Chiral Magnetic Effect and Cosmic Magnetic Fields



Based on the old and recent works with I.Antoniadis, A.Boyarsky, J.Fröhlich, J.Harvey, M.Shaposhnikov, J.Wells

- Phys. Rev. Lett. 108 (2012) 031301 [arXiv:1109.3350]
- Phys. Rev. Lett. **109** (2012) 111602 [arXiv:1204.3604]
- Nucl. Phys. B 824 (2010) 296 [arXiv:0901.0639]
- Nucl. Phys. B 793 (2008) 246 [arXiv:0708.3001]
- Phys. Rev. D 72 (2005) 085011 [arXiv:hep-th/0507098]
 Appala Phys. 201 (2002) 1 [arXiv:hep-th/0202154]
- Annals Phys. 301 (2002) 1 [arXiv:hep-th/0203154]

Outline

- In this talk I argue that the conventional description of relativistic magnetized plasma is incomplete.
- There exists a subtle quantum effect that has macroscopic manifestation in the low-energy classical hydrodynamic equations
- This effect may be important in "real life" whenever you have relativistic magnetized plasma with high densities
 - Early Universe
 - Neutron stars
 - Astrophysical jets
 - Quark-gluon plasma
- I will describe early Universe application and its connection with BSM problems

Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^{\mu}\partial_{\mu} - \mathbf{m})\psi = \begin{pmatrix} -\mathbf{m}^{0} & i(\partial_{t} + \vec{\sigma} \cdot \vec{\nabla}) \\ i(\partial_{t} - \vec{\sigma} \cdot \vec{\nabla}) & -\mathbf{m}^{0} \end{pmatrix} \begin{pmatrix} \psi_{L} \\ \psi_{R} \end{pmatrix} = 0$$

where $\gamma_5 \psi_{R,L} = \pm \psi_{R,L}$ and $\gamma_5 = i \gamma_0 \gamma_1 \gamma_2 \gamma_3$. In the above basis (Peskin & Schroeder conventions)':

$$\gamma_5 = \begin{pmatrix} -1 & 0\\ 0 & 1 \end{pmatrix}$$

• Number of left $N_L = \int d^3x \, \psi_L^{\dagger} \psi_L$ and right $N_R = \int d^3x \, \psi_R^{\dagger} \psi_R$ particles is conserved independently

 $N_L + N_R$ and $N_L - N_R$ are conserved independently in the free theory

• Gauge interactions respects chirality ($D_{\mu} = \partial_{\mu} + eA_{\mu}$)...

$$\begin{pmatrix} -m^{0} & i(D_{t} + \vec{\sigma} \cdot \vec{D}) \\ i(D_{t} - \vec{\sigma} \cdot \vec{D}) & -m^{0} \end{pmatrix} \begin{pmatrix} \psi_{L} \\ \psi_{R} \end{pmatrix} = 0$$

■ ... but the difference of left and right-movers is **not conserved**...

$$\frac{d(N_L - N_R)}{dt} = \int d^3 \vec{x} \left(\partial_\mu j^5_\mu\right) = \frac{e^2}{4\pi^2} \int d^3 \vec{x} \vec{E} \cdot \vec{B}$$

• ... once the quantum corrections are taken into account:



$$\frac{d(N_L - N_R)}{dt} \propto \frac{d}{dt} \int d^3x \, \vec{A} \cdot \vec{B}$$

Magnetic helicity or Chern-Simons number

Introduce magnetic helicity:

$$\mathcal{H} \equiv \int_V d^3x \vec{A} \cdot \vec{B}$$

■ In terms of B_k^{\pm} (left and right circular polarized modes of B_k)

$$\mathcal{H}_k = \frac{k}{2\pi^2} (|B_k^+|^2 - |B_k^-|^2) \text{ and } \rho_k = \frac{k^2}{(2\pi)^2} (|B_k^+|^2 + |B_k^-|^2)$$

so that

$$\mathcal{H} = \int dk \, \mathcal{H}_k \qquad \rho = \int dk \, \rho_k$$

From its definition

$$\frac{d\mathcal{H}}{dt} = -\frac{2}{V} \int_{V} d^{3}x \, \vec{E} \cdot \vec{B}$$



Chiral anomaly at finite fermion densities



Chiral anomaly for degenerate Fermi gas

$$\Delta N_{L,R} = \int_{t_i}^{t_f} dt \, \dot{N}_{L,R}(t) = \mp \int dt \frac{\alpha}{\pi} \int d^3x \, E \cdot B$$

Nielsen & Ninomiya (1983);

• The energy change: $\delta \mathcal{E} = \delta N_L \mu_L + \delta N_R \mu_R = \frac{\alpha}{2\pi} \mu_5 \int d^3x A \cdot B$

Rubakov (1986)

• Free energy density:
$$\delta \mathcal{F} = \int d^3x \frac{\alpha}{2\pi} \mu_5 A \cdot B$$

• The free energy for static magnetic fields $\mathcal{F}[\vec{A}]$ has the form

$$\mathcal{F}[A] = \frac{1}{2} \int d^3k \, A_i(\vec{k}) \Pi_{ij}(k) A_j(-\vec{k}) + \mathcal{O}(A^3)$$

(magnetic field $\vec{B} = \nabla \times \vec{A}$)

• Polarization operator in plasma Π_{ij} should be rotation invariant and gauge invariant (i.e. transversal: $k_i \Pi_{ij} = 0$). The most general form:

$$\Pi_{ij}(\vec{p}) = (k^2 \delta_{ij} - k_i k_j) \Pi_1(k^2) + \underbrace{i \epsilon_{ijn} k^n \Pi_2(k^2)}_{\text{parity-even part}} + \underbrace{i \epsilon_{ijn} k^n \Pi_2(k^2)}_{\text{parity-odd part}}$$

here and below we will speak only about $\Pi_2(0)$ that we denote simply by Π_2

■ In vacuum
$$\Pi_{\mu\nu}(k) = (\eta_{\mu\nu}k^2 - k_{\mu}k_{\nu})\Pi(k^2)$$

In plasma with the different number of left and right particles



Vilenkin (1978)

Redlich & Wijewardhana (1985);

Fröhlich et al. (1998–2001)

This diagram is related to axial anomaly



 Chiral Magnetic Effect is an electric current in the presence of the chiral chemical potential µ₅:

 $\vec{j} = \frac{\delta \mathcal{F}}{\delta \vec{A}} = \frac{2\alpha}{\pi} \mu_5 \vec{B}$

• Let us compare this with the Ohmic current $\vec{j}_{Ohm} = \sigma \vec{E}$

Indeed, let us compare this w consider transformation w.r.t. timereversal symmetry T:

Rubakov'86;

Cheianov, Fröhlich, Alexeev'98;

Fröhlich & Pedrini'00,'02;

Fukushima, Kharzeev, Warringa'08

Son & Surowka'09 In coordinate space $\Pi_2 \neq 0$ leads to a Chern-Simons term:

$$\mathcal{F}[A] = \frac{1}{2} \int d^3x \left(\vec{B}^2 + \underbrace{\Pi_2 \vec{A} \cdot \vec{B}} \right)$$

- The Chern-Simons term
 - contains less derivatives than $(\nabla \times A)^2$
 - can be both positive and negative
- The matrix Π_{ij} has a negative eigenvalue for

 $k < \Pi_2 \Longrightarrow$ **Instability!**

The unstable mode will have a form

$$\vec{A}(\vec{x}) = A_0 \Big(\cos(kz), \sin(kz), 0 \Big)$$

For this configuration the magnetic field

$$\vec{B}(\vec{x}) = -k\vec{A}(\vec{x})$$

(maximally helical configuration)

- On this configuration $\vec{B}^2 = k\vec{A}\cdot\vec{B}$ and are homogeneous
- The effective action:

$$\mathcal{F}[A] = \frac{1}{2} \int d^3x \left(k^2 - k\Pi_2\right) A_0^2 < 0$$

for $k < \Pi_2 \implies$ increasing A_0 we decrease the free energy

- The presence of difference of chemical potential of left and right fermions leads to additional terms in the effective Lagrangian for electromagnetic fields – Chern-Simons term
- As a result Maxwell equations contain current, proportional to µ₅ Kharzeev'11 — MHD turns into chiral MHD:

$$\operatorname{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\operatorname{Chiral magnetic effect}$$

$$\operatorname{curl} \vec{B} = \sigma \vec{E} + \underbrace{\frac{2\alpha}{\pi} \mu_5 \vec{B}}_{\pi}$$

Vilenkin (1978)

Redlich & Wijewardhana (1985);

Fröhlich et al. (1998–2001)

Joyce & Shaposhnikov (1997)

In addition, μ_5 should be allowed to **become dynamical**:

$$rac{d(N_L-N_R)}{dt} \propto rac{d\mu_5}{dt} \propto rac{lpha}{\pi} \int d^3x \, ec{E} \cdot ec{B} \; .$$

New degree of freedom



• Without *B* chirality flipping reactions drive $\mu_5 \rightarrow 0$ ($\mu_5 = \mu_0 e^{-\Gamma_{\text{flip}}t}$)

• Without μ_5 finite conductivity drives $B \to 0$ ($B_k = B_0 e^{-\frac{k^2 t}{\sigma}}$)

• Maxwell equations with μ_5 are unstable:



Exponential growth for $k < \frac{\alpha}{\pi}\mu_5$ (for one of the circular polarizations depending on the sign of μ_5) — generation of helical magnetic fields

$$B_{\pm} = B_0 \exp\left(-rac{k^2}{\sigma}t \pm rac{lpha k\mu_5}{\pi \sigma}t
ight)$$

■ Consider sharply peaked at *k*₀ maximally helical field





Oleg Ruchayskiy

CHIRAL ANOMALY AND MAGNETIZED PLASMA





Process continues while $\rho_B \gg \Gamma_{\rm flip}$



Continuous initial spectrum with $\mathcal{H}_k \propto k$ and fraction of magnetic energy density 5×10^{-5} (blue) or 5×10^{-4} (green). Red – evolution without flip

Evolution of helicity spectrum



Process continues while $\Gamma_B \gg \Gamma_{\text{flip}}$ (recall that $\Gamma_B \propto \rho_B$)



Oleg Ruchayskiy

Change in magnetic helicity is coupled to the chiral chemical potential $\mu_5(t)$

Change of magnetic helicity

$$\left(\mathsf{Change of } \mu_L - \mu_R
ight)$$

- 1. Change in magnetic helicity excites μ_5 and then evolution of already existing magnetic fields occurs differently
- 2. Presence of μ_5 excites helical magnetic fields

$$\left(\begin{array}{c} \text{Change of } \mu_L - \mu_R \end{array} \right) \implies \left(\begin{array}{c} \text{Change of magnetic helicity} \end{array} \right)$$

Consequences for the "real world" physics?

Can be important in "real life" whenever you have relativistic magnetized plasma

- Early Universe (Cosmological magnetic fields)
 - Many mechanisms to generate helical magnetic fields at electroweak epoch in the early Universe. Usually they survival against dissipation is problematic.
 - New mechanism of generation of cosmological magnetic fields?
- Neutron stars
- Astrophysical jets
- Quark-gluon plasma

Consequences: Magnetic Field in the Early Universe

Magnetic fields in the Universe

- Our Universe today is magnetized: (earth, stars, galaxies, clusters of galaxies)
- Magnetic fields in the spiral (i.e. rotating) galaxies can be generated by turbulent effect from tiny primordial seeds ("dynamo mechanism")
- Magnetic fields in elliptical galaxies?
- Even intergalactic medium seems to be filled with magnetic fields Neronov & Vovk'10;
- Are we observing the evidence of process in the very early Universe?
 Dolag et al.'10 Tavecchio et al.'11

Magnetic fields in the early Universe

- Magnetic fields affect every important process in the early Universe: Many, many
 - Change the nature of electroweak phase transition
 - Affect baryogenesis
 - Leave its imprints in production of gravity waves
 - Affect BBN
 - Leave its imprints in CMB
 - Affect structure formation
 - Could have played a role of seeds of galactic magnetic fields
- Many mechanisms of generation of primordial magnetic fields exist, usually associated with violent events (at inflation, electroweak transition, QCD transition, etc.)
- It is commonly believed that noticeable magnetic fields are not generated in the Universe filled with Standard Model particles

works... Recent reviews: Widrow et al.'11; Kandus et al.'11; Yamazaki et al.'12

Evidence for magnetic fields in voids?



Neronov & Vovk, Science (2010);

Dolag et al. (2010);

Tavecchio et al. (2011)

Survival of helical fields



Generation of cosmic magnetic field

For magneto-genesis — baryo-genesis connection see e.g. many works by Vachaspati, Joyce & Shaposhnikov, Cornwall, Tashiro et al.



What can create μ_5 in the early Universe?

■ Although $\left(\frac{m_e}{80 \text{ TeV}}\right)^2 \sim 10^{-17}$ chirality flipping reactions are in thermal equilibrium for T < 80 TeV and drive $\mu_L - \mu_R$ to zero exponentially fast (suppression of at least e^{-1000} over one Hubble time)







Right components of neutrinos?!

Recall gauge charges of neutrinos and Higgs:

- $\nu_{e,\mu,\tau}$: upper component of the SU(2) doublet, U(1)_Y charge = -1
- Higgs boson: SU(2) doublet, U(1)_Y charge = 1
- Dirac mass term: $(m_{\text{Dirac}})_{\alpha I} = \langle H \rangle F_{\alpha I} \, \bar{\nu}_{\alpha} \, N_I$
 - N_I new particles, *right-handed* fermions, I = 1, 2, ..., N
 - $F_{\alpha I}$ Yukawa matrix, Size $3 \times \mathcal{N}$ (Neutrino Yukawa matrix needs not be square. One can have any number of sterile neutrinos $\mathcal{N} = 1, 2, 3, 4, 5, ...$)
- (Right-chiral neutrinos N_I carry no charge under the SM interactions ("sterile neutrinos")
- Neutral leptons can have Majorana mass term:

$$(M_M) = M_{IJ}\bar{N}_I^c N_J \quad I, J = 1, 2\dots \mathcal{N}$$

Scale of sterile neutrino masses?



Mass of sterile neutrinos is not determined by neutrino oscillations!



Sterile neutrinos behave as **superweakly interacting** heavy neutrinos with a smaller Fermi constant $\vartheta \times G_F$

This mixing strength or mixing angle is

$$\vartheta_{e,\mu,\tau}^2 \equiv \frac{|M_{\rm Dirac}|^2}{M_{\rm Majorana}^2} = \underbrace{\frac{\mathcal{M}_{\rm active}}{M_{\rm sterile}}}_{M_{\rm sterile}} \approx 5 \times 10^{-11} \left(\frac{1 \, {\rm GeV}}{M_{\rm sterile}}\right)$$

Properties of sterile neutrino

For example, 1 GeV sterile neutrino has Yukawa coupling $\mathcal{O}(10^{-8})$ and mixing angle $\vartheta^2 \sim 5 \times 10^{-11}$

- This number means that for each 2 × 10¹⁰ decays of, say, *D*-meson that produces v_µ there will be 1 right-chiral neutrino with the mass 1 GeV produced!
- Can be searches
 - "Rare event" experiments
 - Early Universe where densities of particles were very high
- Previous "rare event" searches have never reached the sensitivity dictated by the see-saw mechanism

Neutrino Minimal Standard Model (ν MSM)

- Two neutrino mass splitting \Rightarrow need (at least) two sterile neutrinos
- Leptogenesis? Possible, but they need to be heavy (10^{12} GeV) or quasi-degenerate in mass (and then as light as MeV)
- Are they Dark matter? \Rightarrow No way! Very short lifetime
- Third sterile neutrino?
 → Yes! Great DM (but two other should prepare for it correct initial conditions)

		N mass	v masses	eV v anoma– lies	BAU	DM	M _H stability	direct search	experi– ment
	GUT see–saw	^{10–16} 10 GeV	YES	NO	YES	NO	NO	NO	_
	EWSB	²⁻³ 10 GeV	YES	NO	YES	NO	YES	YES	LHC
	ν MSM	keV – GeV	YES	NO	YES	YES	YES	YES	a'la CHARM
	v scale	eV	YES	YES	NO	NO	YES	YES	a'la LSND

Review: Boyarsky, O.R., Shaposhnikov Ann. Rev. Nucl. Part. Sci. (2009), [0901.0011]

Parameter space of heavy sterile neutrinos



If we detect sterile neutrinos with masses in GeV range, we can check directly if they can explain **both** for neutrino flavour oscillations and **baryon asymmetry of the Universe**



A dedicated experiment

W. Bonivento, A. Boyarsky, H. Dijkstra, U. Egede, M. Ferro-Luzzi, B. Goddard, A. Golutvin, D. Gorbunov, R. Jacobsson, J. Panman, M. Patel, **O. Ruchayskiy**, T. Ruf, N. Serra, M. Shaposhnikov, D. Treille

Proposal to Search for Heavy Neutral Leptons¹**at the SPS**

Expression of Interest. Submitted to the CERN SPS council on October 7, 2013

[arXiv:1310.176]



Parameter space of heavy neutral leptons



Magenta – dedicated experiments, see the Expression of Interest [arXiv:1310.176]. Also in Proposal to European Strategy Preparatory Group[arXiv:1301.5516]



O.R. with Lesgourgues, Viel PRL 2009

Review: [0901.0011]

Black points – allowed by Ly- α forest data

Only resonantly produced sterile neutrinos are consistent with all data \Rightarrow to precise the properties of sterile neutrino Dark Matter **one should predict the value of lepton asymmetry** L_6

Early Universe with Sterile Neutrinos



- At *T* just below 100 GeV lepton asymmetry in the left neutrino sector is generated very fast. Weak reactions transform left-electron sector appears and right electrons follow via chirality flip $\Rightarrow \mu_5$ between left and right electrons appears
- Evolution of the difference of chemical potential:



• ... and instability quickly develops: $B \propto e^{\lambda_k(t)}$ with

$$\lambda_k(T) \sim \frac{k(\alpha\mu_5 - k)}{\sigma} \frac{M_{Pl}}{T^2}$$



- Sterile neutrinos with mass \sim GeV have small Yukawas ($\sim 10^{-8}$)
- They are out-of-equilibrium in the early Universe and can generate significant lepton number



See Tashiro et al. (2012) & (2013) for other connections between leptogenesis and magneto-genesis



Dispersion relations get modified:

$$E^2 = (|\vec{p}|^2 \pm \mu_5)^2 + m^2$$

• The resulting μ_5 is proportional to the **asymmetry** of all fermions, running in the loops: $\mu_5 \propto G_F(n-\bar{n})$



Boyarsky, Shaposhnikov **O.R.** [1204.3604]

$$\Pi_2 = \frac{\alpha}{2\pi} G_F \times (c_1 \text{ baryon number} + c_2 \text{ lepton numbers}) \neq 0$$

Summary: axions and chiral anomaly at finite temperature

- Chern-Simons term should be added to the Standard Model Lagrangian and finite densities of lepton and/or baryon number
- System with chiral anomaly when put at non-zero temperature necessarily contains an effective axion degree of freedom in its spectrum!
- This additional IR degree of freedom is the difference of chemical potentials of left and right particles should me made dynamical and significantly affects evolution.
- Evolution of such systems is not described by the standard MHD equations (as was previously believed) but rather by chiral MHD

Thank you for your attention!