{1,2,3\}$

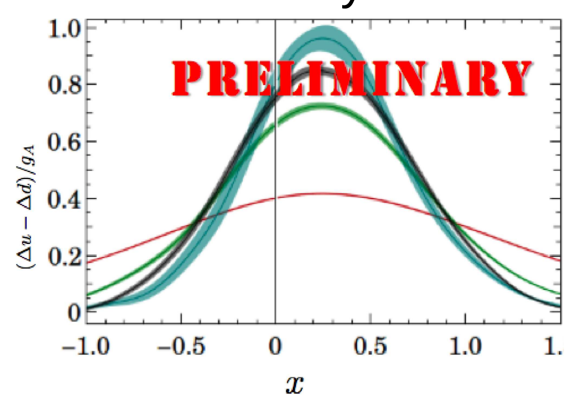


PDF's are recovered by taking the $P_z \to \infty$ limit.

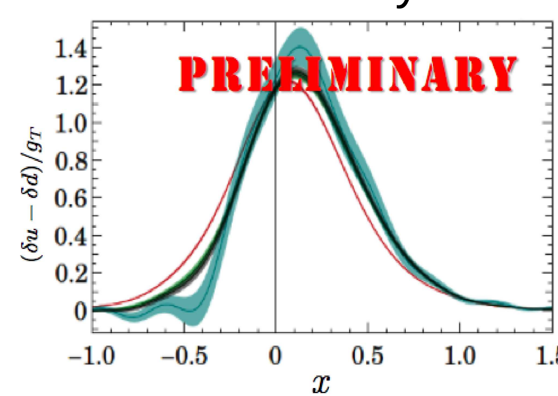
Isovector quark composition



helicity



transversity



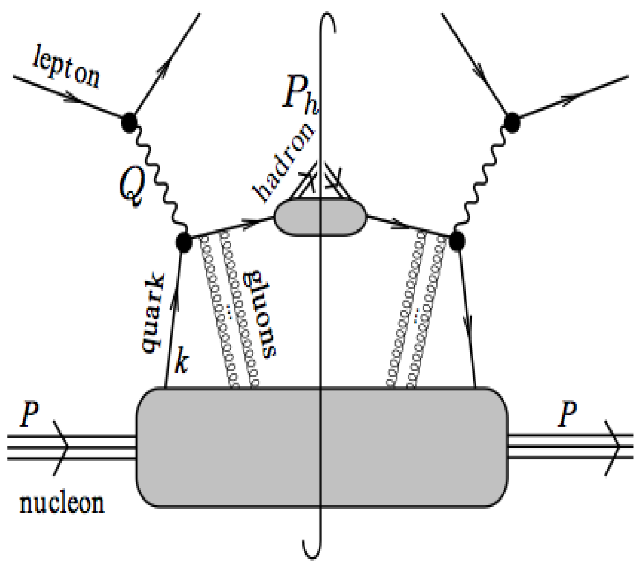
Transverse momentum dependent PDF's

Those distribution functions encode information about the distribution of transverse momentum among partons in a hadron: they can be extracted from semi-inclusive deep inelastic scattering or the Drell-Yan processes.

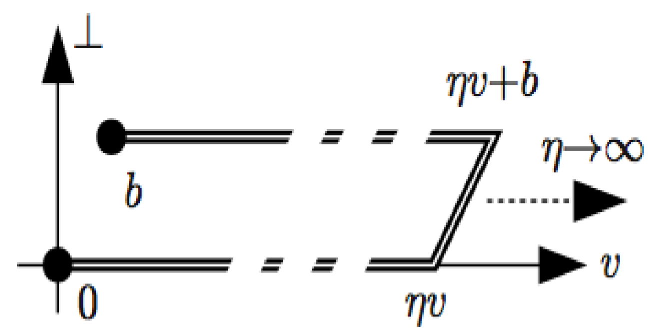
$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) = \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi)P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)}$$

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2} \langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

The "soft factor" $\mathcal{S}$ cancels divergences introduced by the gauge connection $\mathcal{U}$.



$l + N(P) \to l' + h(P_h) + X$



includes SIDIS final state effects

light-cone limit: $\tilde{\xi} = \frac{P \cdot v}{m_N |v|} \to \infty$

$$\Phi^{[\gamma^+]}(x,\boldsymbol{k}_\mathrm{T};P,S,\ldots) = f_1 - \left[\frac{\epsilon_{ij}\,\boldsymbol{k}_i\,\boldsymbol{S}_j}{m_N}\; f_{1T}^{\perp}\right]_{\text{odd}}$$

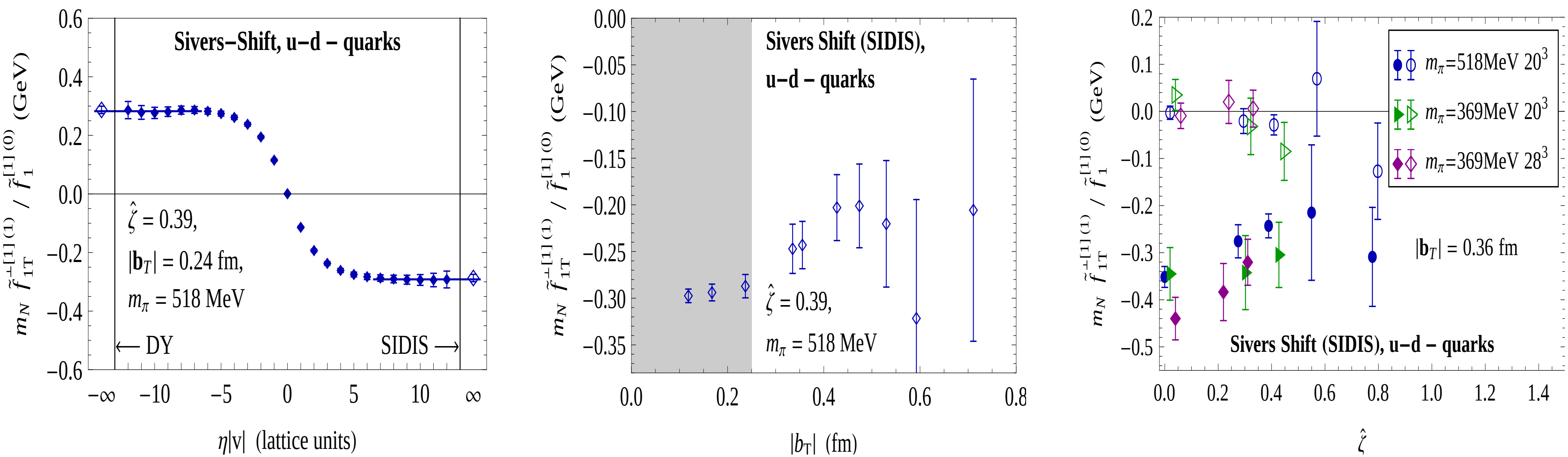
$$\Phi^{[\gamma^+\gamma^5]}(x,\boldsymbol{k}_\mathrm{T};P,S,\ldots) = \Lambda\, g_1 + \frac{\boldsymbol{k}_\mathrm{T}\cdot\boldsymbol{S}_\mathrm{T}}{m_N}\; g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]}(x,\boldsymbol{k}_\mathrm{T};P,S,\ldots) = \boldsymbol{S}_i\; h_1 + \frac{(2\boldsymbol{k}_i\boldsymbol{k}_j - \boldsymbol{k}_\mathrm{T}^2\delta_{ij})\boldsymbol{S}_j}{2m_N^2}\; h_{1T}^{\perp} + \frac{\Lambda\boldsymbol{k}_i}{m_N} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij}\boldsymbol{k}_j}{m_N} h_1^{\perp}\right]_{\text{odd}}$$

The presence of final states effects breaks the invariance under time reversal. Extraction of T-odd TMD's

Sivers shift: $$\langle \boldsymbol{k}_y\rangle(\boldsymbol{b}_\mathrm{T}^2;\ldots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)} \xrightarrow{\boldsymbol{b}_\mathrm{T}^2\to 0} \left.\frac{\int d^2\boldsymbol{k}_\mathrm{T}\,\boldsymbol{k}_y\;\Phi^{[\gamma^+]}(x,\boldsymbol{k}_\mathrm{T},P,S;\ldots)}{\int d^2\boldsymbol{k}_\mathrm{T}\;\Phi^{[\gamma^+]}(x,\boldsymbol{k}_\mathrm{T},P,S;\ldots)}\right|_{\boldsymbol{S}_\mathrm{T}=(1,0)}$$

[M. Engelhardt *et al*, '12]



$$\Phi^{[\gamma^+]}(x,\boldsymbol{k}_{\rm T};P,S,\ldots) = f_1 - \left[\frac{\epsilon_{ij}\,\boldsymbol{k}_i\,\boldsymbol{S}_j}{m_N}\;f_{1T}^{\perp}\right]_{\rm odd}$$

$$\Phi^{[\gamma^+\gamma^5]}(x,\boldsymbol{k}_{\rm T};P,S,\ldots) = \Lambda\,g_1 + \frac{\boldsymbol{k}_{\rm T}\cdot\boldsymbol{S}_{\rm T}}{m_N}\;g_{1T}$$

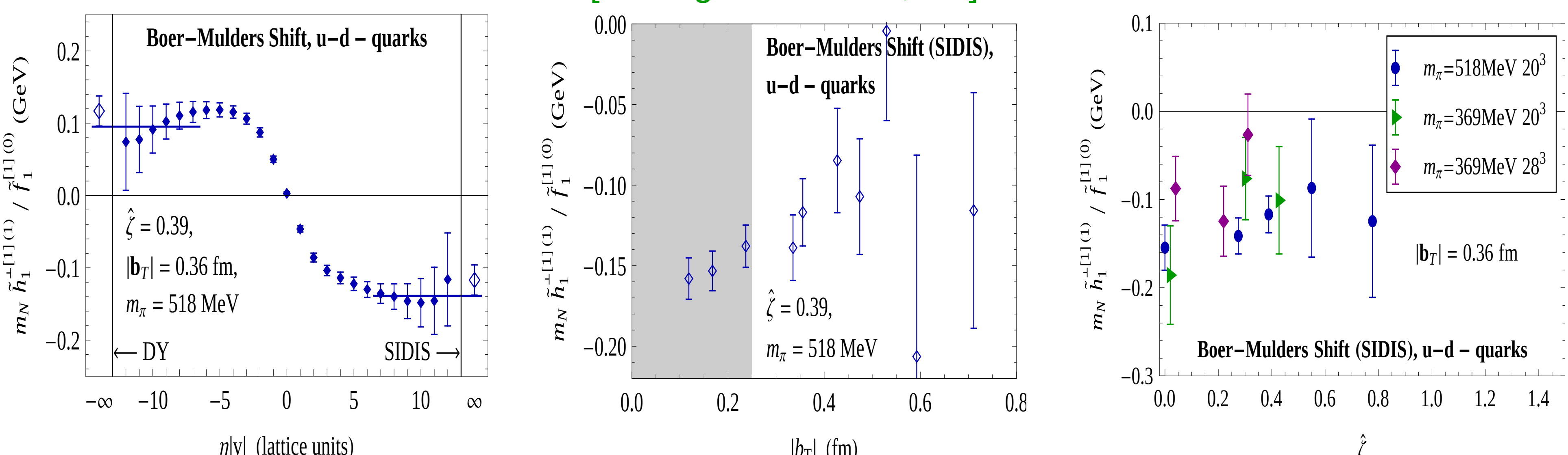
$$\Phi^{[i\sigma^{i+}\gamma^5]}(x,\boldsymbol{k}_{\rm T};P,S,\ldots) = \boldsymbol{S}_i\;h_1 + \frac{(2\boldsymbol{k}_i\boldsymbol{k}_j - \boldsymbol{k}_{\rm T}^2\delta_{ij})\boldsymbol{S}_j}{2m_N^2}\;h_{1T}^{\perp} + \frac{\Lambda\boldsymbol{k}_i}{m_N}h_{1L}^{\perp} + \left[\frac{\epsilon_{ij}\boldsymbol{k}_j}{m_N}h_1^{\perp}\right]_{\rm odd}$$

The presence of final states effects breaks the invariance under time reversal. Extraction of T-odd TMD's

Boer-Mulders shift:

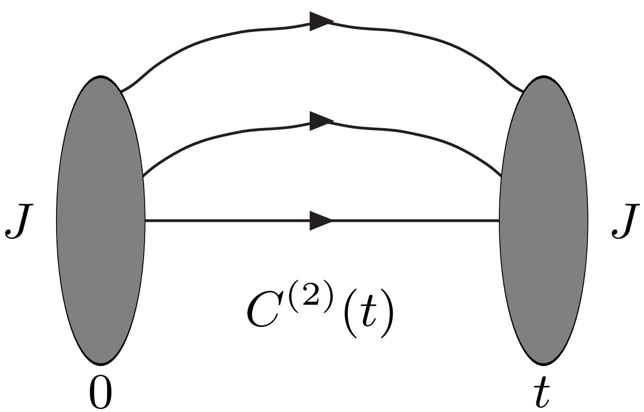
$$\langle \boldsymbol{k}_y^{BM}\rangle(\boldsymbol{b}_{\rm T}^2;\ldots) \equiv m_N\,\frac{\tilde{h}_1^{\perp[1](1)}(\boldsymbol{b}_{\rm T}^2;\ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_{\rm T}^2;\ldots)} \xrightarrow{\boldsymbol{b}_{\rm T}^2\to 0} \left.\frac{\int d^2\boldsymbol{k}_{\rm T}\;\boldsymbol{k}_y\;\;\Phi^{[\sigma^{x,+}]}(x,\boldsymbol{k}_{\rm T},P,S;\ldots)}{\int d^2\boldsymbol{k}_{\rm T}\qquad\Phi^{[\gamma^+]}(x,\boldsymbol{k}_{\rm T},P,S;\ldots)}\right|_{\boldsymbol{S}_{\rm T}=(1,0)}$$

[M. Engelhardt *et al*, '12]



Multihadron states and nuclear potential

Extracting masses of bound states is straighforward if they do not decay strongly:

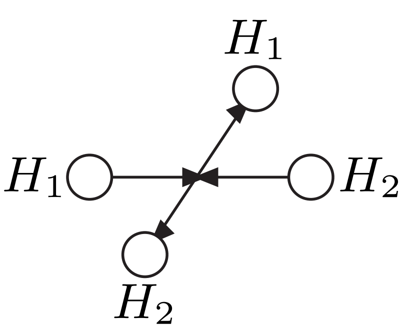


$$C^{(2)}_{JJ}(t) = \mathrm{Tr}\left[\left(\tfrac{1\pm\gamma_0}{2}\right)\sum_{\vec{x}}\langle\Omega|\mathcal{T}[J(\vec{x},t)J^\dagger(0)]|\Omega\rangle\right]$$
$$= \sum_n \frac{\mathcal{Z}_n^2\, e^{-E_n\, t}}{2E_n}$$
$$\langle\Omega|J|n(0,s)\rangle = \mathcal{Z}_n u(0,s) \quad \langle n(0,s)|m(0,s')\rangle = 2E_n\delta_{mn}\delta_{ss'}$$
$$C^{(2)}_{JJ}(t) \xrightarrow{(E_1-E_0)t\gg 1} \frac{\mathcal{Z}_0^2\, e^{-E_0\, t}}{2E_0}$$

However the situation is quite different if wide states (Δ, Roper state, ...) or multihadron states (NN, ...), often produced in experiments, are considered.

☹ One can not compute scattering properties from infinite volume Euclidean simulations [L. Maiani and M. Testa, '90].

☺ Measuring in the elastic region the phase shift δ_l of a 2-hadrons scattering by a finite size method gives the missing information [M. Lüscher, '86, '91]



$$E^*_{\mathrm{free}} = \sqrt{m^2_{H_1} + p_0^{*2}} + \sqrt{m^2_{H_2} + p_0^{*2}} \quad \vec{p_0^*} = 2\pi/L\,\vec{n} \quad \vec{n}\in Z^3$$
$$p^{*2} = \tfrac{1}{4s}\left(s-(m_{H_1}+m_{H_2})^2\right)\left(s-(m_{H_1}-m_{H_2})^2\right) \quad \vec{p^*} = 2\pi/L\,\vec{q}$$

$$\tan\delta_l(q) = \frac{\pi^{3/2} q}{Z_{00}(1,q^2)} \qquad Z_{00}(s,q^2) = \frac{1}{\sqrt{4\pi}} \sum_{n\in Z^3} (n^2 - q^2)^{-s}$$

Relativistic Breit-Wigner form for the scattering amplitude at a resonance $(M_R,\ \Gamma_R)$:

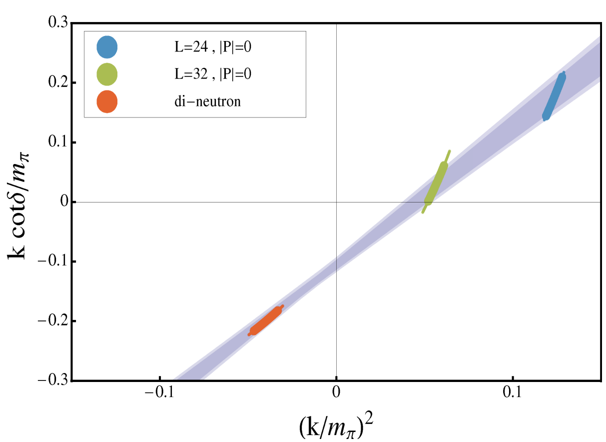
$$a_l = \frac{-\sqrt{s}\Gamma_R(s)}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)} \qquad \tan\delta_l(q) = \frac{\sqrt{s}\Gamma_R(s)}{M_R^2 - s}$$

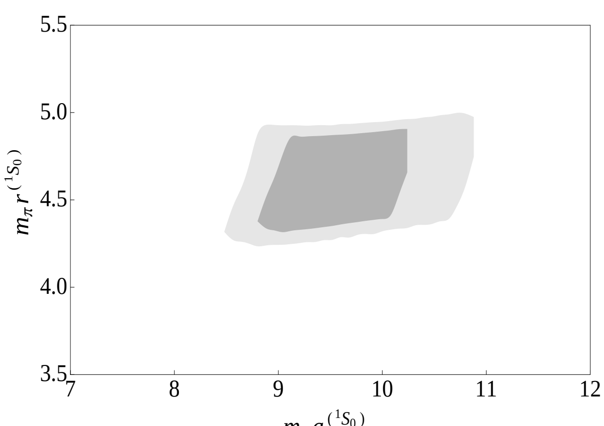
Application: extraction of NN phase shift parameters in the $SU(3)$ limit [S. Beane *et al*, '13]

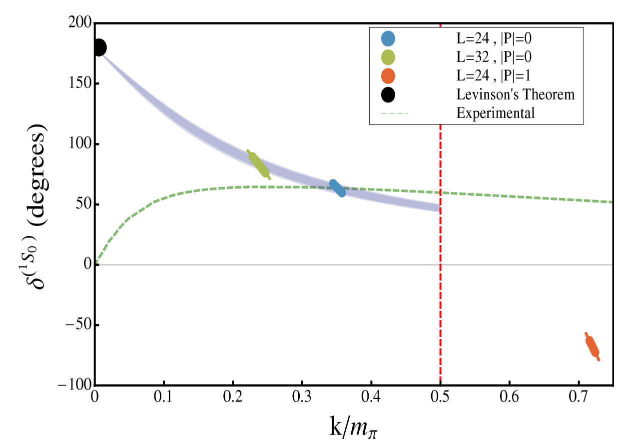
Existence of a bound state in the ${}^1S_0$ channel with a binding energy $B_{NN} = 15.9(2.7)(2.7)(0.2)$ MeV.

Analysis of correlators with smeared or local interpolating fields of nucleon-nucleon state with a total momentum $|P| = 0$ or $|P| = 1$.

Power series for the phase shift: $k\cot\delta = -\frac{1}{a} + \frac{1}{2} r|\vec{k}|^2 + P|\vec{k}|^4 + \mathcal{O}(|\vec{k}|^6)$.
a: scattering length; r: effective range; P: shape parameter







$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \qquad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm} \qquad P \sim 0$$

$$\tan\delta_l(q) = \frac{\pi^{3/2} q}{Z_{00}(1,q^2)} \quad Z_{00}(s,q^2) = \frac{1}{\sqrt{4\pi}} \sum_{n\in Z^3} (n^2 - q^2)^{-s}$$

Relativistic Breit-Wigner form for the scattering amplitude at a resonance $(M_R,\ \Gamma_R)$:

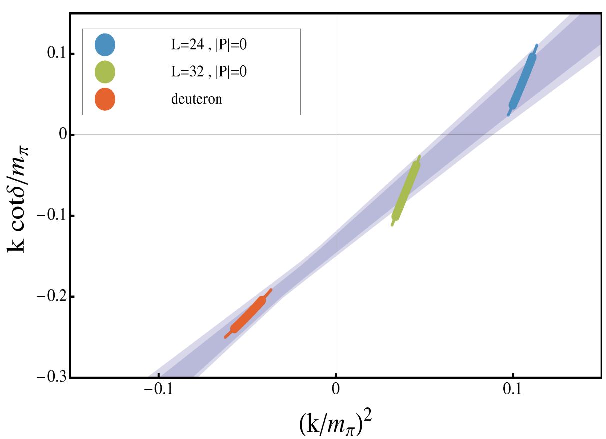
$$a_l = \frac{-\sqrt{s}\Gamma_R(s)}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)} \quad \tan\delta_l(q) = \frac{\sqrt{s}\Gamma_R(s)}{M_R^2 - s}$$

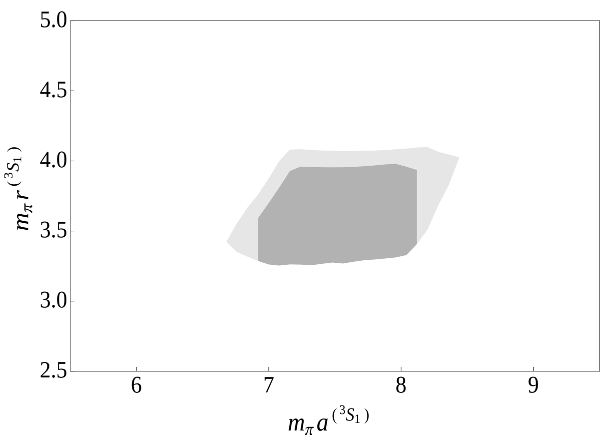
Application: extraction of NN phase shift parameters in the $SU(3)$ limit [S. Beane *et al*, '13]

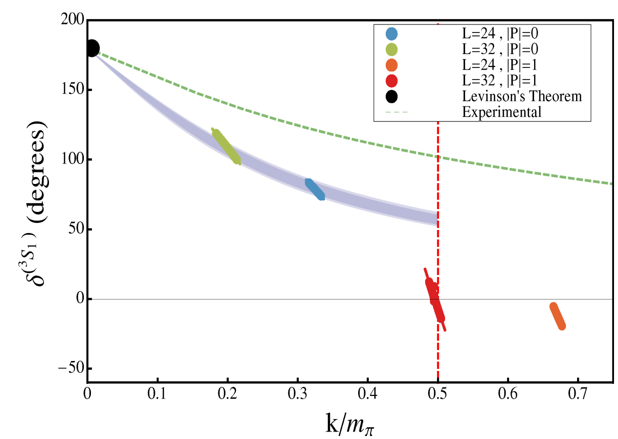
Existence of a bound deuteron in the ${}^3S_1$ channel with a binding energy $B_d = 19.5(3.6)(3.1)(0.2)$ MeV.

Analysis of correlators with smeared or local interpolating fields of nucleon-nucleon state with a total momentum $|P| = 0$ or $|P| = 1$.

Power series for the phase shift: $k\cot\delta = -\frac{1}{a} + \frac{1}{2} r|\vec{k}|^2 + P|\vec{k}|^4 + \mathcal{O}(|\vec{k}|^6)$.
a: scattering length; r: effective range; P: shape parameter







$$a^{({}^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad r^{({}^3S_1)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm} \quad P \sim 0$$

Since a couple of years people have tried to extract the interaction potential between nucleons from a Schrödinger equation approach [S. Aoki *et al*, '10, '12]

$$\left[\frac{|\vec{p}|^2}{2\mu} - H_0\right]\psi_p(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r'})\psi_p(r')$$

Approximate potential: $U(\vec{r},\vec{r'}) = V_c(\vec{r})\delta(\vec{r}-\vec{r'}) + \mathcal{O}(\nabla_r^2/\Lambda^2)$

$$\begin{aligned} V_c(\vec{r}) &\simeq \frac{|\vec{p}|^2}{2\mu} + \lim_{t\to\infty}\frac{1}{2\mu}\frac{\nabla_r^2 C_{NN}(\vec{r},t)}{C_{NN}(\vec{r},t)} \\ &= \frac{|\vec{p}|^2}{2\mu} + \frac{1}{2\mu}\frac{\nabla_r^2(e^{-E_0 t}\psi(\vec{r})A_0^\dagger)}{e^{-E_0 t}\psi(\vec{r})A_0^\dagger} \\ &= \frac{|\vec{p}|^2}{2\mu} + \frac{1}{2\mu}\frac{\nabla_r^2\psi(\vec{r})}{\psi(\vec{r})} \end{aligned}$$

Issues: assumption of a saturation by the ground state, systematics introduced by the gradient expansion of the potential tricky to estimate

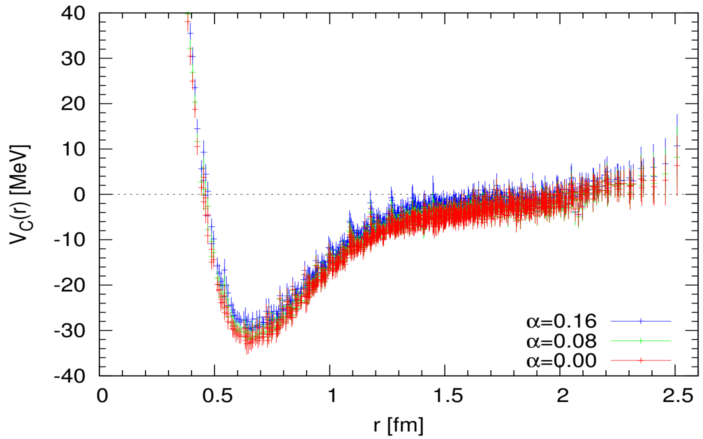
"time-dependent" Schrödinger-like equation:

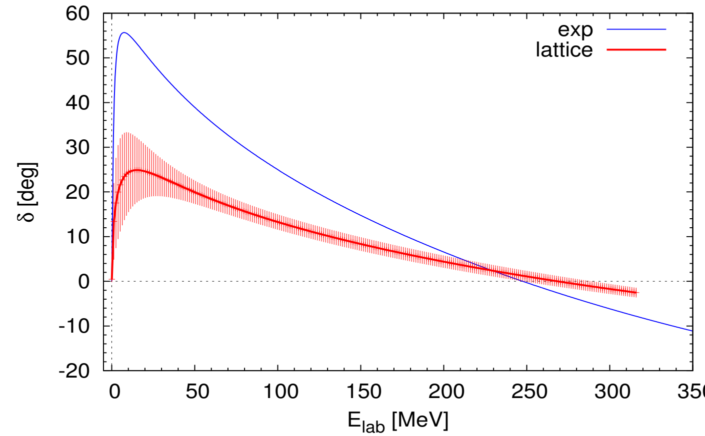
$$\left[\frac{1}{4M}\partial_t^2 - \partial_t - H_0\right]R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}'R(\vec{r}',t) \quad R(\vec{r},t) = \frac{C_{NN}(\vec{r},t)}{[C_N(t)]^2}$$

Issue: assumption that only elastic states contribute to $C_{NN}(\vec{r},t)$

$$V_c(\vec{r}) \simeq \frac{1}{M}\frac{\nabla_r^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial_t R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{4M}\frac{\partial_t^2 R(\vec{r},t)}{R(\vec{r},t)}$$

[S. Aoki *et al*, '12]





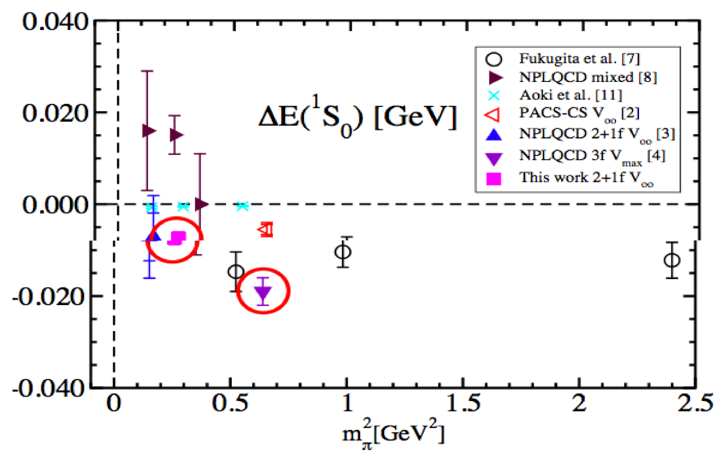
One injects the potential $V_c(\vec{r})$ in a Schrödinger equation; phase shifts are extracted from the wave functions.

NN system: scattering state or bound state? $\Delta E = \sqrt{M^2 + \vec{k}^2} - 2M$

scattering state: $\Delta E = -\frac{4\pi a}{ML^3}\left[1 + \mathcal{O}\left(\frac{a}{L}\right)\right]$

bound state: $\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right) + \ldots,\ \kappa^2 = -k^2,\ \gamma = \sqrt{M_\Lambda^\infty B_H^\infty}$

Contradictory results between Lüscher method [T. Yamazaki *et al*, '11; S. Beane *et al*, '13] and Schrödinger-like equation method [S. Aoki *et al*, '12].



Isospin breaking effects

With the present level of precision (a few %), evaluating low energy hadronic matrix elements and checking the reliability of effective theories like χPT needs to take into account isospin breaking effects, from QED ($e_u \neq e_d$) and from mass terms in the QCD Lagrangian ($m_u \neq m_d$).

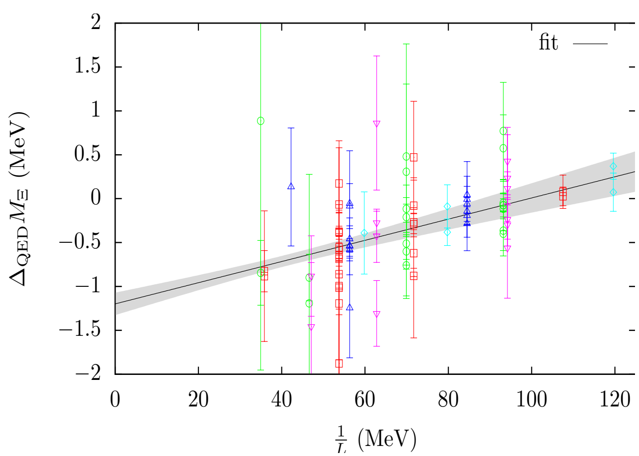
Isospin breaking generates a rich phenomenology:

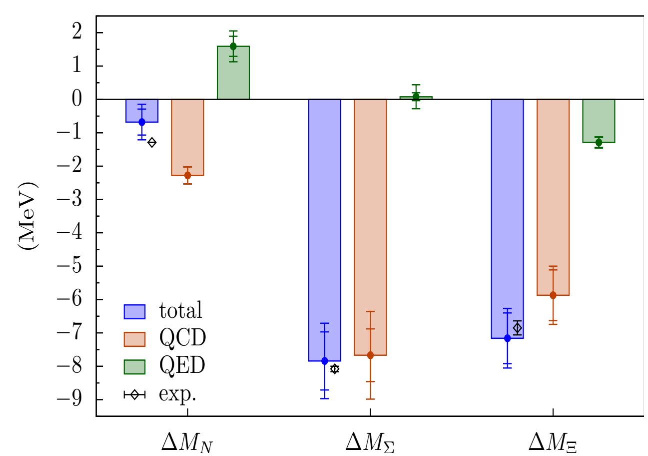
$m_n - m_p = [m_n - m_p]^{\rm QCD} + \underbrace{[m_n - m_p]^{\rm QED}}_{<0} > m_e \longrightarrow$ the hydrogen atom is stable (no electron capture)

Lattice simulation performed within QCD+(q)QED directly:

$U^{\rm QCD} \rightarrow U^{\rm QCD+(q)QED} = e^{ieA_\mu^{\rm QED}}$, $A_\mu^{\rm QED}$ obtained by solving Maxwell equations with periodic boundary conditions and removing the zero mode in the QED action.

Finite volume effects are pretty large: QED is a long-range unconfined theory. Promising results for the octet baryon spectrum [Sz. Borsanyi *et al*, '13]





They can be included through a “reweighting” of the pure isosymmetric QCD ensembles, that corresponds to a matching of the theories $\{\alpha, \alpha_s, m_u, m_d\}$ and $\{0, \alpha_s^0 \hat{m}, \hat{m}\}$ and an expension in $m_u - m_d$ and α.

$$\langle O(g) \rangle = \frac{\int dA e^{-S_{\text{gauge}}(A)} dU e^{-\beta S_{\text{gauge}}(U)} \Pi_f \det(D_f[U, A, g]) O[U, A, g]}{\int dA e^{-S_{\text{gauge}}(A)} dU e^{-\beta S_{\text{gauge}}(U)} \Pi_f \det(D_f[U, A, g])}$$

$$\langle O(g^0) \rangle = \frac{\int dU e^{-\beta^0 S_{\text{gauge}}(U)} \Pi_f \det(D_f[U, g^0) O[U]}{\int dU e^{-\beta^0 S_{\text{gauge}}(U)} \Pi_f \det(D_f[U, g^0])}$$

$$R[U, A, g] = e^{-(\beta - \beta^0) S_{\text{gauge}}[U]} \; r[U, A, g] \quad r[U, A, g] = \prod_f \frac{\det[D_f[U, A, g]}{\det[D_f[U, g^0]}$$

$$\langle O \rangle^A = \frac{\int dA \; e^{-S_{\text{gauge}}[A]} O[A]}{\int dA \; e^{-S_{\text{gauge}}[A]}}$$

$$\langle O \rangle^g = \frac{\langle RO \rangle^{A, g^0}}{\langle R \rangle^{A, g^0}} = \frac{\Big\langle \, \langle \; R[U, A, g] \; O[U, A, g] \; \rangle^A \; \Big\rangle^{g^0}}{\Big\langle \, \langle \; R[U, A, g] \; \rangle^A \; \Big\rangle^{g^0}}$$

Leading corrections in Δm_{ud} and α are computed through the operator ΔO

$$\Delta O \sim \left\{ e^2 \frac{\partial}{\partial e^2} + \left[g_s^2 - (g_s^0)^2\right] \frac{\partial}{\partial g_s^2} + \left[m_f - m_f^0\right] \frac{\partial}{\partial m_f} \right\} O(g) \Big|_{g=g^0}$$

$$\begin{aligned}
\Delta O &= \left\langle \Delta(RO) \right\rangle^{A,g^0} - \left\langle \Delta R \right\rangle^{A,g^0} \left\langle O \right\rangle^{g^0} \\
&= \left\langle \Delta O[U,A,g]\big|_{g=g^0} \right\rangle^{A,g^0} + \Big\{ \left\langle \Delta\left(RO - O\right)[U,A,g]\big|_{g=g^0} \right\rangle^{A,g^0} \\
&\qquad - \left\langle \Delta R[U,A,g]\big|_{g=g^0} \right\rangle^{A,g^0} \left\langle O[U,g^0] \right\rangle^{g^0} \Big\}
\end{aligned}$$

Example for a 2-pt correlator:

$$C_{HH}(t,g) = \langle\, O_H(t)\, O_H^\dagger(0)\, \rangle^g \;=\; Z_H^2 e^{-tM_H} \;+\; \cdots \;\; e^{M_H} = \frac{C_{HH}(t-1,g)}{C_{HH}(t,g)} \;+\; \cdots$$

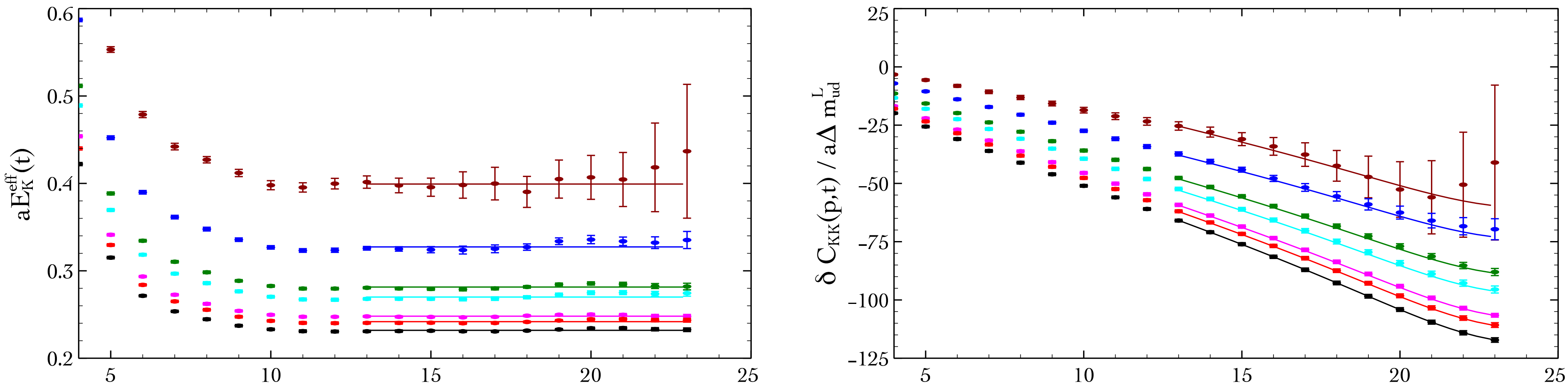
The expansion Δ reads

$$C_{HH}(t,g) = C_{HH}(t,g^0)\left[1 + \frac{\Delta C_{HH}(t)}{C_{HH}(t,g^0)} + \ldots\right] \;\; \Delta M_H = M_H - M_H^0 = -\partial_t \frac{\Delta C_{HH}(t)}{C_{HH}(t,g^0)} + \ldots$$

$$\Delta C_{HH}(t)/C_{HH}(t,g) = \Delta\left(\frac{Z_H^2}{2E_H}\right) / \left(\frac{\mathcal{Z}_{\mathcal{H}}^{\in}}{2E_H}\right) - t\Delta M_H$$

Application to kaon and nucleon physics [G. M. de Divitiis *et al*, '11]

$a = 0.085$ fm, $L = 2$ fm, $m_\pi \sim 300$ MeV

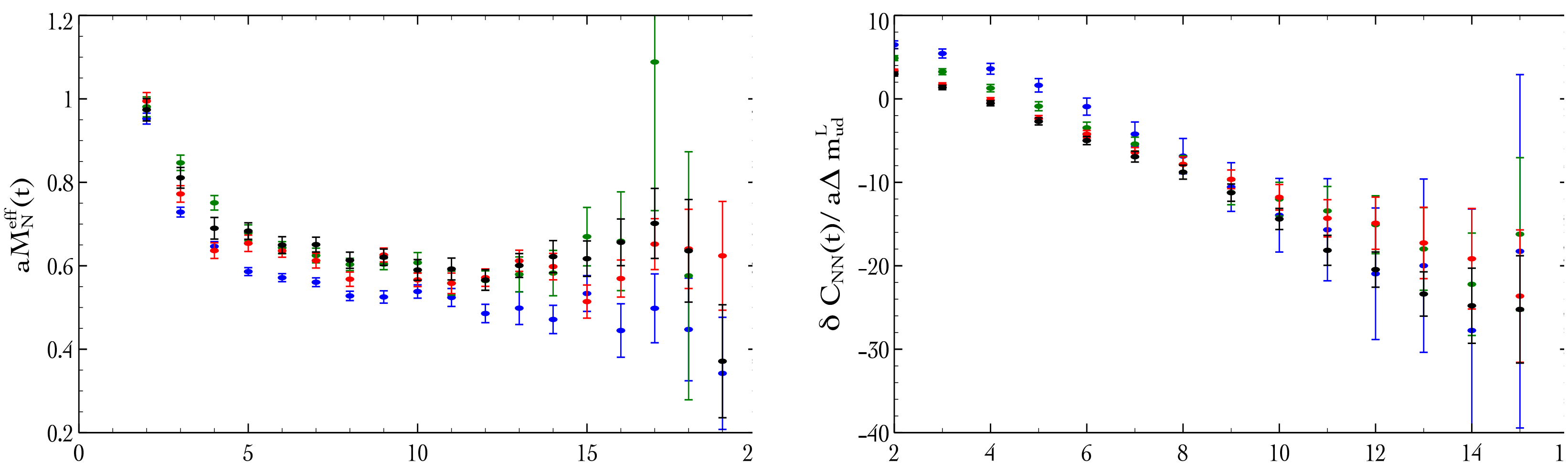


$$\Delta M_K^2 \equiv m_{K^+}^2 - m_{K^0}^2 = (m_{K^+}^2 - m_{K^0}^2)^{\rm QCD} + \Delta_K^\gamma \quad (m_{K^+}^2 - m_{K^0}^2)^{\rm QCD} \propto (m_d - m_u)$$

Correction to Dashen theorem: $\Delta_K^\gamma = (1+\epsilon)\Delta_\pi^\gamma$

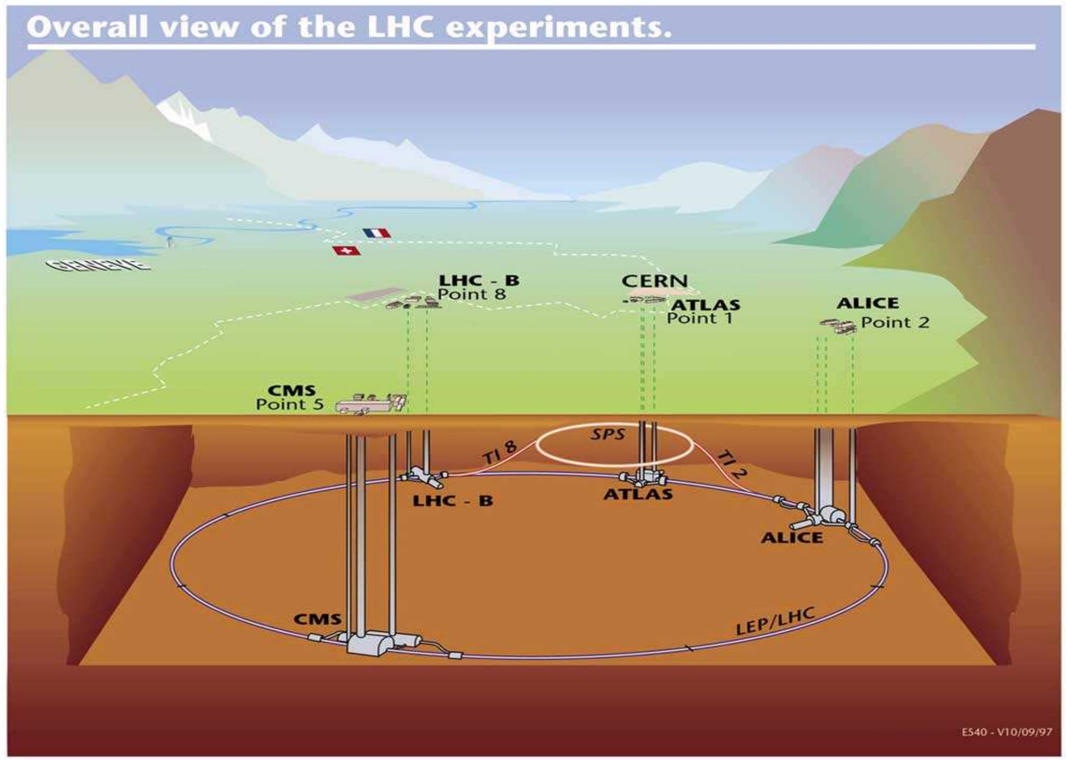
From $\epsilon(\overline{\rm MS}, 2\,{\rm GeV}) = 0.79(18)(18)$: $[m_d - m_u]^{\rm QCD}\,(\overline{\rm MS}, 2\,{\rm GeV}) = 2.39(8)(17)$ MeV

Nucleon mass $a = 0.085$ fm, $L = 2$ fm, $m_\pi \sim 350$ MeV



$$[M_n - M_p]^{\rm QCD}\,(\overline{\rm MS},\ 2{\rm GeV}) = 2.9(6)(2) \qquad (1)$$

New Physics in the baryon sector

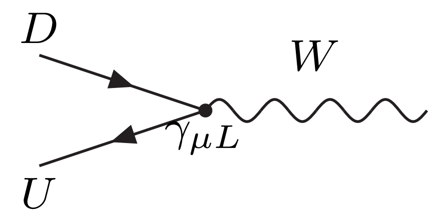


LHC is working very well, a lot of forthcoming data will be analysed to try to give an answer to important questions (hierarchy problem, ...). Lattice QCD is a powerful tool to bring theoretical ingredients that are necessary as soon as bound states of quarks and gluons are involved in processes under study.

Standard Model in the flavour sector

3 families of quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$

Quarks are coupled to charged weak bosons by a left-handed current.



Quark flavour eigenstates $\neq$ quark weak eigenstates; the flavour mixing is described by the Cabibbo-Kobayashi-Maskawa mechanism, the only source of CP violation.

$$\begin{pmatrix} \text{d'} \\ \text{s'} \\ \text{b'} \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} \text{d} \\ \text{s} \\ \text{b} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} \text{d} \\ \text{s} \\ \text{b} \end{pmatrix}$$

$V_{ij} \sim \mathcal{O}(1)$

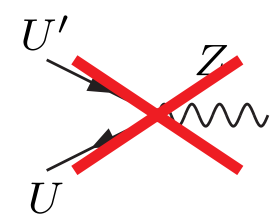
$V_{ij} \sim \mathcal{O}(\lambda)$

$V_{ij} \sim \mathcal{O}(\lambda^2)$

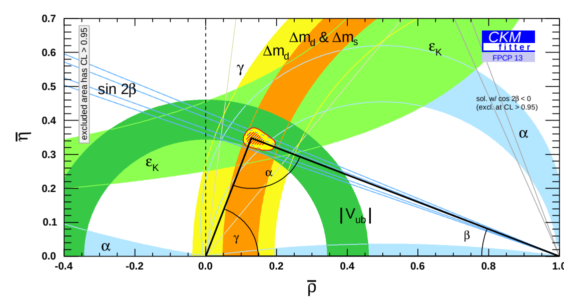
$V_{ij} \sim \mathcal{O}(\lambda^3)$

$\lambda \sim 0.22$

Unitarity of the CKM matrix: Glashow - Iliopoulos - Maiani mechanism, no Flavour Changing Neutral Current at tree level.

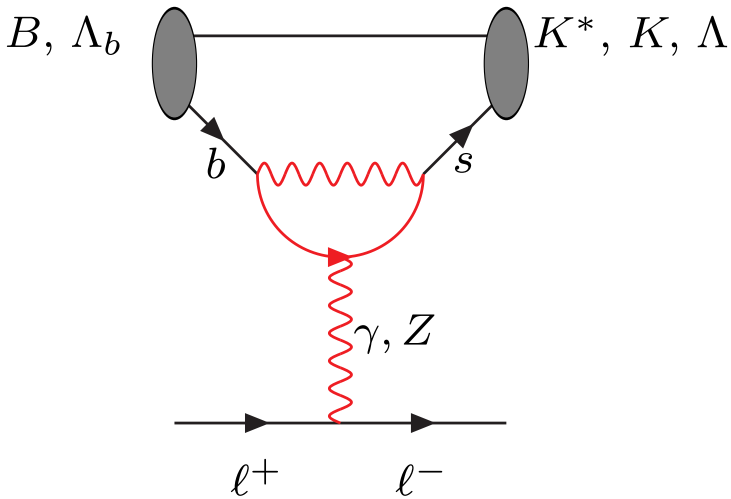


6 unitarity triangles: flavour physics constraints on sides and angles.



$b \to s$ transitions

Those processes are among the most important to test SM extensions. $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$ rare events offer a rich set of constraints on New Physics scenarios.



$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=7,9,10,S,P} (C_i O_i + C_i' O_i')$$

$$O_7^{(')} = \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu} P_{L(R)} b\, F^{\mu\nu}$$
$$O_9^{(')} = \frac{\alpha}{4\pi} \bar{s}\gamma_{\mu\ L(R)} b\, \bar{l}\gamma^\mu l$$
$$O_{10}^{(')} = \frac{\alpha}{4\pi} \bar{s}\gamma_{\mu\ L(R)} b\, \bar{l}\gamma^\mu \gamma^5 l$$
$$O_S^{(')} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l} l$$
$$O_P^{(')} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l}\gamma^5 l$$

– 3 form factors $T_{1,2,3}(q^2)$ associated to $\langle K^*(\epsilon_{(\lambda)}, k)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle$

– 2 form factors $f_{+,0}(q^2)$ associated to $\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle$

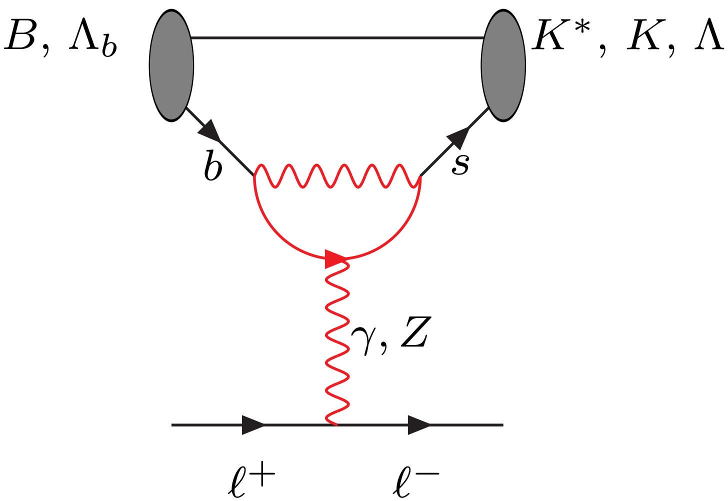
– 1 form factor $f_0(q^2)$ associated to $\langle K(k)|\bar{s}b|B(p)\rangle$

– 1 form factor $f_T(q^2)$ associated to $\langle K(k)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle$

– in HQET, 2 form factors $F_{1,2}(p'\cdot v)$ associated to $\langle\Lambda(p', s')|\bar{s}\Gamma h|\Lambda_h(v, 0, s)\rangle$

$b \to s$ transitions

Those processes are among the most important to test SM extensions. $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$ rare events offer a rich set of constraints on New Physics scenarios.



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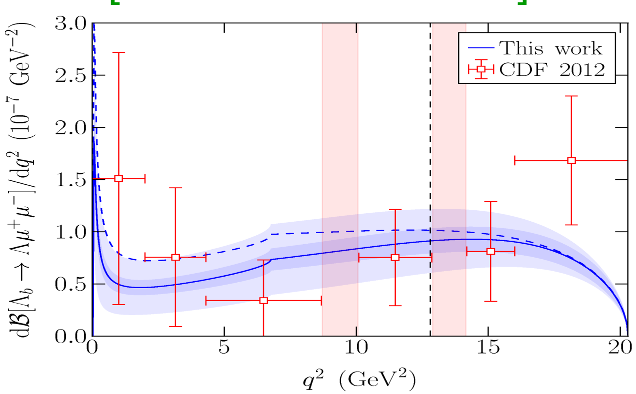
$$O_S^{(\prime)} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l}l$$
$$O_P^{(\prime)} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l}\gamma^5 l$$

$\Lambda_b \to \Lambda\ell^+\ell^-$: the matching of HQET to QCD is applied to compute the partial widths. A smooth interpolation is applied in q^2 except in regions of the phase space where long-distance effects are large (charmonium resonances)

[W. Detmold *et al* '12]



So far, no sign of NP seen in $\Lambda_b \to \Lambda\ell^+\ell^-$.
LHCb data are analysed to confirm that statement.

Outlook

- Lattice community does make an important effort to compute from first principles of quantum field theory hadronic quantities with a competitive accuracy with respect to experimental measurements.
- We provide theoretical inputs to improve the understanding of the dynamics governing nucleon physics: form factors, moments of parton distribution functions.
- Exploratory studies are led to measure directly the PDF's, including TMD's, and the interaction potential between nucleons. Isospin breaking effects are also taken into account.
- With the excellent luminosity at LHCb, there is some hope that baryon physics can constrain NP scenarios in the flavour sector, especially from the rare $b \to s$ transition $\Lambda_b \to \Lambda l^+ l^-$.