Cold atoms: three-body recombination into deep dimers: universality

D.V. Fedorov,
M. Mikkelsen, A.S. Jensen, N.T. Zinner

Aarhus Universitet

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Universality in low-energy scattering

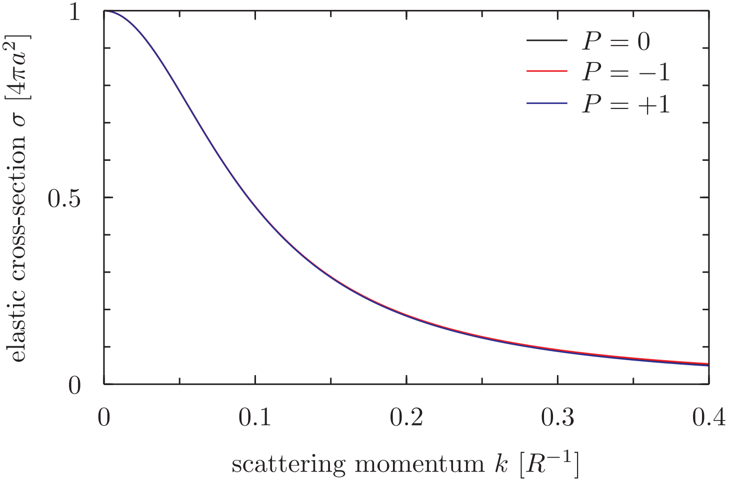

Figure: An illustration of elastic cross-section, σ, for potentials wih the same effective range R and scaterring length $a = 10R$, but with different shape parameter P.

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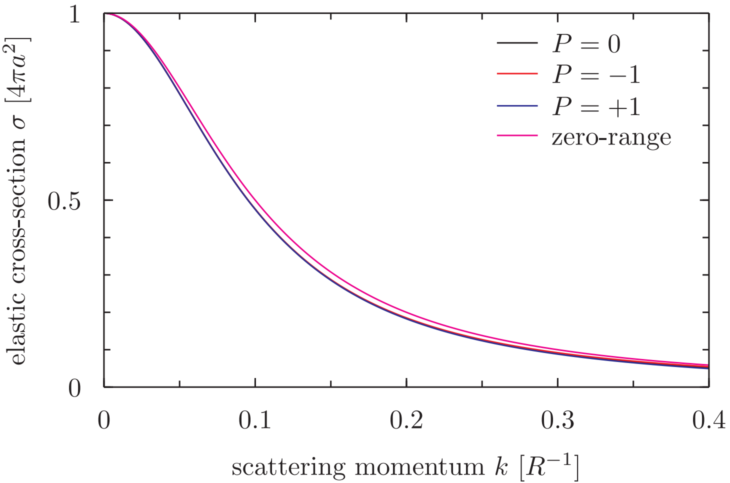

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$a \gg R$: universal dimer (weakly bound two-body state)

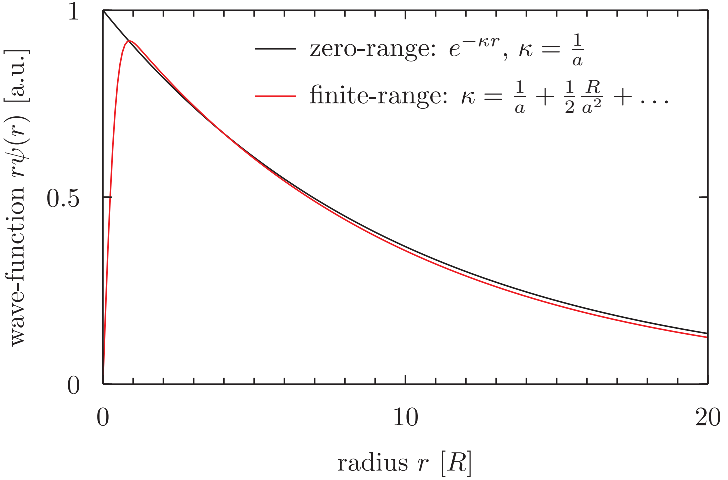

Figure: Wave-function of a dimer with $a = 10R$.

Three-body recombination into shallow dimer

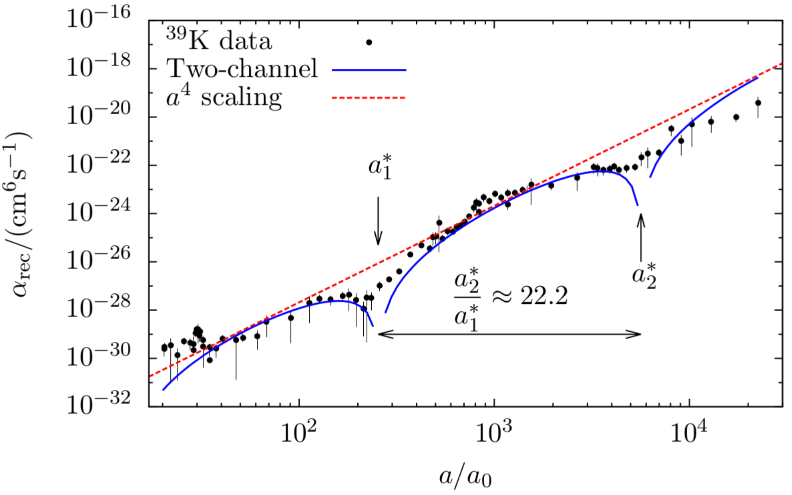

Figure: The recombination coefficient α_{rec} for ^{39}K as function of the scattering length a; experimental data from [Zaccanti 2009].

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$$\big(K_\rho + \epsilon_0\big) f_0 = E f_0$$

Hyperspheric adiabatic method

Slow variable: hyperradius:

$$m\rho^2 = \sum_i m_i (\mathbf{r}_i - \mathbf{R}_{c.m.})^2$$

$\epsilon_n(\rho)$ is universal in the limit $\rho \gg R$ and (semi)analytic.

Thomas/Efimov effect and logarithmic scaling

In the region $R \ll \rho \ll a$:

$$-u'' - \frac{\frac{1}{4} + s^2}{\rho^2} u = 0,$$

where $u = \rho^{5/2} f_0$.

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$$u = \rho^{1/2} \exp\left(\pm is \log\left(\frac{\rho}{R}\right)\right)$$

Recombination into shallow dimer

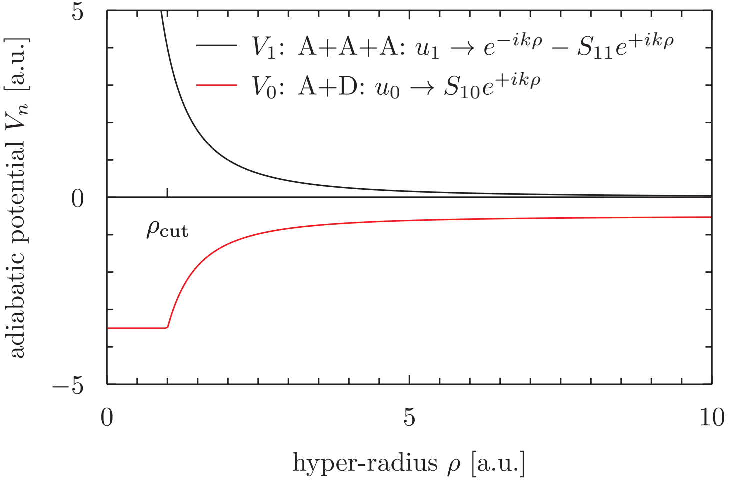

Figure: Schematic illustration of the two lowest adiabatic potentials for a three-body system with a shallow dimer. The recombination rate is propotional to $1 - |S_{11}|^2$.

Recombination into deep dimers

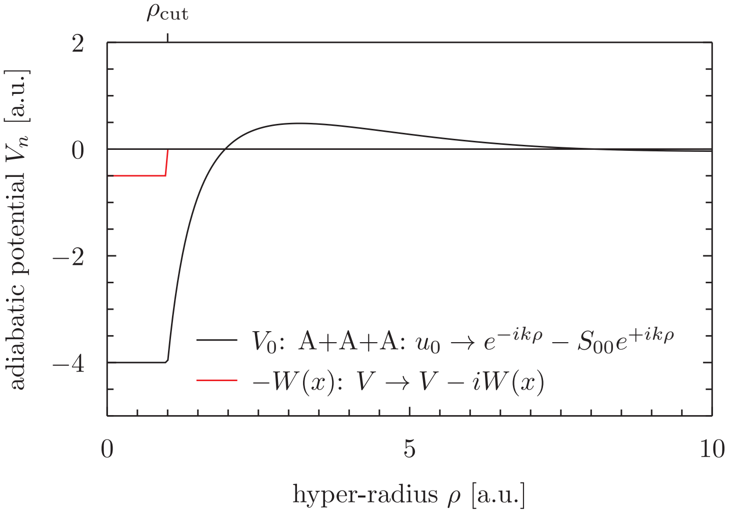

Figure: Schematic illustration of the lowest adiabatic potential for a three-body system without a shallow dimer. The recombination rate is propotional to $1 - |S_{00}|^2$.

Recombination into deep dimers: quasi-continuum

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Adding imaginary potential $-iW$ leads to appearance of widths,

$$E \to E - i\langle W \rangle = E - i\Gamma/2$$

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Recombination in a gas with two types of particles

$$\dot{N}_1 = -\frac{3}{3!}\frac{\Gamma_{111}}{\hbar}N_1^3 - \frac{2}{2}\frac{\Gamma_{112}}{\hbar}N_1^2 N_2 - \frac{1}{2}\frac{\Gamma_{122}}{\hbar}N_1 N_2^2$$

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Temperature dependence

$$\alpha_{\text{rec}}(a;\,T) = \frac{1}{2T^3}\int E^2 e^{-\frac{E}{T}}\,\alpha_{\text{rec}}(a,E)\,dE$$

$$\alpha_{\text{rec}}(a;\,T) = \frac{\sum_k e^{-\frac{E_k}{T}}\,\alpha_{\text{rec}}(a,E_k)}{\sum_k e^{-\frac{E_k}{T}}}$$

Cs-Cs-Cs

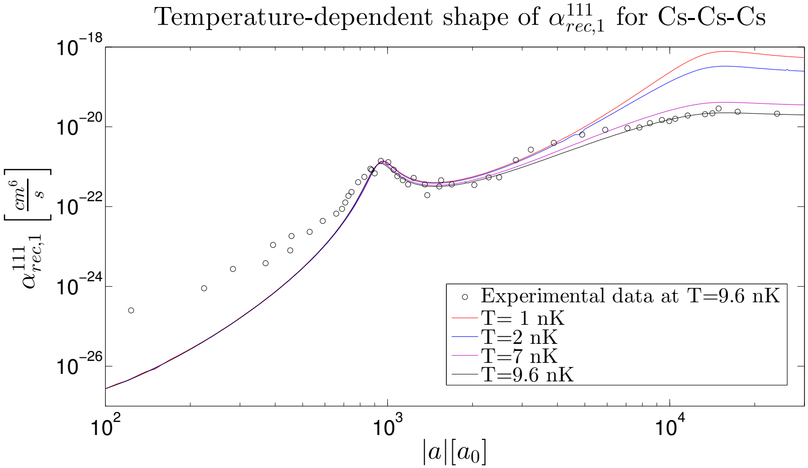

Figure: Recombination rate for Cs-Cs-Cs as function of scattering length. Experimental data from [Berninger 2011, Huang 2014].

Cs-Cs-Li (preliminary)

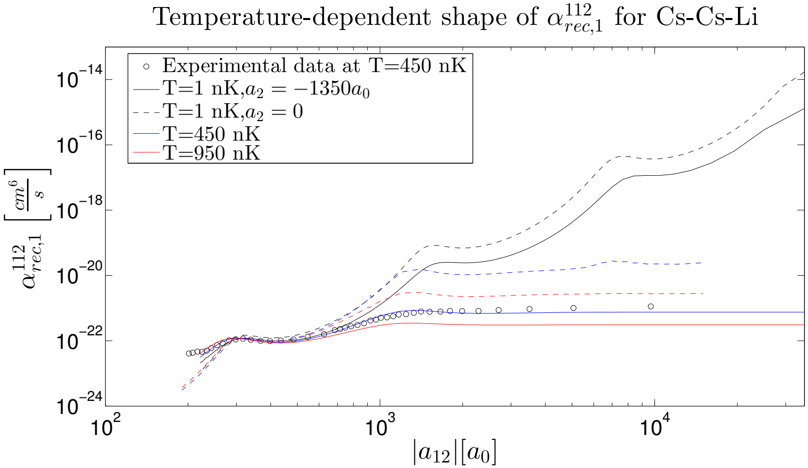

Figure: Recombination rate for Cs-Cs-Li as function of Cs-Li scattering length. Experimental data from [Pires 2014].