A New First-Principle Calculation of Field-Dependent RF Surface Impedance of BCS Superconductor

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Based on the Ph.D. thesis supervised by C. E. Reece and M. J. Kelley
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Cavity Performance

All measured at 2.0K

H. Pandamsee: “Two Major Open Physics Topic in RF Superconductivity”
Theories for Q Explanation

What’s the theoretical limit?

- W. Weingarten, SRF2009.

What happened on the low field increase?
What’s the best performance we can experimentally achieve?
Will the theory and experiments agree with each other?
**Mattis-Bardeen Theory**

The electron states distribution and probability of occupation at $T<T_c$, from BCS theory by minimizing the free energy:

$$ h_k = \frac{1}{2} \left[ 1 - \left( \frac{\epsilon_k}{E_k} \right) \right] $$

$$ f_k = \frac{1}{e^{\beta E_k} + 1} = f(E_k) $$

Applying these to the matrix elements of single-particle scattering operator, and then to the anomalous skin effect theory:

$$ R \propto \int_{\Delta}^{\infty} \left[ f(E) - f(E + \hbar \omega) \right] g(E) dE $$

The “golden rule”

- “Theory of Superconductivity” by J. Bardeen, L. N. Cooper and J. R. Schrieffer
- “Theory of the Anomalous Skin Effect in Normal and Superconducting Metals” by D. C. Mattis and J. Bardeen
- “The Surface Impedance of Superconductors and Normal Conductors: The Mattis-Bardeen Theory” by J. P. Turneaure, J. Halbritter, and H. A. Schwettman
States with a net current flow can be obtained by taking a pairing \((k_1^{\uparrow}, k_2^{\downarrow})\) with \(k_1 + k_2 = 2q\), and \(2q\) the same for all virtual pairs” – quoted from BCS theory.
Cooper pair and moving Cooper pair

With total momentum $2q$ for all Cooper pairs. (Energies are based on Nb with selected parameters)

$\varepsilon_k = \frac{1}{2}mv_k^2 - \varepsilon_F$

$\varepsilon_{k+q} = \frac{1}{2}m(v_k + v_s)^2 - \varepsilon_F = \varepsilon_k + \varepsilon_s + \varepsilon_{\text{ext}}$

$\varepsilon_{-k+q} = \frac{1}{2}m(v_k - v_s)^2 - \varepsilon_F = \varepsilon_k + \varepsilon_s - \varepsilon_{\text{ext}}$

$\varepsilon_{\text{ext}} = mv_kv_s\cos\alpha = p_Fv_s x$
Consequence of moving Cooper pairs

Modified density of states and probability of occupation at $T < T_c$:

$$h_k = \frac{1}{2} \left(1 - \frac{\varepsilon_k + \varepsilon_s}{E_k}\right)$$

$$f_{-k+q} = \begin{cases} f(E_{-k+q}), k > k_F & \text{For electron} \\ f(E_{k+q}), k < k_F & \text{For hole} \end{cases}$$

For electron

$$f_{k+q} = \begin{cases} f(E_{k+q}), k > k_F & \text{For electron} \\ f(E_{-k+q}), k < k_F & \text{For hole} \end{cases}$$

For hole

Plots with $P_F V_s = \Delta/2$ and $T/T_c = 0.97$

Density of states and distribution function with moving cooper pairs, angle averaged

Density of states and distribution function with moving cooper pairs, angle-dependent

Consequence of moving Cooper pairs

After following the same analytical derivation as M-B but with new distributions, then coding and obtaining numerical solution of resulting challenging quadruple integral, one obtains:

Surface resistance, $R_s$, (red line) and reactance, $X_s$, (blue dashed line) versus Cooper pair velocity and corresponding magnetic field for Nb at 2 K and 1.5 GHz.

The “golden rule”
In extreme anomalous limit and low temperature approximation

\[ R \propto \int_\Delta \left[ f(E) - f(E + \hbar \omega) \right] g(E) dE \]

Absorb photon

\[
\begin{align*}
E & \quad \leftrightarrow \quad E + \hbar \omega \\
\text{Relax, release energy, cause } R_s
\end{align*}
\]

Why is \( R_s \) decreasing?

- **Source**: angle between \( V_F \) (any direction) and \( V_s \) cause energy split with angle dependence.
- **Consequence**: While the energy relaxation happens from high energy to low energy in Mattis-Bardeen theory, it is possible this process also happens from low energy to high energy. While this procedure “borrows” energy from those from high energy to low energy, the net effect still obey the 2\(^{nd}\) law of thermodynamics.

Note that \( P_F V_s \gg \hbar \omega \) could happen, the overlap between red and purple could be significant.
Theory vs Experiment

Calc for:
\( \lambda = 32 \text{ nm} \)
\( \xi = 40 \text{ nm} \)
\( \Delta/T_c = 1.85 \)
mfp = 50 nm

Why increasing?

Mattis-Bardeen

Parameter Survey

See tomorrow’s poster: TUP011

Frequency dependent at different temperature

Temperature dependent

Energy gap dependent

Coherence length dependent

Penetration depth dependent

Mean free path dependent
Summary

- Previous surface impedance calculations are available only for the low field limit.

- A field-dependent derivation of the Mattis-Bardeen theory of SRF surface impedance has been developed.

- The extended range of gradients is treated for the first time.

- Field-dependent $R_s$ agreement with experiment with recent clean heat-treated Nb with unusual surface loading is excellent, and we are ready to look closer.

- The reduction in resistance with increasing field is seen to be an intrinsic effect.
  
  - For type-I, and type-II under $H_{c1}$.
  
  - What is going to happen between $H_{c1}$ and $H_{c2}$?