Stability and collapse of a trapped degenerate dipolar Bose or Fermi gas

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PLAN

- Degenerate Bose and Fermi gases
- Dipolar atoms
- Collapse above a critical dipolar strength in trapped degenerate Bose and Fermi systems
- Binary Bose-Fermi ^{164}Dy-^{161}Dy mixture
- Collapse dynamics of the trapped mixture
- Concluding remarks

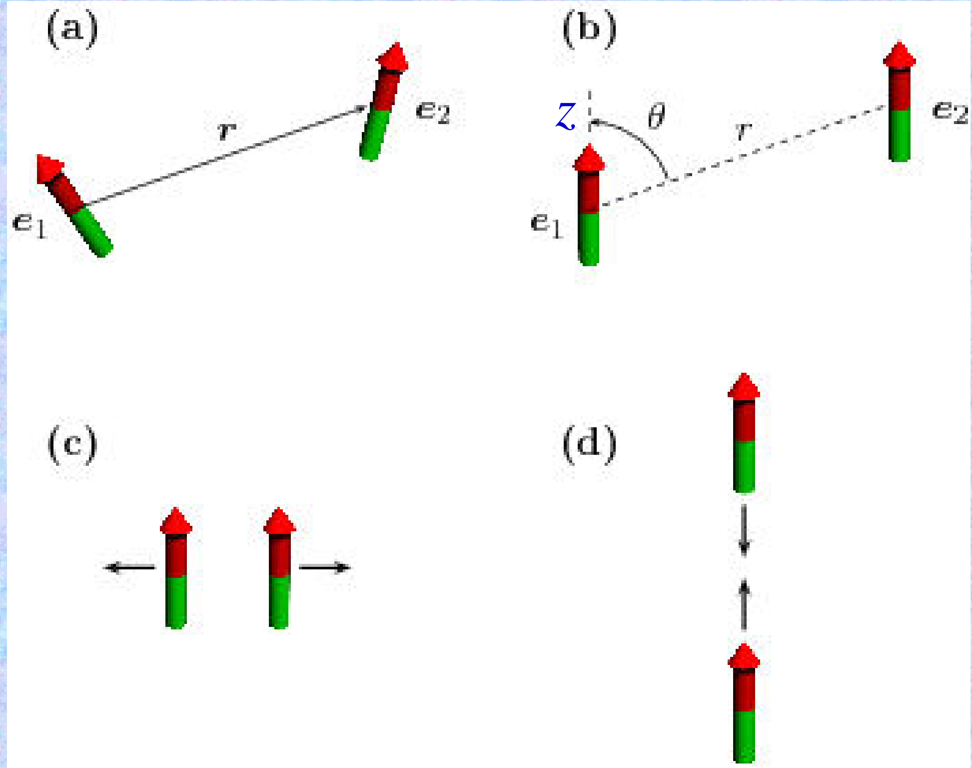
Stability of trapped nondipolar Bose and Fermi gases

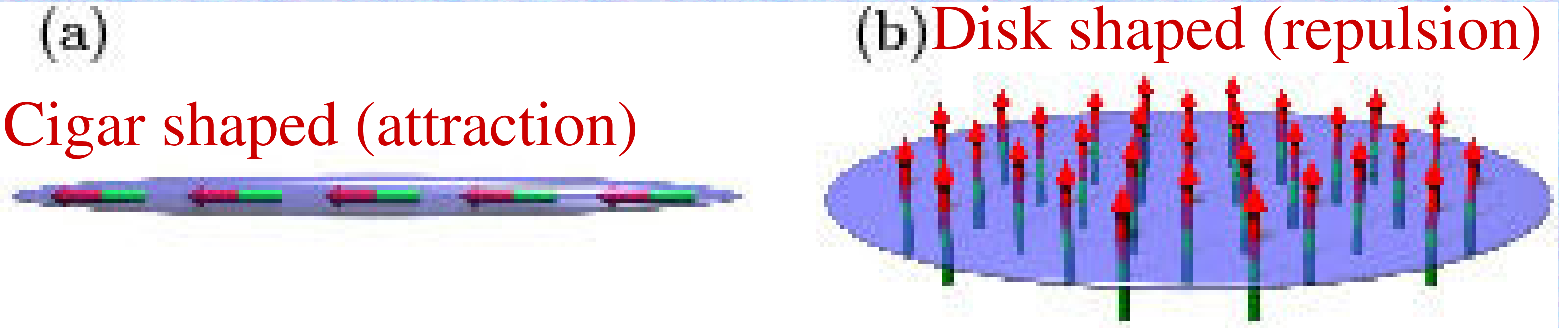
- The interaction is determined by the sign of scattering length *a*.
- Fermi and repulsive Bose gases are absolutely stable.
- Instability and collapse take place in attractive Bose gas above a critical strength.

Trapped degenerate Dipolar Bose and Fermi gas

- We consider always an axially-symmetric trap along z axis.
- The degenerate bosons and fermions are always spin-polarized along z axis.

Dipolar interaction

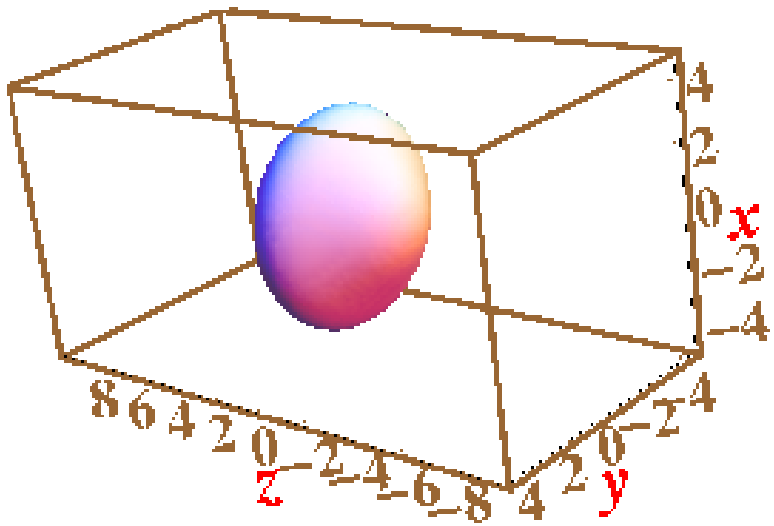


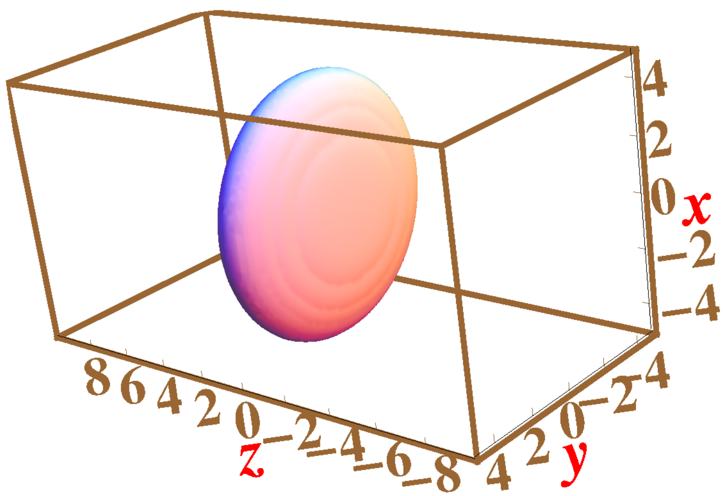


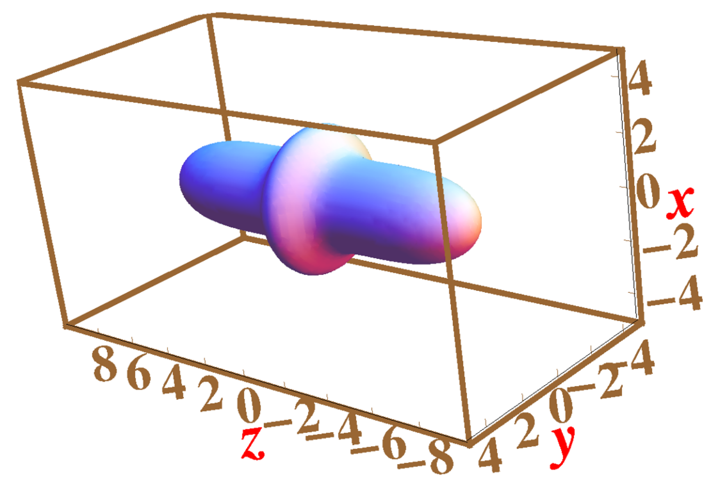
Trapped dipolar Bose or Fermi gas: cigar shape

- A cigar-shaped Bose or Fermi system placed along the polarization z direction will have added attraction among the aligned atomic dipoles.
- If the radial trap and contact repulsion are weak, with the increase of dipolar interaction the system becomes cigar shaped and with further increase of dipolar interaction the system collapses on the polarization axis.

Change of shape of BEC as the dipolar interaction is increased in a dipolar BEC



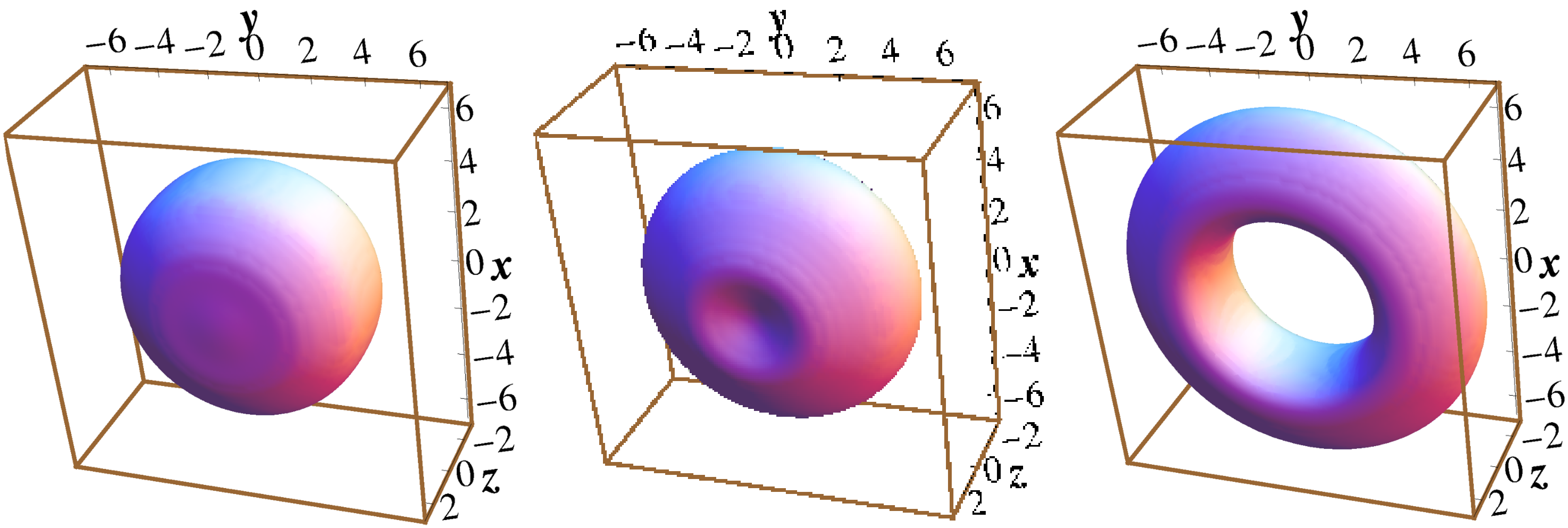




Trapped dipolar Bose or Fermi gas: disk shape

- As polarized dipoles in a plane perpendicular to the polarization direction repel, the dipolar interaction is repulsive in disk-shape and should enhance stability.
- However, for strong axial trap, with the increase of dipolar interaction the system passes through a biconcave shape to a ring and, with further increase, collapses on the perimeter of the ring

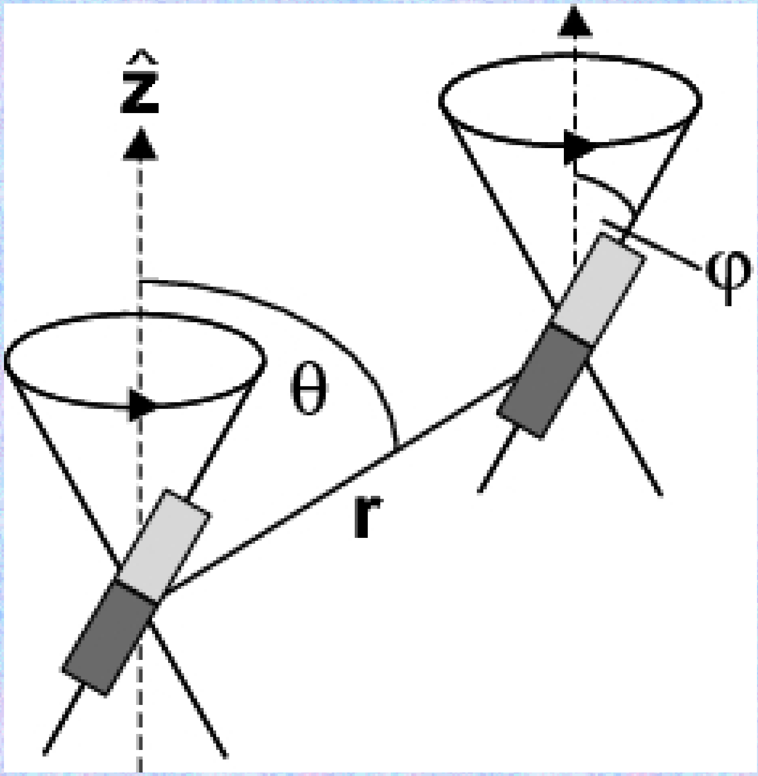
Change of shape of BEC as the dipolar interaction is increased in a dipolar BEC



Dipolar interaction

$$U_{dd}(\mathrm{r}) = 3a_{dd}\frac{(1-3\cos^2\theta)}{\mathrm{r}^3};a_{dd} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

Strongly anisotropic
Magnetic/Electric Dipole-Dipole Interactions

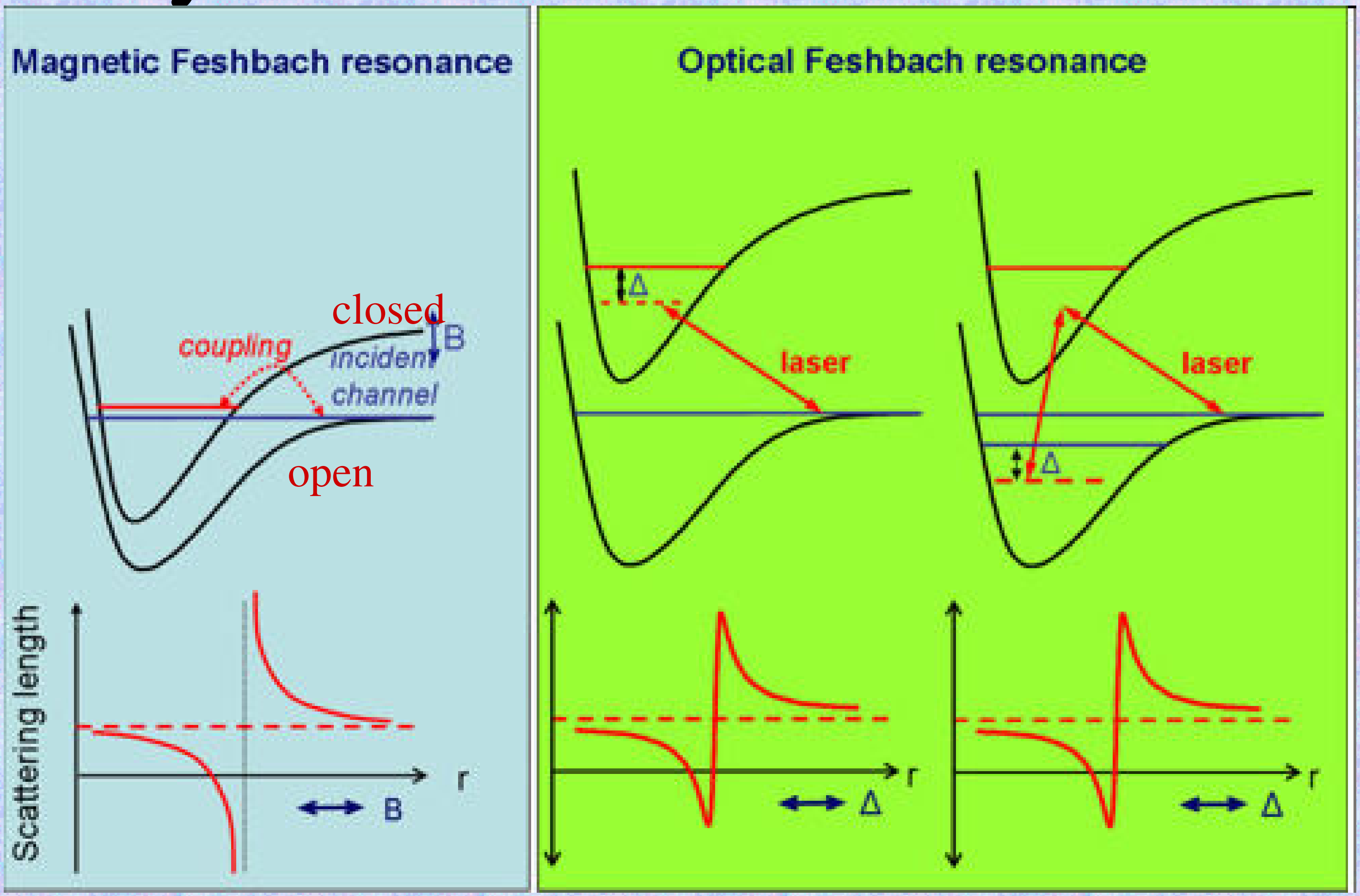


μ= magnetic moment
μ_0=permeability of vacuum

Dipolar interaction can be tuned

to a smaller value by a rotating orienting field.

Tuning of short-range interaction by a Feshbach resonance



BECs of ^{52}Cr (Griesmaier/Pfau 2005) and ^{164}Dy and ^{168,161}Er (Lu/Lev, 2011)

Dipole moment $\mu(^{52}\mathrm{Cr}) = 6\mu_B$,	$a_{dd} = 15\ a_0$
$\mu(^{164}\mathrm{Dy}) = 10\mu_B$	$a_{dd} = 133\ a_0$
$\mu(^{161}\mathrm{Dy}) = 10\mu_B$	$a_{dd} = 131\ a_0$
$\mu(^{168}\mathrm{Er}) = 7\mu_B$	$a_{dd} = 67\ a_0$
$\mu(^{87}\mathrm{Rb}) = 1\mu_B$	$a_{dd} = 0.69\ a_0$

μ_B = Bohr Magneton

a_0 = Bohr radius

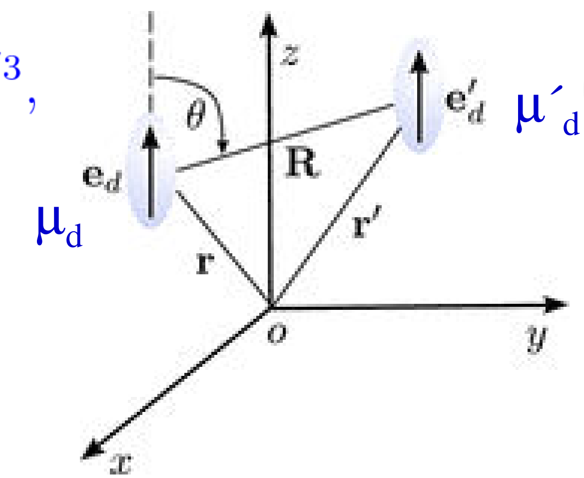
Generalized Gross-Pitaevskii Equation (mean-field equation) for the BEC

$$i\hbar\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \Big[-\frac{\hbar^2\nabla^2}{2m} + \frac{m\omega^2}{2}(\lambda^{-2/3}\rho^2 + \lambda^{4/3}z^2)$$

$$+\frac{4\pi\hbar^2 aN}{m}|\phi(\mathbf{r},t)|^2 + N\frac{\mu_0\mu_d^2}{4\pi}\int V_{\mathrm{dd}}(\mathbf{r}-\mathbf{r}')|\phi(\mathbf{r}',t)|^2 d\mathbf{r}'\Big]\phi(\mathbf{r},t), \qquad (1)$$

$$V_{\mathrm{dd}}(\mathbf{R}) = \frac{(1-3\cos^2\theta)}{R^3}, \quad \mathbf{R} = \mathbf{r}-\mathbf{r}', \qquad \theta = \widehat{z\mathbf{R}} \qquad (2)$$

$$\int |\phi(\mathbf{r})|^2 d\mathbf{r} = 1, \quad \lambda = \omega_z/\omega_\rho, \quad \omega = (\omega_\rho^2\omega_z)^{1/3}, \qquad (3)$$



In dimensionless form

Introduce scale

$$a_{dd} = \frac{m\mu_0\mu_d^2}{12\pi\hbar^2}$$

to get

$$i\hbar\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \Big[-\frac{\nabla^2}{2} + \frac{1}{2}(\lambda^{-2/3}\rho^2 + \lambda^{4/3}z^2) + 4\pi a N|\phi(\mathbf{r},t)|^2$$
$$+ 3a_{\rm dd}N\int V_{\rm dd}(\mathbf{r}-\mathbf{r}')|\phi(\mathbf{r}',t)|^2 d\mathbf{r}'\Big]\phi(\mathbf{r},t).$$

energy in $\hbar\omega$, length in $l_0 \equiv \sqrt{\hbar/m\omega}$, time in ω^{-1}.

Hydrodynamical mean-field Equation for degenerate and Superfluid Fermi gas: Collisional Hydrodynamics

Based on a hydrodynamic description of the trapped degenerate dipolar gas of N single-component fermions each of mass m, we derived the following mean-field equation for this system

$$\mu\sqrt{n(\mathbf{r})} = \Big[-\frac{\hbar^2\nabla^2}{8m} + \frac{m\omega^2}{2}(\lambda^{-2/3}\rho^2 + \lambda^{4/3}z^2) + \frac{\hbar^2}{2m}[6\pi^2 N n(\mathbf{r})]^{2/3} + N\frac{\mu_0\mu_d^2}{4\pi}\int V_{\mathrm{dd}}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}')d\mathbf{r}'\Big]\sqrt{n(\mathbf{r})},$$

$n(\mathbf{r})$ = density

and written in dimensionless units

$$\mu\sqrt{n(\mathbf{r})} = \Big[-\frac{\nabla^2}{8} + \frac{1}{2}(\lambda^{-2/3}\rho^2 + \lambda^{4/3}z^2) + \frac{1}{2}[6\pi^2 N n(\mathbf{r})]^{2/3} + 3a_{dd}N\int V_{\mathrm{dd}}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}')d\mathbf{r}'\Big]\sqrt{n(\mathbf{r})},$$

Local Density Approximation: use scaling length $\bar{l} \equiv N^{1/6} l_0$, $l_0 = \sqrt{\hbar/m\omega}$, scaled quantities $\bar{\rho} = \rho/\bar{l}$, $\bar{z} = z/\bar{l}$, $\bar{\mathbf{r}} = \mathbf{r}/\bar{l}$, $\bar{R} = R/\bar{l}$, $\varepsilon_{\rm dd} \equiv 3N^{1/6}(a_{\rm dd}/l_0) = 3N^{1/3} a_{\rm dd}/\bar{l}$, $\bar{\mu} = \mu/(N^{1/3}\hbar\omega)$, $\bar{n}(\mathbf{r}) = n(\mathbf{r})\bar{l}^3$,

$$\bar{\mu} = \frac{1}{2}(\lambda^{-2/3}\bar{\rho}^2 + \lambda^{4/3}\bar{z}^2) + \frac{1}{2}[6\pi^2\bar{n}(\mathbf{r})]^{2/3}$$
$$+ \varepsilon_{\rm dd}\int \frac{1 - 3\cos^2\theta}{\bar{R}^3}\bar{n}(\mathbf{r}')d\mathbf{r}' - \beta\varepsilon_{\rm dd}^2[\bar{n}(\mathbf{r})]^{4/3},$$

with $\int d\bar{\mathbf{r}}\bar{n}(\mathbf{r}) = 1$, where $\beta = 0$ for Hartree, $\beta = 28(6\pi^2)^{1/3}/135$ for Hartree-Fock (HF), and $\beta = 28(6\pi^2)^{1/3}/135 + 14/9$ for HF+Brueckner-Goldstone correlation. This N-independent scaled equation gives both the chemical potential and the density.

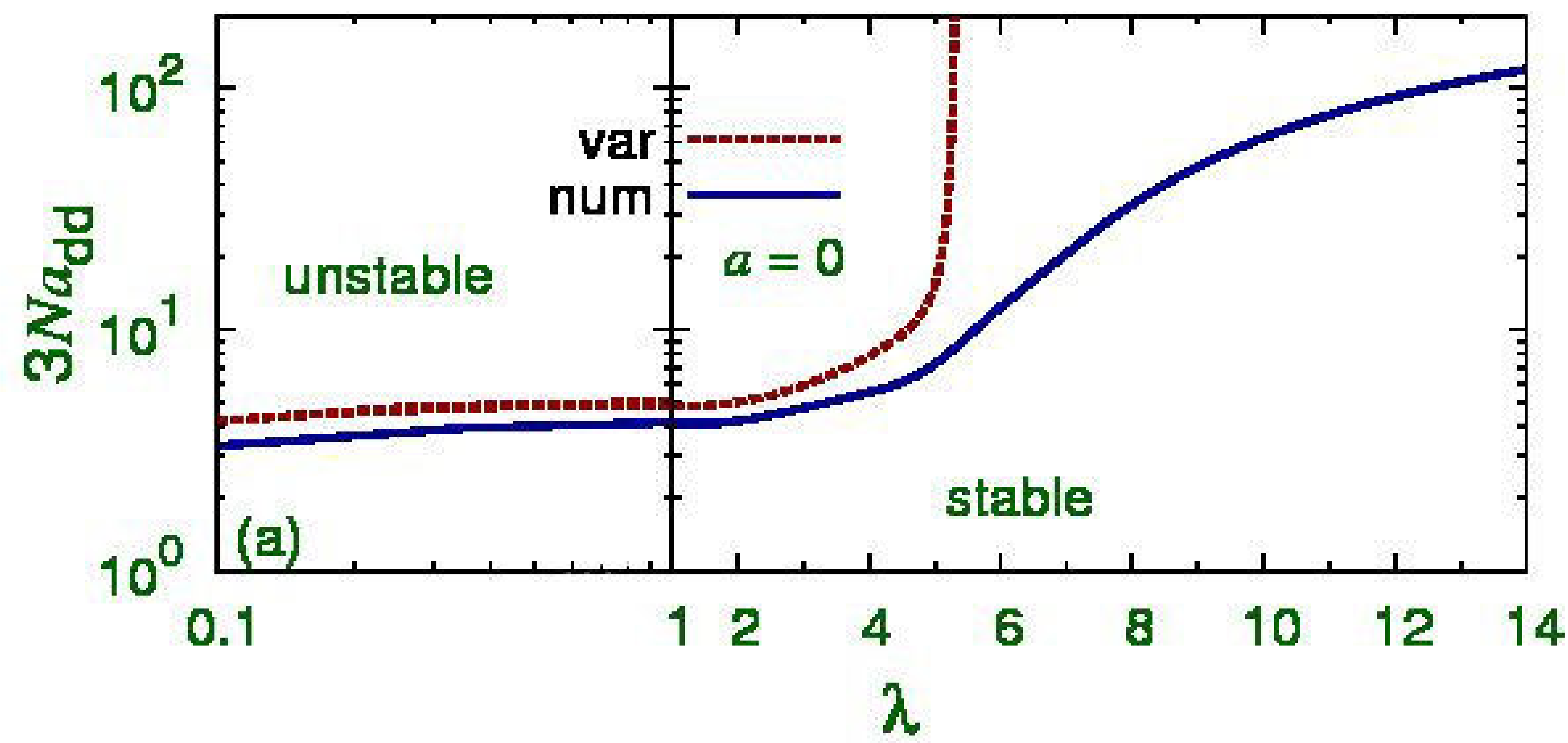
Numerical calculation

- The three-dimensional GP equation is solved numerically by split-step Crank-Nicolson method without further approximation.
- Fortran and C programs for nondipolar GP Eq.

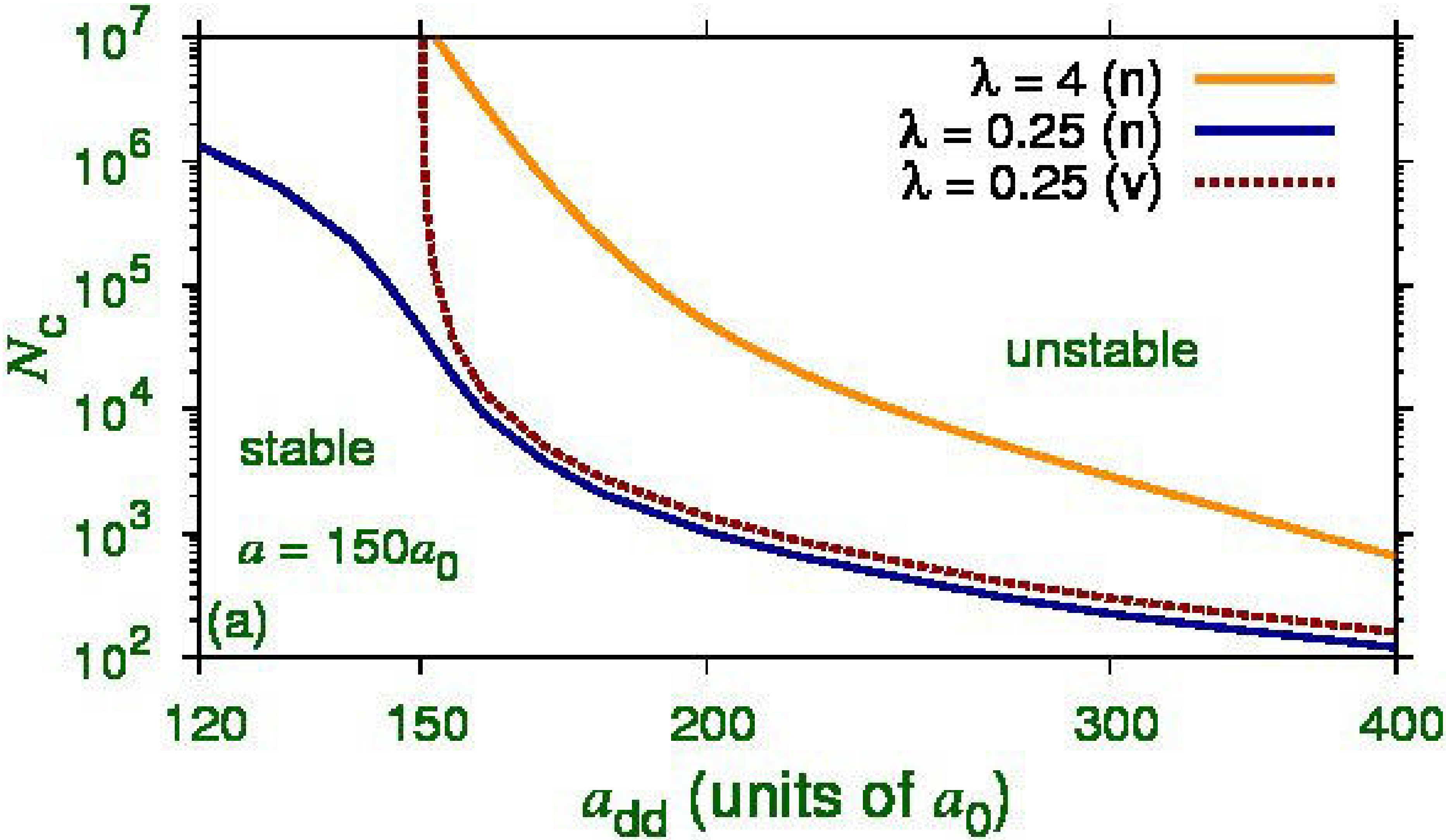
  Comput. Phys. Commun. 180 (2009) 1888
  Comput. Phys. Commun. 183 (2012) 2021
- Fortran and C programs for dipolar GP Eq.,

  Kumar, Young-S, Vudragovic, Balaz, Muruganandam, and Adhikari, Submitted to Comput. Phys. Commun.

Stability phase plot in a dipolar Bose gas



Stability phase plot (dipolar bosons)

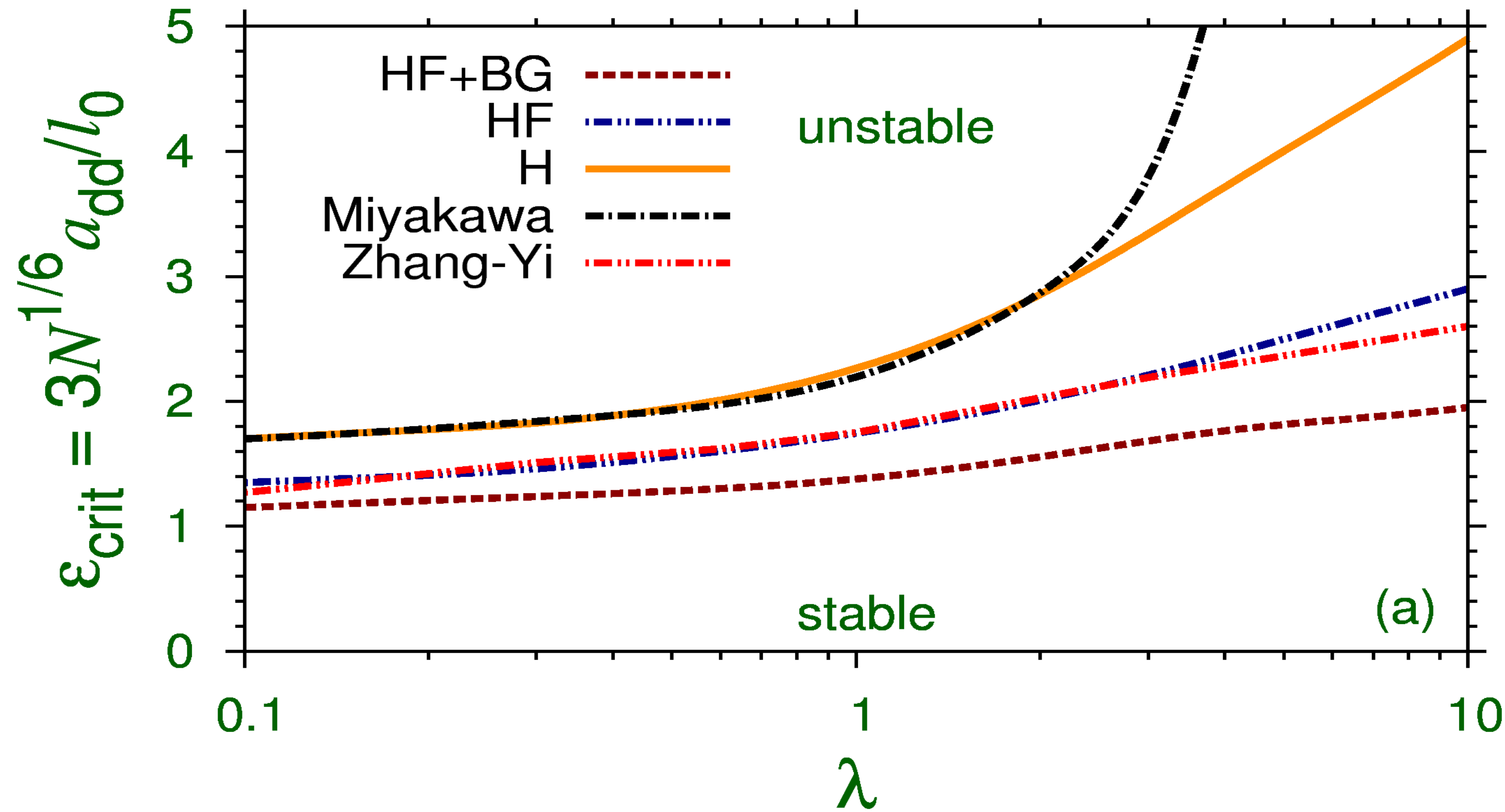


Stability phase plot for fermions

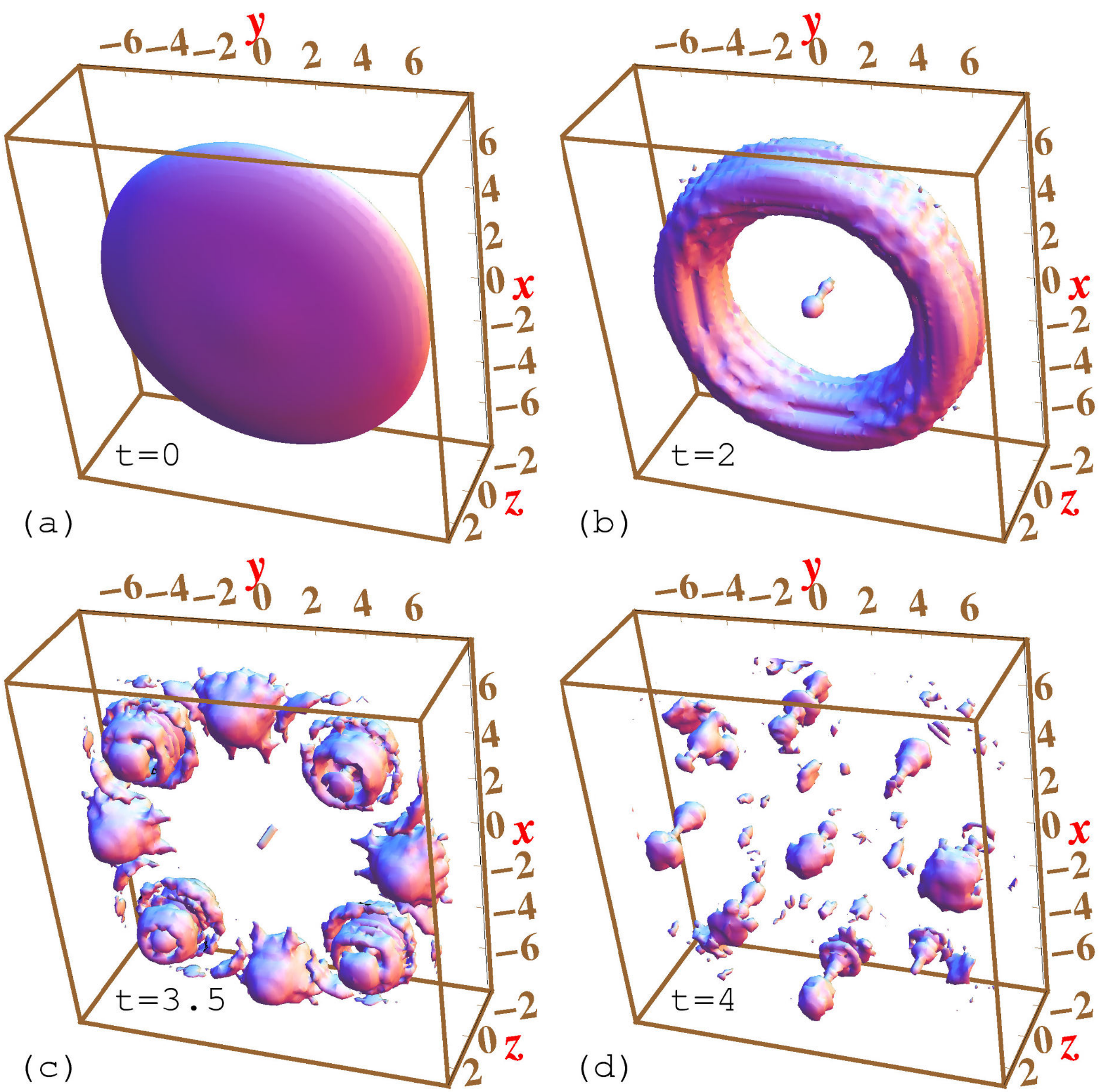
H (Hartree), HF (Hartree-Fock), BG (Brueckner-Goldstone)

Miyakawa et al, 2008 Phys. Rev. A 77 061603 (2008) HF

Zhang and Yi, Phys. Rev. A 80 053614 ()2009) HF



Collapse dynamics in a disk-shaped dipolar Bose gas



13000 ^{164}Dy atoms

$\omega = 2\pi X 60$ Hz, $\lambda = 8$,

a_{dd} changed from $16a_0$ to $40a_0$ at t=0

a = 0

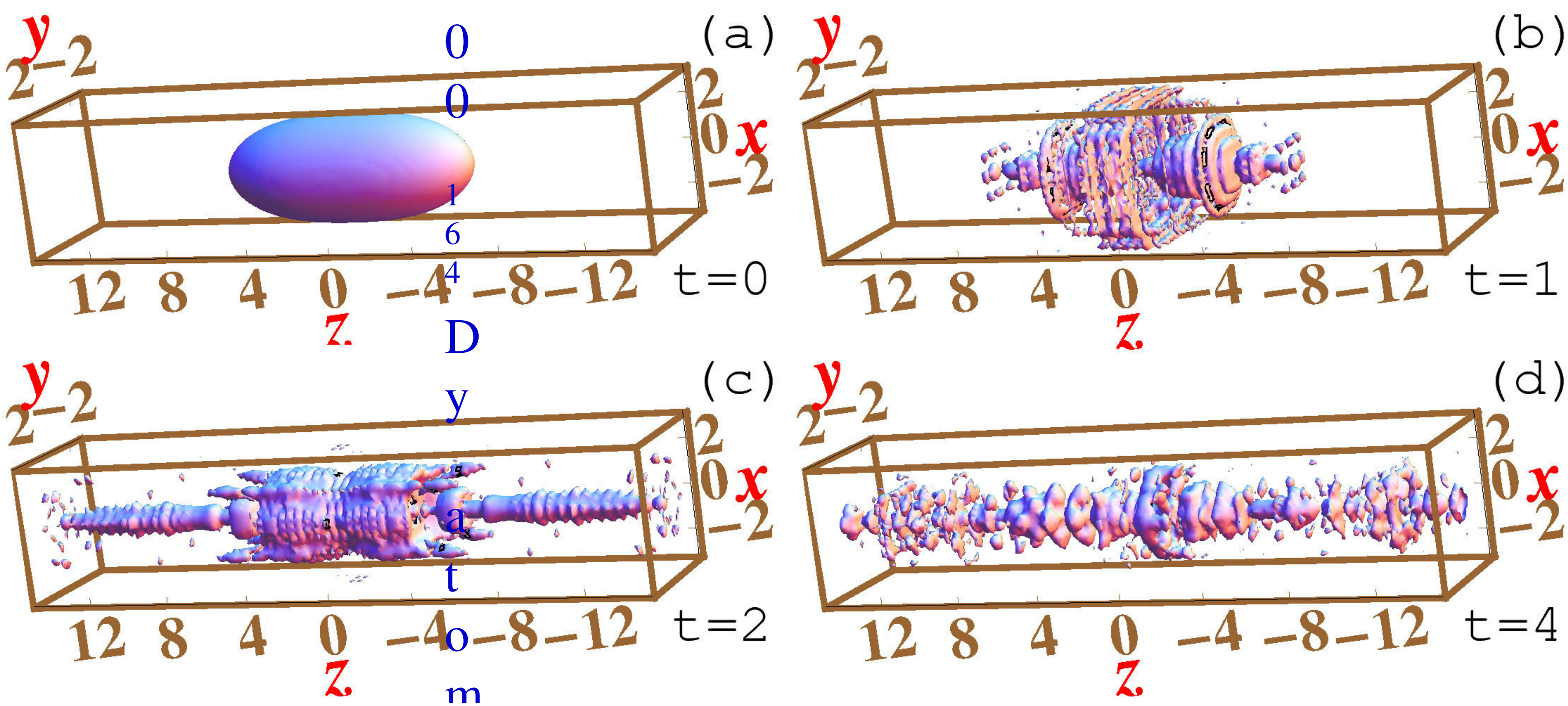
Collapse dynamics in a cigar-shaped dipolar Bose gas

1300 ^{164}Dy atoms

$\omega = 2\pi X60$ Hz, $\lambda = 1/8$, $a = 0$

a_{dd} changed from $16a_0$ to $48a_0$ at $t=0$

13000 ^{164}Dy atom



Binary Bose-Fermi ^{164}Dy-^{161}Dy mixture

$$i\hbar\frac{\partial\Phi_b(\mathbf{r},t)}{\partial t} = \Bigg[-\frac{\hbar^2}{2m_b}\nabla^2 + \frac{1}{2}m_b\omega_b^2\left(\frac{\rho^2}{\lambda_b^{2/3}} + z^2\lambda_b^{4/3}\right) + \frac{4\hbar^2\pi a_b}{m_b}|\Phi_b|^2 + \frac{2\pi\hbar^2}{m_R}a_{bf}|\Phi_f|^2$$
$$+ \frac{\mu_0\mu_b^2}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_b(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\mu_0\mu_b\mu_f}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_f(\mathbf{r}')|^2 d\mathbf{r}'\Bigg]\Phi_b(\mathbf{r},t), \tag{1}$$

$$i\hbar\frac{\partial\Phi_f(\mathbf{r},t)}{\partial t} = \Bigg[-\frac{\hbar^2}{8m_f}\nabla^2 + \frac{1}{2}m_f\omega_f^2\left(\frac{\rho^2}{\lambda_f^{2/3}} + z^2\lambda_f^{4/3}\right) + \frac{2\pi\hbar^2}{m_R}a_{bf}|\Phi_b|^2$$
$$+ \frac{\mu_0\mu_b\mu_f}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_b(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\mu_0\mu_f^2}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_f(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\hbar^2(6\pi^2 N_f|\Phi_f|^2)^{2/3}}{2m_f}\Bigg]\Phi_f(\mathbf{r},t), \tag{2}$$

$$\int|\Phi_i(\mathbf{r})|^2 d\mathbf{r} = N_i, i = b, f, \qquad \lambda_i = \frac{\omega_{zi}}{\omega_{\rho i}}, \quad \omega_i = (\omega_{zi}\omega_{\rho i}^2)^{1/3}, \quad m_R = \frac{m_b m_f}{m_b + m_f}, \tag{3}$$

$$a^{(b)}{}_{dd} = \frac{\mu_0 {\mu_b}^2 m_b}{12\pi\hbar^2}; a^{(f)}{}_{dd} = \frac{\mu_0 {\mu_f}^2 m_f}{12\pi\hbar^2}; a^{(bf)}{}_{dd} = \frac{\mu_0\mu_b\mu_f m_R}{6\pi\hbar^2}$$

Dimensionless form

$$
i\frac{\partial \phi_b(\mathbf{r},t)}{\partial t} = \Big[-\frac{\nabla^2}{2} + \frac{1}{2}\Big(\frac{\rho^2}{\lambda_b^{2/3}} + \lambda_b^{4/3} z^2\Big) + g_{bf}|\phi_f|^2 + g_{dd}^{(b)} \int V_{dd}(\mathbf{R})|\phi_b(\mathbf{r}',t)|^2 d\mathbf{r}' + g_b|\phi_b|^2
$$

$$
+ g_{dd}^{(bf)} \int V_{dd}(\mathbf{R})|\phi_f(\mathbf{r}',t)|^2 d\mathbf{r}' - ik_3^{(bb)} N_b^2 |\phi_b|^4 - ik_3^{(bf)} N_b N_f |\phi_b|^2 |\phi_f|^2 \Big] \phi_b(\mathbf{r},t), \tag{8}
$$

$$
i\frac{\partial \phi_f(\mathbf{r},t)}{\partial t} = \Big[- m_{bf}\frac{\nabla^2}{8} + \frac{m_w}{2}\Big(\frac{\rho^2}{\lambda_f^{2/3}} + \lambda_f^{4/3} z^2\Big) + g_{dd}^{(f)} \int V_{dd}(\mathbf{R})|\phi_f(\mathbf{r}',t)|^2 d\mathbf{r}'
$$

$$
+ \frac{m_{bf}}{2}(6\pi^2 N_f |\phi_f|^2)^{2/3} + g_{fb}|\phi_b|^2 + g_{dd}^{(fb)} \int V_{dd}(\mathbf{R})|\phi_b(\mathbf{r}',t)|^2 d\mathbf{r}' - ik_3^{(bf)} N_b^2 |\phi_b|^4 \Big] \phi_f(\mathbf{r},t), \tag{9}
$$

$$
\int |\phi_i(\mathbf{r})|^2 d\mathbf{r} = 1, \quad m_{bf} = \frac{m_b}{m_f}, \quad m_w = \frac{\omega_f^2}{(m_{bf}\omega_b^2)}, \quad g_b = 4\pi a_b N_b, \quad g_{dd}^{(b)} = 3N_b a_{dd}^{(b)},
$$

$$
g_{bf} = \frac{2\pi m_b a_{bf} N_f}{m_R}, \quad g_{fb} = \frac{2\pi m_b a_{bf} N_b}{m_R}, \quad g_{dd}^{(f)} = 3N_f a_{dd}^{(f)} m_{bf}, \quad g_{dd}^{(bf)} = 3N_f a_{dd}^{(bf)} \frac{m_b}{2m_R},
$$

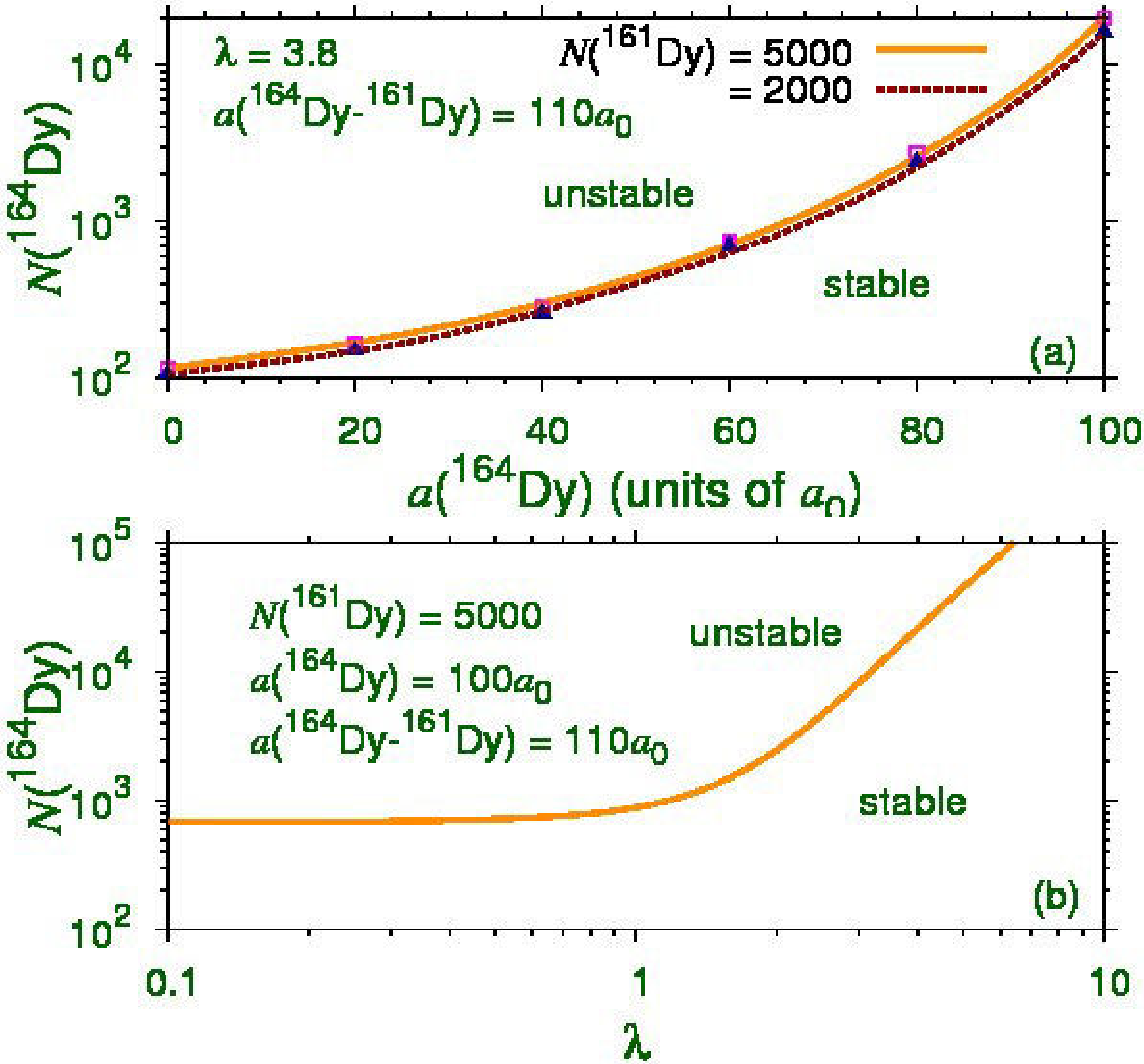
$$
g_{dd}^{(fb)} = 3N_b a_{dd}^{(bf)} \frac{m_b}{2m_R}, \quad l_0 = \sqrt{\frac{\hbar}{\omega_b m_b}} \quad t_0 = \omega_b^{-1}, \quad k_3^{bf} = \frac{K_3^{bf}}{\omega_b l_0^6}, \quad k_3^{bb} = \frac{K_3^{bb}}{\omega_b l_0^6}.
$$

Parameters

The binary dipolar ^{164}Dy-^{161}Dy boson-fermion mixture
For ^{161}Dy, $f_f = \{180, 200, 720\}$ Hz.
Mean frequency $\omega_f = 2\pi \times 296$ Hz and $\lambda_f = 3.795$
For ^{164}Dy, $f_b = \{195, 205, 760\}$ Hz.
Mean frequency $\omega_b = 2\pi \times 312$ Hz and $\lambda_b = 3.801$
Oscillator lengths $l_b = l_0 = 0.443$ μm, $l_f = 0.459$ μm,
From experiments of B. L. Lev (Stanford University).
$a_{dd}(^{164}\text{Dy}) = 133a_0$, $a_{dd}(^{161}\text{Dy}) = 131a_0$, $a_{dd}(^{164}\text{Dy-}^{161}\text{Dy}) = 132a_0$
Use split-step Crank-Nicolson approach with
$\Delta x = \Delta y = \delta z = 0.1 - 0.2, \quad \delta t = 0.001 - 0.003$

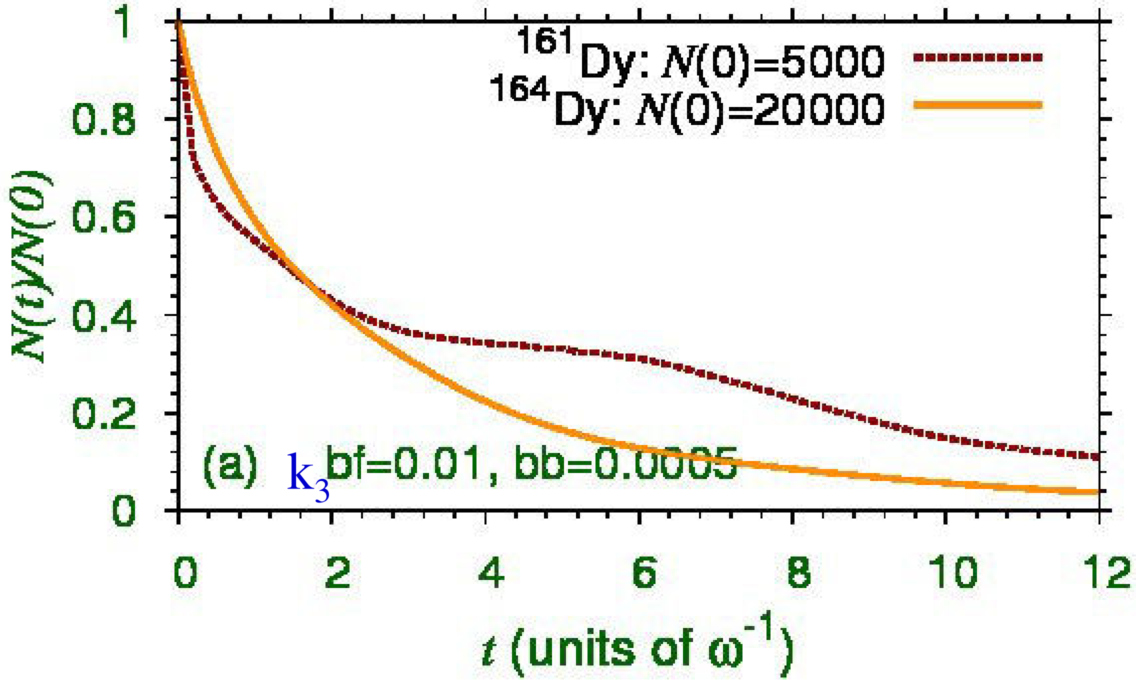
$a(^{164}\text{Dy}) = 100a_0$

$K_3^{(bb)} = 7.5\text{X}10^{-27}$ cm^6/s; $K_3^{(bf)} = 1.5\text{X}10^{-25}$ cm^6/s;


λ = 3.8
a(164Dy-161Dy) = 110a0
N(161Dy) = 5000
= 2000
unstable
stable
(a)
N(164Dy)
a(164Dy) (units of a0)
N(161Dy) = 5000
a(164Dy) = 100a0
a(164Dy-161Dy) = 110a0
unstable
stable
(b)
λ

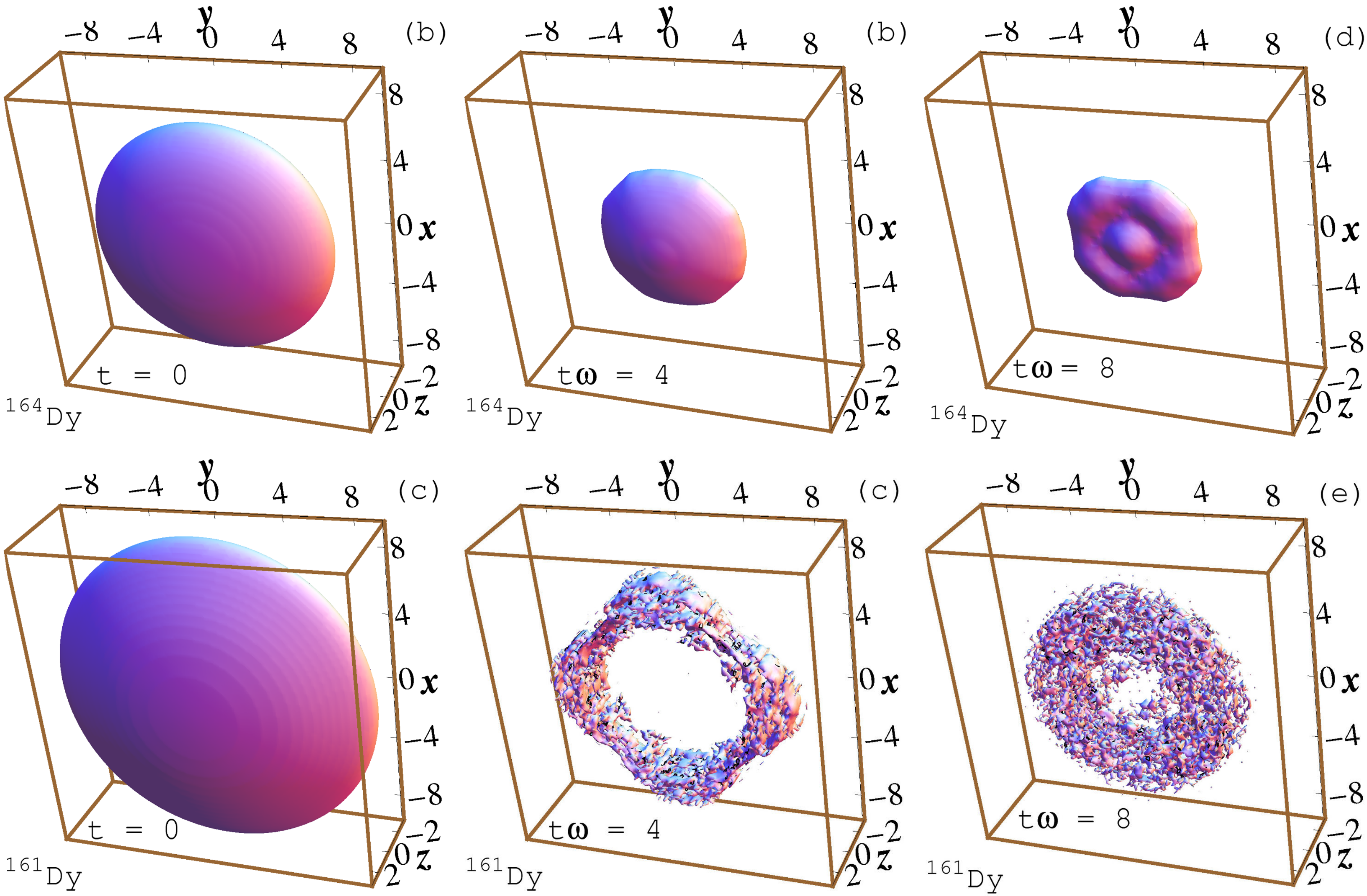
Collapse started by jumping $a_{12} = 100a_0$ to $-100a_0$

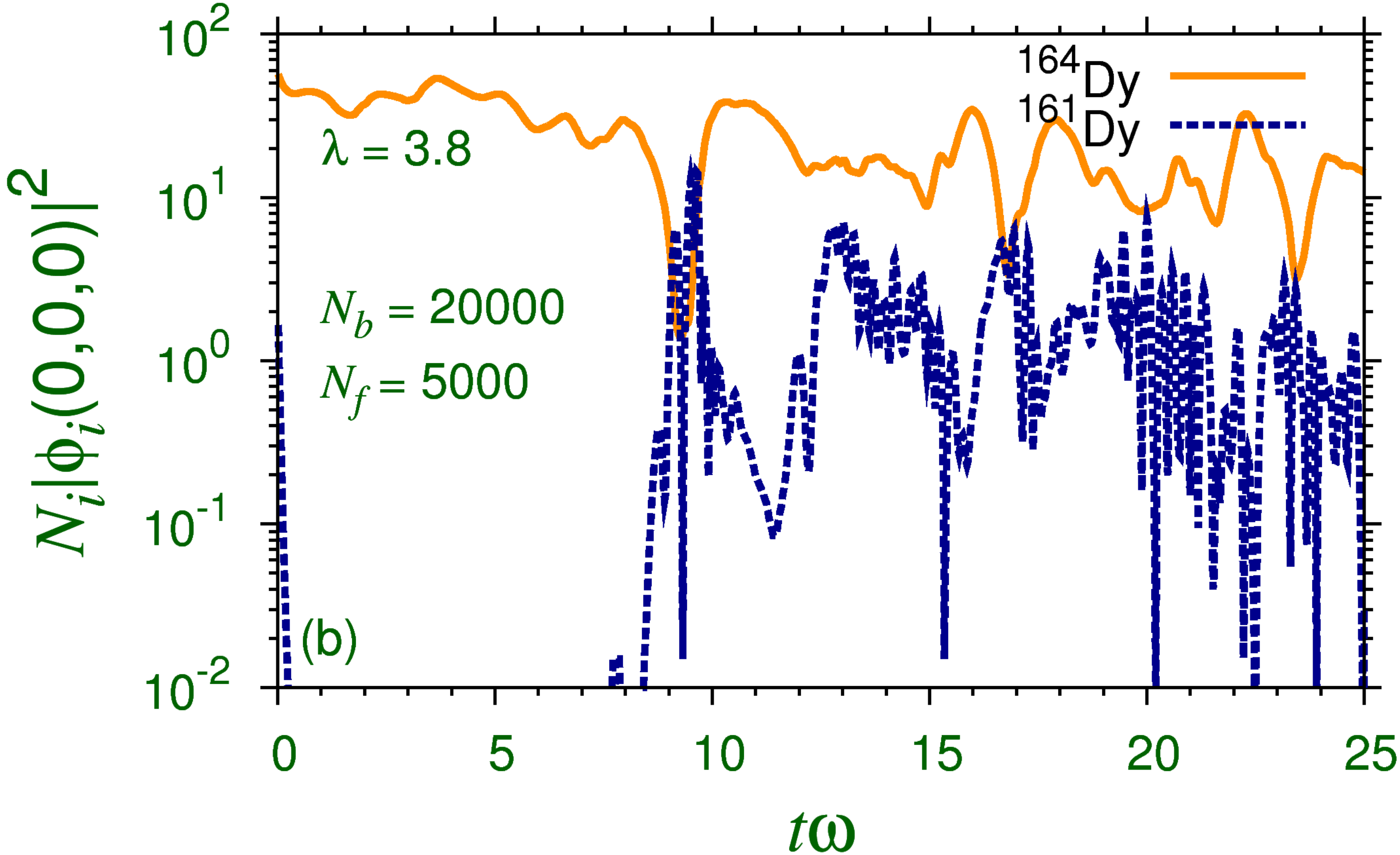
•



$K_3^{(bb)} = 7.5X10^{-27}$ cm^6/s; $K_3^{(bf)} = 1.5X10^{-25}$ cm^6/s;

Strong dipolar effect in fermionic collapse





$N_i|\phi_i(0,0,0)|^2$
$t\omega$
$\lambda = 3.8$
$N_b = 20000$
$N_f = 5000$
(b)
^{164}Dy
^{161}Dy
10^2
10^1
10^0
10^{-1}
10^{-2}
0
5
10
15
20
25

Summary & Conclusion

- Instability of degenerate dipolar Bose and Fermi systems.
- Unlike in nondipolar fermions, collapse can take place in trapped dipolar fermions.
- We studied statics and dynamics of collapse in a binary ^{164}Dy-^{161}Dy mixture.
- Experiments needed to verify theory.

Gross-Pitaevskii Equation: Dipolar atom

$$\left[-\frac{1}{2}\nabla^2 + U + g_3 |\Psi(\mathrm{r})|^2 + \int d\mathrm{r}' \mathrm{U}_{\mathrm{dd}}(\mathrm{r} - \mathrm{r}')\Psi(\mathrm{r}')\right]\Psi(\mathrm{r}) = \mu\Psi(\mathrm{r})$$

$$\mathrm{U}_{\mathrm{dd}}(\mathrm{r}) = 3Na_{dd}(1 - 3\cos^2\vartheta)/r^3, \text{ For Cr } a_{dd} = 15a_0$$

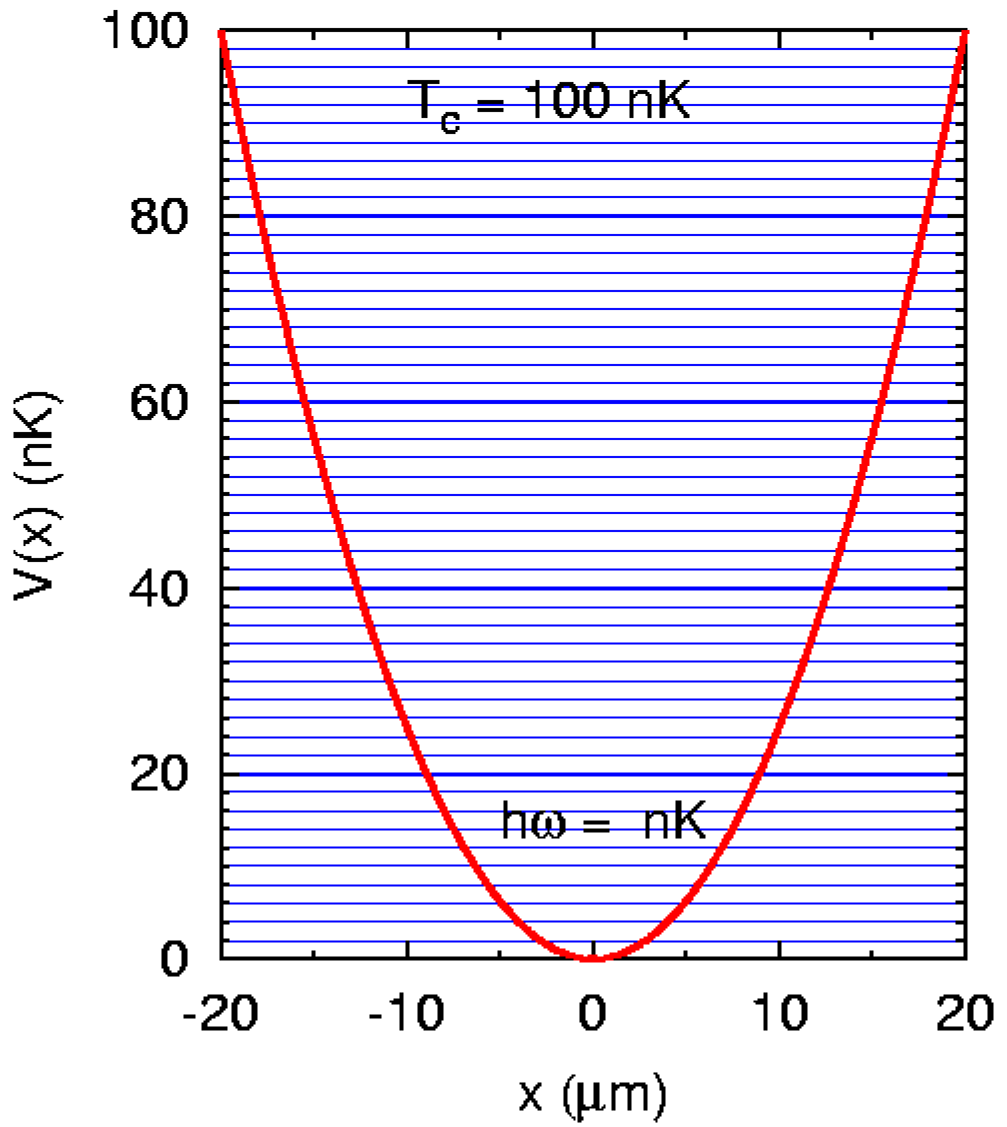
$$\int |\Psi|^2 d^3r = N, \quad g_3 = 4\pi aN,$$

$$U = V(x) + V(y) + V(z)$$

$$V(x) = \sum_{i=1}^{2} \frac{4\pi^2 s_i}{\lambda_i^2} \sin^2\left(\frac{2\pi x}{\lambda_i}\right)$$

$s_1 = s_2 = 2$, $\lambda_1 = 5$, $\lambda_2 = 0.862\lambda_1$

Harmonic trap and quantum statistics



Bose-Einstein Condensate (BEC)

Uncertainty relation:

$$\Delta x \Delta p \approx h,$$

$$\Delta p = m \Delta \mathrm{v}$$

$$\Delta x \Delta \mathrm{v} \approx h / m$$

Mass (BEC) = $10^8 \sim 10^{10}$ X Mass (electron)

Makes the experimetal realization much easier

Soliton-soliton Interaction

- Frontal collision at medium to high velocities.
- Numerical simulation in 3D shows that the two dipolar solitons pass through each other.
- Molecule formation at low velocities. Soliton_2
- If two solitons in 3D are kept side-by-side at rest, due to long range dipolar interaction they attract and slowly move towards each other. They penetrate, coalese and never come out and form a soliton molecule.

Generalized Gross-Pitaevskii Equation (mean-field equation) for the BEC

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = -\left[\frac{\hbar^2}{2m}\nabla^2 + V_{trap} + \frac{4\pi\hbar^2 aN}{m}\left|\psi(r,t)\right|^2 \right]\psi(r,t)$$

$$+ \frac{3\hbar^2 a_{dd} N}{m}\int U_{dd}(r-r')\left|\psi(r',t)\right|^2 dr' \psi(r,t)$$

$$= \mu\psi(r,t)$$

$$V_{trap} = \frac{1}{2} m\omega^2[(x^2+y^2)+\lambda z^2]$$

Generalized Hydrodynamical Equation (mean-field equation) for degenerate and Superfluid Fermi gas

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = -\left[\frac{\hbar^2}{2m} \nabla^2 + V_{trap} + \frac{4\pi\hbar^2 aN}{m} \left|\psi(r,t)\right|^2 \right] \psi(r,t)$$

$$+ \frac{3\hbar^2 a_{dd} N}{m} \int U_{dd}(r - r') \left|\psi(r',t)\right|^2 dr' \psi(r,t)$$

$$= \mu \psi(r,t)$$

$$V_{trap} = \frac{1}{2} m \omega^2 [(x^2 + y^2) + \lambda z^2]$$

Variational Equations

Lagrangian density

$$\mathcal{L} = \frac{i}{2}(\phi\phi_t^\star - \phi^\star\phi_t) + \frac{1}{2}|\nabla\phi|^2 + \frac{\rho^2}{2}|\phi|^2 + 2\pi a N|\phi|^4 + \frac{N}{2}|\phi|^2 \int U_{dd}(\mathbf{r}-\mathbf{r}')|\phi(\mathbf{r}')|^2 d\mathbf{r}'. \quad (1)$$

Gaussian ansatz

$$\phi(\mathbf{r},t) = \exp(-\rho^2/2w_\rho^2 - z^2/2w_z^2 + i\alpha\rho^2 + i\beta z^2)/(w_\rho\sqrt{w_z}\pi^{3/4})$$

Effective Lagrangian L (per particle)

$$L \equiv \int \mathcal{L}\, d\mathbf{r} = \left(w_\rho^2\dot{\alpha} + \frac{1}{2}w_z^2\dot{\beta} + 2w_\rho^2\alpha^2 + w_z^2\beta^2\right) + \frac{1}{2}\left(\frac{1}{w_\rho^2} + \frac{1}{2w_z^2} + w_\rho^2\right) + \mathcal{E}_{\rm dip}, \quad (2)$$

with $\mathcal{E}_{\rm dip} = N[a - a_{dd}f(\kappa)]/(\sqrt{2\pi}w_\rho^2 w_z)$, $f(\kappa) = [1 + 2\kappa^2 - 3\kappa^2 d(\kappa)]/(1-\kappa^2)$, $d(\kappa) = (\text{atanh}\sqrt{1-\kappa^2})/\sqrt{1-\kappa^2}$, $\kappa = w_\rho/w_z$. Equations for the widths

$$\ddot{w}_\rho + w_\rho = \frac{1}{w_\rho^3} + \frac{1}{\sqrt{2\pi}}\frac{N}{w_\rho^3 w_z}\left[2a - a_{dd}g(\kappa)\right], \quad (3)$$

$$\ddot{w}_z = \frac{1}{w_z^3} + \frac{1}{\sqrt{2\pi}}\frac{2N}{w_\rho^2 w_z^2}\left[a - a_{dd}h(\kappa)\right], \quad (4)$$

with $g(\kappa) = [2 - 7\kappa^2 - 4\kappa^4 + 9\kappa^4 d(\kappa)]/(1-\kappa^2)^2$, $h(\kappa) = [1 + 10\kappa^2 - 2\kappa^4 - 9\kappa^2 d(\kappa)]/(1-\kappa^2)^2$.

Mean-field Gross-Pitaevskii Equation

Full quantum many-body Hamiltonian:

$$H = \int \psi^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ex}(\mathbf{r}) \right] \psi(\mathbf{r})$$

$$+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \psi(\mathbf{r}) \psi(\mathbf{r}'),$$

$\psi^\dagger(\mathbf{r})$ and $\psi(\mathbf{r}')$ are boson creation and destruction operators ar $\mathbf{r}$ and $\mathbf{r}'$, and V the interaction.

Simple mean-field description:

$$\psi(\mathbf{r}, t) = \phi(\mathbf{r}, t) + \chi(\mathbf{r}, t), \qquad \phi(\mathbf{r}, t) = \langle \psi(\mathbf{r}, t) \rangle,$$

$\chi(\mathbf{r}, t)$ contains quantum fluctuations and $\phi(\mathbf{r}, t)$ is a classical field, often called an order parameter or wave function of the condensate.

Heisenberg equation for field ψ:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = [\psi, H]$$

$$= [T + V_{ex}(\mathbf{r}) + \int d\mathbf{r}'\psi^{\dagger}(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}',t)]\psi(\mathbf{r},t),$$

In mean-field theory replace field ψ by wave function ϕ. At zero energy in a dilute gas binary collision is important and controlled by the S-wave scattering length. It is proper to replace

$$V(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}') \equiv \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}-\mathbf{r}'),$$

Mean-field Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{ex}(\mathbf{r}) + g|\phi(\mathbf{r},t)|^2\right)\phi(\mathbf{r},t),$$

$$\int d\mathbf{r}|\phi|^2 = N.$$

- Application in nonlinear optics

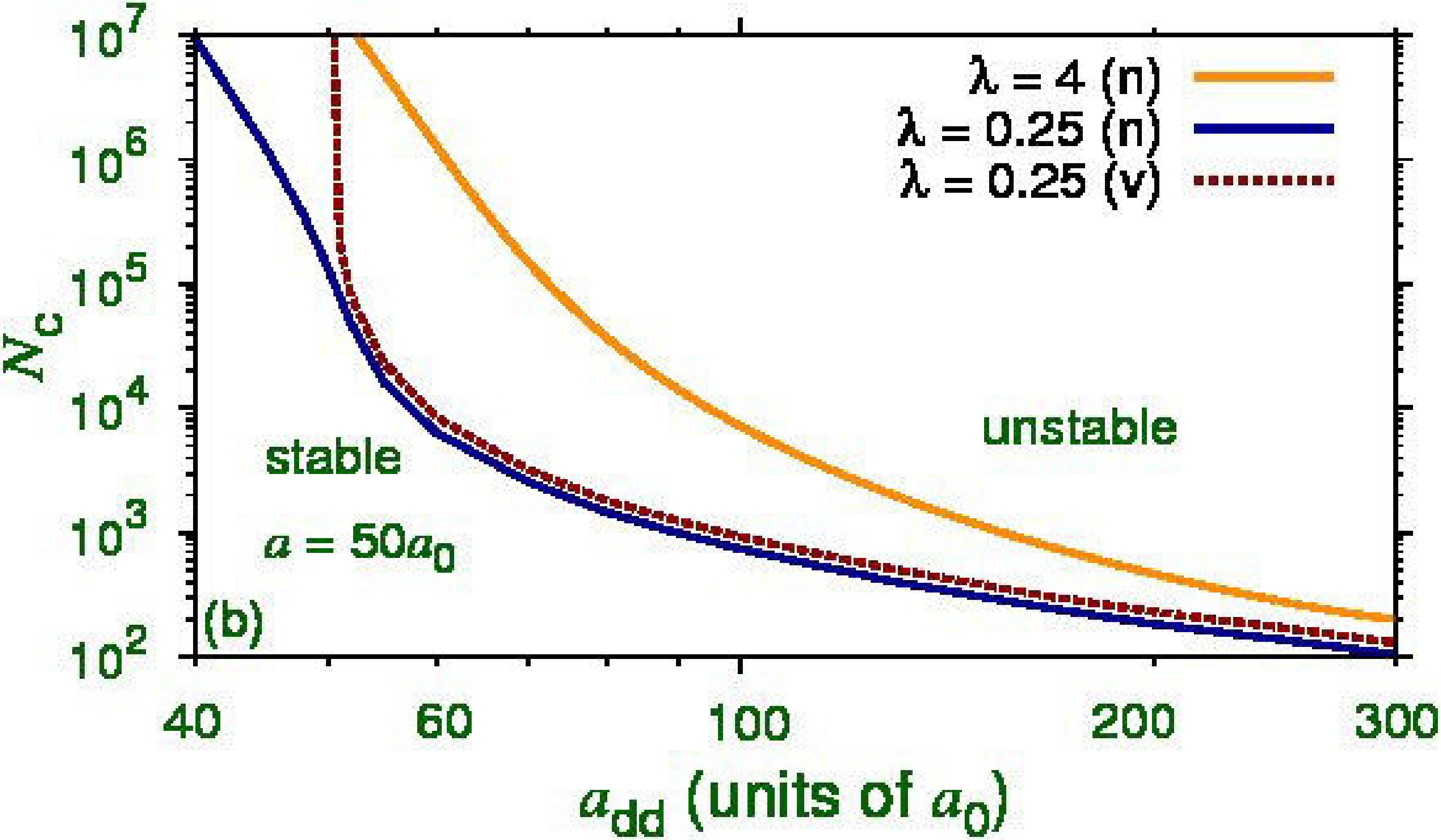
- This potential gives scattering length a in the Born approximation

Trapped degenerate Dipolar gas

Cigar shape

- If the radial trap is weak, with the increase of dipolar interaction the system becomes cigar shaped and with further increase of dipolar interaction the system collapses on the polarization axis.

Stability phase plot

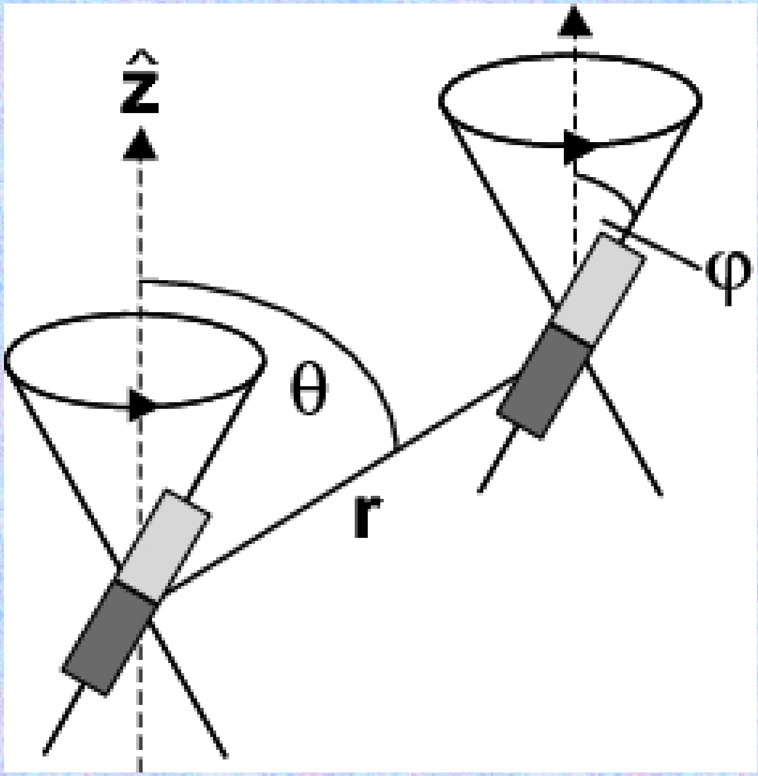


Tuning of dipolar interaction by rotating orienting field

$$U_{dd}(R) = 3a_{dd}\frac{(1-3\cos^2\theta)}{R^3}\frac{(3\cos^2\varphi-1)}{2};\; a_{dd} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

Strongly anisotropic Magnetic/Electric Dipole-Dipole Interactions

$$\mathrm{B(t)} = \mathrm{B}[\cos(\varphi)\mathbf{z} + \sin(\varphi)(\cos(\Omega \mathrm{t})\mathbf{x} + \sin(\Omega \mathrm{t})\mathbf{y})]$$



Trapped degenerate Dipolar gas

Disk shape

- However, these disk-shaped degenerate Bose and Fermi systems collapse for the net dipolar interaction above a critical value.
- If the axial trap is strong, with the increase of dipolar interaction the system passes through a biconcave shape to a ring and with further increase collapses on the perimeter of the ring.

Static Dipole-Dipole Interactions

Magnetic dipole-dipole interaction: the magnetic moments of the atoms are aligned with a strong magnetic field [Goral, Rzazewski, and Pfau, 2000]

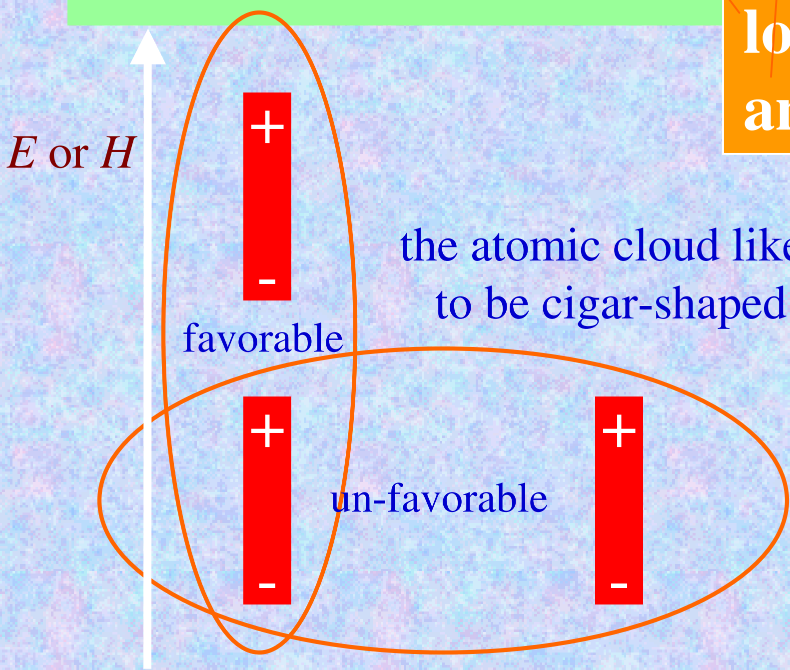
$$U_{dd}(r) = \frac{\beta \mu_0 \mu^2}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

Electrostatic dipole-dipole interaction: (i) permanent electric moments (polar molecules); (ii) electric moments induced by a strong electric field *E* [Yi and You 2000; Santos, Shlyapnikov, Zoller and Lewenstein 2000]

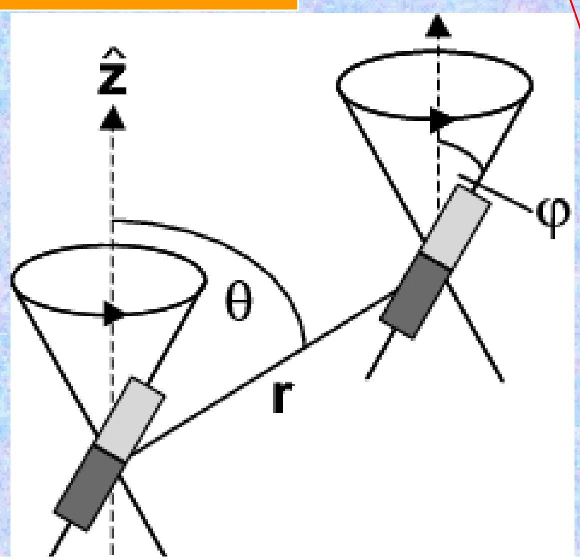
$$U_{dd}(r) = \frac{\beta \alpha^2}{4\pi\varepsilon_0}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

long-range + anisotropic

tunability



the atomic cloud likes to be cigar-shaped



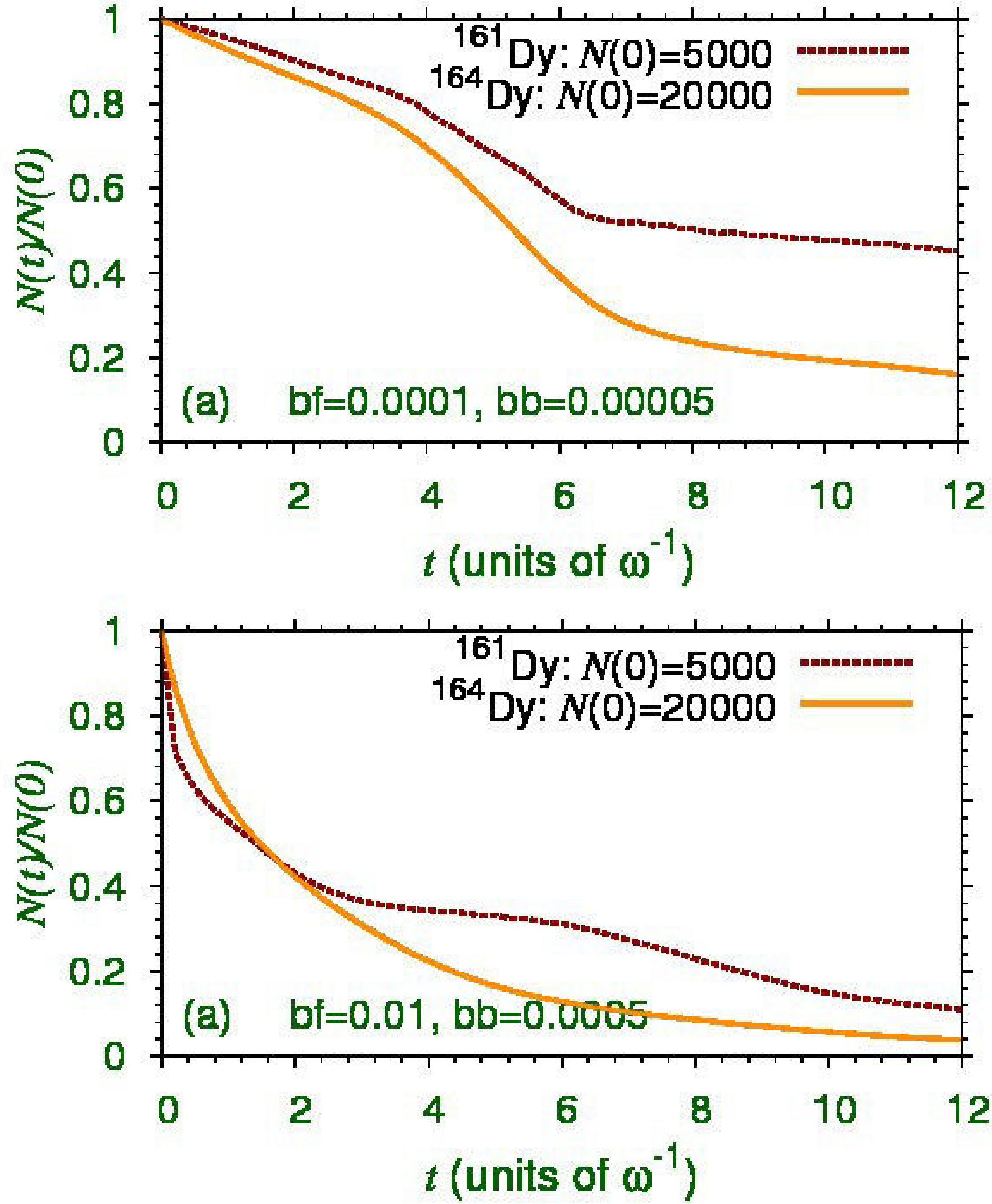
Binary Bose-Fermi ^{164}Dy-^{161}Dy mixture

$$
\begin{aligned}
i\hbar\frac{\partial\Phi_b(\mathbf{r},t)}{\partial t} =& \Big[-\frac{\hbar^2}{2m_b}\nabla^2 + \frac{1}{2}m_b\omega_b^2\left(\frac{\rho^2}{\lambda_b^{2/3}} + z^2\lambda_b^{4/3}\right) + \frac{4\hbar^2\pi a_b}{m_b}|\Phi_b|^2 + \frac{2\pi\hbar^2}{m_R}a_{bf}|\Phi_f|^2 \\
&+ \frac{\mu_0\mu_b^2}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_b(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\mu_0\mu_b\mu_f}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_f(\mathbf{r}')|^2 d\mathbf{r}' \Big]\Phi_b(\mathbf{r},t), \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
i\hbar\frac{\partial\Phi_f(\mathbf{r},t)}{\partial t} =& \Big[-\frac{\hbar^2}{8m_f}\nabla^2 + \frac{1}{2}m_f\omega_f^2\left(\frac{\rho^2}{\lambda_f^{2/3}} + z^2\lambda_f^{4/3}\right) + \frac{2\pi\hbar^2}{m_R}a_{bf}|\Phi_b|^2 \\
&+ \frac{\mu_0\mu_b\mu_f}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_b(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\mu_0\mu_f^2}{4\pi}\int V_{dd}(\mathbf{R})|\Phi_f(\mathbf{r}')|^2 d\mathbf{r}' + \frac{\hbar^2(6\pi^2 N_f|\Phi_f|^2)^{2/3}}{2m_f}\Big]\Phi_f(\mathbf{r},t), \qquad (2)
\end{aligned}
$$

$$
\int |\Phi_i(\mathbf{r})|^2 d\mathbf{r} = N_i, i = b, f, \qquad \lambda_i = \frac{\omega_{zi}}{\omega_{\rho i}}, \quad \omega_i = (\omega_{zi}\omega_{\rho i}^2)^{1/3}, \quad m_R = \frac{m_b m_f}{m_b + m_f}, \qquad (3)
$$

$$
a^{(b)}{}_{dd} = \frac{\mu_0\mu_b{}^2 m_b}{12\pi\hbar^2}; a^{(f)}{}_{dd} = \frac{\mu_0\mu_f{}^2 m_f}{12\pi\hbar^2}; a^{(bf)}{}_{dd} = \frac{\mu_0\mu_b\mu_f m_R}{6\pi\hbar^2}
$$



^{161}Dy: $N(0)$=5000
^{164}Dy: $N(0)$=20000
$N(t)/N(0)$
(a) bf=0.0001, bb=0.00005
t (units of ω^{-1})
^{161}Dy: $N(0)$=5000
^{164}Dy: $N(0)$=20000
$N(t)/N(0)$
(a) bf=0.01, bb=0.0005
t (units of ω^{-1})

Results of calculations

- The three-dimensional GP equation is solved numerically by split-step Crank-Nicolson method without further approximation.
- Fortran programs for GP Eq. published in Comput. Phys. Commun. 180 (2009) 1888-1912
- Results are compared with Gaussian variational approximation.

Collapse dynamics: Collapse and explosion