Vacuous falsehoods: how sure can we be that the desired vacuum of our model is stable?

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in collaboration with
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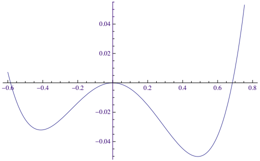
Annecy-le-Vieux,
November 14th, 2013

Even tree-level potentials for single scalars have in general multiple minima:



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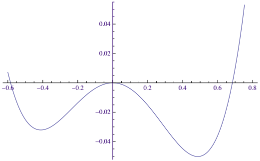
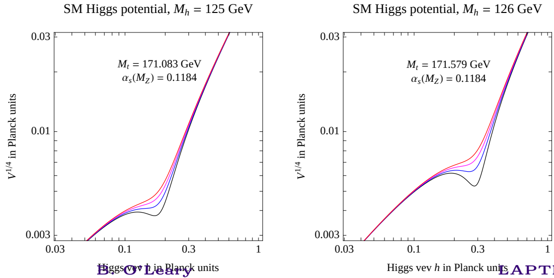


Top loops create extra minima for high Higgs field values in SM:



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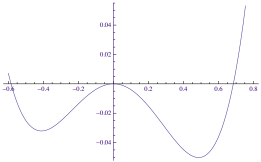
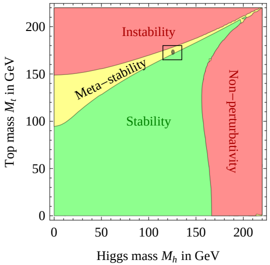


SM is probably metastable!



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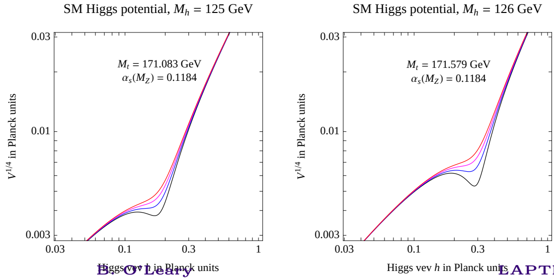

Figs. 2, 3, 4: Degrassi *et al.*, JHEP 1208 (2012)

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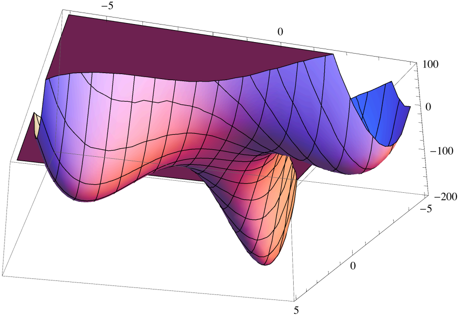

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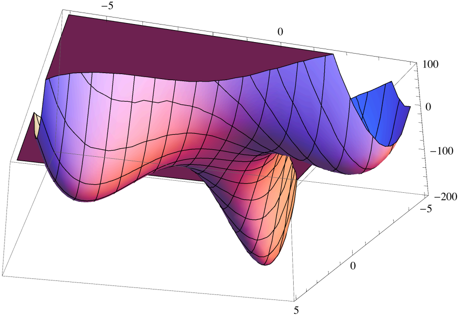

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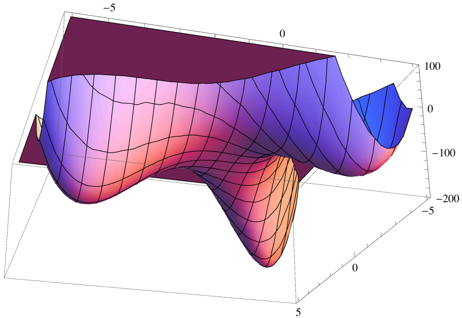

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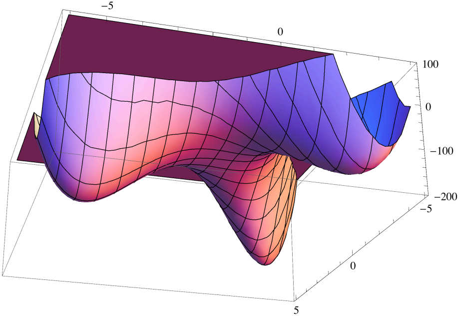

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- Charge- and/or color-breaking (CCB) minima (VEVs for charged or colored scalars)?
- Desired VEV combination may not be global minimum (even non-CCB if there are enough VEVs required)

Models with potentials that could develop CCB minima

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- $m^2_{\text{scalar}}(Q_{\text{GUT}}) = M_0^2$
- $m_{\text{gaugino}}(Q_{\text{GUT}}) = M_{1/2}$
- $[\text{scalar-scalar-scalar factor}](Q_{\text{GUT}}) = A_0$

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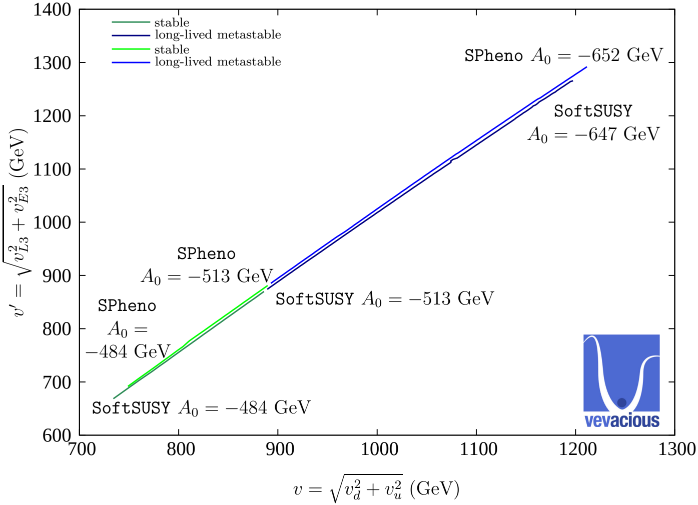
Minima could develop where $v_{\tilde{\tau}_L} v_{\tilde{\tau}_R} \left(A_\tau v_d - \mu v_u\right)$ gets more negative than "$m^2 v^2 + \lambda v^4$" is positive

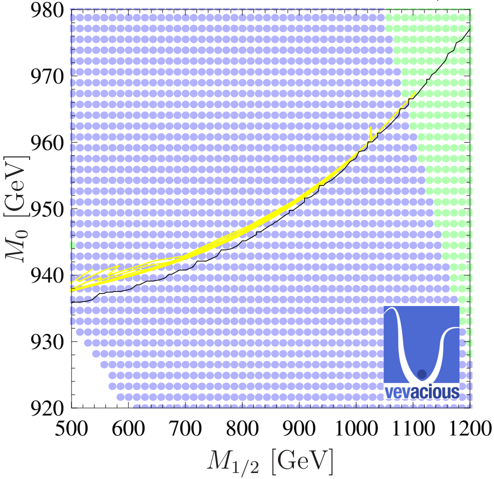
Camargo-Molina, BO'L, Porod, Staub, arXiv:1309.7212

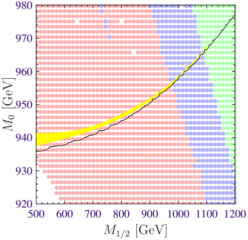


$m_0 = 400$ GeV, $M_{1/2} = 300$ GeV, $\tan\beta = 50$, $\mu > 0$

CCB restricts $\tilde{\tau}$ co-annihilation

$A_0 = +3$ TeV, $\tan\beta = 40$, $\mu > 0$; (`arXiv:1309.7212`)



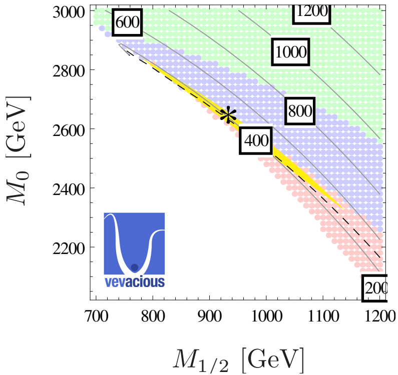


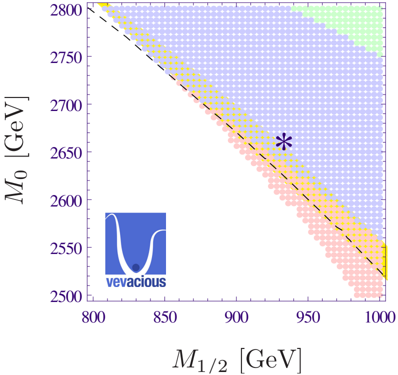
(Red $\tau_{\text{tunnel}} < 3$ Gy if misinterpreting `CosmoTransitions` $T \neq 0$K action as $T = 0$K action...)

blue: metastable ($\tau_{\text{tunnel}} > 3$ Gy); green: stable
yellow region: correct relic density; black: $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$

CCB restricts $\tilde{t}$ co-annihilation

$A_0 = -6.444$ TeV, $\tan\beta = 8.52$, $\mu < 0$
$m_{\tilde{t}_1}$ (GeV) contours (`arXiv:1309.7212`)

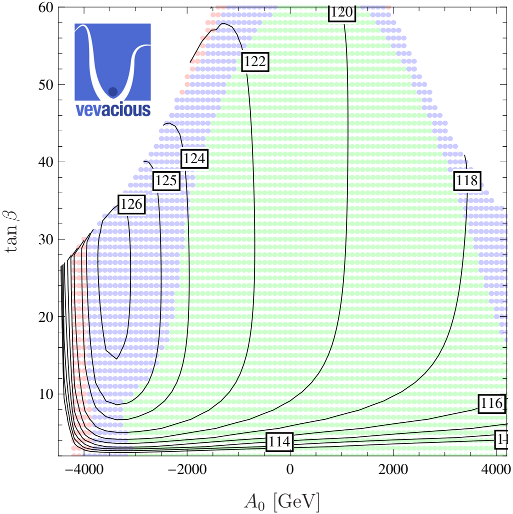




red/blue: metastable ($\tau_{\text{tunnel}} < / > 3$ Gy); green: stable
yellow region: correct relic density; dashed black for $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$

CCB restricts region with correct m_h

$M_0 = M_{1/2} = 1$ TeV, $\mu > 0$; m_h (GeV) contours
colors as before (1309.7212)



Analytic conditions

$$V^{\text{tree}}(H_d = v_d/\sqrt{2}, H_u = v_u/\sqrt{2}, \tilde{\tau}_L = v_{\tilde{\tau}_L}/\sqrt{2}, \tilde{\tau}_R = v_{\tilde{\tau}_R}/\sqrt{2})$$
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- $|(Y_\tau v_u \mu)/(\sqrt{2}m_\tau)| < 56.9\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} + 57.1(m_{\tilde{\tau}_L} + 1.03m_{\tilde{\tau}_R}) - 1.28 \times 10^4\text{GeV} + \frac{1.67\times10^6\text{GeV}^2}{m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}} - 6.41 \times 10^6\text{GeV}^3(\frac{1}{m_{\tilde{\tau}_L}^2} + \frac{0.983}{m_{\tilde{\tau}_R}^2})$ [“numeric”]

(“GUT”: Ellwanger, Rausch de Traubenberg, Savoy, Nucl. Phys. B**492**
“A_τ”, “A_t”: Alvarez-Gaumé, Polchinski, Wise, Nucl. Phys. B**221**;
“numeric”: Kitahara, Yoshinaga, arXiv:1303.0461, JHEP)

SPS4 ($M_0 = 400\text{GeV}, M_{1/2} = 300\text{GeV}, \tan\beta = 50, |\mu| > 0$,
$A_0 = 0\text{GeV}$) but with $A_0 \to$ ...

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A_0	generator	v_d	v_u	$v_{\tilde{\tau}_L}$	$v_{\tilde{\tau}_R}$
-484	`SPheno`	184	726	409	558
-484	`SoftSUSY`	181	712	394	540
-513	`SPheno`	269	851	540	701
-513	`SoftSUSY`	274	846	532	694
-652	`SPheno`	485	1110	819	999
-647	`SoftSUSY`	481	1097	800	981

$v_d : v_{\tilde{\tau}_L} : v_{\tilde{\tau}_R} : v_u \neq 1 : 1 : 1 : 0$ at CCB minimum

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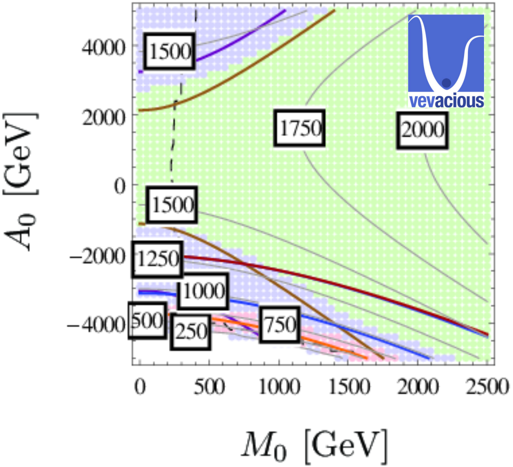
$v_d : v_{\tilde{\tau}_L} : v_{\tilde{\tau}_R} : v_u \neq 1 : 1 : 1 : 0$ at CCB minimum
$\Rightarrow$ not on line of "A_τ"!

Sometimes it looks like analytic conditions do well

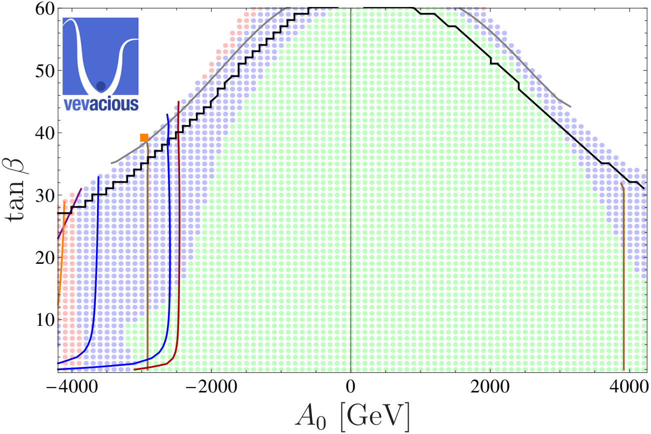
$M_{1/2} = 1$ TeV, $\tan\beta = 10$, $\mu > 0$; $m_{\tilde{\tau}_1}$ (GeV) contours (1309.7212)



Brown: "GUT"; Purple: "A_τ"; Orange: "A_t"; Dashed black: $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$
Dark red: "A_t" with small $v_{\tilde{b}}$ (Casas, Lleyda, Munoz, Nucl. Phys. B**471**)
Bright blue: "A_t" for $\tan\beta \to \infty$ (range) (Le Mouël, Phys. Rev. D**64**)

Analytic conditions do not always do well

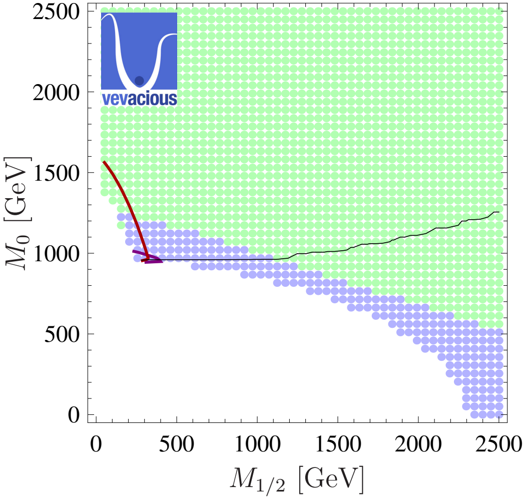
$M_{1/2} = 1000$ GeV, $m_0 = 1000$ GeV, $\mu > 0$ (1309.7212)



Brown: "GUT"; Purple: "A_τ"; Orange: "A_t"
Grey: "numeric"; Dark red: "A_t" with small $v_{\tilde{b}}$
Bright blue: "A_t" for $\tan\beta \to \infty$ (range); Black: $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$

Analytic conditions can completely fail

$A_0 = +3$ TeV, $\tan\beta = 40$, $\mu > 0$ (1309.7212)



Purple: "A_τ"; Dark red: improved "A_t"; Black: $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$

vevacious

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http://vevacious.hepforge.org/

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Before `Vevacious`: only implemented on a model-by-model basis, at tree level!

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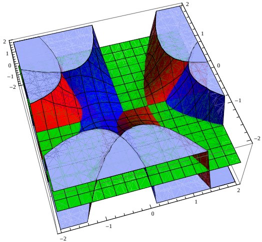
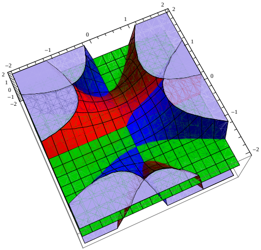
Now we deform f_s to f_t keeping track of the roots.



$\rightarrow$



Red: real part of f
Blue: imaginary part of f

Horizontal-ish axis: $\mathrm{Re}(z)$
Vertical-ish axis: $\mathrm{Im}(z)$

$(1-t)f_s + tf_t = z^3 + (2t-1)z$

- $t = 0$, roots at $z = -1, 0, 1$
- $t = 0.01$, roots at $z = -0.98995, 0, 0.98995$

How homotopy continuation works

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- $t = 0.99$, roots at $z = -0.98995i, 0, 0.98995i$
- $t = 1$, roots at $z = -i, 0, i$

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Animation time! (I hope...)

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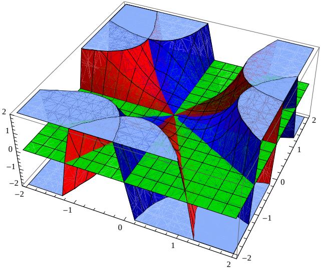

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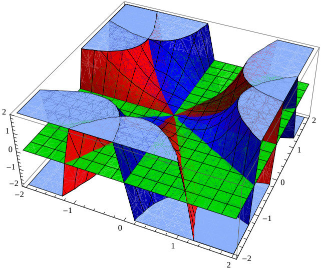

Way around is to take $(1-t)f_s + tcf_t$
where c is a complex constant.
Random $c \rightarrow$ *path crossing* (due to repeated root) happens only on set of measure zero.

Take tree-level potential SPS1a as *real* function of two *real* variables ϕ_d, ϕ_u

Analytically continue to complex function of two complex variables $\rightarrow$ four real variables $\phi'_d = x_d + iy_d$, $\phi'_u = x_u + iy_u$

Not same as original potential in terms of complex fields!
$\phi'^2_d = x^2_d - y^2_d + 2ix_d y_d$

Starting tadpoles (units of GeV): $\phi'^3_d = (200)^3$, $\phi'^3_u = (200)^3$
Complex factor $c = e^{1.23i}$

Starting root $\phi'_d = \phi'_u = -100 - 173.205i$

t	ϕ'_d	ϕ'_u
0.01	-97.6572 - 172.323 i	-100.297 - 173.262 i
0.5	5.92983 - 65.991 i	-111.973 - 175.893 i
0.99	-23.6607 - 3.37601 i	-233.192 - 26.8802 i
1.0	-25.0021 - 0.0 i	-243.781 - 0.0 i

The full set of solutions (ϕ'_d, ϕ'_u):

$$
\begin{array}{rrcrr}
-100-173.205i, & -100-173.205i & \to & -25, & -243\\
-100-173.205i, & 200+0i & \to & +25, & +243\\
-100-173.205i, & 100+173.205i & \to & 0, & 0\\
200+0i, & -100-173.205i & \to & -1516.79i, & -155.562i\\
200+0i, & 200+0i & \to & \text{``}\infty\text{''}, & \text{``}\infty\text{''}\\
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\end{array}
$$

"∞": target system does not have full amount of terms and thus not maximal amount of roots: some starting roots must go on divergent paths.

Note $\phi_d = -1516.79i, \phi_u - 155.562i$ is *not* an extremum of the potential, just a solution of the analytically continued tadpoles.

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- Evaluating stability of a parameter point depends on model

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- Creating model file with `SARAH` takes minutes: `MakeVevacious[]`

Outlook

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- Major bottleneck right now is `CosmoTransitions`: 5 minutes zero temperature, 10 hours non-zero temperature... Can this stage be optimized better? (Currently, tree-level is acceptably fast.)

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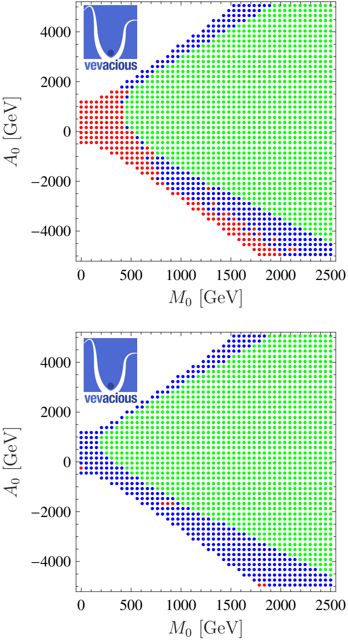
Thank you for your attention!

Backup slides

- $\Gamma/$ volume $= Ae^{-B/\hbar}(1 + \mathcal{O}(\hbar))$
- A is solitonic solution, should be $\sim$ energy scale of potential
- $B \sim$ ([surface tension]/[energy density difference])^3 for small energy density differences ("thin wall" bubbles)
- B very strongly dependent on energy barrier for large depth differences ("thick wall" bubbles)

Scale and loop order dependence: halving Q



Scale and loop order dependence: doubling Q