A solution to the cosmic ray anisotropy problem

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CRISM 2014, Montpellier
27 June, 2014
Where do cosmic rays come from?
Why anisotropy? (I)

\[ \frac{\partial n_i}{\partial t} - \nabla \cdot \left( D_{xx} \cdot \nabla n_i - \vec{u} n_i \right) - \frac{\partial}{\partial p} p^2 D_{pp} \frac{1}{\partial p} p^2 n_i - \frac{\partial}{\partial p} \left( \frac{dp}{dt} n_i - \frac{p}{3} \left( \nabla \cdot \vec{u} \right) n_i \right) \]

\[ = q + \sum_{i<j} \left( c \beta n_{\text{gas}} \sigma_{j \rightarrow i} + \gamma \tau_{j \rightarrow i} \right) n_j - \left( c \beta n_{\text{gas}} \sigma_i + \gamma \tau_{i} \right) n_i , \]

\[ \nabla n_{\text{CR}} \]

asymmetric source distribution \( \rightarrow \nabla n_{\text{CR}} \).
Why anisotropy? (II)

\[ |\delta| = \frac{\phi_{\text{max}} - \phi_{\text{min}}}{\phi_{\text{max}} + \phi_{\text{min}}} = \frac{\phi(\vec{r} + \lambda) - \phi(\vec{r} - \lambda)}{\phi(\vec{r} + \lambda) + \phi(\vec{r} - \lambda)} \approx \frac{2\lambda|\nabla \phi(\vec{r})|}{2\phi(\vec{r})} = \frac{3D}{c} \frac{|\nabla \phi(\vec{r})|}{\phi(\vec{r})} \]

\[ \ddot{\delta} = \frac{3D}{c} \frac{\nabla n_{\text{CR}}}{n_{\text{CR}}} \]
Experimental situation

factor 30
Figure 2. Anisotropy amplitude for ten random realizations of sources in the cylindrical model, assuming $\delta = 1/3$ and a SN rate $R = 1/100$ yr $^{-1}$ (left) and $R = 1/30$ yr $^{-1}$ (right). The halo size is $H = 4$ kpc. The injections spectrum is assumed to have a slope (with cutoff) such that $\gamma + \delta = 2.67$. The data points are from $20 - 22$. Both panels of figure 2 show very clearly the strong dependence of the strength of anisotropy on the specific realization of source distribution, thereby also disproving the naive expectation that the anisotropy should grow only as a function of energy with the same slope as the diffusion coefficient $D(E)$. Whenever the small scale contribution is not negligible, the observed anisotropy can in fact be a non-monotonic function of energy, with dips and bumps, and with wide energy regions in which it is flat with energy, quite like what the data show at energies $E < 10^{15}$ GeV. It is interesting however that none of our realizations of the source distribution leads to anisotropies as low as the one suggested by the data in the energy region $10^{15} - 10^{16}$ GeV (contributed by the EASTOP experiment). Data in this region are in fact somewhat puzzling because the years of observation suggest that the Compton-Getting effect [25] leads to an average of anisotropy close to the lowest expected limit. The Compton-Getting anisotropy is estimated to be between $3 \times 10^{-4}$ and $10^{-3}$ depending on the velocity with which the Earth moves with respect to the CR scattering centers. This velocity is not known and the above estimates refer to a velocity range from a minimum of $\sim 20$ km/s to a maximum of $\sim 250$ km/s, corresponding to the motion of the solar system through the Galaxy [26]. It is clear that the measured anisotropy between $10^{15}$ and $10^{16}$ GeV is only marginally consistent with a velocity of few tens of km/s at most.

We also checked the effects of decreasing further the source rate, which could be the case if the bulk of CRs does not come from standard SNe but rather from rarer events, like for example an especially energetic sub-sample of SNe or GRBs. The resulting anisotropy is somewhat larger at low energies, on average: the data can still be easily reproduced at the low and high energies, but the central, more problematic region is now more extended, in general, to the left than in figure 2, approximately ranging from few $\times 10^{4}$ to $10^{6}$.

In figure 2 we adopted a diffusion coefficient scaling with $E^{1/3}$. The energy dependence of the diffusion coefficient is however the subject of an ongoing debate: given $D(E) \propto E^\delta$ it is controversial whether $\delta$ is 1/3, 1/2, 0.6 or even larger (see [27] and references therein). The all-particle spectrum alone, while giving some indications that $\delta = 1/3$ could be preferable (see Paper I), does not allow one to really clinch the question. This is because the all-particle spectrum only depends on the combination $\delta + \gamma$. In principle the $B/\text{Cr}$ ratio would allow a direct measurement of $\delta$, if it could be measured data sufficiently high –11– Blasi & Amato, JCAP 01 (2012) 011.
Measured vs. predicted diffusion coefficient

\[ D = D_0 (\mathcal{R}/GV)^\delta \]

from fitting B/C:

\[ \delta = 0.33 \Rightarrow D_0 \simeq 4.0 \times 10^{28} \text{ cm}^2 \text{s}^{-1} \quad \text{for } z_{\text{max}} = 4 \text{ kpc} \]

\[ \delta = 0.55 \Rightarrow D_0 \simeq 2.3 \times 10^{28} \text{ cm}^2 \text{s}^{-1} \]

from quasi-linear theory:

turbulence spectrum \( W(k) \propto k^{-q} \) where \( kW(k) \sim \delta B^2(k) \)

\[ D_{||} \sim r_g \left( \frac{r_g}{L} \right)^{q-1} \left( \frac{\delta B}{B_0} \right)^2 \]

falls short of measured values (for \( B_0 = 4 \mu \text{G} \) and \( L = 100 \text{ pc} \))

\[ D_{||,0} = 4.3 \times 10^{27} \text{ cm}^2 \text{s}^{-1} \left( \frac{\delta B}{B_0} \right)^{-2} \quad \text{for } 2 - q = \delta = 0.33 \]

\[ D_{||,0} = 1.6 \times 10^{26} \text{ cm}^2 \text{s}^{-1} \left( \frac{\delta B}{B_0} \right)^{-2} \quad \text{for } 2 - q = \delta = 0.5 \]
Conclusion I

- can be used to constrain spectral index of diffusion coefficient:

- poor agreement for $\delta = 1/3$

- for $\delta = 0.6$, it’s even worse

Blasi & Amato, JCAP 01 (2012) 011
Conclusion II

- maybe the predicted global gradient is too large
- also in disagreement with gamma-ray data

- vary diffusion coefficient with galacto-centric radius
  
  - \( D_{||} \propto \left( \frac{\delta B}{B_0} \right)^{-2} \) but \( D_{\perp} \propto \left( \frac{\delta B}{B_0} \right)^2 \)
  
  - turbulence level follows source density \( q(r) \)
  
  - in the inner Galaxy escape is dominated by perpendicular diffusion
  
  - simulated by \( D \propto q(r)^\tau \)

Evoli et al., PRL 108 (2012) 211102
Conclusion II

Evoli et al., PRL 108 (2012) 211102
Ensemble averaging

- distribution function $f(\vec{x}, \vec{p}, t)$ develops under influence of $\delta B(\vec{x})$ and $\delta E(\vec{x})$

- we predict only the ensemble average $\langle f(\vec{x}, \vec{p}, t) \rangle$ for ensemble averaged force term

- usually, this is determined from Gaussian random B-field, characterised by $W(k)$

- we live in one particular realisation of random magnetic field!

$\rightarrow$ deviations from ensemble average
Anisotropic diffusion

- decompose distribution function
  \[ f_0(\vec{x}, p, \mu, t) \equiv F(\vec{x}, p, t) + g(\vec{x}, p, \mu, t) \]

- dipole = first harmonic of anisotropic part
  \[ |\vec{\delta}| = \frac{\int_{-1}^{1} d\mu \mu g(\mu)}{\int_{-1}^{1} d\mu f_0} = \ldots = -\frac{1}{4v} \frac{\partial F/\partial z}{F} D_{||} \]

→ amplitude depends on gradient along background B-field
→ orientation not in direction of gradient but of background B-field

- can this help decrease the dipole amplitude?
Numerical approach
Numerical approach

1. set up large scale gradient at time $(t_0 - \Delta t)$: $f(\vec{x}, \vec{p}, t_0 - \Delta t) = \ldots$

2. back-track large number of particles $i \in N$ for time $\Delta t$:
   $$\{\vec{x}_i(t_0), \vec{p}_i(t_0)\} \rightarrow \{\vec{x}_i(t_0 - \Delta t), \vec{p}_i(t_0 - \Delta t)\}$$

3. Liouville’s theorem:
   $$df = 0 \quad \Rightarrow \quad f(\vec{x}_{\text{obs}}, \vec{p}_i(t_0)) = f(\vec{x}_i(t_0 - \Delta t), \vec{p}_i(t_0 - \Delta t))$$
check: diffusion coefficient

- average over large number of trajectories and B-field realisations
- isotropic diffusion coefficient in agreement with “theory”
check: diffusion coefficient

- average over large number of trajectories and B-field realisations
- anisotropic diffusion coefficients in agreement with quasi-linear theory
w/o background B-field
w/o background B-field

1 PeV
w/o background B-field
w/o background B-field
w/o background B-field
w/ background B-field @ 0°
w/ background B-field @ 60°
w/ background B-field @ 90°

1 PeV
w/ background B-field @ 0°
w/ background B-field @ 90°
w/ background B-field
w/ background B-field
w/ background B-field

angle between CR gradient and B-field = 90 deg

- KASCADE
- Akeno
- Adelaide
- EAS-TOP 2003
- EAS-TOP 1996
- Mt Norikura
- Tibet AS-gamma
- Milagro
- IceCube
- IceTop
- EAS-TOP

- 10 random realisations
- ensemble average, isotropic

$\text{dipole anisotropy}$

$E [\text{eV}]$
Conclusion

1. local diffusion not in ensemble average but in particular realisation of B-field
2. relative orientation between B-field and gradient

considerable variations  B-field and gradient @ 90°  finding sources?