A solution to the cosmic ray anisotropy problem

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Stanford University

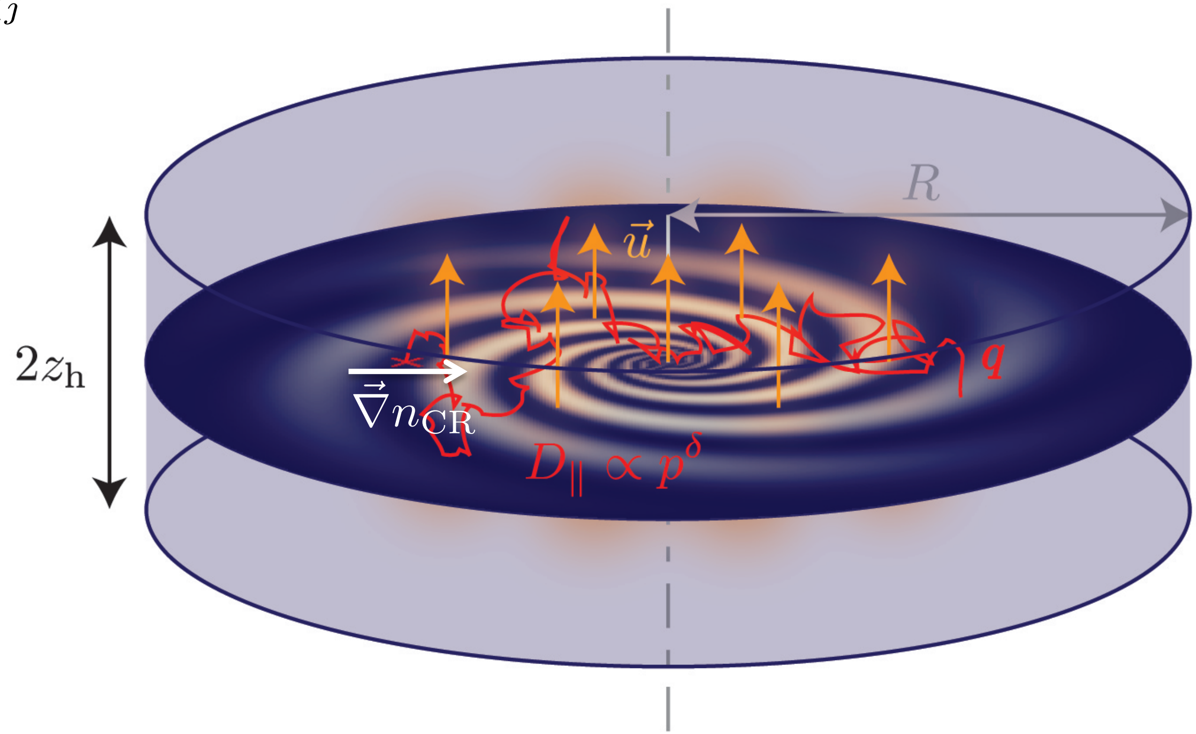
KIPAC


OPERA – COMEDIE
TRAGEDIE
COMEDIE


CR THEORY
SPECTRUM
COMPOSITION
ANISOTROPY
DIFFUSE BKG
Where do cosmic rays come from?

Why anisotropy? (I)

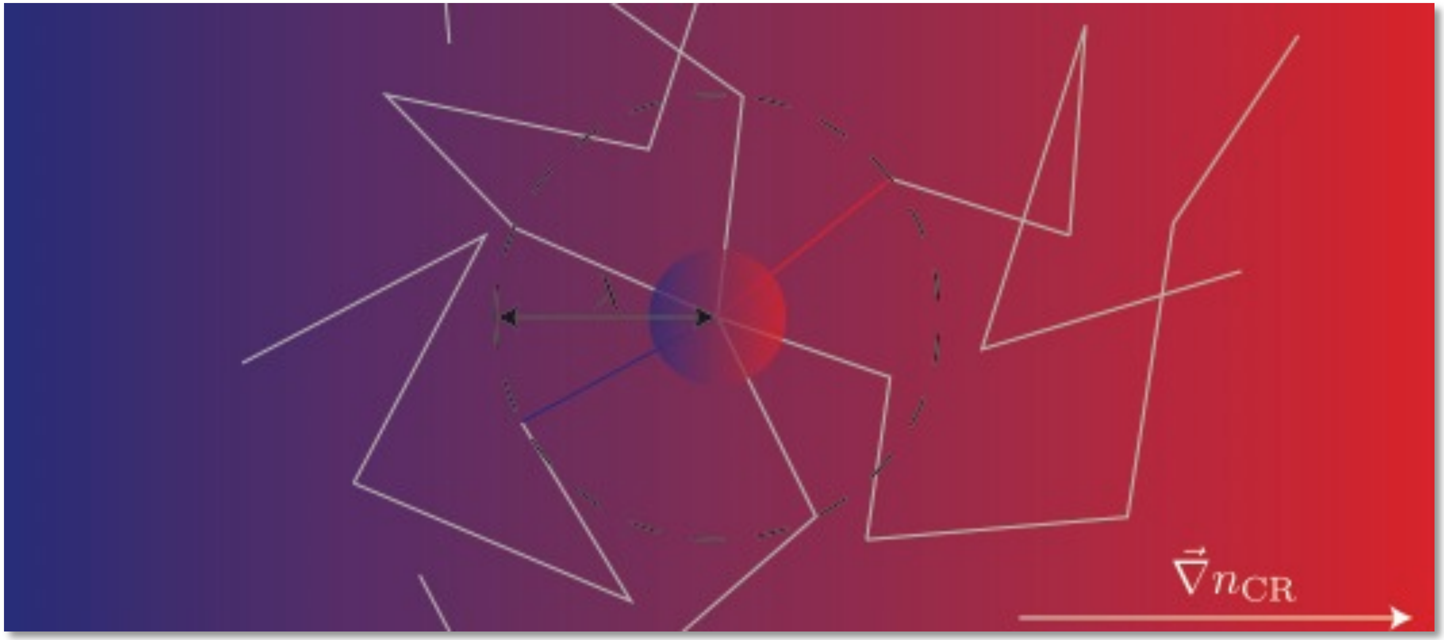
$$\frac{\partial n_i}{\partial t} - \vec{\nabla}\cdot\left(D_{xx}\cdot\vec{\nabla}n_i - \vec{u}\,n_i\right) - \frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial}{\partial p}\frac{1}{p^2}n_i - \frac{\partial}{\partial p}\left(\frac{\mathrm{d}p}{\mathrm{d}t}n_i - \frac{p}{3}\left(\vec{\nabla}\cdot\vec{u}\right)n_i\right)$$
$$= q + \sum_{i<j}\left(c\,\beta\,n_{\rm gas}\,\sigma_{j\to i} + \gamma\,\tau^{-1}_{j\to i}\right)\,n_j - \left(c\,\beta\,n_{\rm gas}\,\sigma_i + \gamma\,\tau_i^{-1}\right)\,n_i\,,$$



asymmetric source distribution ➔ $\vec{\nabla}n_{\rm CR}$.

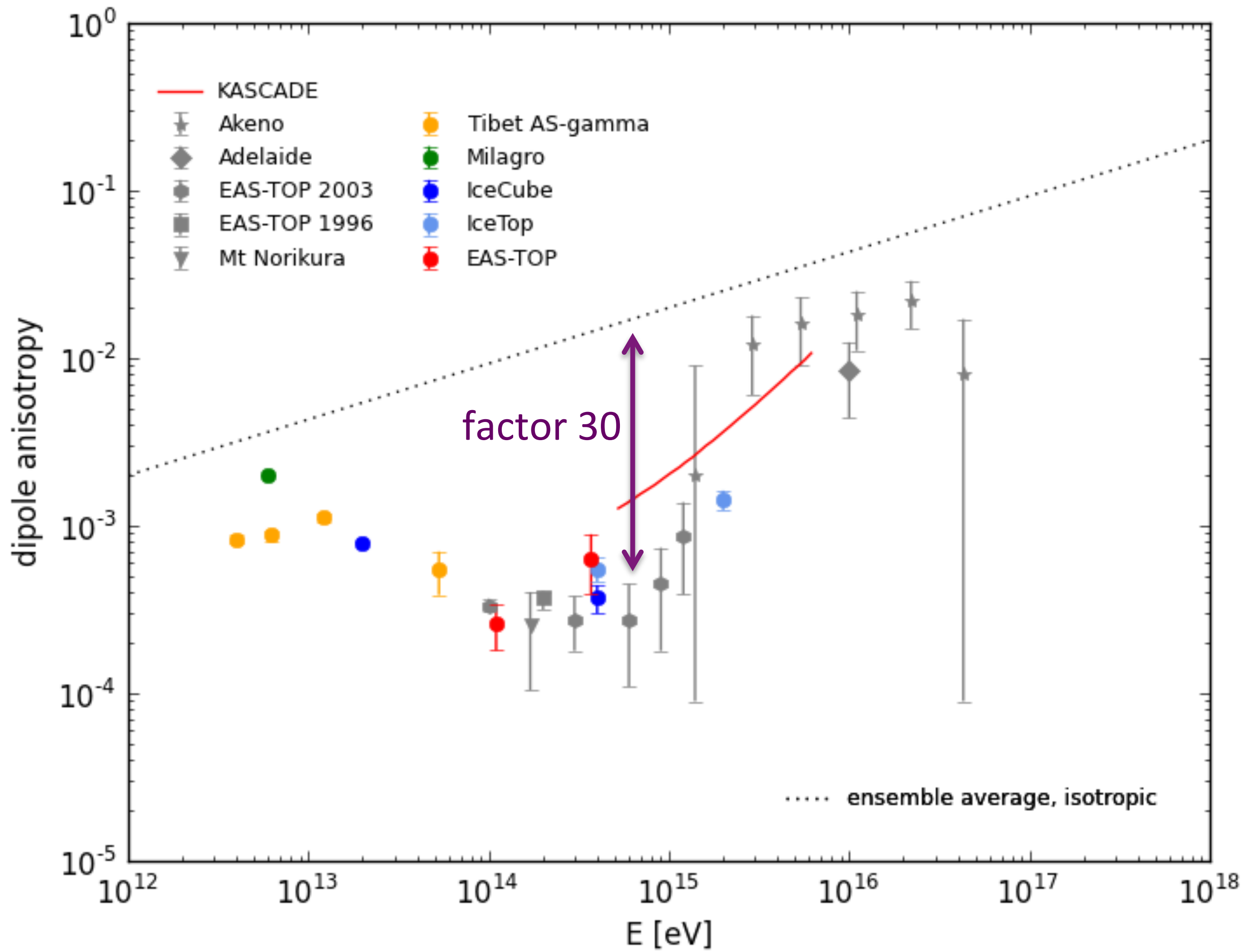
Why anisotropy? (II)

$$|\delta| = \frac{\phi_{\max} - \phi_{\min}}{\phi_{\max} + \phi_{\min}} = \frac{\phi(\vec{r} + \vec{\lambda}) - \phi(\vec{r} - \vec{\lambda})}{\phi(\vec{r} + \vec{\lambda}) + \phi(\vec{r} - \vec{\lambda})} \simeq \frac{2\lambda |\nabla \phi(\vec{r})|}{2\phi(\vec{r})} = \frac{3D}{c} \frac{|\nabla \phi(\vec{r})|}{\phi(\vec{r})}$$

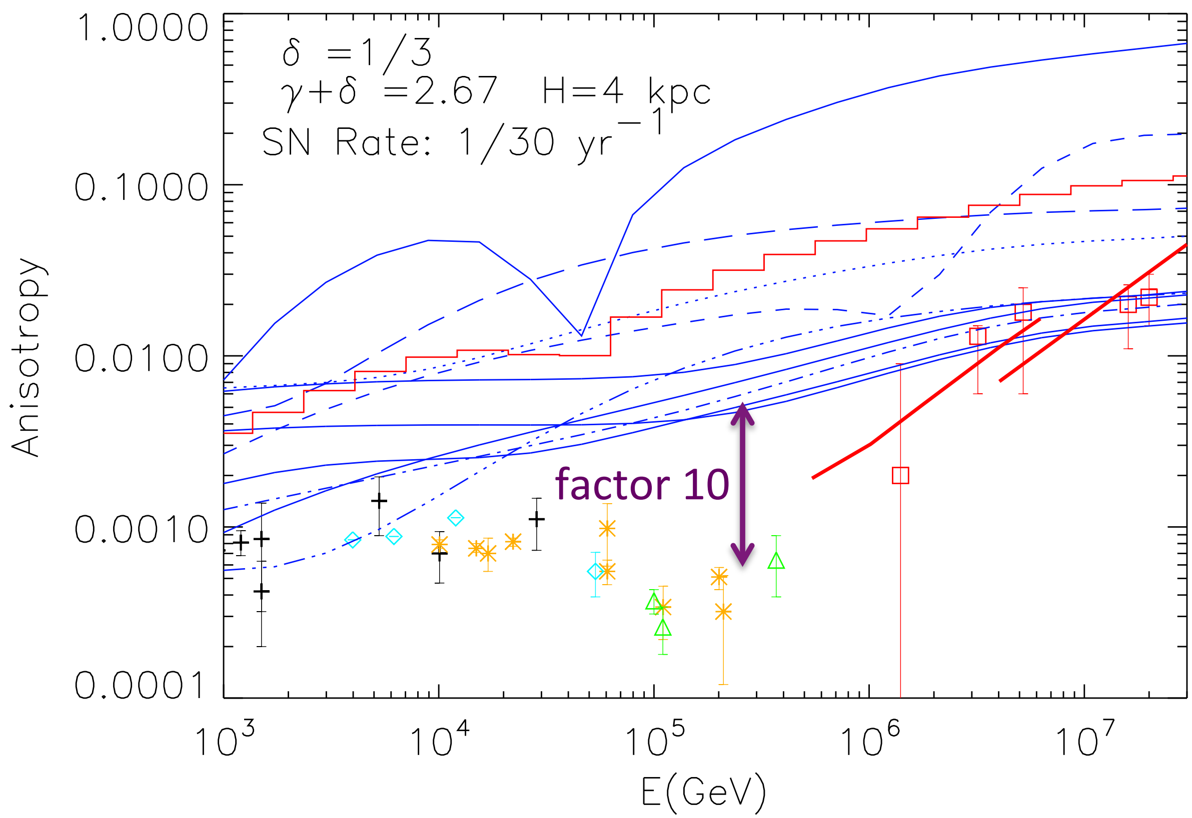


$$\vec{\delta} = \frac{3D}{c} \frac{\vec{\nabla} n_{\rm CR}}{n_{\rm CR}}$$

Experimental situation



Source stochasticity



Blasi & Amato, *JCAP* **01** (2012) 011

Measured vs. predicted diffusion coefficient

$$D = D_0(\mathcal{R}/\mathrm{GV})^\delta$$

from fitting B/C:

$$\delta = 0.33 \Rightarrow D_0 \simeq 4.0 \times 10^{28}\,\mathrm{cm}^2\mathrm{s}^{-1}$$

$$\delta = 0.55 \Rightarrow D_0 \simeq 2.3 \times 10^{28}\,\mathrm{cm}^2\mathrm{s}^{-1}$$

for $z_{\mathrm{max}} = 4\,\mathrm{kpc}$

from quasi-linear theory:

turbulence spectrum $W(k) \propto k^{-q}$ where $kW(k) \sim \delta B^2(k)$

$$D_{||} \sim r_g \left(\frac{r_g}{L}\right)^{q-1} \left(\frac{\delta B}{B_0}\right)^2$$

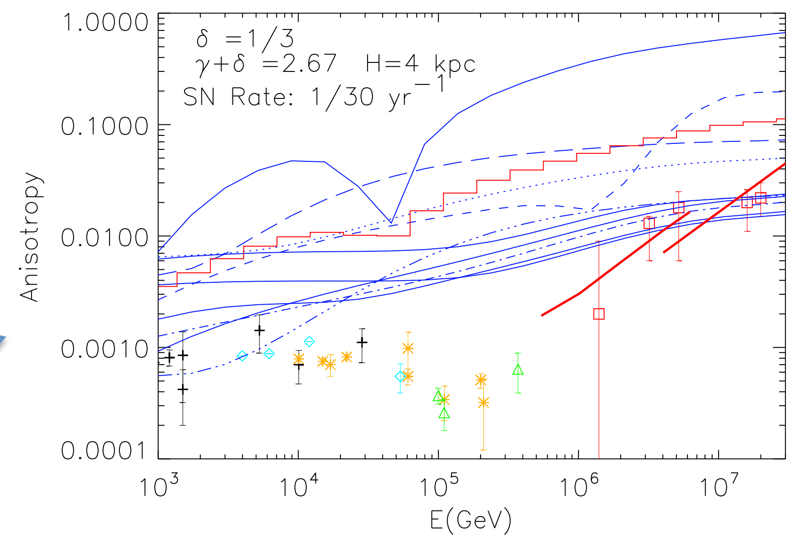
falls short of measured values (for $B_0 = 4\,\mu\mathrm{G}$ and $L = 100\,\mathrm{pc}$)

$$D_{||,0} = 4.3 \times 10^{27}\,\mathrm{cm}^2\mathrm{s}^{-1} \left(\frac{\delta B}{B_0}\right)^{-2} \quad \text{for } 2 - q = \delta = 0.33$$

$$D_{||,0} = 1.6 \times 10^{26}\,\mathrm{cm}^2\mathrm{s}^{-1} \left(\frac{\delta B}{B_0}\right)^{-2} \quad \text{for } 2 - q = \delta = 0.5$$

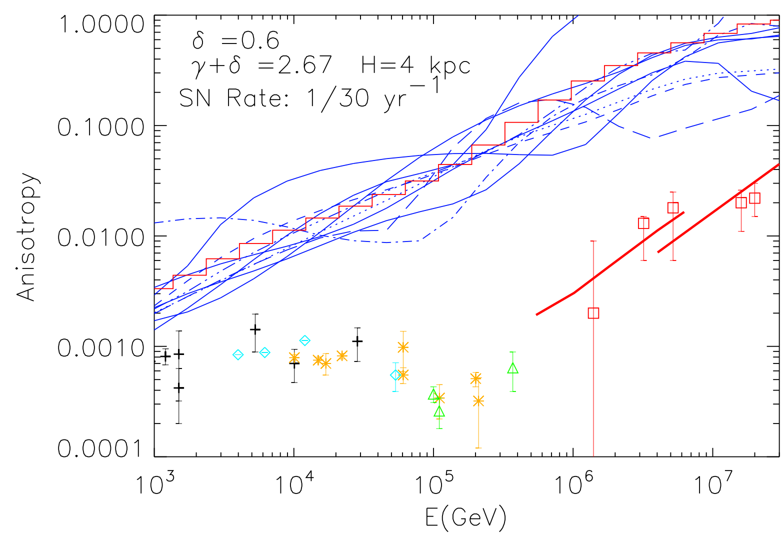
Conclusion I

- can be used to constrain spectral index of diffusion coefficient:



- poor agreement for $\delta = 1/3$

- for $\delta = 0.6$,
  it's even worse



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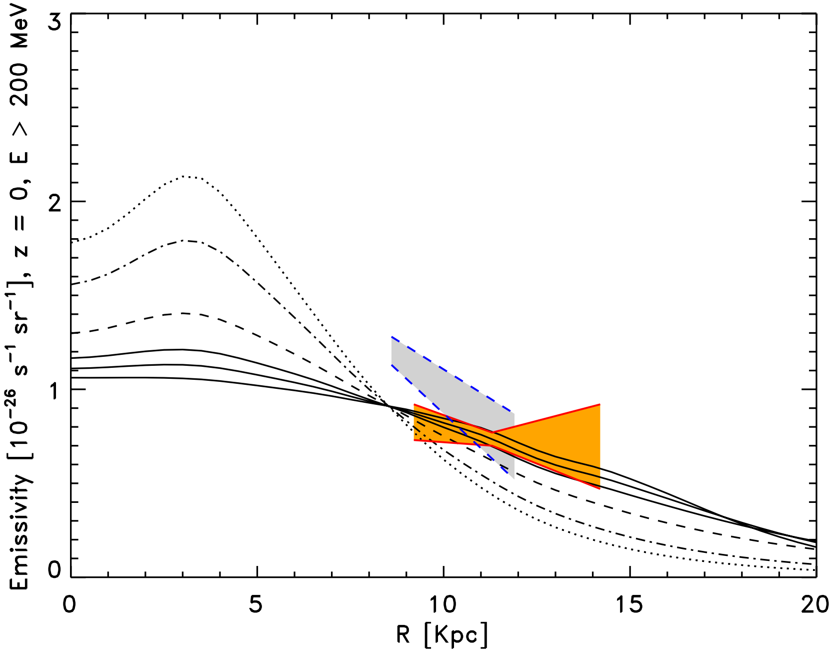
Conclusion II

- maybe the predicted global gradient is too large
- also in disagreement with gamma-ray data

- vary diffusion coefficient with galacto-centric radius
  - $D_{||} \propto \left(\frac{\delta B}{B_0}\right)^{-2}$ but $D_\perp \propto \left(\frac{\delta B}{B_0}\right)^{2}$
  - turbulence level follows source density $q(r)$
  - in the inner Galaxy escape is dominated by perpendicular diffusion
  - simulated by $D \propto q(r)^\tau$

Evoli *et al.*, *PRL* 108 (2012) 211102

Conclusion II



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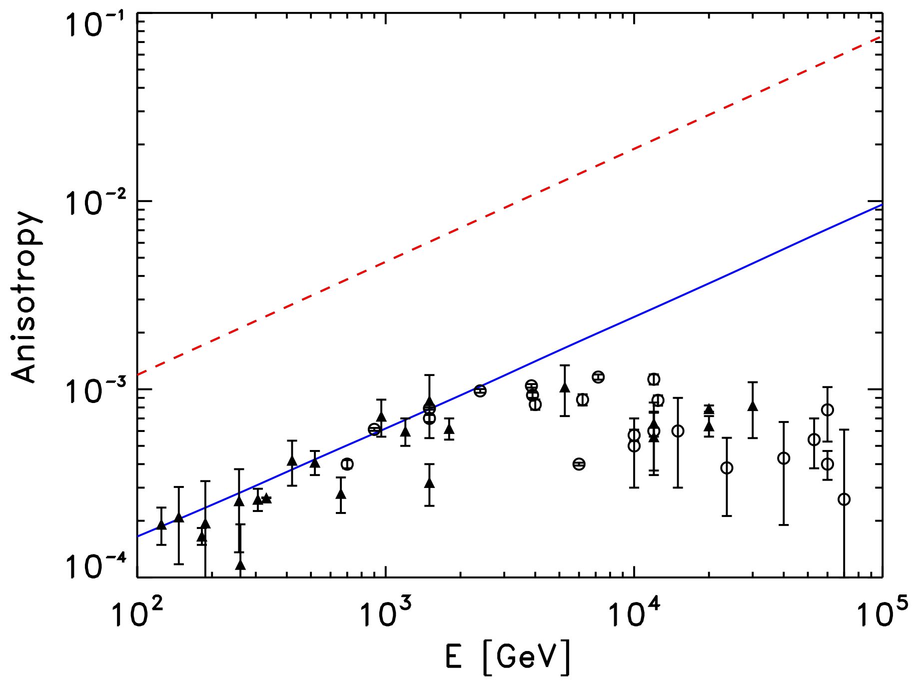
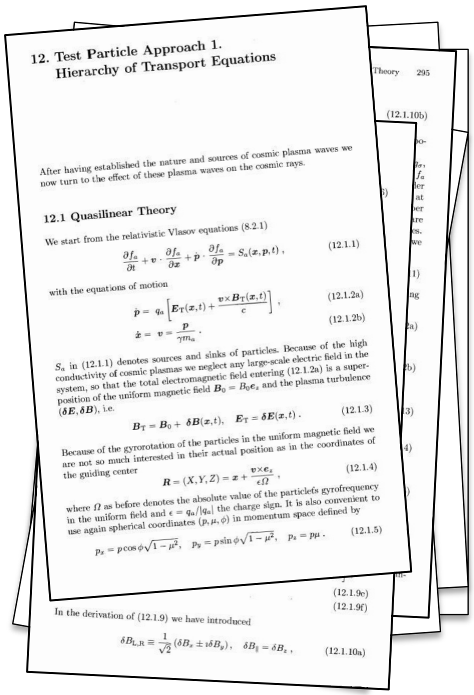


Ensemble averaging

Schlickeiser, *Cosmic Ray Astrophysics*



12. Test Particle Approach 1.
Hierarchy of Transport Equations

After having established the nature and sources of cosmic plasma waves we now turn to the effect of these plasma waves on the cosmic rays.

12.1 Quasilinear Theory

We start from the relativistic Vlasov equations (8.2.1)

$$\frac{\partial f_a}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_a}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f_a}{\partial \boldsymbol{p}} = S_a(\boldsymbol{x}, \boldsymbol{p}, t) , \tag{12.1.1}$$

with the equations of motion

$$\dot{\boldsymbol{p}} = q_a \left[\boldsymbol{E}_{\mathrm{T}}(\boldsymbol{x}, t) + \frac{\boldsymbol{v} \times \boldsymbol{B}_{\mathrm{T}}(\boldsymbol{x}, t)}{c} \right] , \tag{12.1.2a}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{v} = \frac{\boldsymbol{p}}{\gamma m_a} . \tag{12.1.2b}$$

S_a in (12.1.1) denotes sources and sinks of particles. Because of the high conductivity of cosmic plasmas we neglect any large-scale electric field in the system, so that the total electromagnetic field entering (12.1.2a) is a superposition of the uniform magnetic field $\boldsymbol{B}_0 = B_0 \boldsymbol{e}_z$ and the plasma turbulence $(\boldsymbol{\delta E}, \boldsymbol{\delta B})$, i.e.

$$\boldsymbol{B}_{\mathrm{T}} = \boldsymbol{B}_0 + \boldsymbol{\delta B}(\boldsymbol{x}, t), \quad \boldsymbol{E}_{\mathrm{T}} = \boldsymbol{\delta E}(\boldsymbol{x}, t) . \tag{12.1.3}$$

Because of the gyrorotation of the particles in the uniform magnetic field we are not so much interested in their actual position as in the coordinates of the guiding center

$$\boldsymbol{R} = (X, Y, Z) = \boldsymbol{x} + \frac{\boldsymbol{v} \times \boldsymbol{e}_z}{\epsilon \Omega} , \tag{12.1.4}$$

where Ω as before denotes the absolute value of the particle's gyrofrequency in the uniform field and $\epsilon = q_a/|q_a|$ the charge sign. It is also convenient to use again spherical coordinates (p, μ, ϕ) in momentum space defined by

$$p_x = p \cos\phi \sqrt{1-\mu^2}, \quad p_y = p \sin\phi \sqrt{1-\mu^2}, \quad p_z = p\mu . \tag{12.1.5}$$

In the derivation of (12.1.9) we have introduced

$$\delta B_{\mathrm{L,R}} \equiv \frac{1}{\sqrt{2}} (\delta B_x \pm \imath \delta B_y), \quad \delta B_\parallel = \delta B_z , \tag{12.1.10a}$$

- distribution function $f(\vec{x}, \vec{p}, t)$ develops under influence of $\delta B(\vec{x})$ and $\delta E(\vec{x})$

- we predict only the ensemble average $\langle f(\vec{x}, \vec{p}, t) \rangle$ for ensemble averaged force term

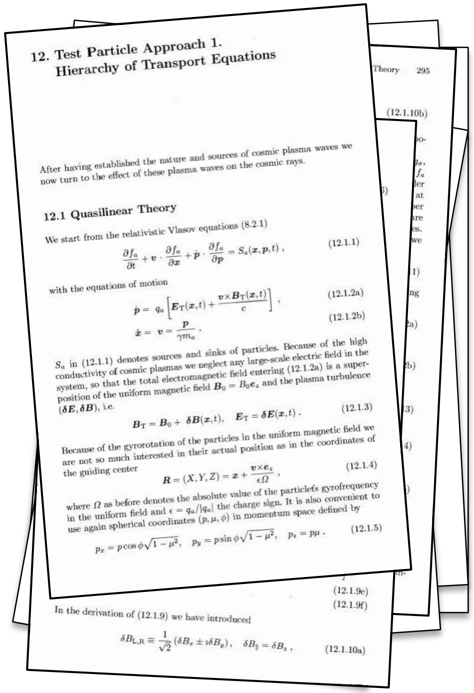
- usually, this is determined from Gaussian random B-field, characterised by $W(k)$

- we live in **one particular realisation** of random magnetic field!

→ deviations from ensemble average

Anisotropic diffusion

Schlickeiser, *Cosmic Ray Astrophysics*



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(12.1.9e)
(12.1.9f)

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Theory 295

(12.1.10b)

- decompose distribution function

$$f_0(\vec{x}, p, \mu, t) \equiv F(\vec{x}, p, t) + g(\vec{x}, p, \mu, t)$$

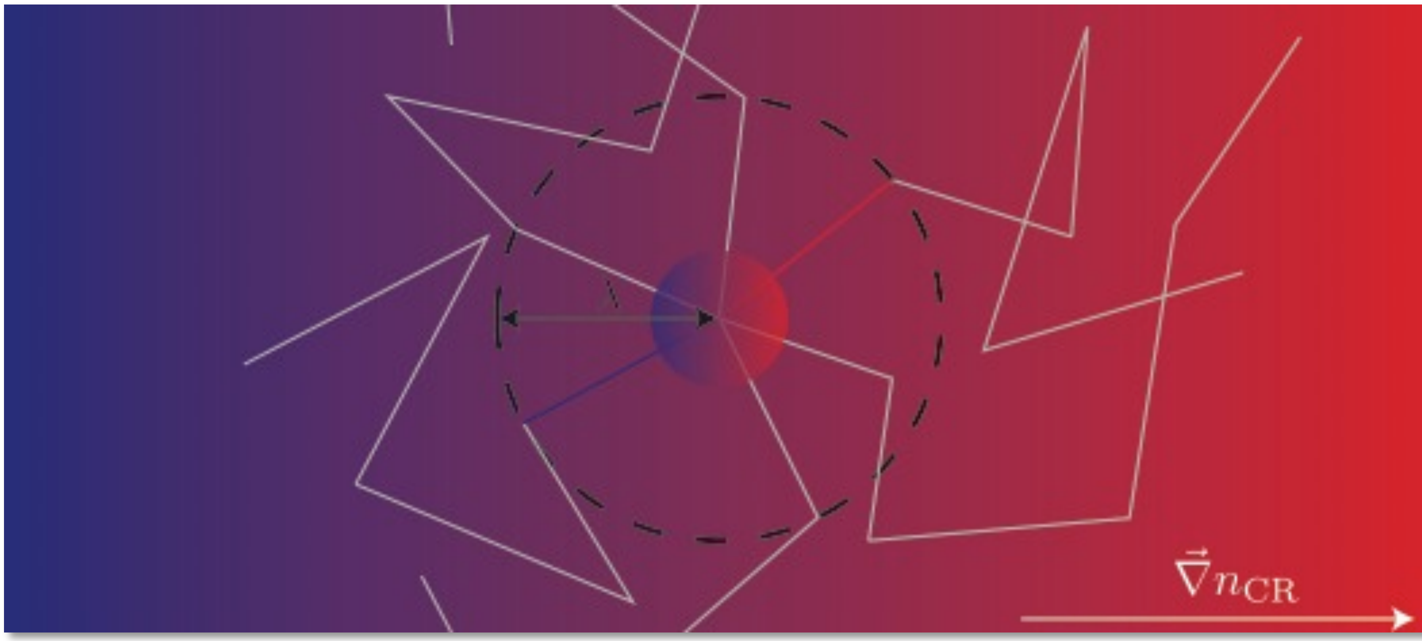
- dipole = first harmonic of anisotropic part

$$|\vec{\delta}| = \frac{\int_{-1}^{1} \mathrm{d}\mu \, \mu g(\mu)}{\int_{-1}^{1} \mathrm{d}\mu \, f_0} = \ldots = -\frac{1}{4v} \frac{\partial F / \partial z}{F} D_{||}$$

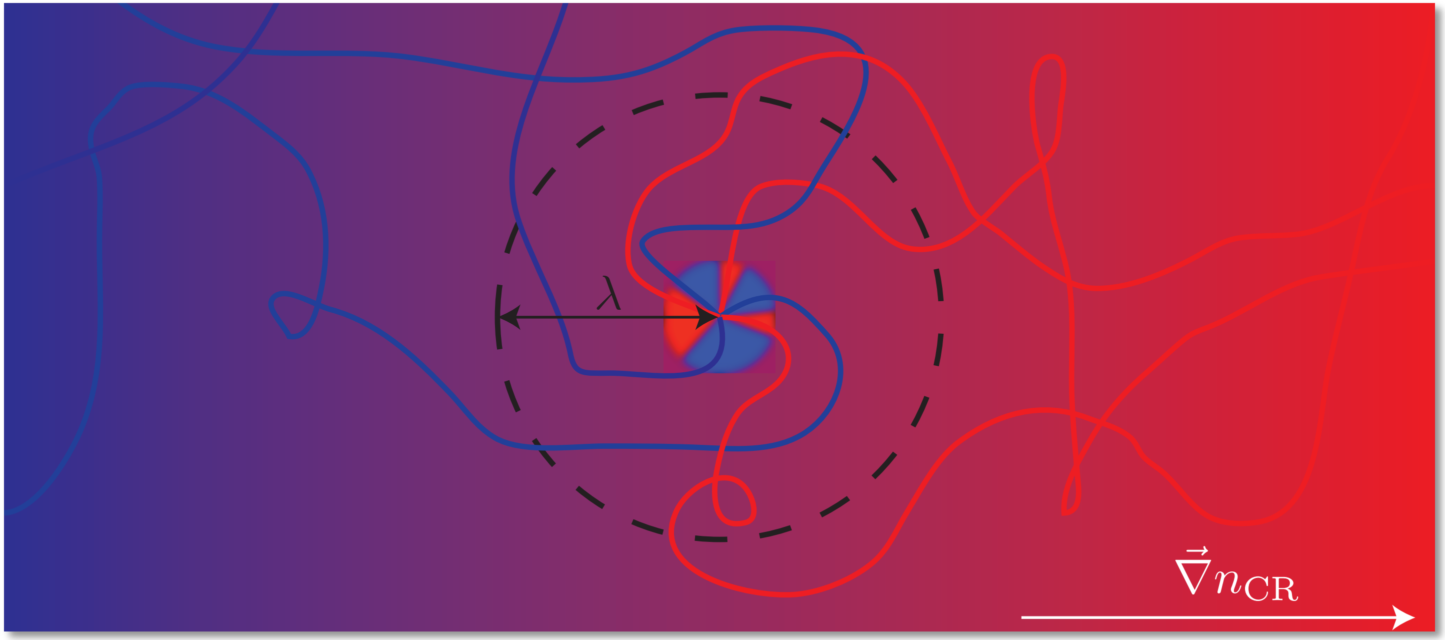
→ amplitude depends on gradient *along* background B-field
→ orientation not in direction of gradient but of background B-field

- can this help decrease the dipole amplitude?

Numerical approach



Numerical approach



1. set up large scale gradient at time $(t_0 - \Delta t)$: $f(\vec{x}, \vec{p}, t_0 - \Delta t) = \ldots$

2. back-track large number of particles $i \in N$ for time Δt:

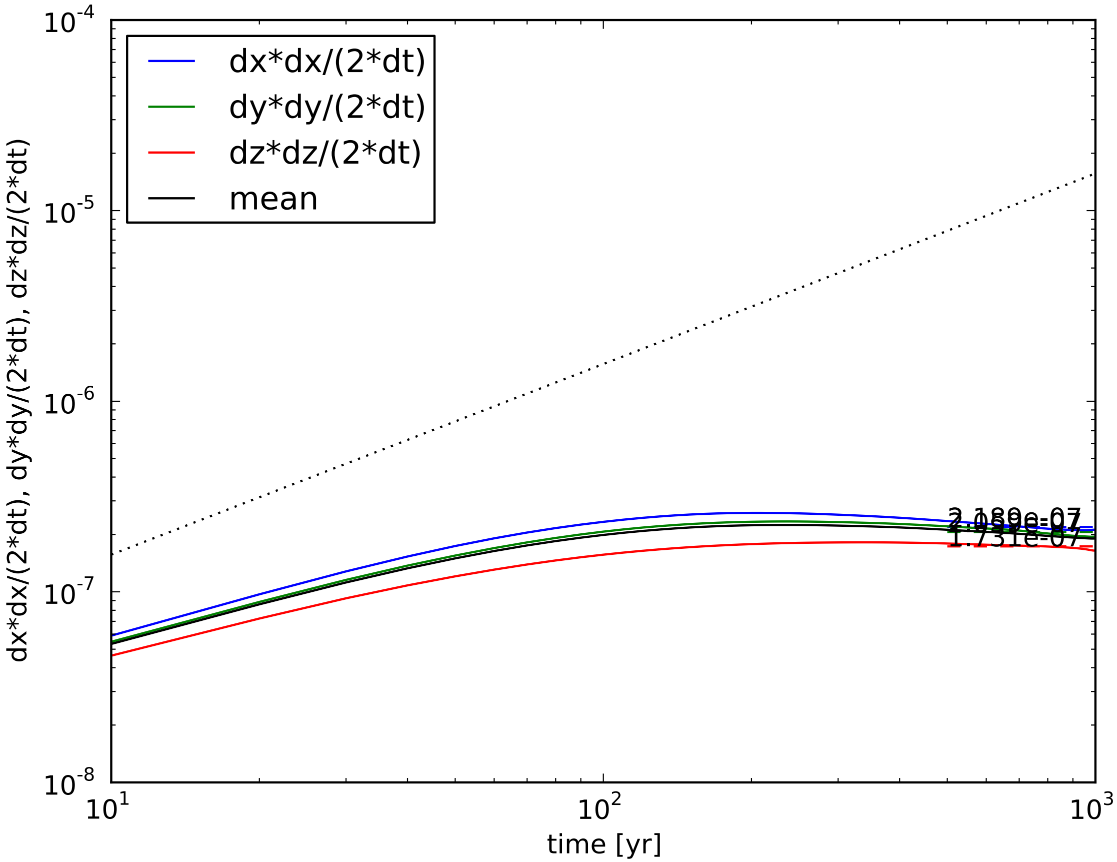
$$\{\vec{x}_i(t_0), \vec{p}_i(t_0)\} \quad \rightarrow \quad \{\vec{x}_i(t_0 - \Delta t), \vec{p}_i(t_0 - \Delta t)\}$$

3. Liouville's theorem:

$$df = 0 \quad \Rightarrow \quad f(\vec{x}_{\mathrm{obs.}}, \vec{p}_i(t_0)) = f(\vec{x}_i(t_0 - \Delta t), \vec{p}_i(t_0 - \Delta t))$$

check: diffusion coefficient

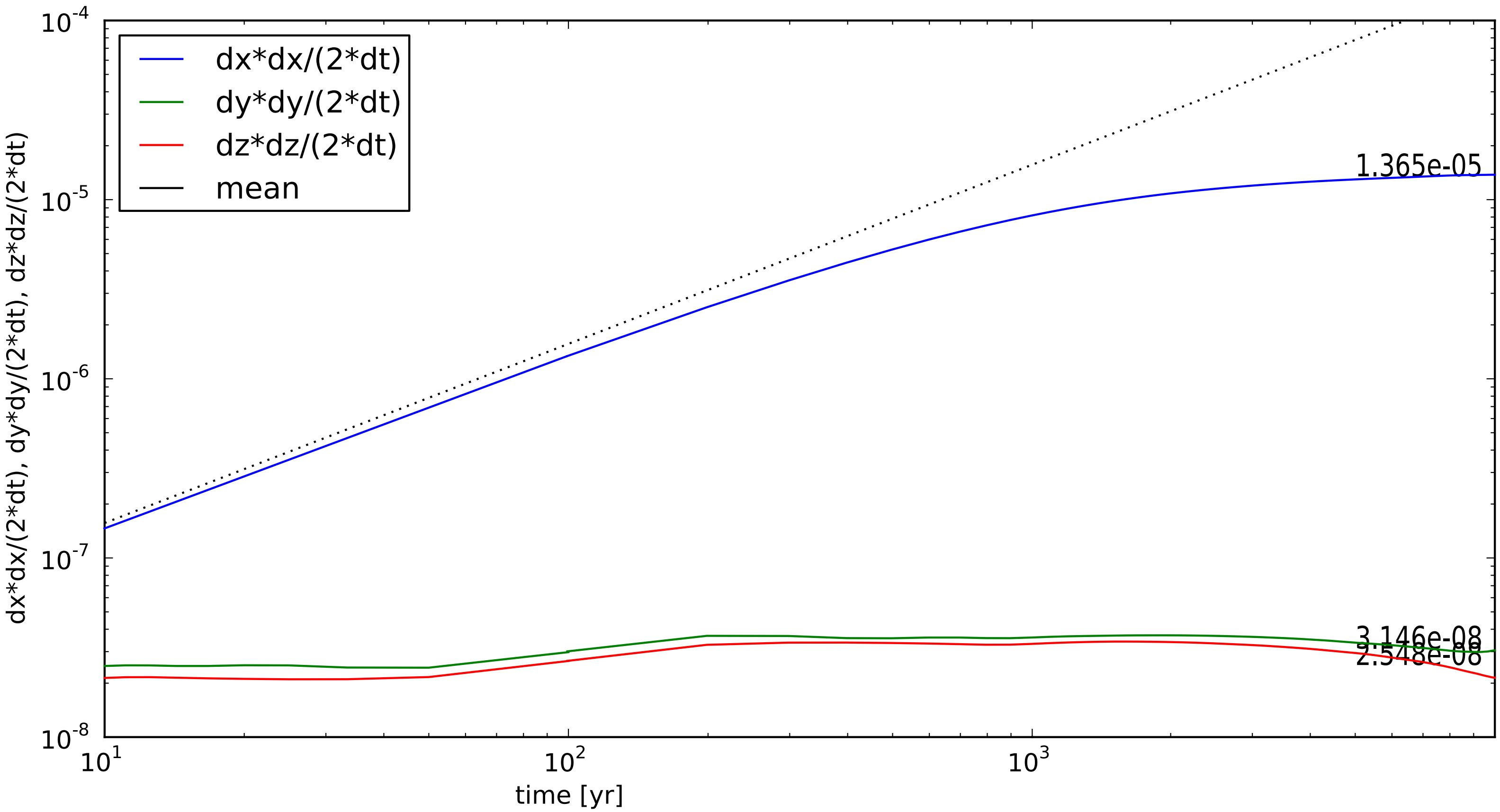
- average over large number of trajectories and B-field realisations
- isotropic diffusion coefficient in agreement with “theory”



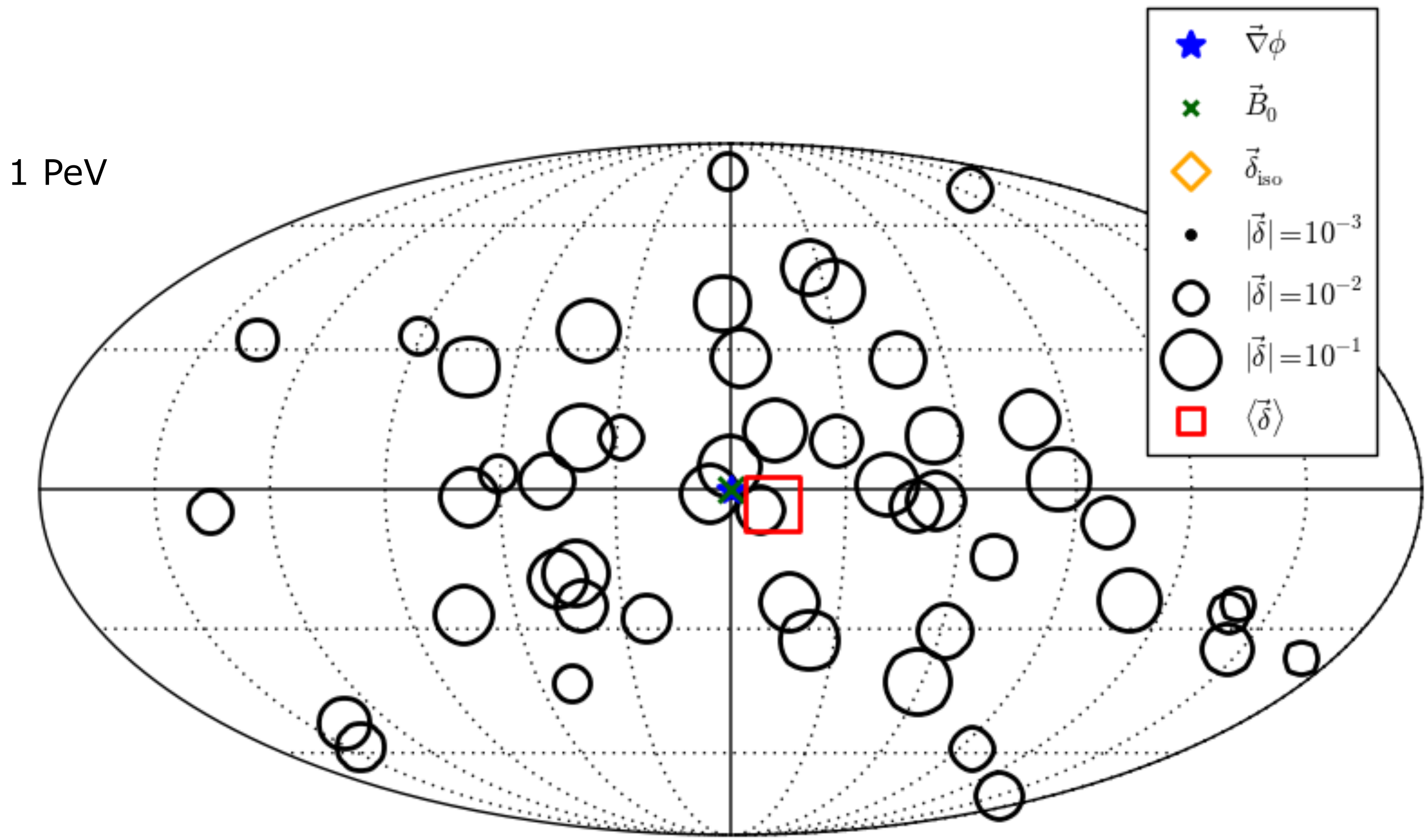
check: diffusion coefficient

- average over large number of trajectories and B-field realisations

- anisotropic diffusion coefficients in agreement with quasi-linear theory



w/o background B-field



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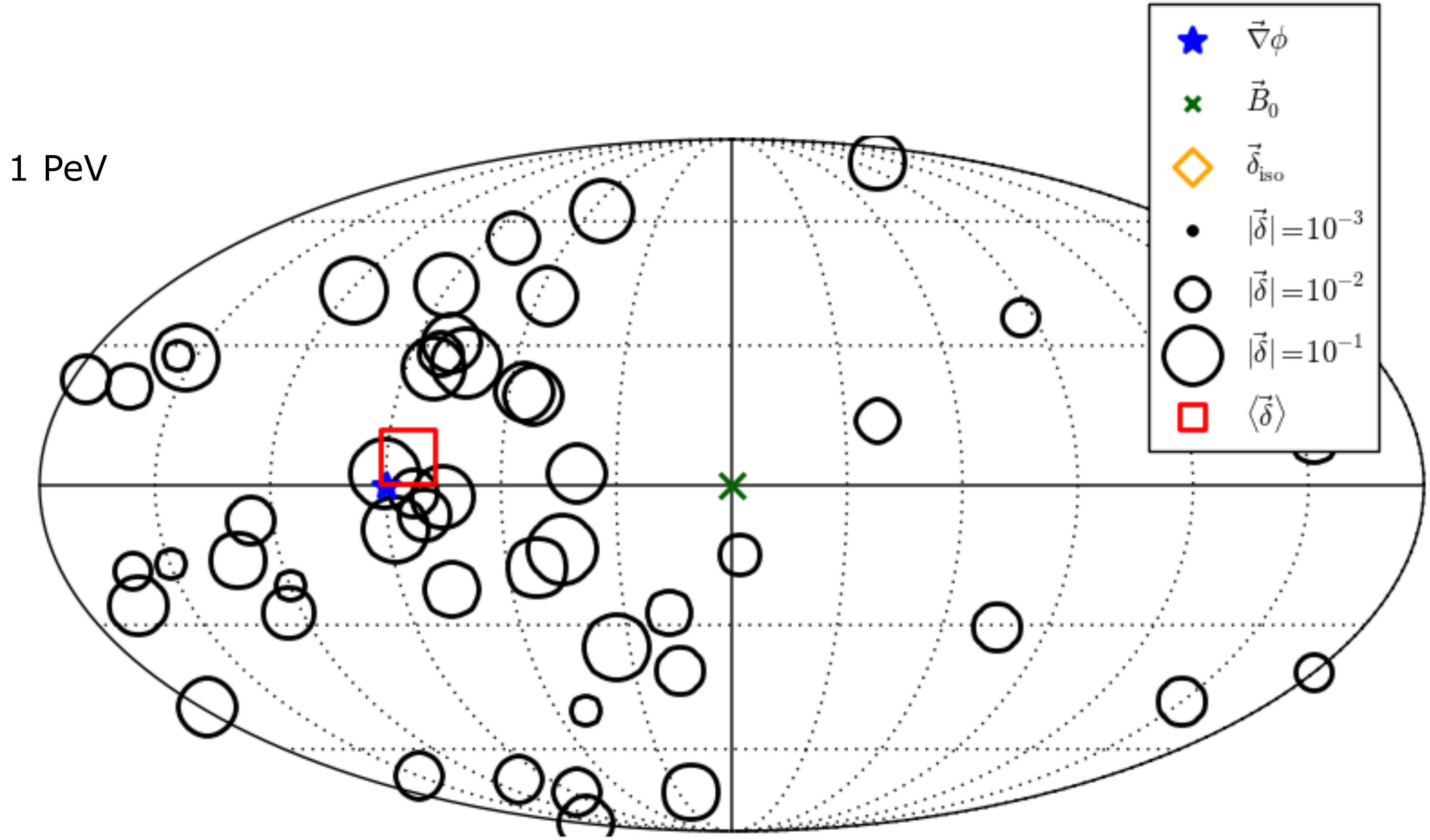

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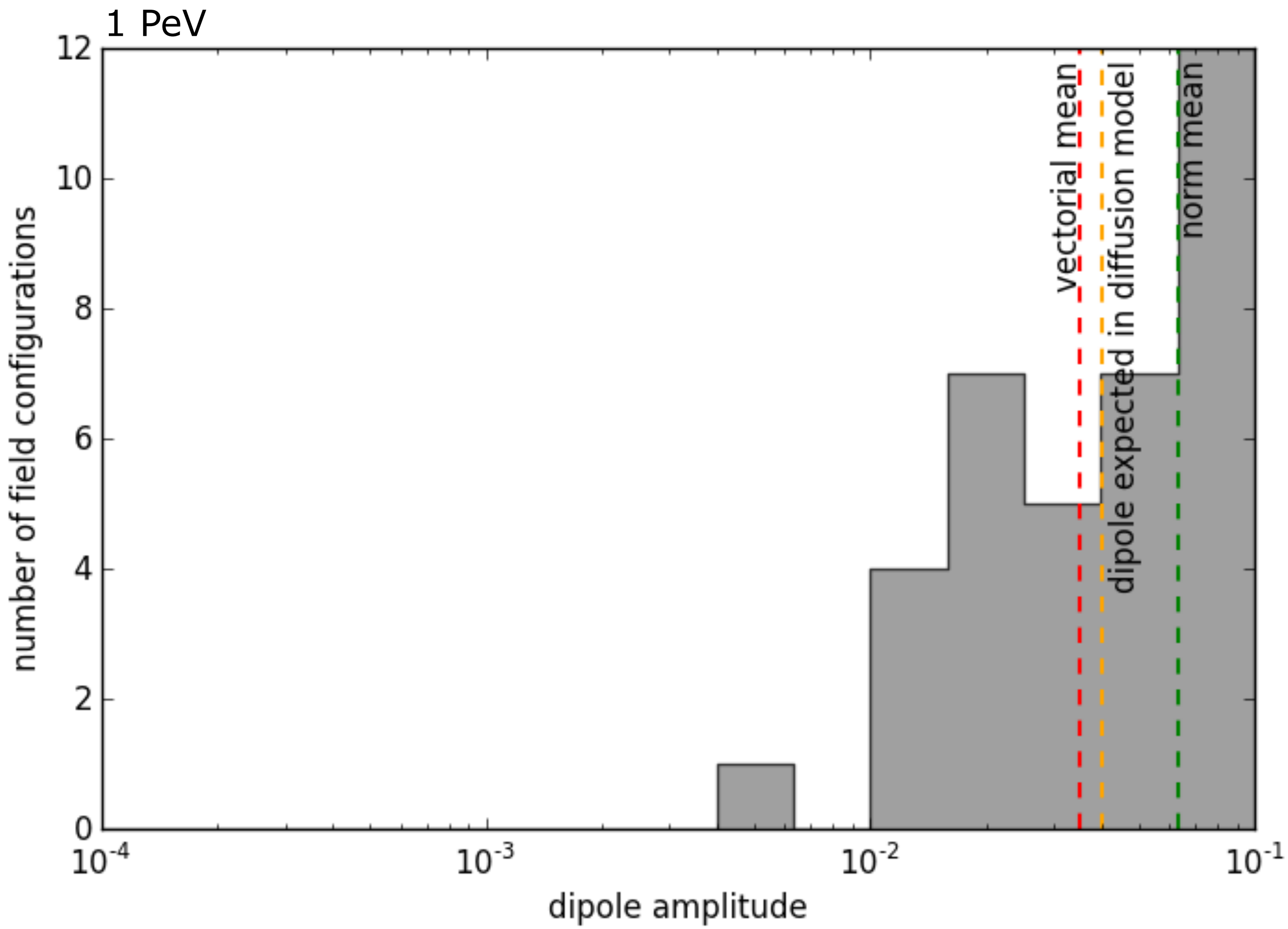

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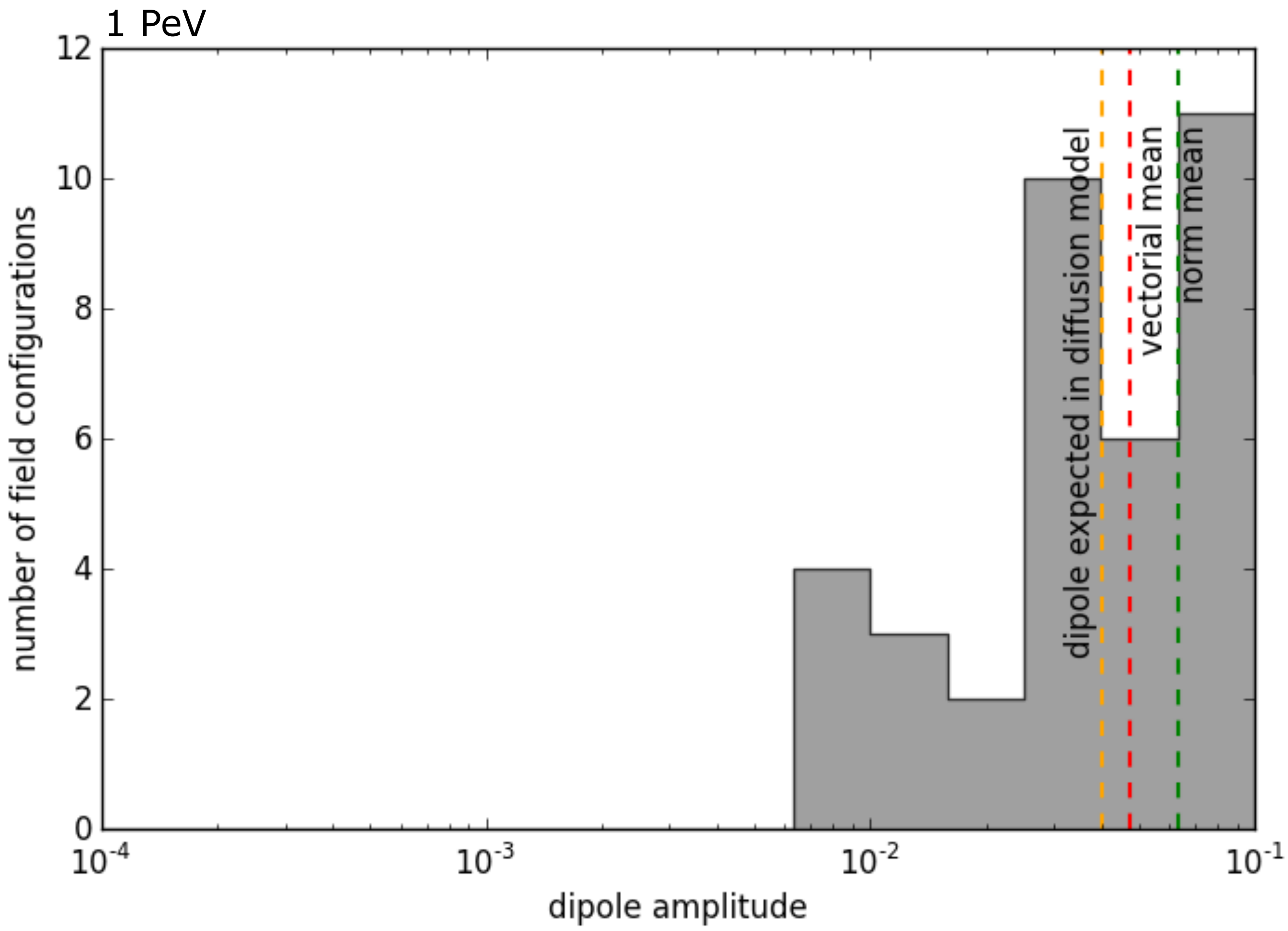

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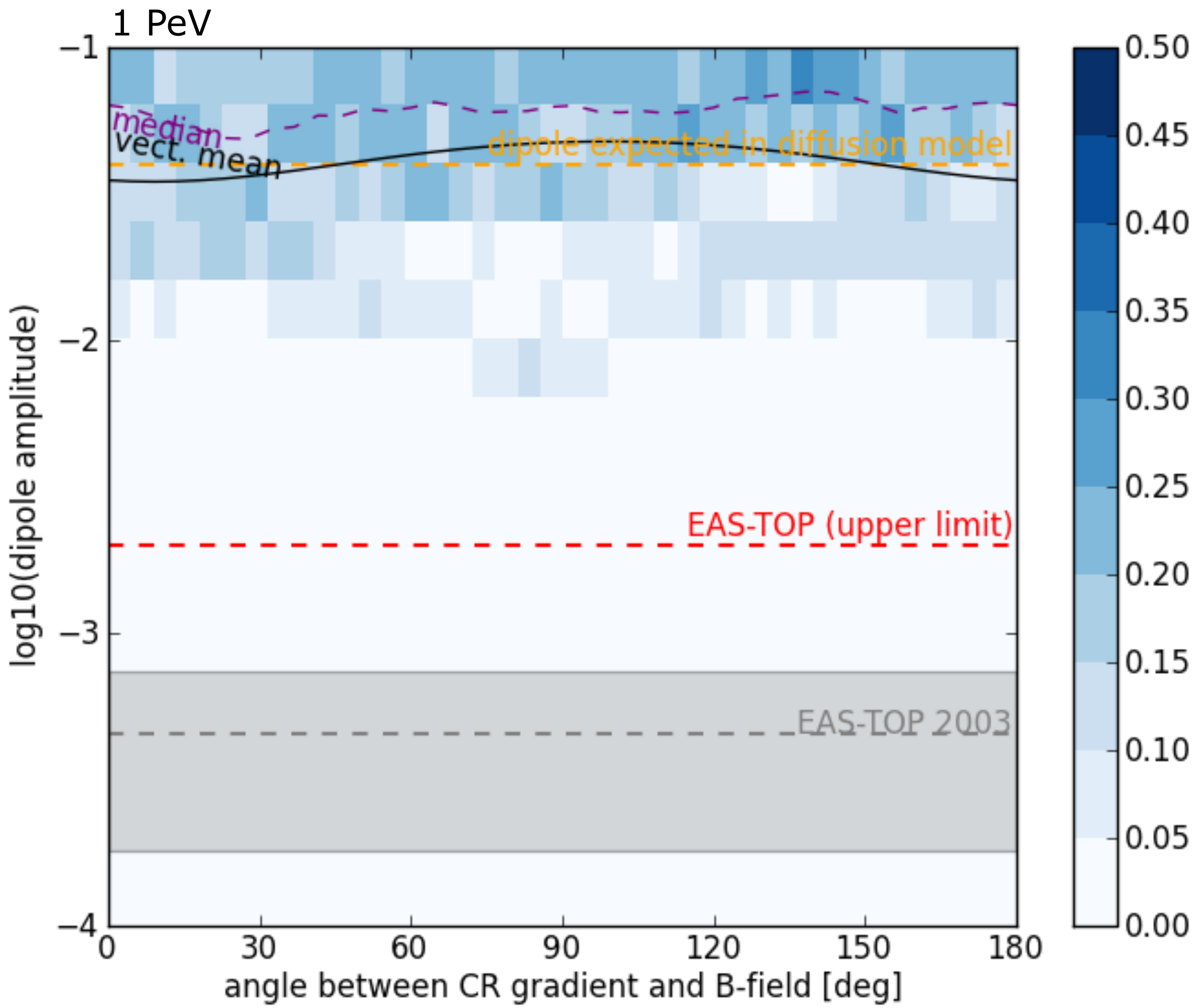
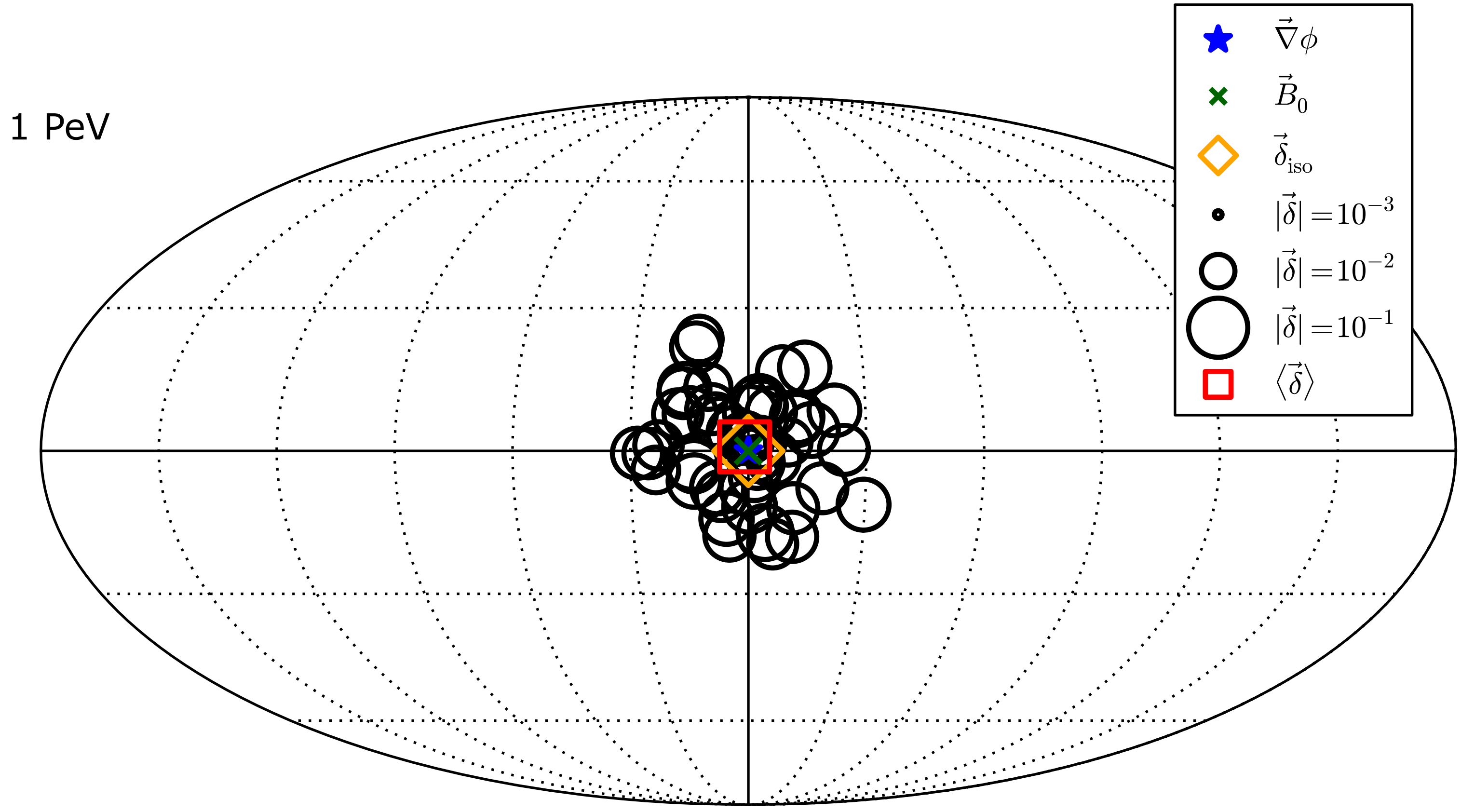
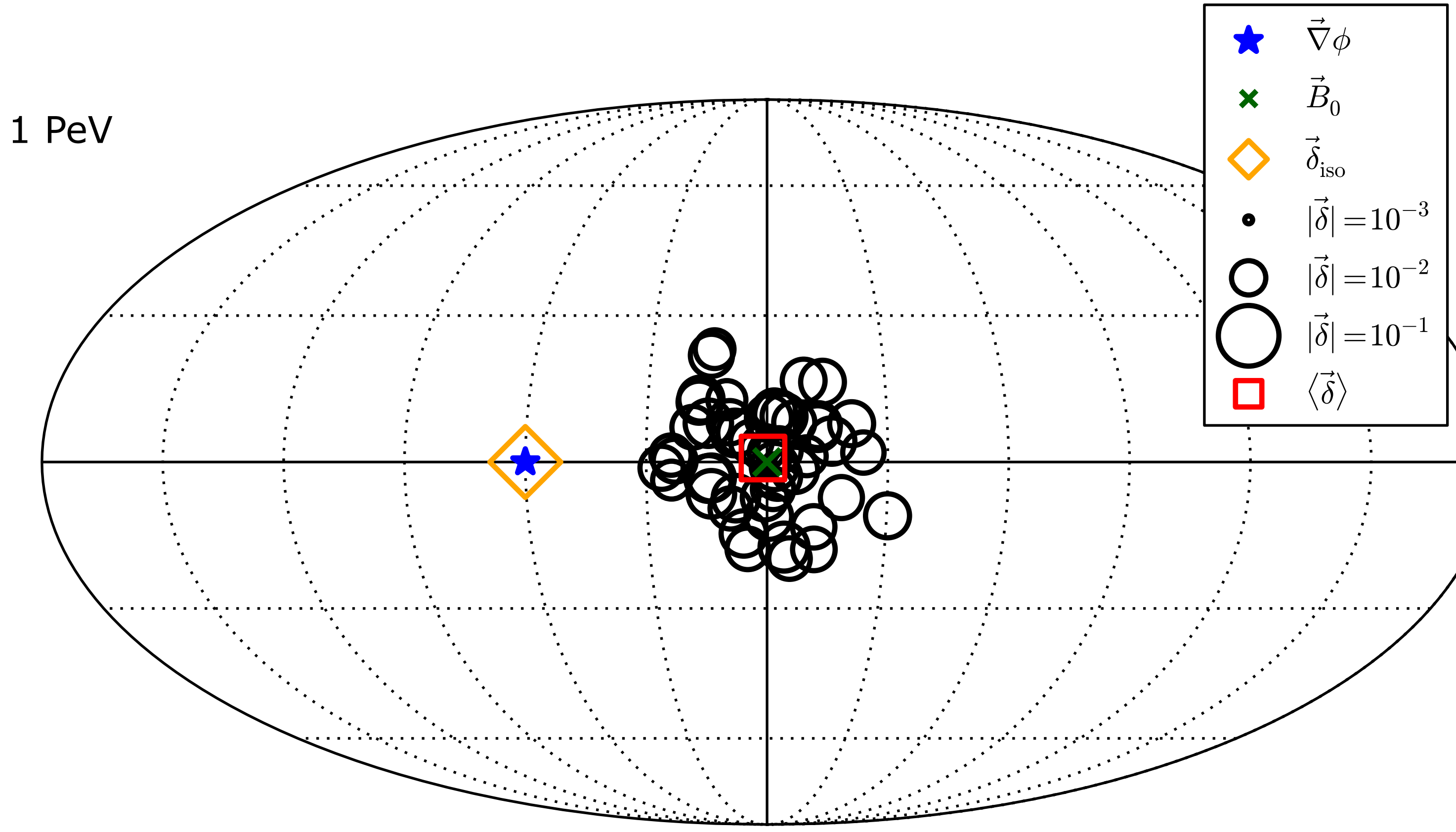


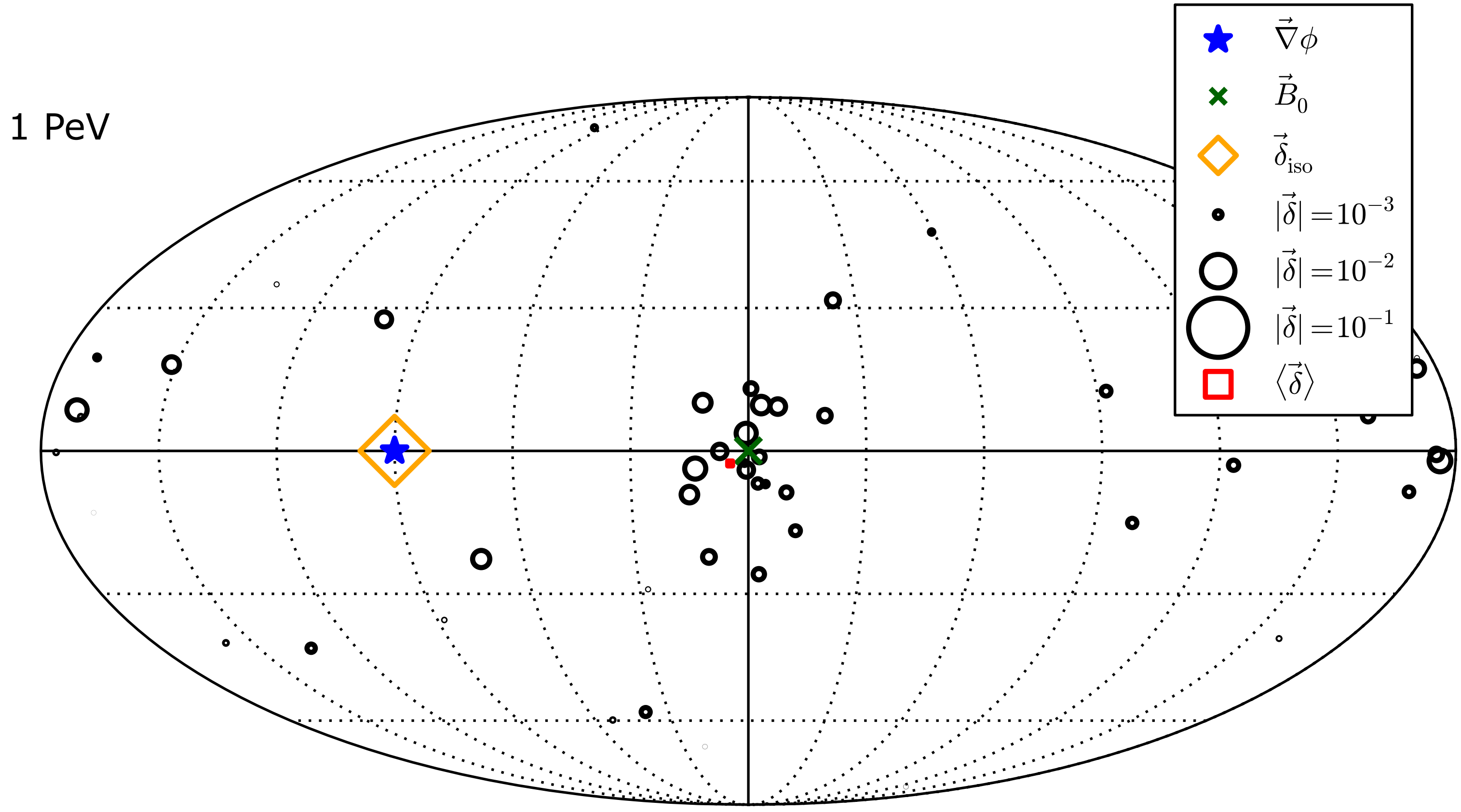
w/ background B-field @ 0°



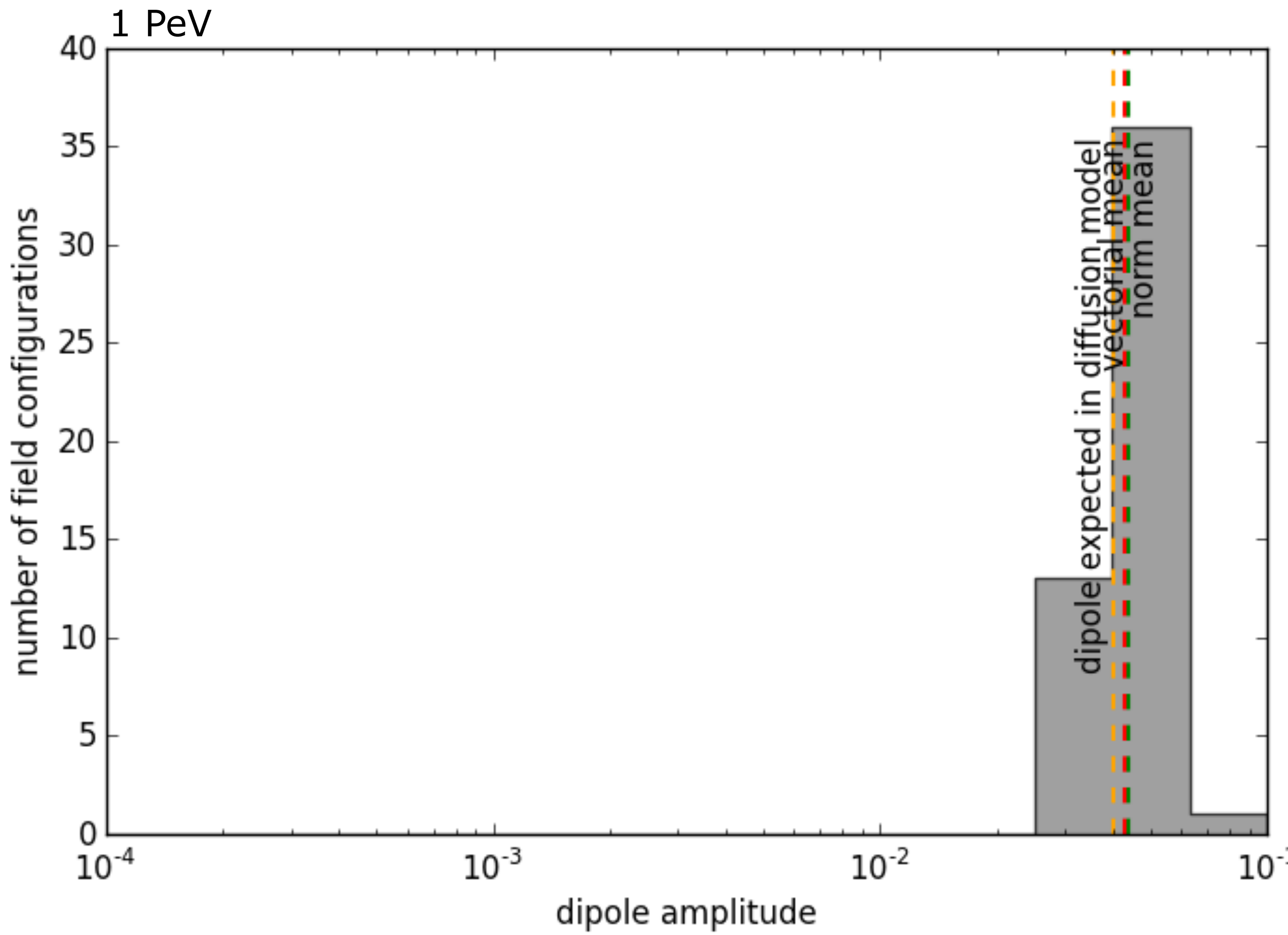
w/ background B-field @ 60°



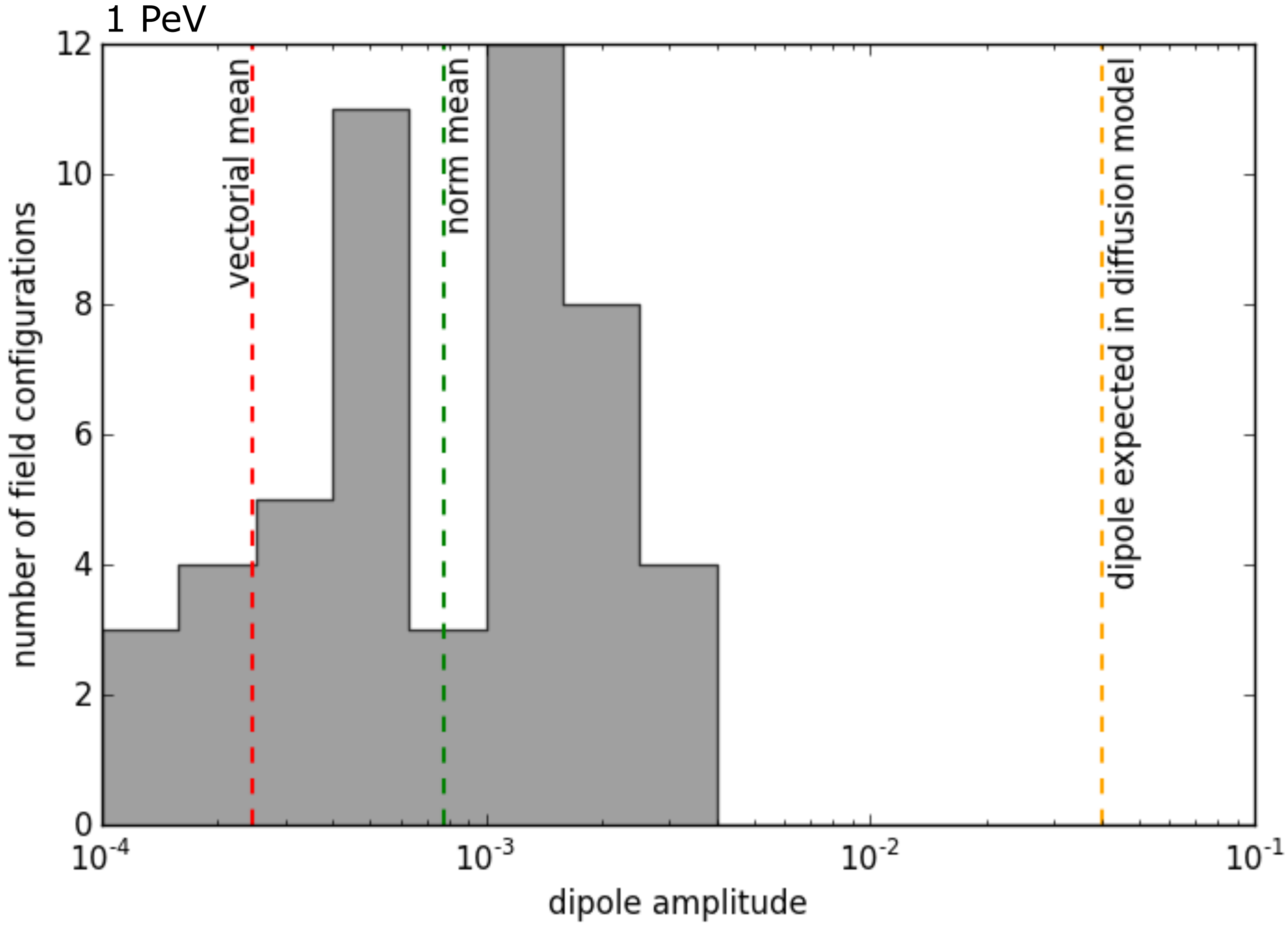
w/ background B-field @ 90°



w/ background B-field @ 0°



w/ background B-field @ 90°



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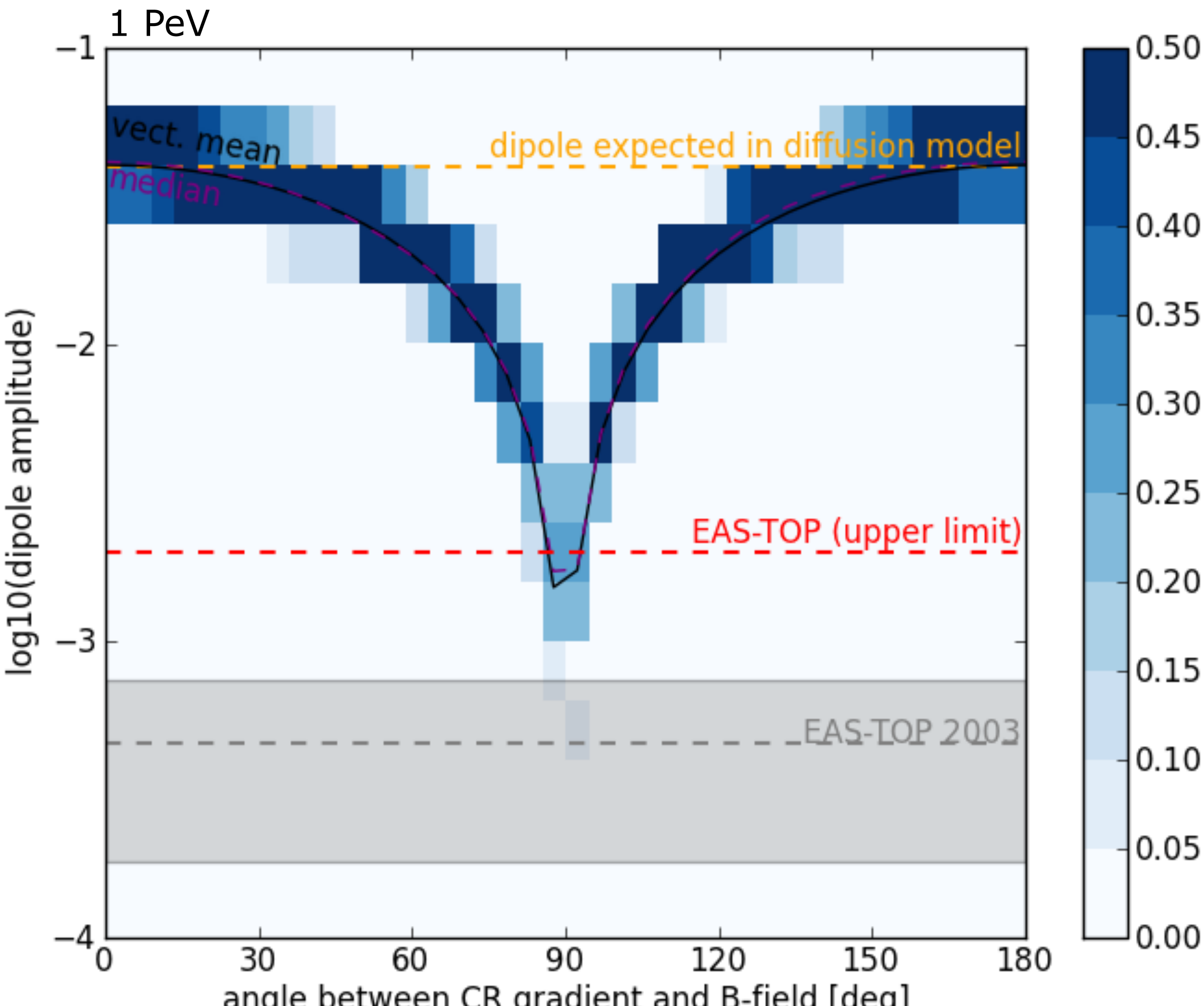

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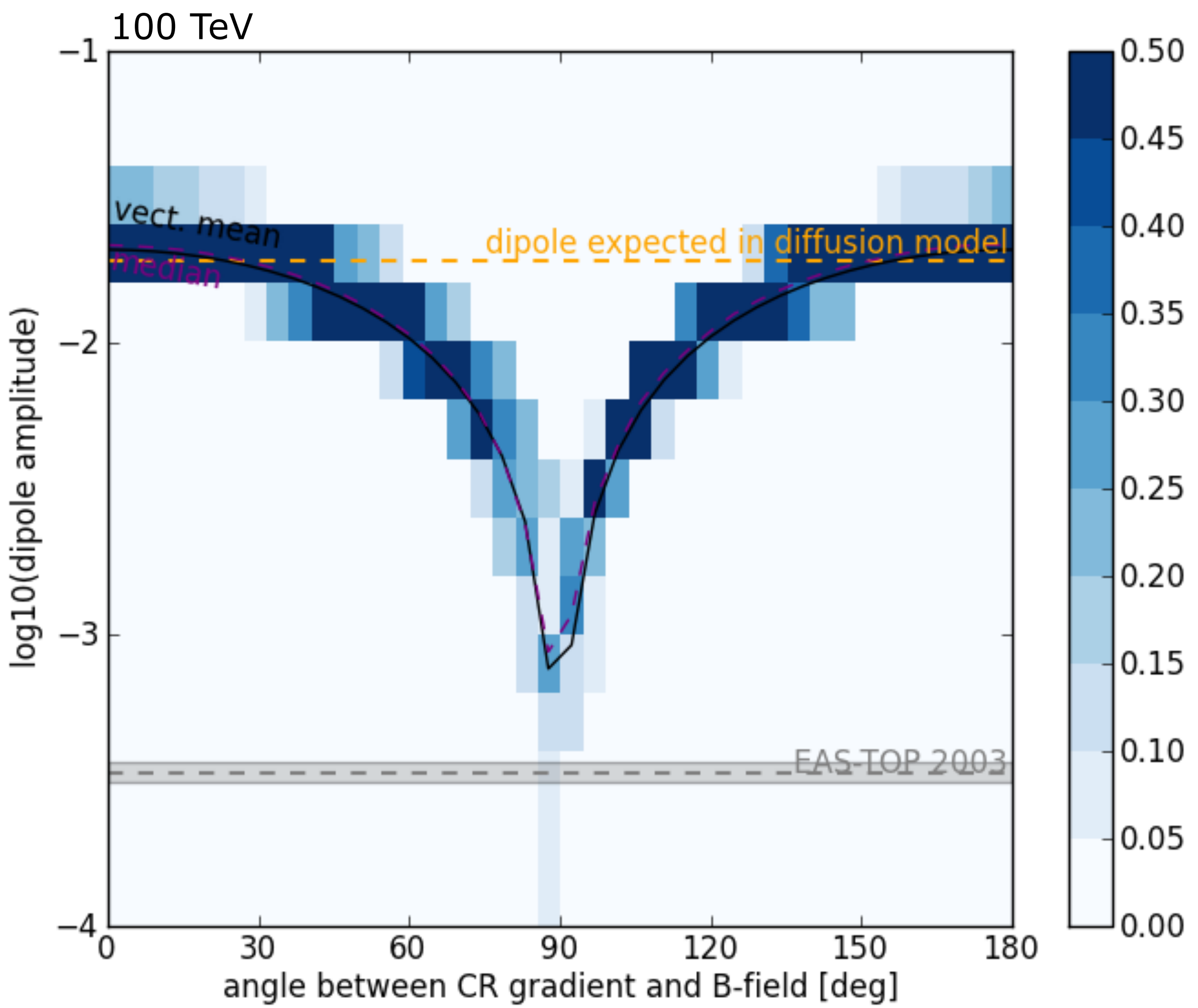

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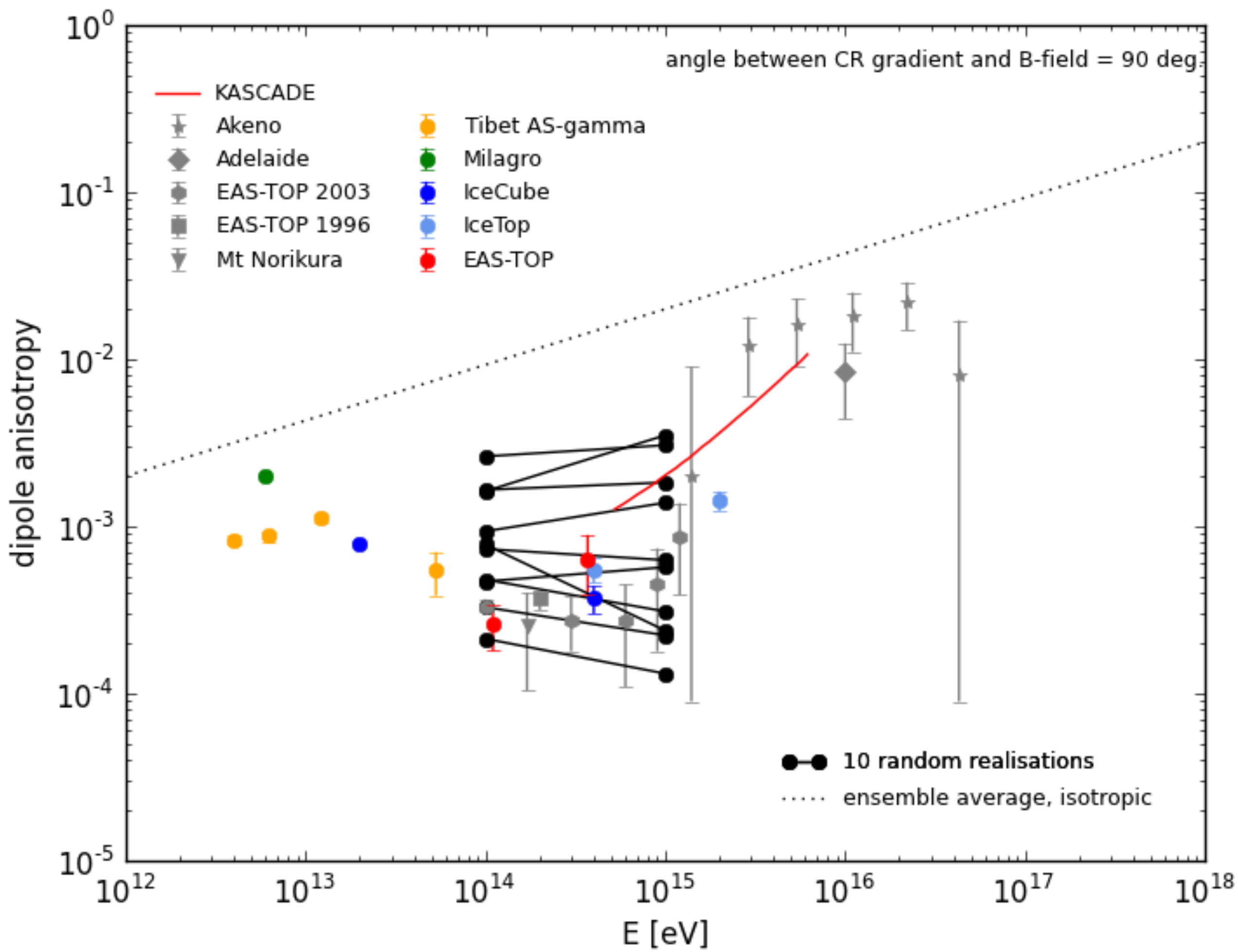
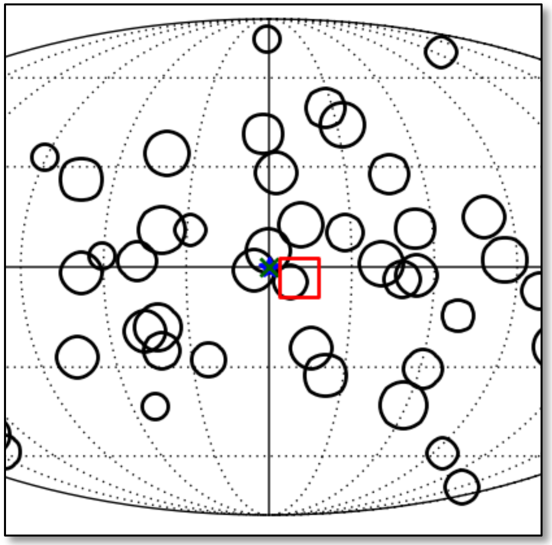
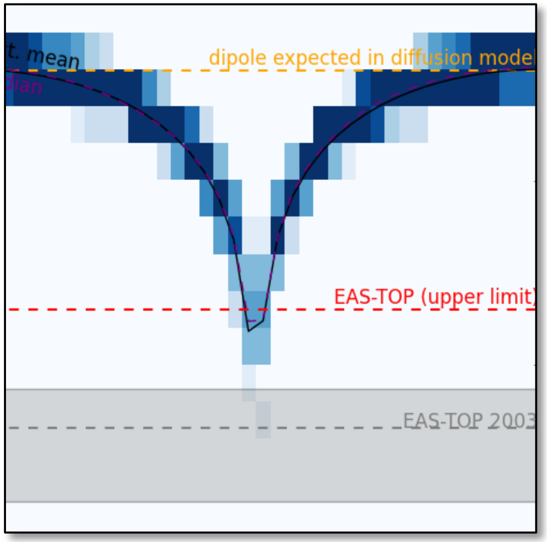


Conclusion

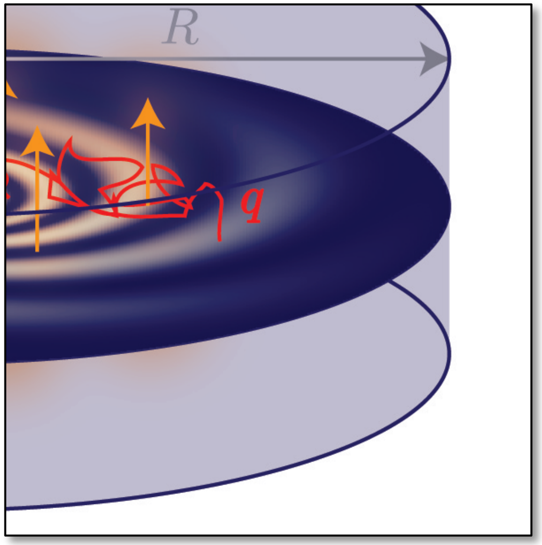
1. local diffusion not in ensemble average but in particular realisation of B-field
2. relative orientation between B-field and gradient



considerable variations



B-field and gradient @ 90°



finding sources?