Instabilities of Cosmic Rays

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Importance of the Problem

- Propagation: confinement time, formation of halos, penetration into clouds, predict ionization rates, synchrotron, γ-ray, & inverse Compton emission.
- Acceleration: efficiency of DSA, stochastic acceleration.
- Collisionless momentum & energy deposition in background medium; feedback.

Instability Landscape

Microscopic Instabilities

Resonant

Nonresonant

Cosmic Ray Hydrodynamics

Macroscopic instabilities

Parker

Drury

Cosmic Ray Feedback

Gyroresonant Pitch Angle Scattering

For transverse magnetic fluctuations of frequency ω & parallel wavenumber k,

$$\omega - kv\mu \pm n\omega_c = 0,$$

where $\mu \equiv \mathbf{v} \cdot \mathbf{B}/vB$ & $n = \pm 1$ is the strongest resonance. For $|\omega/kvu| \ll 1$ the resonance condition is

$$\mu = \mp \frac{qB}{cp} \equiv \frac{p_1}{p}.$$

Summing over many uncorrelated scatterings, the mean scattering rate ν is

$$\nu = \frac{\pi}{4}\omega_c k_r \frac{\delta B_k^2}{B^2}.$$

Fokker-Planck Equation

For $\omega_c^{-1} \ll \nu^{-1} \ll \tau_{dyn}$, a Fokker-Planck eqn. for the gyor-averaged distribution function holds

$$\frac{\partial f}{\partial t} + \mu v \hat{\mathbf{n}} \cdot \nabla f = \frac{df}{dt}|_{coll};$$

$$\frac{df}{dt}|_{coll} = \sum_{\pm} \frac{\partial}{\partial \mu} \left[\frac{1-\mu^2}{2} \nu_{\pm} \frac{\partial f}{\partial \mu} + \nu_{\pm} \hat{\mathbf{n}} \cdot \mathbf{w}_{\pm} m \gamma \frac{\partial f}{\partial p} \right],$$

with $\pm \equiv$ direction of wave propagation & $\mathbf{w}_{\pm} \equiv \mathbf{v} \pm \mathbf{v_w}$.

Momentum Transfer

Multiply Fokker-Planck eqn. by $p\mu$ & integrate over momentum space.

$$\frac{\partial}{\partial t}(n_c p_D) = -\nabla_{\|} P_c - n_{cr}\nu(v_D - v_w).$$

- Scattering drives f toward isotropy in the frame of the waves
- In a steady state, acceleration down the pressure gradient balances friction.
- Waves propagating in both directions contribute to ν

Energy Transfer: I

Work with the μ averaged Fokker-Planck eqn. for F

$$\frac{\partial F}{\partial t} + \frac{1}{3p^2}\frac{\partial}{\partial p}(p^3\mathbf{u})\cdot\nabla F - (\nabla\cdot\mathbf{u})\frac{p}{3}\frac{\partial F}{\partial p} = \frac{dF}{dt}|_{coll};$$

$$\frac{dF}{dt}|_{coll} = \nabla_\parallel D_\parallel \nabla_\parallel F + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\right)\frac{\partial F}{\partial p},$$

where $\mathbf{u} \equiv \mathbf{v} + \langle\frac{3}{2}(1-\mu^2)\frac{\nu_+\mathbf{w}_+ - \nu_-\mathbf{w}_-}{\nu_+ + \nu_-}\rangle$, $D_\parallel \equiv v^2\langle\frac{1-\mu^2}{2(\nu_+ + \nu_-)}\rangle$, $D_{pp} \equiv 4m^2\gamma^2 w^2\langle\frac{1-\mu^2}{2}\frac{\nu_+\nu_-}{\nu_+ + \nu_-}\rangle$.

- u is a mean (fluid plus wave) velocity.
- D_{pp} represents 2nd order Fermi acceleration & requires waves traveling in both directions.

Energy Transfer: II

Multiply by particle energy $\epsilon \sim cp$ & integrate over momentum space. Ignoring Fermi term for now,

$$\frac{\partial U_c}{\partial t} + \mathbf{u} \cdot \nabla U_c = -\frac{4}{3} U_c \nabla \cdot \mathbf{u} + \nabla \cdot \kappa_{\mathbf{c}} \cdot \nabla U_c$$

as we'd expect for a relativistic, diffusive gas with velocity $\mathbf{u} = \mathbf{v} + \mathbf{v_W}$.

Energy Transfer: III

Combine with the ideal fluid equations to yield an energy transport equation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{P_g}{\gamma_g - 1} + \frac{P_c}{\gamma_c - 1}\right) = -\nabla \cdot \mathbf{F} + Q;$$

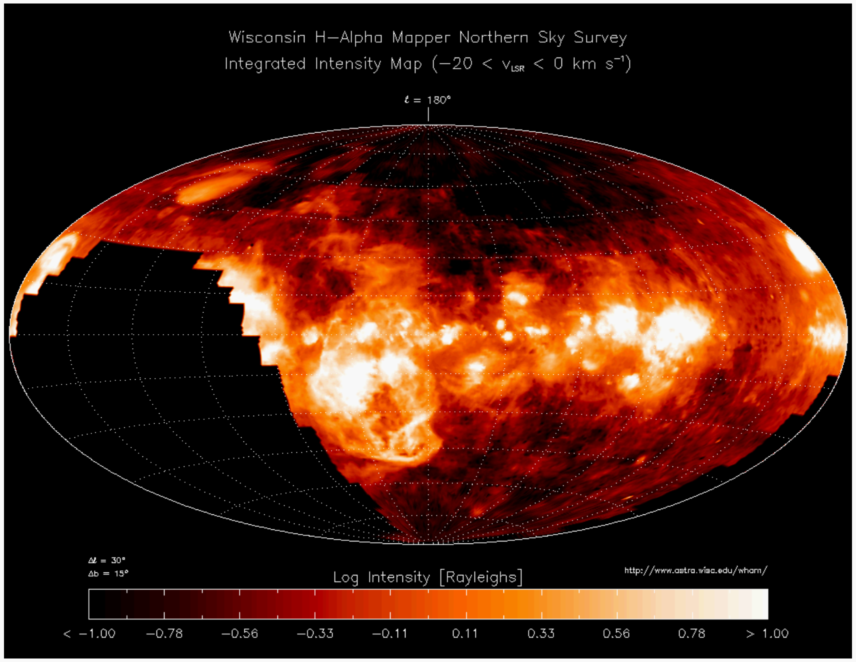
$$\mathbf{F} = \mathbf{v}\left(\frac{1}{2}\rho v^2 + \frac{\gamma_g P_g}{\gamma_g - 1}\right) + \mathbf{u}\frac{\gamma_c P_c}{\gamma_c - 1} - \frac{\kappa_c \nabla P_c}{\gamma_c - 1},$$

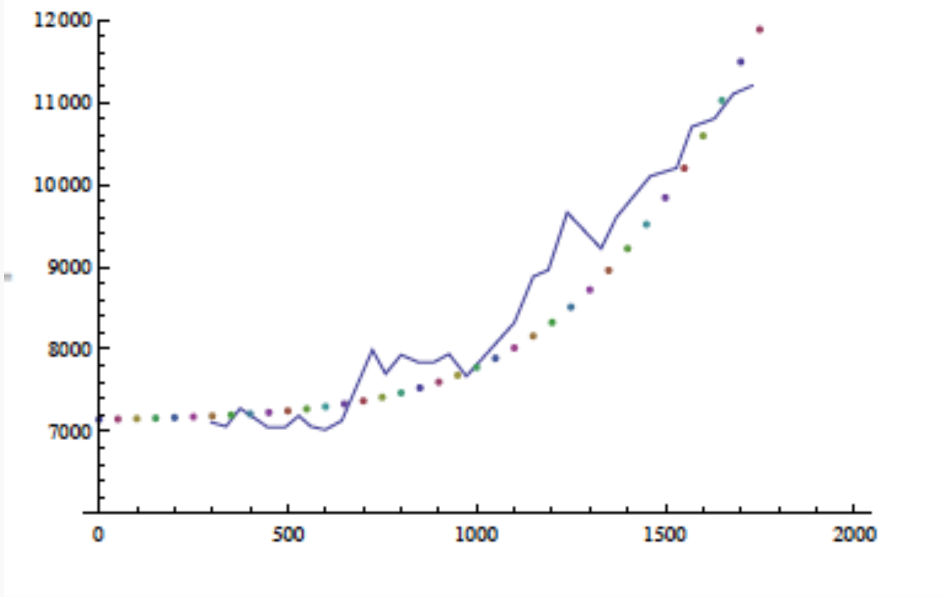
$$Q = \mathbf{v_w} \cdot \nabla P_c.$$

What is v_w?

- Self confinement picture: Alfven waves amplifed by super-Alfvenic cosmic rays flowing down their pressure gradient.
  - $Q = |v_A \nabla_{||} P_c|$.
- Extrinsic turbulence picture: Waves produced by an MHD turbulent cascade.
  - For balanced turbulence, $Q = 0$ & there is some Fermi acceleration.
  - For imbalanced turbulence, could be similar to self confinement picture.
- Long recognized that self confinement only works up to a certain energy.

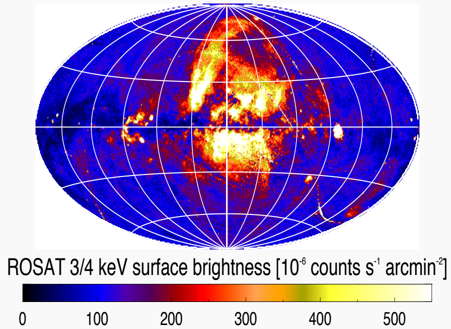
Cosmic Ray Heating of Diffuse Interstellar Gas

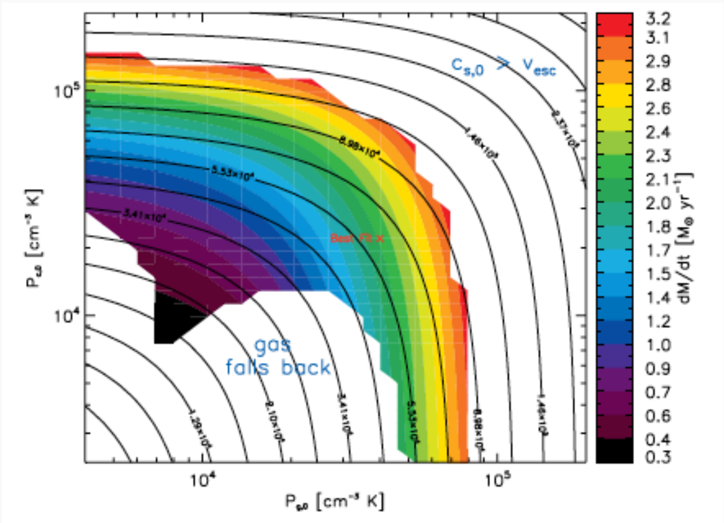


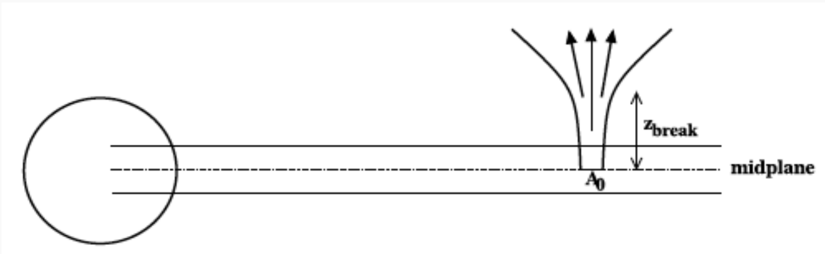


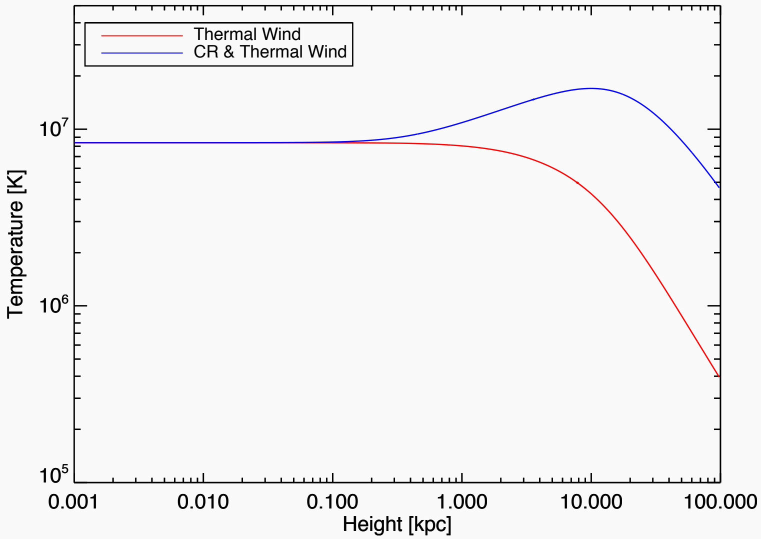
Left: Galactic Ha emission, showing a thick layer of warm ionized gas. Right: Model of thermal equilibrium , including cosmic ray heating (Wiener et al. in preparation)

Galactic Wind









<u>Top left</u>: Soft x-ray sky,
<u>Bottom left</u>: Magnetic flux tube geometry.
<u>Top right</u>: Domains of flow, with mass loss rates
<u>Bottom right</u>: Gas temperature with & without cosmic ray heating.

Everett et al. 2008 ApJ

Streaming Instability

Resonance condition

$$k \sim \frac{qB}{p\mu}$$

Higher energies resonate with longer wavelength waves, but as $\mu \to 0$, $k \to \infty$. Instability growth rate

$$\Gamma_c \sim \omega_{cp} \frac{n_c(> p_1)}{n_i} \left(\frac{v_D}{v_A} - 1 \right).$$

Because of powerlaw energy spectrum, Γ_c declines with cosmic ray energy; for $f(p) \propto p^{-a}$,$n_{cr}(> p_1) \propto p_1^{3-a}$. Linearly polarized waves are unstable to drift anisotropy; circularly polarized waves to pressure anisotropy.

Damping Balances Growth

- Ion - neutral friction in weakly ionized regions: *typically eliminates coupling under mean Milky Way conditions*.
- Nonlinear Landau damping on thermal ions: especially important in hot regions.
- Shearing apart of wave packets by background turbulence.

Marginal stability condition plus relationship between anisotropy & pressure gradient determines pressure profile & diffusivity κ_c. Self confinement works for $E < 100 - 200$ GeV for average Milky Way conditions.

Partially Ionized Regions

- Are cosmic rays repelled from high B regions (e.g. molecular clouds) by magnetic mirroring?
- Can diffusive shock acceleration operate in molecular gas?
- Do cosmic rays transfer momentum & energy to cool gas in galactic outflows?
- Figure of merit:

$$R_\Gamma \equiv \frac{\Gamma_c}{\Gamma_{in}}$$

Alfven Waves in Weakly Ionized Gas

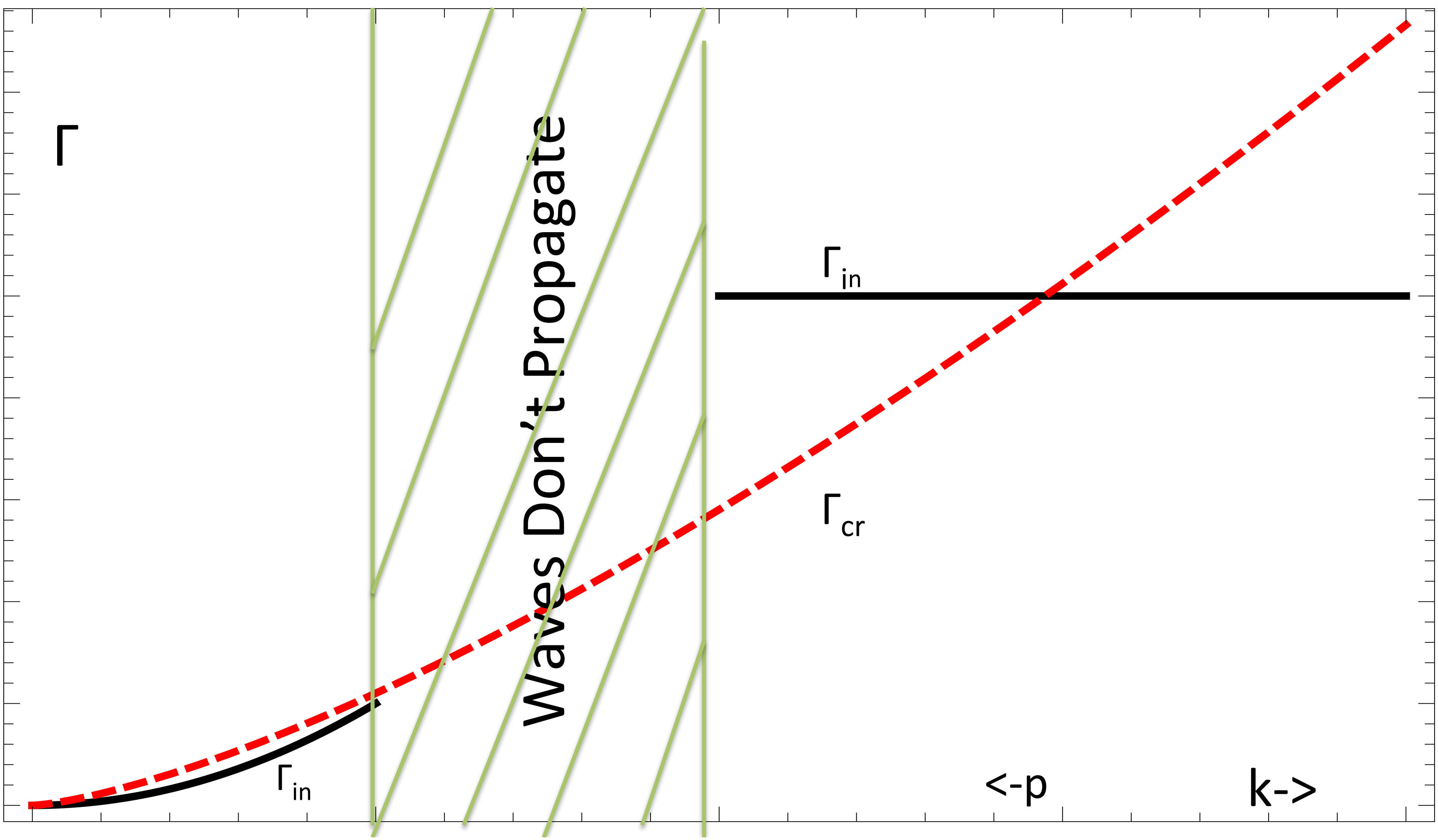
- Strongly coupled: $\omega/k \sim B/\sqrt{4\pi\rho} \equiv kv_A$; $\Gamma_{in} \sim \omega^2/(2\nu_{ni})$.
- Weakly coupled: $\omega/k \sim B/\sqrt{4\pi\rho_i} \equiv kv_{Ai}$; $\Gamma_{in} \sim \nu_{in}/2$.
- Strong coupling for $k < k_l \equiv 2\nu_{ni}/v_A$; weak coupling for $k > k_u \equiv \nu_{in}/(2v_{Ai})$.
- Nonpropagation for $k_l < k < k_u$.

Instability Regimes

Galactic molecular gas: $n_i \sim 10^{-5} \mathrm{n}_n^{1/2}$.

- $p_1(k_u)/m_p c \sim 1.5 \times 10^4 B_\mu^2 / n_n^{5/4}$: scattering waves are weakly coupled w. maximal Γ_{in}.
- $p_1(k_l)/m_p c \sim 3.4 \times 10^5 B_\mu^2 / n_n$; scattering waves are strongly coupled w. weaker $\Gamma_{in} \propto k^2$.
- For $p_1(k_u) < p_1 < p_1(k_l)$, waves do not propagate.

Trapping in Partially Ionized Gas



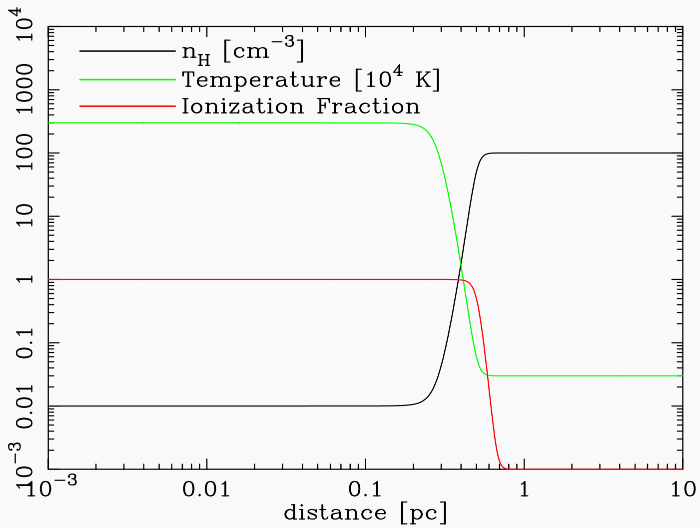
A Simple Cloud Model

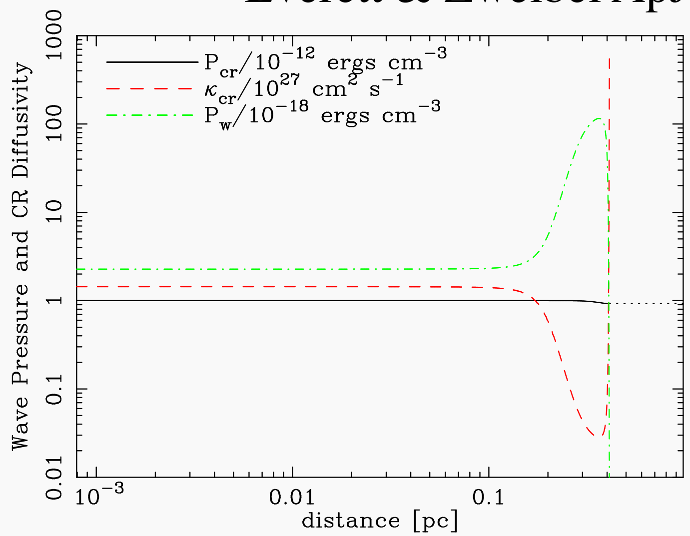
J. Everett & EZ 2011

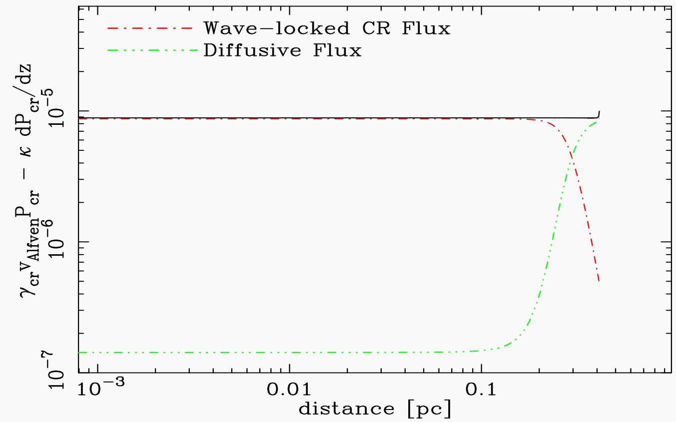
- 1D setup w. simple tanh profiles prescribed for ρ, T, x_i, constant B
- Solve steady state equations for
  - P_{cr} with advection, diffusion, & collisional losses
  - P_w with excitation by cosmic ray streaming & collisional damping
  - Imposed intercloud cosmic ray pressure gradient appropriate to a galactic wind
- Typically, coupling of cosmic rays to cloud breaks down inside a thin skin, although increased B, P_{cr}, and x all increase the coupled region.

Cosmic Ray Coupling to Clouds

Everett & Zweibel ApJ 2011







Top left: Model cloud setup. Top Right: Cosmic ray & wave pressure vs. depth. Bottom right: Transition from advection to diffusion, followed by free streaming -> **No force on the bulk of the cloud** .

DSA in Molecular Clouds

- Figure of merit is

$$R_\Gamma \sim \frac{\omega_{cp}}{\nu_{in}} \frac{n_{cr}}{n_i} \frac{v_S}{v_A} \sim 10^{10} \frac{n_{cr}}{n_i} \frac{v_S}{100km/s} \left(\frac{\rho_i}{\rho_n} \right)^{1/2}$$

- Conditions for excitation can easily be met *once cosmic ray acceleration is already efficient.*

Coupling in Galactic Outflows

Problem: Galactic outflows are multiphase, & include both hot & cool gas.

- Figure of merit is

$$R_\Gamma \sim \frac{\omega_{cp}}{\nu_{in}} \frac{n_{cr}}{n_i} \sim 10^7 \frac{B_\mu}{n_n} \frac{n_{cr}}{n_i}$$

- In M82, $B_\mu \sim 200 - 300$, $n_{cr} \sim 100 - 200 n_{crMW} \rightarrow$ excitation can overcome damping.

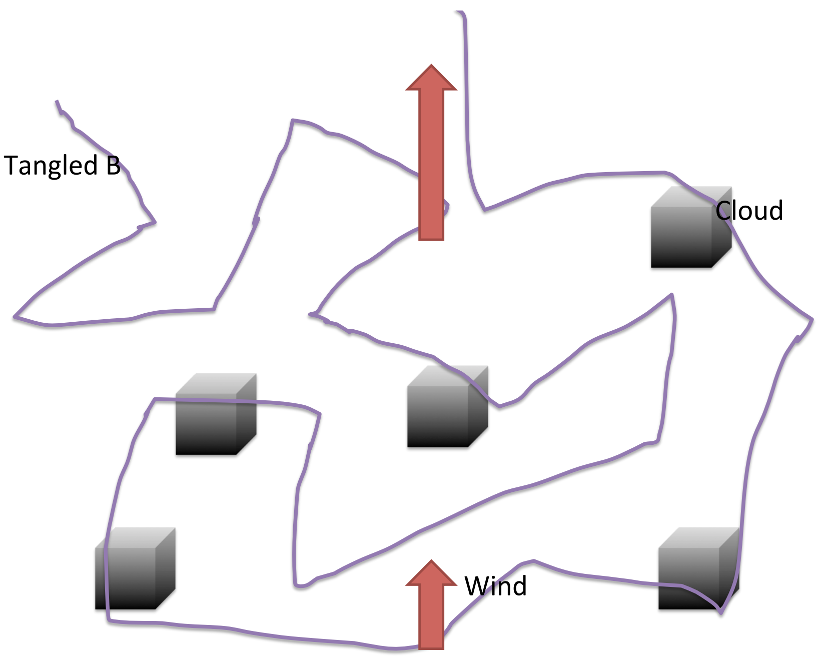
Mirroring

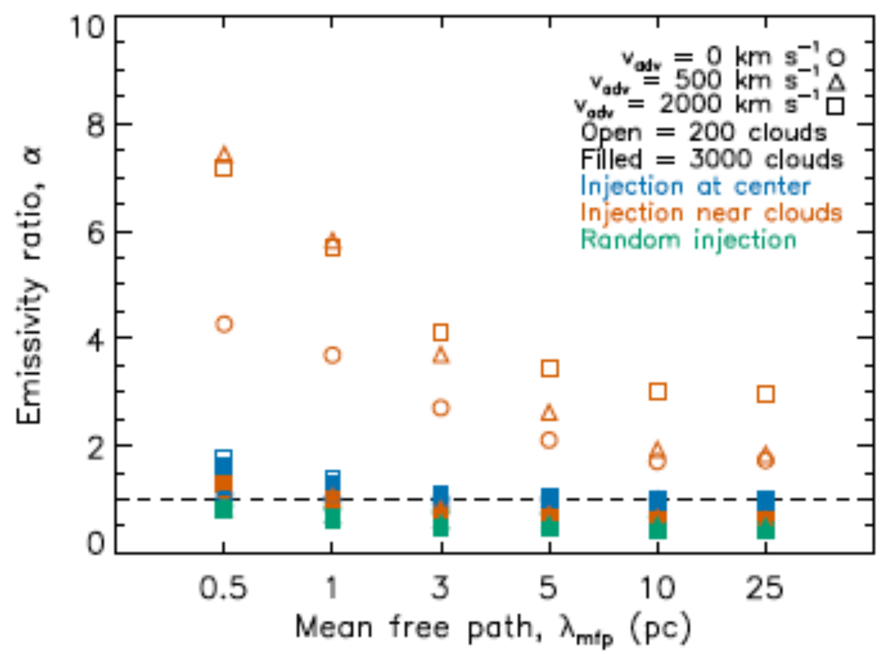
Particles “feel" the mirror force only if their distribution is anisotropic:

$$\frac{df}{dt}_{\nabla B} = -\frac{p(1-\mu^2)}{2m}\frac{d\ln B}{ds}\frac{\partial f}{\partial \mu}$$

- Does scattering prevent mirroring in molecular clouds?
- Can pressure ansiotropic destabilize waves & increase scattering?

Do Cosmic Rays Sample the Mean ISM Density?





Boettcher et al. 2013

Fits to the Gamma Ray Spectrum of M82

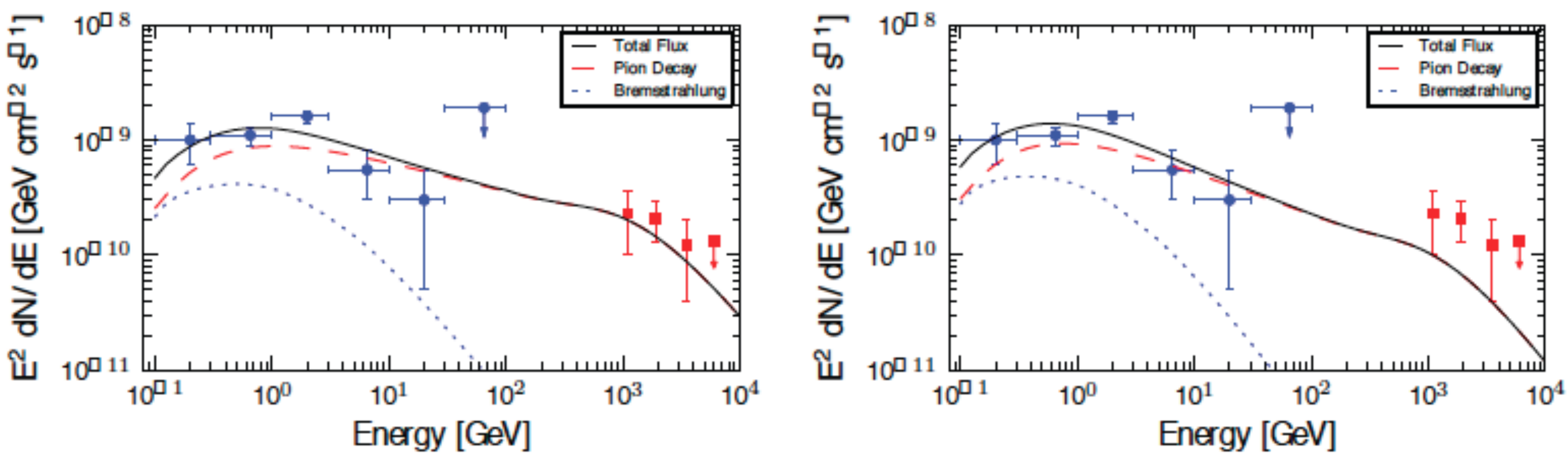


FIG. 5.— γ-ray spectra. *Left:* γ-ray spectrum with parameters $p = 2.1$, $M_{mol} = 4 \times 10^8$ $M_\odot$, $B = 275$ μG, $v_{adv} = 500$ km s^{-1}, $n_{ion} = 100$ cm^{-3}. This spectrum is the best fit to the γ-ray data. *Right:* γ-ray spectrum with parameters $p = 2.2$, $M_{mol} = 4 \times 10^8$ $M_\odot$, $B = 275$ μG, $v_{adv} = 400$ km s^{-1}, $n_{ion} = 150$ cm^{-3}. This is the γ-ray spectrum that corresponds to the best radio fit for a spectral index of $p = 2.2$. The solid black lines represent the total γ-ray flux, the dashed red lines represent the contribution from neutral pion decay, and the dotted blue lines represent the contribution from bremsstrahlung. γ-ray data include: Ackermann et al. (2012) (*Fermi* - blue circles), Acciari et al. (2009) (VERITAS - red squares). Data with downward arrows represent upper limits for both Fermi and VERITAS data.

YEGZ ApJ 2013

Fitting Parameters

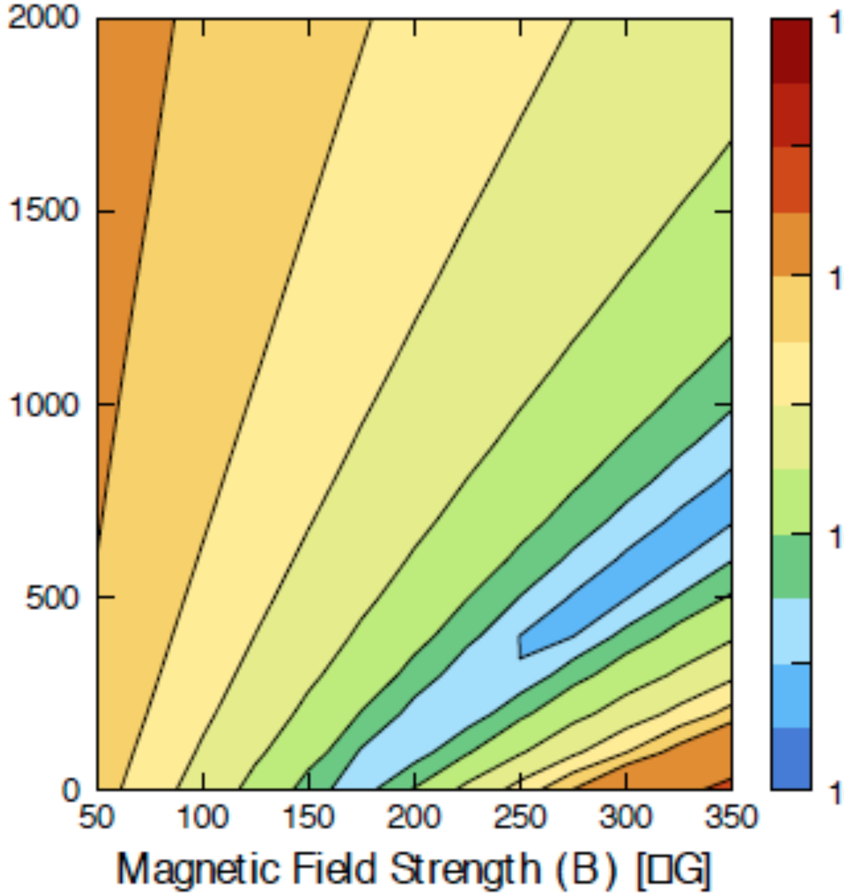


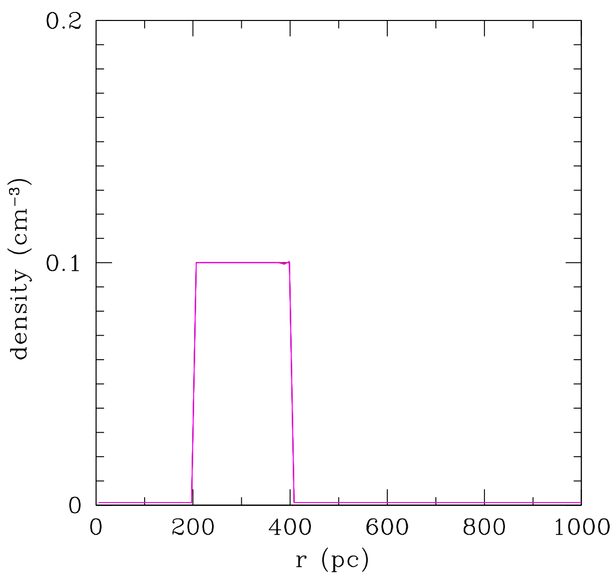
TABLE 3
BEST-FIT MODEL PARAMETERS

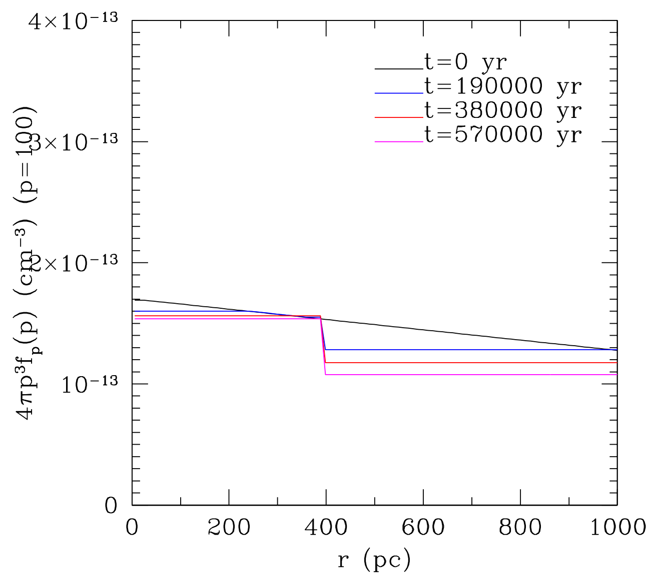
Physical Parameters	Best-Fit Value
Magnetic Field Strength (B)	275 □G
Advection (Wind) Speed (v_{adv})	500 km s^{-1}
Ionized Gas Density (n_{ion})	100 cm^{-3}
Spectral Index (p)	2.1
Molecular Gas Mas (M_{mol})	4×10^8 $M_\odot$

NOTE. — Results for $□^2_{radio} = 22.6$, $□^2_\gamma = 9.6$

M82 is an excellent electron calorimeter
And a ~50% proton calorimeter.

Bottleneck Effect





If v_A has a minimum, the adiabatic relation requires P_{cr} to have a maximum, which would have cosmic rays streaming up their density gradient. A plateau forms instead. From Wiener et al. 2014

Wrinkles in the Picture

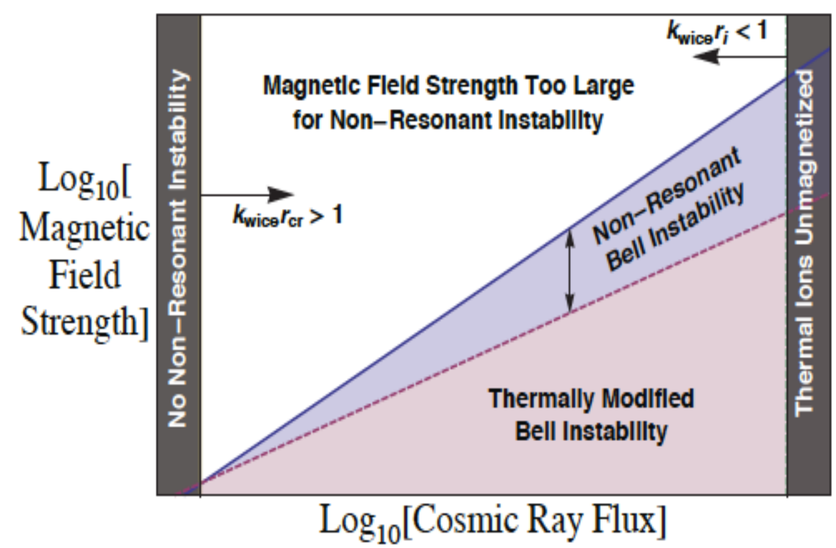
- Breakdown of diffusion theory as $\mu \to 0$; does $v_D \to v_A$? *addressed with mirroring by Felice & Kulsrud 1991*.
- Drastically modified waves and growth rates for $U_c/U_B > c/v_D$.
- Modification of Alfven wave dispersion relation for $\beta > (c/v_i)^2$.

Extrinsic Turbulence

- MHD cascade is too anisotropic to scatter cosmic rays efficiently; compressive waves must be generated.
- If turbulence is balanced, cosmic rays extract energy from the background rather than donating energy to it.
- Long confinement times & low anisotropy requirements can be met, but requires a separate theory for the turbulence.

Nonresonant Instabilities

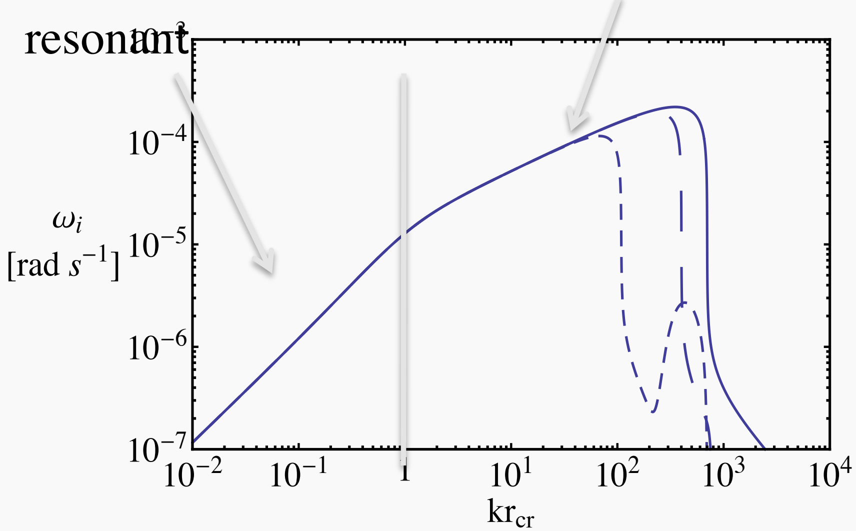
- When $U_{cr}/U_B > c/v_D$ there is a new, nonresonant instability driven by the electron current that compensates the cosmic ray current.
- Conditions are met at shocks, and possibly in young galaxies.



Rapid Growth to Nonlinear Amplitude

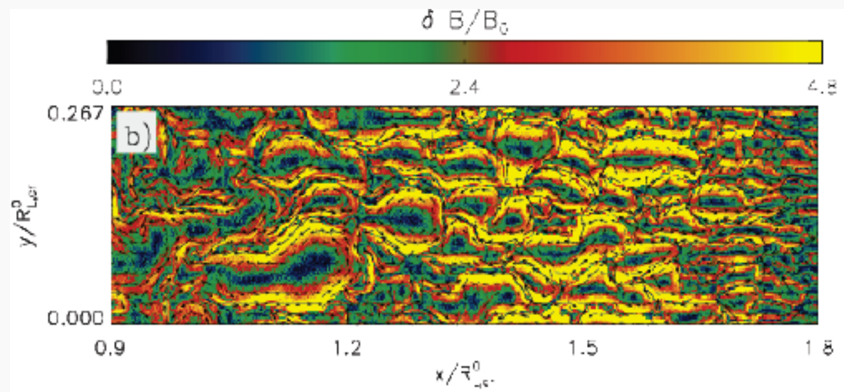
nonresonant

resonant



Linear growth rates (Zweibel & Everett 2010)

PIC simulation showing magnetic field growth in a shock layer.



Riquelme & Spitkovsky 2010

Simulations suggest that the magnetic field can be amplified, producing the observed thin synchrotron rims, increasing the acceleration rate, producing a new saturated state.

Conclusions

- The "classical" theory of cosmic ray self confinement leads to a hydrodynamic description of cosmic rays that describes their coupling to the thermal gas.
- The theory breaks down in weakly ionized clouds except at high cosmic ray fluxes and/or strong ambient magnetic fields.
- Complete understanding of cosmic ray propagation & coupling to thermal gas will require progress in turbulence theory.