A New First-Principle Calculation of Field-Dependent RF Surface Impedance of BCS Superconductor

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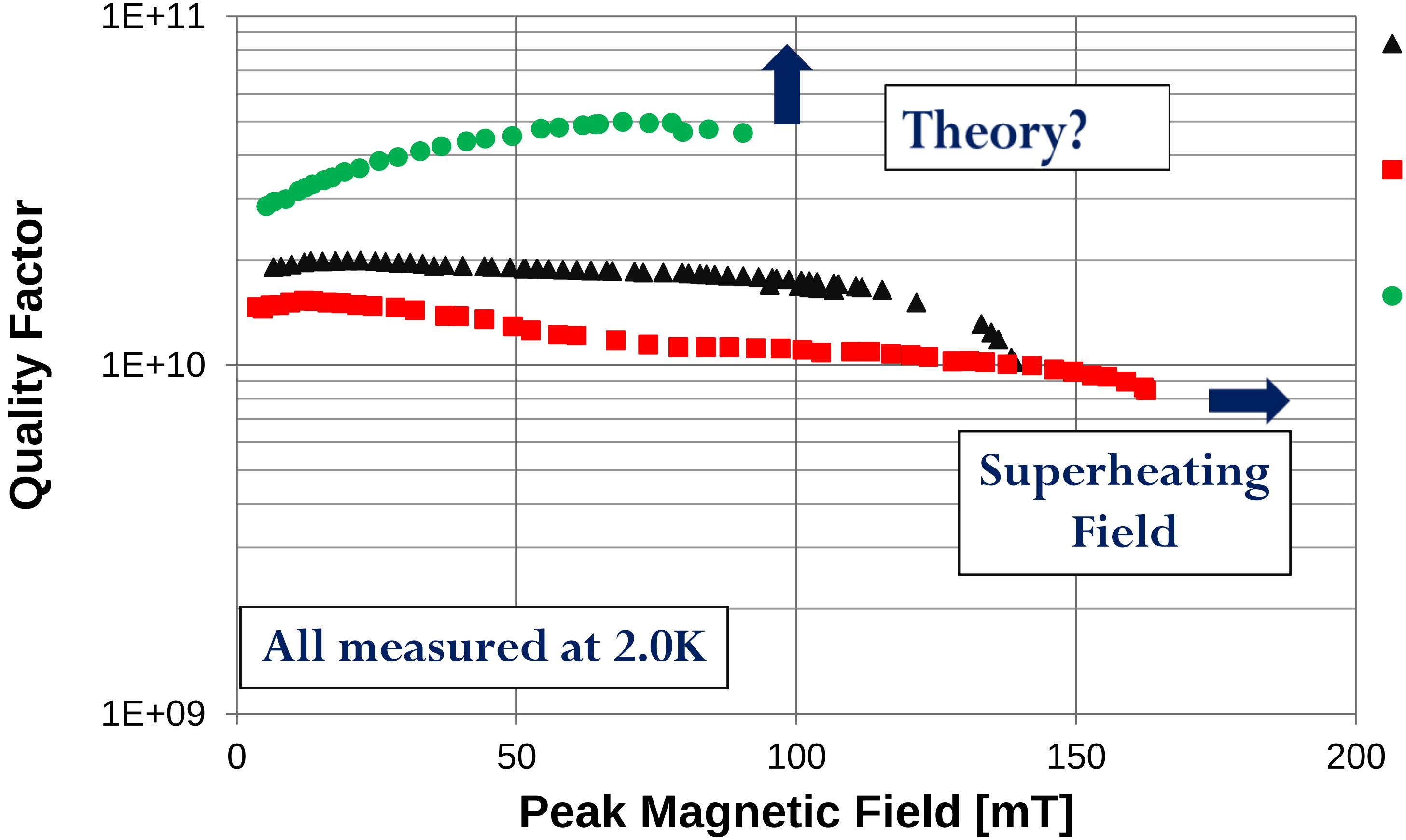
Cavity Performance



- P. Dhakal, G. Ciovati, G. R. Myneni, in: IPAC 2012.
- C. E. Reece *et al.*, in PAC2005
- C. E. Reece, and H. Tian, in LINAC2010

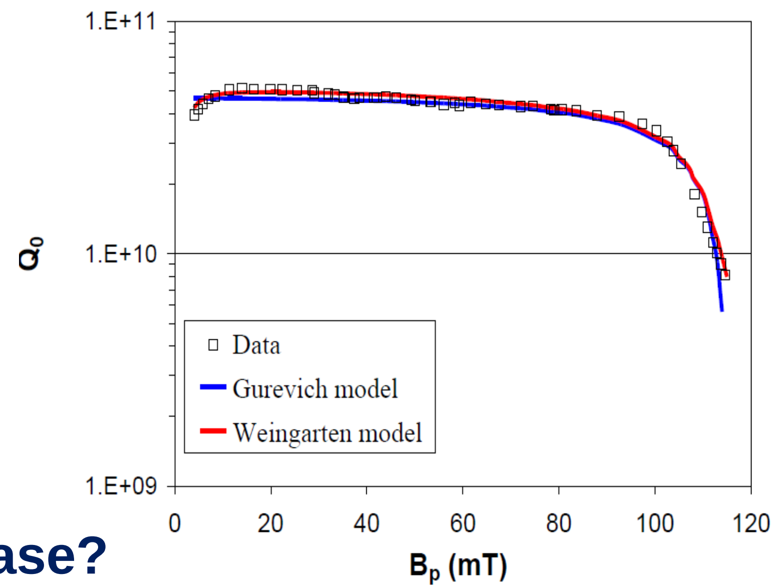
H. Pandamsee: "Two Major Open Physics Topic in RF Superconductivity"

Theories for Q Explanation

What's the theoretical limit?

	Q-Slope Fit	Q-Slope before baking (EP ≡ BCP)	Q-Slope Improvem[t] after baking	Q-Slope after baking (EP < BCP)	No change after 4 y. air exposure	Exceptional Results (BCP)	Q-Slope unchanged after HF chemistry	TE_{011} Q-slope after baking	Quench EP > BCP	BCP Quench unchanged after baking	Argum[t] Validity	Fund[al] Disagreem[t] Exper. ≠ Theory
Magnetic Field Enhancem[t]	Y simulat. code	~~N $\beta_m \neq B_{C2}^S \neq$~~	Y $B_{C2}^S \uparrow$	Y lower β_m	-	N high β_m	-	-	Y lower β_m	N $B_{C2}^S \uparrow$	Y	D_1
Interface Tunnel Exchange	Y E^8	N $\beta* \neq$	Y $Nb_2O_{5-y} \downarrow$	Y lower $\beta*$	N $Nb_2O_{5-y} \uparrow$	N high $\beta*$	~~N new Nb_2O_{5-y}~~	~~N improv[t]~~	-	-	Y	D_2
Thermal Feedback	Y parabolic	Y ≅ thermal properties	Y $R_{BCS} \downarrow R_{res} \uparrow$	N ≅ therm. propties	-	-	-	-	-	-	N C coeff.[t]	-
Magnetic Field Dependence of Δ	Y expon[tial]	N $B_{C2}^S \neq$	Y $B_{C2}^S \uparrow$	Y higher B_{C2}^S	-	-	-	-	-	-	~~N thin film~~	D_1
Segregation of Impurities	?	N segregation ≠	N only O diffusion	Y surface ≠	-	Y good cleaning	N chemistry	-	-	-	Y	-
Bad S.C. Layer Interstitial Oxygen $Nb_{4-6}O$	?	Y NC layer	Y O diffusion	N	N interstitial re-appears	-	~~N new bad layer~~	-	Y higher B_{C2}^S	N $B_{C2} \downarrow$	Y	D_1

- B. Visentin, Thin Films SRF 2006.
- A. Gurevich and G. Ciovati, Phys. Rev. B 77, 104501 2008
- W. Weingarten, SRF2009.
- G. Ciovati, SRF2009.



What happened on the low field increase?

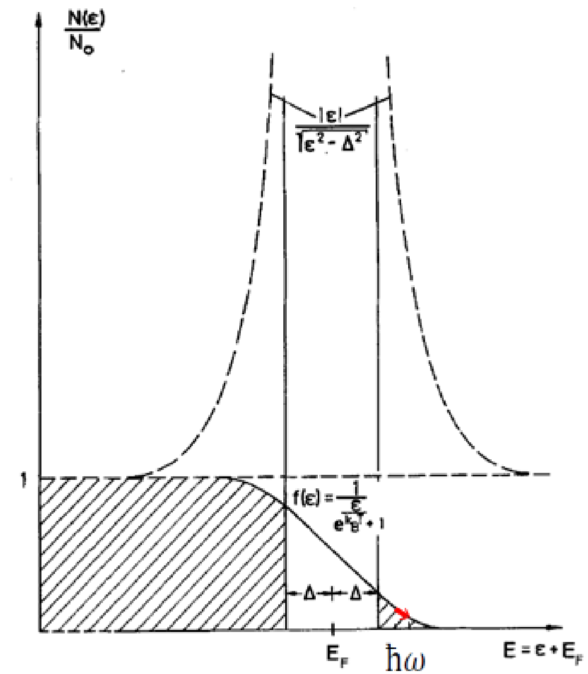
What's the best performance we can experimentally achieve?

Will the theory and experiments agree with each other?

Mattis-Bardeen Theory

The electron states distribution and probability of occupation at $T<T_c$, from BCS theory by minimizing the free energy :

$$h_k = \frac{1}{2}[1-(\epsilon_k/E_k)], \quad f_k = \frac{1}{e^{\beta E_k}+1} = f(E_k).$$



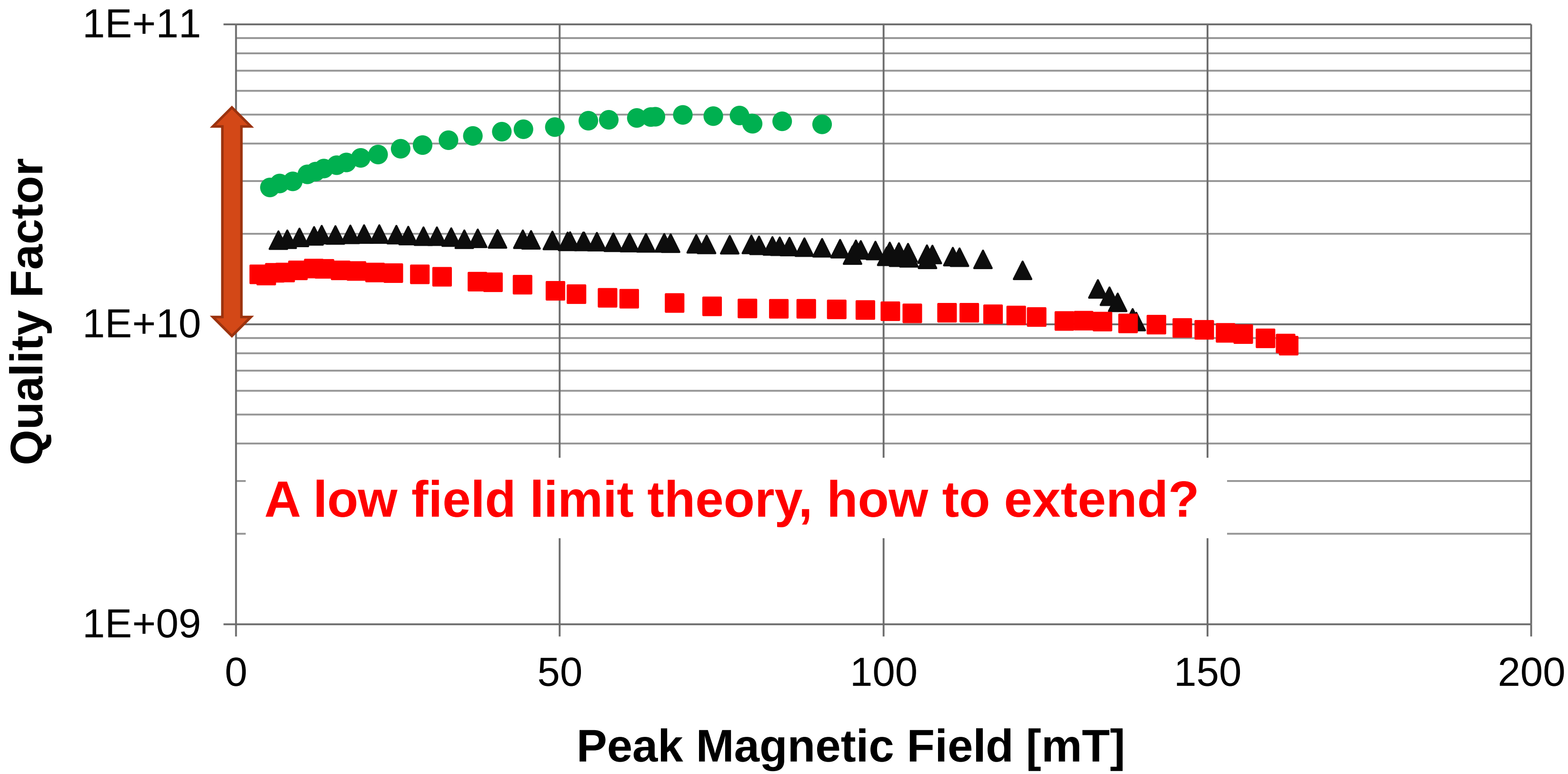
Applying these to the matrix elements of single-particle scattering operator, and then to the anomalous skin effect theory:

$$R \propto \int_{\Delta}^{\infty} [f(E) - f(E+\hbar\omega)] g(E) dE$$

The "golden rule"

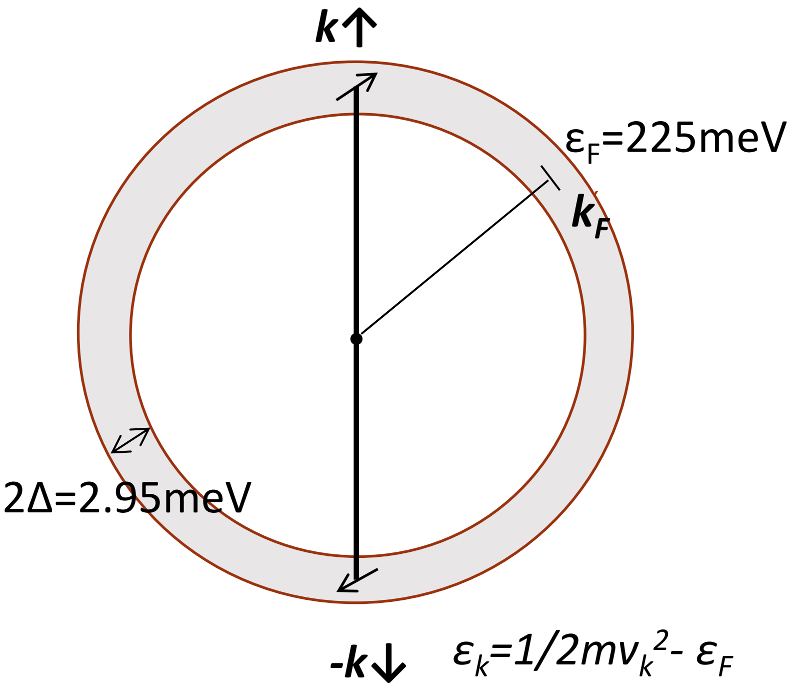
- "**Theory of Superconductivity**" by J. Bardeen, L. N. Cooper and J. R. Schrieffer
- "**Theory of the Anomalous Skin Effect in Normal and Superconducting Metals**" by D. C. Mattis and J. Bardeen
- "**The Surface Impedance of Superconductors and Normal Conductors: The Mattis-Bardeen Theory**" by J. P. Turneaure, J. Halbritter, and H. A. Schwettman

Mattis-Bardeen Theory (Continued)



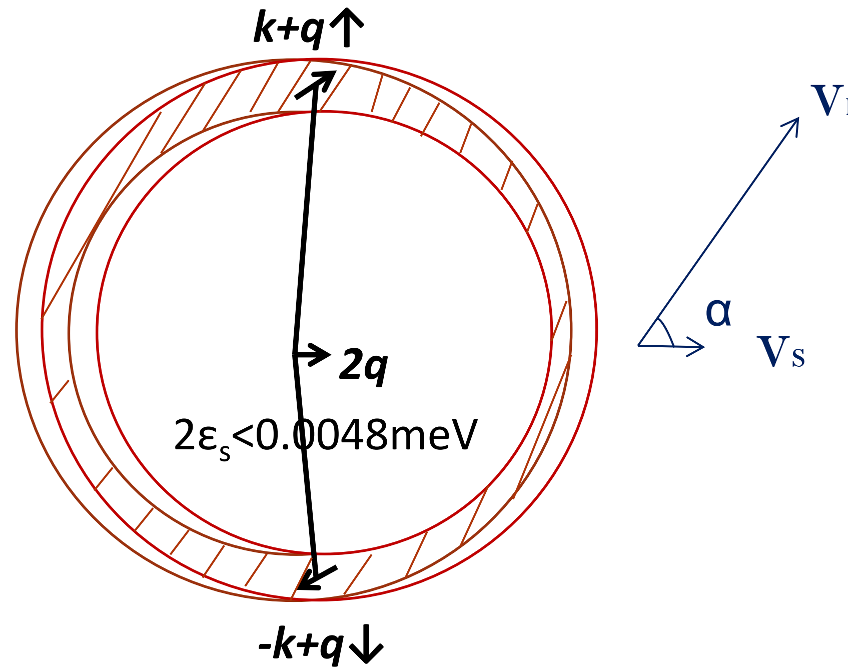
"**<u>States with a net current flow can be obtained by taking a pairing (*k1*↑, *k2*↓) with *k1*+*k2* =2q, and 2q the same for all virtual pairs</u>**" – quoted from **BCS theory**

Cooper pair and moving Cooper pair



$\varepsilon_k = 1/2mv_k^2 - \varepsilon_F$

Energy split appears in Cooper pair with angle dependence



$\varepsilon_{k+q} = 1/2m(v_k + v_s)^2 - \varepsilon_F = \varepsilon_k + \varepsilon_s + \varepsilon_{ext}$

$\varepsilon_{-k+q} = 1/2m(v_k - v_s)^2 - \varepsilon_F = \varepsilon_k + \varepsilon_s - \varepsilon_{ext}$

$\varepsilon_{ext} = mv_k v_s \cos\alpha = p_F v_s x$

With total momentum 2***q*** for **all** Cooper pairs. (Energies are based on Nb with selected parameters)

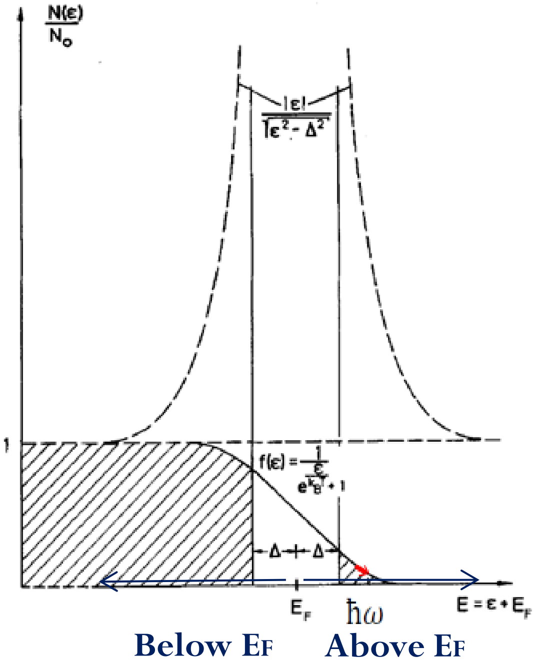
Consequence of moving Cooper pairs

Modified density of states and probability of occupation at $T<T_c$:

$$h_k = \frac{1}{2}\left(1 - \frac{\varepsilon_k + \varepsilon_s}{E_k}\right)$$

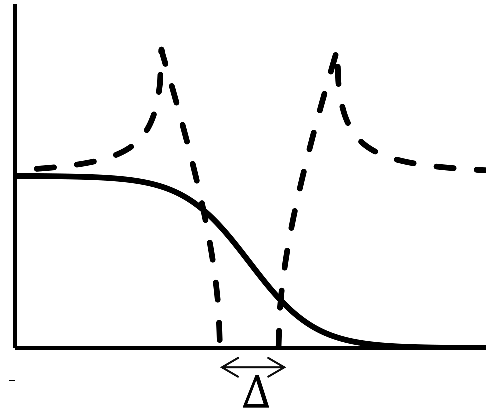
$$f_{-k+q\downarrow} = \begin{cases} f(E_{-k+q\downarrow}), k > k_F & \text{For electron} \\ f(E_{k+q\uparrow}), \ k < k_F & \text{For hole} \end{cases}$$

$$f_{k+q\uparrow} = \begin{cases} f(E_{k+q\uparrow}), k > k_F & \text{For electron} \\ f(E_{-k+q\downarrow}), \ k < k_F & \text{For hole} \end{cases}$$

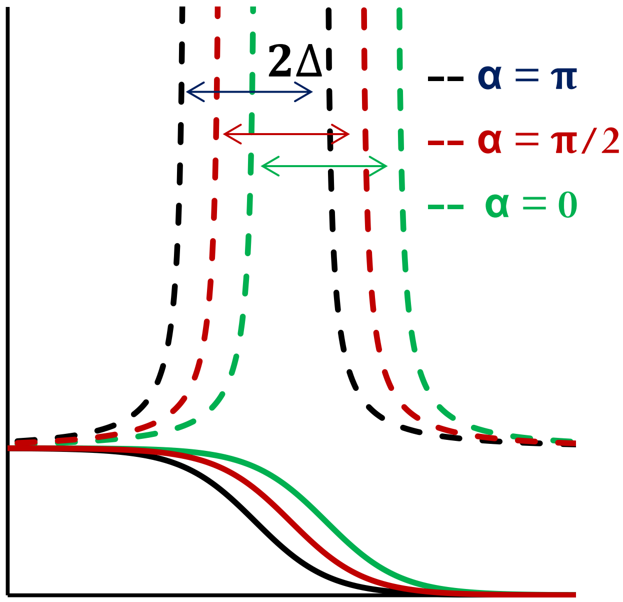


Low field limit density of states and distribution function

Plots with $P_F V_s = \Delta/2$ and $T/T_c = 0.97$



Density of states and distribution function with moving cooper pairs, angle averaged

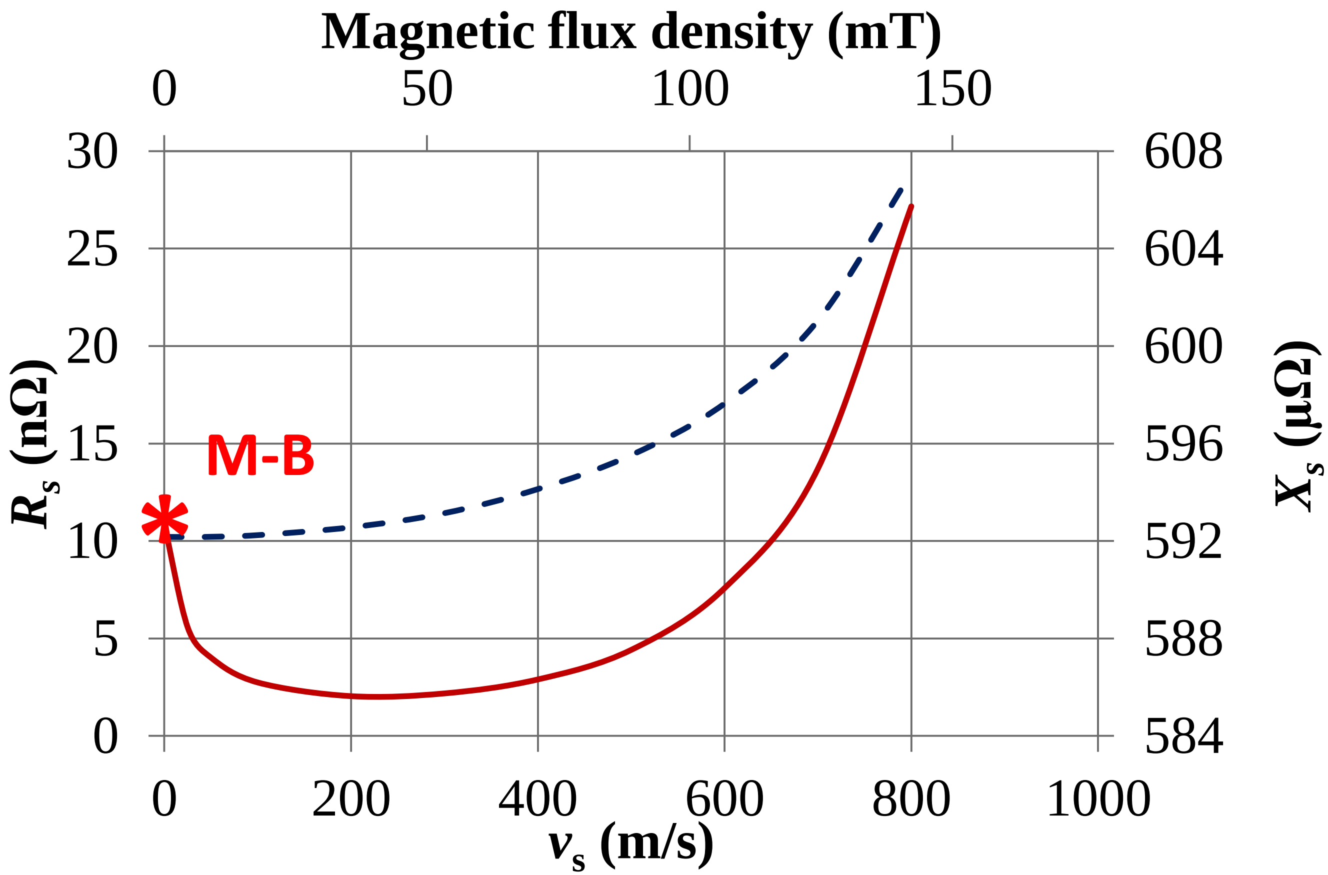


Density of states and distribution function with moving cooper pairs, angle-dependent

B.P. Xiao, C.E. Reece, M.J. Kelley, *Physica C* **490** (2013) 26–31

Consequence of moving Cooper pairs

After following the same analytical derivation as M-B but with new distributions, then coding and obtaining numerical solution of resulting challenging quadruple integral, one obtains:



Surface resistance, R_s, (**red line**) and reactance, X_s, (**blue dashed line**) **versus Cooper pair velocity** and corresponding magnetic field for Nb at 2 K and 1.5 GHz.

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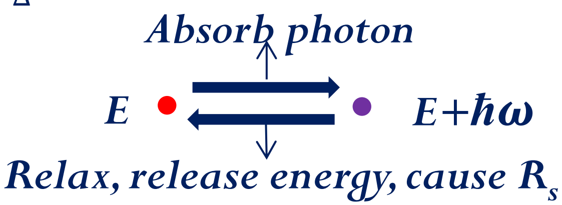
Explanation with new “Golden Rule”

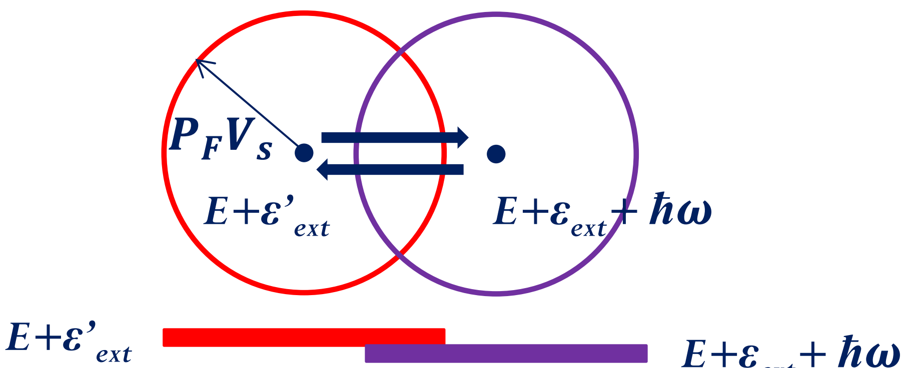
The “golden rule”

In extreme anomalous limit and low temperature approximation

$$R \propto \int_{\Delta}^{\infty} [f(E) - f(E + \hbar\omega)]g(E)dE$$



$$R \propto \int_{\Delta}^{\infty} [f(E+\varepsilon_{ext}+\hbar\omega) - f(E+\varepsilon'_{ext})][f(\varepsilon_{ext})+f(-\varepsilon_{ext})]g(E,\alpha,\alpha')dE$$



Why is R_s decreasing?

- **Source: angle between V_F (any direction) and V_s cause energy split with angle dependence.**
- **Consequence: While the energy relaxation happens from high energy to low energy in Mattis-Bardeen theory, it is possible this process also happens from low energy to high energy. While this procedure “borrows” energy from those from high energy to low energy, the net effect still obey the 2nd law of thermodynamics.**

Note that $P_F V_S >> \hbar\omega$ could happen, the overlap between red and purple could be significant.

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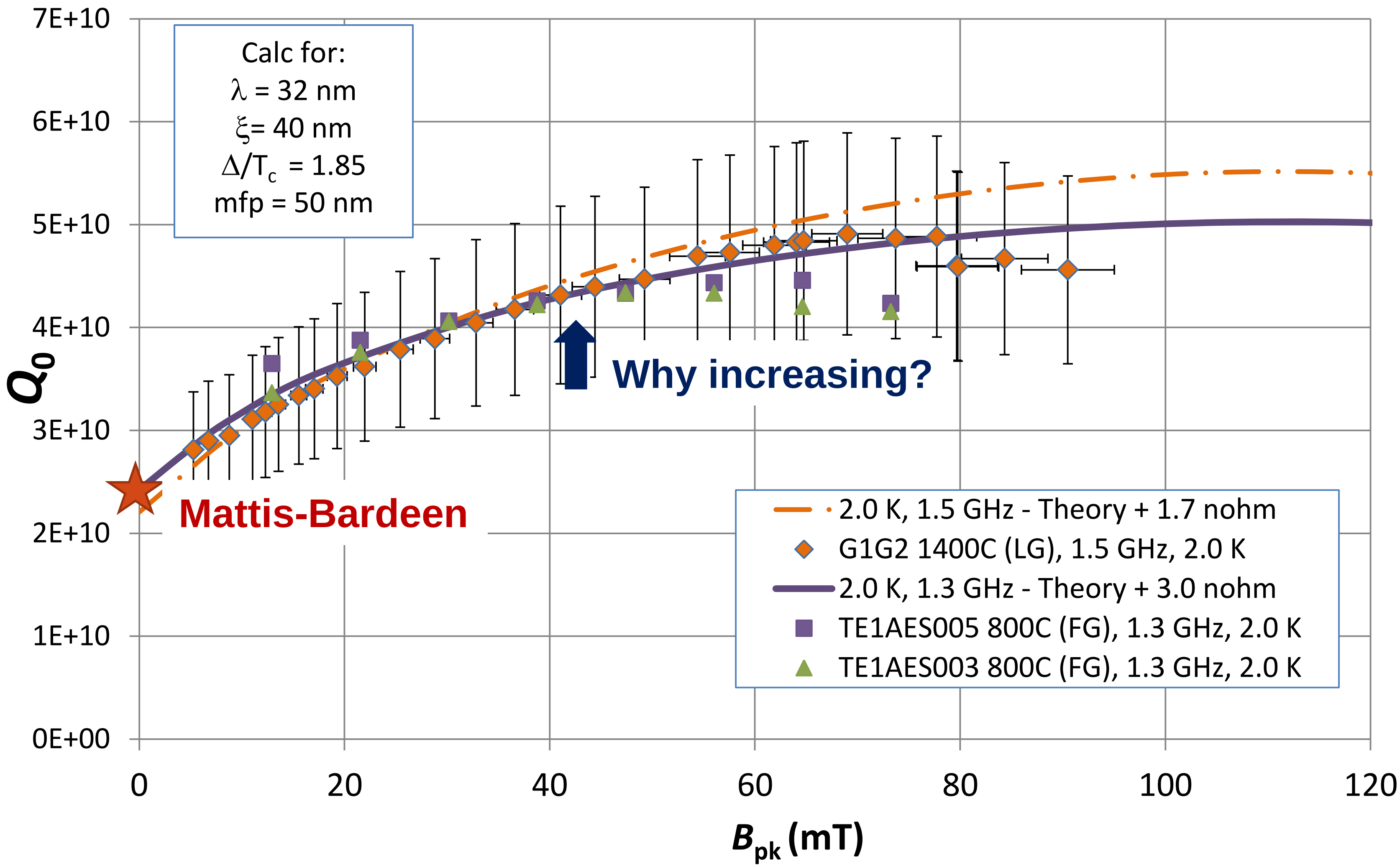
Theory vs Experiment

P. Dhakal, *et al.*, PRST-AB, 2013. **16**(4): p. 042001.
A. Grassellino, *et al.*, *Supercon. Sci.and Tech.*, 2013. **26**(10): p. 102001.



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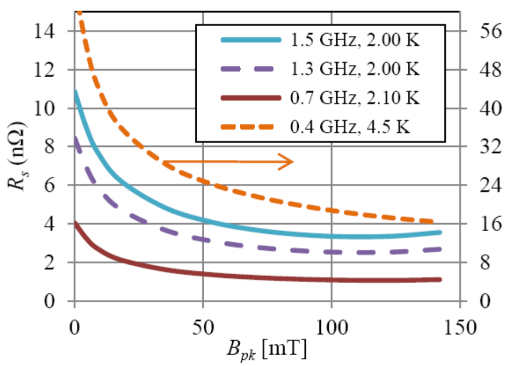
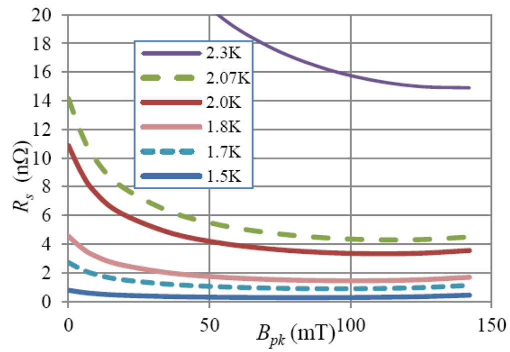
Parameter Survey

See tomorrow's poster: TUP011

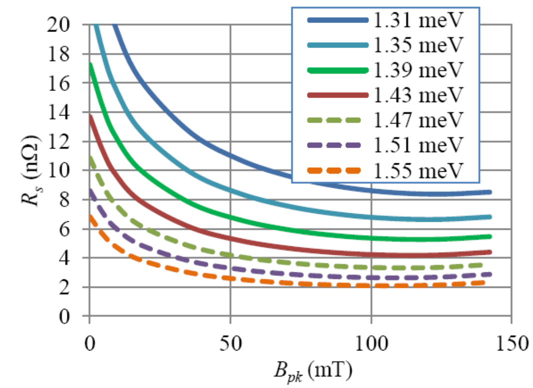
Frequency dependent at different temperature



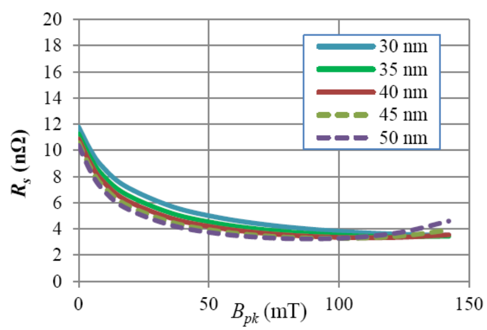
Temperature dependent



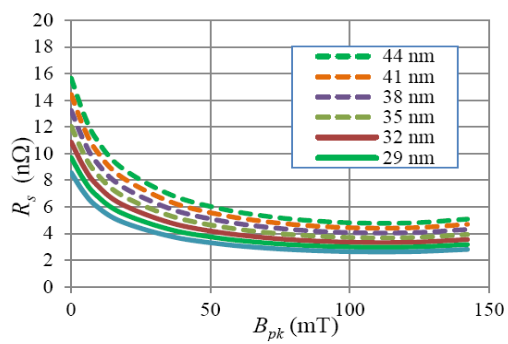
Energy gap dependent



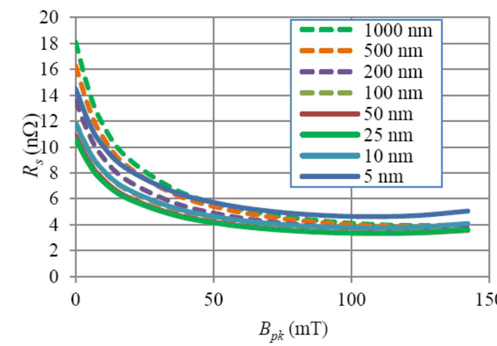
Coherence length dependent



Penetration depth dependent



Mean free path dependent



Summary

- Previous surface impedance calculations are available only for the low field limit.
- A field-dependent derivation of the Mattis-Bardeen theory of SRF surface impedance has been developed.
- The extended range of gradients is treated for the first time.
- Field-dependent R_s agreement with experiment with recent clean heat-treated Nb with unusual surface loading is excellent, and we are ready to look closer.
- The reduction in resistance with increasing field is seen to be an intrinsic effect.
  - For type-I, and type-II under H_{c1}.
  - What is going to happen between H_{c1} and H_{c2}?