Cosmic-Ray Diffusion in Magnetized Turbulence

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- Diego Dominici (State University of New York at New Paltz)
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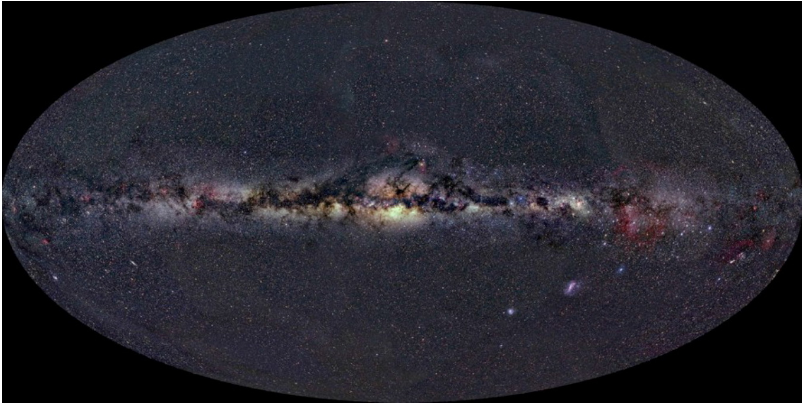
I.
Introduction

The background

Propagation of cosmic rays

- Photons $\longrightarrow$ (almost) direct path to observer
- Protons $\longrightarrow$ permament scattering and deflections



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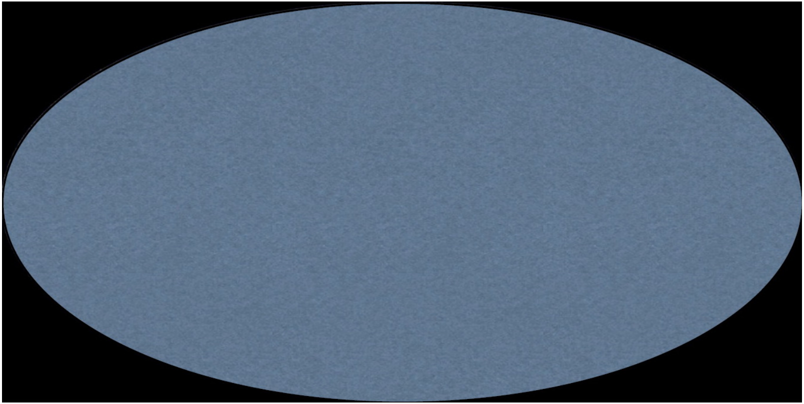

The background

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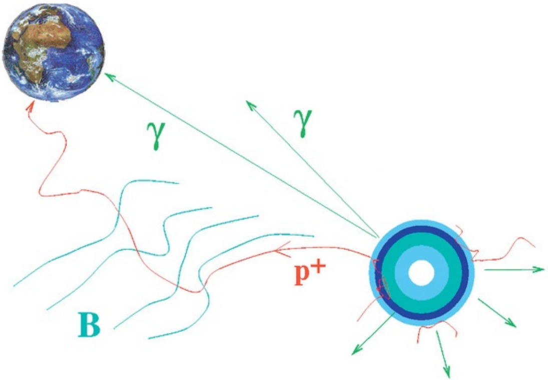
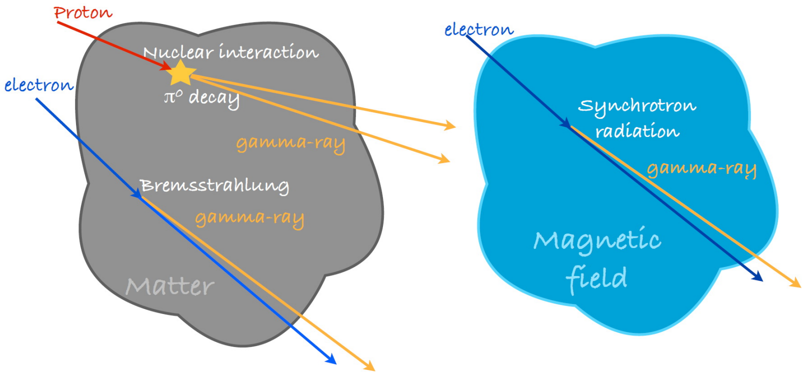


The interactions

The main processes

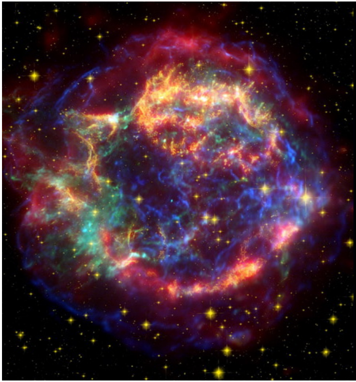
- . . . with matter ⟶ gamma radiation
- . . . with magnetic fields ⟶ gamma radiation

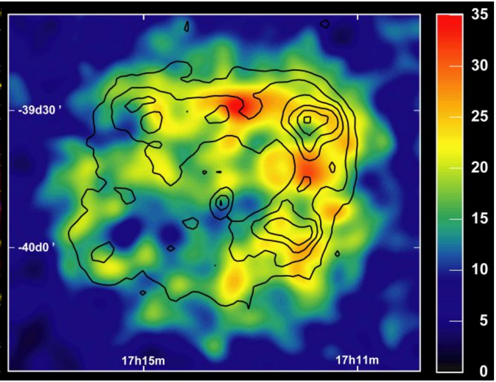


The interactions

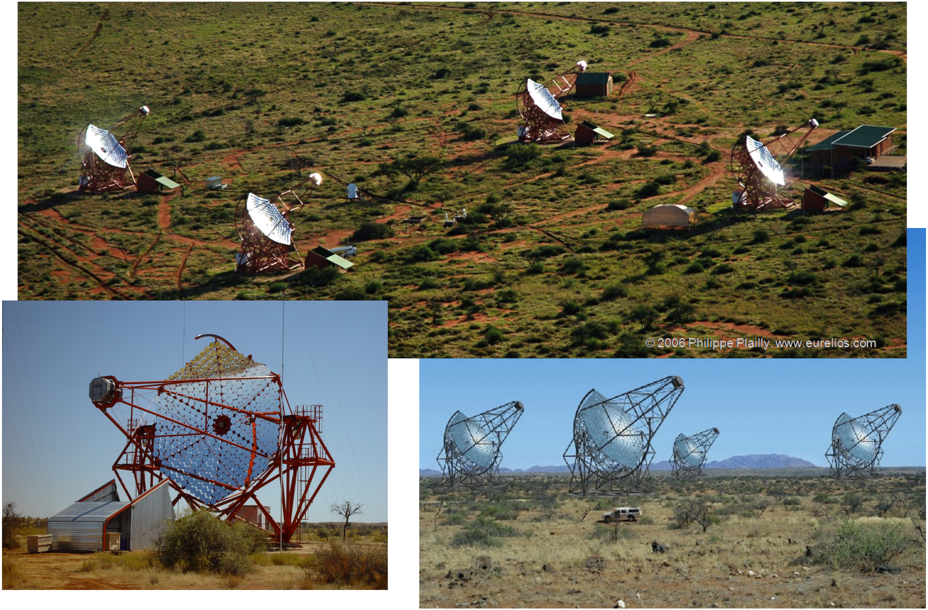
The main principles

- . . . with matter $\longrightarrow$ gamma radiation
- . . . with magnetic fields $\longrightarrow$ gamma radiation





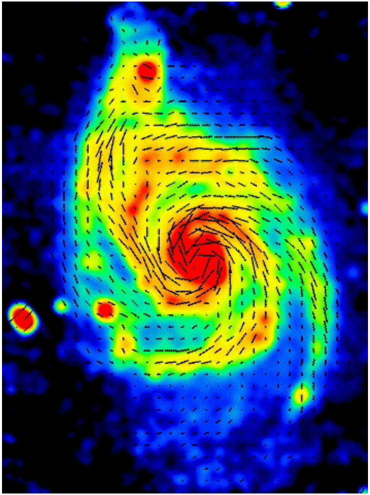
H.E.S.S. telescope



The problem

Magnetic fields in space

- Magnetic fields are omnipresent[1]
  - Galactic magnetic fields
  - Interplanetary magnetic fields
- Field strengths typically $\mu G - nT$
- In most cases: two components
  1. Regular, large-scale



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The problem

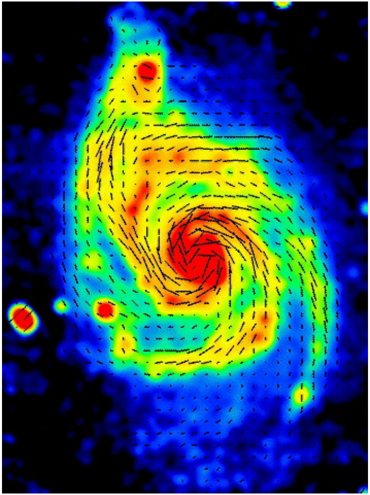
Magnetic fields in space

- Magnetic fields are omnipresent[1]
  - Galactic magnetic fields
  - Interplanetary magnetic fields
- Field strengths typically μG – nT
- In most cases: two components
  1. Regular, large-scale
  2. Turbulent, small-scale

  ☞ comparable field strengths!
- Usual assumption

$$\boldsymbol{B} = {\color{green}B_0\,\hat{\boldsymbol{e}}_z} + {\color{red}\delta\boldsymbol{B}(\boldsymbol{r},t)}$$

homogeneous turbulent



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II.
Diffusion-Convection Problems

The ansatz

Can we do a diffusion-convection description?

- Distribution function: solve a *transport equation*[1]

$$\frac{\partial f}{\partial t} - S = \nabla \cdot \left(\kappa_{nj} \cdot \nabla f - \boldsymbol{v}\, f \right) + \frac{\partial}{\partial \boldsymbol{p}} \left(p^2 D_p \frac{\partial}{\partial \boldsymbol{p}} \frac{f}{p^2} - \dot{\boldsymbol{p}}\, f \right) + \ldots$$


E. N. Parker


A. Fick

[1] Parker, *Planet. Space Sci.* 13, 9 (1965)
[2] RCT, Shalchi, & Schlickeiser, *Astrophys. J.* 685, L165 (2008)
[3] Shalchi, RCT, & Rempel, *Plasma Phys. Contr. Fusion* 53, 105016 (2011)
[4] RCT & Shalchi, *Astrophys. J.* 744, 125 (2012)

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- Diffusion tensor: approximation required

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa_\perp & \kappa_\mathrm{A} & 0 \\ -\kappa_\mathrm{A} & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

- Three main effects
  1. $\kappa_\parallel$: Diffusion *along*[2] $\boldsymbol{B}$
  2. $\kappa_\perp$: Diffusion *across*[3] $\boldsymbol{B}$
  3. κ_A: Drift effects[4]

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The analytical calculation

Important parameter: parallel mean-free path

- Averaging[1,2] over all pitch-angles $\mu = \cos\angle(\boldsymbol{v}, \boldsymbol{B}_0)$

$$\lambda_{||} \propto \kappa_{||} \propto \int_{-1}^{1} \mathrm{d}\mu \, \frac{\left(1-\mu^2\right)^2}{D_{\mu\mu}(\mu)}$$

[1] Hasselmann & Wibberenz, *Z. Geophys.* 34, 353 (1968)
[2] Earl, *Astrophys. J.* 193, 231 (1974)

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- Taylor-Green-Kubo formula for the Fokker-Planck coefficient

$$D_{\mu\mu} = \int_0^\infty \mathrm{d}t \; \langle \dot{\mu}(t)\, \dot{\mu}^\star(0) \rangle$$

- From the equation of motion (Newton-Lorentz eq.)

$$\dot{\mu} = \frac{\partial}{\partial t}\left(\frac{v_\parallel}{v}\right) \overset{\text{static}}{=} \frac{\dot{v}_\parallel}{v} = \frac{\Omega}{v}\left(v_x \frac{\delta B_y}{B_0} - v_y \frac{\delta B_x}{B_0}\right)$$

  1. unknown velocity components $v_{x,y}$
  2. unknown position in $\delta B_{x,y}(\boldsymbol{r}, t)$

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The microphysics

Resonant wave-particle interactions

- Quasi-linear theory[1]
  - $z(t) = v\mu t$
  - sharp resonance

[1] Jokipii, *Astrophys. J.* 146, 480 (1966)
[2] Owens, *Astrophys. J.* 191, 235 (1974)
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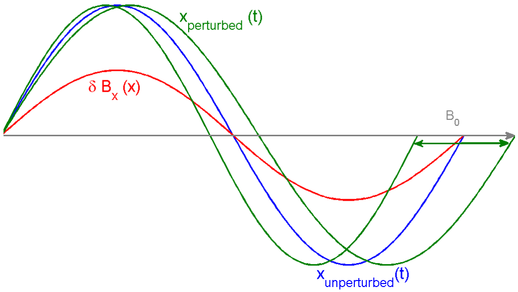
The microphysics

Resonant wave-particle interactions

- Quasi-linear theory[1]
  - $z(t) = v\mu t$
  - sharp resonance
- Reality
  - stochastic motion
  - resonance broadening[2,3]



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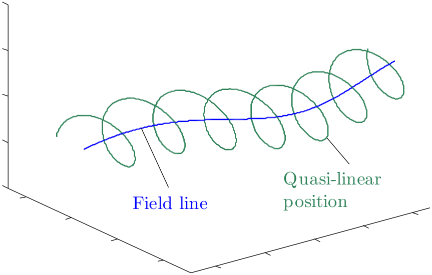
The standard cases I

Example: Second-order QLT[1,2]

- Parallel diffusion
  - stochastic particle orbits using QLT
  - describe resonance broadening



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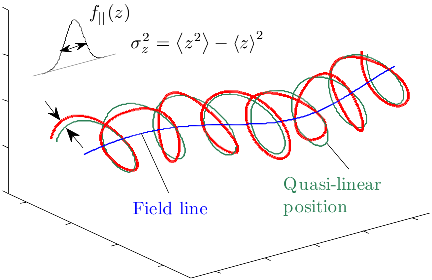

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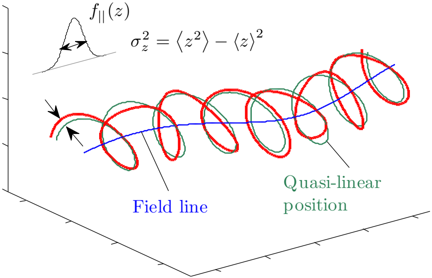
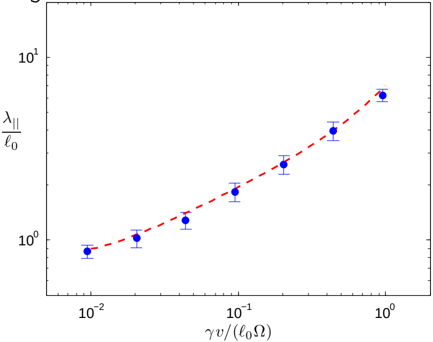
Example: Second-order QLT[1,2]

- Parallel diffusion
    - stochastic particle orbits using QLT
    - describe resonance broadening
- Agreement with simulations[1]





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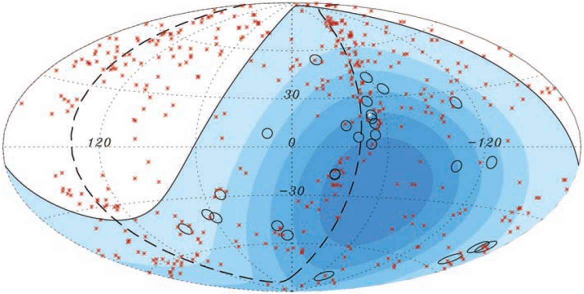
The consequence

Calculation: quasi-linear vs. non-linear

- Hillas: no confinement of high-energy particles[1] if $v_{||} > \Omega L_{\max}$
- Extragalactic origin if $E \gtrsim 10^{17}$ eV
  - ☞ does the deflection[2] allow for a correlation with AGNs?[3]



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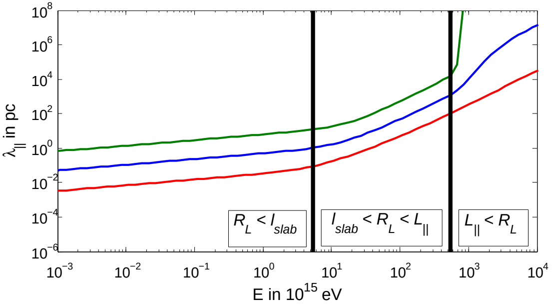

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The standard cases II

Non-linear guiding center theory

- Assume that particles follow field lines
  - ☞ write the field line equation as

$$\mathrm{d}x = \frac{\delta B_x}{B_0}\,\mathrm{d}z$$

[1] Matthaeus, Qin, Bieber, & Zank, *Astrophys. J.* 590, L53 (2003)
[2] le Roux et al., *Astrophys. J.* 716, 671 (2010)

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$$v_x = \frac{\delta B_x}{B_0} v_z$$

- Multiply by v_x at some other time
  - ☞ sixth-order correlation function

$$\kappa_\perp \propto \int \mathrm{d}^3 k \left\langle v_z(t)\, v_z(0) \quad \delta B_x(t)\, \delta B_x(0) \quad e^{i\boldsymbol{k}\cdot(\boldsymbol{x}(t)-\boldsymbol{x}(0))} \right\rangle$$

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- Conventional non-linear guiding center (NLGC) theory[1,2]
  1. Split into three second-order correlation functions
  2. Assume diffusive behavior
  3. Calculate diffusion coefficient

[1] Matthaeus, Qin, Bieber, & Zank, *Astrophys. J.* 590, L53 (2003)
[2] le Roux et al., *Astrophys. J.* 716, 671 (2010)

The (non-)diffusivity

Turbulent particle transport can be non-Markovian

- General behavior: $\left\langle (\Delta x)^2 \right\rangle \propto t^{\alpha+1}$ or $\kappa \propto t^{\alpha}$
  ☞ *"diffusion"* requires $\alpha = 0$!
- Three cases:

---

[1] RCT & Shalchi, *J. Geophys. Res.* 115, A03104 (2010)
[2] Jokipii, Kóta, & Giacalone, *Geophys. Res. Lett.* 20, 1759 (1993)
[3] Qin, Matthaeus, & Bieber, *J. Geophys. Res.* 29, 1048 (2002)

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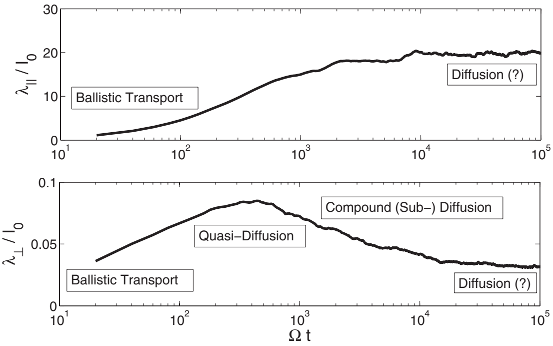
  ☞ *"diffusion"* requires $\alpha = 0$!

- Three cases:

1. Ballistic regime
   - free streaming
   - $\alpha = 1$
2. Parallel
   - diffusion[1]
   - $\alpha \approx 0$
3. Perpendicular:
   - subdiffusion[1–3]
   - $-\frac{1}{2} \leqslant \alpha < 0$



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The (non-)diffusivity

Example: unified non-linear theory[1]

- Calculate 4^{th}-order correlation as

$$\langle \ldots \rangle = \frac{1}{4} \int_{-1}^{1} \mathrm{d}\mu \int_{-1}^{1} \mathrm{d}\mu' \int \mathrm{d}^3 r \; \ldots \; f(\mu, \boldsymbol{r}, t)$$

  with a *Fokker-Planck* solution $f(\mu, \boldsymbol{r}, t)$

- Creative mathematical procedures required[2]

1 Shalchi, *Astrophys. J.* 720, L127 (2010)
2 Lerche & RCT, *Phys. Plasmas* 18, 082305 (2011)
3 Shalchi, RCT, & Rempel, *Plasma Phys. Contr. Fusion* 53, 105016 (2011)

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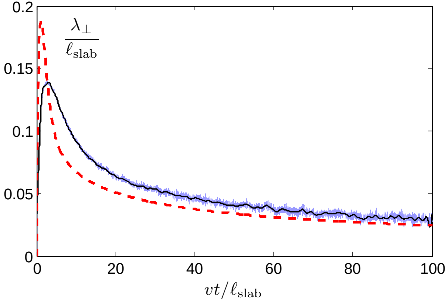
  with a *Fokker-Planck* solution $f(\mu, \boldsymbol{r}, t)$

- Creative mathematical procedures required[2]
- Time dependent diffusion
- Agreement with numerical test-particle simulations[3]



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The alternatives

Magnetic field-line random walk

- Compound diffusion[1,2]
- Bohm diffusion

Scaling relations

- Simple energy and field strength dependence[3]
- Estimates based on decorrelation mechanisms[4]

Other approaches

- Percolation theory[5]
- Markov processes[6] and Lévy walks[7]

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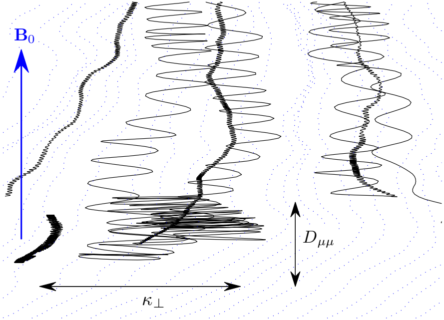
[1] Webb et al., *Astrophys. J.* 651, 211 (2006)
[2] RCT, Shalchi, & Schlickeiser, *Astrophys. J.* 672, 642 (2008)
[3] Reinecke, Moraal, & McDonald, *J. Geophys. Res.* 98, 9417 (1993)
[4] Hauff et al., *Astrophys. J.* 711, 997 (2010)
[5] Isichenko, *Rev. Mod. Phys.* 64, 961 (1992)
[6] Lemons, *Phys. Plasmas* 19, 012306 (2012)
[7] Zimbardo et al., *Astrophys. J.* 778, 35 (2013)

III.
Electromagnetic Turbulence

The approach

Turbulence model

- Fully developed turbulence
- Required:
  1. energy spectrum[1]
  2. geometry[2,3]

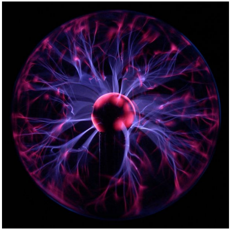


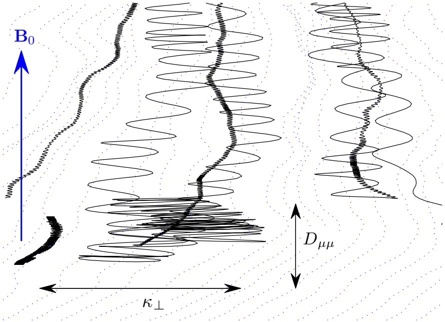
[1] Shalchi, Bieber, Matthaeus, & Schlickeiser, *Astrophys. J.* 642, 230 (2006)
[2] RCT & Lerche, *J. Math. Phys.* 54, 053303 (2013)
[3] Alouani-Bibi & le Roux, *Astrophys. J.* 781, 93 (2014)

The approach

Turbulence model

- Fully developed turbulence
- Required:
  1. energy spectrum
  2. geometry
  3. dynamical behavior





- Dynamical behavior[1]:
  1. instabilities[2]
  2. (damped) waves
  3. intermittency[3]

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[2] RCT & Lerche, *J. Math. Phys.* 54, 053303 (2013)
[3] Alouani-Bibi & le Roux, *Astrophys. J.* 781, 93 (2014)

The spectrum

Turbulence model

- Correlation function

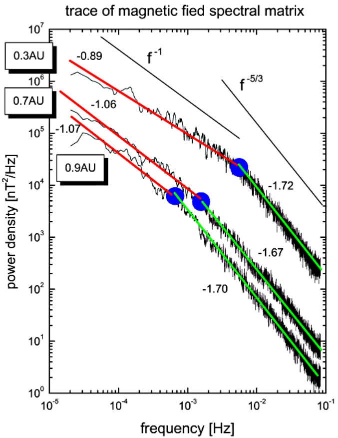
$$\left\langle \delta B_j(\boldsymbol{r}, t)\, \delta B_n(\boldsymbol{r}', t') \right\rangle$$

- Fourier transformation

$$\left\langle \delta B_j(\boldsymbol{k})\, \delta B_n(\boldsymbol{k}') \right\rangle$$

☞ plus Corrsin hypothesis[3]

Measurements[1,2]



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[2] Wicks et al., *Astrophys. J.* 778, 177 (2013)
[3] RCT & Shalchi, *Phys. Plasmas* 17, 122313 (2010)

The spectrum

Turbulence model

- Correlation function

$$\langle \delta B_j(\mathbf{r}, t)\, \delta B_n(\mathbf{r}', t') \rangle$$

- Fourier transformation

$$\langle \delta B_j(\mathbf{k})\, \delta B_n(\mathbf{k}') \rangle$$

☞ plus Corrsin hypothesis[3]

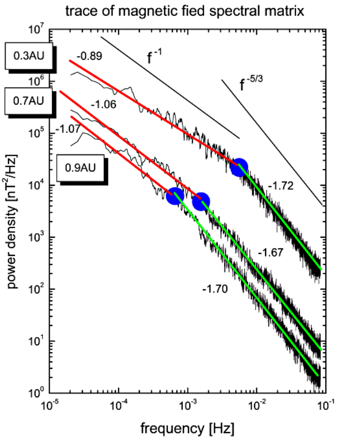
- Expressible as

$$\frac{G(\mathbf{k})}{8\pi k^2} \left(\delta_{jn} - \frac{k_j k_n}{k^2} \right)$$

- Measurements provide $G(|\mathbf{k}|)$

☞ Solar wind: Kolmogorov[1]

Measurements[1,2]



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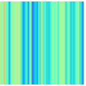
The geometry

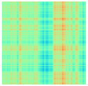
Basic analytical models

- Need to know the geometry $G(k_{\|}, k_{\perp})$
  ☞ particle transport requires integration over $\boldsymbol{k}$
- Basic geometries

Slab: depends only on $k_{\|}$
☞ $\delta \boldsymbol{B}(z)$ varies along $\boldsymbol{B}_0$



2D: depends only on $k_{\perp}$
☞ $\delta \boldsymbol{B}(x, y)$ varies $\perp$ to $\boldsymbol{B}_0$



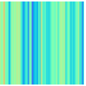
Composite: superposition of slab and 2D
☞ "quasi-3D" turbulence

The geometry

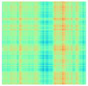
Basic analytical models

- Need to know the geometry $G(k_{\|}, k_{\perp})$
  - ☞ particle transport requires integration over $\boldsymbol{k}$
- Basic geometries

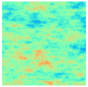
Slab: depends only on $k_{\|}$
☞ $\delta\boldsymbol{B}(z)$ varies along $\boldsymbol{B}_0$



2D: depends only on $k_{\perp}$
☞ $\delta\boldsymbol{B}(x, y)$ varies $\perp$ to $\boldsymbol{B}_0$



Composite: superposition of slab and 2D
☞ "quasi-3D" turbulence



Isotropic: no preferred direction
☞ $\delta\boldsymbol{B}$ independent of θ and ϕ

Others: e. g., Goldreich-Sridhar
☞ perpendicular-parallel energy exchange

The simulation

How to obtain transport coefficients numerically?

- Trace trajectories of test particles[1–4]
  ☞ use relations $(\Delta z)^2 \propto \kappa \propto \lambda$
- Time-dependent ("running") diffusion coefficients

$$\text{e. g.,}\quad \kappa_{\|}(t) = \frac{1}{2}\,\frac{\mathrm{d}}{\mathrm{d}t}\left\langle(\Delta z(t))^2\right\rangle \approx \frac{1}{2t}\left\langle(\Delta z(t))^2\right\rangle$$

[1] Giacalone & Jokipii, *Astrophys. J.* 520, 204 (1999)
[2] Zimbardo, Veltri, & Pommois, *Phys. Rev. E* 61, 1940 (2000)
[3] RCT, *Comput. Phys. Commun.* 181, 71 (2010)
[4] Laitinen, Dalla, & Marsh, *Astrophys. J. Lett.* 773, L29 (2013)

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- Generate artificial turbulence
  - ☞ superposition of plane waves[1,3]

$$\delta \boldsymbol{B}(\boldsymbol{r}) \propto \Re\mathfrak{e} \sum_{n=1}^{N} \hat{\boldsymbol{e}}' \sqrt{G(k_n)}\, \cos\left(k_n z' - \omega(k_n)t + \beta_n\right)$$

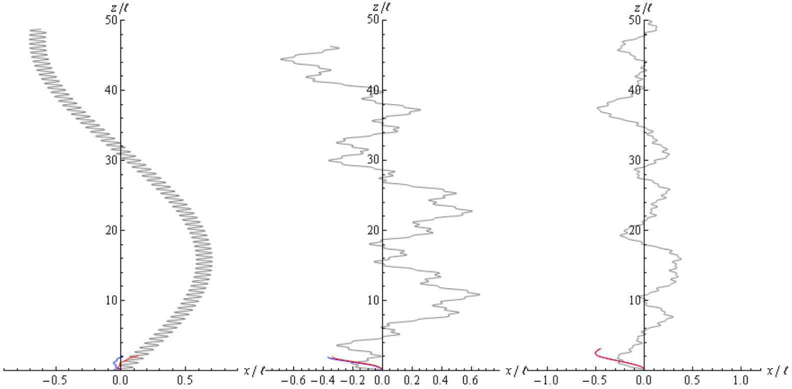
- Turbulence power spectrum $G(k) \propto k^{-5/3}$

1 Giacalone & Jokipii, *Astrophys. J.* 520, 204 (1999)
2 Zimbardo, Veltri, & Pommois, *Phys. Rev. E* 61, 1940 (2000)
3 RCT, *Comput. Phys. Commun.* 181, 71 (2010)
4 Laitinen, Dalla, & Marsh, *Astrophys. J. Lett.* 773, L29 (2013)

The side note

How many wave modes do we need?

- Investigate diffusive behavior with a (quasi) Lyapunov technique[1]
- A minimum of 16 wave modes is required

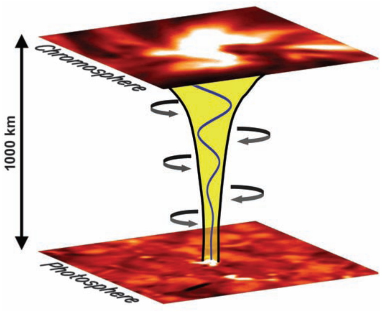


[1] RCT & Dosch, *Phys. Plasmas* 20, 022302 (2013)

The dynamics

Turbulent electric fields

- Include[1] (MHD) plasma waves:
  1. Alfvén waves
     $$\omega = \pm v_A k_\parallel$$
  2. Fast magnetosonic waves
  3. Whistler waves[2]
- Alfvén speed $v_A = B_0/\sqrt{4\pi\rho}$



[1] RCT, Shalchi, & Schlickeiser, *J. Phys. G* 32, 1045 (2006)
[2] Vocks et al., *Astrophys. J.* 627, 540 (2005)
[3] Petrosian, *Space Sci. Rev.* 173, 535 (2012)

The dynamics

Turbulent electric fields

- Include[1] (MHD) plasma waves:
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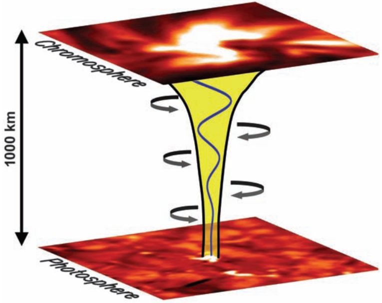
     $$\omega = \pm v_A k_\parallel$$
  2. Fast magnetosonic waves
  3. Whistler waves[2]
- Alfvén speed $v_A = B_0/\sqrt{4\pi\rho}$
- Faraday: turbulent electric fields
  $$\delta \boldsymbol{B} = \frac{c}{\omega(\boldsymbol{k})}\, \boldsymbol{k} \times \delta \boldsymbol{E}$$
- Diffusion in momentum space: Fermi-like acceleration

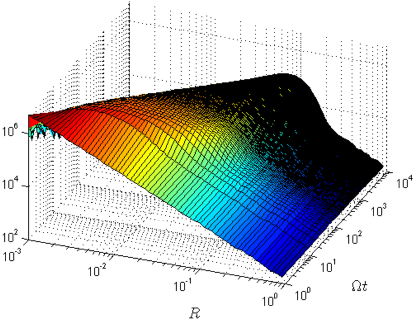


[1] RCT, Shalchi, & Schlickeiser, *J. Phys. G* 32, 1045 (2006)
[2] Vocks et al., *Astrophys. J.* 627, 540 (2005)
[3] Petrosian, *Space Sci. Rev.* 173, 535 (2012)

The dynamics

Stochastic acceleration

- Evolution of a velocity distribution function $f \propto p^{-a}$

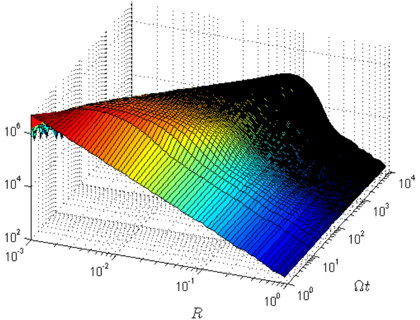


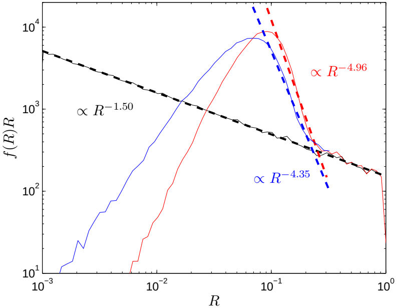
[1] RCT, *Plasma Phys. Contr. Fusion* 52, 045016 (2010)
[2] RCT, Lerche, & Kruse, *Astron. Astrophys.* 555, A101 (2013)

The dynamics

Stochastic acceleration

- Evolution of a velocity distribution function $f \propto p^{-a}$
- Momentum diffusion[1] mostly near $v = v_A$
  - ☞ modified spectral index[2]



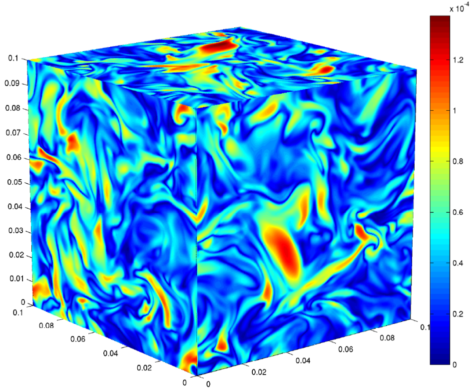


[1] RCT, *Plasma Phys. Contr. Fusion* 52, 045016 (2010)
[2] RCT, Lerche, & Kruse, *Astron. Astrophys.* 555, A101 (2013)

The fully numerical approach

Diffusion in MHD turbulence

- 2-step procedure:[1–3]
  1. evolution of MHD turbulence[4]
  2. trace test-particle trajectories



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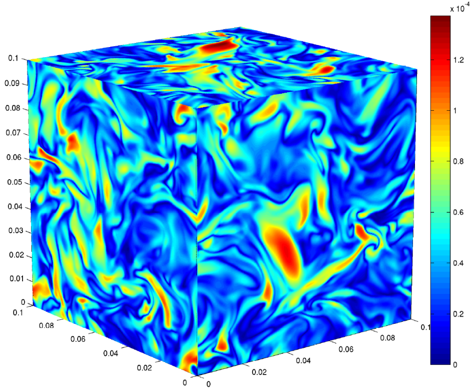
[1] Beresnyak, Yan, & Lazarian, *Astrophys. J.* 728, 60 (2011)
[2] Lange & Spanier, *Astron. Astrophys.* 546, A51 (2012)
[3] Nakwacki & Peralta-Ramos, ArXiv:1312.7822 (2014)
[4] Müller, in *Interdisciplinary Aspects of Turbulence*, Berlin:Springer, p. 223 (2009)
[5] RCT & Triptow, *Astrophys. Space Sci.* 348, 133 (2013)

The fully numerical approach

Diffusion in MHD turbulence

- 2-step procedure:[1–3]
  1. evolution of MHD turbulence[4]
  2. trace test-particle trajectories
- Advantages:
  - plasma instabilities[3,5]
  - dynamical turbulence
- But: limited resolution

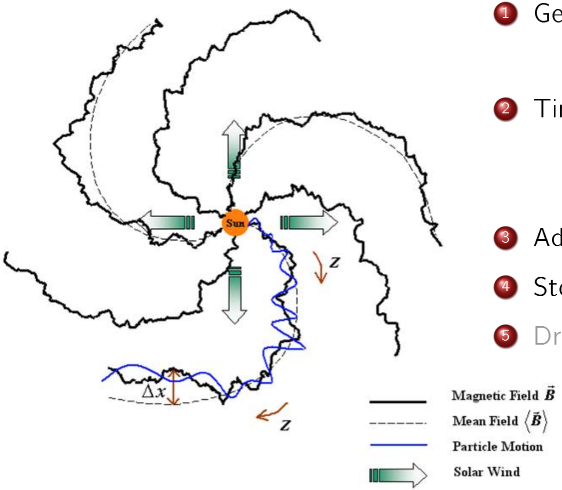


[1] Beresnyak, Yan, & Lazarian, *Astrophys. J.* 728, 60 (2011)
[2] Lange & Spanier, *Astron. Astrophys.* 546, A51 (2012)
[3] Nakwacki & Peralta-Ramos, ArXiv:1312.7822 (2014)
[4] Müller, in *Interdisciplinary Aspects of Turbulence*, Berlin:Springer, p. 223 (2009)
[5] RCT & Triptow, *Astrophys. Space Sci.* 348, 133 (2013)

IV.
Solar Wind

The reality

Additional effects in the Solar wind

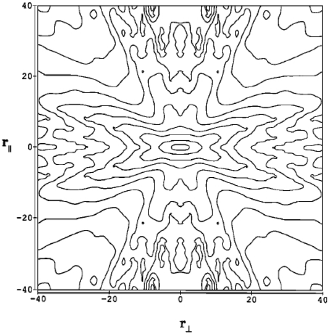


1. Geometry?
   - parallel/perpendicular
2. Time dependence?
   - Solar wind
   - intermittence
3. Adiabatic focusing?
4. Stochastic acceleration?
5. Drift motions?

The geometry

"Maltese cross" model

- Solar wind measurements[1] "justify" the slab/2D composite model[2]

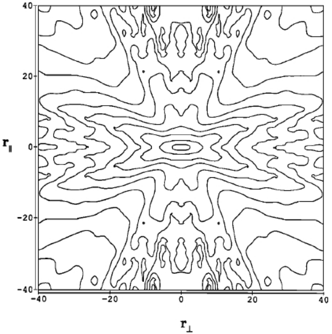


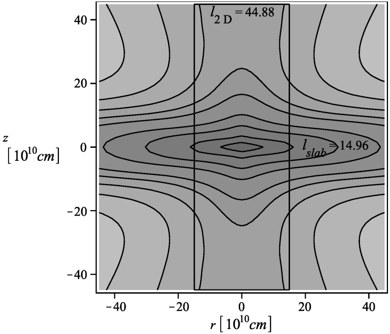
[1] Matthaeus, *J. Geophys. Res.* 95, 673 (1990)
[2] Bieber et al., *J. Geophys. Res.* 101, 2511 (1996)
[3] Weinhorst & Shalchi, *MNRAS* 403, 287 (2010)
[4] Rausch & RCT, *MNRAS* 428, 2333 (2013)

The geometry

"Maltese cross" model

- Solar wind measurements[1] "justify" the slab/2D composite model[2]
- Fit model allows for 1D→3D interpolation[3,4]





[1] Matthaeus, *J. Geophys. Res.* 95, 673 (1990)
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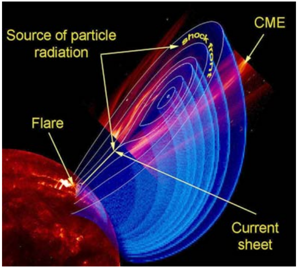
"Space weather"

Anisotropy-time profiles[1,2]

- Model measured profiles
  - time resolved
  - pitch-angle resolved

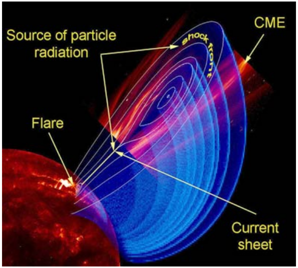


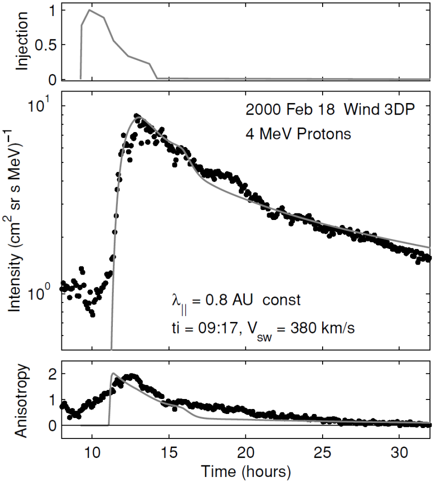
[1] Dröge & Kartavykh, *Astrophys. J.* 693, 69 (2009)
[2] Saíz et al., *Astrophys. J.* 672, 650 (2008)

"Space weather"

Anisotropy-time profiles[1,2]

- Model measured profiles
  - time resolved
  - pitch-angle resolved
- Fit to a diffusion solution





[1] Dröge & Kartavykh, *Astrophys. J.* 693, 69 (2009)
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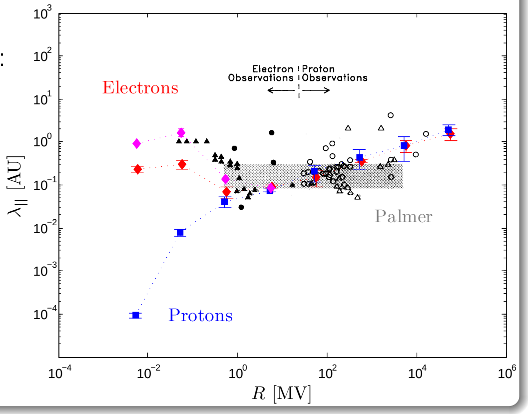
The comparison

Dissipation and the particle mass

- Low-energetic Solar cosmic rays
  - ☞ 10 keV to 100 GeV
- Palmer consensus range[2]:
  - ☞ agreement[2]

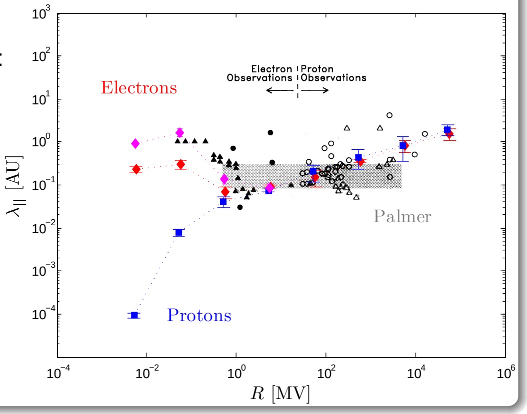


[1] RCT & Shalchi, *J. Geophys. Res.* 118, 642 (2013)
[2] Bieber, Matthaeus, et al., *Astrophys. J.* 420, 294 (1994)

The comparison

Dissipation and the particle mass

- Low-energetic Solar cosmic rays
  - ☞ 10 keV to 100 GeV
- Palmer consensus range[2]:
  - ☞ agreement[2]
- Turbulence model[1]
  - dissipation: electrons vs. protons
  - composite: Alfvén waves + 2D component



[1] RCT & Shalchi, *J. Geophys. Res.* 118, 642 (2013)
[2] Bieber, Matthaeus, et al., *Astrophys. J.* 420, 294 (1994)

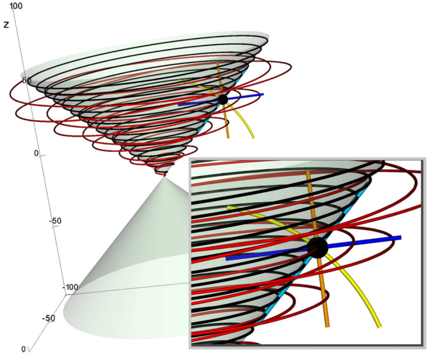
The large scales

Curved mean field

- Global transformation[1]
  - ☞ field-aligned diffusion tensor

$$\kappa_{\mathrm{global}} = A \cdot \kappa \cdot A^{T}$$

- Useful for SDE methods



[1] Effenberger et al., *Astrophys. J.* 750, 108 (2012)
[2] Parker, *Astrophys. J.* 128, 664 (1958)
[3] He & Wan, *Astrophys. J.* 747, 38 (2012)
[4] RCT et al., *J. Geophys. Res.* 116, A02102 (2011)

The large scales

Curved mean field

- Global transformation[1]
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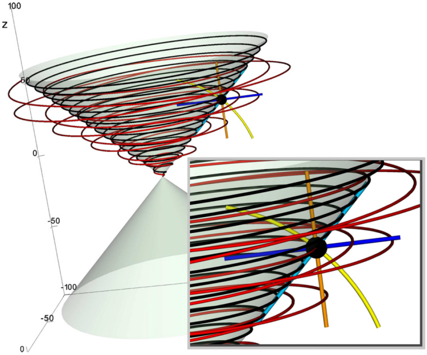
$$\boldsymbol{\kappa}_{\text{global}} = \mathsf{A} \cdot \boldsymbol{\kappa} \cdot \mathsf{A}^{\mathsf{T}}$$

- Useful for SDE methods
- Alternative: *focusing length L*

$$L^{-1} = \nabla \cdot \frac{\boldsymbol{B}}{B} \approx \frac{1}{B} \frac{\partial B}{\partial z}$$

- Applications:
  1. magnetic bottles
  2. Parker spiral[2–4]



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[3] He & Wan, *Astrophys. J.* 747, 38 (2012)
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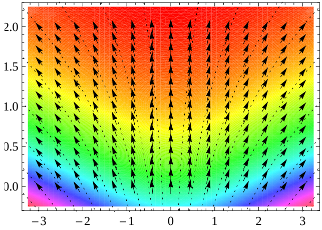
The large scales

Adiabatic focusing

- Test analytical results[1]
  - ☞ assume $L = \text{const}$ so that[2]

$$B_{\{x,y\}} \approx B_0 \, \frac{\{x,y\}}{2L} \, e^{-z/L}$$

$$B_z \approx B_0 \, e^{-z/L}$$



[1] Shalchi, *Astrophys. J.* 728, 113 (2011)
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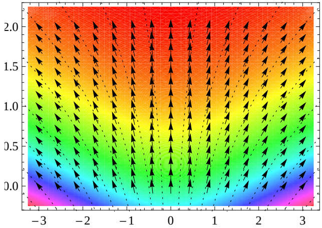
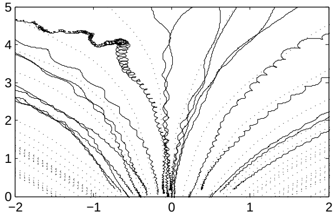
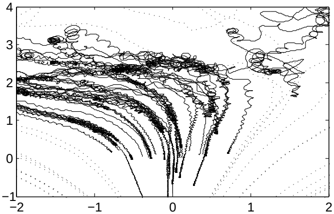


- Turbulence strength: *relative*. . . or *absolute*?



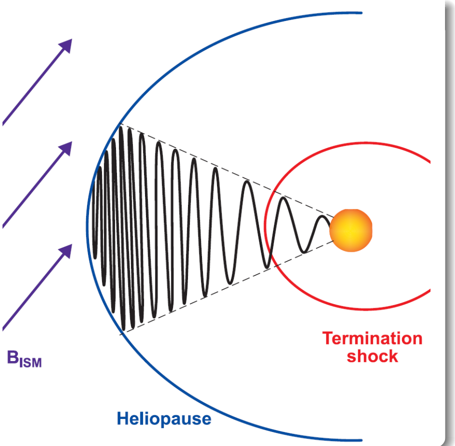


[1] Shalchi, *Astrophys. J.* 728, 113 (2011)
[2] RCT, Dosch, & Lerche, *Astron. Astrophys.* 545, A149 (2012)

The outer heliosphere

"Piled up" Parker spiral

- Sectored magnetic field[1,2]
  - ☞ outer heliosphere



[1] Florinski et al., *Astrophys. J.* 754, 31 (2012)
[2] Laitinen, Dalla, & Kelly, *Astrophys. J.* 749, 103 (2012)
[3] Lazarian & Opher, *Astrophys. J.* 703, 8 (2009)
[4] Bian & Kontar, *Phys. Rev. Lett.* 110, 151101 (2013)

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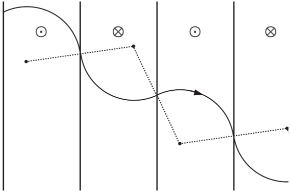
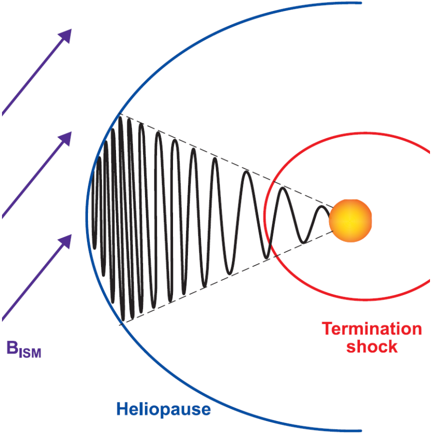


- Quasi-diffusive drift motion
- Magnetic reconnection[3]
  - ☞ particle acceleration?[4]



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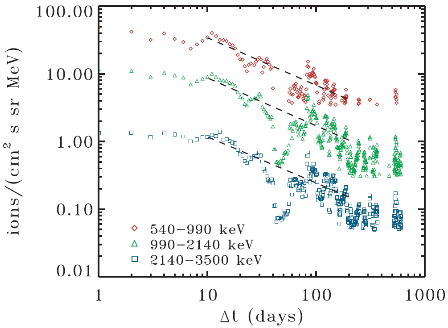
3 Lazarian & Opher, *Astrophys. J.* 703, 8 (2009)
4 Bian & Kontar, *Phys. Rev. Lett.* 110, 151101 (2013)

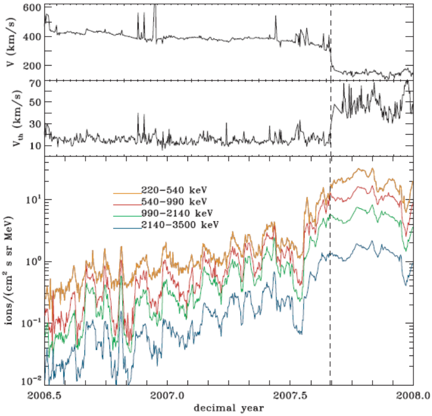
The outer heliosphere

Comparison with observations

- Ions at the termination shock[1]
- Super-diffusive behavior

☞ $\langle (\Delta x)^2 \rangle \propto t^{1.3}$



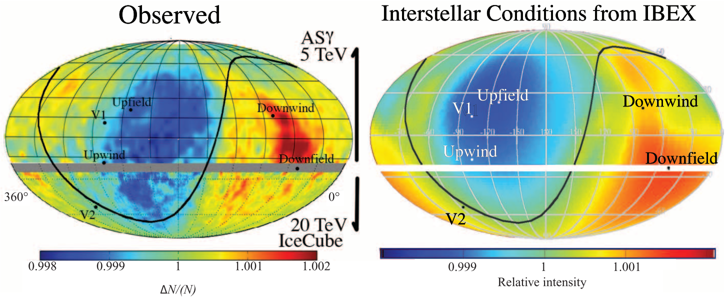


[1] Perri & Zimbardo, *Astrophys. J.* 693, L118 (2009)

The local interstellar medium

Cosmic-ray anisotropy[1]

- Cosmic rays as a diagnostic tool[1]
  ☞ requires a reliable transport model
- *"Local wiggle"* in the interstellar magnetic field[2,3]



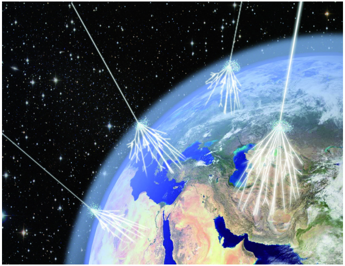
[1] Schwadron et al., *Science* 343, 988 (2014)
[2] Opher et al., *Science* 316, 875 (2007)
[3] Jokipii, *Science* 316, 839 (2007)

V.
Summary & Outlook

Summary & outlook

Transport theory: turbulence matters!

- "Standard turbulence"
  1. Parallel: SOQLT ✓
  2. Perpendicular: UNLT ✓
- Additional turbulence effects
  1. Plasma waves ✓
  2. Intermittency, . . . ?



Summary & outlook

Transport theory: turbulence matters!

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  1. Plasma waves ✓
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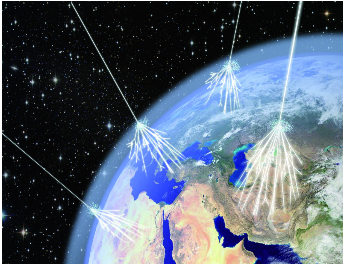
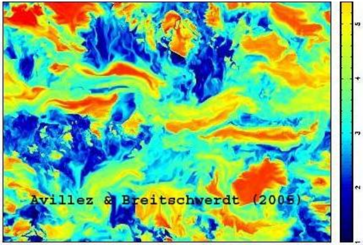


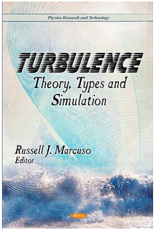


Future

- Curvature, special geometries
- Anisotropy time profiles
- Coupling with ISM simulations
- GPU accelerated simulations

The conclusion

Review papers



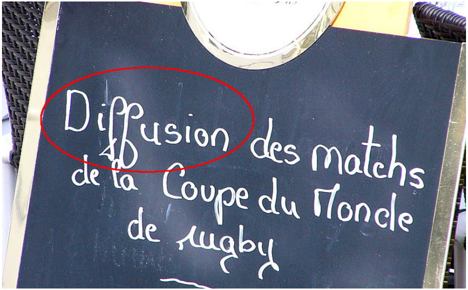
"On Cosmic Rays and Astrophysical Turbulence"

in *Turbulence: Theory, Types and Simulation*
ed. Russell J. Marcuso, New York: Nova Publishers (2012)
pp. 365-406

"Cosmic wave-particle interactions:
astrophysical magnetic turbulence and high-energy particles"

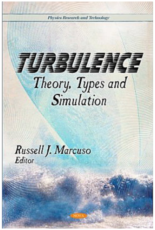
Astronomische Nachrichten <u>335</u>,
pp. 501–506 (2014)

Rugby
Diffusion



The conclusion

Review papers



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pp. 365-406

"Cosmic wave-particle interactions:
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Astronomische Nachrichten 335,
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. . . and *"alternative"* diffusion

Rugby
Diffusion