Leptonic CP-Violation and Leptogenesis

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After the high precision measurement of $\sin\theta_{13} = 0.15$ (Daya Bay, RENO), one of the next most important goals of the future research in neutrino physics - determine the status of the CP symmetry in the lepton sector.

All compelling $\nu-$ oscillation data is compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}(x)\,, \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

The current "reference scheme": 3-ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj}\,|\nu_j>, \quad \nu_j: \ m_j \neq 0; \quad l = e, \mu, \tau; \quad n = 3;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}(x), \quad \nu_{j\mathrm{L}}(x): \ m_j \neq 0; \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: the 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_\tau) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U, m_j^2 - m_k^2)$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j– Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j– Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48\ (2.44) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425\ (0.437)$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234\ (0.0239)$, NH (IH).

F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- $\mathrm{sgn}(\Delta m^2_{\mathrm{atm}}) = \mathrm{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\mathrm{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\mathrm{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

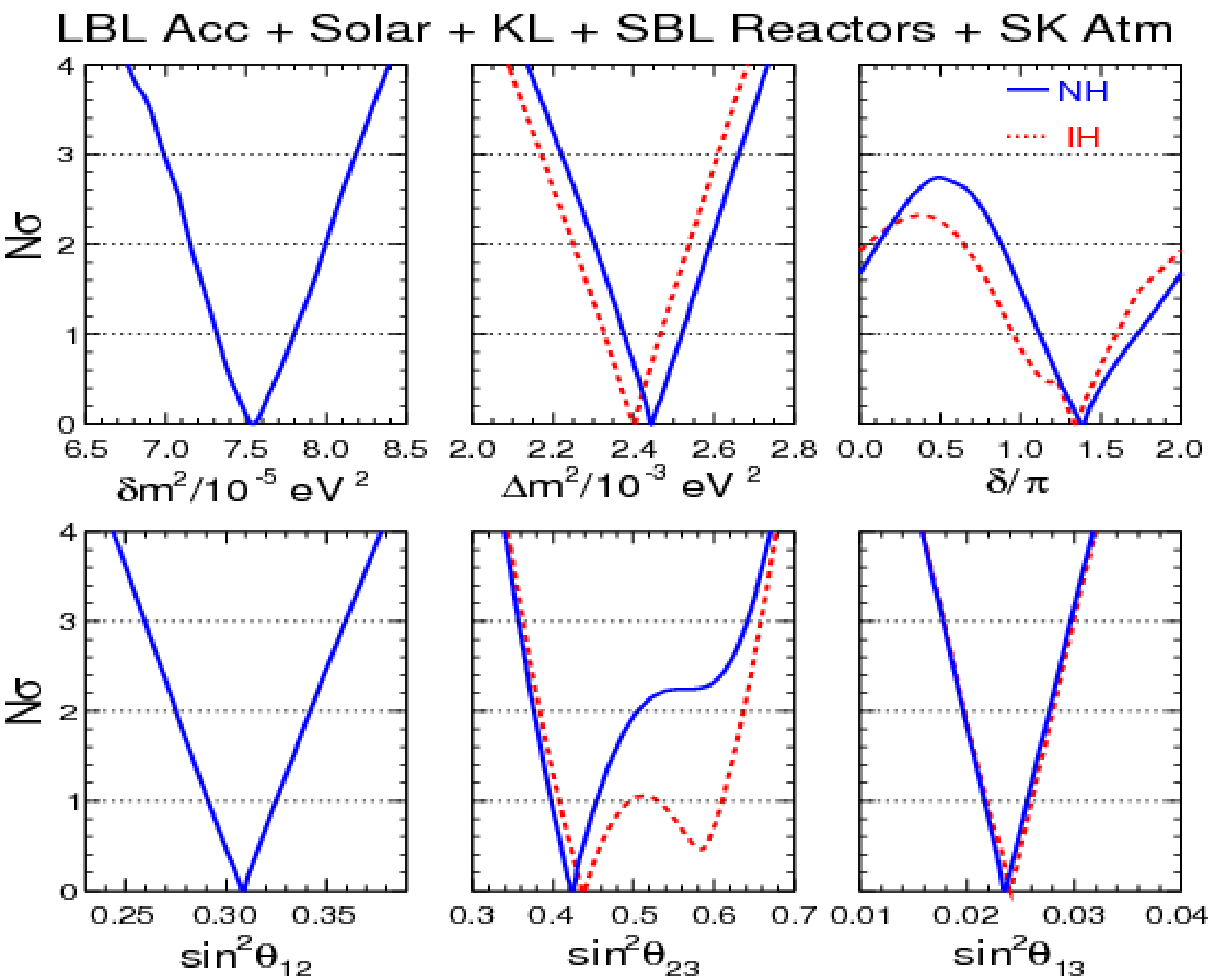
$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\mathrm{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

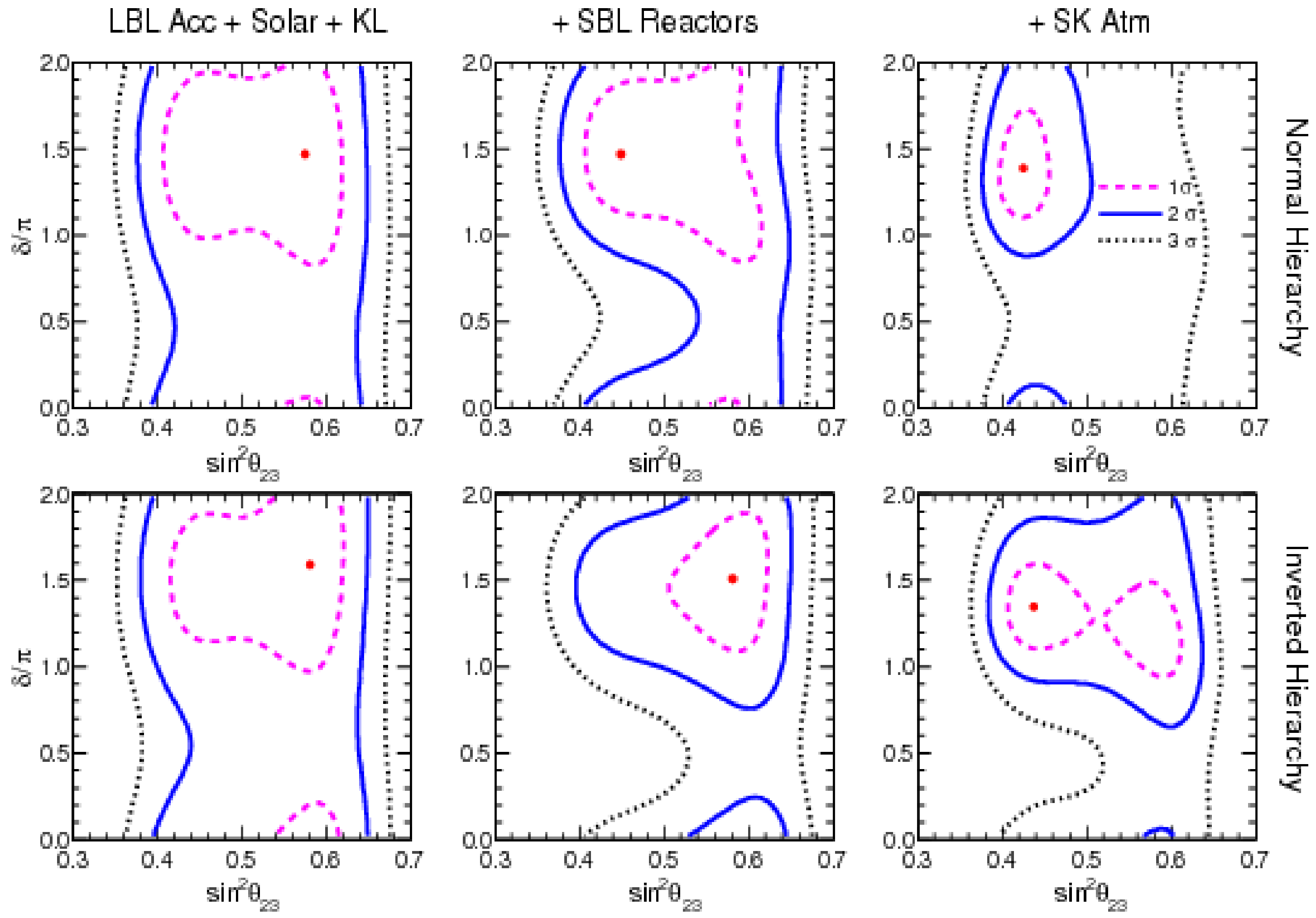
- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241\ (0.0244)$, NH (IH).

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}\ \text{eV}^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28\ (3\sigma)$,
- $|\Delta m^2_{31(32)}| \cong 2.48\ (2.44) \times 10^{-3}\ \text{eV}^2$, $\sin^2\theta_{23} \cong 0.425\ (0.437)$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234\ (0.0239)$, NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$, $1\sigma(\sin^2\theta_{23}) = 14\%$;
- $1\sigma(\sin^2\theta_{13}) = 10\%$,
- $3\sigma(\Delta m^2_{21}):\ (6.99 - 8.18) \times 10^{-5}\ \text{eV}^2$; $3\sigma(\sin^2\theta_{12}):\ (0.259 - 0.359)$;
- $3\sigma(|\Delta m^2_{31(23)}|):\ 2.19(2.17) - 2.62(2.61) \times 10^{-3}\ \text{eV}^2$;
  $3\sigma(\sin^2\theta_{23}):\ 0.331(0.335) - 0.637(0.663)$;
- $3\sigma(\sin^2\theta_{13}):\ 0.0169(0.0171) - 0.0313(0.0315)$.

Fogli *et al.*, arXiv:1205.5254v3



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

Large $\sin\theta_{13} \cong 0.15$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13}\,\sin\delta| \gtrsim 0.1$$

S. Pascoli, S.T.P., A. Riotto, 2006.

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!)

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987
  - $\left|< m >\right|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ ,\quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ ,\qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger, S. Pakvasa et al.,1980.

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ ,\quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l':\quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

3ν — mixing:

$$A^{(l,l')}_{\text{CP}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})\ ,\quad l \neq l' = e, \mu, \tau$$

$$A^{(l,l')}_{\text{T}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

$$A^{(e,\mu)}_{\text{T(CP)}} = A^{(\mu,\tau)}_{\text{T(CP)}} = -A^{(e,\tau)}_{\text{T(CP)}}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa *et al.*, 1980

In vacuum: $A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin\left(\frac{\Delta m^2_{21}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E}L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\mathrm{CP}: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\mathrm{CPT}: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \; l \neq l'$$

In matter with constant density: $\mathrm{A}^{(e,\mu)}_{\mathrm{T}} = \mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} \mathrm{F}^{\mathrm{mat}}_{\mathrm{osc}}$

$$\mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} = \mathrm{J}^{\mathrm{vac}}_{\mathrm{CP}}\; \mathrm{R}_{\mathrm{CP}}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

P. Harrison, S. Scott, 2000

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U_{e2}^*\,U_{\mu 1}^*\right\}\ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\left\{U_{e1}U_{e3}^*\right\},\quad S_2 = \text{Im}\left\{U_{e2}U_{e3}^*\right\}\quad \text{(not unique); or}$$
$$S_1' = \text{Im}\left\{U_{\tau 1}U_{\tau 2}^*\right\},\quad S_2' = \text{Im}\left\{U_{\tau 2}U_{\tau 3}^*\right\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\left\{U_{e1}U_{e3}^*\right\} \neq 0$ and $\text{Re}\left\{U_{e1}U_{e3}^*\right\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Predictions for the CPV Phase δ

Theories with $U \sim U_{\text{TBM,BM,GR,...}}$
(+ "minimal" correcting U_ℓ giving $\sin\theta_{13} \cong 0.15$, $\sin^2\theta_{23} \cong 0.4$, $\sin^2\theta_{12} \cong 0.31$):

$$\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

For $U \sim U_{\text{TBM}}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \quad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

Models with $U \sim U_{\text{BM}}$:

$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

The next most important steps are:

- determination of the status of the CP symmetry in the lepton sector;
- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or min(m_j)).

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on $^3\mathrm{H} \rightarrow ^3\mathrm{He} + e^- + \bar{\nu}_e$:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Cosmological and astrophysical data imply (depending on the model complexity and the input data used):

$$\sum_j m_j \leq (0.3 - 1.3) \text{ eV} \quad (95\% \text{ C.L.})$$

K.N. Abazajian *et al.*, arXiv:1103.5083

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + ΛCDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04) \text{ eV}.$$

NH: $\sum_j m_j \leq 0.05$ eV (3σ);

IH: $\sum_j m_j \geq 0.10$ eV (3σ).

These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q, \; q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\begin{aligned}
\mathcal{L}^{\text{lep}}(x) &= \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,, \\
\mathcal{L}_{\text{CC}} &= -\frac{g}{\sqrt{2}}\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,, \\
\mathcal{L}_{\text{Y}}(x) &= \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,, \\
\mathcal{L}_{\text{M}}^{\text{N}}(x) &= -\frac{1}{2}M_i\,\overline{N_i}(x)\,N_i(x)\,.
\end{aligned}$$

ψ_{lL} - LH doublet, $\psi_{lL}^{\text{T}} = (\nu_{lL}\ \ l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\ \ v = 174\ \text{GeV},\ \ v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\ \lambda^T\,M_R^{-1}\,\lambda = U_{\text{PMNS}}^{*}\ m_\nu^{\text{diag}}\ U_{\text{PMNS}}^{\dagger}\ .$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\text{PMNS}})^\dagger / v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

$$m_\nu \simeq v^2 \; \lambda^T M_R^{-1} \lambda = U_{\mathsf{PMNS}}^* \, m_\nu^{\mathsf{diag}} \, U_{\mathsf{PMNS}}^\dagger \, ,$$

$$\lambda \equiv Y_\nu$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\text{PMNS}})^\dagger / v_u$, all at M_R;

R-complex, $R^T R = 1$.

J.A. Casas and A. Ibarra, 2001

$D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

Models: R - CP conserving ($SU(5) \times T'$); CPV parameters in R determined by the CPV phases in U (class of A_4 models).
Texture zeros in Y_ν: CPV parameters in R and U - related.

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j, \;\; C(\overline{N}_k)^T = N_k, \;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$U_{\rm CP}\, N_j(x)\, U^{\dagger}_{\rm CP} \;=\; \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i\,,$$
$$U_{\rm CP}\, \nu_k(x)\, U^{\dagger}_{\rm CP} \;=\; \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho_k^\nu = \pm i\,.$$

CP-invariance:

$$\lambda^*_{jl} \;=\; \lambda_{jl}\, (\eta_j^{NCP})^*\, \eta^l\, \eta^{H*}\,, \quad j = 1,2,3, \;\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i, \;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\lambda^*_{jl} \;=\; \lambda_{jl}\, \rho_j^N\,, \;\; \rho_j^N = \pm 1\,,$$
$$U^*_{lj} \;=\; U_{lj}\, \rho_j^\nu\,, \;\; \rho_j^\nu = \pm 1\,,$$
$$R^*_{jk} \;=\; R_{jk}\, \rho_j^N\, \rho_k^\nu, \quad j,k = 1,2,3, \;\; l = e,\mu,\tau,$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \;\equiv\; R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,,$$
$$CP: \quad P^*_{jkml} \;=\; P_{jkml}\, (\rho_j^N)^2\, (\rho_k^\nu)^2\, (\rho_m^\nu)^2 = P_{jkml}\,, \;\; {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at "high" E.

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \qquad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- **B number non-conservation.**
- **Violation of C and CP symmetries.**
- **Deviation from thermal equilibrium.**

Leptogenesis

- The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.

- When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.

- If $\Gamma(N_1 \to \Phi^- \ell^+) \neq \Gamma(N_1 \to \Phi^+ \ell^-)$, a lepton asymmetry will be generated.

- Wash-out processes, like $\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, **etc.** tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final resullt is a net (non-zero) lepton asymmetry.

- This lepton asymmetry is then converted into a baryon asymmetry by $(B+L)$ **violating but** $(B-L)$ **conserving** sphaleron processes which exist within the SM (at $T \gtrsim M_{\text{EWSB}}$).

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B:

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

2. solve the Boltzmann (or similar) equation to account for the wash-out of the asymmetry:

$$Y_{\mathsf{L}} = \kappa\, \varepsilon$$

where $\kappa = \kappa(\widetilde{m})$ is the "efficiency factor", $\widetilde{m}$ is the "the wash-out mass parameter" - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_{\mathsf{B}} = \frac{c_S}{g_*} \kappa \varepsilon$$

Baryon number violation in the SM

Instanton and Sphaleron processes

SU(2) instantons lead to (leading order) to effective 12 fermion $(B+L)$ nonconserving, but $(B-L)$ conserving, interactions:

$O(B+L) = \Pi_i q_{Li} q_{Li} q_{Li} l_{Li}$

These would induce $\Delta B = \Delta L = 3$ processes:

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \rightarrow \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at T = 0 the probability of such processes is $\Gamma/V \sim e^{-4\pi/\alpha} \sim 10^{-165}$.

't Hooft, 1976

At finite T, the transitions proceed via thermal fluctuations (over the barrier) with an unsuppressed probability (due to sphaleron (static) configurations - saddle "points" of the field energy of the $SU(2)$ gauge - Higgs field system):

$$\Gamma/V \sim \alpha^4 T^4.$$

Kuzmin, Rubakov, Shaposhnikov, 1985;
Arnold et al., 1987 and 1997.

Sphaleron processes are efficient (in the case of interest) at

$$T_{\rm EW} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Can generate $B \neq 0$, $L \neq 0$ at $T < T_{\rm EW} (< 10^{12}$ GeV$)$ from $(B-L)_0 \neq 0$ (with $(B-L) = const.$).

Harvey, Turner, 1990

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma: \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \;\; \varepsilon\, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ– efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

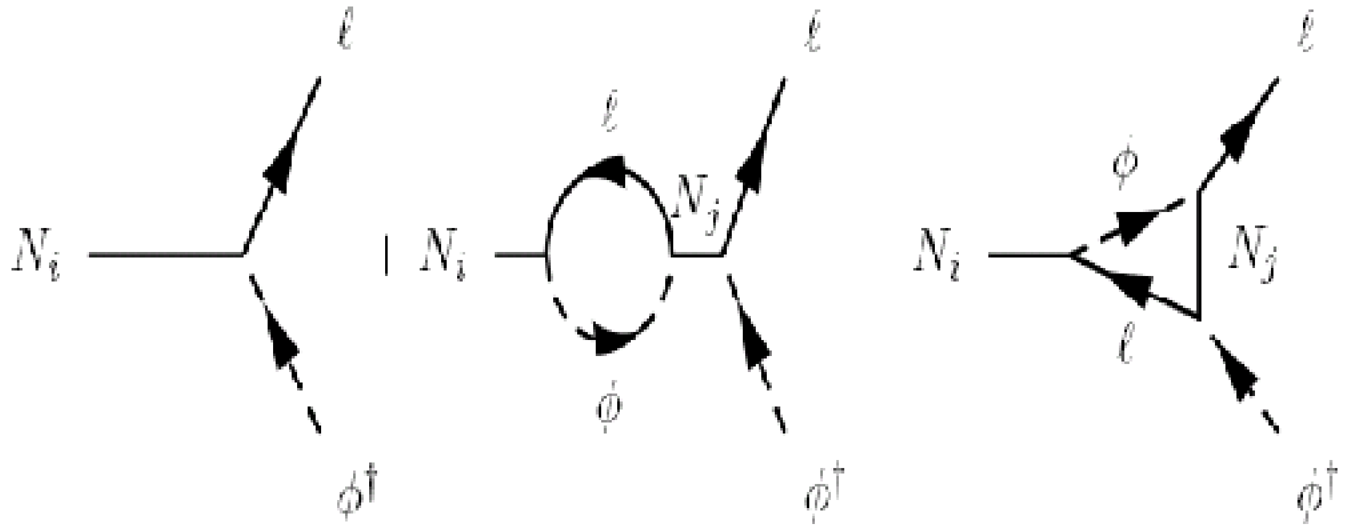
$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \, \ell^+) - \Gamma(N_1 \to \Phi^+ \, \ell^-)}{\Gamma(N_1 \to \Phi^- \, \ell^+) + \Gamma(N_1 \to \Phi^+ \, \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\quad \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004



N_i
ℓ
$\phi^\dagger$
N_i
ℓ
ϕ
N_j
ℓ
$\phi^\dagger$
N_i
ϕ
ℓ
N_j
ℓ
$\phi^\dagger$

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau \,.$$

The "one-flavor" approximation - $\mathbf{Y}_{e,\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l \; H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k |R_{1k}|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k |R_{1k}|^2 \,.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \to (\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+$; $(\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+ \to N_1$;

$\tau_L^- + \Phi^0 \to \tau_R^-$, $\tau_L^- + \tau_L^+ \to N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1\ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\, v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*}\left(\epsilon_2\,\eta\left(\frac{417}{589}\,\widetilde{m_2}\right) + \epsilon_\tau\,\eta\left(\frac{390}{589}\,\widetilde{m_\tau}\right)\right),$$

$$\eta\left(\widetilde{m_l}\right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1}.$$

$Y_B = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0,$

$$\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j) R_{1j} R_{1k} \mathrm{Im}\left(U_{\tau j}^* U_{\tau k}\right)}{\sum_j m_j |R_{1j}|^2}, \; R_{1j}R_{1k} = \pm |R_{1j}R_{1k}|, \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j) |R_{1j} R_{1k}| \mathrm{Re}\left(U_{\tau j}^* U_{\tau k}\right)}{\sum_j m_j |R_{1j}|^2}, \; R_{1j}R_{1k} = \pm i |R_{1j}R_{1k}|
\end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\mathrm{Im}\left(U_{\tau j}^* U_{\tau k}\right) \neq 0$, $\mathrm{Re}\left(U_{\tau j}^* U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right) \right)$$

$m_1 \ll m_2 \ll m_3$, $M_1 \ll M_{2,3}$; $R_{12}R_{13}$ – real; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}}|R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi$, $\delta = 0$: $\mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0$, CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2.8 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.2}\right) \left(\frac{M_1}{10^9\ \text{GeV}}\right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \qquad |J_{CP}| \gtrsim 2.0 \times 10^{-2}$$

The requirement $\sin\theta_{13} \gtrsim 0.09$ (0.11) - compatible with the Daya Bay result: $\sin\theta_{13} \cong 0.15$.

$|\sin\theta_{13}\sin\delta| \gtrsim 0.11$ implies $|\sin\delta| \gtrsim 0.7$ - compatible with $\delta \cong 3\pi/2$.

$\sin\theta_{13} \cong 0.15$ and $\delta \cong 3\pi/2$ imply relatively large (observable) CPV effects in neutrino oscillations: $J_{CP} \cong -3.5 \times 10^{-2}$.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05\ \text{eV}}\right) \left(\frac{M_1}{10^9\ \text{GeV}}\right) .$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m^2_{\odot}/|\Delta m^2_{\rm A}| \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

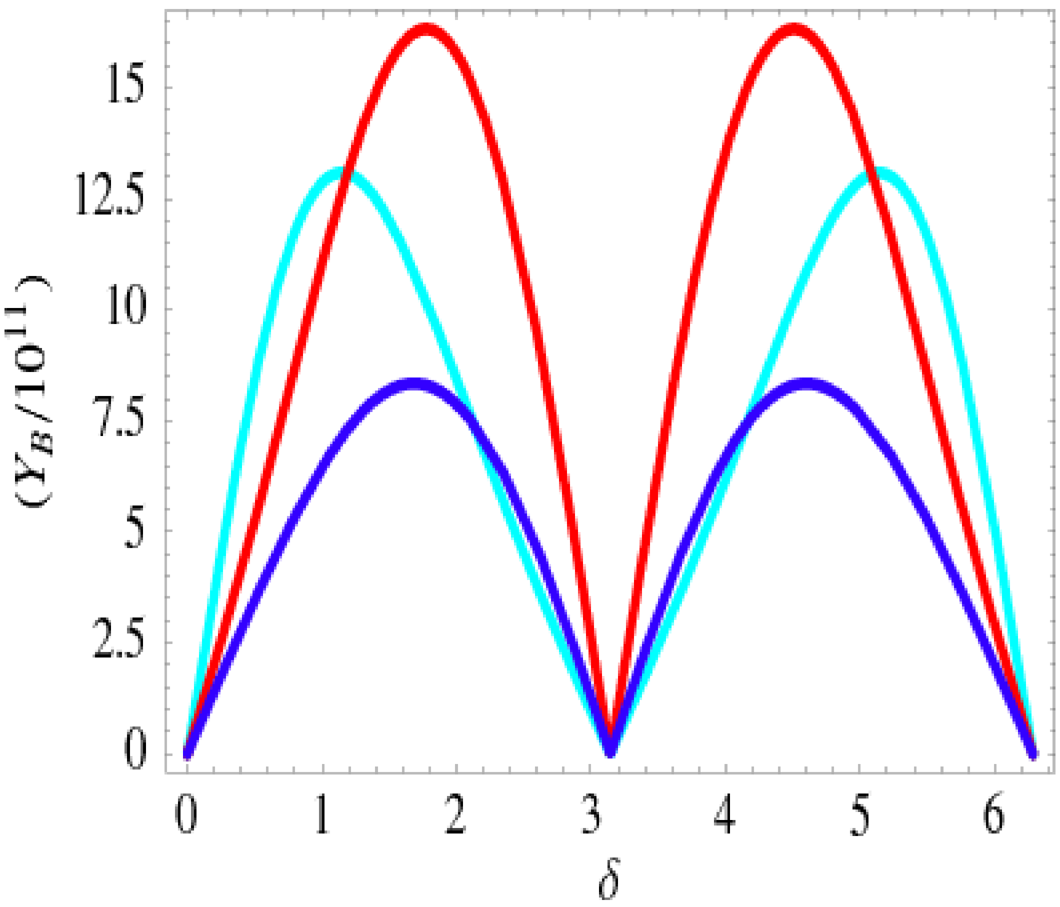
$$|Y_B| \cong 8.1 \times 10^{-12}\, |s_{13}\sin\delta| \left(\frac{M_1}{10^9\ \text{GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,.$$

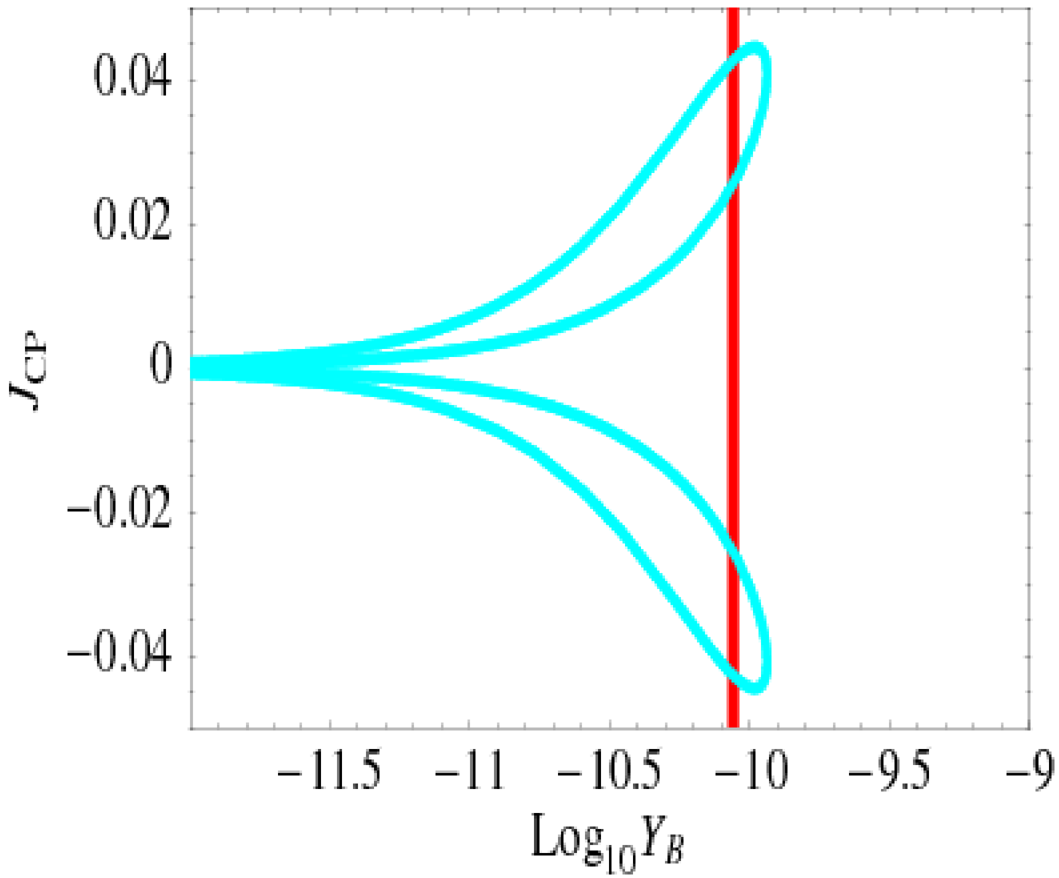
The lower limit corresponds to

$$|J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$



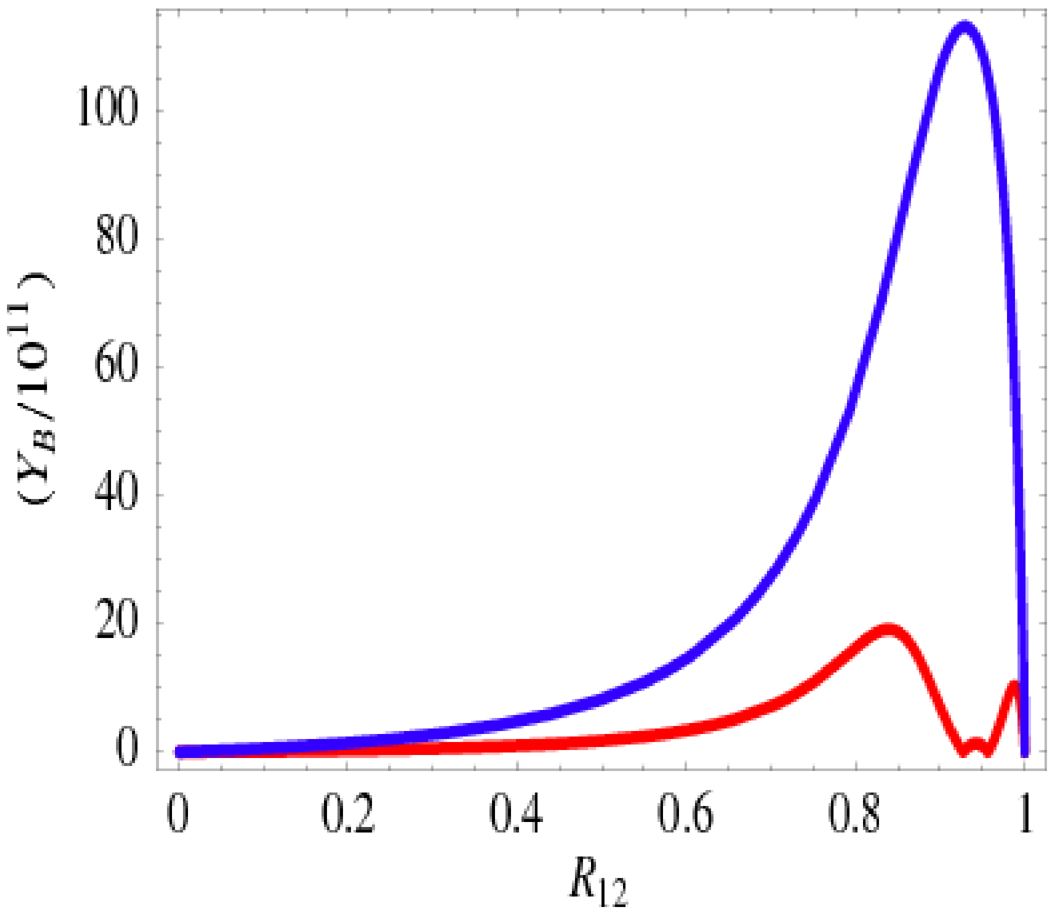
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

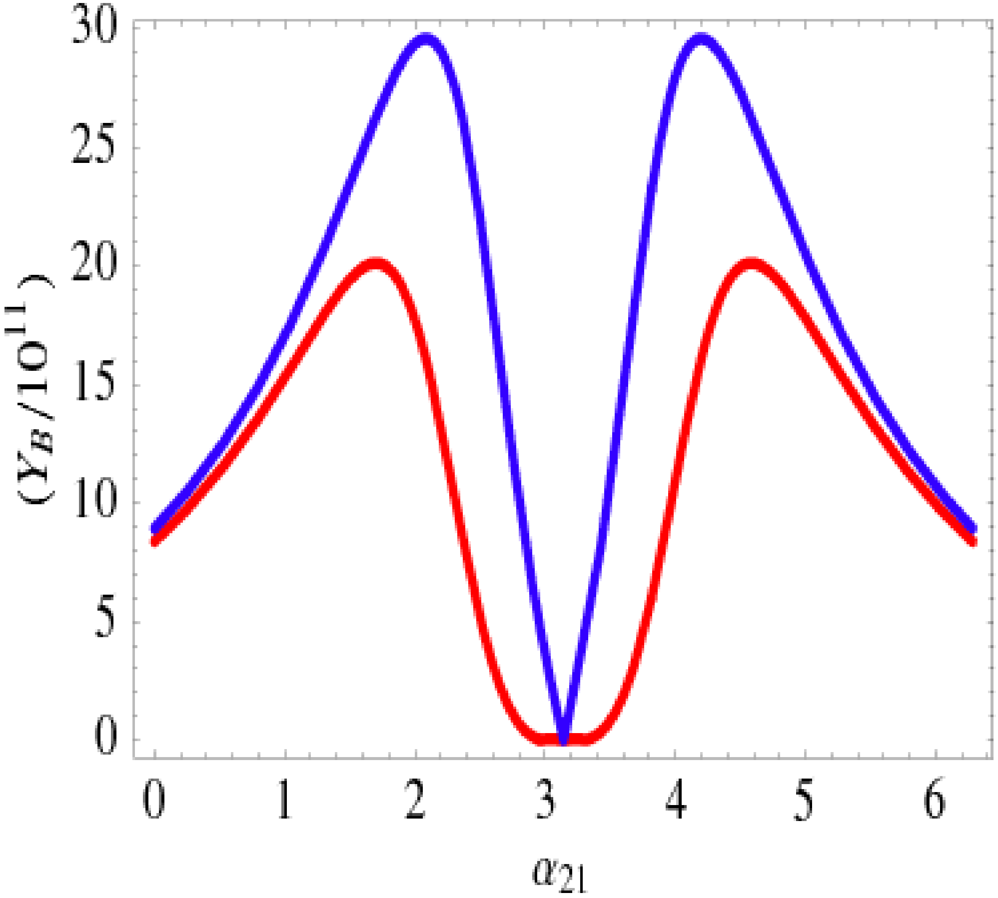


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0$ (2π);
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$ (-1) $(\beta_{23} = 0$ (π), $\kappa' = +1)$;
The red region denotes the 2σ allowed range of Y_B.

S. Pascoli, S.T.P., A. Riotto, 2006.

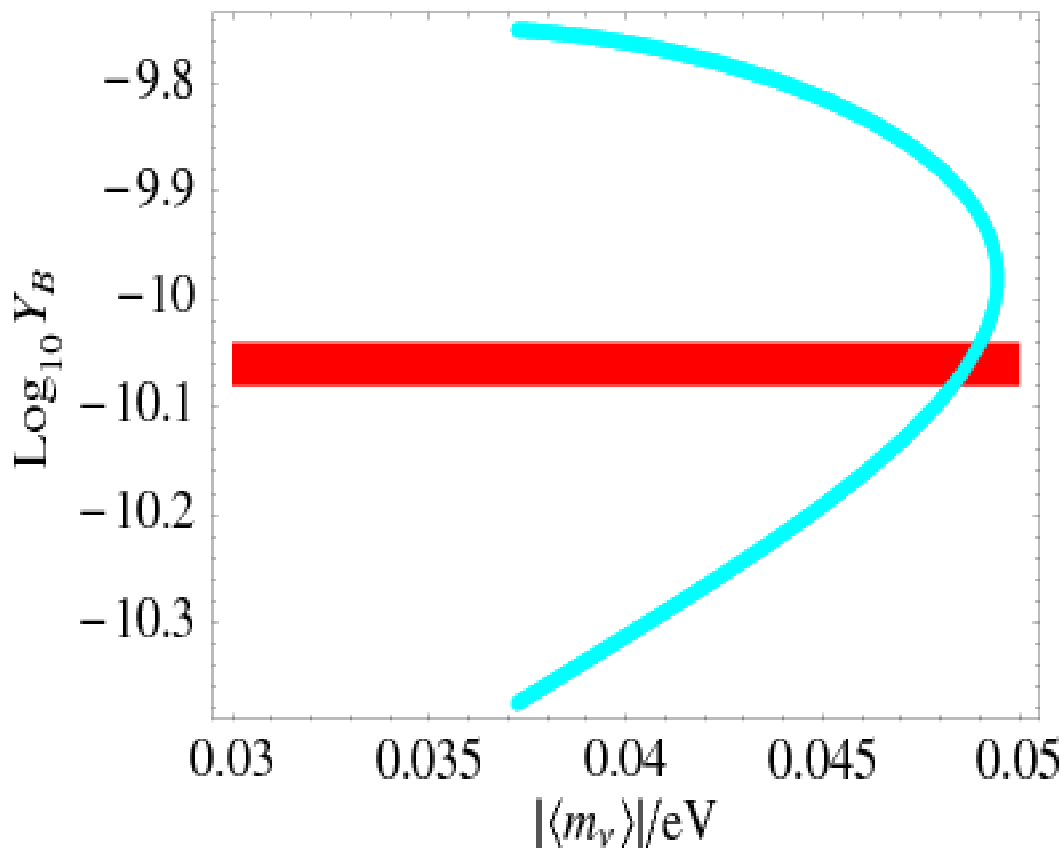


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign $(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)} R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

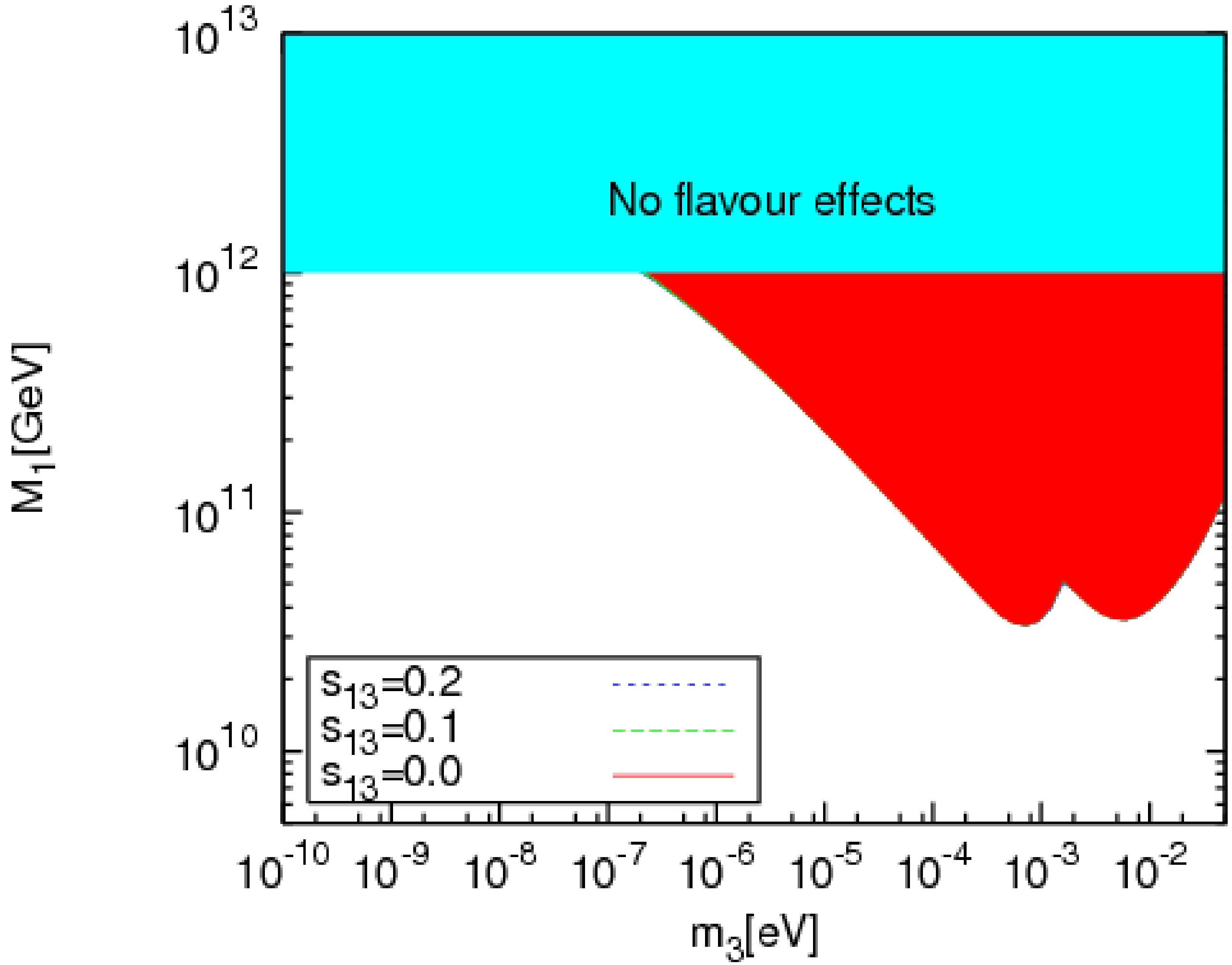
$$|\sin\theta_{13} \sin\delta|, \;\; \sin\theta_{13} \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{\rm CP}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 <$ $(\ll)$ $m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11} R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0;$

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Complex R: $\varepsilon_{1l} \neq 0$, CPV from U and R

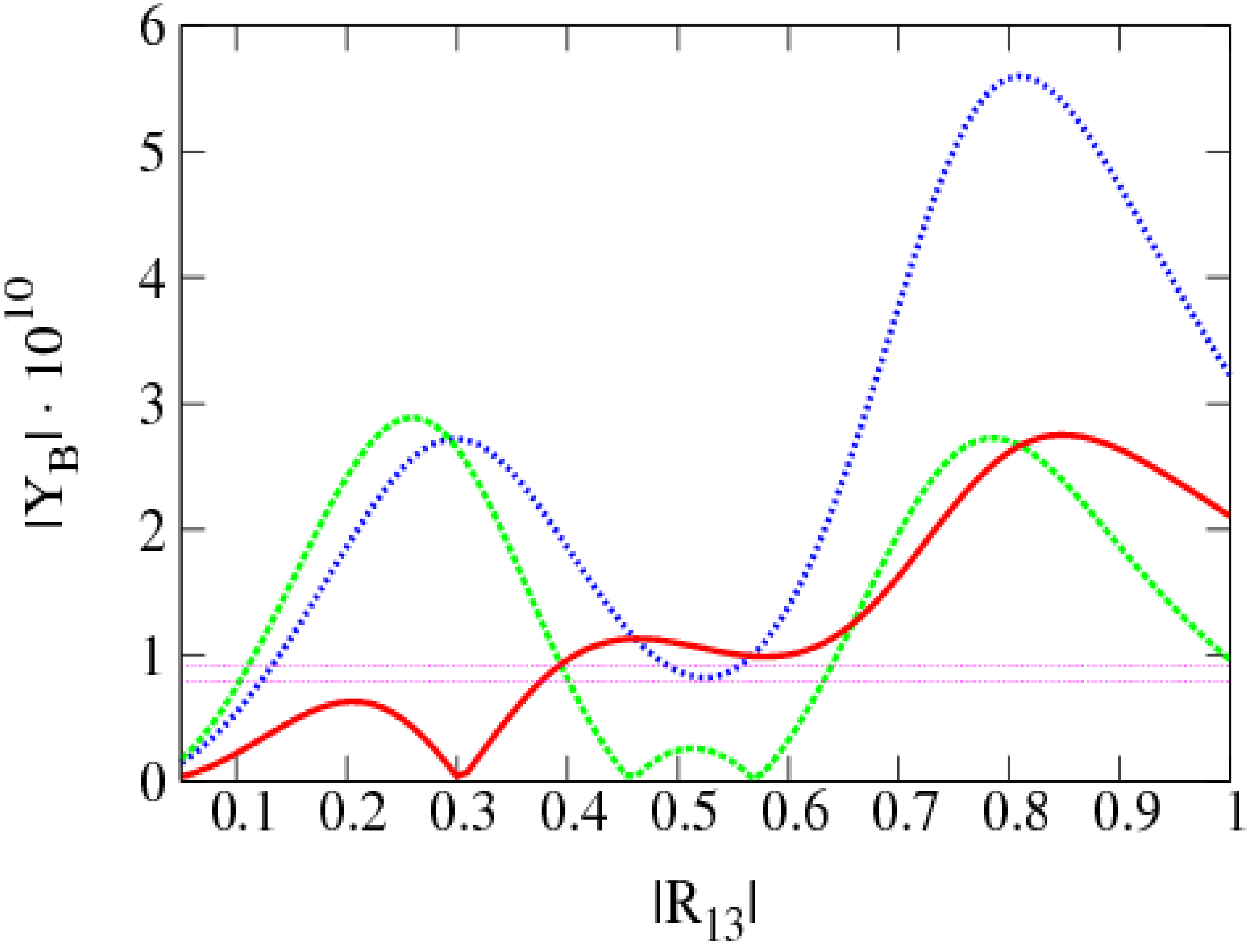
$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2 e^{i2\varphi_{12}} + |R_{13}|^2 e^{i2\varphi_{13}} = 1\,,$$

$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0: \ \mathrm{sgn}(\sin 2\varphi_{12}) = -\mathrm{sgn}(\sin 2\varphi_{13})\,.$$

$$\cos 2\varphi_{12} = \frac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}\,, \quad \sin 2\varphi_{12} = \pm\sqrt{1-\cos^2 2\varphi_{12}}\,,$$

$$\cos 2\varphi_{13} = \frac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}\,, \quad \sin 2\varphi_{13} = \mp\sqrt{1-\cos^2 2\varphi_{13}}\,.$$



$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0.2$, $\delta = 0$, $\sin^2\theta_{23} = 0.64$, $|R_{12}| \cong 1$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\rm HE}|$ (R CPV, blue), $|Y_B^0 A_{\rm MIX}|$ (U CPV, green), total $|Y_B|$ (red line)

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis (contd.)

E. Interesting case: CPV due to the Majorana phases in U_{PMNS} and the R-phases

$m_3 \ll m_1 < m_2$ (IH), $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $\text{Im}(R_{13}^2) = 0$.

$R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$:

$|R_{11}|^2 e^{i2\varphi_{11}} + R_{12}^2 e^{i2\varphi_{12}} + R_{13}^2 = 1$,

$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0$.

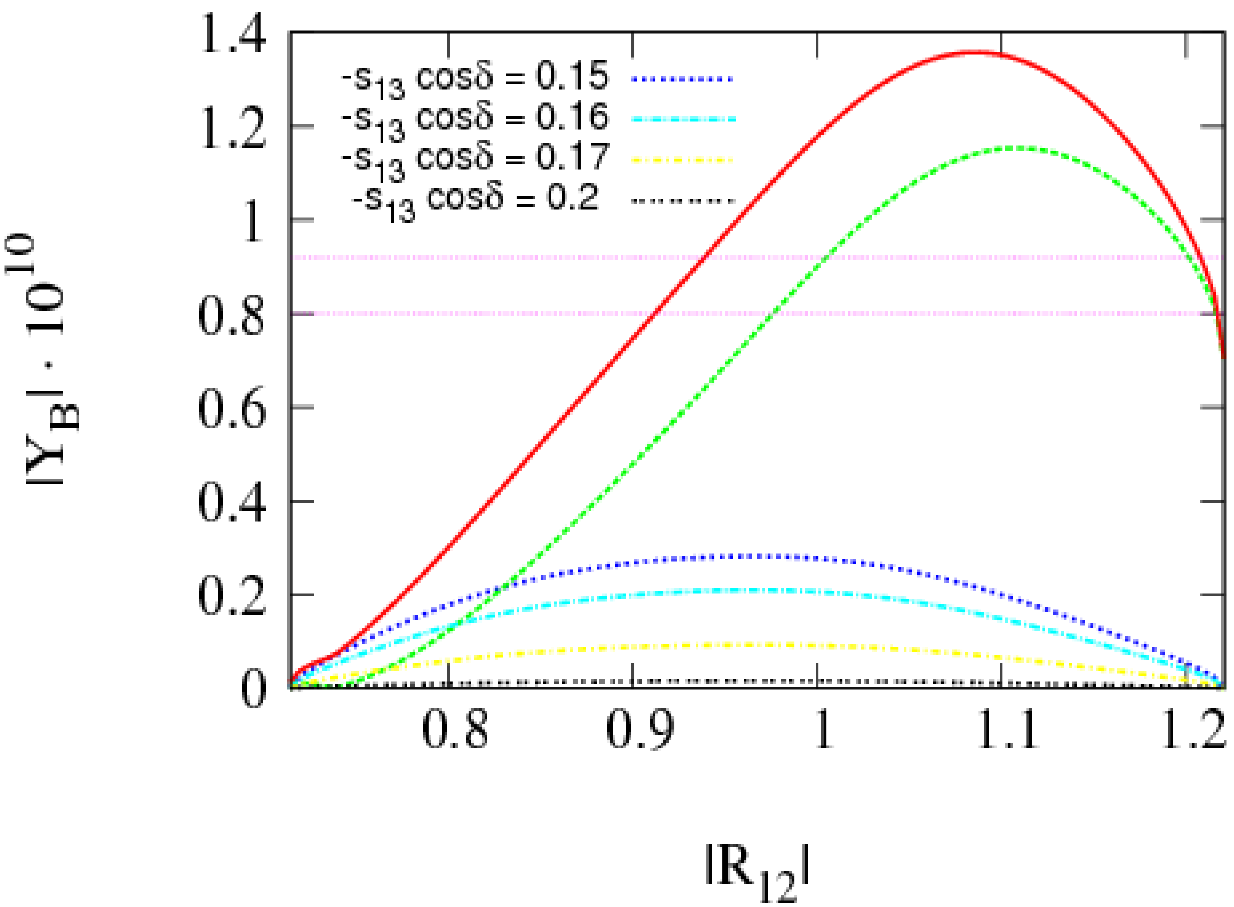
$|Y_B^0 A_{\text{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11}) \left(|U_{\tau 1}|^2 - |U_{\tau 2}|^2\right)$ - can be suppressed:

$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2) s_{23}^2 - 4 s_{12} c_{12} s_{23} c_{23} s_{13} \cos\delta \cong -0.20 - 0.92\, s_{13} \cos\delta$.

$\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 0.5$: $(-\sin\theta_{13}\cos\delta) \gtrsim 0.15$

($\sin^2\theta_{12} = 0.38$, $\sin^2\theta_{23} = 0.36$: $0.06 \lesssim (-\sin\theta_{13}\cos\delta) \lesssim 0.12$)

E. Molinaro, S.T.P., 2008, 2010.



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV,
$\alpha_{21} = \pi/2$, $(-s_{13}\cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\rm HE}|$ (R CPV, blue), $|Y_B^0 A_{\rm MIX}|$ (U CPV, green), total $|Y_B|$ (red line).

E. Molinaro, S.T.P., 2008

The preceding results: for

$|R_{13}|^2 |\sin(2\tilde{\varphi}_{13})| << \min(|R_{11,12}|^2 |\sin(2\tilde{\varphi}_{11,12})|$.

Results for arbitrary complex R_{13}:

the "high energy" contribution to the BAU is subdominat (or strongly suppressed) for, e.g., $M_1 = 10^{11}$ GeV and arbitrary $\arg(R_{13}) \equiv \tilde{\varphi}_{13}$ if

- for $|R_{11}| < 0.5$, $|R_{13}|$ satisfies $|R_{13}| \lesssim |R_{11}|$;
- for $0.5 \lesssim |R_{11}| < 1$ we have $|R_{13}| < 0.5$;
- and if for $|R_{11}| > 1$ we have $|R_{13}| < |R_{11}|/2$.

In each of these cases we can have successful leptogenesis due to the contribution to the baryon asymmetry associated with the Majorana CP violating phase(s) in the neutrino mixing matrix.

Conclusions

The see-saw mechanism provides a link between the ν-mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in $U_{\rm PMNS}$ can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

Dirac and Majorana CPV may have the same source.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

These results underline further the importance of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.

SUPPORTING SLIDES

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U_{lj}^{*} \, |\nu_j> , \quad \nu_j : \; m_j \neq 0 ; \quad l = e, \mu, \tau ; \quad n \geq 3 ;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj} \, \nu_{j\mathrm{L}}(x) , \quad \nu_{j\mathrm{L}}(x) : \; m_j \neq 0 ; \quad l = e, \mu, \tau .$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \ldots$) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\ldots} \sim 1$ eV;
in this case $\nu_{e(\mu)} \to \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly", data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly");

ii) $M_{4,5,\ldots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
$M_{4,5,\ldots} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

We can also have, in principle:

$m_4 \sim 1$ eV ($\nu_{e(\mu)} \to \nu_S$), $m_5 \sim 5$ keV (DM), $M_6 \sim (10 - 10^3)$ GeV (seesaw).

- Data (relativistic ν's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).

Standard Theory: ν_l, $\tilde{\nu}_l$ - $\nu_{lL}(x)$;

$\nu_{lL}(x)$ form doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau\,.$$

- No (compelling) evidence for existence of (relativistic) ν's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)

If ν_R, $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l, $\tilde{\nu}_l$: ν_R, $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν's can play crucial role
i) in the generation of $m(\nu) \neq 0$,
ii) in understanding why $m(\nu) \ll m_l, m_q$,
iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = const.$, $l = e, \mu, \tau$;
$L \equiv L_e + L_\mu + L_\tau = const.$

The current "reference scheme": 3-ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj} |\nu_j>\,, \quad \nu_j : \; m_j \neq 0\,; \quad l = e, \mu, \tau\,; \quad n = 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x) : \; m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: the 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_\tau) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U, m_j^2 - m_k^2)$$

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathrm{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1; \quad C^{-1}\gamma_\alpha C = -\gamma_\alpha^T$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\mathrm{el}} = 0,\ L_l = 0,\ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

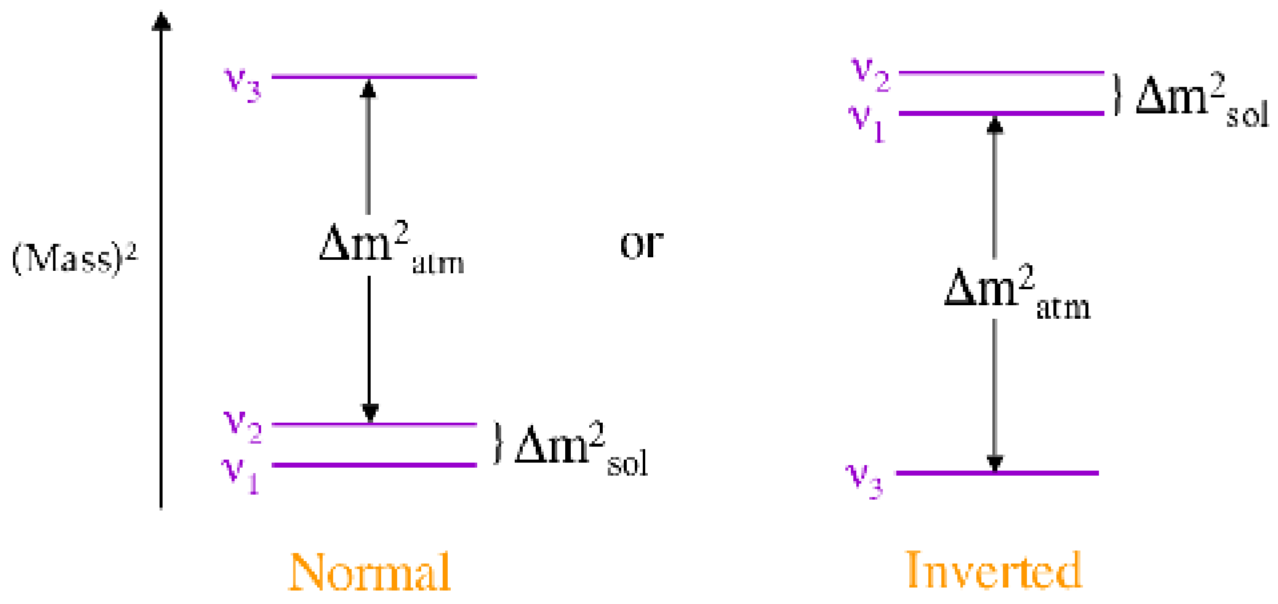
$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

The (Mass)^2 Spectrum

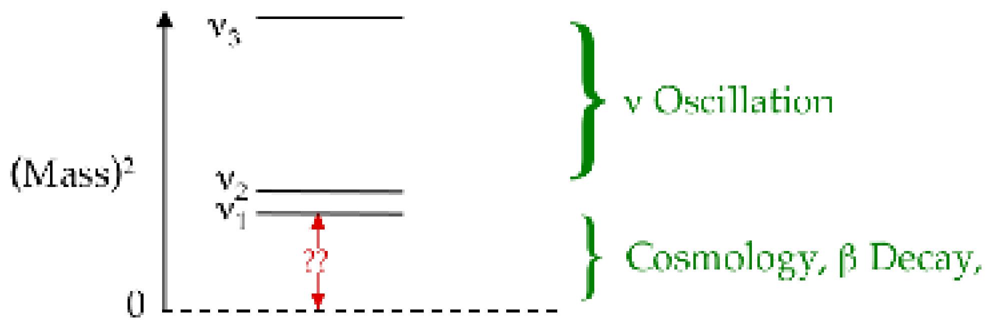


$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

Due to B. Kayser

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

Due to B. Kayser

Predictions for the CPV Phase δ

Models with $U \sim U_{\text{TBM}}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \qquad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

Models with $U \sim U_{\text{BM}}$:

$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20, \quad \theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 - 0.10.$
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\rm lep}^{\dagger}(\theta^{\ell}_{ij}, \psi)\; Q(\phi, \varphi) U_{\rm TBM,BM,LC}\; P(\alpha_{21}, \alpha_{31}),$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\rm lep}^{\dagger}(\theta^{\ell}_{ij}, \psi)$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC}\; P(\alpha_{21}, \alpha_{31})$ - from diagonalization of the ν mass matrix;
- $Q(\phi, \varphi)$, - from diagonalization of both the l^- and ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

Predictions for δ

Assume:

- $U_{PMNS} = U_{\rm lep}^{\dagger}(\theta_{ij}^{\ell}, \psi)\ Q(\phi, \varphi) U_{\rm TBM,BM}\ P(\alpha_{21}, \alpha_{31})$,
- $U_{\rm lep}^{\dagger}$ - minimal, such that

i) $\sin\theta_{13} \cong 0.16$; BM: $\sin^2\theta_{12} \cong 0.31$;
ii) $\sin^2\theta_{23}$ can deviate significantly (by more than $\sin^2\theta_{13}$) from 0.5 (b.f.v. = 0.42-0.43) .

From i), ii) + $m_e << m_\mu << m_\tau$:

$$U_{\rm lep}^{\dagger}(\theta_{ij}^{\ell}, \psi) = R_{12}(\theta_{12}^{\ell})\, R_{23}(\theta_{23}^{\ell})\,,\quad Q(\phi, \varphi) = {\rm diag}(1, e^{i\phi}, 1)$$

Leads to $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$ - new sum rules for δ!

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

For $U_{\rm TBM}$:

$$\cos\delta = \frac{\tan\theta_{23}}{3\sin 2\theta_{12}\sin\theta_{13}}\left[1 + \left(3\sin^2\theta_{12} - 2\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]$$

For $U_{\rm TBM}$ + b.f.v. of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \qquad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

T' model of lepton flavour: $U_{\rm TBM}$, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

For U_{BM}:

$$\cos\delta = -\frac{1}{2\sin\theta_{13}}\cot 2\theta_{12}\tan\theta_{23}\left(1-\cot^2\theta_{23}\sin^2\theta_{13}\right).$$

For U_{BM} + b.f.v. of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or min(m_j));
- determination of the status of the CP symmetry in the lepton sector.

Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) - far-reaching implications for the program of reserach in neutrino physics:

- For the determination of the type of $\nu-$ mass spectrum (or of $\text{sgn}(\Delta m^2_{\text{atm}})$) in neutrino oscillation experiments.

- For understanding the patern of the neutrino mixing and its origins (symmetry, etc.?).

- For the predictions for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).

Large $\sin\theta_{13} \cong 0.15$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

Improved β energy resolution requires a ***BIG*** β spectrometer.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Porto
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Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N

9

Mass and Hierarchy from Cosmology