STRONGLY-INTERACTING FEW-FERMION SYSTEMS IN A TRAP

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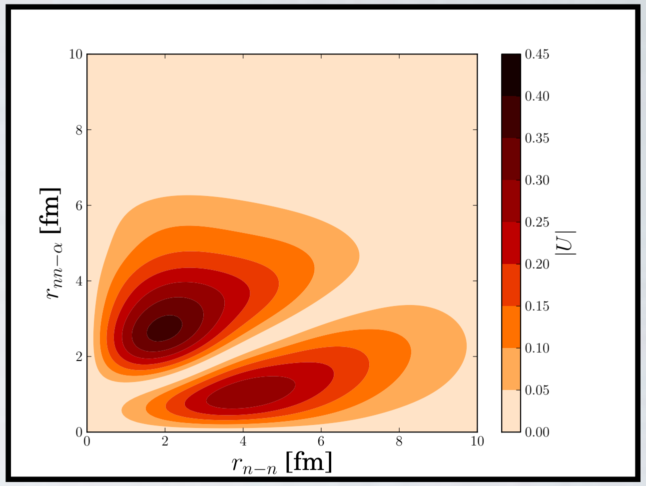
Outline

- INTRO
- PART I: Tunable open quantum systems
- PART II: From few to many
- CONCLUSION

INTRODUCTION:
few-body physics with trapped atoms

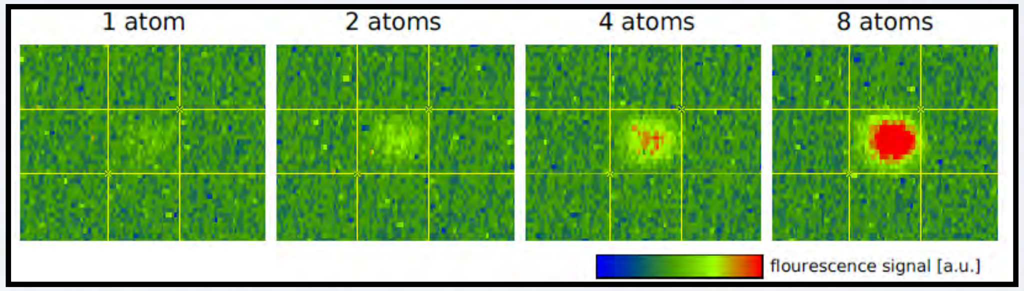
A tunable few-body system

- Few-fermion systems in nature
  - Atoms, nuclei, …
  - Limited tunability of interaction

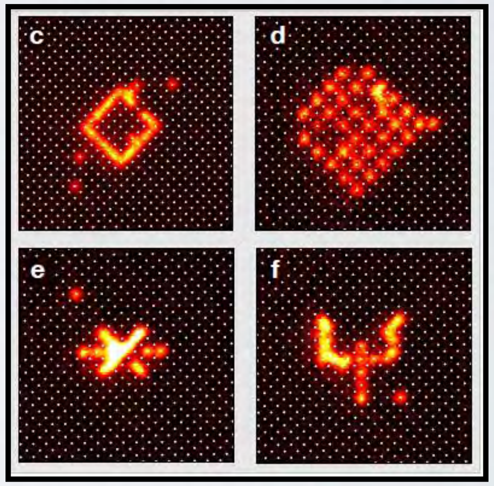


D. Sääf and CF, Phys. Rev. C 89 (2014) 011303R

- Artificial quantum systems
  - Atomic clusters
  - Quantum dots
  - Ultracold atoms



F. Serwane et al., Science 332 (2011) 336

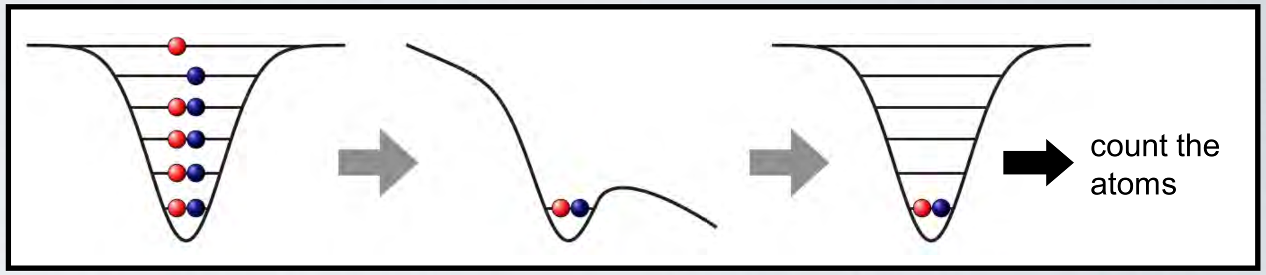


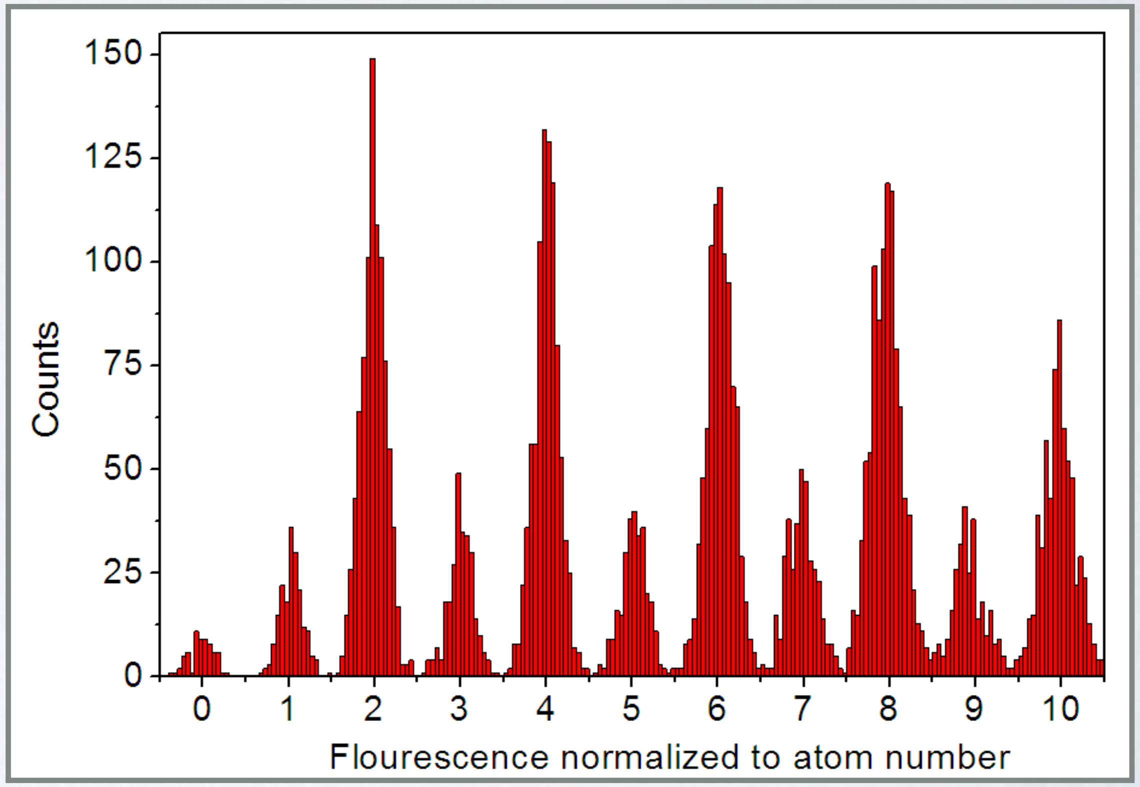
C. Weitenberg et al., Nature 471 (2011) 319

- **Tunable few- and many-body quantum systems are becoming a reality**

High-fidelity preparation

- 2-component mixture in reservoir T=250nK
- Superimpose microtrap
- Scattering
  ⇒ thermalization
- Switch off reservoir
- 1-10 atoms can be distinguished with high fidelity (> 99%)





F. Serwane et al., Science 332 (2011) 336

Universality in 1d-systems

- Tunability of interaction via magnetic Feshbach resonance

- Cold and dilute systems
  $\rho R^3 \ll 1$

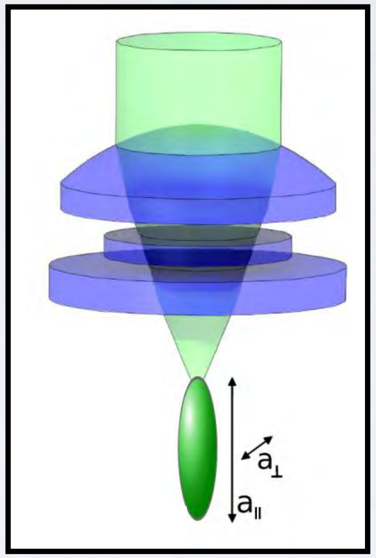
- Detailed knowledge of the interaction is not needed

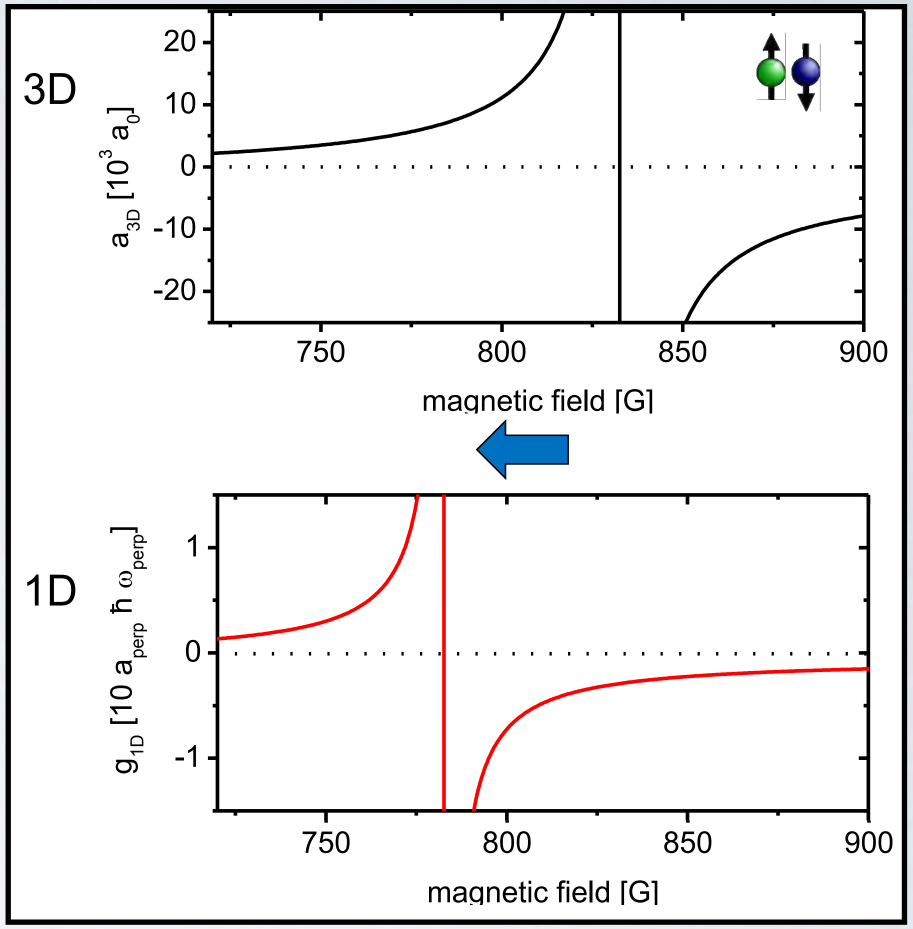
$$k \cot \delta = -\frac{1}{a}$$

- Radially strongly confined
  Aspect ratio $(\omega_{\parallel} / \omega_{\perp})$ 1:10

- Effectively 1D system with contact interaction.

Figures from G. Zürn, Heidelberg



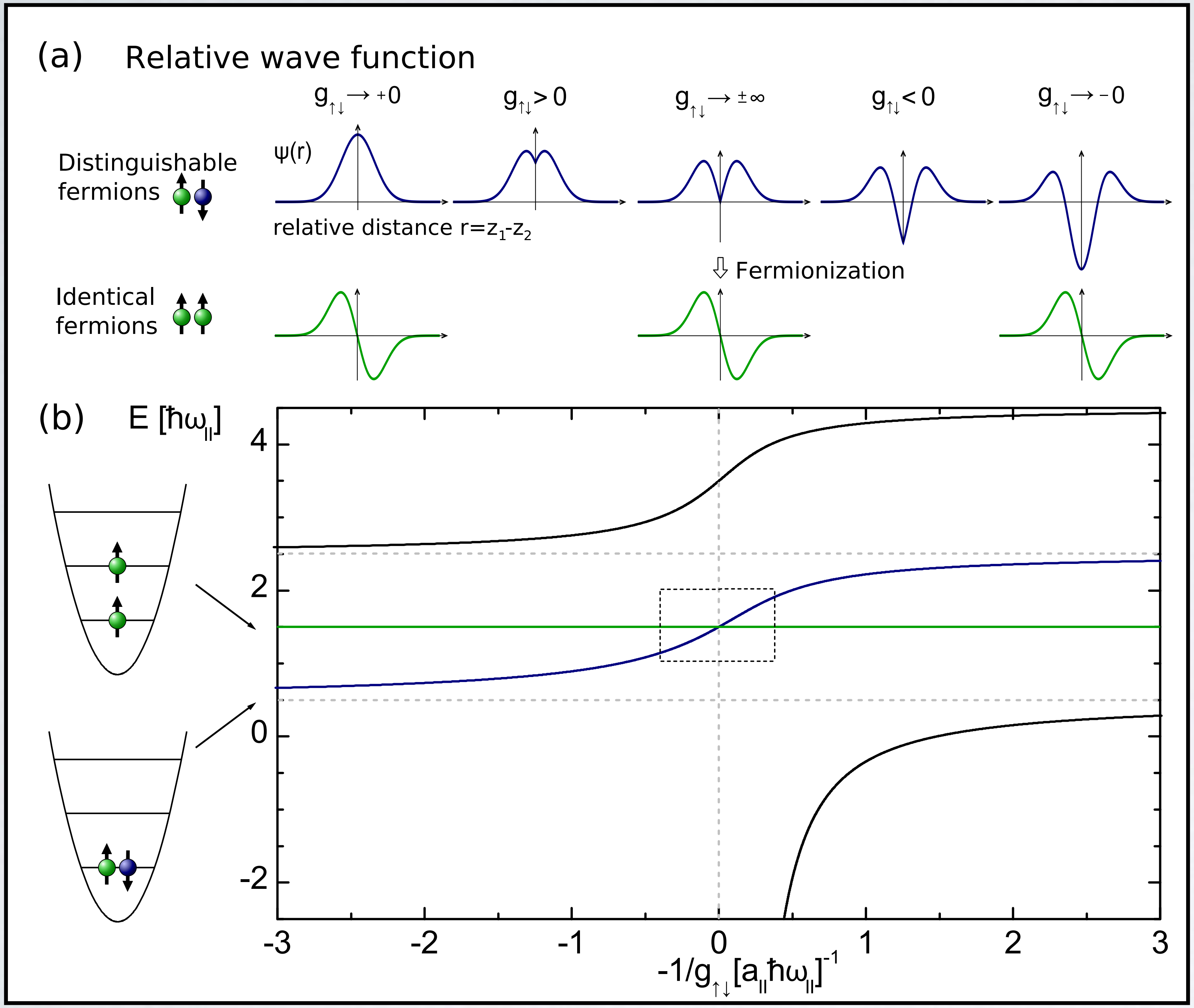


G. Zürn et al., PRL 108 (2012)

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{m a_{\perp}^2} \frac{1}{1 - C a_{3D}/a_{\perp}}$$

M. Olshanii, PRL 81 (1998) 938

Busch model for two particles



From: G. Zürn et al., Phys. Rev. Lett. 108, 075303 (2012).

- Zero-range interaction
- Parabolic trapping potential
- Energy spectrum given by the Busch formula

$$\frac{\Gamma(-E/2+1/4)}{\Gamma(-E/2+3/4)} = -\frac{2}{g}$$

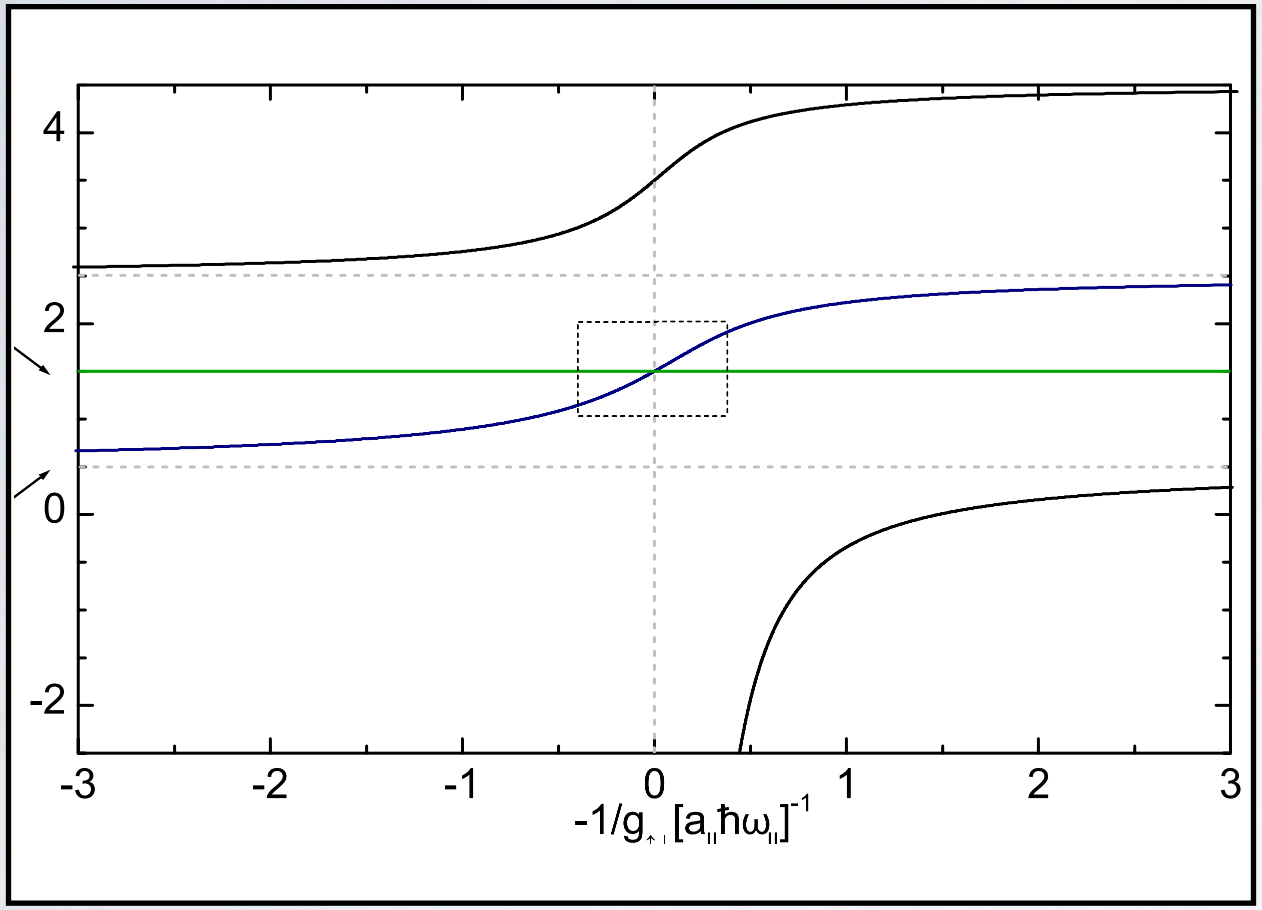
- Analytical expressions for wave functions

T. Busch et al., Found. Phys. 28 (1998) 549.

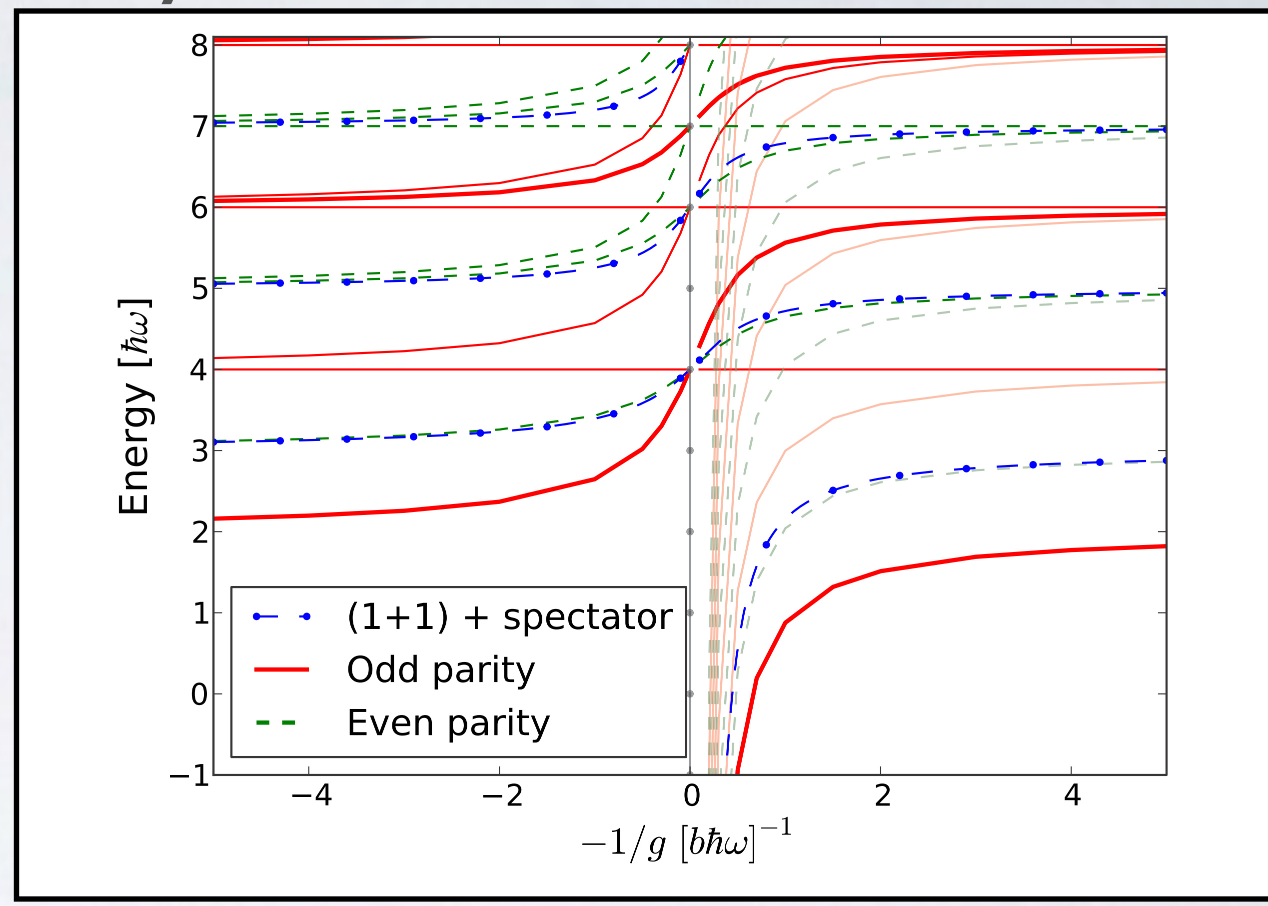
Foundations of Physics, Vol . 28, No. 4, 1998

Energy spectrum

1+1 system



1+2 system



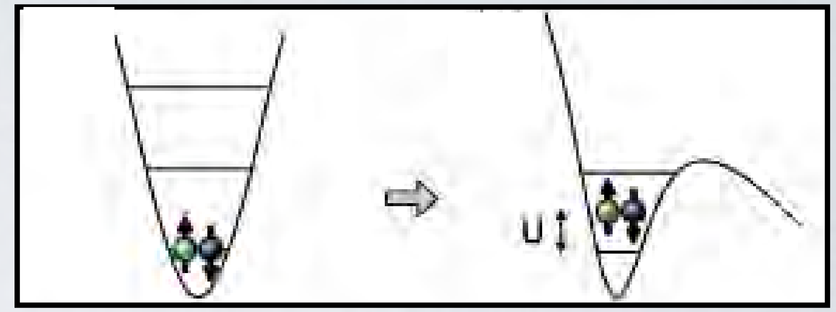
- Effective interaction approach with exact diagonalization
- Studied up to 1+9 particles

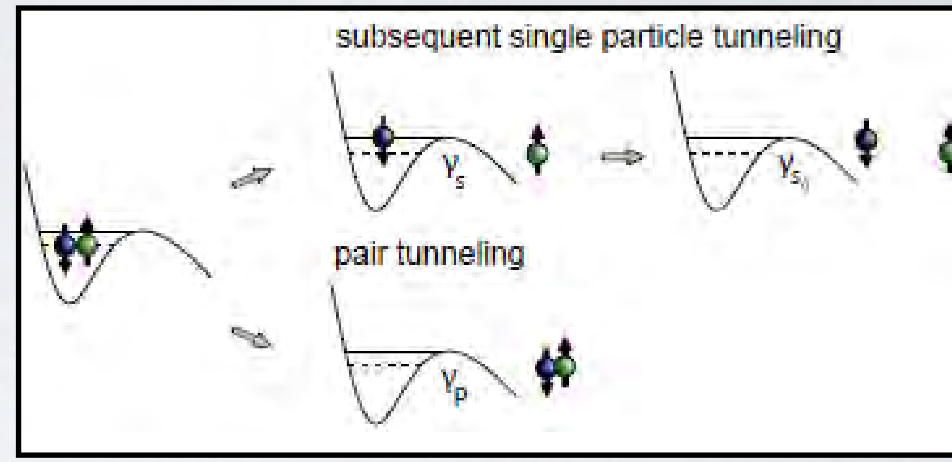
J. Lindgren et al, New J. Phys. 16 (2014) 063003.

Tunable Open Quantum systems

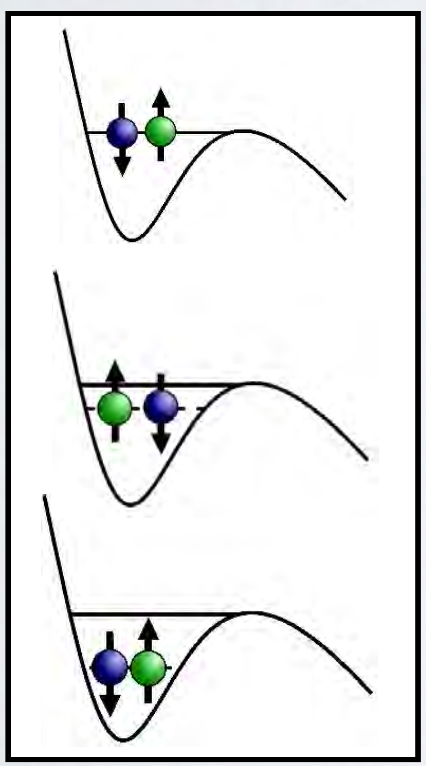
Tunable open quantum systems

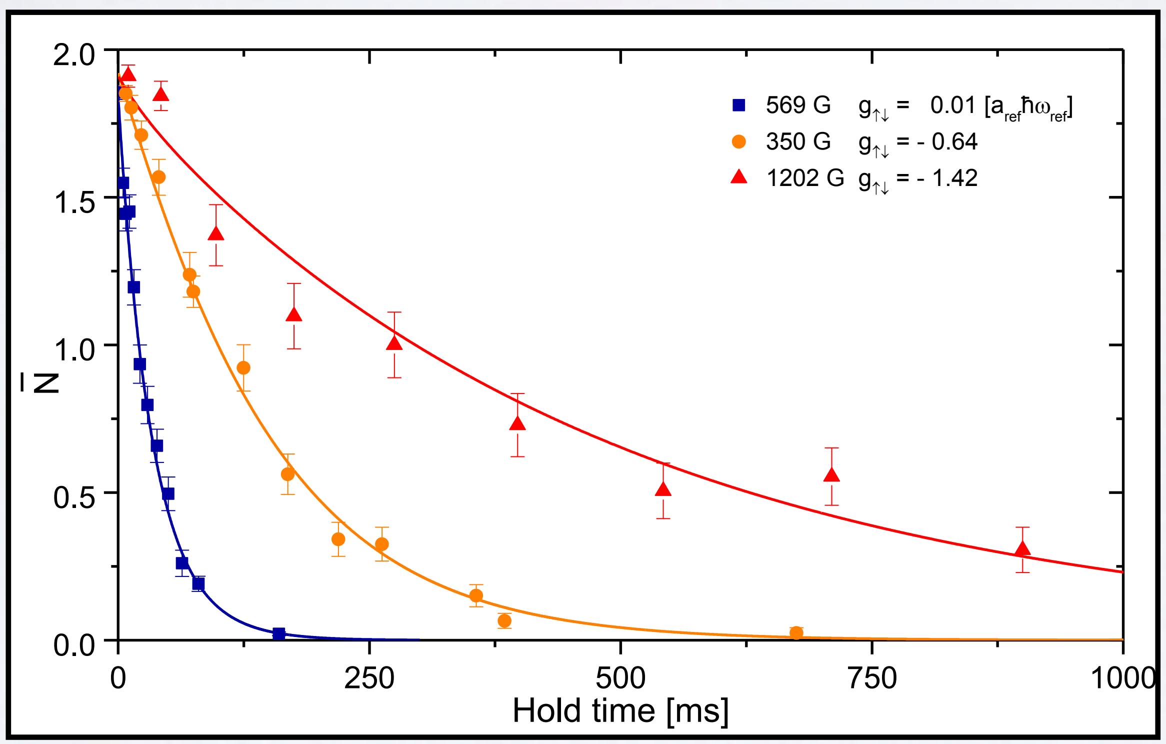
- Interacting atoms in an open trap.
- How do the two atoms tunnel out?
- How is the decay mechanism affected by the “pairing” interaction?





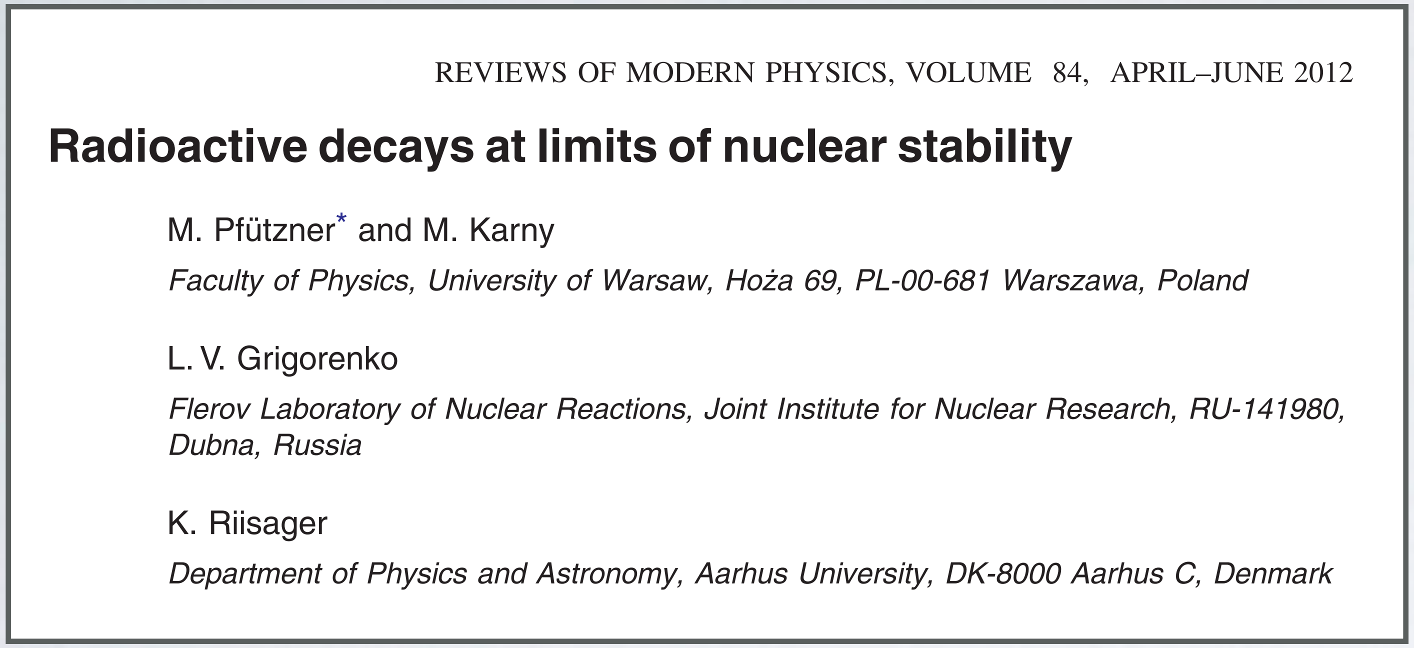
From: G. Zürn et al., PRL 108 (2012) 075303.





G. Zürn et al., PRL 111 (2013) 175302

Two-proton radioactivity


REVIEWS OF MODERN PHYSICS, VOLUME 84, APRIL–JUNE 2012

Radioactive decays at limits of nuclear stability

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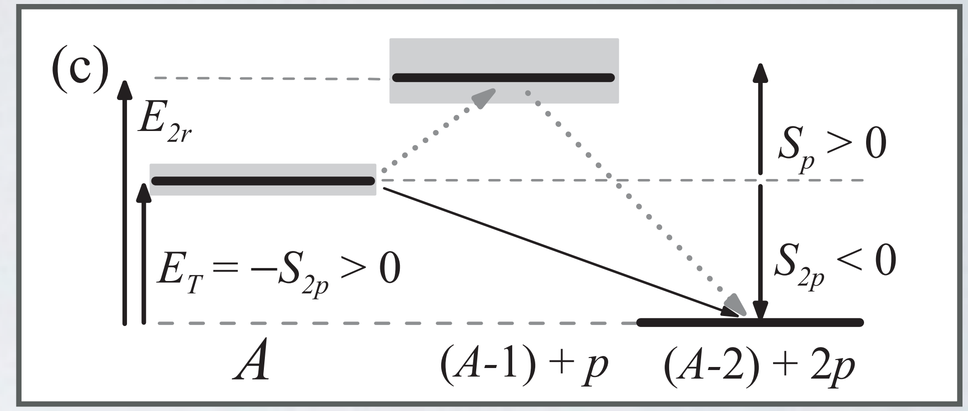
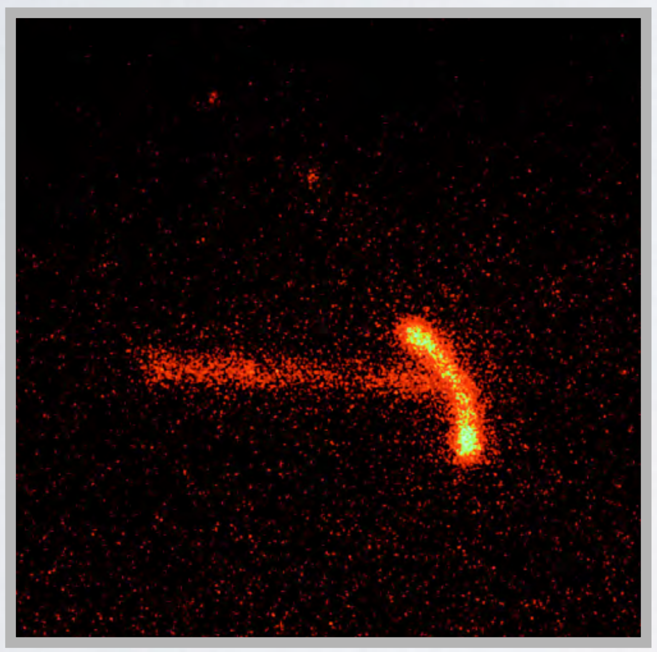
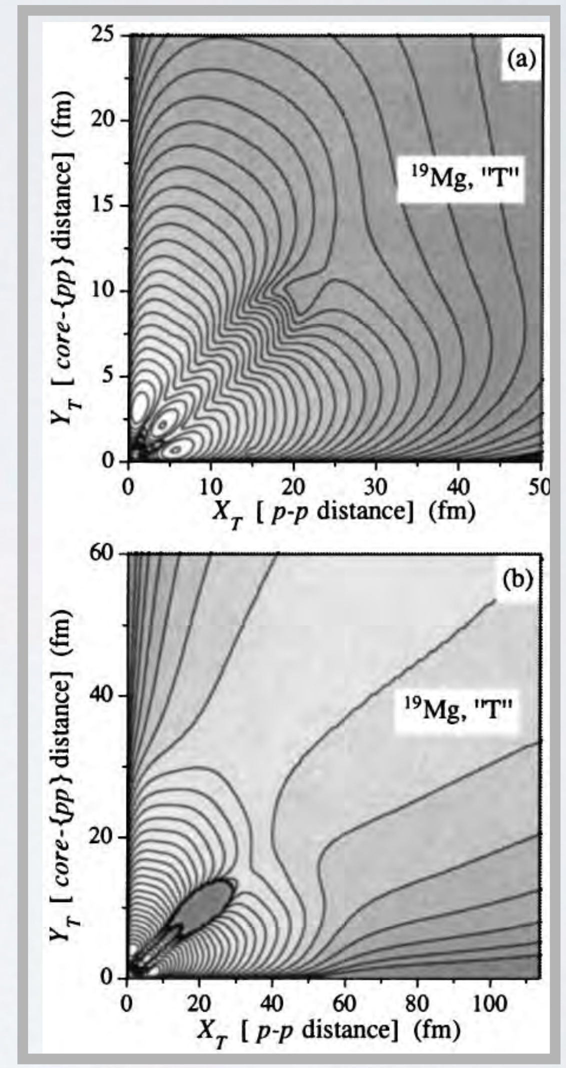




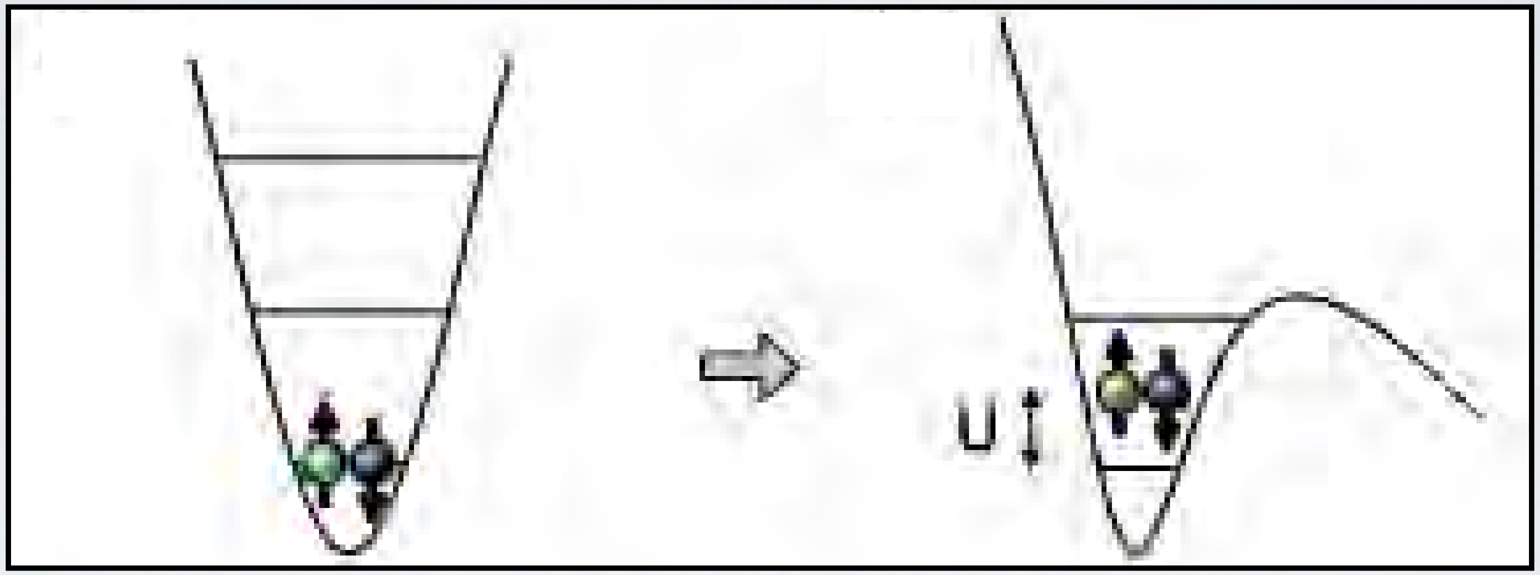
K. Miernik et al., PRL 99 (2007) 192501.

Hot topic in the description of physics of exotic nuclei



L. Grigorenko et al., PRL 85 (2000) 22.

Closed versus **open** quantum systems



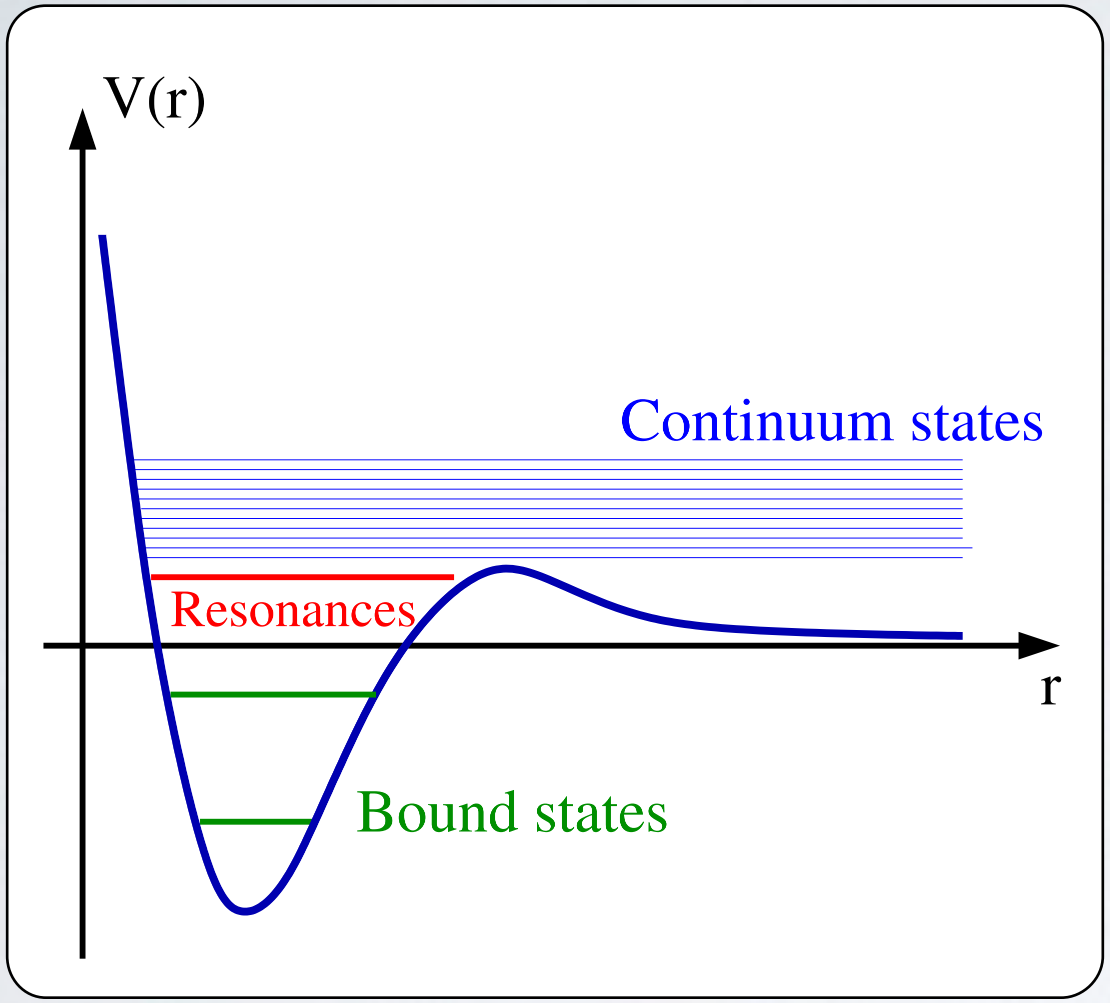
- **Closed** quantum system
- Well described in a discrete real-energy basis
- Newton completeness relation

$$\sum_b |u_b\rangle\langle u_b| + \int_0^{+\infty} dk |u_k\rangle\langle u_k| = 1$$

Bound states — Real-energy continuum states

- **Open** quantum system
- Strongly affected by the vicinity of the continuum of decay channels
- Spectrum can contain bound, resonance and scattering states

Gamow states



- How to describe decay in a (quasi-) stationary formalism?
- Complex energy eigenstates

$$\tilde{E} = E_r - i\frac{\Gamma}{2}$$

- Gamow states

$$\Psi(r,t) = e^{-\frac{i\tilde{E}t}{\hbar}}\psi(r)$$

$$|\Psi(r,t)| \sim e^{-\frac{\Gamma t}{2\hbar}}e^{kr}, \quad r \to \infty$$

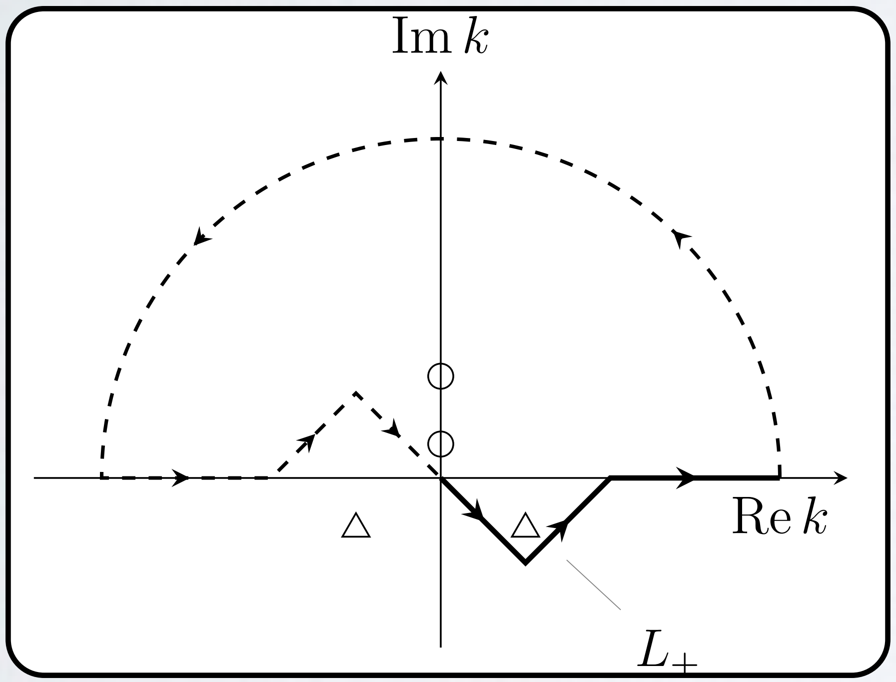
G. Gamow, Z. Phys. 51 (1928) 204

Rigged Hilbert space

- How to include resonances in the complete set of states?

$$\sum_b |u_b\rangle\langle u_b| + \int_0^{+\infty} dk |u_k\rangle\langle u_k| + \sum_{\text{res}} |u_{\text{res}}\rangle\langle u_{\text{res}}| = 1 + \sum_{\text{res}} |u_{\text{res}}\rangle\langle u_{\text{res}}| \neq 1$$

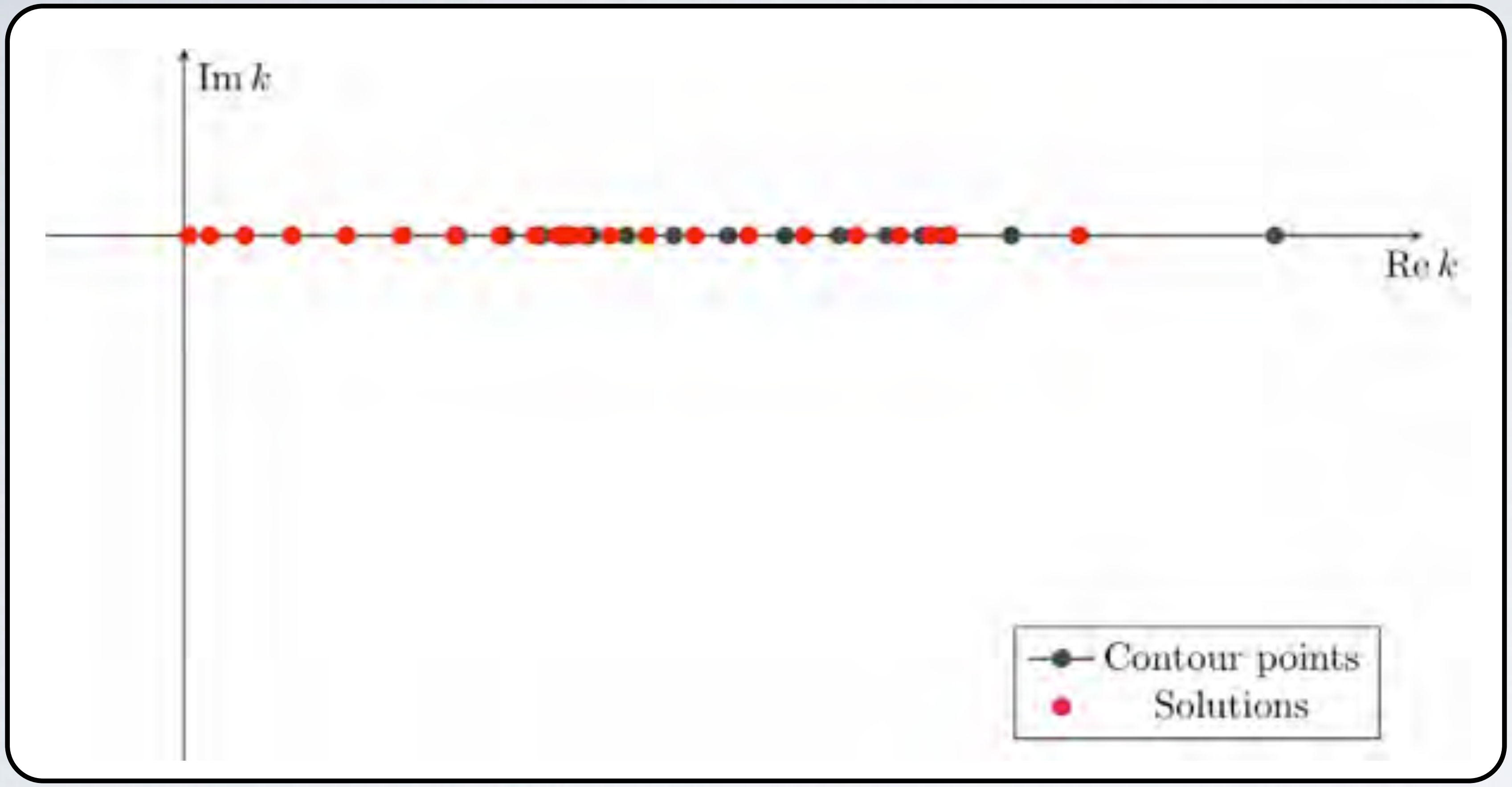
- Instead, the Berggren completeness relation



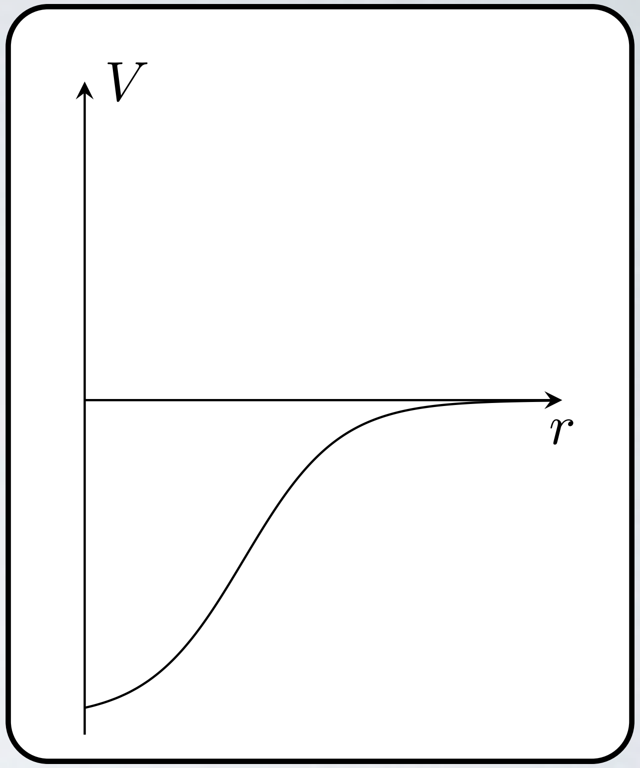
Gamow states and completeness
- T. Berggren, Nucl. Phys A **109**(1968)265; NPA**389**(1982)261
- T. Lind, Phys. Rev. C **47**(1993)1903

$$\sum_{n=b,r} |u_n\rangle\langle \bar{u}_n| + \frac{1}{\pi}\int_{L_+} |u(k)\rangle\langle u^*(k)| dk = 1$$

Complex-momentum basis



Courtesy: O. Embréus, V. Ericsson, P. Granström, and N. Wireklint



Resonance at:
k_r=0.17-0.036i fm^{-1}

- **Black** circles correspond to complex-momentum plane-wave states

Scattering solutions

$$u(r) \to C^+ H^+_{l,\eta}(kr) + C^- H^-_{l,\eta}(kr)$$

Resonance solutions

$$u_n(r) \to C^+ H^+_{l,\eta}(k_n r)$$

Gamow shell model

(i) discretization of continuum contour

$$\sum_{n=b,r} |u_n\rangle\langle u_n| + \sum_i |u_{k,i}\rangle\langle u_{k,i}| \approx 1$$

(ii) construction of many-body basis

$$|\mathrm{SD}_i\rangle = |u_{i1}, \ldots, u_{iA}\rangle$$

(iii) construction of Hamiltonian matrix (complex symmetric matrix)

$$\langle \mathrm{SD}_i | H | \mathrm{SD}_j \rangle$$

(iv) many-body spectrum contains: bound, resonant and continuum states

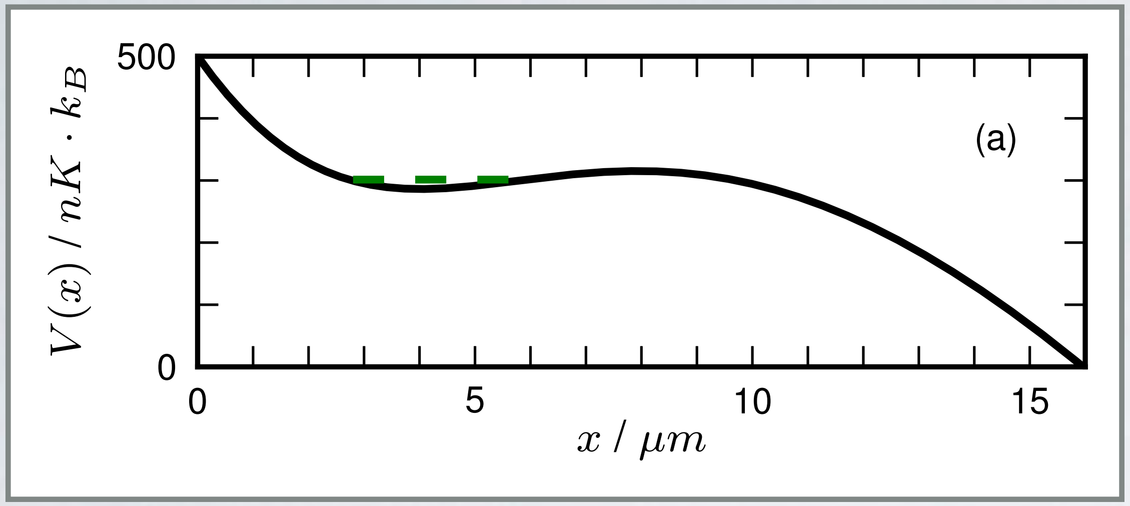
Gamow Shell Model
- N. Michel et al, PRL 89 (2002) 042502; PRC67 (2003) 054311; PRC70 (2004) 064313; JPG (2009) 013101
- G. Hagen et al, PRC71 (2005) 044314
- J. Rotureau et al, PRL 97 (2006) 110603
- G.Papadimitriou et al, PRC(R) 84 (2011) 051304

- Pole approximation is 0th order approximation:

$$H^{\mathrm{p.a.}}|\Psi^{\mathrm{p.a.}}\rangle = E^{\mathrm{p.a.}}|\Psi^{\mathrm{p.a.}}\rangle$$

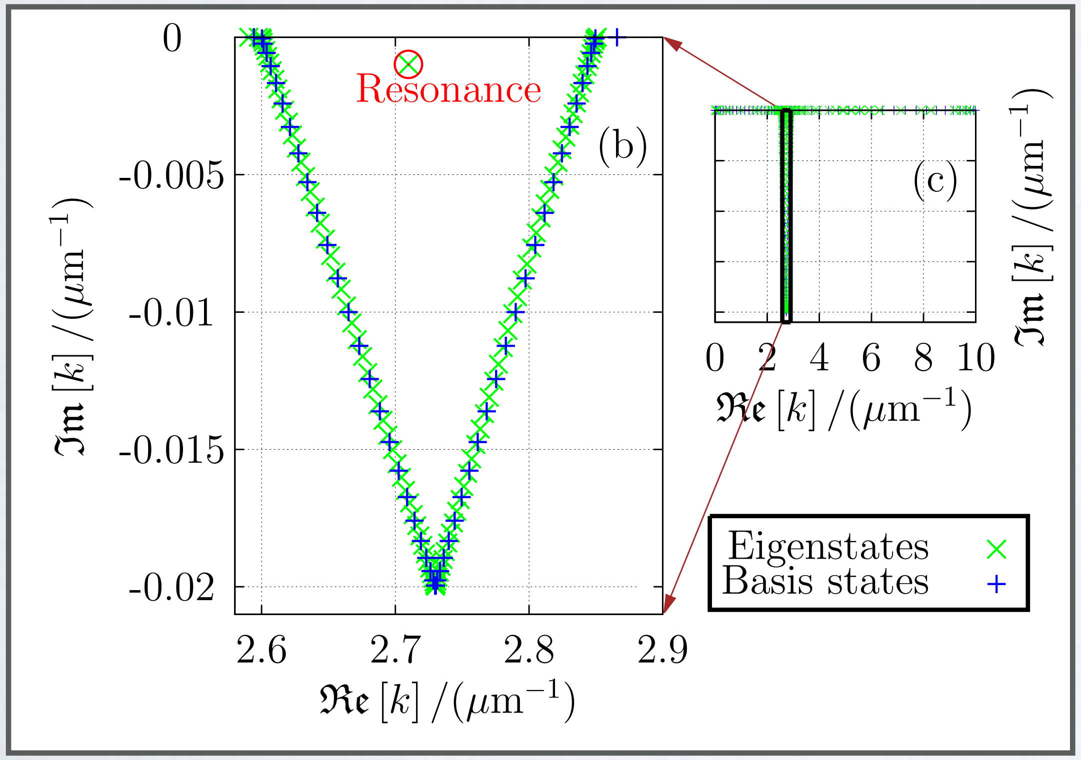
- Many-body resonance (or bound state) has large overlap:

Application to trapped ultracold atoms



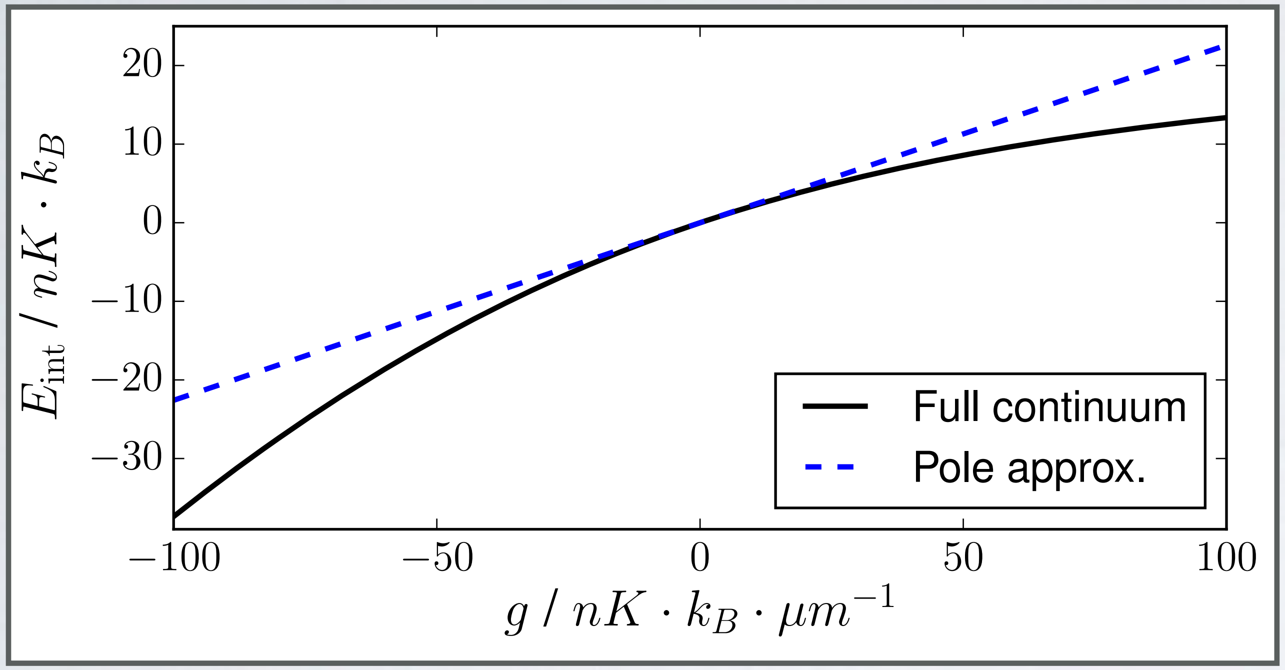
$$V(x) = pV_0\left(1 - \frac{1}{1+(x/x_R)^2}\right) - c_{B|\text{state}\rangle}\mu_B B' x$$

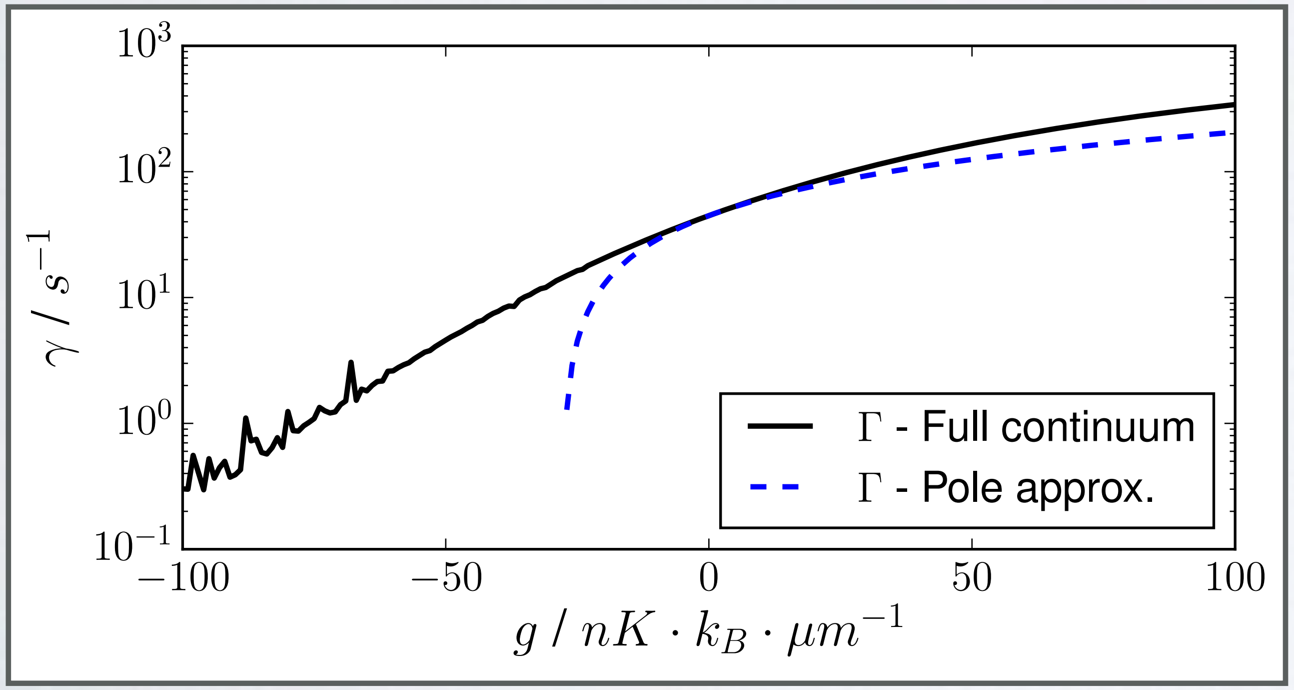
Parameter	Value	Designation
V_0	$3.326\mu\text{K}\cdot\text{k}_\text{B}$	Potential depth.
z_R	$9.975\mu\text{m}^2$	Rayleigh range of trapping beam.
μ_B	$6.717\,138\,8\cdot 10^5\,\mu\text{K}\cdot\text{k}_\text{B}/\text{T}$	Bohr magneton.
B'	$18.92\cdot 10^{-8}\,\text{T}/\mu\text{m}$	Magnetic field gradient.
$c_{B\|\text{state}\rangle}$	≈ 1	



- Resolution of the Schrödinger equation in the Berggren Basis
- Calculated decay rates off by almost a factor two(!) compared to values extracted from experiment [by G. Zürn et al., PRL 111 (2013).]
- Let's come back to this discrepancy.

Two-particle states





- Interaction energy

$$\mathfrak{Re}(E_2) \equiv 2E_r + E_{\text{int}}$$

- Pole approximation

$$|\Psi_2\rangle_{\text{p.a.}} = |u_{\text{res}}(1), u_{\text{res}}(2)\rangle$$

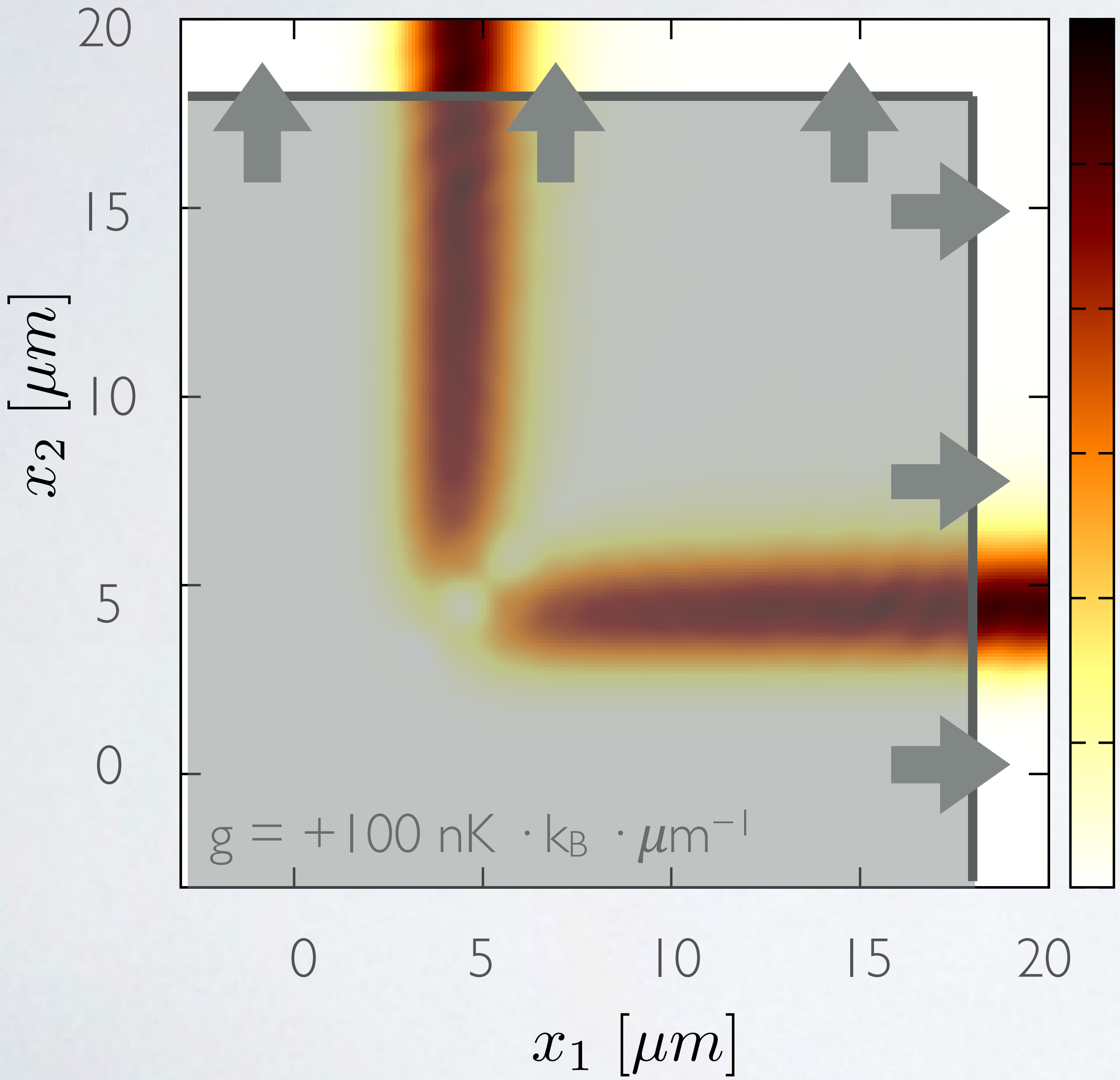
- Tunneling rate

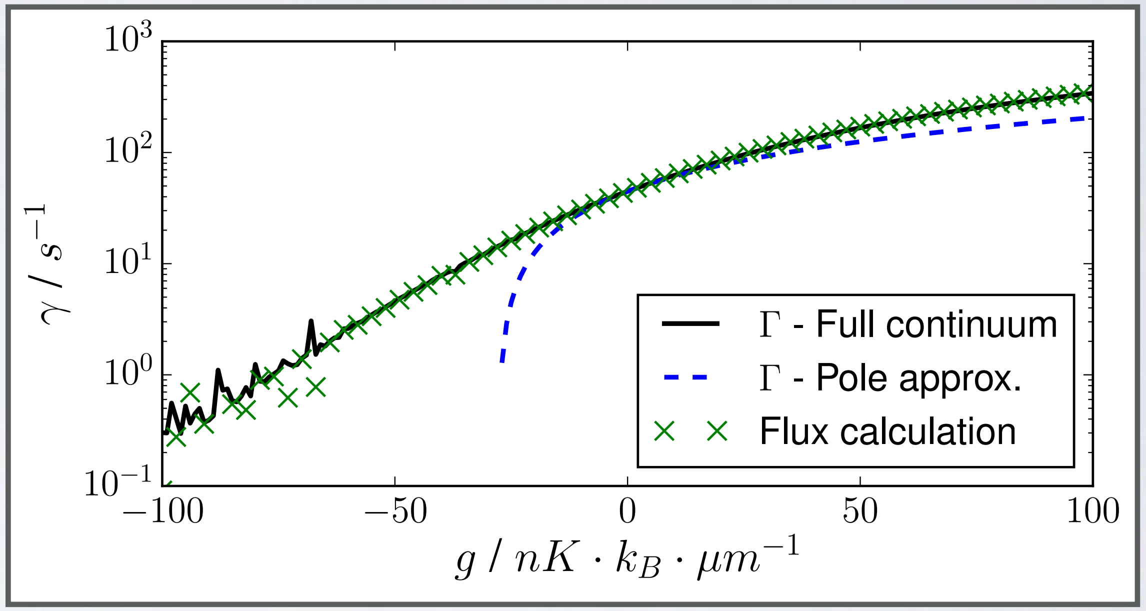
$$\gamma = \underbrace{-2\mathfrak{Im}(E_2)}_{=\Gamma_2} / \hbar$$

Decay rate from particle flux

$$\hbar\gamma = \frac{j}{N}\left(= \frac{\sum_i j(x_1,x_2)|_{x_i=x_{\text{outside}}}}{\iint_0^x dx_1 dx_2 N(x_1,x_2)} \right)$$

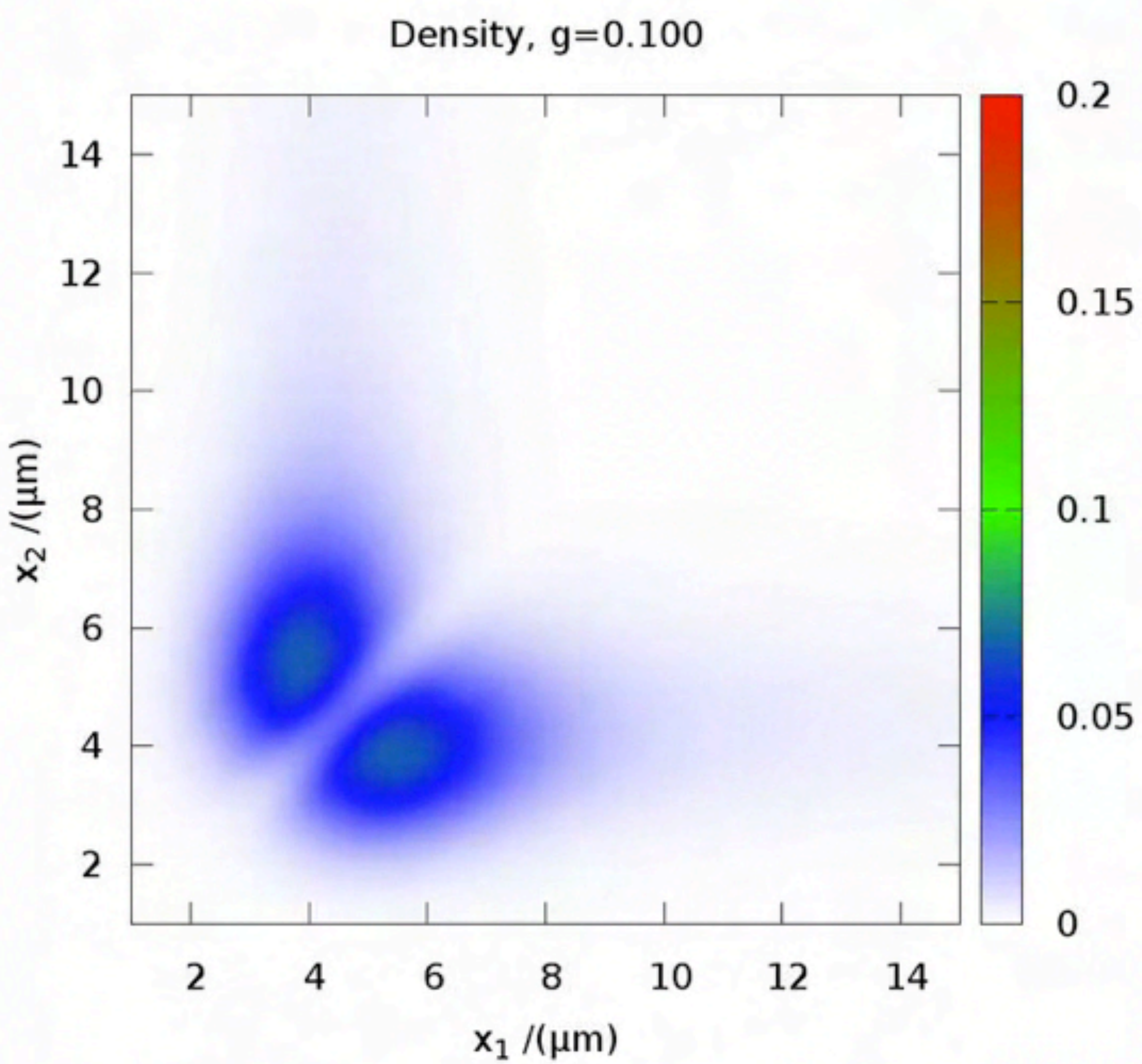
$j(x_1, x_2)$



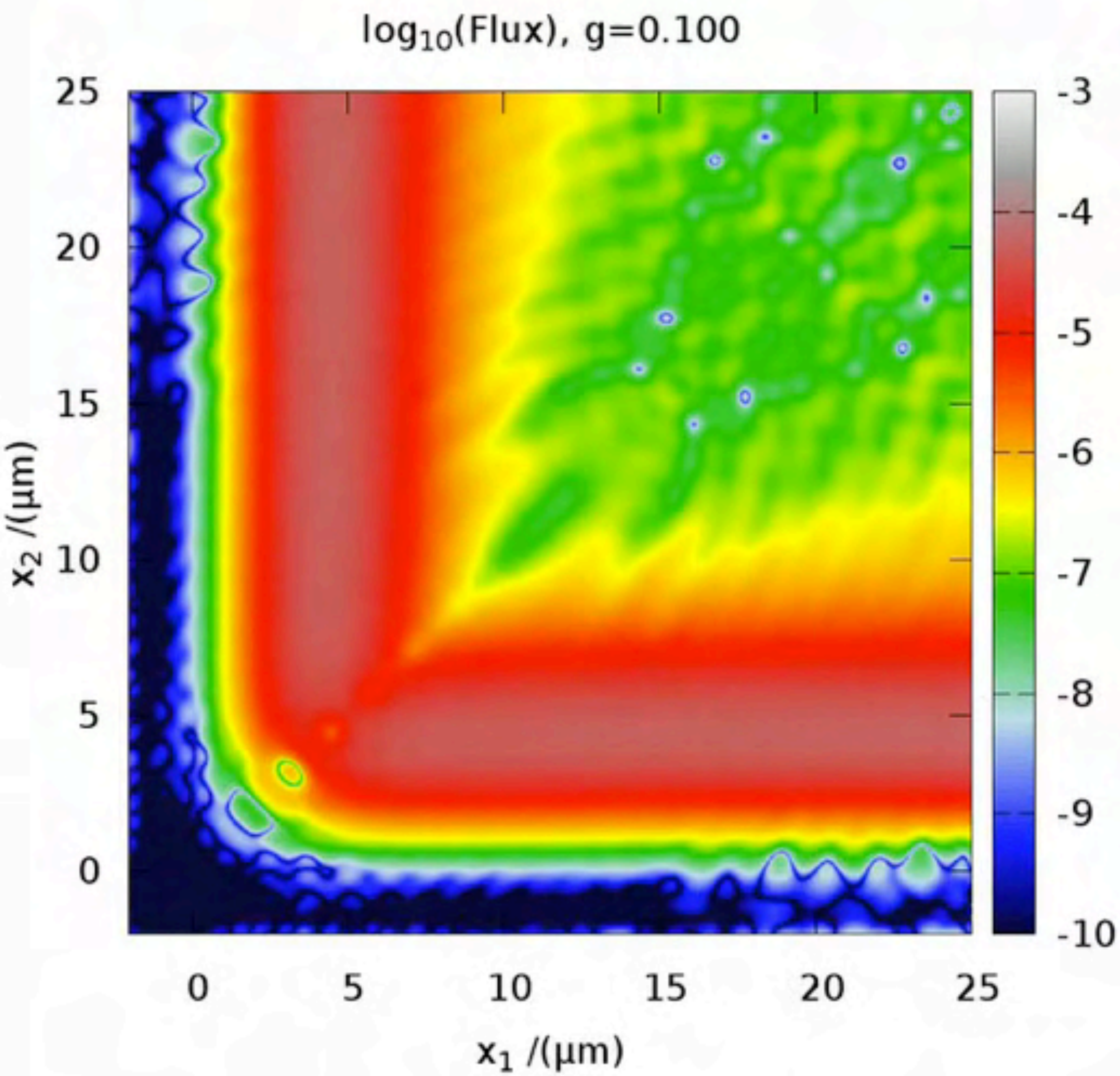


- Decay rate from **flux and width calculations agree**
- Indicates that decay is indeed **exponential**

Density distribution



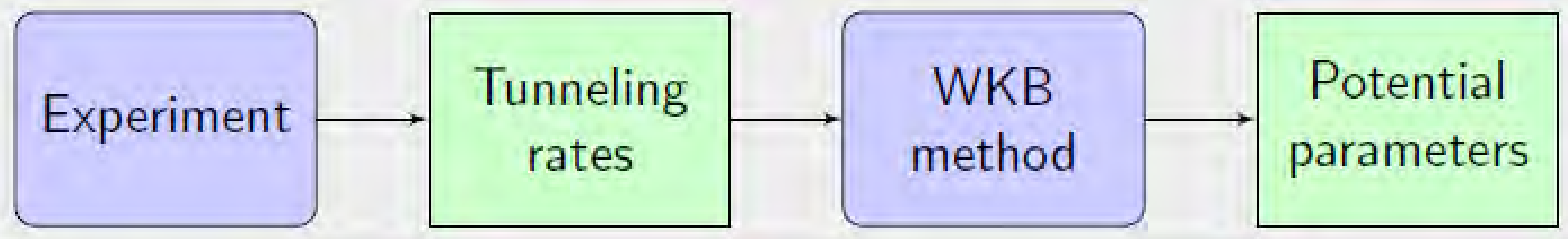
Flux distribution



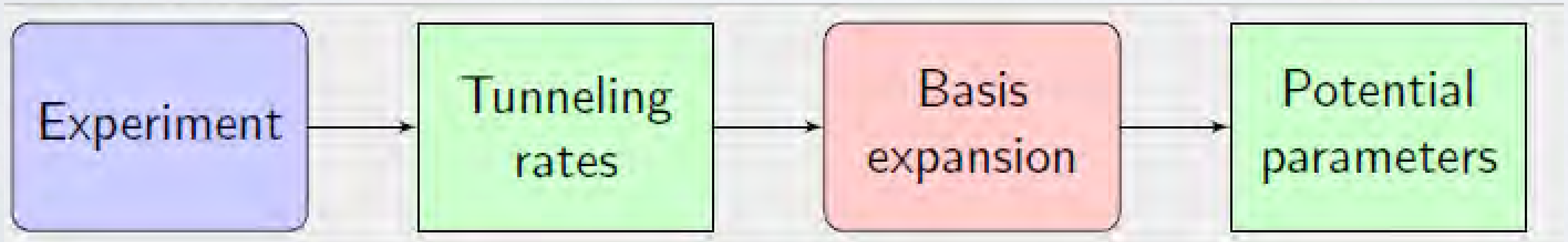
Comparison with tunneling experiment

- In the experiment: change g ⇒ change B'
  I.e., the single-particle trap potential changes
- In addition. change B' ⇒ change $c_B(1,2)$
  I.e., the two fermions see slightly different traps.

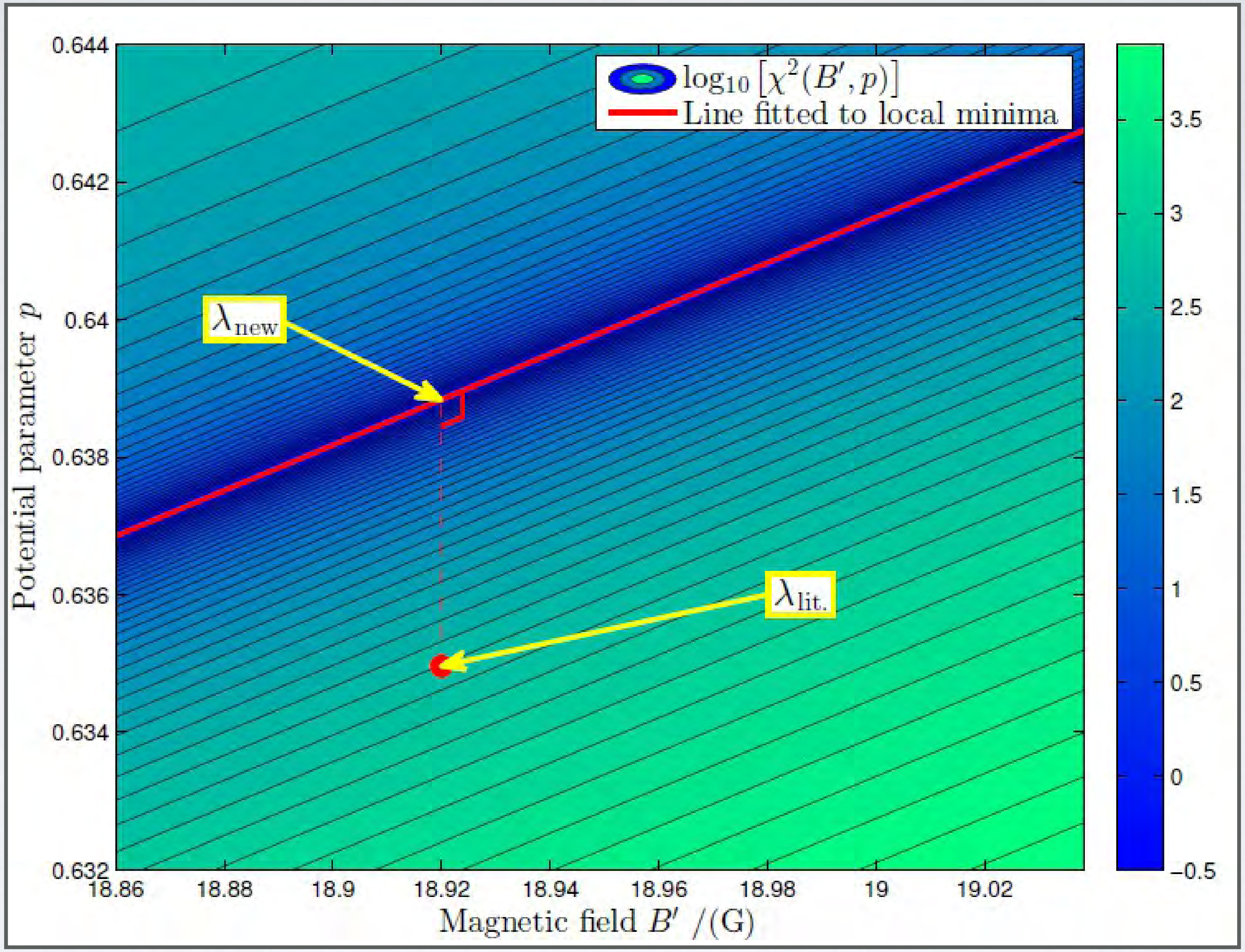
Some **input** parameters are extracted from experimental results using **WKB method**



We want to refit experimental results without **WKB**



Refit trap potential



Tunneling of two atoms

g $/(\mathrm{nK}\cdot k_B\cdot \mu m)$	$E_{\mathrm{int}}^{\mathrm{calc}}$ $/(\mathrm{nK}\cdot k_B)$	$\gamma_{\mathrm{total}}^{\mathrm{calc}}$ $/(s^{-1})$	$E_{\mathrm{int}}^{\mathrm{WKB}}$ $/(\mathrm{nK}\cdot k_B)$	$\gamma_{\mathrm{total}}^{\mathrm{exp}}$ $/(s^{-1})$
−30.969 33	−8.45	19.19	−3.093 ± 0.228	22.20 ± 1.0
−41.527 05	−12.10	12.54	−4.167 ± 0.391	13.84 ± 1.0
−45.046 30	−13.59	25.81	−4.753 ± 0.326	9.70 ± 0.3
−99.946 47	−37.02	(0.44)	(−10.418 ± 1.107)	2.14 ± 0.2
−104.169 56	−39.28	(0.56)	(−10.646 ± 0.912)	1.93 ± 0.1
−110.504 19	−42.79	(1.12)	(−11.623 ± 0.814)	1.23 ± 0.1
−123.877 31	−50.55	(0.34)	(−13.283 ± 0.977)	0.51 ± 0.0

Berggren basis

Exp. + WKB

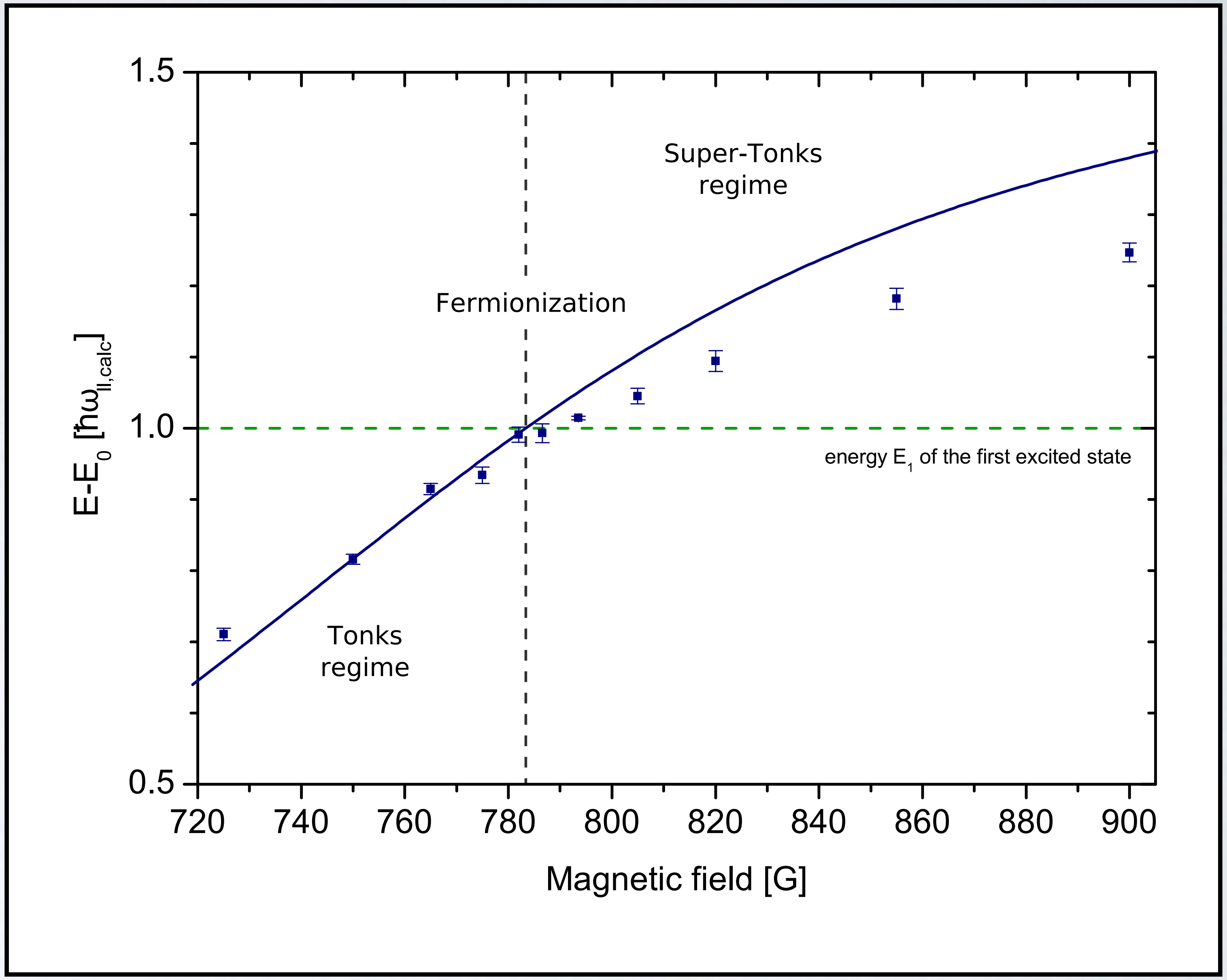
For each g, the single particle potential is slightly different

g	−0.703 85	−30.969 33	−41.527 05	−45.046 30
$c_{B\vert\uparrow\rangle}$	1.00457	1.00407	1.00356	1.00311
$c_{B\vert\downarrow\rangle}$	0.99968	0.99806	0.99512	0.98989

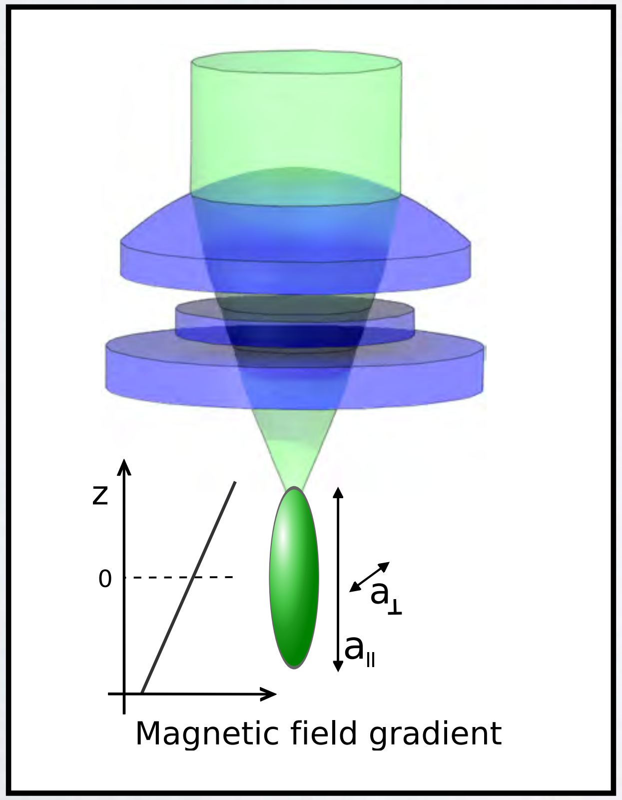
From Few to Many

Heidelberg Experiment

- 1d system with repulsively interacting bosonic gases (Tonks- Girardeau regime)
- Two-component fermionic systems using hyperfine states of ^{6}Li
- 1:10 asymmetric opto-magnetic trap.



From: G. Zürn et al., Phys. Rev. Lett. 108, 075303 (2012).



Model and energy spectrum

J. Lindgren et al, New J. Phys. 16 (2014) 063003.

- Adapting tools and methodology from our research on many-nucleon systems we studied 1+N systems, with N up to 9.
- Short-range interaction $$g \propto \frac{a_{3d}}{1 - C a_{3d}/a_{\perp}}$$
- Hamiltonian $$H = \sum_{i_\sigma} \left(\frac{p^2}{2} + \frac{1}{2} x_{i_\sigma}^2 \right) + g \sum_{i_\sigma, j_{\tilde{\sigma}}} \delta(x_{i_\sigma} - x_{j_{\tilde{\sigma}}})$$ with $\sigma = \pm, \tilde{\sigma} = -\sigma$
- Unitary transformation to obtain an effective interaction

Effective interaction

T. Busch et al., Found. Phys. 28 (1998) 549.

- Busch solution
  - Energy spectrum $$\frac{\Gamma(-E/2+1/4)}{\Gamma(-E/2+3/4)} = -\frac{2}{g}$$
  - Wave functions $$\phi(r) = Are^{-\frac{r^2}{2b^2}} U\left(\frac{3/4 - E/2}{\hbar\omega}, \frac{3}{2}, \frac{r^2}{b^2}\right)$$

- Unitary transformation formed with the energies and eigenvectors in the infinite Hilbert space. $$H^{(2)} = X^\dagger E^{(2)} X$$

- Effective interaction in truncated two-body space

$$H_P^{\mathrm{eff}} = \frac{X_P^\dagger}{\sqrt{X_P^\dagger X_P}} E_P^{(2)} \frac{X_P}{\sqrt{X_P^\dagger X_P}}$$

J. Rotureau, EPJ D 67 (2013) 153
J. Lindgren et al, New J. Phys. 16 (2014) 063003.

Effective interaction (cont'd)

- Two-body harmonic oscillator states, with basis truncation N_{max} that defines P-space
- Two-body energies reproduced in P (by construction)
- Eigenfunctions converge to "true" eigenfunctions as P grows

- Resulting effective interaction used in the many-body calculation.
- Many-body basis: Slater determinants composed of harmonic oscillator single-particle states.

The No-Core Shell Model

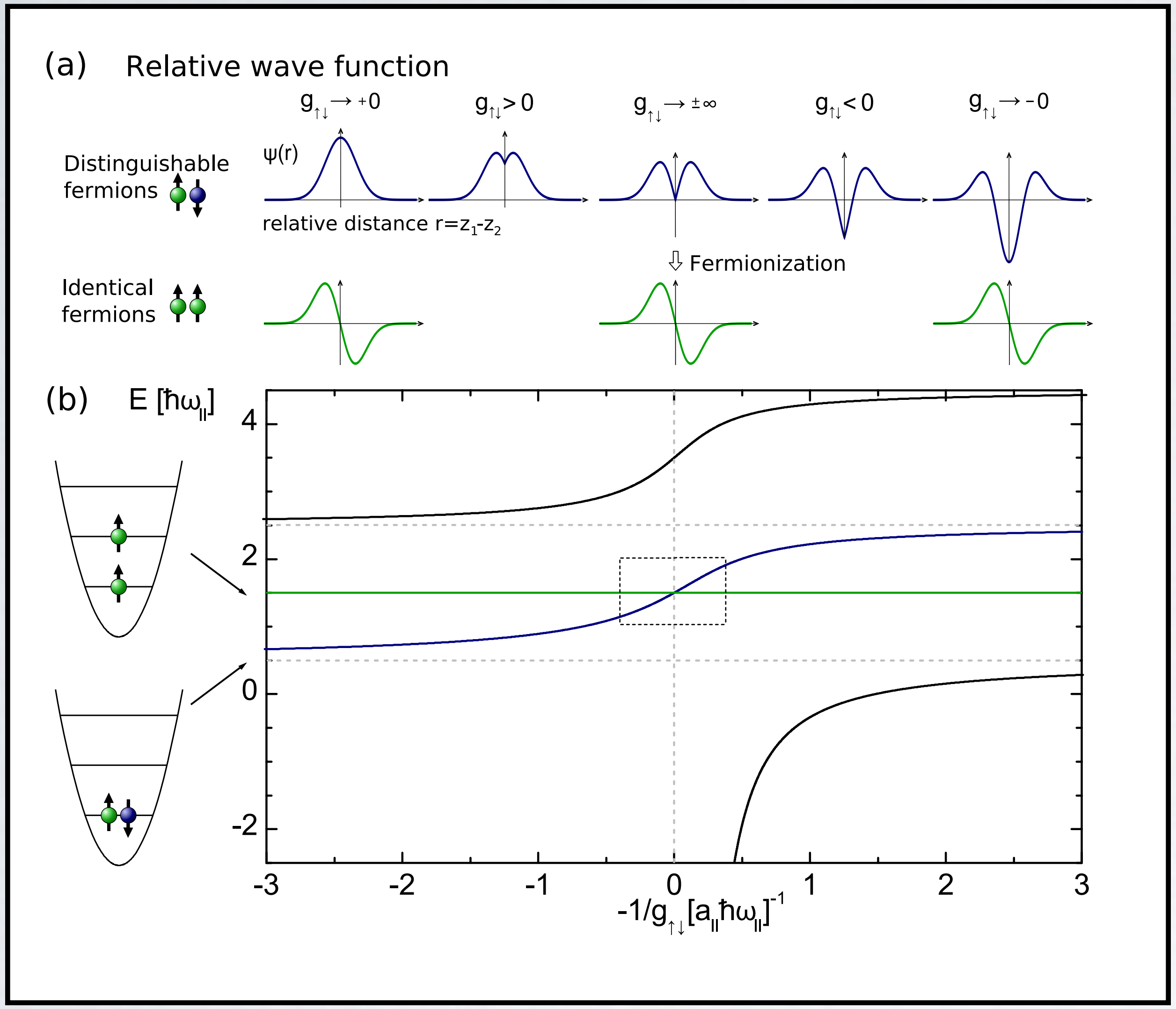
- Many-body Schrödinger equation
  - *A*-nucleon wave function
  - Non-relativistic, point nucleons
- Hamiltonian:

$$H_A = \frac{1}{A}\sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

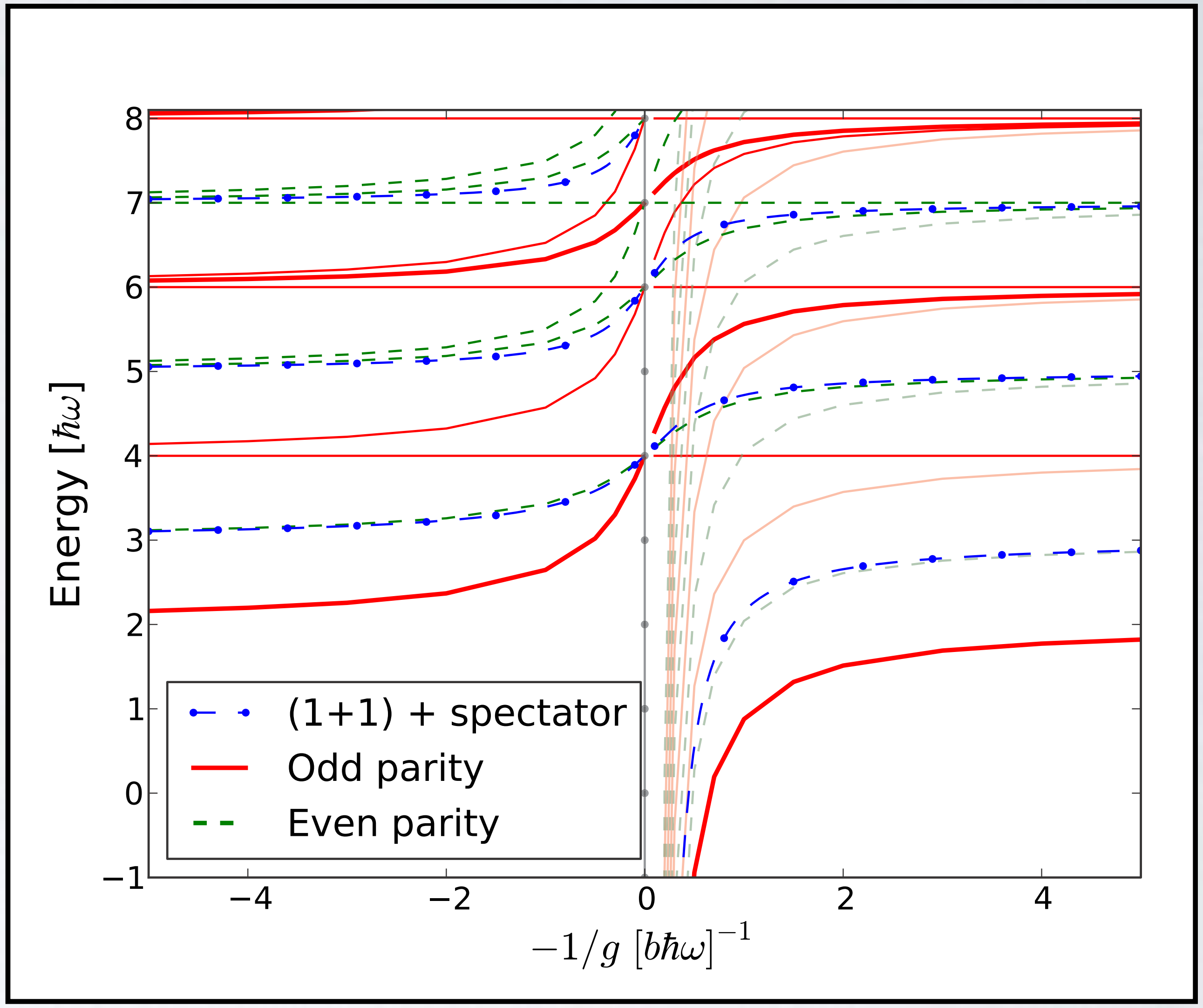
- Many-body basis: Slater determinants composed of harmonic oscillator single-particle states
- Respects translational invariance and includes full antisymmetrization

Energy spectrum

1+1 system



1+2 system

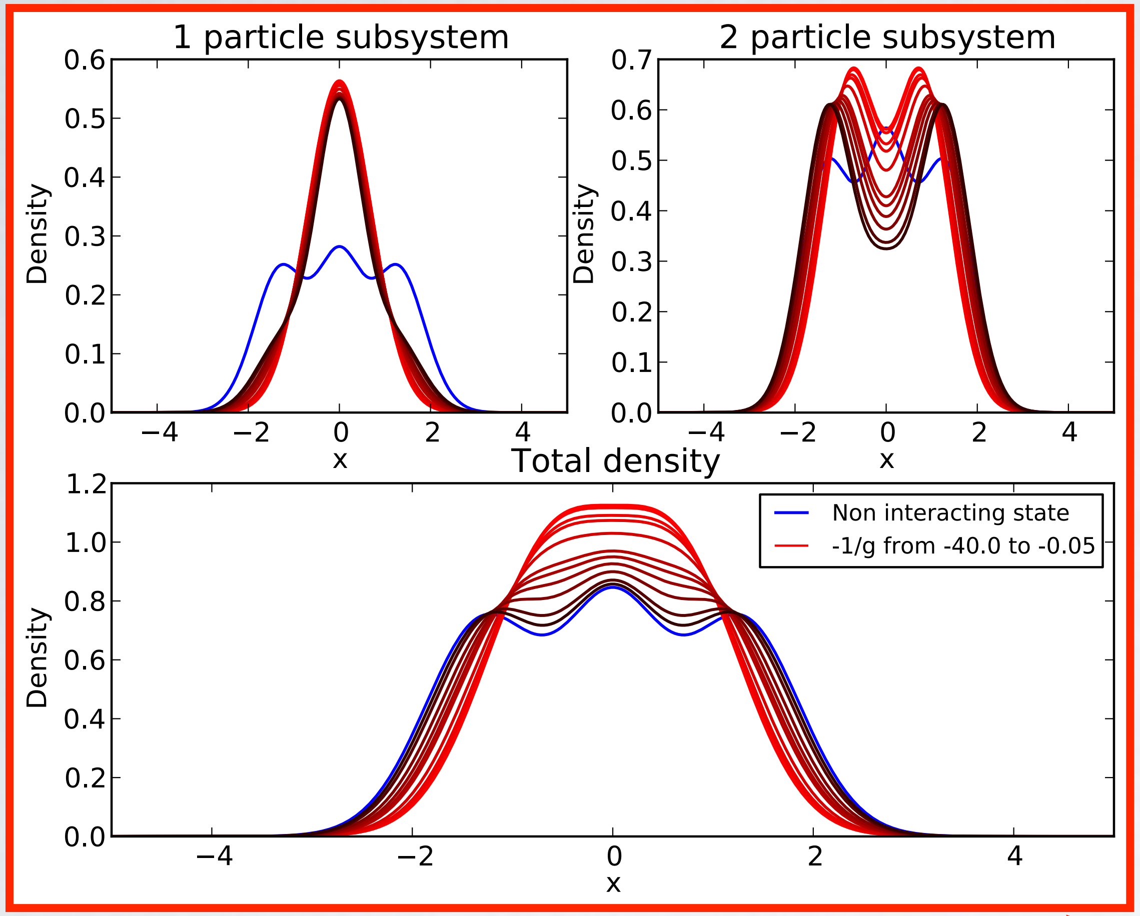


J. Lindgren et al, New J. Phys. 16 (2014) 063003.

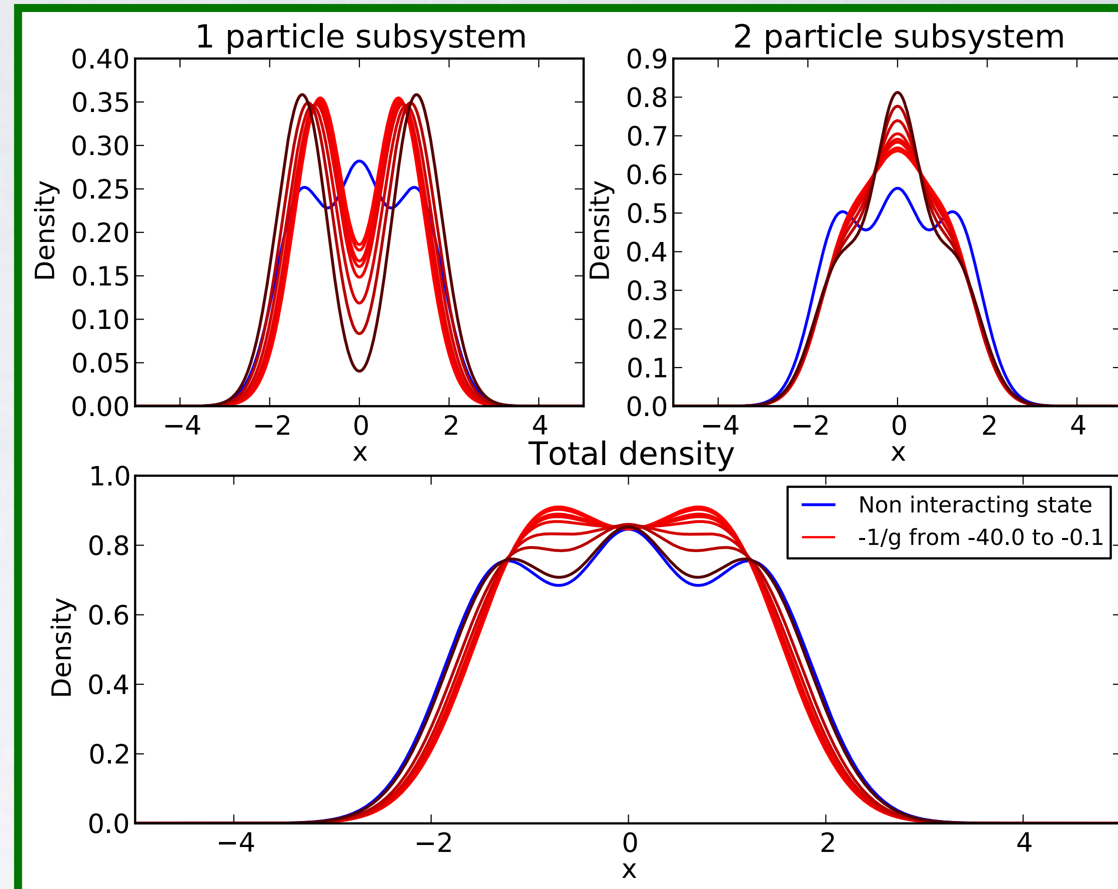
S.E. Gharashi and D. Blume, PRL 111 (2013) 045302

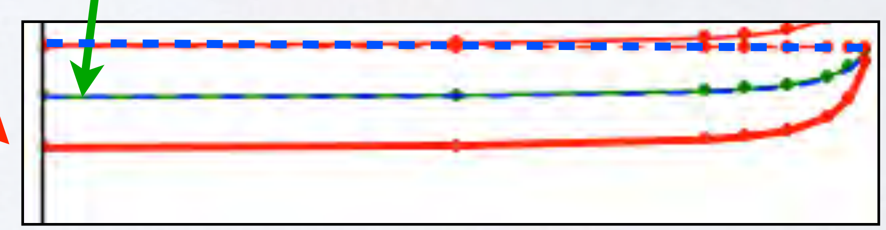
Densities

Ground state



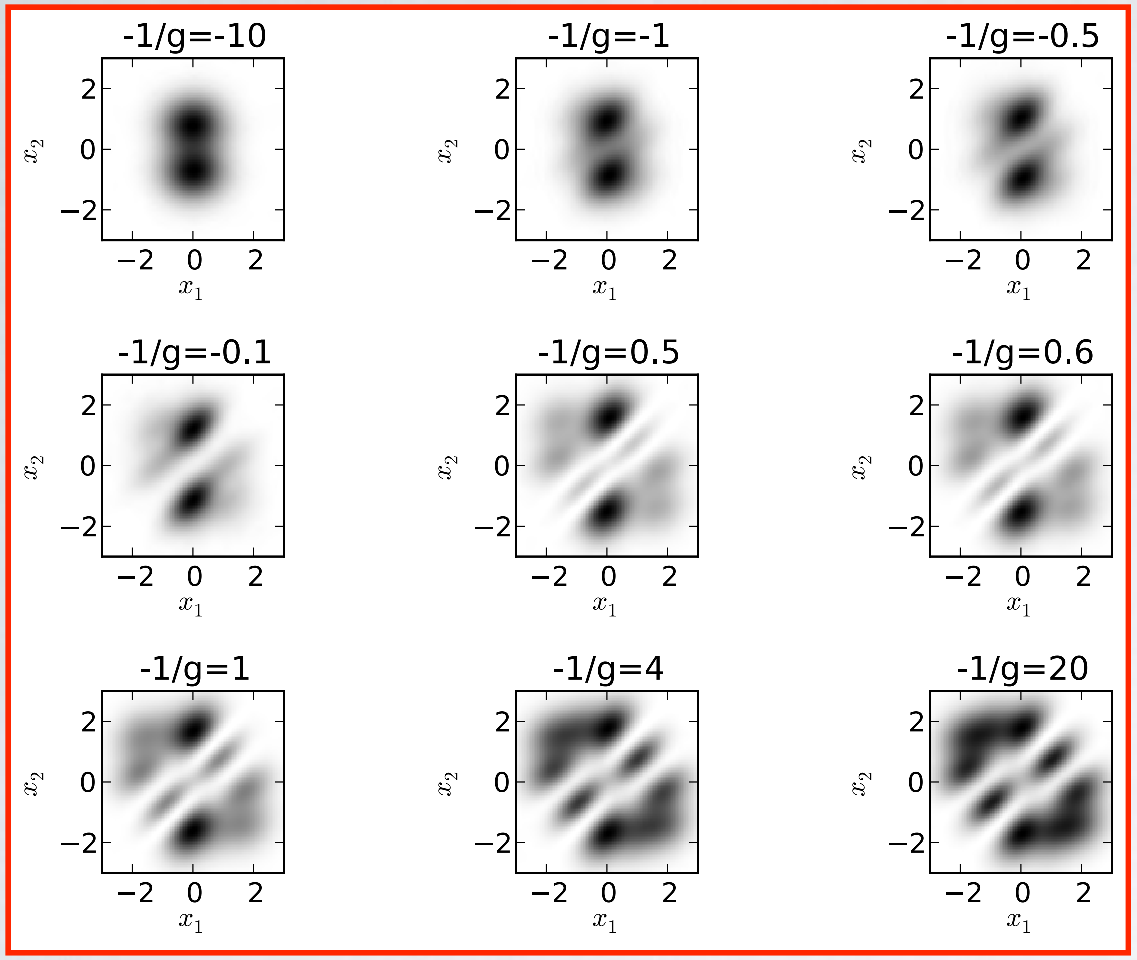
1st excited state

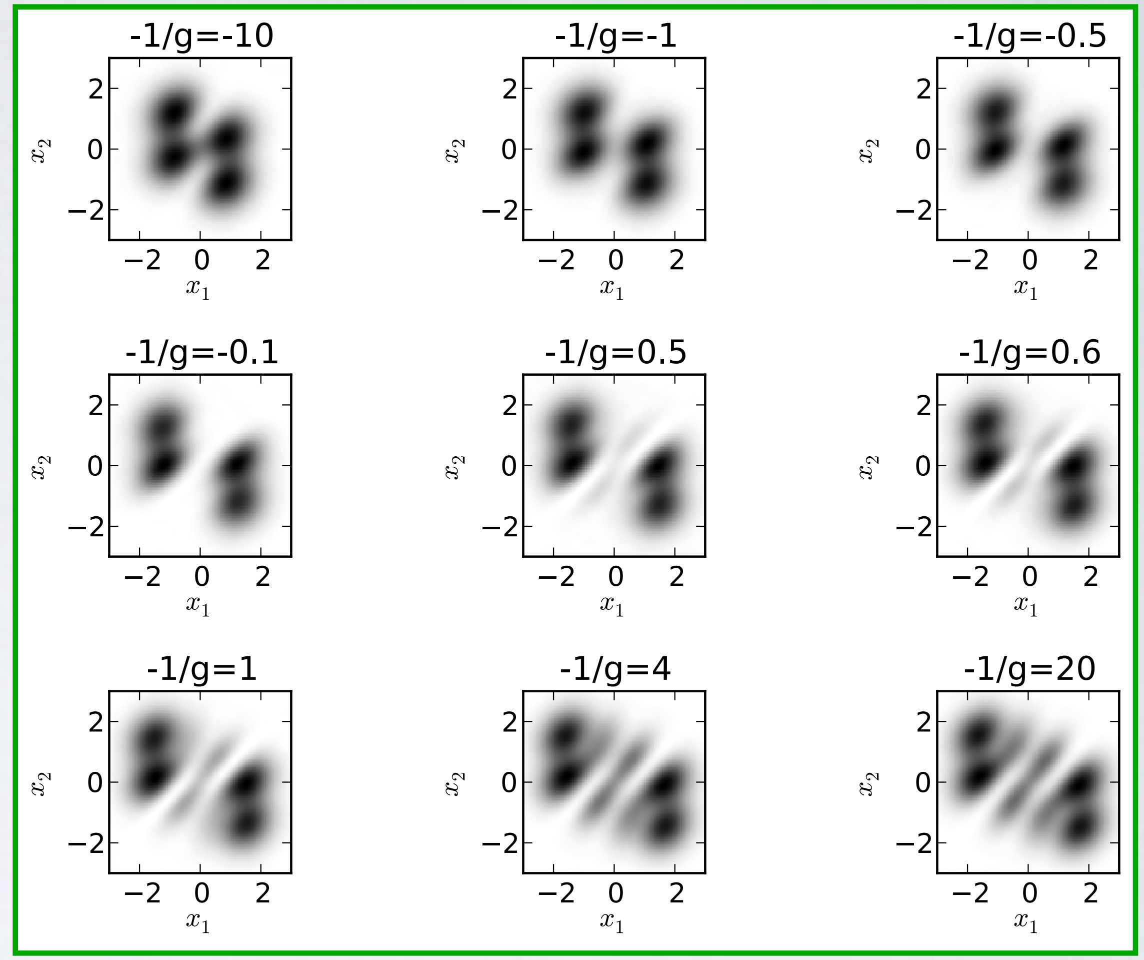




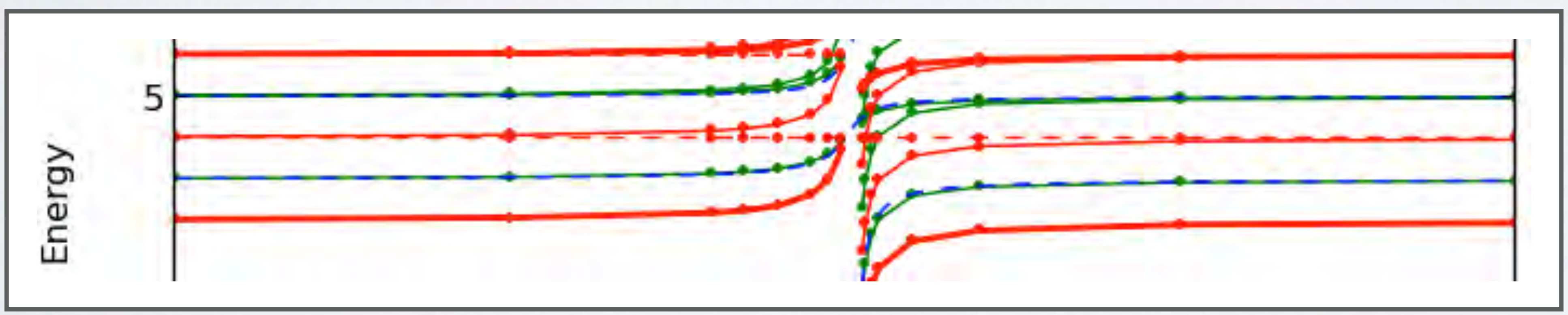
- Repulsive interaction
- From **weak** to **strong**
- **Non-interacting** state corresponds to the three-fermion ground state in HO trap
- At $-1/g \rightarrow 0$, these are degenerate

Densities and correlation densities





skip



For a 1 + N particle system the correlation density can be interpreted as the conditional density of the N particle system given that the single particle is measured at a certain position

From few to many

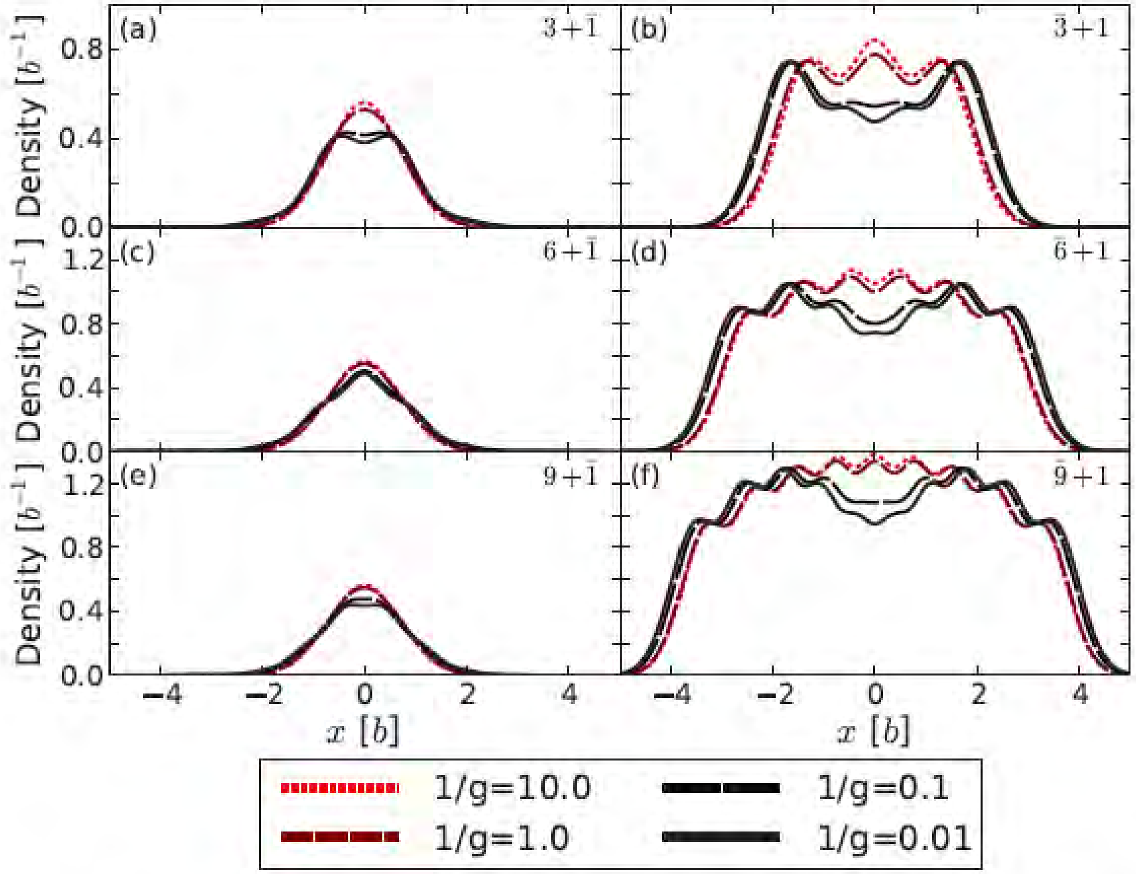


Figure 4. Spin-resolved densities for the 3+1, 6+1, and 9+1 systems, cf. Fig. 3. Panels (a), (c) and (e) show the distribution of the impurity particle, while panels (b), (d) and (f) show the majority density.

- ❖ Ground-state densities for impurity (left) and majority (right) particles.
- ❖ From **few** to **many**, the spin separation persists. Few-body precursor of Stoner ferromagnetism

Conclusion

Summary

- Adapted methods from nuclear physics to the study of ultracold few-atom systems in traps
    - Development of a tunneling theory for two particles using the Berggren basis (OQS). Can be extended to many-body systems.
    - Derivation of effective interaction for few-atom systems in HO trap ⇒ Can handle very large interaction strengths (usually very difficult for numerical methods.)
- Universal physics questions can be studied with these tunable quantum systems.