Arnaldo Gammal

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Critical Stability
2014-Santos

Guarujá and Santos beaches ~100km from Sao Paulo

Arnaldo Gammal- Theory on BEC

E. G. Khamis (post-doc)
K. Piacentini (PhD-sandwich at Amherst)
H. Fabrelli Ferreira (MS)
Robinson Pompeu (UG)

weekly JC with professors
Antonio Piza and Emerson Passos

Collaborations

-T. Frederico ITA (S. Jose dos Campos)
-L. Tomio IFT-UNESP (Sao Paulo)
-F. Kh. Abdullaev (Malaysia)
-B. Malomed (Israel)
-Tito Mendonça (IST-Portugal)
-B. Capo-Grosso (Oklahoma)

Outline

- Review of two previous works
- BEC past obstacle
- Experimental results
- Supersonic flow
- Oscillating attractive-repulsive obstacle

Critical number of atoms trapped BEC a<0

$$N_{cr} = k a_{ho}/|a| \ ,$$

P.A. Ruprecht, M.J. Holland, K. Burnett, and M. Edwards, Phys. Rev. A **51, 4704 (1995). Spherical symmetry k=0.575**

J.L. Roberts, N.R. Claussen, S.L. Cornish, E.A. Donley, E.A.Cornell, and C.E. Wieman, PRL **86,** 4211 (2001).

E. A. Donley *et a*l., Nature **412**, 295 (2001). BOSENOVA

assymetry factor λ=6.80/17.35   k=0.46(6)

PHYSICAL REVIEW A, VOLUME 64, 055602

Critical number of atoms for attractive Bose-Einstein condensates with cylindrically symmetrical traps

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[1]*Instituto de Física Teórica, Universidade Estadual Paulista, 01405-900 São Paulo, Brazil*
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(Received 12 Apri$\omega_\rho=\omega_x=\omega_y$ 10 October 2001)

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{m}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)-\frac{4\pi\hbar^2|a|}{m}|\Psi(\vec{r},t)|^2\right]\Psi(\vec{r},t).$$

$$\Psi(\vec{r},t)=e^{-i\mu t/\hbar}\psi(\vec{r}) \qquad \omega_\rho=\omega_x=\omega_y \qquad \lambda\equiv\frac{\omega_z}{\omega_\rho}$$

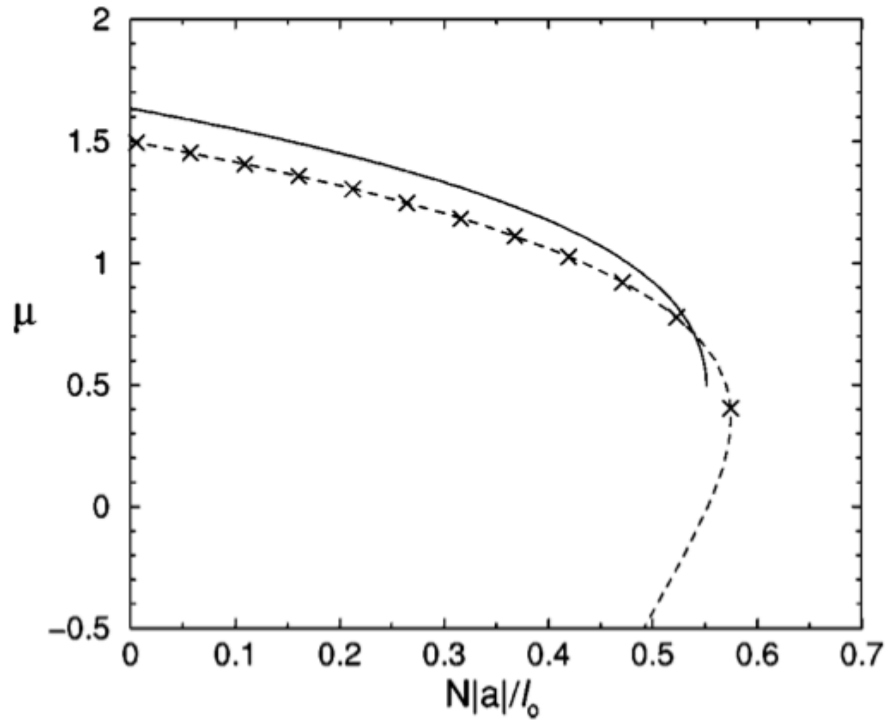

FIG. 1. The chemical potential μ is given in units of $\hbar\bar{\omega}$, as a function of $N|a|/l_0$. Results with spherical symmetry ($\lambda=1$), in dashed line and with $\times$, are compared with results using $\lambda=6.80/17.35$ (solid line). Dashed line was obtained using shooting-Runge-Kutta method, while the $\times$ and the solid line were obtained by propagation in imaginary time.

For the experimental value assymetry we got theoretical value k=0.550

The experimental value was revised to k=0.547(48)

N. R. Claussen, S.J.J.M.F. Kokkelmans,
S. T. Thompson, E. A. Donley, and C. E. Wieman, PRA **67**, 060701R (2003)

PHYSICAL REVIEW A, VOLUME 61, 051602(R)

Liquid-gas phase transition in Bose-Einstein condensates with time evolution

A. Gammal,[1] T. Frederico,[2] Lauro Tomio,[1] and Ph. Chomaz[3]
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[2]*Departamento de Física, Instituto Tecnológico da Aeronáutica, CTA, 12228-900, São José dos Campos, Brazil*
[3]*GANIL, Boîte Postal 5027, F-14021 Caen Cedex, France*
(Received 30 September 1999; published 30 March 2000)

$$\mu\psi(\vec{r}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}\omega^2 r^2 - N\frac{4\pi\hbar^2|a|}{m}|\psi(\vec{r})|^2 + \lambda_3 N^2|\psi(\vec{r})|^4\right]\psi(\vec{r})$$

$$g_3 \equiv \lambda_3 \hbar\omega \left[\frac{m}{4\pi\hbar^2 a}\right]^2$$

$$\beta \equiv \frac{\mu}{\hbar\omega}$$

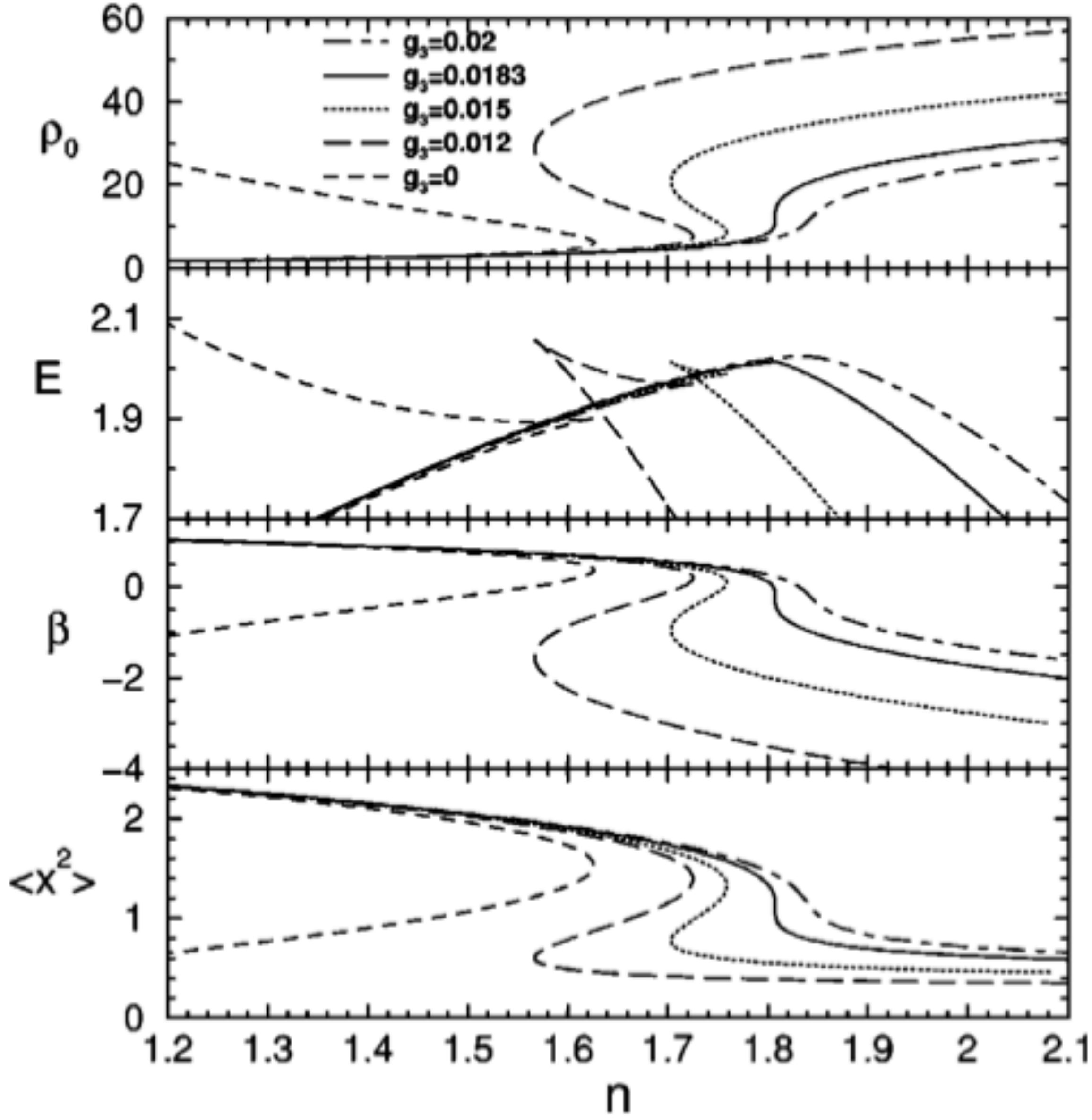

FIG. 1. Central density ρ_0, total energy E, chemical potential β, and average square radius $\langle x^2 \rangle$, in dimensionless units, as functions of the reduced number of atoms n. The three-body strengths g_3 are given in the upper frame.

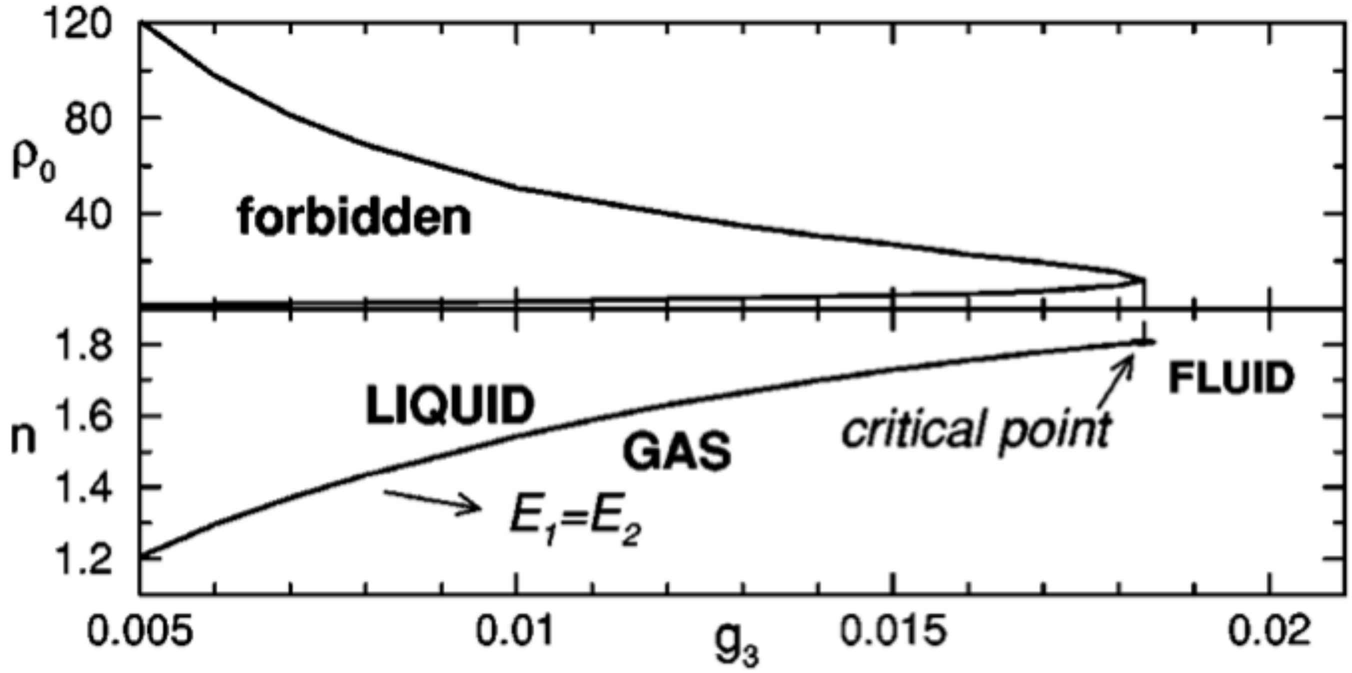

FIG. 2. Phase diagram of the Bose condensate, for the central density ρ_0 and the reduced number of atoms n, in dimensionless units.

Oscillating attractive-repulsive obstacle in supersonic Flow in BEC

Eduardo G. Khamis[1,2], Arnaldo Gammal[2],

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[2]Universidade de São Paulo, São Paulo, Brazil

Outline

- Introduction
- BEC past obstacle
- Experimental results
- Supersonic flow
- Oscillating attractive-repulsive obstacle

Introduction- Classical Fluids

Reynolds Nr.

$$\mathrm{Re} = D v_0 \rho / \eta$$

D = depends of the shape/size of the body [L]
v_0 = flux velocity
η = viscosity

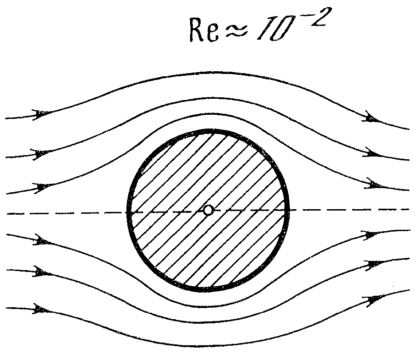


Fig. 9.1. Laminar flow around a cylinder for small Re

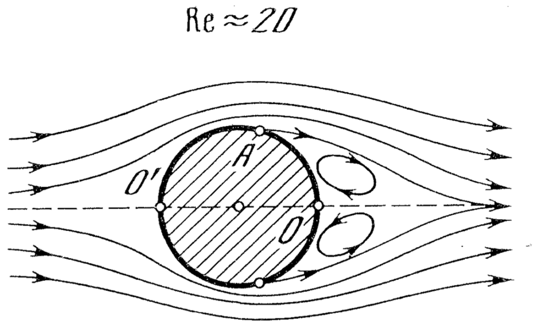


Fig. 9.2. Steady flow past a cylinder with two vortices

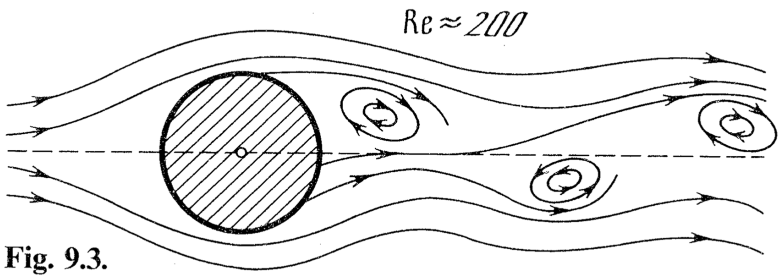


Fig. 9.3.

Fig. 9.3. Illustrating a Karman street

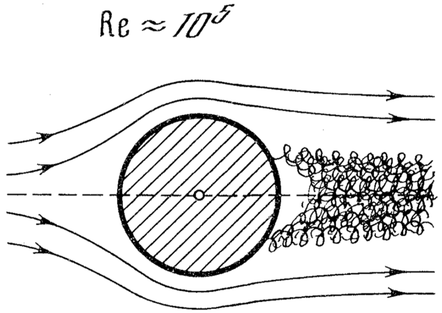
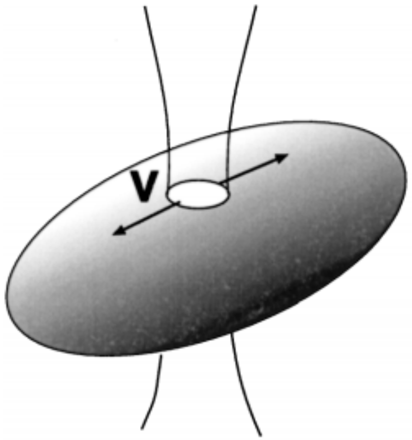


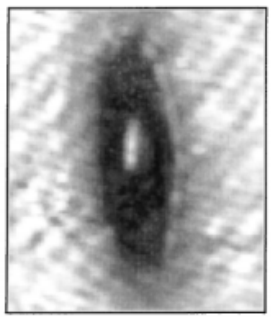
Fig. 9.4. The flow with a fully developed turbulent wake

BEC in general are superfluids, i.e., have no viscosity!

- How behaves a superfluid past an obstacle?

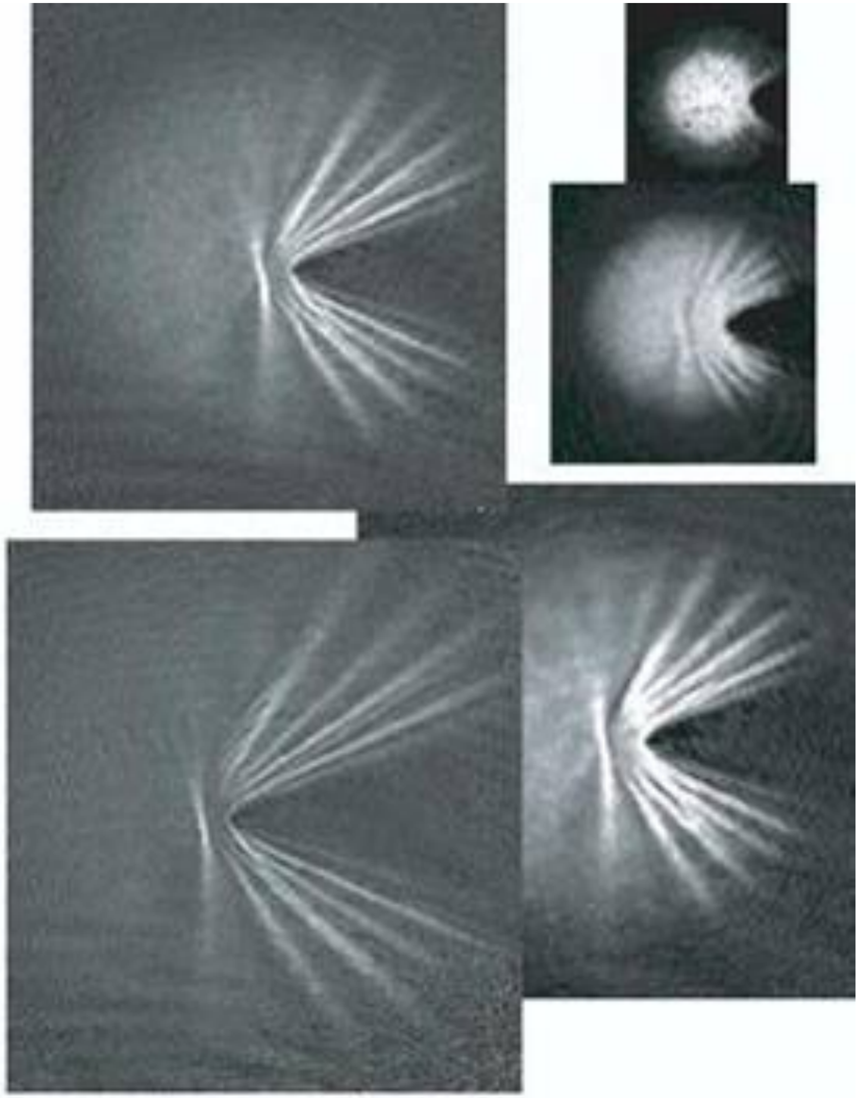
C. Raman, M. K¨ohl, R. Onofrio, D. S. Durfee, C. E. Kuklewicz, Z. Hadzibabic, and W. Ketterle, PRL 83, 2502 (1999).





Large dissipation at $v_c >\sim 0.26\, c_s$

Quantum Fluids-BEC

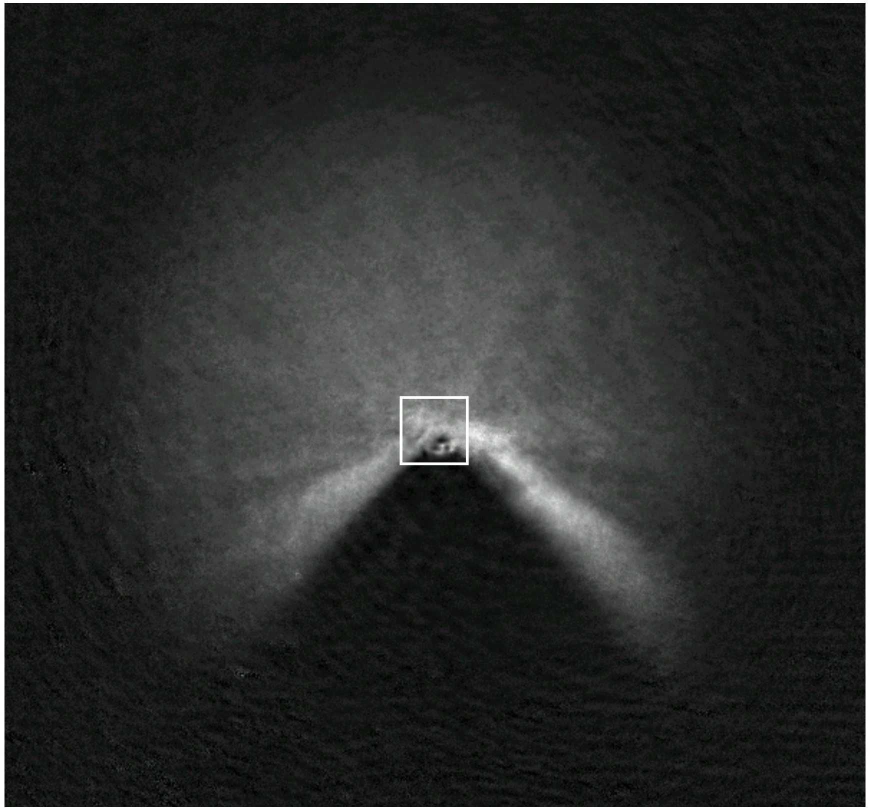


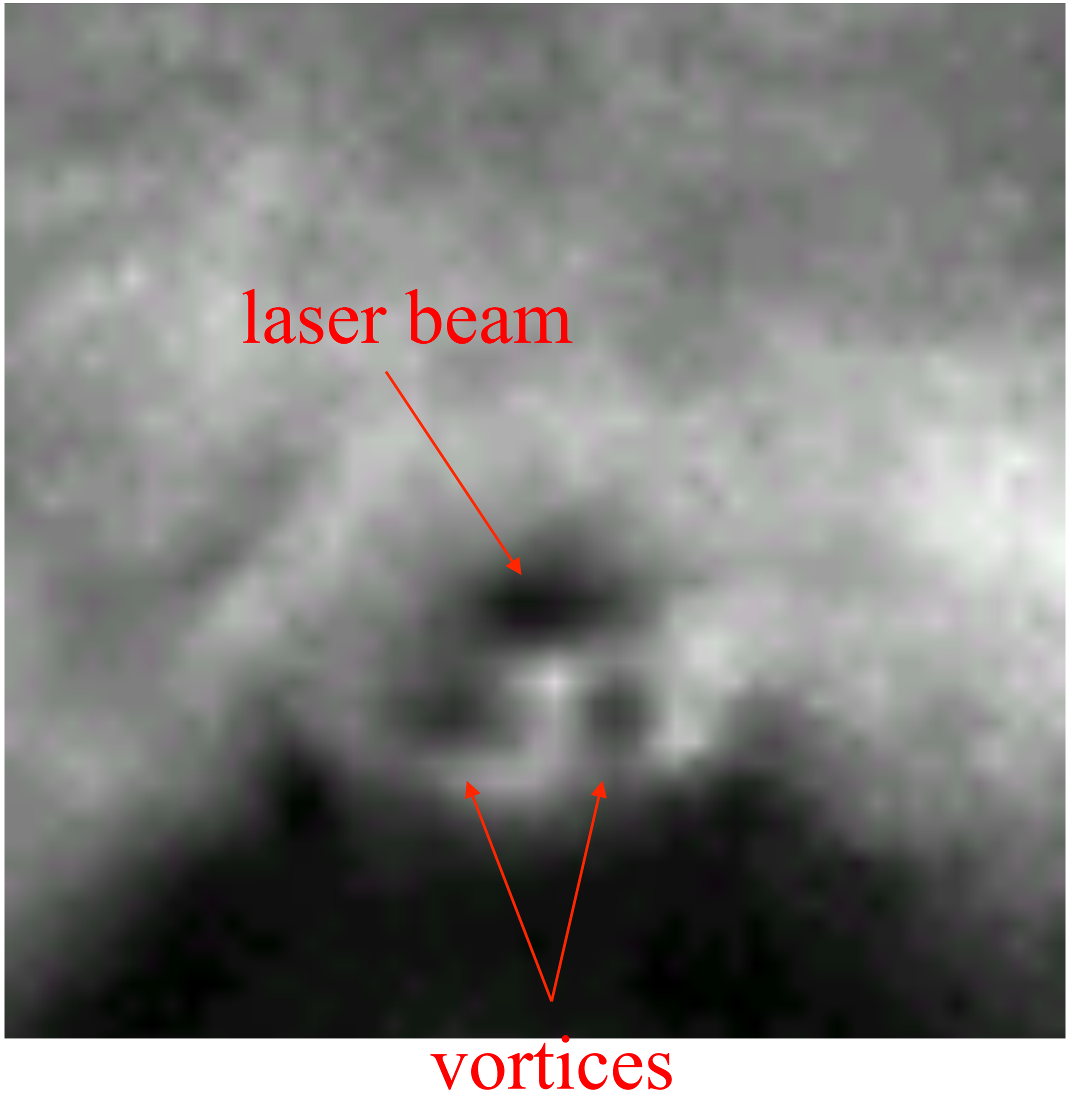
Shocks in the flow of BEC past an obstacle

JILA 2005

Cornell (2005)

Vortices behind a small cylinder



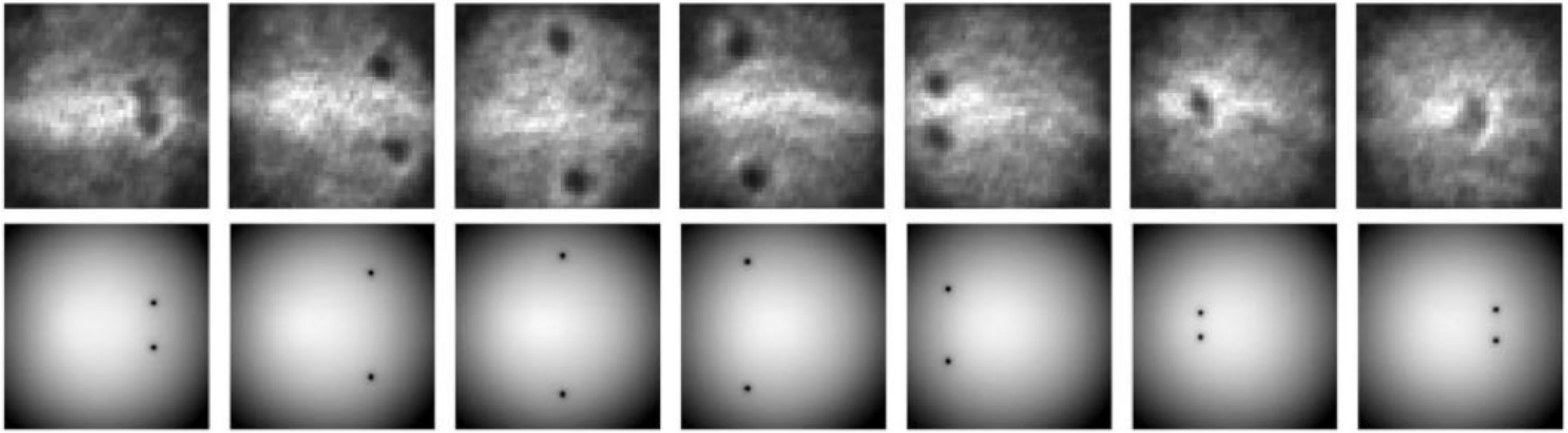


Selected for a Viewpoint in *Physics*
PRL **104**, 160401 (2010) PHYSICAL REVIEW LETTERS week ending 23 APRIL 2010

Observation of Vortex Dipoles in an Oblate Bose-Einstein Condensate

T. W. Neely,[1] E. C. Samson,[1] A. S. Bradley,[2] M. J. Davis,[3] and B. P. Anderson[1,4]

experimental



Simulation with Gross-Pitaevskii eq.

but subsonic!

Gross-Pitaevskii equation

Dynamics of a dilute condensate is described by the Gross-Pitaevskii equation ~1961

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{ext}\psi + g|\psi|^2\psi$$

where $$V_{ext}(\mathrm{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

$$g = \frac{4\pi\hbar^2 a_s}{m}, \qquad \int|\psi|^2 \, d\mathrm{r} = N,$$

Gross-Pitaevskii Eq. in hydrodinamic form

$$\psi = \sqrt{n} e^{i\phi}, \mathrm{v} = \frac{\hbar}{m} \nabla \phi$$

$$\frac{\partial n}{\partial t} + \nabla . \left(n \mathrm{v} \right) = 0$$

$$m \frac{\partial \mathrm{v}}{\partial t} + \nabla \left(\frac{1}{2} m \mathrm{v}^2 + V_{ext} + gn - \frac{\hbar^2}{2m\sqrt{n}} \nabla^2 \sqrt{n} \right) = 0$$

And sound velocity for uniform solution is

$$c_s = \sqrt{gn / m}$$

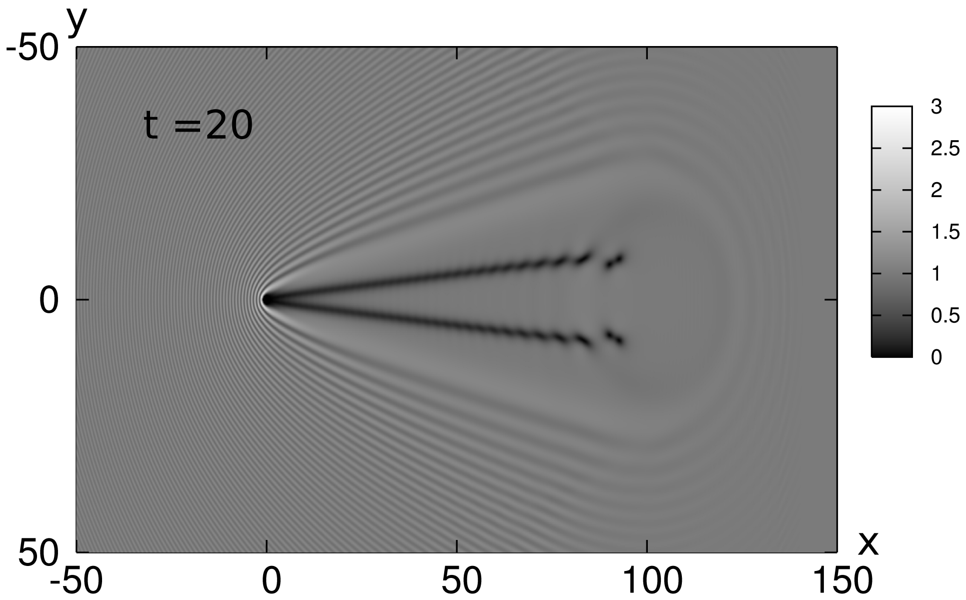
No viscosity
quantum pressure term

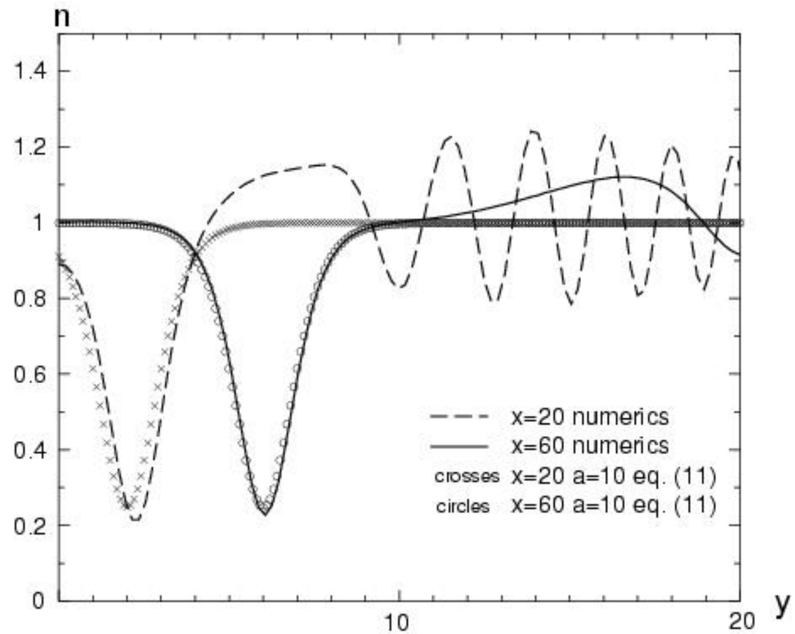
Supersonic Flow in BEC

T. Winiecki, J. F. McCann, and C. S. Adams, PRL(1999)
“vortex street”

G.El, A.G., A.M. Kamchatnov, PRL (2006)

M=5, r=1

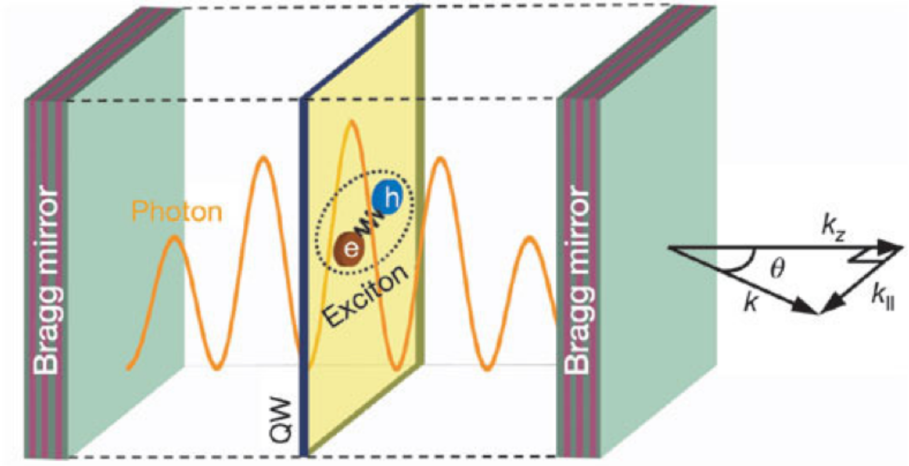




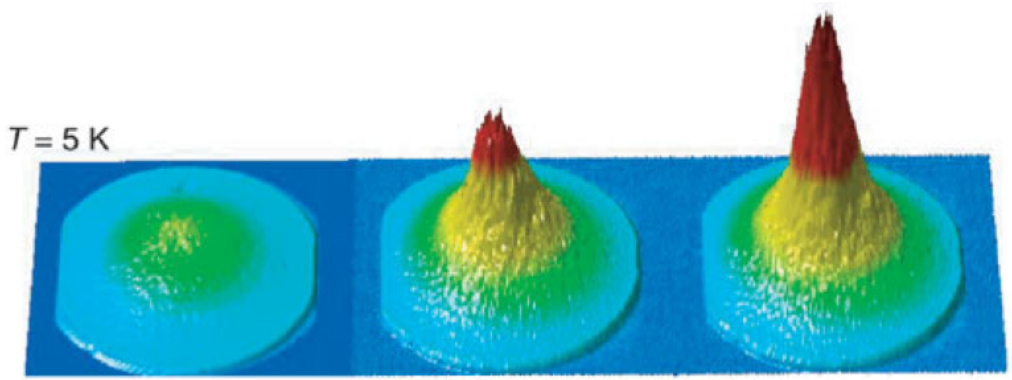
Cutting in x we see dark solitons

Differently from Navier-Stokes, that predicts turbulence for sufficient high velocities, the potential flow in GP-2D at a simple case showed to be *integrable*, which seems a remarkable result!

BEC of exciton-polaritons



J. Kasprzak et al, Nature (2006)

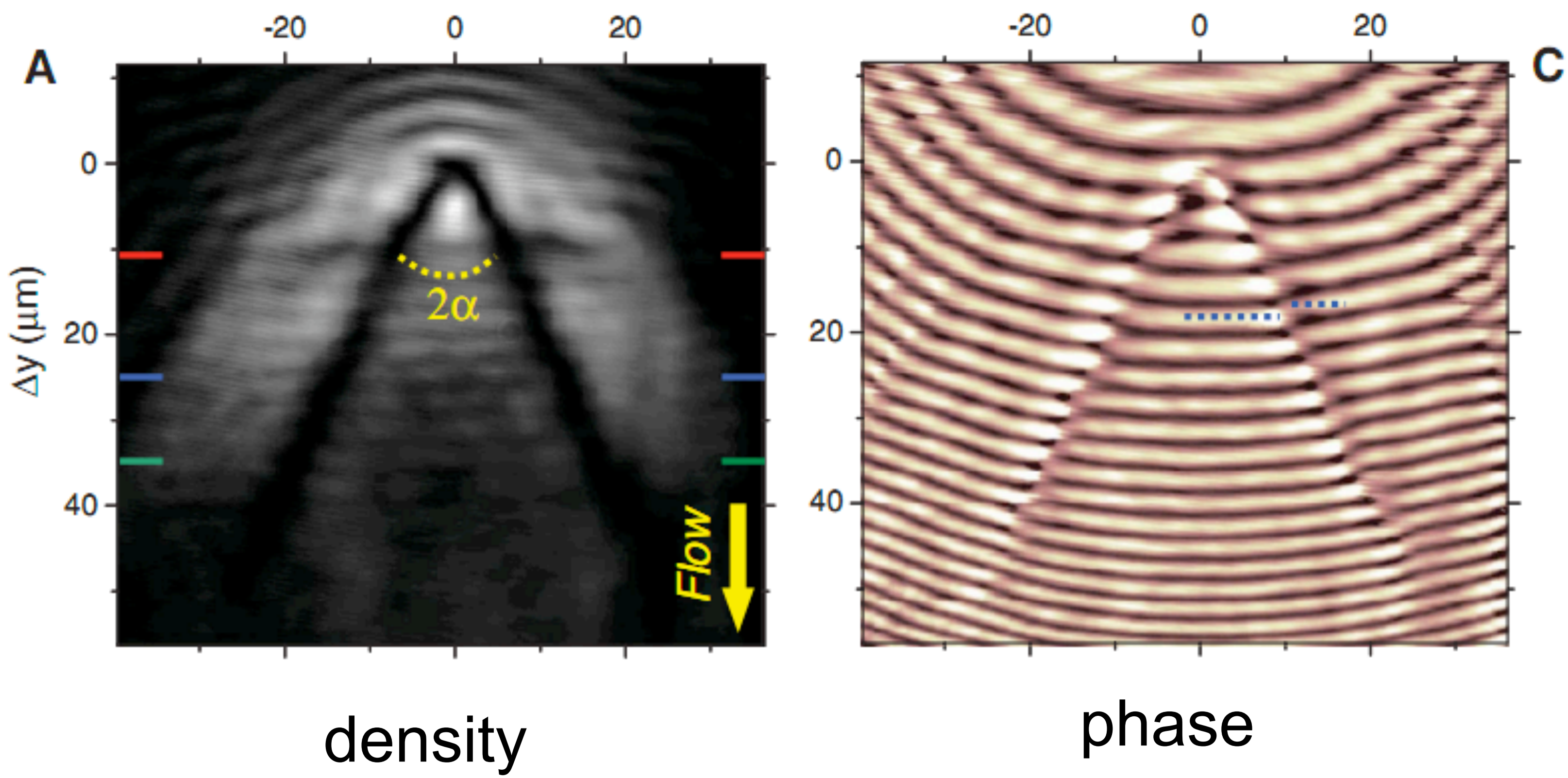


P=0.92kW cm^{-2} 1.67kW cm^{-2} 1.90kW cm^{-2}

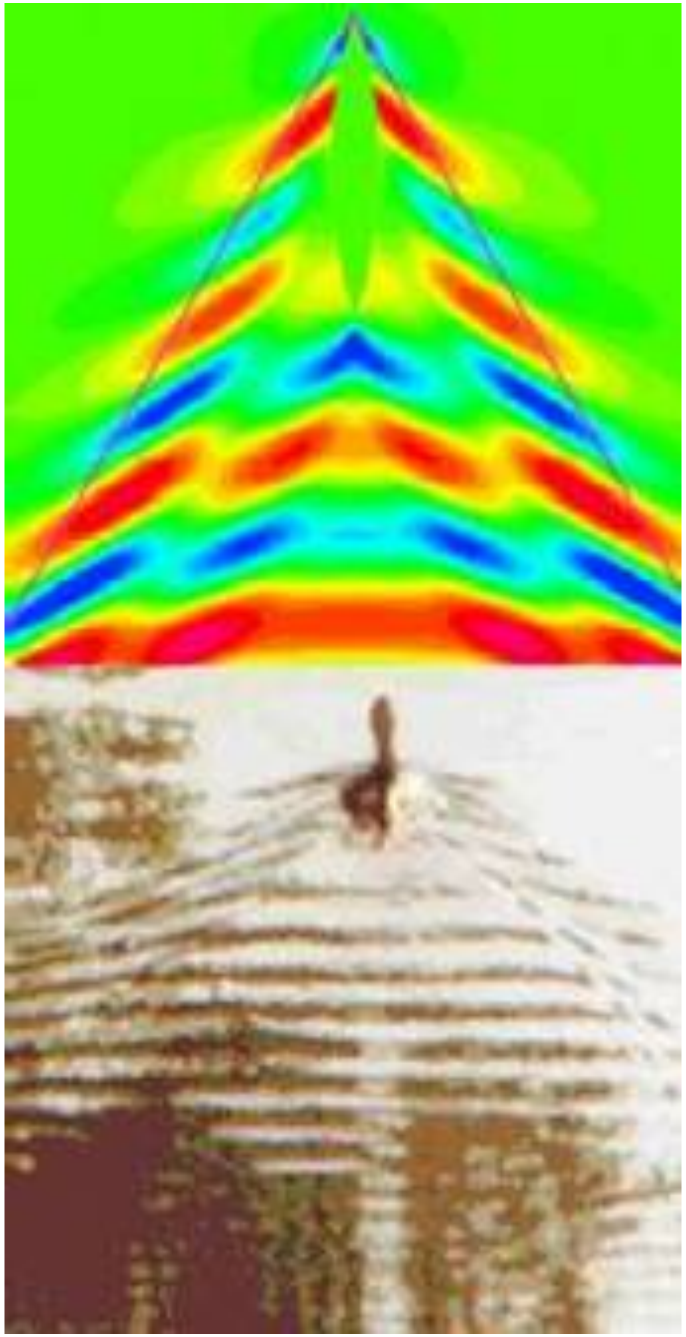
www.sciencemag.org SCIENCE VOL 332 3 JUNE 2011 1167

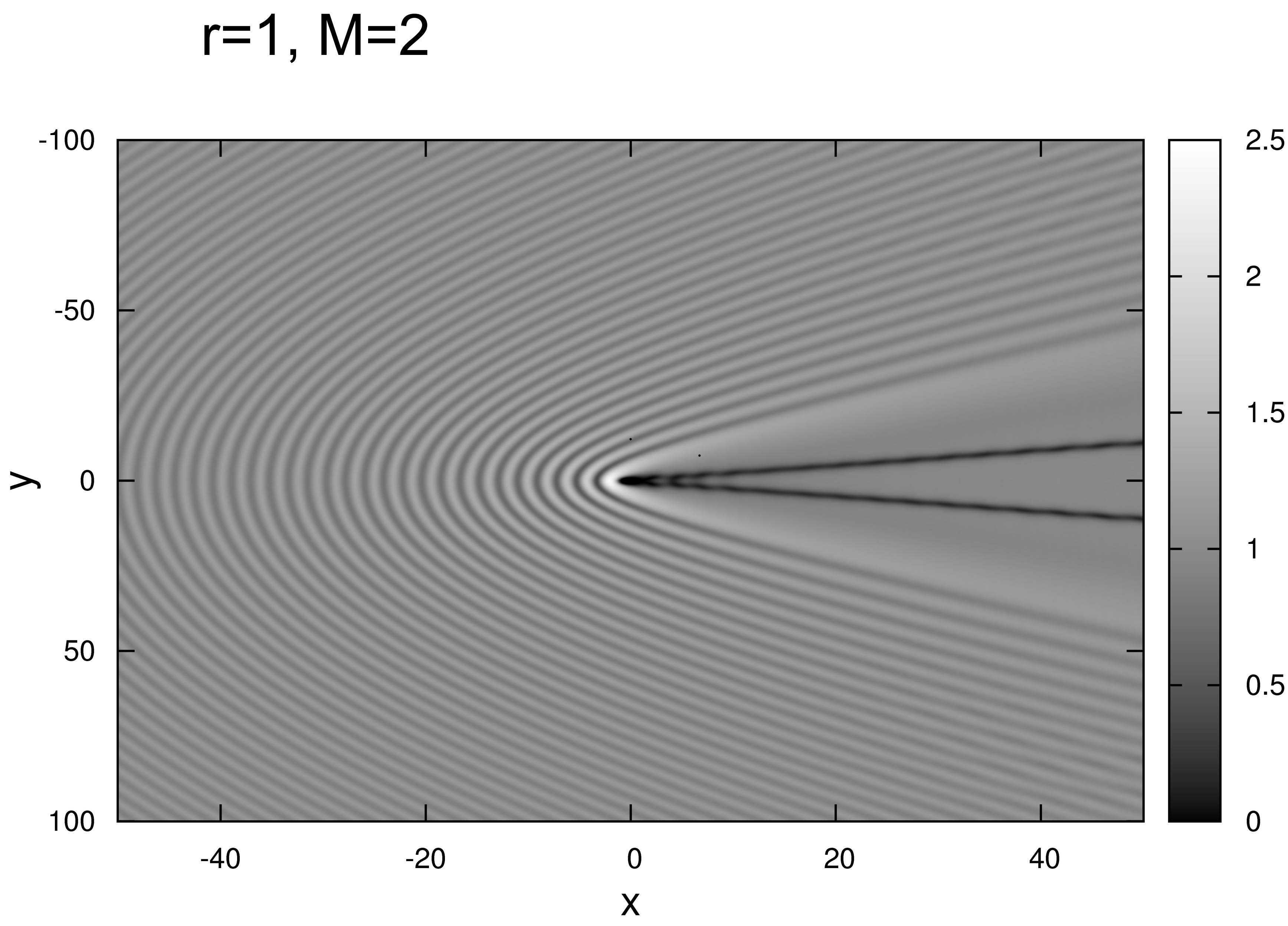
Polariton Superfluids Reveal Quantum Hydrodynamic Solitons

A. Amo,[1,2*] S. Pigeon,[3] D. Sanvitto,[4] V. G. Sala,[1] R. Hivet,[1] I. Carusotto,[5] F. Pisanello,[1,4,6] G. Leménager,[1] R. Houdré,[7] E Giacobino,[1] C. Ciuti,[3] A. Bramati[1*]



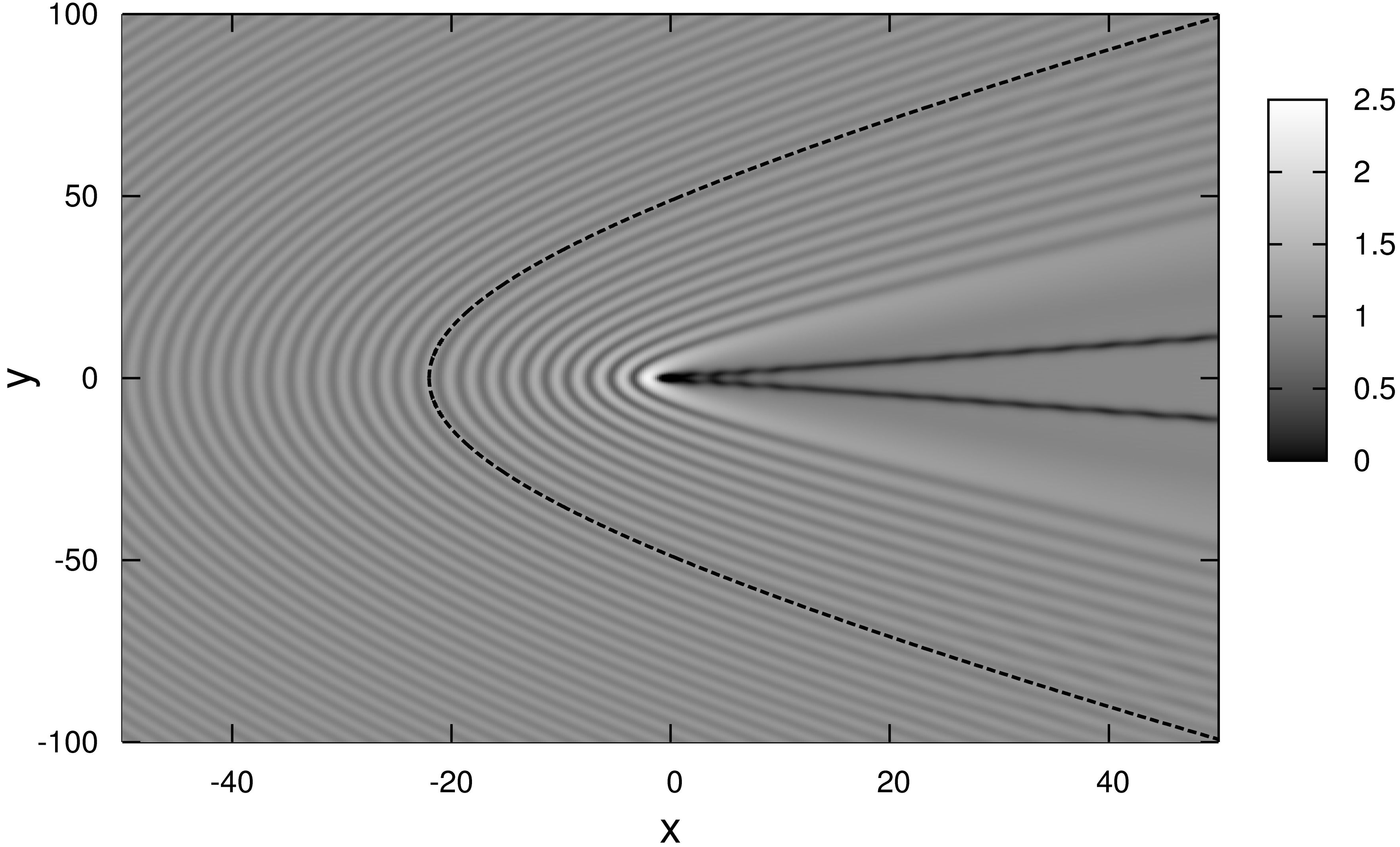
Kelvin bow waves




r=1, M=2
-100
-50
0
50
100
y
-40
-20
0
20
40
x
2.5
2
1.5
1
0.5
0

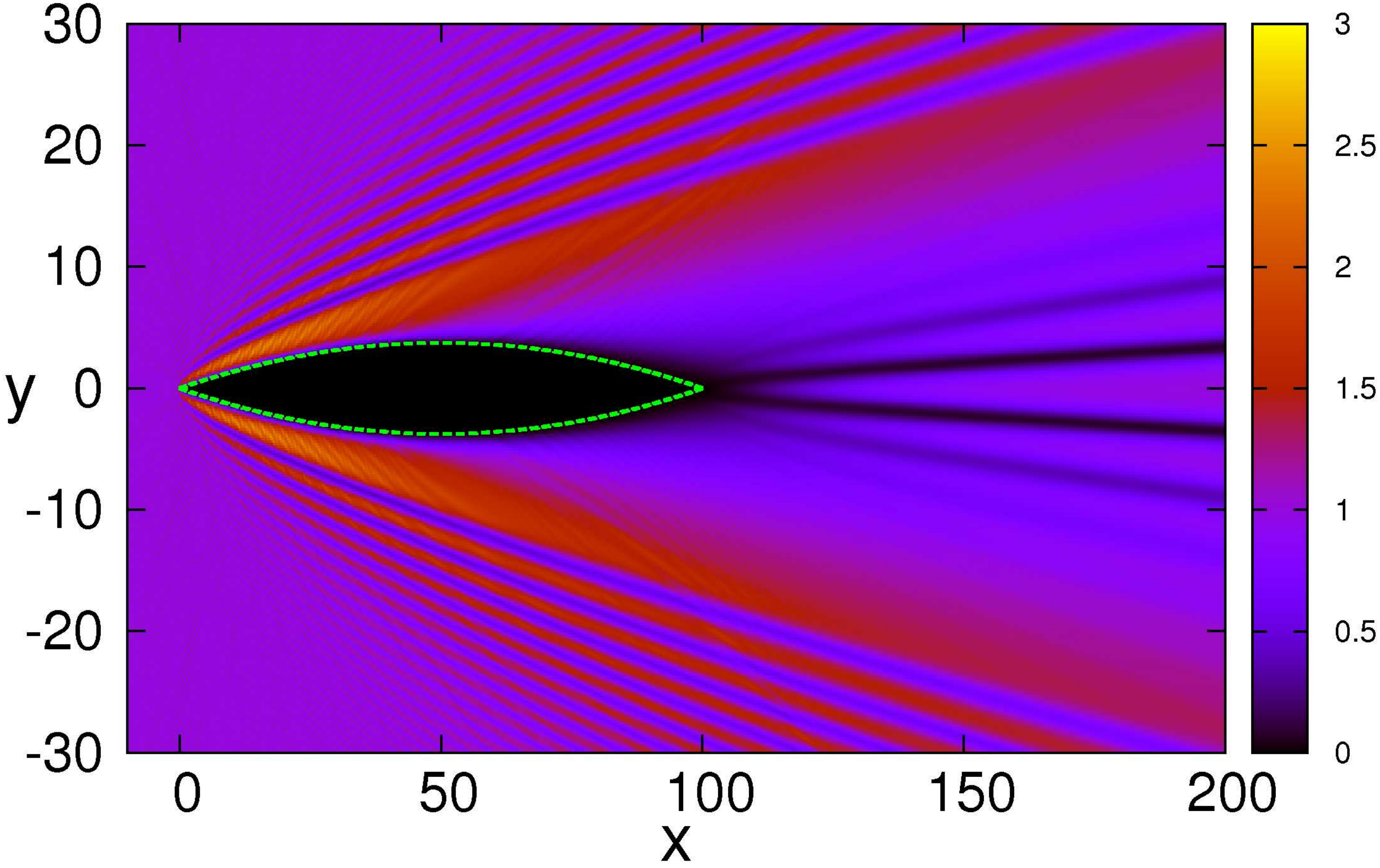
Applying the same techniques developed by
by Lord Kelvin to the GP equation
it is possible to derive analytical shape of the
bow waves profile

r=1, M=2



Yu.G. Gladush, G. El, A. G., A.M. Kamchatnov
PRA (2007)

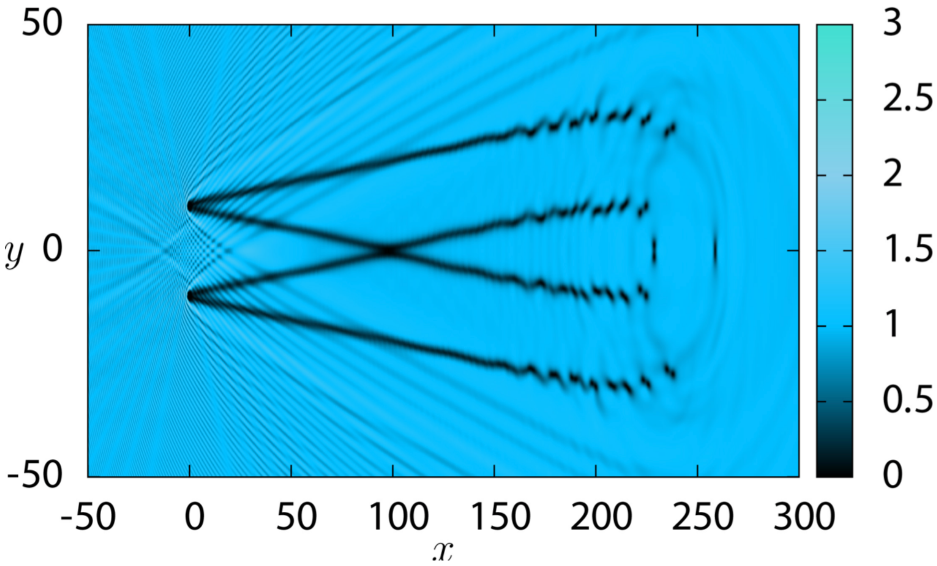
Extended obstacle



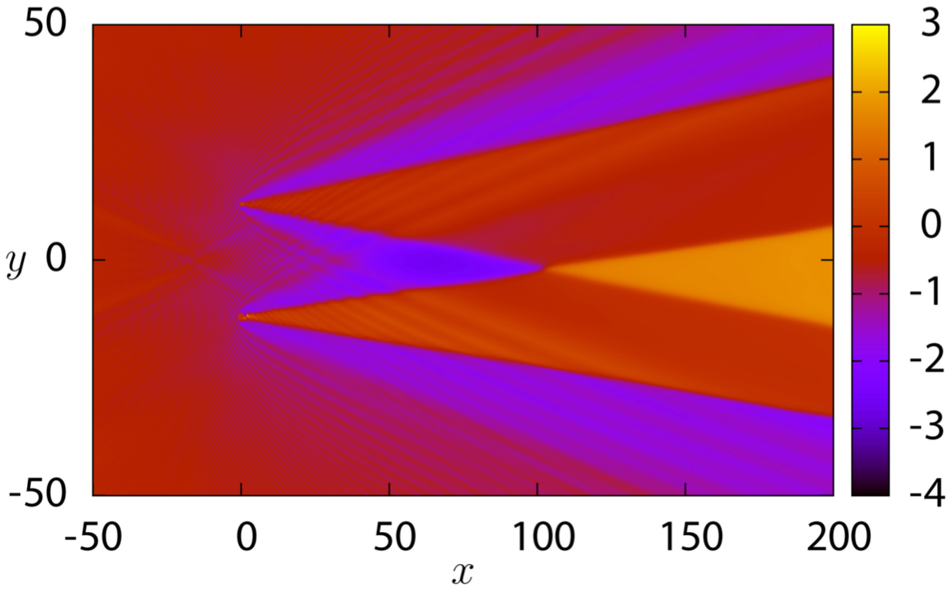
G. A. El, A. M. Kamchatnov, V. V. Khodorovskii, E. S. Annibale, and A. G., PRE 80, 046317 (2009). PRE Kaleidoscope, October 2009.

Collision of oblique solitons with two obstacles

M=5



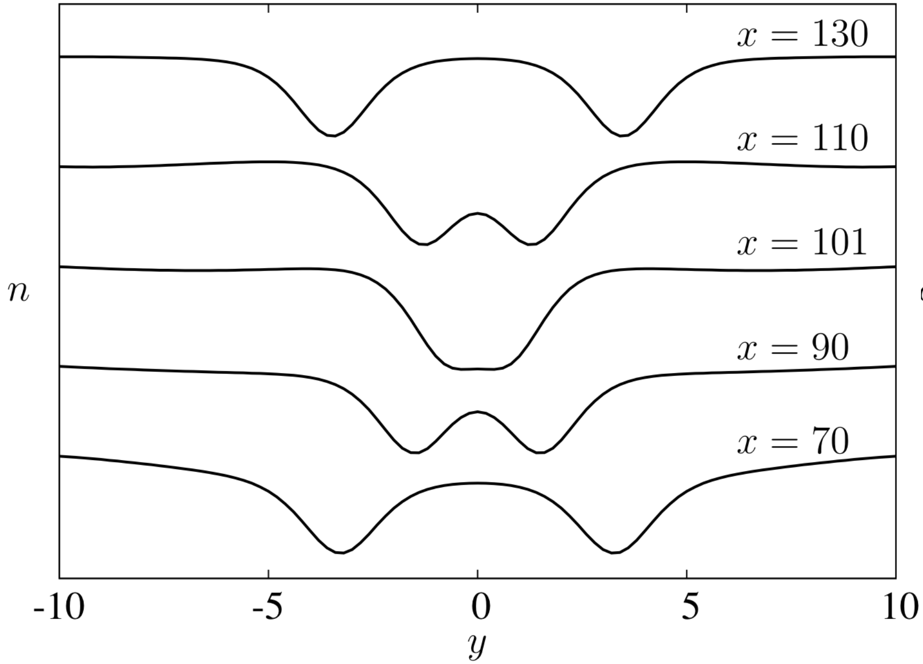
density



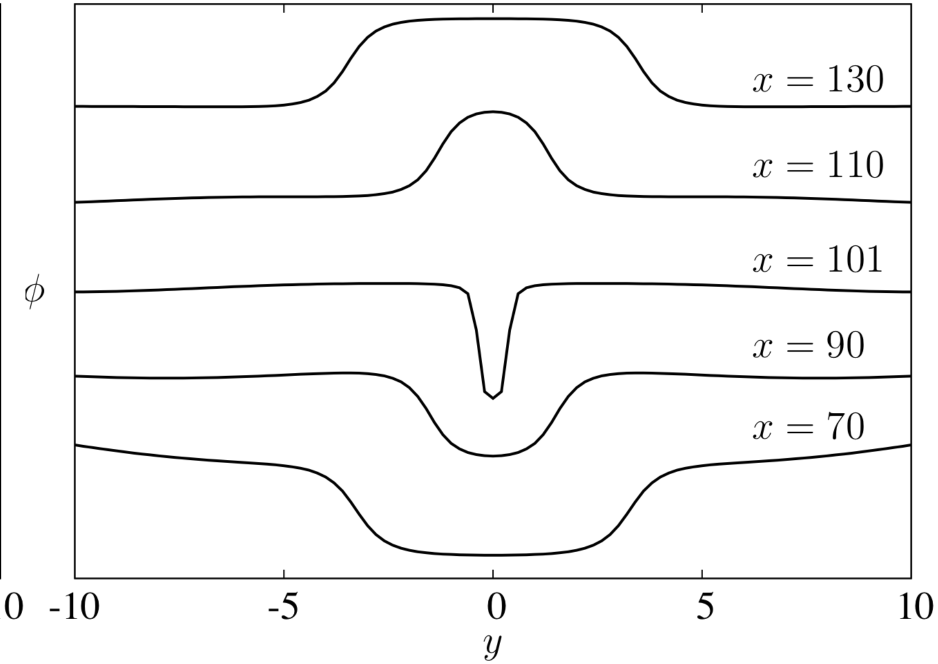
phase

E.S. Annibale, <u>A.G.</u>, PLA 376, 46 (2011)
E.G. Khamis, <u>A.G.</u>, PLA 376, 2422 (2012)

density



phase



Problem almost integrable when M>> and/or θ<<

Atomic BEC

-obstacles are typically repulsive (blue detuned)
but can also be attractive (red detuned)

-attractive obstacles have non-classical counterpart

-motivation: Oscillating attractive-repulsive obstacle
Should depend on frequency

Oscillating attractive-repulsive obstacle in 2D

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + U_{\text{ext}}\,\Psi + \frac{4\pi a\hbar^2}{m}|\Psi|^2\Psi,$$

$$U_{\text{ext}} = U_{\text{trap}}(x,y,z) + U(x+vt,y,z,\Omega t)$$

$$U(x,y,\Omega t) = U_0\cos(\Omega t)\exp\left[\frac{-2(x^2+y^2)}{w_0^2}\right]$$

$$\Psi(x,y,z,t) = \psi(x,y,t)\phi(z)e^{-i\mu_z/\hbar}$$

2D reduction

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\partial_x^2 + \partial_y^2\right)\psi + U\,\psi + g|\psi|^2\psi,$$

$$g = 4\pi a\hbar^2 m^{-1}\int \phi^4(z)dz$$

dimensionless variables $\tilde{x} = x/\xi \quad \tilde{y} = y/\xi$

$$\tilde{t} = gn_0 t/\hbar$$

$$\tilde{U} = U/gn_0$$ n_0 = 2D density

$$\tilde{\Omega} = \Omega\hbar/gn_0 \quad M = v/c_s$$

$$\hbar/gn_0 \sim 0.18 \ \text{ms} \quad \xi \sim 0.3 \ \mu\text{m}$$

Thus $\tilde{\Omega} \sim 1$ means kHz

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\left(\partial_x^2 + \partial_y^2\right)\psi + U\psi + |\psi|^2\psi\,,$$

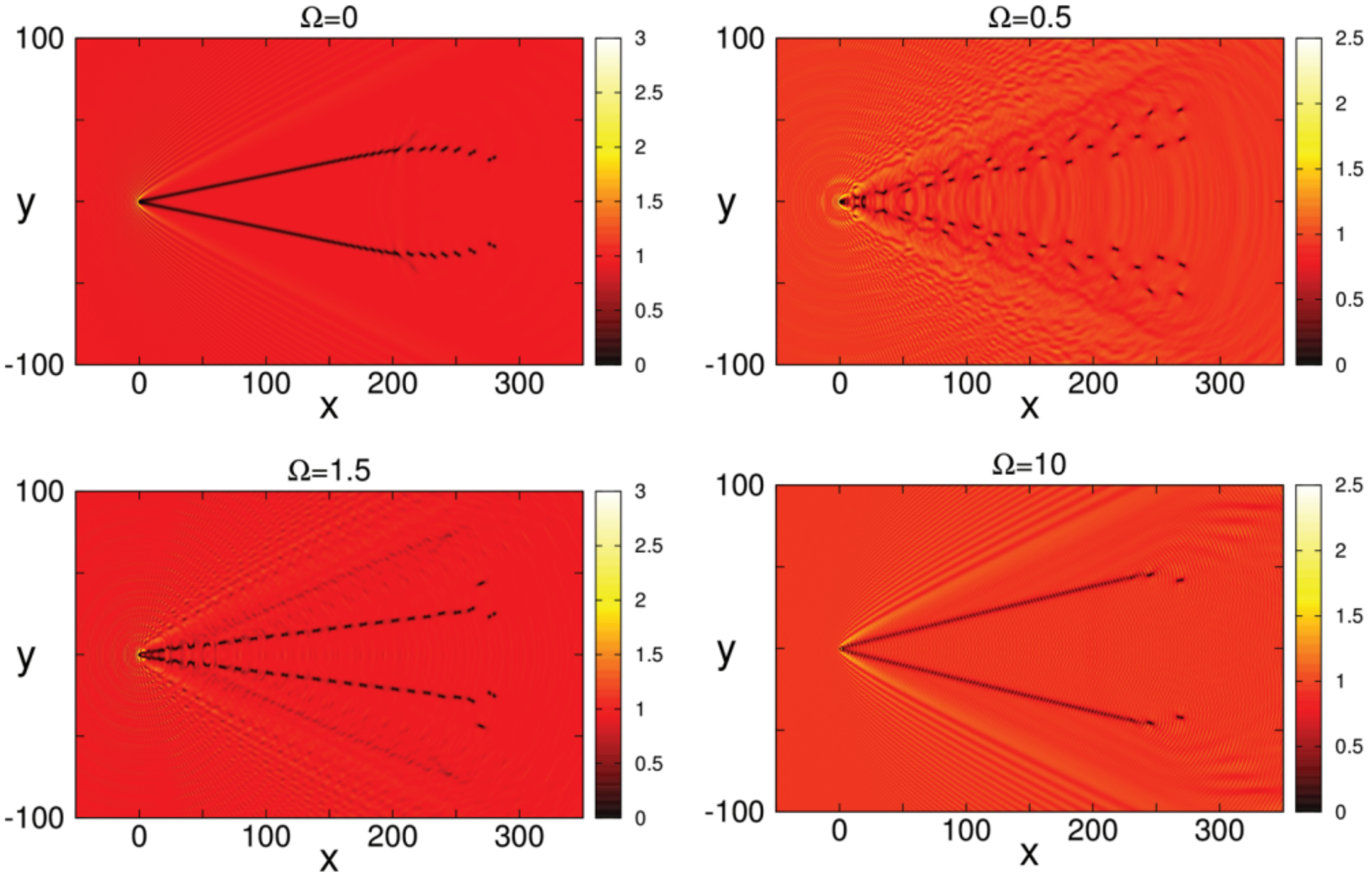
$$U = U(x + Mt, y, \Omega t).$$

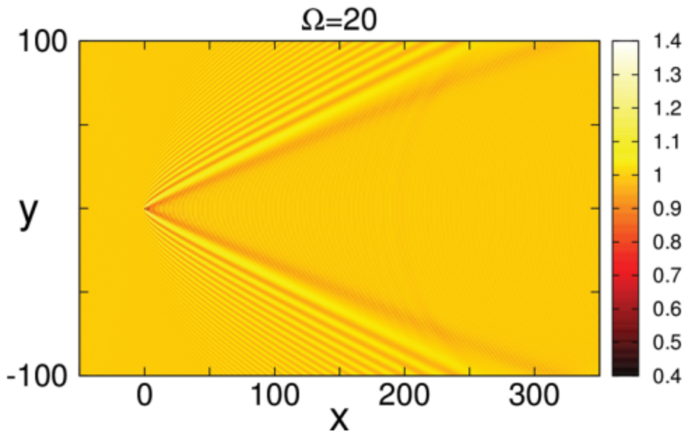
For computational purposes, in Eq. (3) we make a global phase transformation $\psi' = e^{it}\psi$ and later a Galilean transformation $x' = x + Mt$, $t' = t$ leading to

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\left(\partial_x^2 + \partial_y^2\right)\psi - iM\partial_x\psi - \psi + |\psi|^2\psi + U\psi,$$

$$U(x, y, \Omega t) = U_0 \cos(\Omega t) \exp\left[\frac{-2(x^2 + y^2)}{w_0^2}\right]$$

Inside Mach cone, M=3

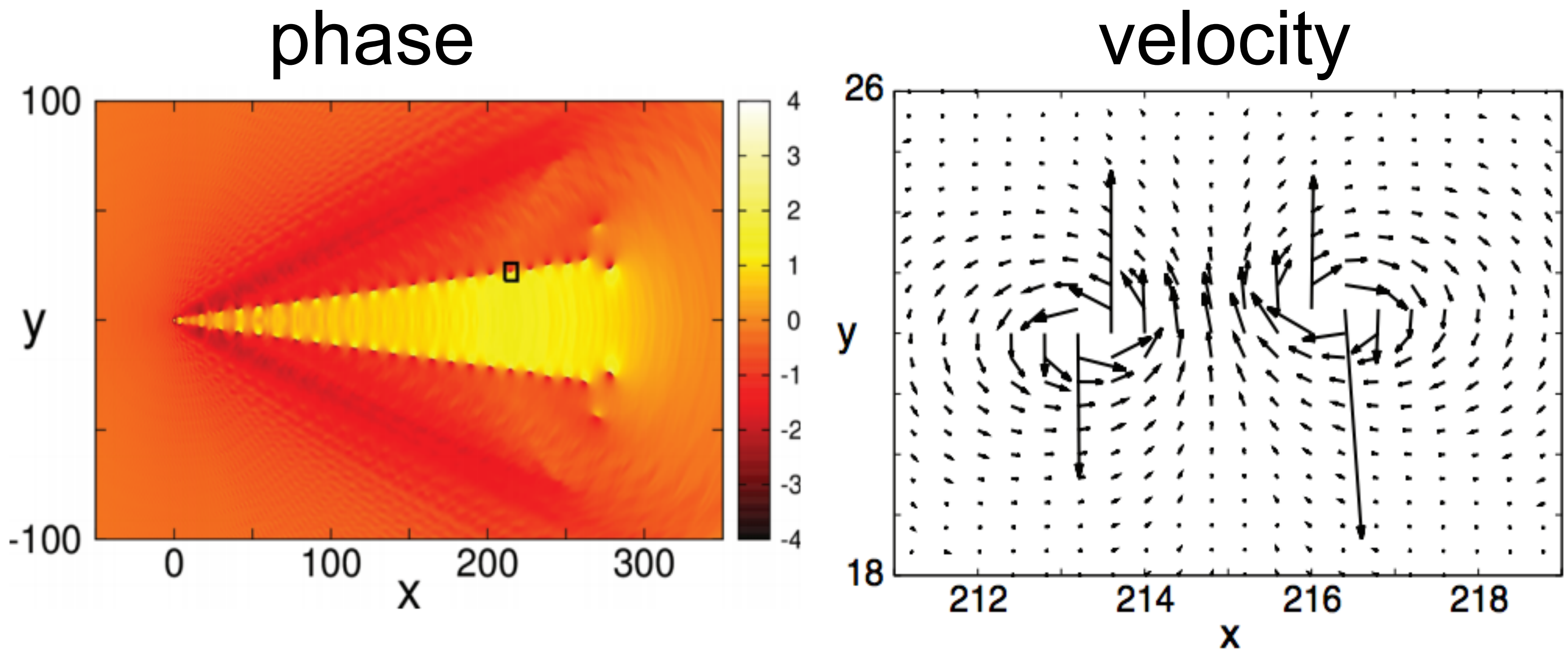




E.G.Khamis, A.G.,
PRA 87, 045601 (2013)

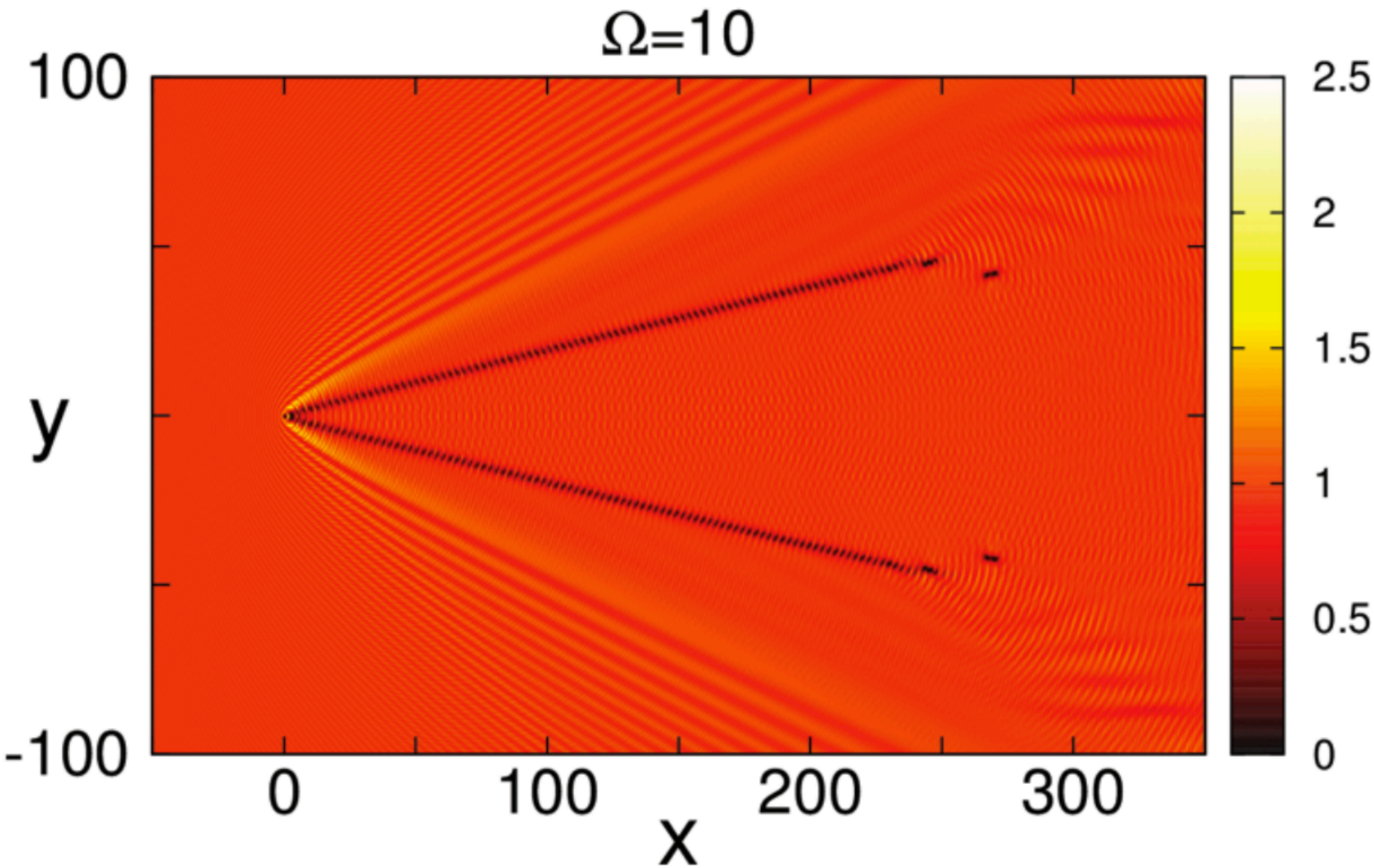
Ω=1.5

PHYSICAL REVIEW A **87**, 045601 (2013)

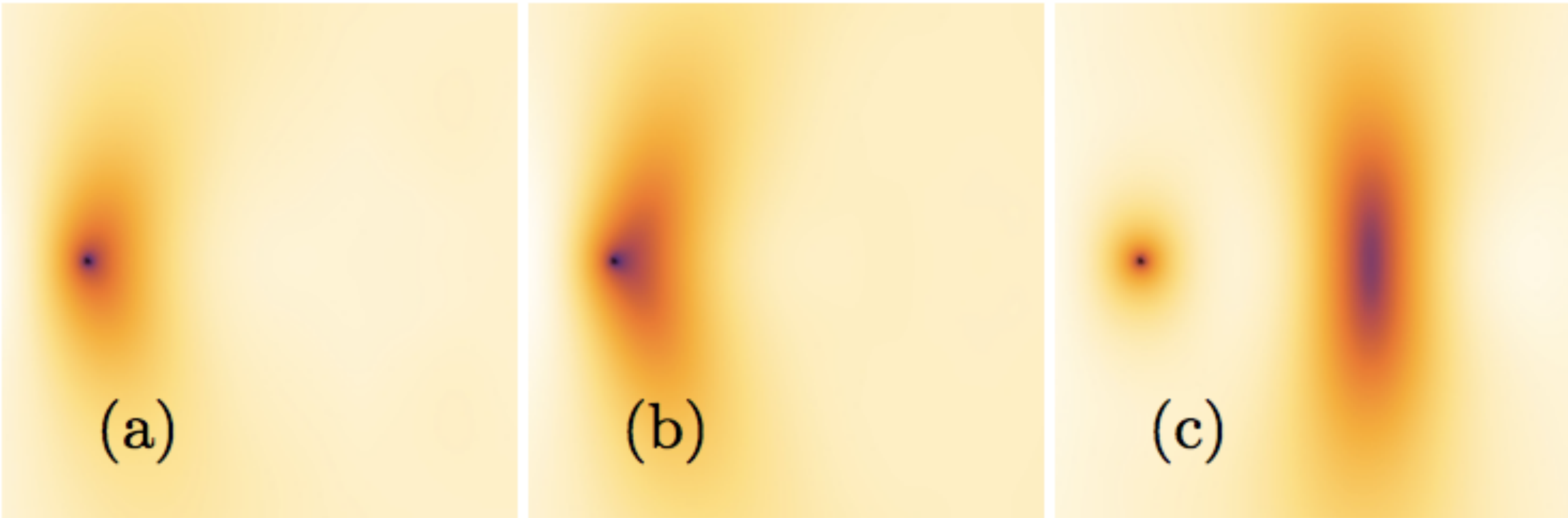


PRA Kaleidoscope March (2013)

E.G.Khamis, A.G., PRA 87, 045601 (2013)



fast oscillation- presence of fragments
not enough time for dipole formation



small obstacles -> rarefaction waves

F. Pinsker and N.G. Berloff, PRA 89, 053605 (2014)

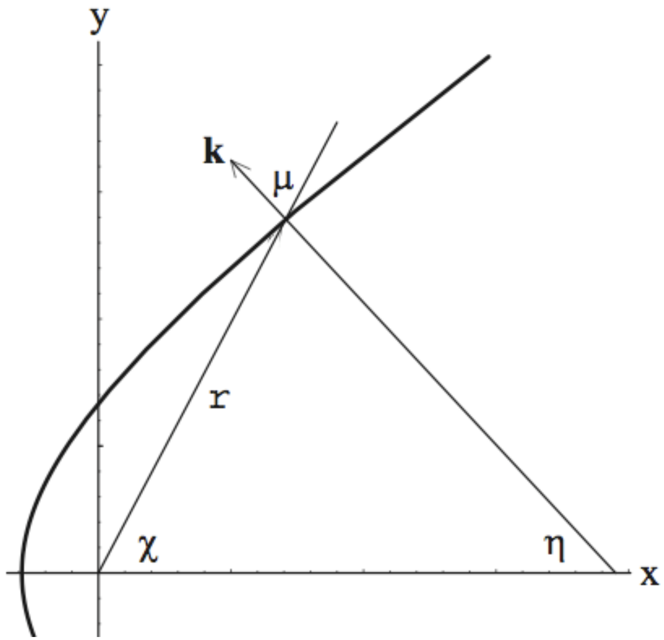
So either fast acting obstacle or small obstacles produces fragments or rarefaction waves instead of vortices.

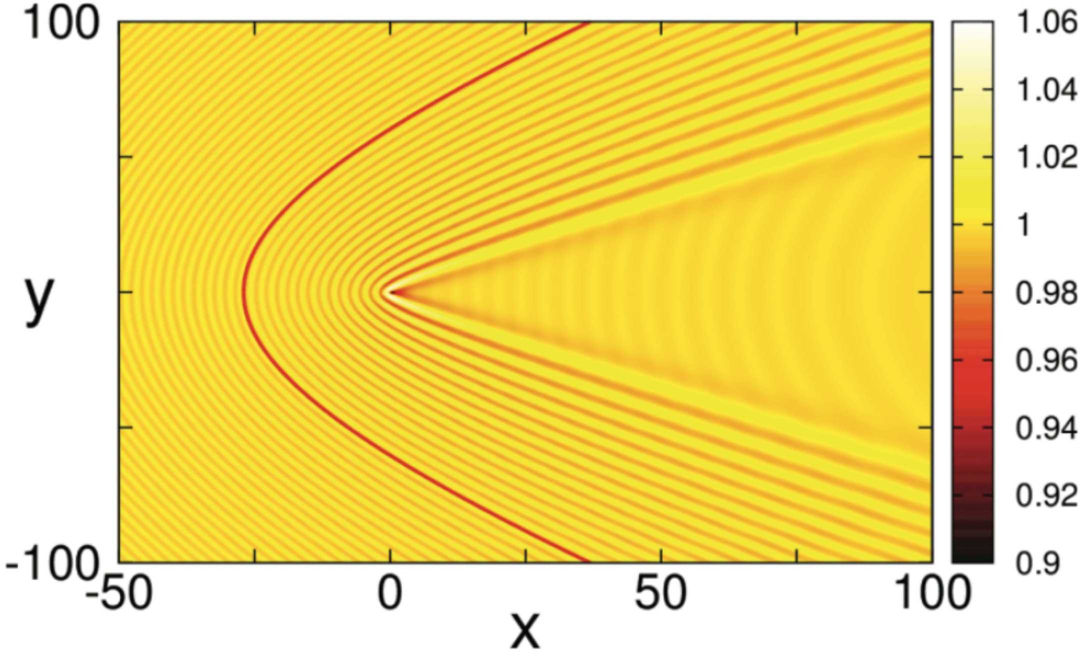
Outside Mach cone-bow waves

$$x = r\ \cos\ \chi = \frac{4\Phi}{k^3} \cos\ \eta(1 - M^2 \cos\ 2\eta),$$

$$y = r\ \sin\ \chi = \frac{4\Phi}{k^3} \sin\ \eta(2M^2 \cos^2 \eta - 1).$$

$$k \equiv 2\sqrt{M^2 \cos^2 \eta - 1},\ \Phi \equiv kr\ \cos\ \mu$$

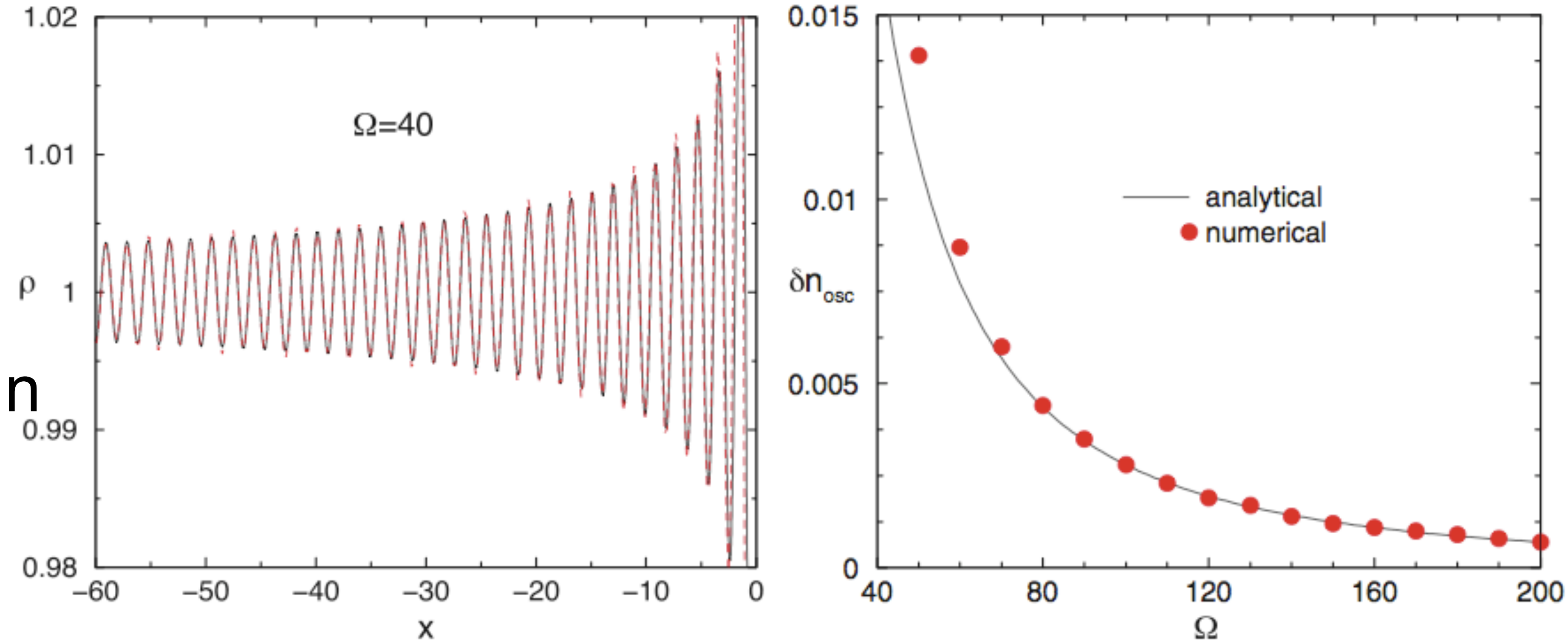




E.G.Khamis, A.G., PRA 87, 045601 (2013)

Fast oscillating obstacle we assume Huygens principle

$$\delta n_{\rm osc} = \frac{V_0 q}{2\pi}\left(\frac{-\Omega k M}{\Omega^2 - k^2 M^2}\right)$$
$$\times\,[\sin(\Phi - \pi/4 + kMT) - \sin(\Phi - \pi/4)].$$



E.G.Khamis, A.G., PRA 87, 045601 (2013)

DRAG FORCE

$$F_x(t) = \int_A dx\, dy\, |\psi|^2 \frac{\partial U}{\partial x}$$

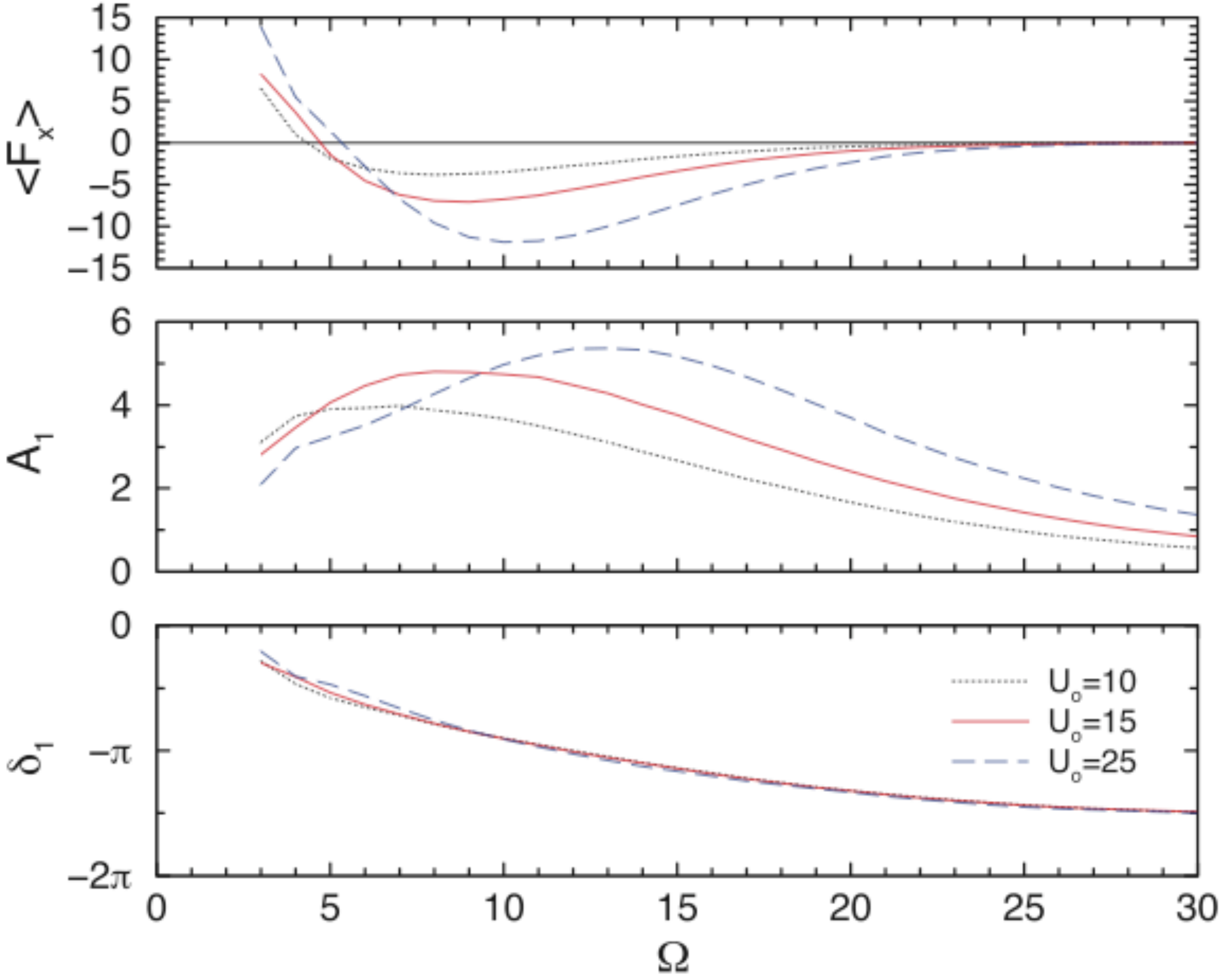
Drag can be shown to have the form

$$F_x = U_0 \cos(\Omega t) R(t)$$

where *R(t)* is the response function.

$$R(t) = A_0 + A_1 \cos(\Omega t + \delta_1) + A_2 \cos(2\Omega t + \delta_2) + \cdots$$

In $\langle F_x \rangle$ only A_1 survive



E.G.Khamis, A.G., PRA 87, 045601 (2013)

Conclusions

-Oscillating attractive-repulsive obstacle in supersonic flow generates different patterns of flow depending on the frequency

-Inside Mach cone, for increasing frequency we have “chopper” with “5 in dice”, vortex dipole street, and fragments

- Fast attractive repulsive oscillating obstacle or small obstacles produces fragments or rarefaction waves instead of vortices.

-vortex dipole ejects secondary radiation when created with energy excess

-Outside Mach cone, for fast oscillations ships waves analytically treated

-Drag force vanishes for intermediate and very high frequencies

Aknowledgements

FAPESP

CNPq

Conselho Nacional de Desenvolvimento Científico e Tecnológico

Thank You

PHYSICAL REVIEW A **82**, 053610 (2010)

Hidden vorticity in binary Bose-Einstein condensates

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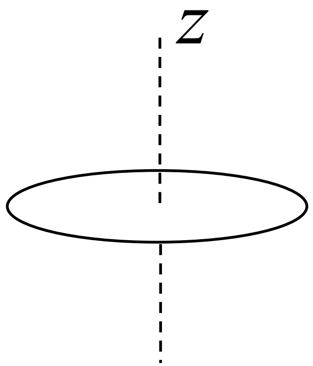
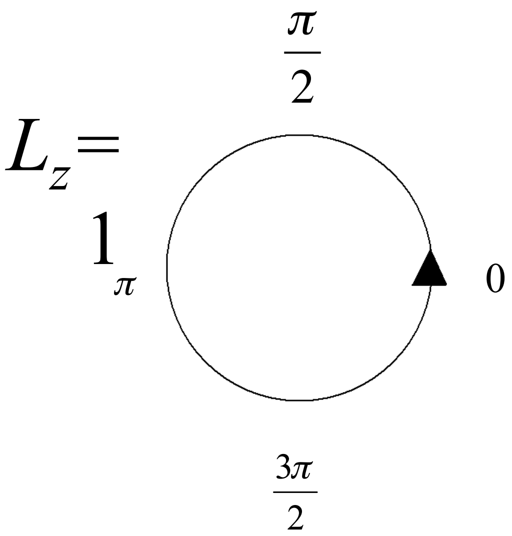
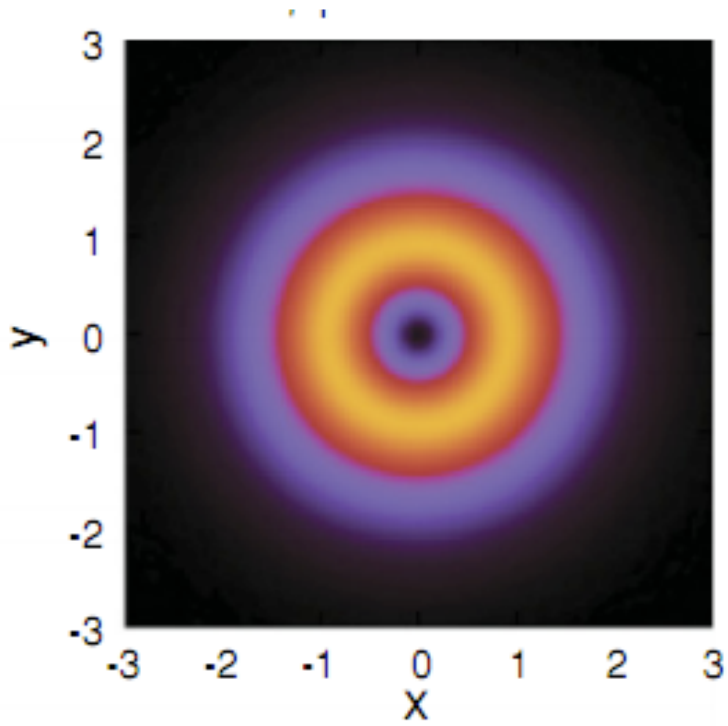
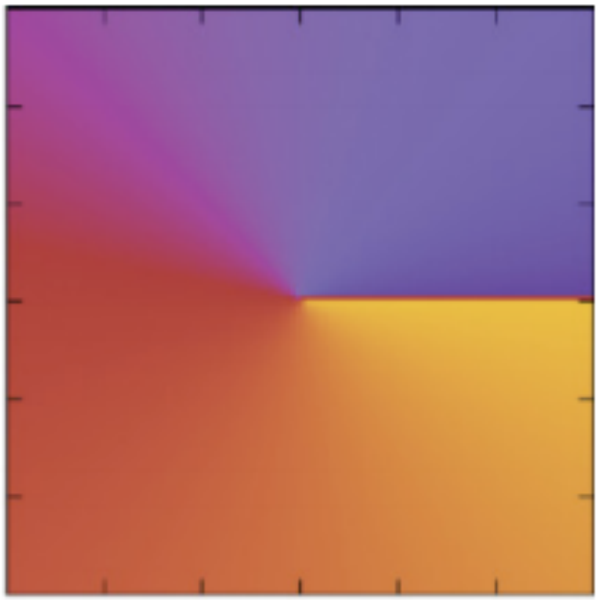






DENSITY



PHASE

rotates
with time

What is hidden vorticity?

- Composition of two counter-rotating vortices of different interacting species such as the total angular momentum is zero.

- Question: if it gets unstable how it happens?

Two coupled 2D Gross-Pitaevskii

$$i\frac{\partial \psi_1}{\partial t} = \Big[-\frac{1}{2}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big) + \frac{1}{2}(x^2 + y^2) - (|\psi_1|^2 + \beta|\psi_2|^2)\Big]\psi_1,$$

$$i\frac{\partial \psi_2}{\partial t} = \Big[-\frac{1}{2}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big) + \frac{1}{2}(x^2 + y^2) - (|\psi_2|^2 + \beta|\psi_1|^2)\Big]\psi_2.$$

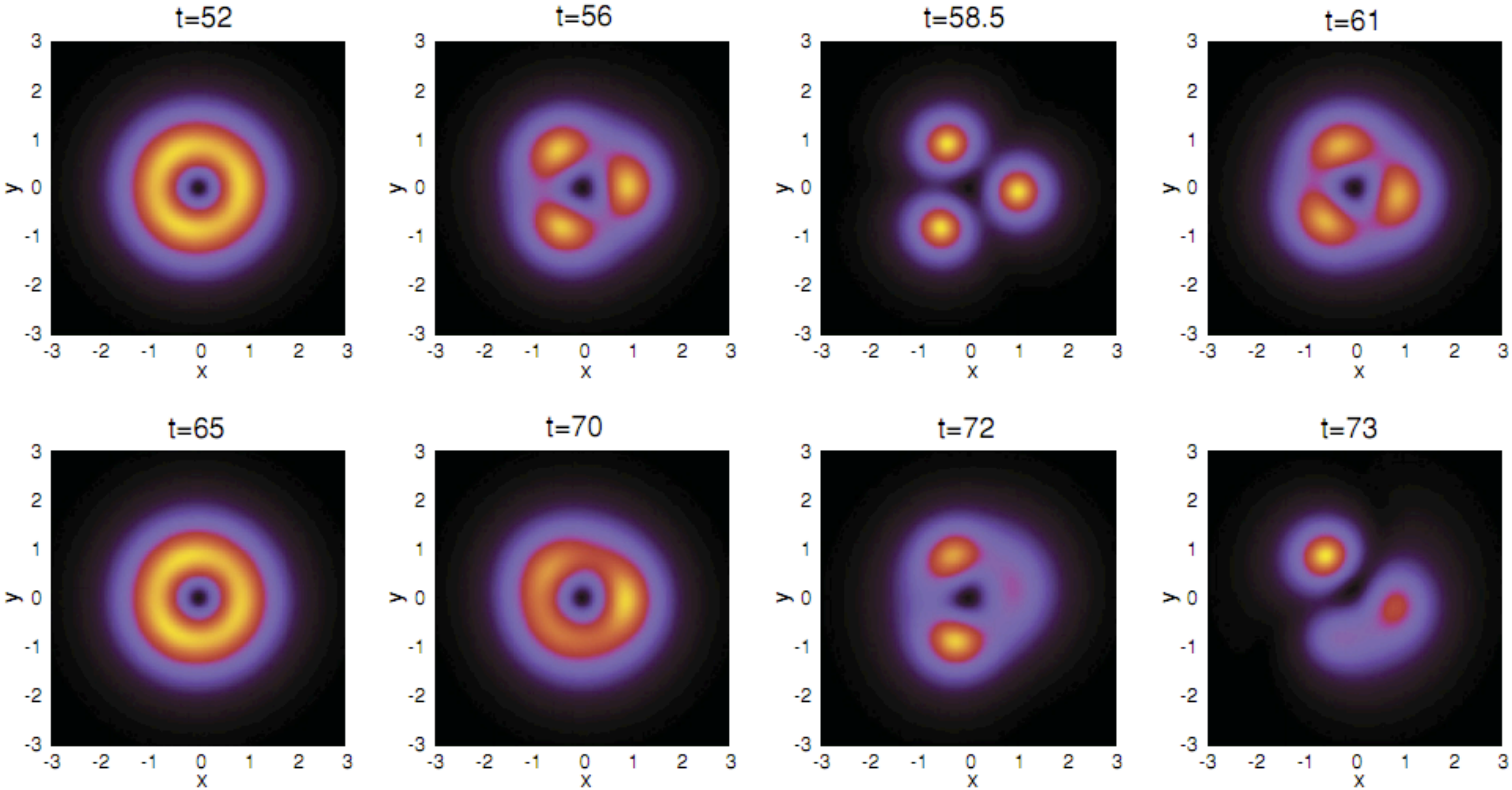


FIG. 8. (Color online) The evolution of an unstable HV state with $N = 8.75$, $\mu = 0.57$. In this case, the sequence of the density plots is shown only for ψ_1, the evolution of the second component being similar.

The stability of the stationary states was investigated by the linearization of the coupled GPEs for perturbed solutions, taken as

$$\psi_{1,2}(r,t) = [R(r) + u_{1,2}(r)e^{-i\omega t - iL\theta} + v_{1,2}^{*}(r)e^{i\omega^{*} t + iL\theta}]e^{-i\mu t + iS_{1,2}\theta}, \tag{10}$$

where integer L is the azimuthal index of perturbation eigenmodes with infinitesimal amplitudes $u_{1,2}(r)$, $v_{1,2}(r)$,

Expand perturbations of the system in terms of axial an
Solution of the Bolyubov-DeGennes equation obtained
for each mode.

$$\begin{pmatrix} D_1^- & -R^2 & -\beta R^2 & -\beta R^2 \\ R^2 & -D_1^+ & \beta R^2 & \beta R^2 \\ -\beta R^2 & -\beta R^2 & D_2^- & -R^2 \\ \beta R^2 & \beta R^2 & R^2 & -D_2^+ \end{pmatrix} U = \omega U, \quad (11)$$

where $U = (u_1, v_1, u_2, v_2)$, and the following set of operators is introduced: $D_m^{\pm} = -\Delta_r^{(L\pm S_m)}/2 + r^2/2 - (2+\beta) R^2 - \mu$, with $\Delta_r^{(M)} \equiv \partial^2/\partial r^2 + (1/r)(\partial/\partial r) - M^2/r^2$. Solutions $u_m(r)$ and $v_m(r)$ of Eq. (11) must exponentially decay at $r \to \infty$ and behave as $r^{|S_m \pm L|}$ at $r \to 0$.

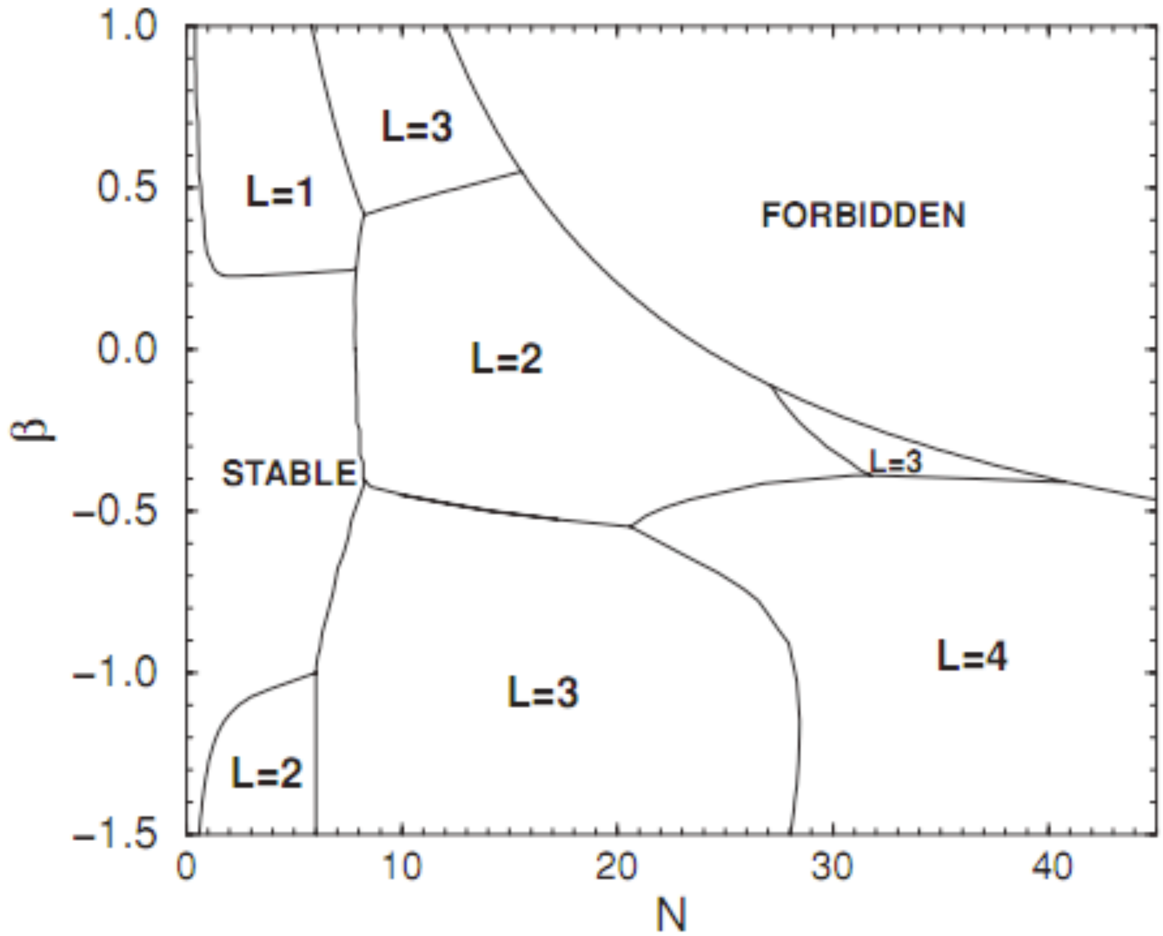


FIG. 3. The stability diagram for symmetric HV modes, in the plane of the norm (of one component) and interaction coefficient. Instability areas are labeled by the azimuthal index of the dominating perturbation eigenmode.

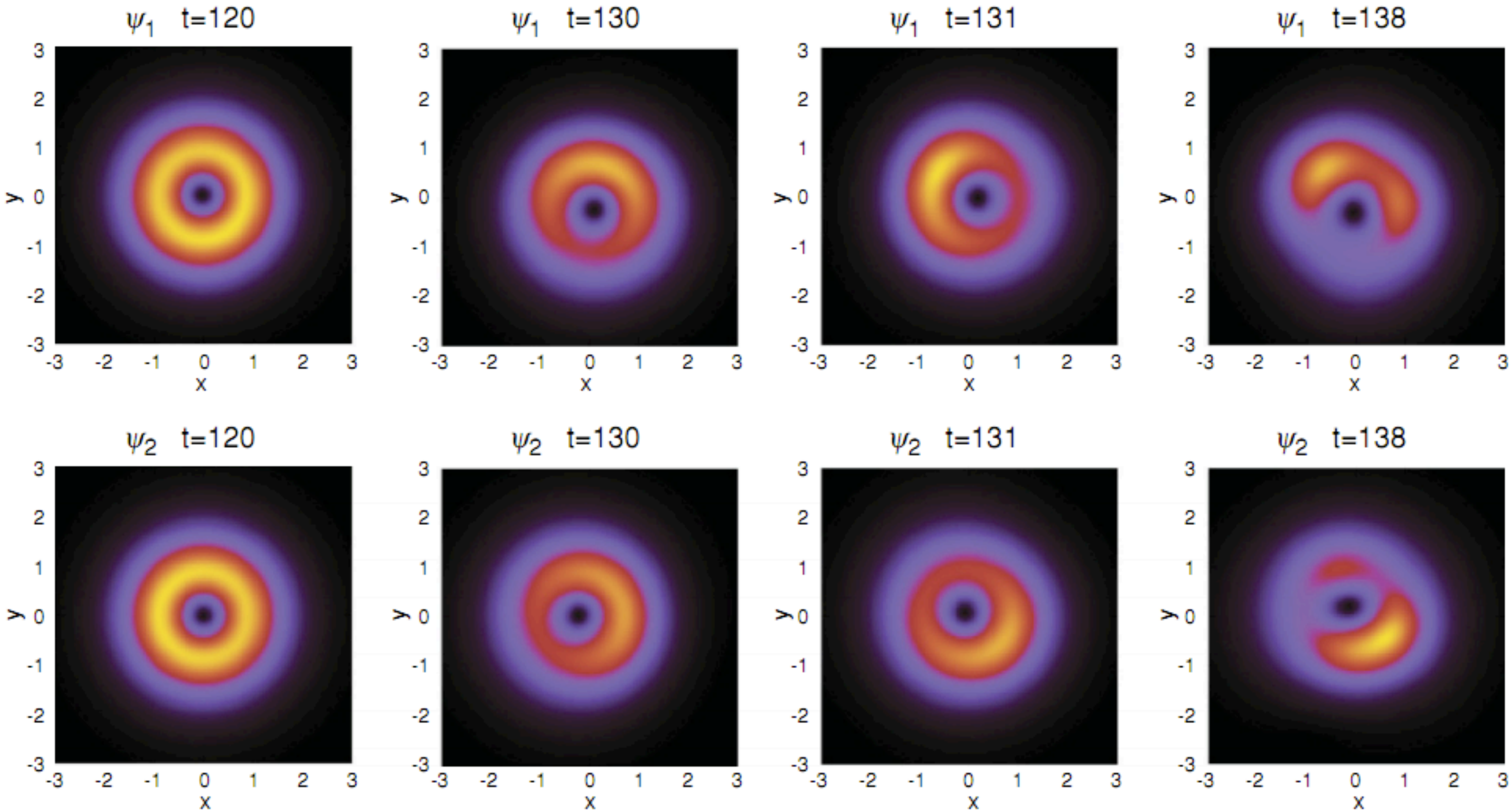


FIG. 6. (Color online) The evolution of the two components ψ_1 and ψ_2 of an unstable HV mode with half-norm $N = 7.5$ and $\mu = 0.83$. Density distributions are displayed at moments of time indicated above the respective panels. In this and two subsequent figures, $\beta = 0.5$ is fixed.

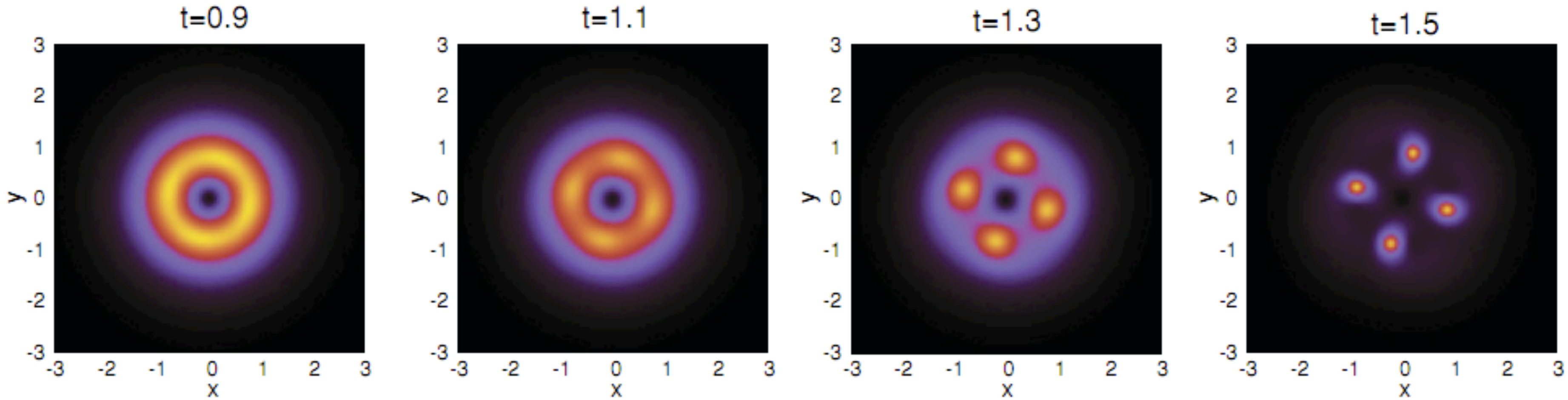


FIG. 9. (Color online) The same as in Fig. 8, but for $\beta = -0.5$ and $N = 34.1$, $\mu = -0.2$. In this case, the HV state quickly splits into a set of four segments.