

# Particle Acceleration in Imbalanced Resistive Turbulence



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# Cross-Helicity in the Solar Wind

- Solar-wind turbulence characteristics are consistent with Alfvénic turbulence
- Positive cross-helicity ( $\mathcal{H}^c = \langle \mathbf{u} \circ \mathbf{b} \rangle$ ) signifies a dominance of outward-propagating Alfvén waves
- However, the magnitude of cross-helicity is not constant throughout the solar wind:

$\mathcal{H}^c$

# Cross-Helicity in the Solar Wind

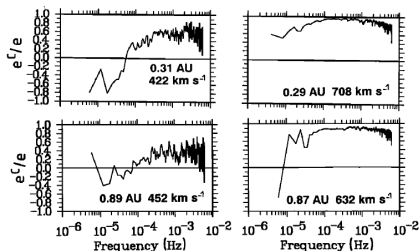


Fig. 4. Normalized cross helicity as a function of heliocentric distance and solar wind flow speed as indicated.

- However, the magnitude of cross-helicity is not constant throughout the solar wind:

$\mathcal{H}^c$

- ▶ decreases at greater radial distances from the sun
- ▶ is greater in the fast wind than in the slow wind
- ▶ decreases in regions with a high energy cascade rate or high compressibility

# Energy Cascade & Cross-Helicity

- In the Elsasser formulation ( $\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}$ ), the incompressible MHD equations are:

$$\partial_t \mathbf{z}^\pm = -[(\mathbf{z}^\mp \mp \mathbf{B}_0) \circ \nabla] \mathbf{z}^\pm + \frac{\nu + \eta}{2} \nabla^2 \mathbf{z}^\pm + \frac{\nu - \eta}{2} \nabla^2 \mathbf{z}^\mp - \nabla p.$$

- Nonlinear interactions (and hence the energy cascade rate  $\varepsilon$ ) disappear if either of  $\mathbf{z}^\pm$  is zero ( $\hat{\sigma}^c = \mathcal{H}^c / \mathcal{E} = \mp 1$ )

# Motivation

- If cross-helicity affects the cascade rate, how does it influence the stochastic heating of charged particles?
  - Dung & Schlickeiser, A&A (1990): heating goes down;  
Chandran et al., ApJ (2010): heating barely affected;  
Beresnyak et al., ApJ (2011): spatial diffusion unaffected
- 
- First step: compare test-particle acceleration in incompressible 3D MHD simulations for
    - ▶ **Balanced** turbulence (zero cross-helicity)
    - ▶ **Strongly imbalanced** turbulence (high cross-helicity)

# MHD code Turbo

- Pseudospectral code for incompressible resistive MHD

$$\partial_t \mathbf{u} = -(\mathbf{u} \cdot \nabla) \mathbf{u} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla] \mathbf{b} + \nu \nabla^2 \mathbf{u} + \mathbf{f}^u - \nabla \check{p},$$

$$\partial_t \mathbf{b} = -(\mathbf{u} \cdot \nabla) \mathbf{b} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla] \mathbf{u} + \eta \nabla^2 \mathbf{b} + \mathbf{f}^b$$

# MHD code Turbo

- Pseudospectral code for incompressible resistive MHD

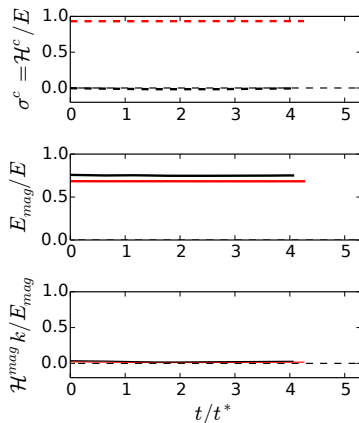
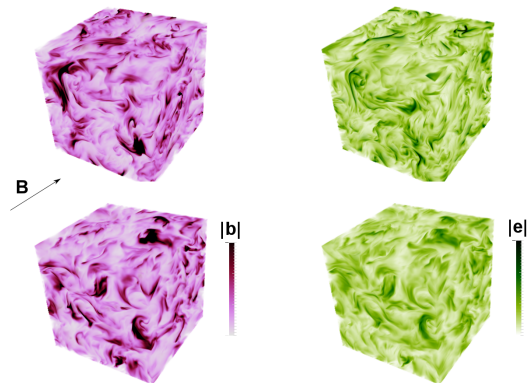
$$\partial_t \mathbf{u} = -(\mathbf{u} \cdot \nabla) \mathbf{u} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla] \mathbf{b} + \nu \nabla^2 \mathbf{u} + \mathbf{f}^u - \nabla \tilde{p},$$

$$\partial_t \mathbf{b} = -(\mathbf{u} \cdot \nabla) \mathbf{b} + [(\mathbf{B}_0 + \mathbf{b}) \cdot \nabla] \mathbf{u} + \eta \nabla^2 \mathbf{b} + \mathbf{f}^b$$

- Tracking of test-particles subject to Lorentz force

$$\ddot{\mathbf{x}}_n = q_n \left( \underbrace{\eta \nabla \times \mathbf{b} - \mathbf{u} \times (\mathbf{B}_0 + \mathbf{b})}_{\mathbf{e}} + \dot{\mathbf{x}}_n \times (\mathbf{B}_0 + \mathbf{b}) \right)$$

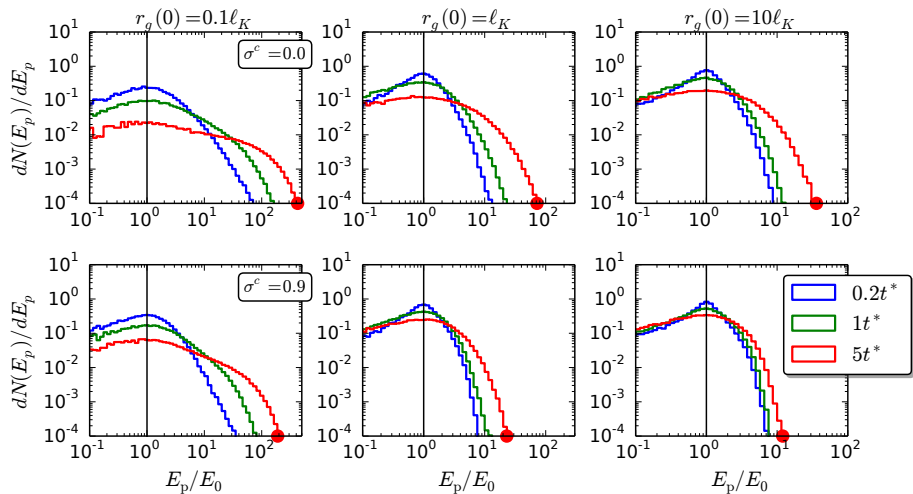
# Balanced vs. imbalanced



- **Balanced** (top) and **imbalanced** (bottom) MHD turbulence with otherwise identical parameters



# Heating in imbalanced turbulence



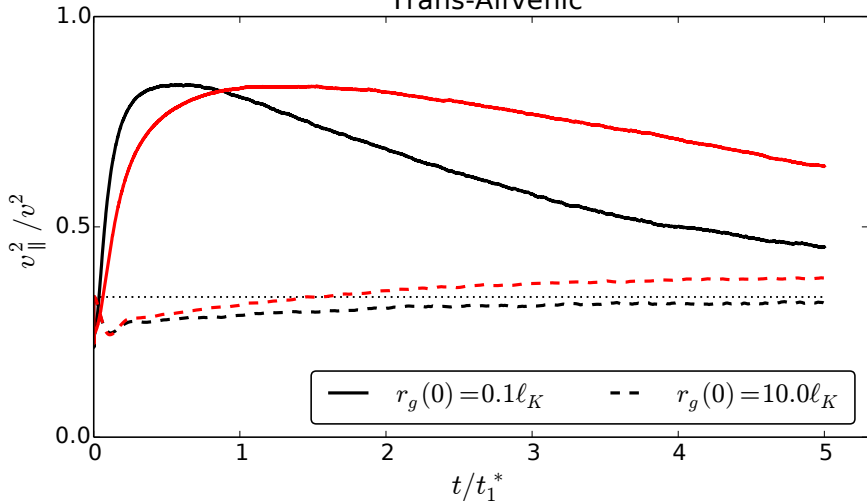
- Particle heating is reduced in imbalanced turbulence

# Two-stage acceleration

- Particles with small gyroradius experience strong unidirectional acceleration in current sheets ( $\mathbf{e}_{\parallel} = \eta \mathbf{j}_{\parallel}$ )
- Pitch-angle scattering and energy gain cause the gyroradius to exceed the transverse extent of the current sheet
- Large-gyroradius particles pass through current sheets too quickly to be accelerated, leaving only  $\mathbf{e}_{\text{mot}} = -\mathbf{u} \times \mathbf{b}$  as acceleration mechanism

# Pitch-angle evolution

Trans-Alfvenic



- Imbalance reduces perpendicular heating



# Quasi-linear momentum diffusion

## Quasi-linear theory

for imbalanced slab turbulence describes particle heating as diffusion in momentum space

$$\frac{\partial}{\partial t} f(p, t) = p^{-2} \frac{\partial}{\partial p} \left[ p^2 D_{pp} \frac{\partial}{\partial p} f(p, t) \right]$$

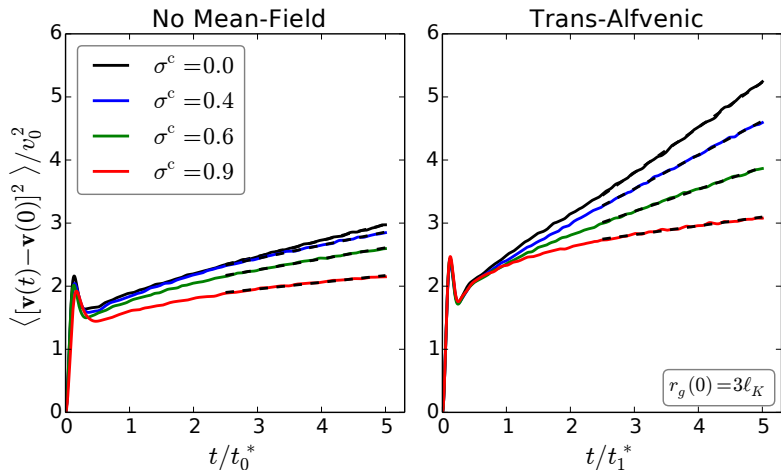
and predicts that momentum diffusion scales as :

$$D_{pp} \sim \frac{p^2}{\tau} \frac{v_A^2}{v^2} \times [1 - (\sigma^c)^2]$$

with the scattering timescale  $\tau \propto r_g^{2-s}$  for  $v \gtrsim v_A$

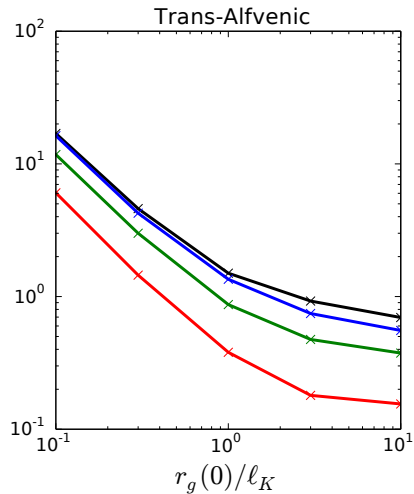
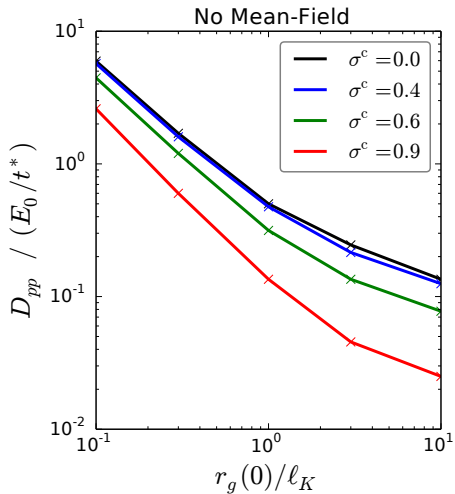


# Momentum diffusion

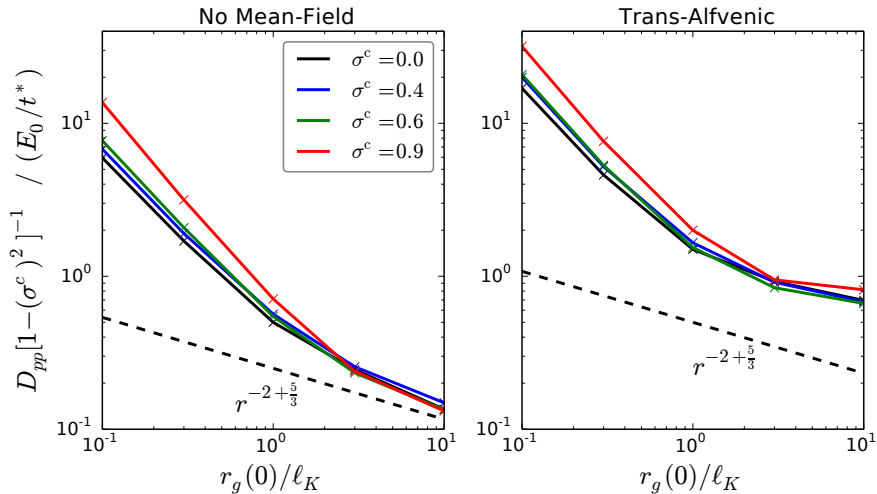


$$D_{pp} = \frac{1}{2} \frac{d}{dt} \langle |\mathbf{v}(t) - \mathbf{v}(0)|^2 \rangle$$

# Heating in imbalanced turbulence

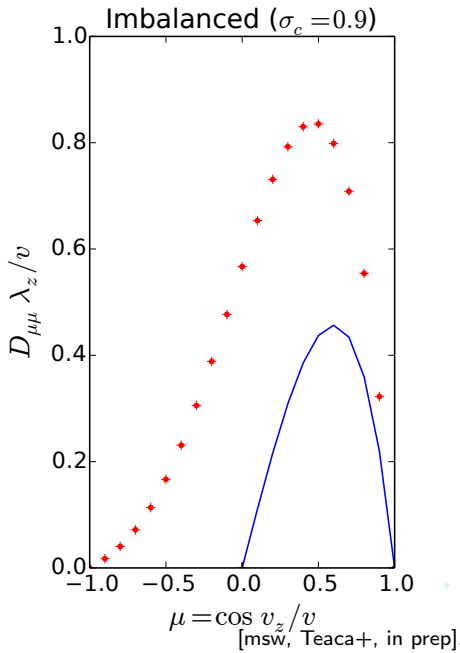
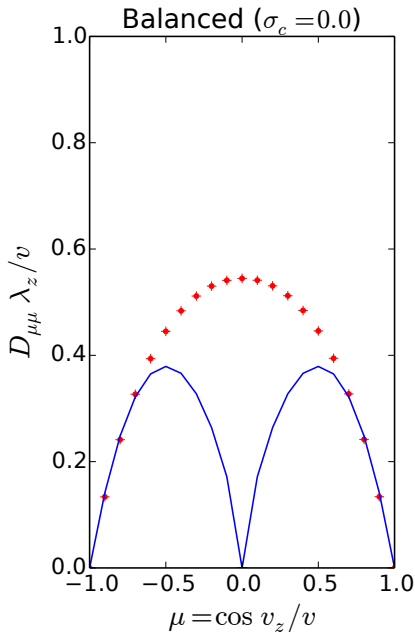


# Heating in imbalanced turbulence



$$D_{pp} \sim [1 - (\sigma^c)^2] r_g^{-2+s}$$

# Pitch-angle diffusion without Ohmic heating





# Summary

- We have compared test-particle acceleration in time-dependent MHD turbulence at various degrees of imbalance, with and without magnetic mean-field
- Strong imbalance (non-zero cross-helicity) inhibits the efficiency of ion heating in MHD turbulence
- At gyroradii in the inertial range, the observed scaling agrees with  $D_{pp} \sim [1 - (\sigma^c)^2] r_g^{-2+s}$
- For smaller gyroradii, momentum diffusion is increased by Ohmic heating (independent of cross-helicity) until the inertial range is reached

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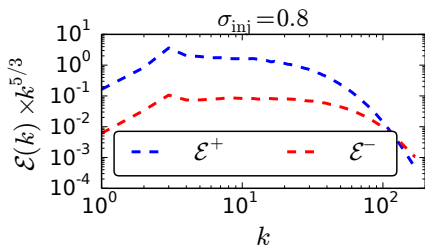
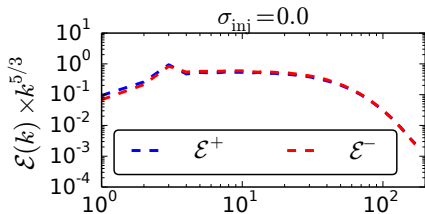
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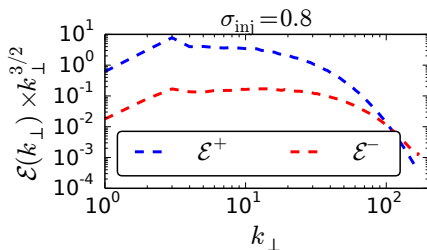
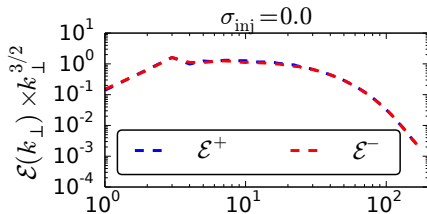
[\[arxiv:1108.2640\]](#)

# Comparison of Elsasser spectra

Isotropic ( $\mathbf{B}_0 = 0$ )



Trans-Alfvénic ( $\mathbf{B}_0 = \langle \mathbf{b}^2 \rangle^{1/2} \hat{\mathbf{z}}$ )



- Steady-state Elsasser energy spectra ( $\mathcal{E}^{\pm} = (\mathbf{u} \pm \mathbf{b})^2/4$ ) both with and without mean-field