Determining the Neutrino Mass Hierarchy

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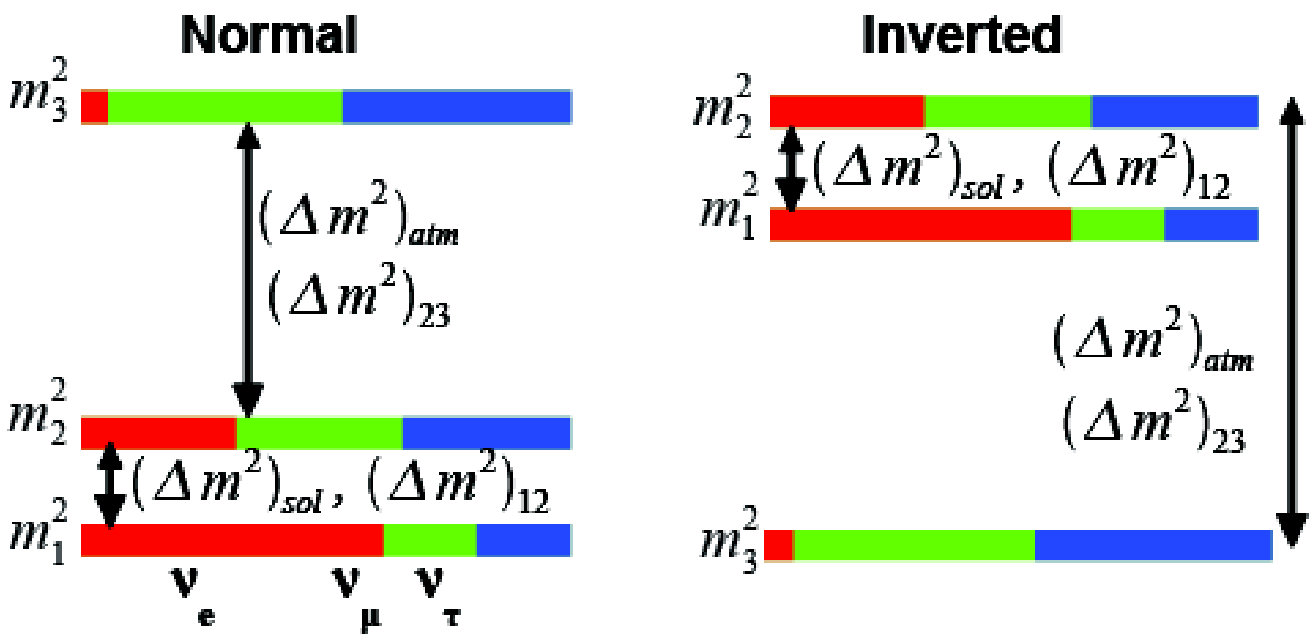
Outline

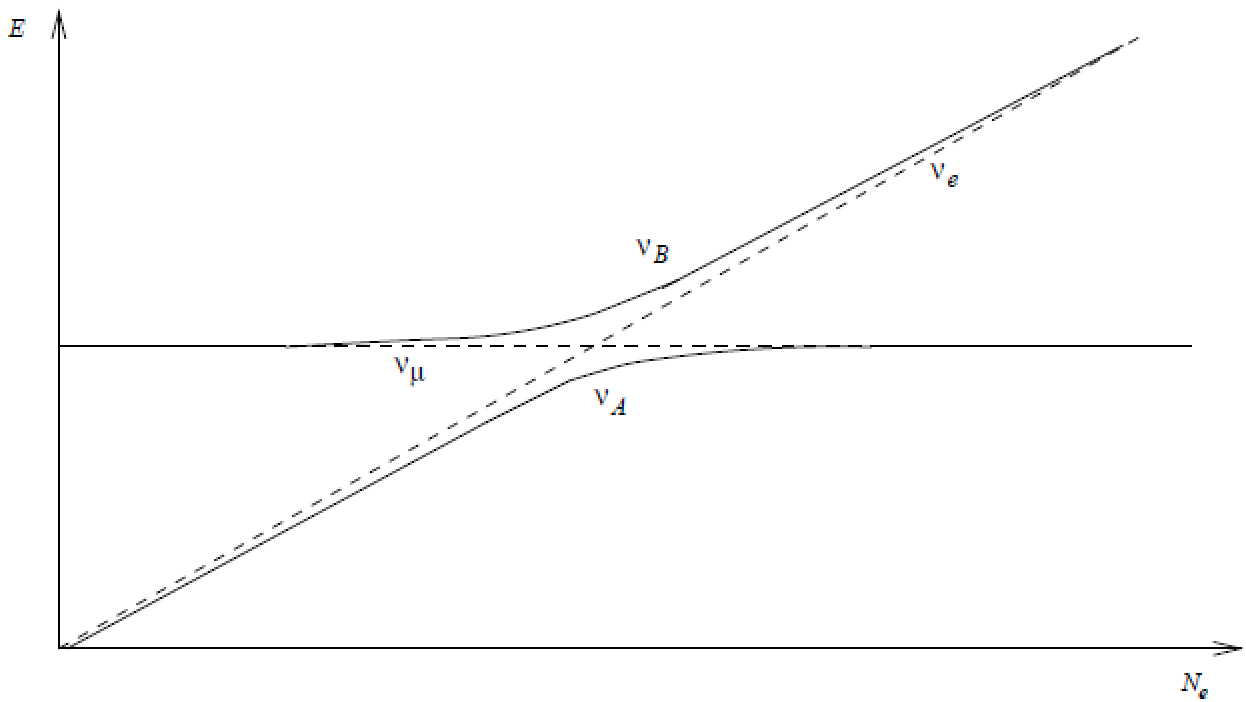
- Motivations
- A Reminder about Hypothesis Testing
- Statistical Issues in MH
- LBNO Study
- Other methods : atmospherics, reactor
- Conclusions

Neutrino Mass Hierarchy



Notice neutrino oscillation in the 2-neutrino regime are not sensitive to MH because
P(nu1->nu2) = sin**2(2th)sin**2(1.27 Delta m**2 L/E)
To gain any sensitivity three neutrino mixing effects needs to be taken into account, and the experiment needs to be sensitive to this
The large theta13 makes this more accessible than previously thought

Solar neutrinos and MSW effect



There are two possible orderings, not four, because the solar splitting is fixed by the MSW effect

Why do we care about the Neutrino Hierarchy ?

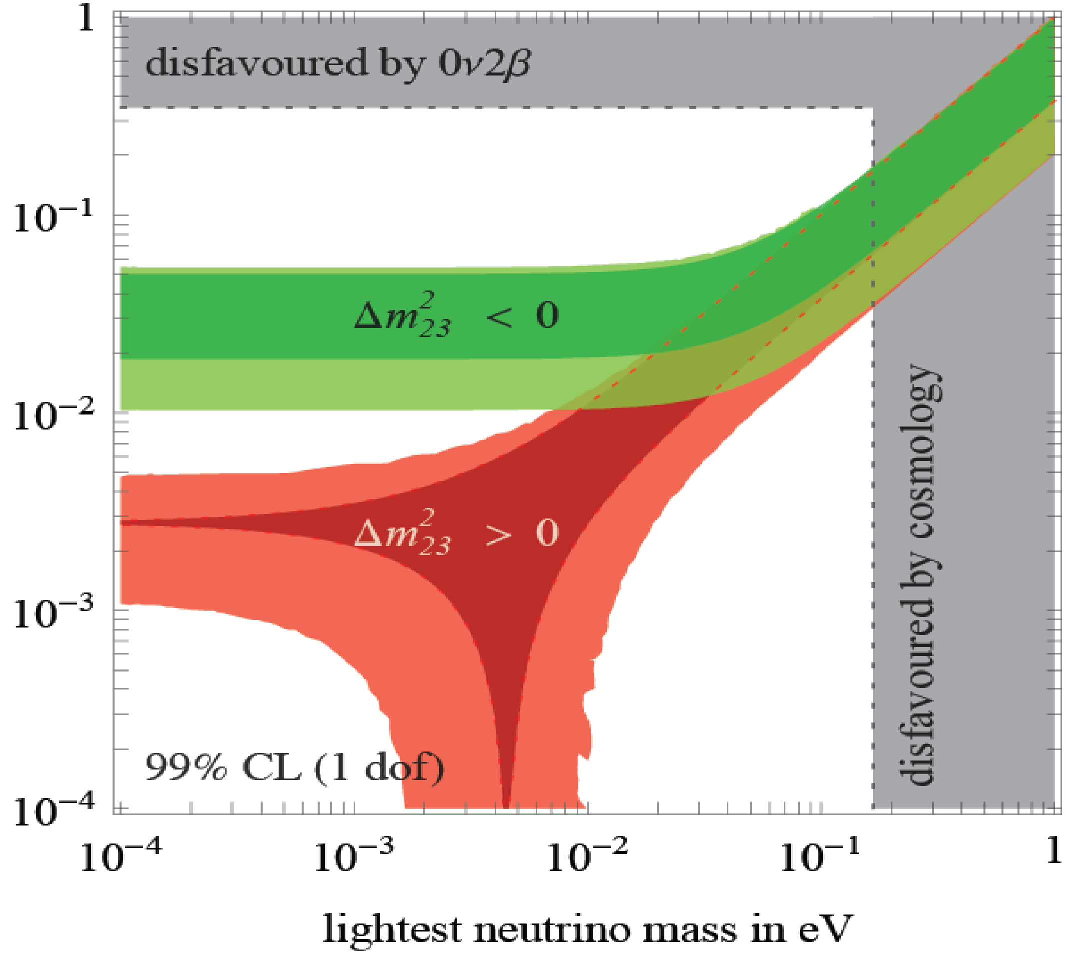
1) Input for model builders

2) Interpretation of double β 0-ν and cosmological measurements

3) Crucial ingredient for PMNS CP violation studies

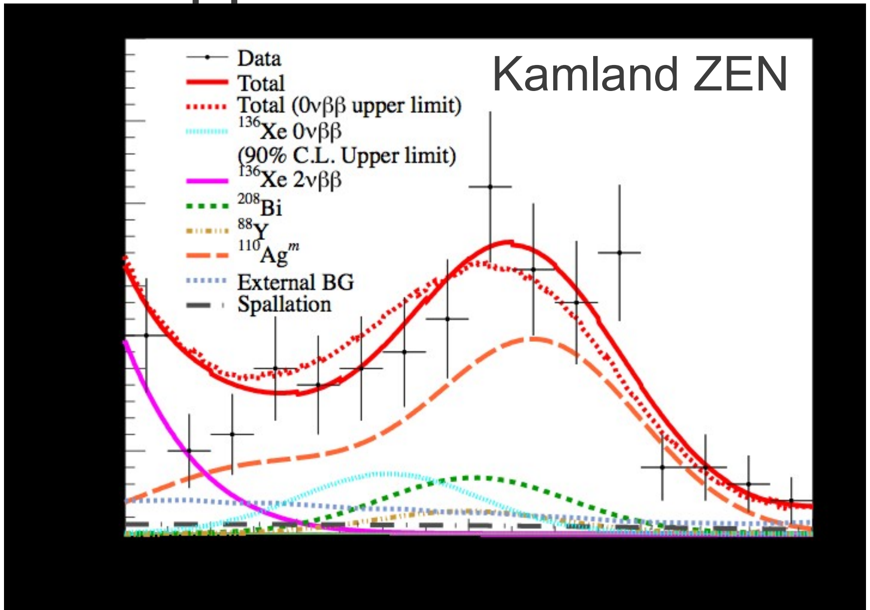
Motivation-2

- Interpreting double beta and cosmology measurements
- Several planned experiment may approach the IH region, below 100 meV

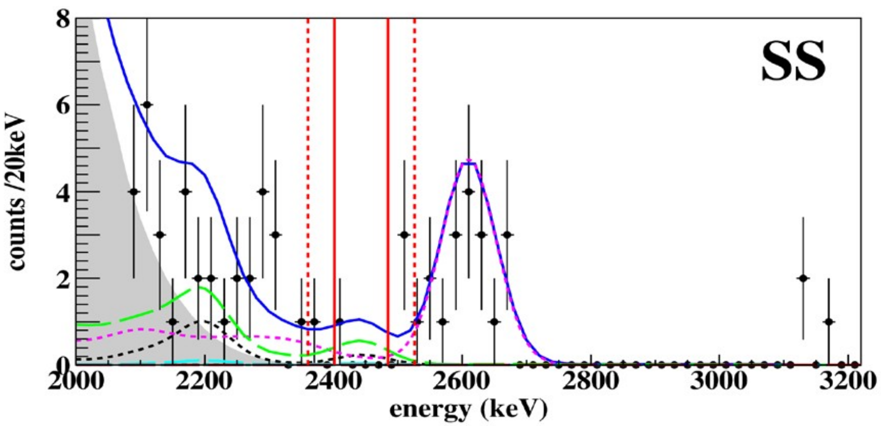


Interpreting double β 0-ν data

- The Klapdor claim should encourage some caution when interpreting data at the limit of the experimental sensitivity
- If an excess is observed in the IH range, knowing (with an independent method) that IH is realized in nature will provide a crucial confirmation
- Cf : theta13 reactor data and T2K numu->nue appearance



EXO Piepke@TAUP



Perspectives in cosmology

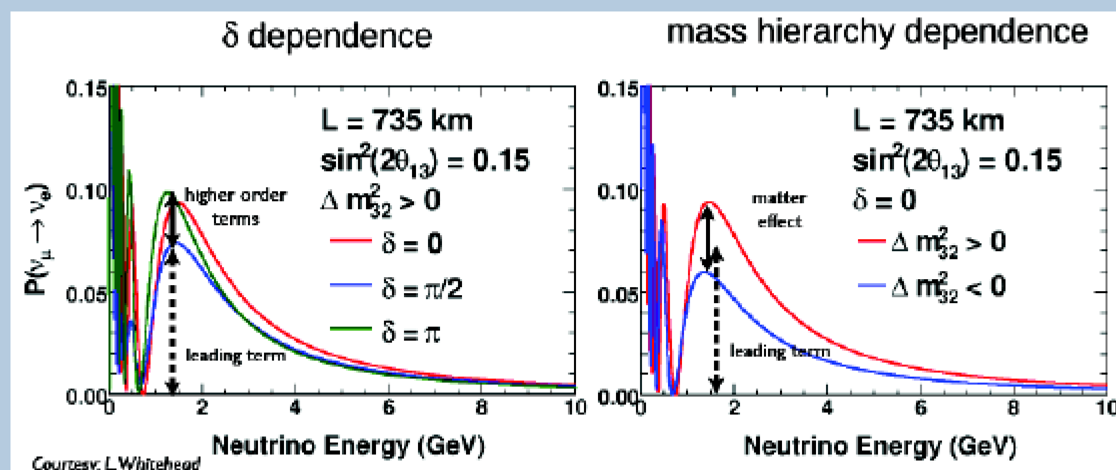
R. Cahn et al, arXiv:1307.5487

	$k_{\rm max}$ [Mpc^{-1}]	$\sigma_{\Sigma m_\nu}$ [eV]	$\sigma_{0.04\ \rm eV}$	Year
P+BigBOSS14+DES	0.07	0.021	1.9	2022
P+Euclid+DES	0.07	0.019	2.1	2026
P+BigBOSS24+DES	0.07	0.019	2.1	2026
P+BB24+Euc+DES	0.07	0.016	2.5	2026
P+BB24+Euc+LSST	0.07	0.014	2.9	$\lesssim 2030$
P+BB14+DES	0.14	0.017	2.4	2022
P+Euclid+DES	0.14	0.015	2.9	2026
P+BB24+DES	0.14	0.015	2.7	2026
P+BB24+Euc+DES	0.14	0.013	3.1	2026
P+BB24+Euc+LSST	0.14	0.011	3.6	$\lesssim 2030$

Motivation-3

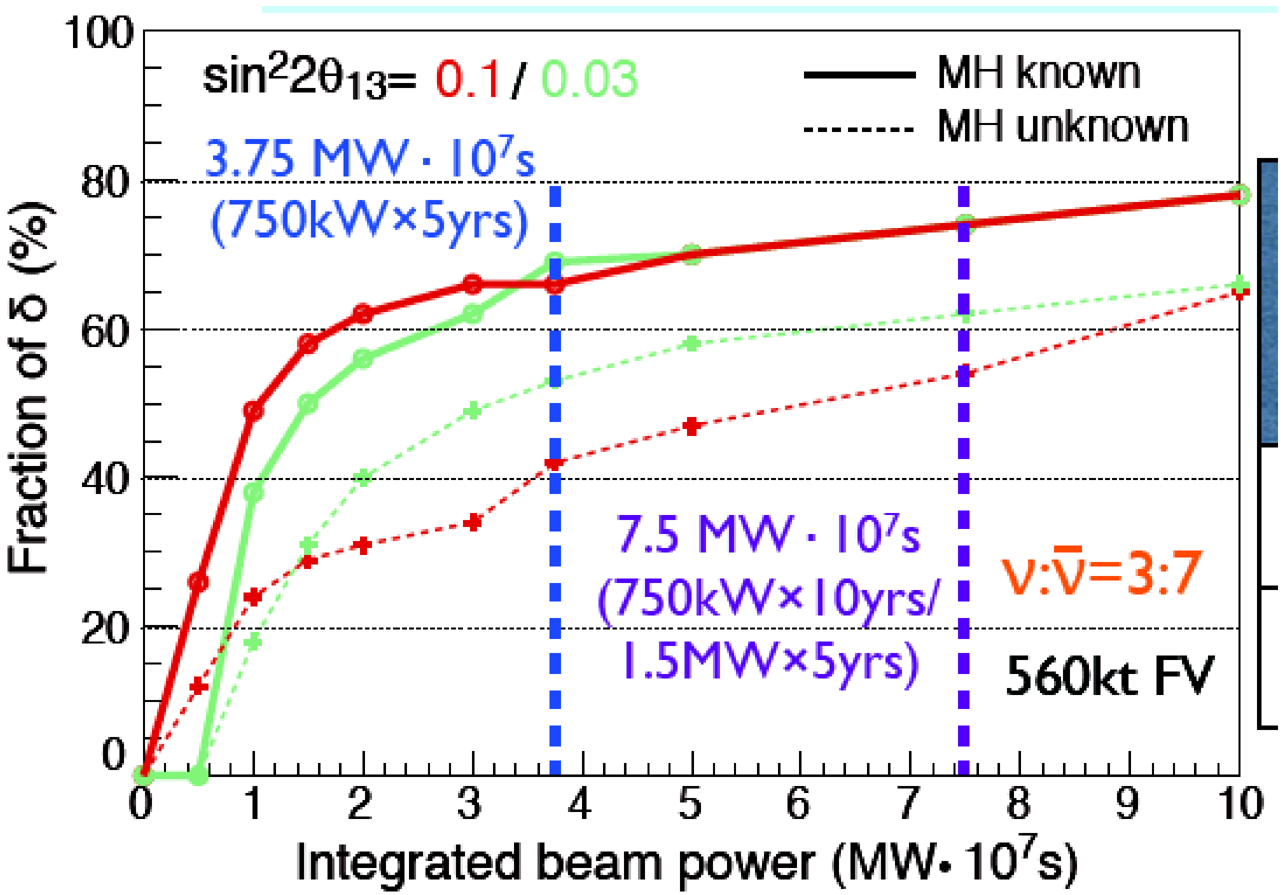
- Disentangling CP from MH

Interplay of CP and Matter Effects



- The simple study of the CP asymmetry is obscured (or enriched) by matter effects (interaction of ν with e in the traversed matter) that mimic a CP effect
- This complication can be seen as a challenge or an opportunity : clean measurement of mass hierarchy

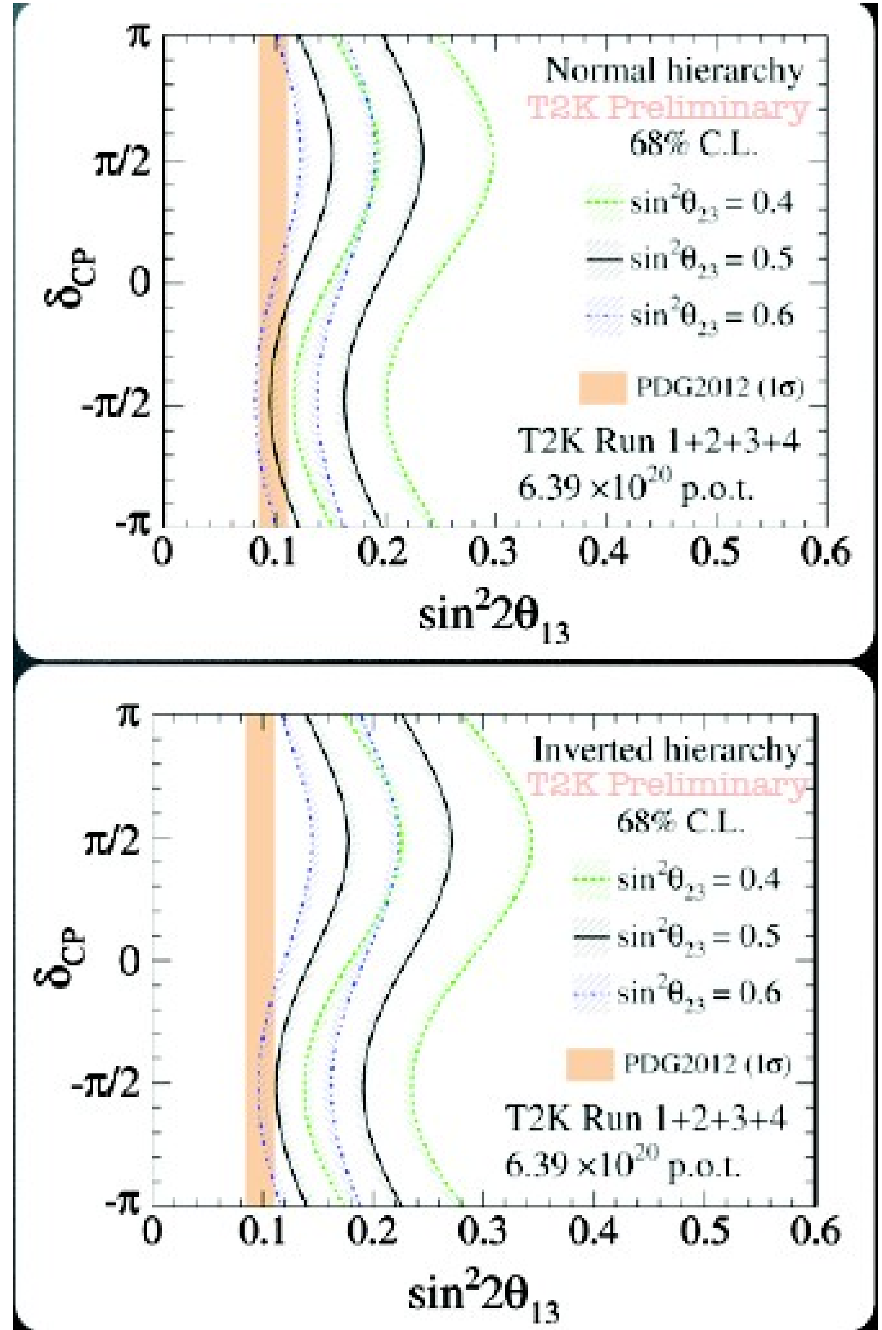
Crucial input for HK



MH knowledge equivalent to ~10 years of HK running

T2K 2013 results

- See talk by Benjamin
- Start excluding delta regions
- Different behaviour according to MH



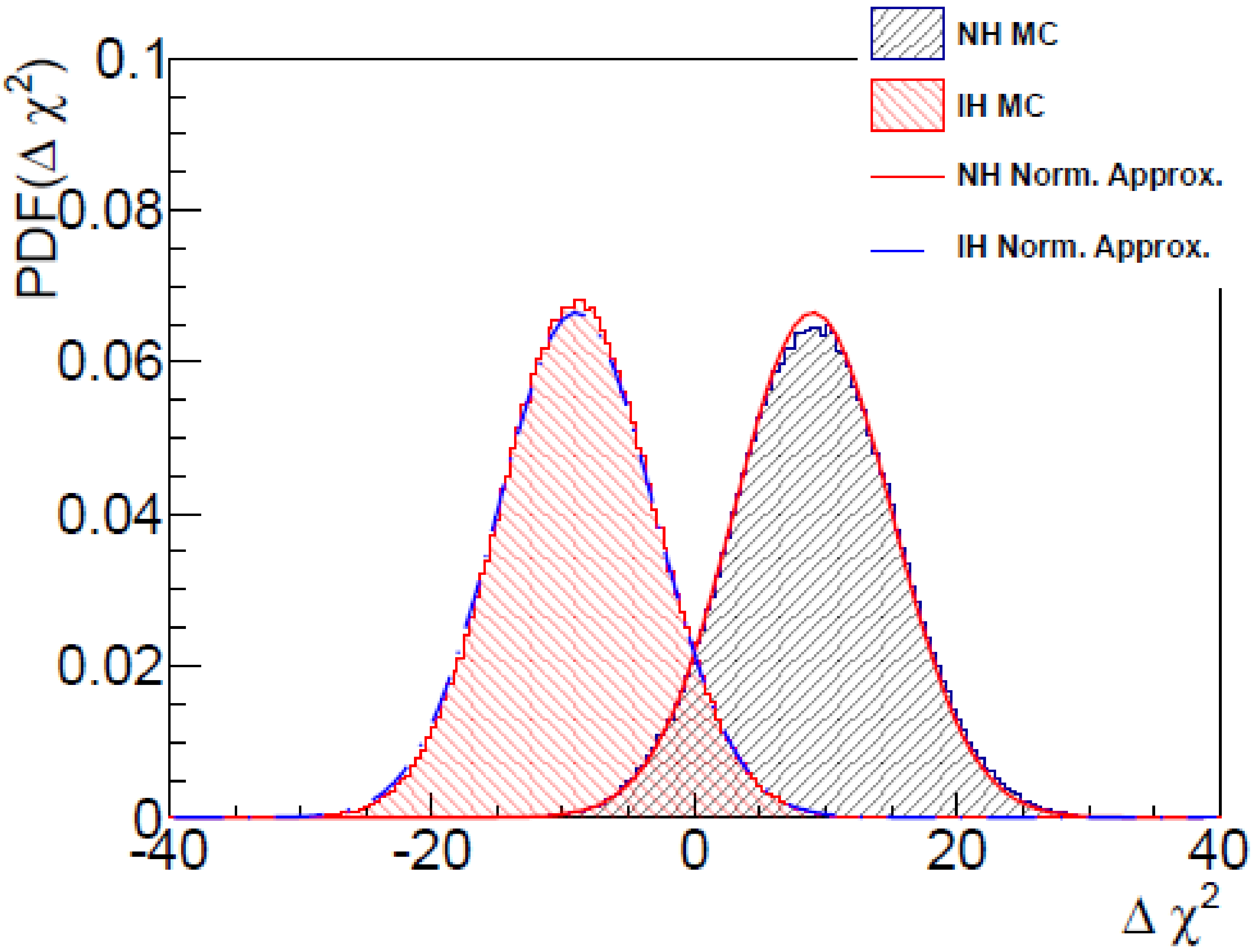
Hypothesis testing-1

- In experimental physics, we often encounter the following question : given a measurement, how well are the data in agreement with a given hypothesis ? How can we choose quantitatively between the default hypothesis H0 and an alternative hypothesis H1 ?
- For instance: H0 = existence of a Higgs boson with MH=125 GeV/c**2, H1= no Higgs
- Or H0=neutrino oscillation with probabilities given by the PMNS model, H1= no oscillation

Hypothesis testing-1

- A measurement consists of n data values X = (x1, x2 …,xn) (eg n of events in each bin of a distribution) and each hypothesis specifies a pdf f(X| H0), f(X|H1) etc
- To measure the agreement between the data and an hypothesis, one constructs a function of the measured variables called a “test statistic” t(X).
- For each of the hypothesis, there is a specific pdf for the statistic t g(t|H0), g(t|H1)

Example


NH MC
IH MC
NH Norm. Approx.
IH Norm. Approx.
PDF(Δ χ²)
0.1
0.08
0.06
0.04
0.02
0
-40
-20
0
20
40
Δ χ²

The Neyman Pearson lemma

- The Neyman-Pearson lemma states that the acceptance region (where we accept H0) with the best purity for a given efficiency is defined by $g(t|H0)/g(t|H1)>c$
- Where c is determined by the required efficiency
- $r=g(t|H0)/g(t|H1)$ is called a likelihood ratio

The Wilks theorem (1937)

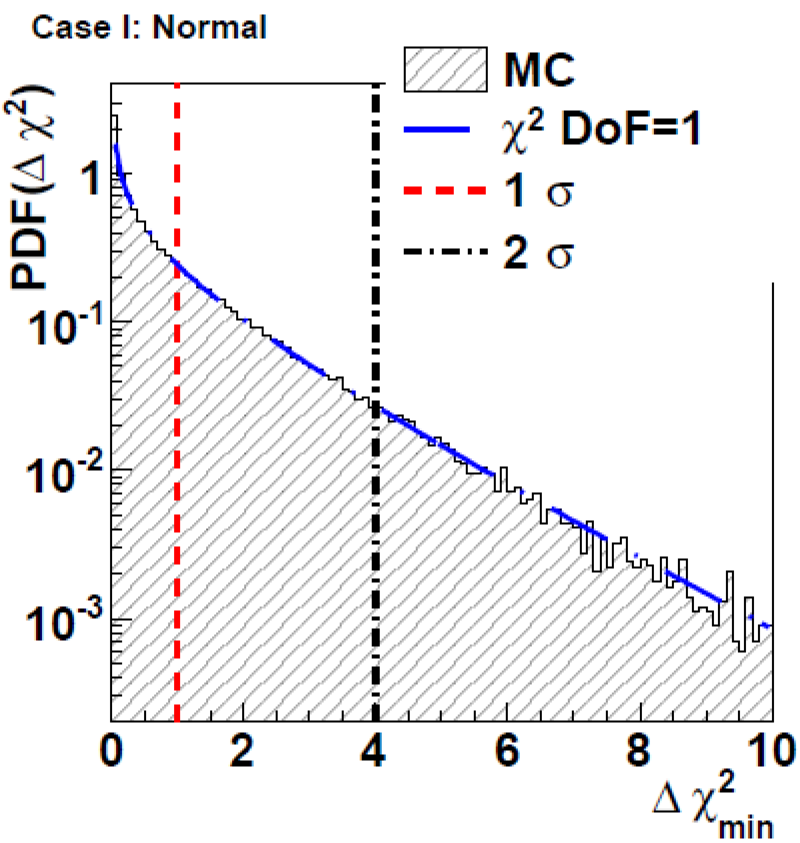
- This is a special case of the hypothesis testing
- Where one hypothesis (H0) consists of a subset ω of all acceptable hypotheses Ω (also called "nested set of hypothesis)
- In the space of n parameters to be fitted (θ_1 ,θ_2..θ_n), H0 is obtained fixing n-m parameters θ_{i+m} = $\theta°_{i+m}$, , θ_n=$\theta°_n$
- Then it can be shown that the likelihood ratio L_ω(H0)/L_Ω(H1) is distributed according to a χ^2 distribution with n-m dof for a large number of events

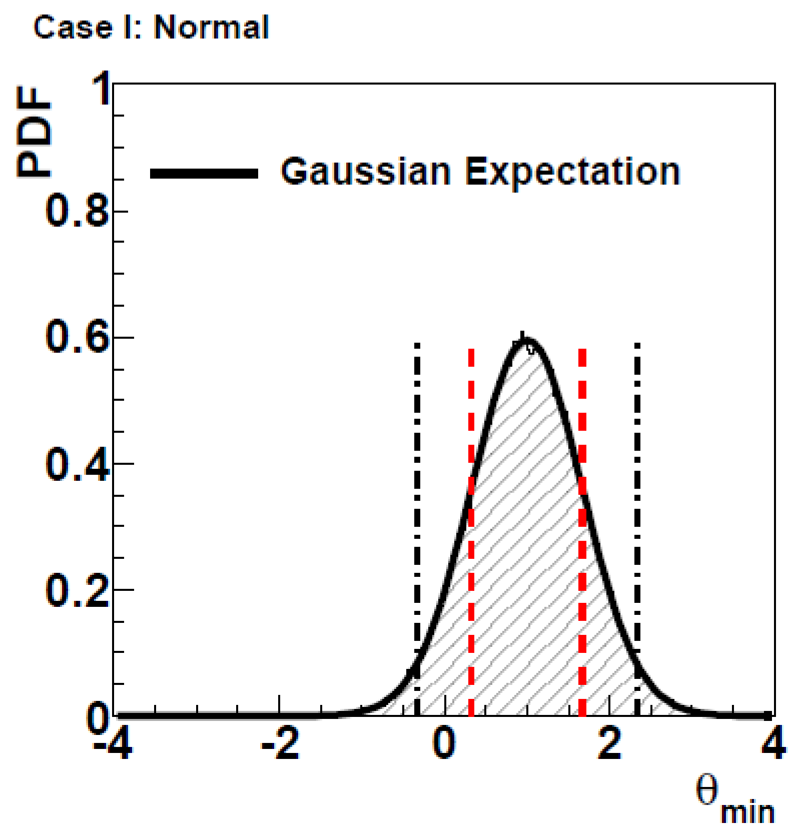
A typical case

- For a single real variable θ, the hypothesis (H0) is $\theta=\theta°$
- Here n=1, n-m=1
- After the measurement, the data are fitted giving θ_{min}
- Then one can accept or reject H0 by studying the likelihood ratio $P(\theta°)/P(\theta_{min})$
- This is equivalent to $\Delta\chi^2 = \chi^2(\theta°)-\chi^2(\theta_{min})$
- This is distributed as a χ^2 with 1 dof
- How far are the data off from the fit value ? From the definition of $\chi^2=(x-x0)^2/\sigma^2$ (x is normally distributed variable) one can simply read the number of σ as $n=\text{sqrt}(\Delta\chi^2)$. This is related to the p-value. Suppose to redo the experiment many times. How often will the χ^2 be as bad as it has been seen or worse ?

Qian et al.

Example





Mass Hierarchy (until recently)

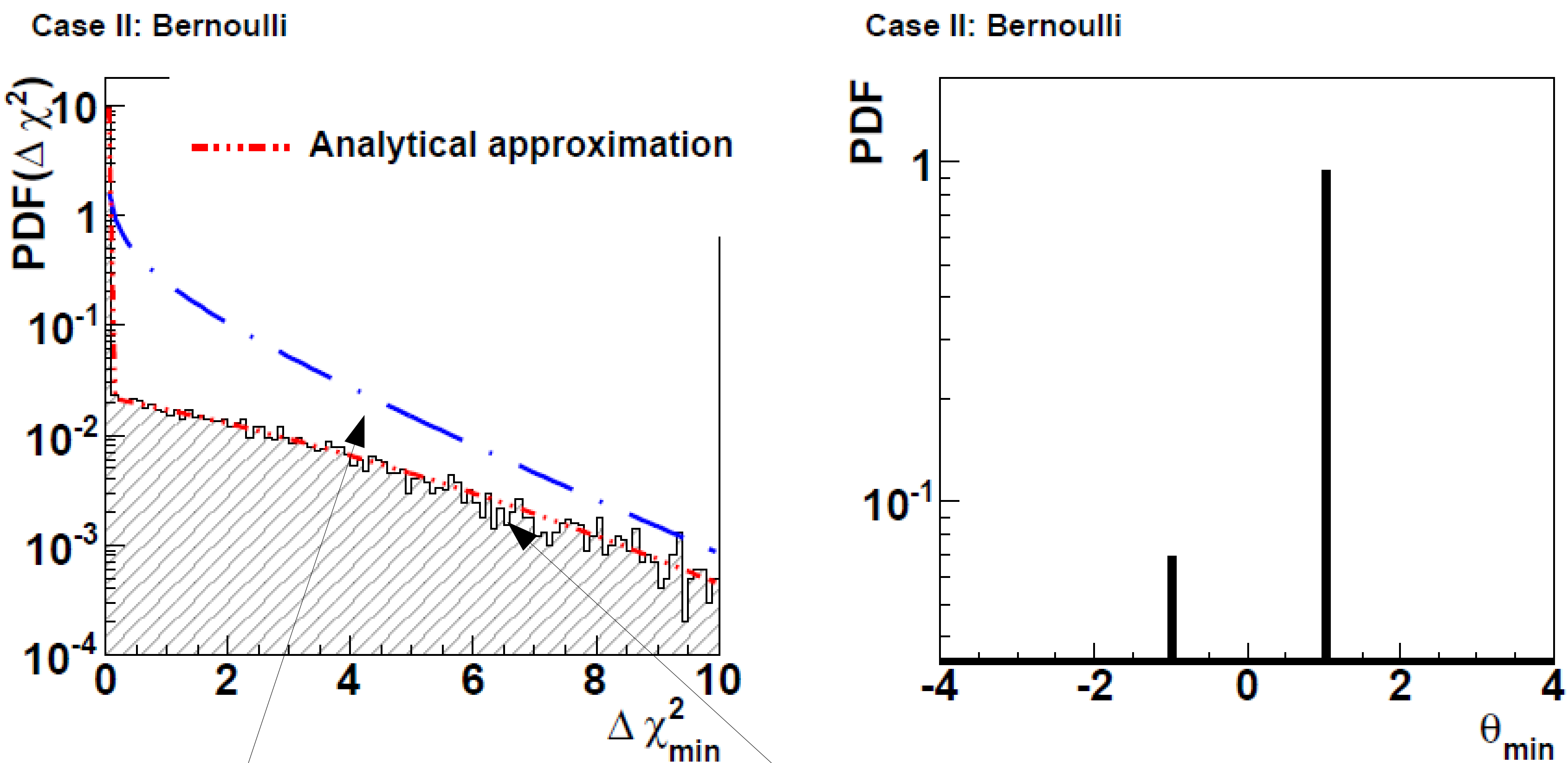
- Until recently the statistical tool to assess the sensitivity consisted of building (say for true NH)
- $\Delta\chi^2 = \chi^2(IH) - \chi^2(NH)$
- Can we apply the Wilks theorem here and interpret $\Delta\chi^2$ as (n_σ)**2 ?

The answer is NO

- X. Qian et al. arXiv:1210.3651v3
- E. Ciuffoli et al. arXiv:1305.5150v2
- F. Capozzi et al. arXiv:1309.1638v1

Qian et al.

Evidence that Wilks does not apply



Expected if Wilks theorem holds

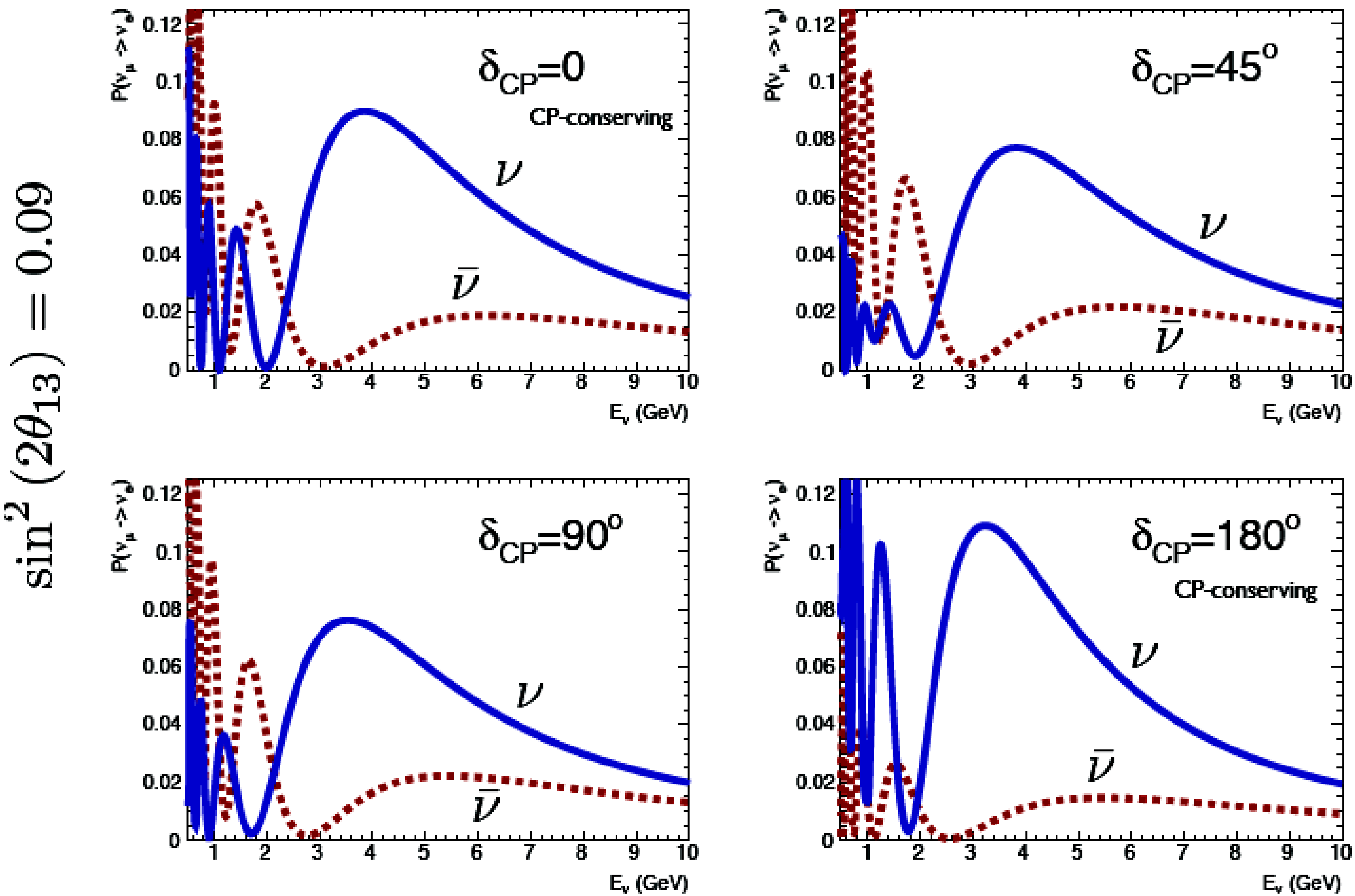
Observed (toys) very different distribution

Interpretation

- Ciuffoli et al demonstrate that $\Delta\chi^2$ is distributed as a gaussian with $\sigma=2\sqrt{\Delta\chi^2}$
- Capozzi builds a continuous variable alpha interpolating between NH and IH. Then, $\Delta\chi^2$ should be measured from alpha=0, where hierarchy information is lost, not from the full $\chi^2(IH)-\chi^2(NH)$. The factor 2 is explained in intuitively easy terms

CERN-Pyhäsalmi: oscillations

★Normal mass hierarchy L=2300 km



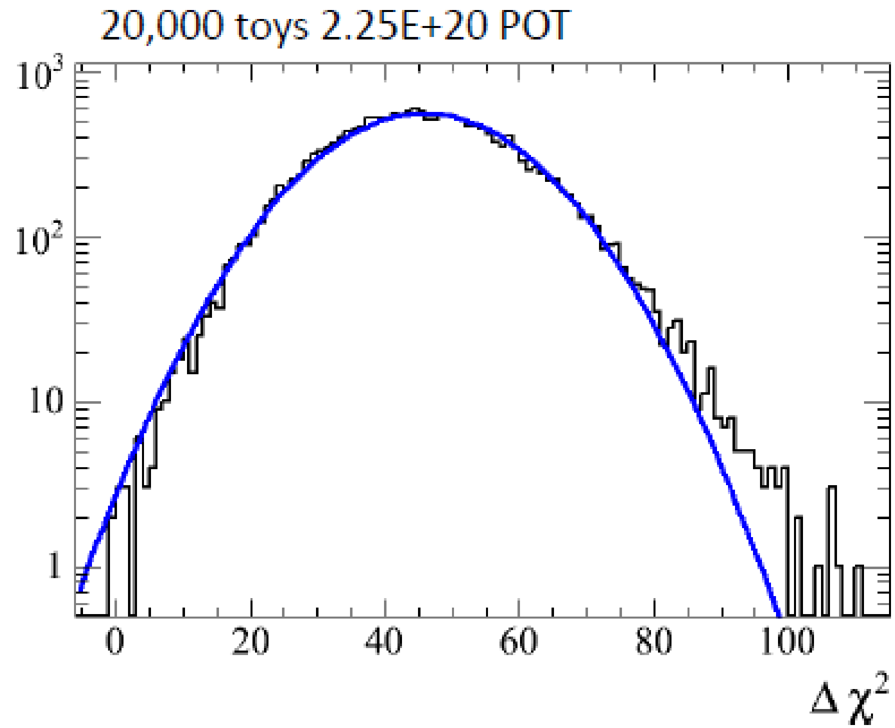
PDF($\Delta\chi^2$) & Significance

- Minimize χ^2 with respect to systematic/oscillation parameters (including δ_{CP}) for each mass hierarchy:
  - $\chi^2_{\min}(NH) = \chi^2_{true}$ ← If the true hierarchy is normal one
  - $\chi^2_{\min}(IH) = \chi^2_{false}$
- Calculate $\Delta\chi^2 = \chi^2_{false} - \chi^2_{true}$ for each toy data set
- If $\Delta\chi^2 < 0$, then the wrong solution is preferred
- Significance:

$$\Pr(\Delta\chi^2 \geq 0) = \int_0^\infty \mathrm{PDF}(\Delta\chi^2)\,\mathrm{d}\Delta\chi^2$$

LBNO

Example: PDF($\Delta\chi^2$) in LBNO for $\delta_{CP} = 90^\circ$



Fit with a Gaussian gives significance of 3.46σ (cf. 3.57σ for Qian et al.)

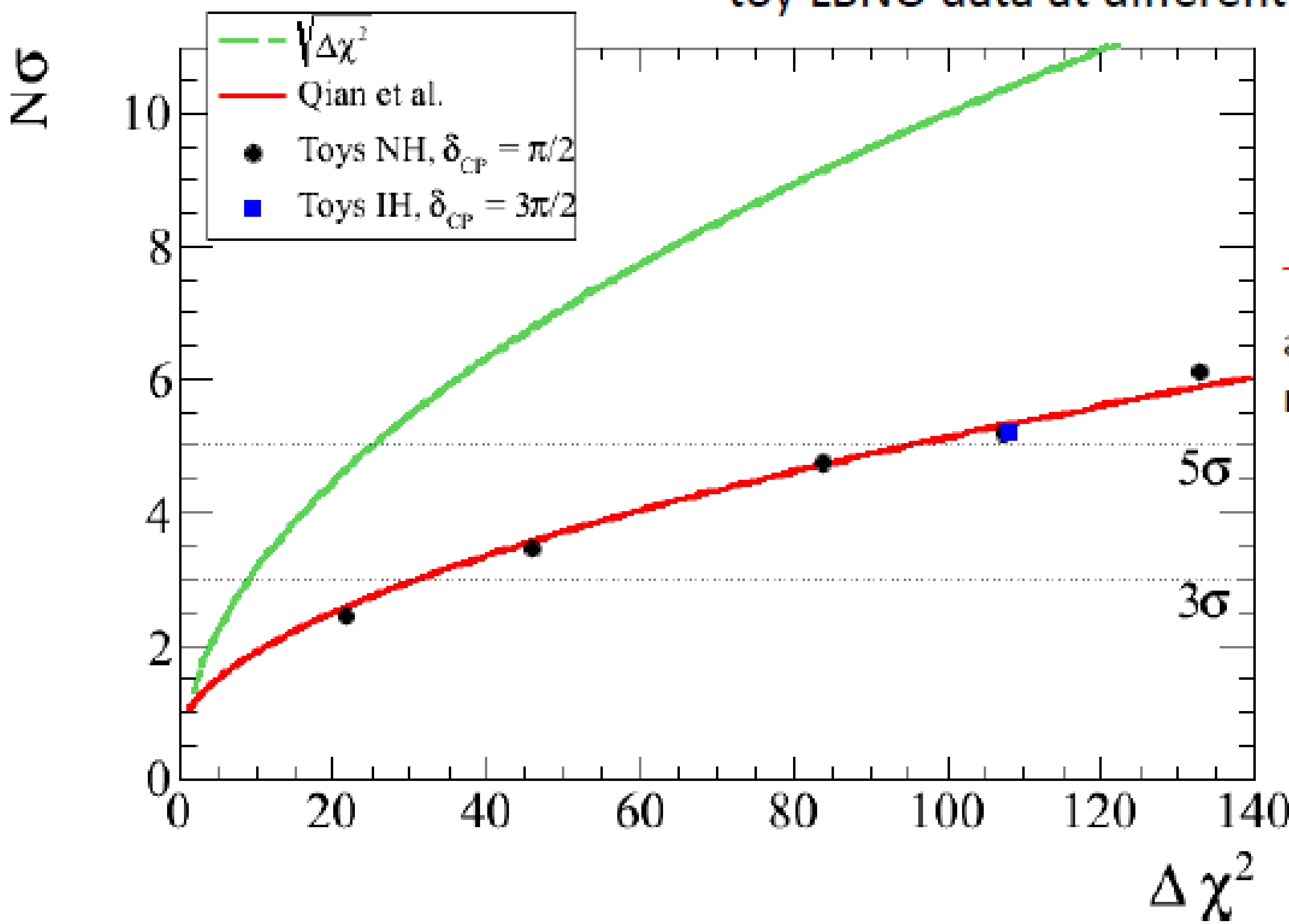
The naïve calculation for $\Delta\chi^2 \sim 45$ gives $\sqrt{45} = 6.7\sigma$

Fit overestimates the negative tail (the distribution is skewed to +ve values).
Skewed gaussian fit → the effect is 6 -- 7%

V. Galymov

Confidence in MH determination

Points are confidence levels estimated from 20,000 toy LBNO data at different exposures

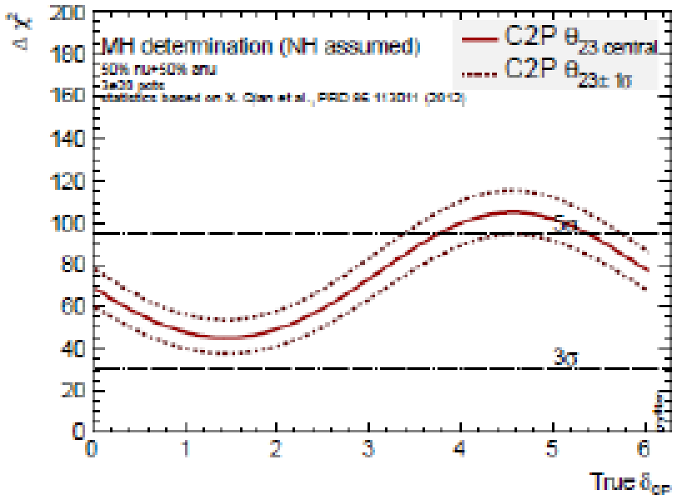


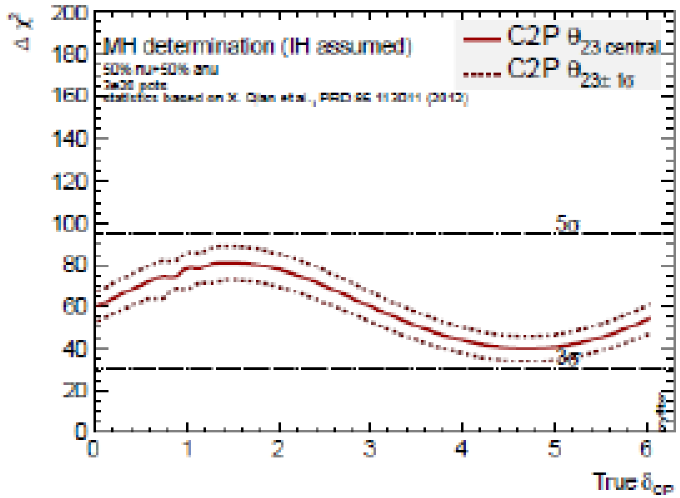
The model of Qian et al., appears to be working reasonable well for LBNO

3σ: $\Delta\chi^2 = 30.96$

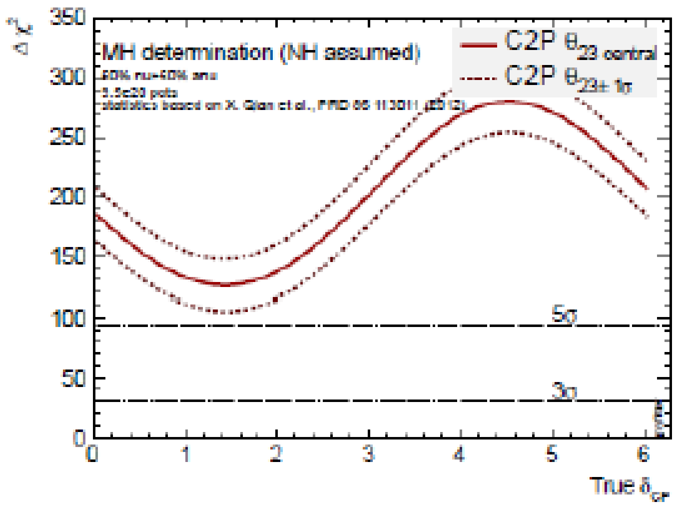
5σ: $\Delta\chi^2 = 94.66$

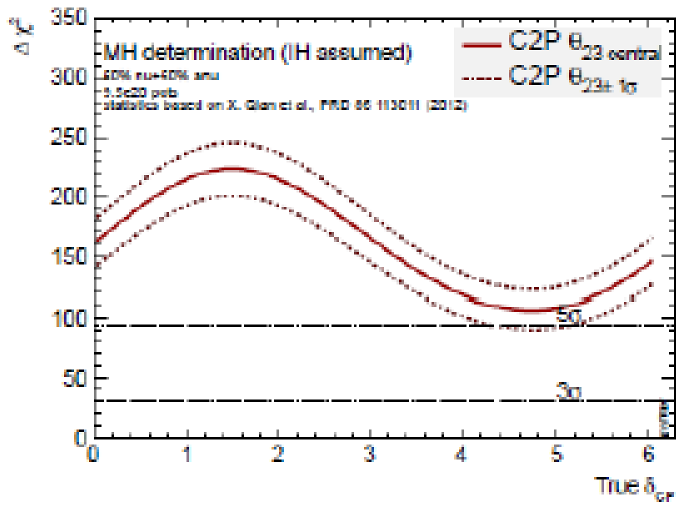
MH determination in LBNO





2-3y run → sensitivity @ 3σ level



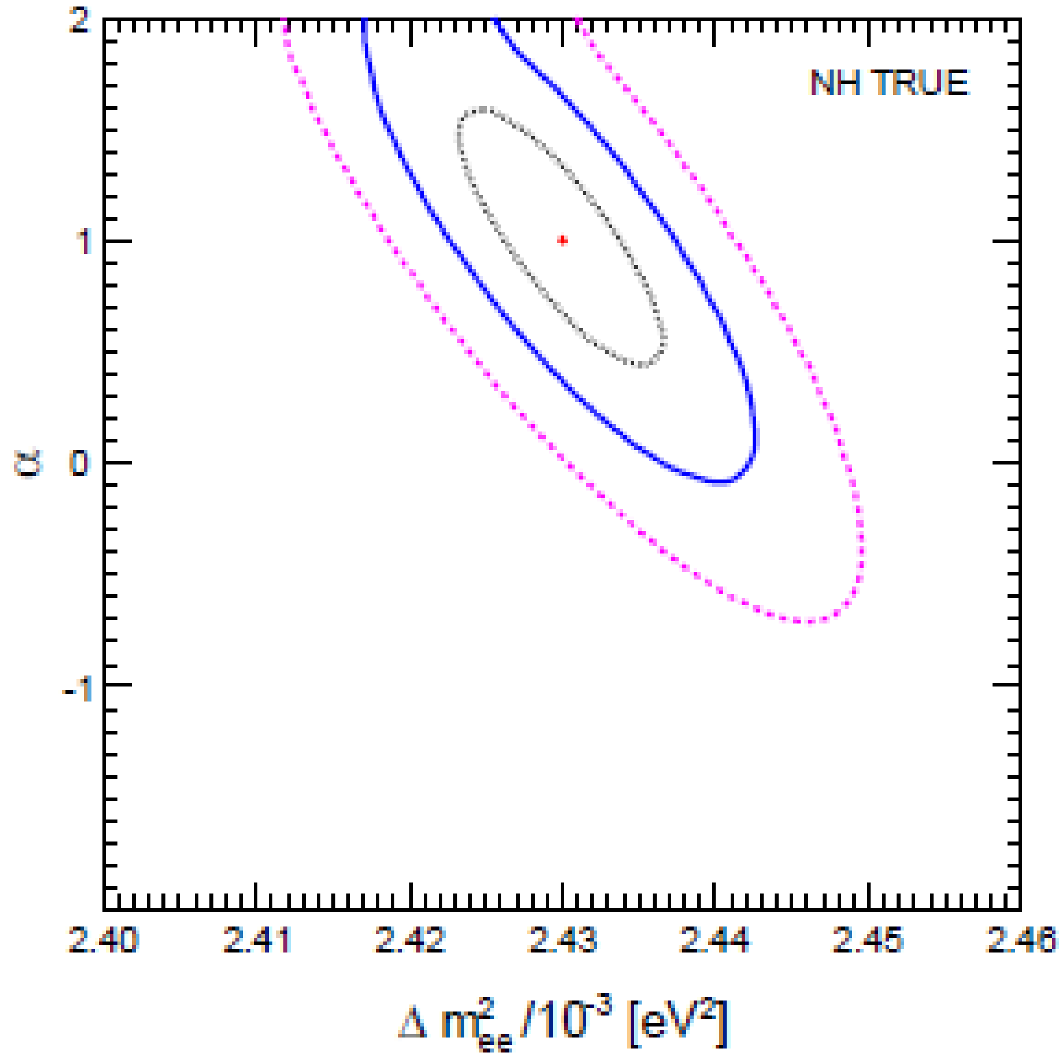


6-7y run → sensitivity @ 5σ level

• F. Capozzi et al. arXiv:1309.1638v1

Other methods

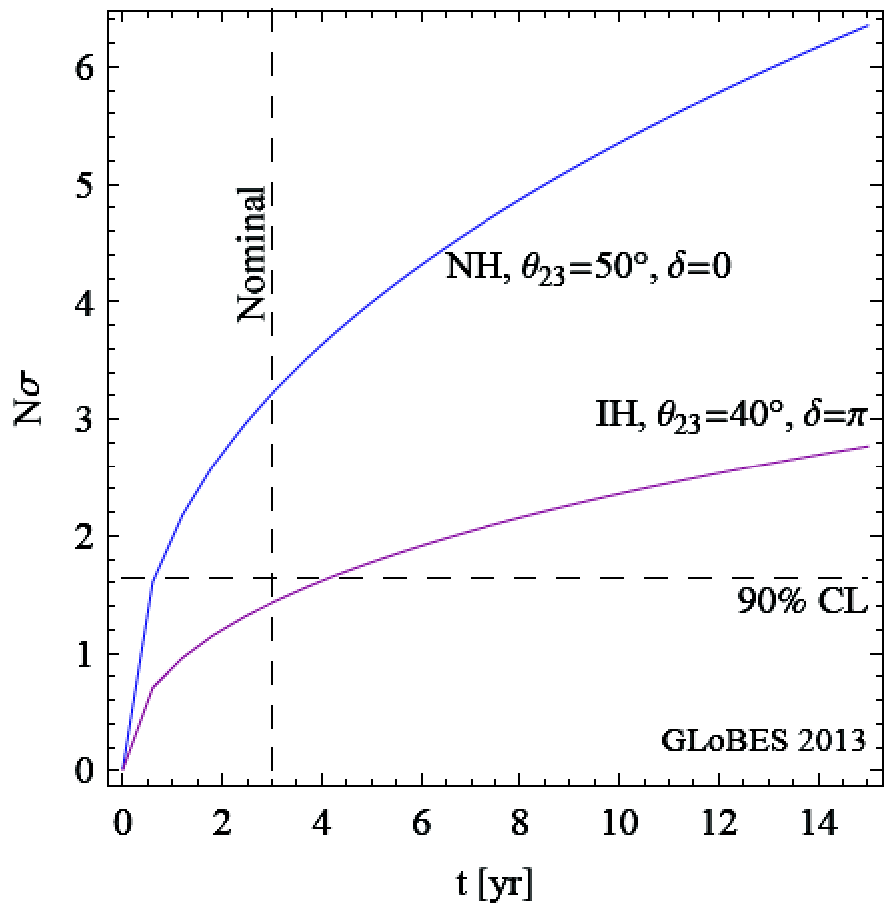
- JUNO



Using correct MH statistics

W. Winter, [arXiv:1305.5539] (2013).

PINGU



Not corrected for MH statistics

Conclusions

- Never apply “widely used statistical recipes” without paying attention to the specific problem
- If any doubt, check with toys
- The sensitivity reported for all the Neutrino Mass Hierarchy determinations needs to be reevaluated. With good approximation the n of sigmas should be divided by 2
- MH will be much more difficult than previously thought
- One good experiment with n sigmas=5 or more is much more valuable than many experiments with 1-2 sigmas