JOINT INSTITUTE FOR NUCLEAR RESEARCH

THE RARE GASES CLASTERS

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Collaborators:

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INTERACTION POTENTIALS

Homogeneous *dimers* : He_2 , Ne_2 , Ar_2 , Kr_2 , Xe_2 , Rn_2 (6)

Heterogeneous *dimers* : He-Ne , He-Ar , He-Kr , He-Xe , He-Rn , Ne-Ar and etc. (15)

Potential models:

– Lennard - Jones [1]:

$$V(r) = 4\,\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

ε – scales the energy and σ – the length scale;

– Tang -Toennies [2]:

$$V(R) = V_{rep} + V_{att} = A\,e^{-bR} - \sum_{n=3}^{N} f_{2n}(bR)\,\frac{C_{2n}}{R^{2n}}$$

where A and b parameters,
the C_{2n} are the dispersion coefficient,
$f_{2n}(bR)$ - the damping function,
which is given by the following expression:

$$f_{2n}(x) = 1 - e^{-x}\sum_{k=0}^{2n}\frac{x^{-k}}{k!}$$

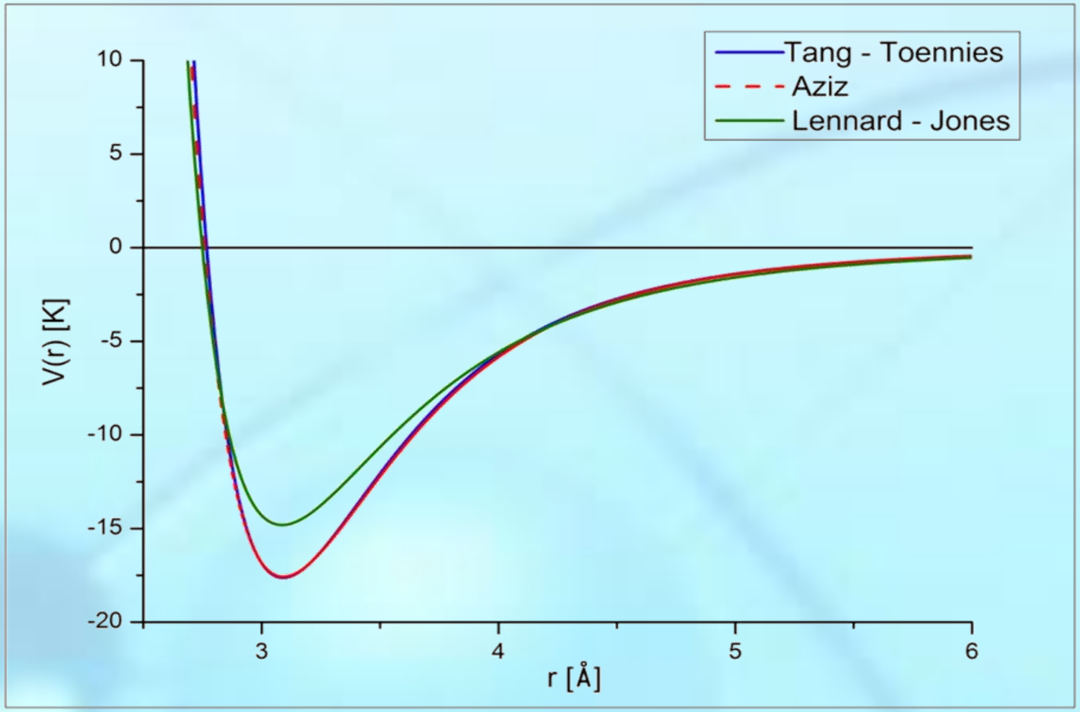
– Aziz [3]: $V(x) = \varepsilon\, V_b(\zeta)$

where $\zeta = x/r_m$, and term $V_b(\zeta)$ has the form:

$$V_b(\zeta) = A\exp(-\alpha\,\zeta + \beta\,\zeta^2) - \left[\frac{c_6}{\zeta^6} + \frac{c_8}{\zeta^8} + \frac{c_{10}}{\zeta^{10}}\right] F(\zeta)$$

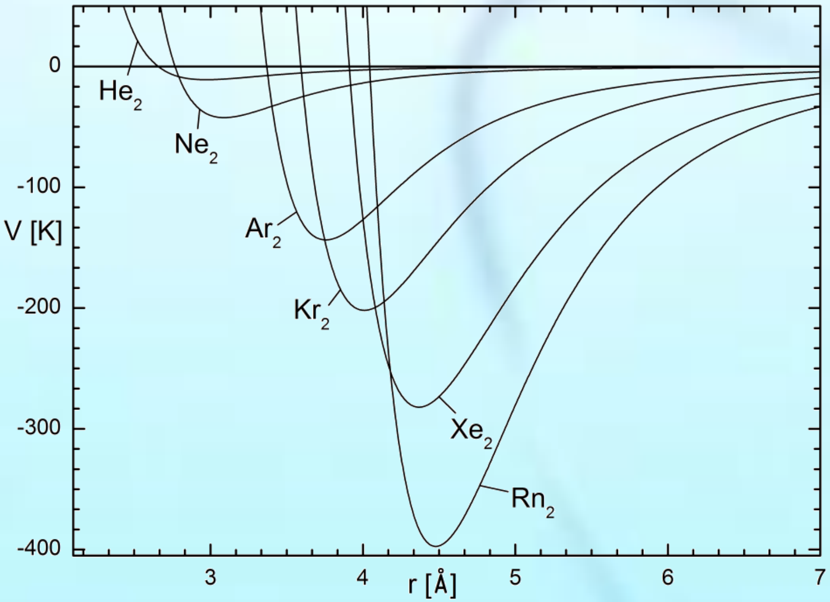
at that x is expressed in the same length units as r_m (for this case they are angstroms).
Function $F(\zeta)$ is given by the expression:

$$F(\zeta) = \begin{cases} \exp[-(D/\zeta - 1)^2], & if\ \zeta \le D, \\ 1, & if\ \zeta > D. \end{cases}$$



[1] D.M. Leither, J.D. Doll, R.M.Whitnell // J.Chem.Phys. **94**, 6644 - 6659 (1991)
[2] K.T. Tang and J.P. Toennies // J.Chem.Phys. **118**, 4976 - 4983,(2003)
[3] R.A. Aziz and M.J. Slaman // J. Chem. Phys. **94**, 8047 - 8053 (1991);
D.A. Barrow, M.J. Slaman, R.A. Aziz // J. Chem. Phys. **91**, 6348-6358 (1989);
R.A. Aziz // J. Chem. Phys. **99**, 4518 - 4525 (1993)

INTERACTION POTENTIALS OF HOMOGENEOUS DIMERS



System	C_6	C_8	C_{10}	A	b, $a.u.^{-1}$
He - He	1,461	14,11	183,6	41,96	2,523
Ne - Ne	6,383	90,34	1536	199,5	2,458
Ar - Ar	64,30	1623	49060	748,3	2,031
Kr - Kr	129,6	4187	155500	832,4	1,865
Xe - Xe	285,9	12810	619800	951,8	1,681
Rn - Rn	420,6	19260	1067000	5565,0	1,824

Tang –Toennies model [1]:

$$V(R) = V_{rep} + V_{att} = A\,e^{-bR} - \sum_{n=3}^{N} f_{2n}(bR)\frac{C_{2n}}{R^{2n}}$$

where A and b parameters,
the C_{2n} *are the dispersion coefficient,*
$f_{2n}(bR)$ - *the damping function,*
which is given by the following expression:

$$f_{2n}(x) = 1 - e^{-x}\sum_{k=0}^{2n}\frac{x^{-k}}{k!}$$

[1] K.T. Tang and J.P. Toennies // J. Chem. Phys. **118**, 4976-4983,(2003)

$^{4}He_2$

Measurements: bond length [3] $< R > = 52 \pm 4\,\overset{o}{A}$

Estimation of the binding energy and scattering length

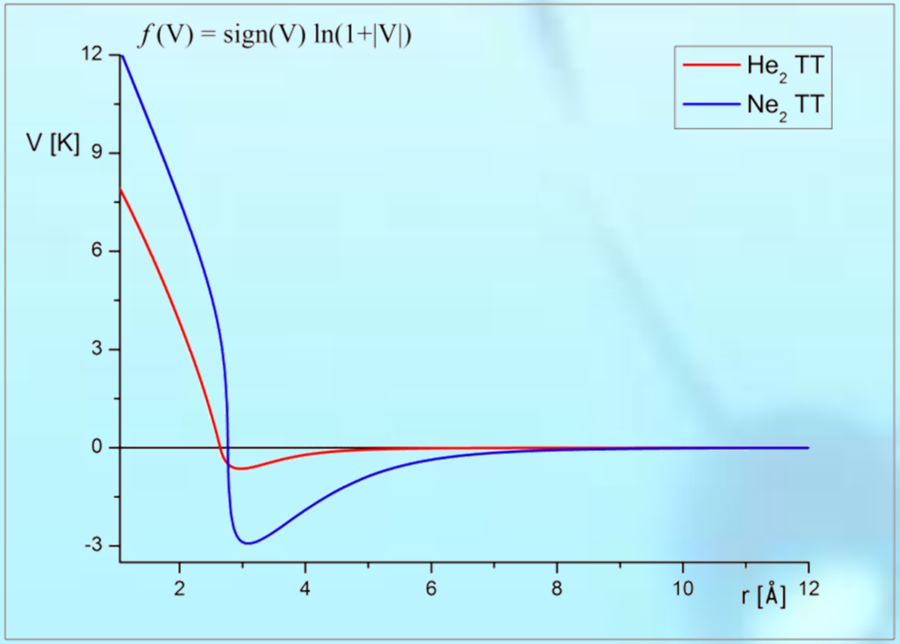
$$\varepsilon_d = 1.1^{+0.3}_{-0.2}\,\mathrm{mK} \qquad l_{sc} = 104^{+8}_{-18}\,\overset{o}{A} \qquad [3]$$

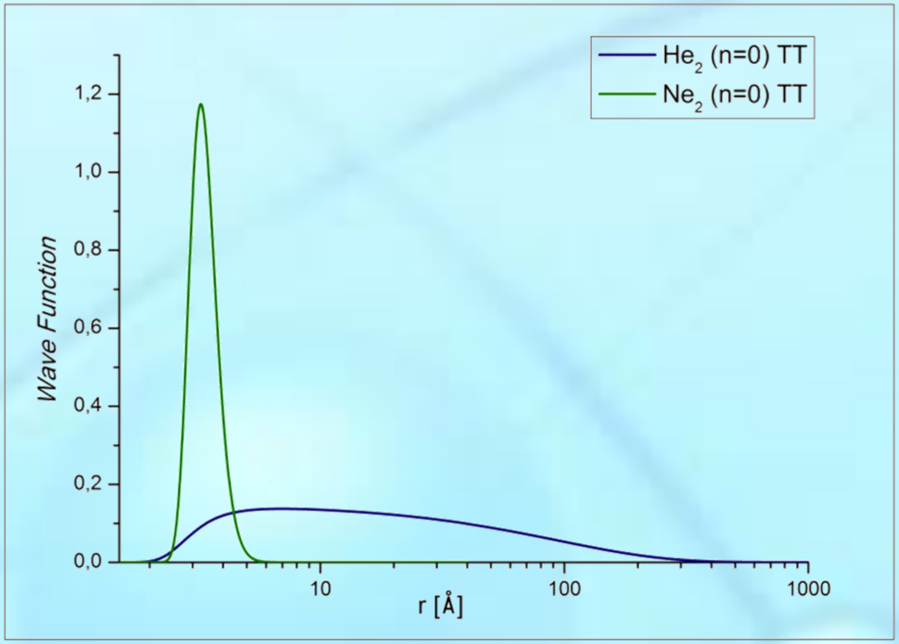
?

$$\varepsilon_d = 1.3^{+0.25}_{-0.19}\,\mathrm{mK} \qquad l_{sc} = 100^{+8}_{-7.9}\,\overset{o}{A} \qquad [6]$$

$^{20}Ne_2$

n	E (K) [5]
0	24,22 ± 0,02
1	4,405 ± 0,02
2	< 0.14



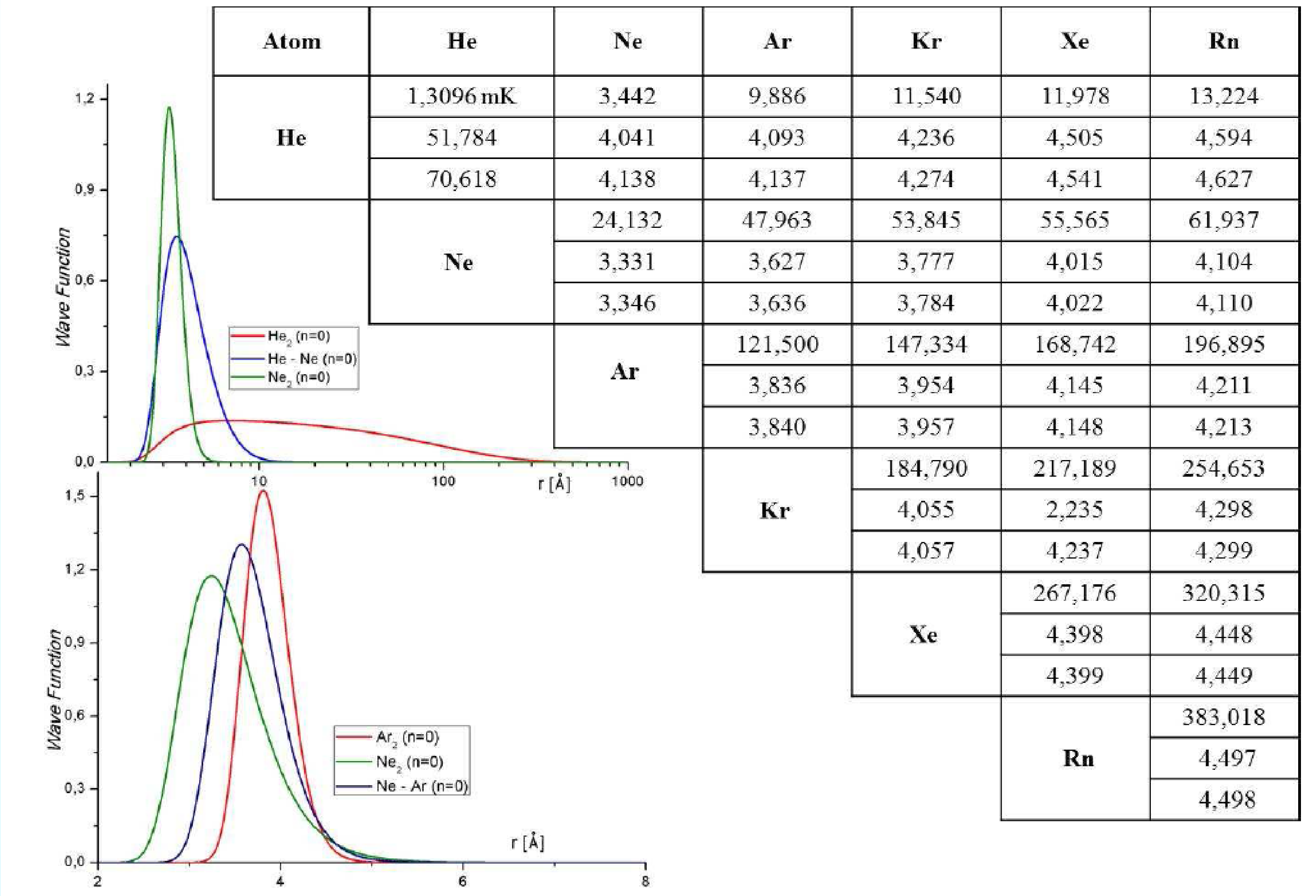


[1] F.Luo et. al.// J. Chem. Phys. 98 (1993) 9687.
[2] W.Schoellkopf et. al.// Science.266 (1994) 1345.
[3] R.Grisenti et. al. // Phys. Rev. Lett. 85 (2000) 2284.
[4] Y.Tanaka,K.Yoshino //J.Chem.Phys.57 (1972) 2964.
[5] A.Wüest, F.Merkt//J.Chem.Phys.118 (2003) 8807.
[6] W.Cencek et.al. // J.Chem.Phys., 136(2012) 224303.

Spectra of homogeneous dimers (TT potentials):

level	He_2 (K)	Ne_2 (K)	Ar_2 (K)	Kr_2 (K)	Xe_2 (K)
0	0,001309	24,1316	121,5004	184,7897	267,1759
1		4,2777	83,7284	153,1110	238,6889
2		0,02215	54,0021	124,8287	212,0169
3			31,8334	99,8756	187,1428
4			16,5115	78,1658	164,0472
5			7,0383	59,5926	142,7075
6			2,1227	44,0234	123,0977
7			0,2823	31,2940	105,1879
8				21,2031	88,9437
9				13,5088	74,3252
10				7,9285	61,2863
11				4,1441	49,7742
12				1,8129	39,7280
13				0,5801	31,0784
14				0,09122	23,7471
15				0,0001393	17,6446
16					12,6781
17					8,7381
18					5,7122
19					3,4831
20					1,9286
21					0,9256
22					0,3511
23					0,08371
24					0,004802

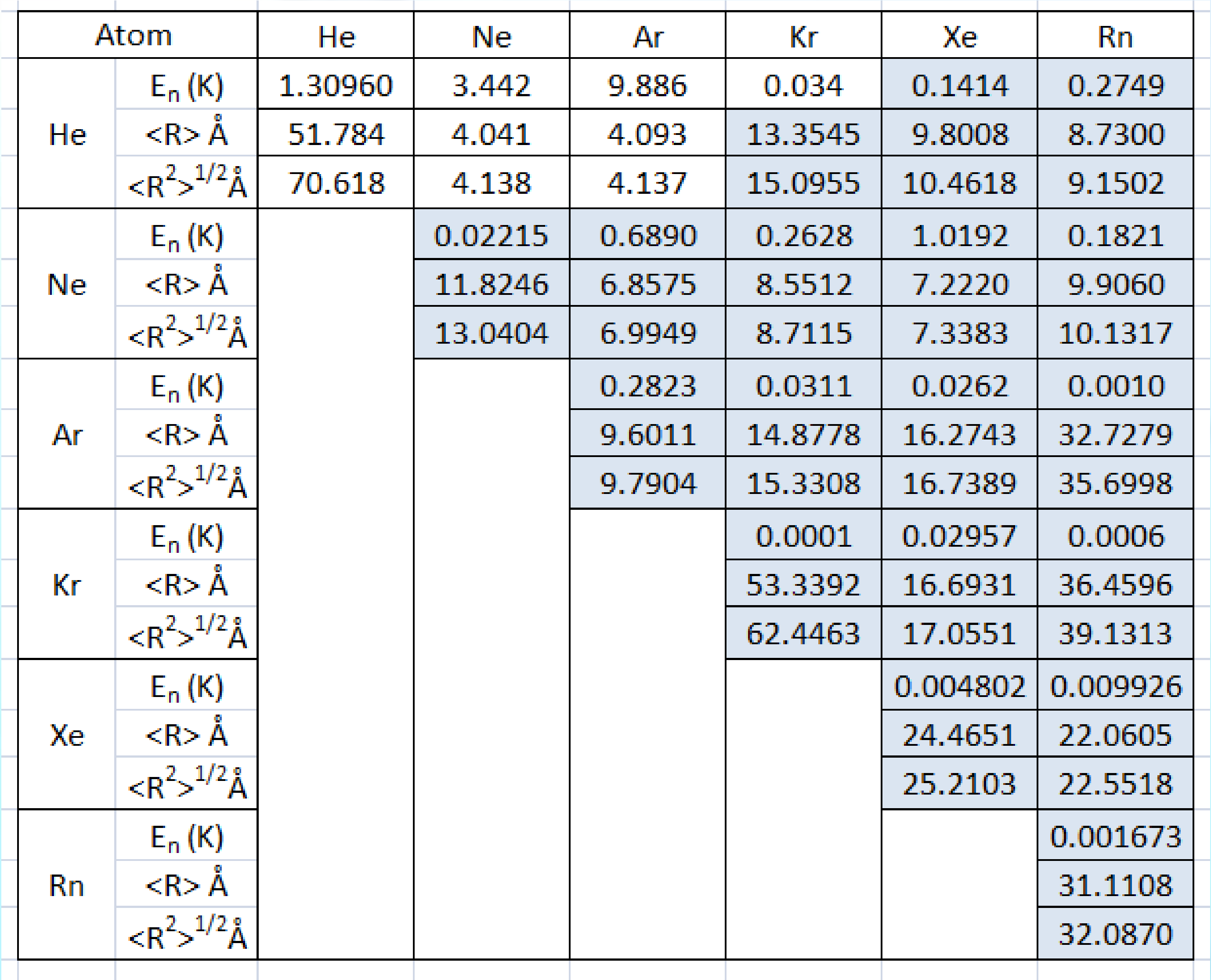
Ground states of heterogeneous dimers:



Atom	**He**	**Ne**	**Ar**	**Kr**	**Xe**	**Rn**
He	1,3096 mK	3,442	9,886	11,540	11,978	13,224
	51,784	4,041	4,093	4,236	4,505	4,594
	70,618	4,138	4,137	4,274	4,541	4,627
	Ne	24,132	47,963	53,845	55,565	61,937
		3,331	3,627	3,777	4,015	4,104
		3,346	3,636	3,784	4,022	4,110
		Ar	121,500	147,334	168,742	196,895
			3,836	3,954	4,145	4,211
			3,840	3,957	4,148	4,213
			Kr	184,790	217,189	254,653
				4,055	2,235	4,298
				4,057	4,237	4,299
				Xe	267,176	320,315
					4,398	4,448
					4,399	4,449
					Rn	383,018
						4,497
						4,498

TABLE. Ground state energy of the heterogeneous rare gases dimers (in K), average distance and mean root square radius (both in Å). FIG. The wave functions of the He_2*, He-Ne and* Ne_2 *dimers (upper) and* Ar_2*,* Ne_2 *and Ne-Ar (lower).*

Ground states of heterogeneous dimers:



Atom		He	Ne	Ar	Kr	Xe	Rn
He	E_n (K)	1.30960	3.442	9.886	0.034	0.1414	0.2749
	$\langle R\rangle$ Å	51.784	4.041	4.093	13.3545	9.8008	8.7300
	$\langle R^2\rangle^{1/2}$ Å	70.618	4.138	4.137	15.0955	10.4618	9.1502
Ne	E_n (K)		0.02215	0.6890	0.2628	1.0192	0.1821
	$\langle R\rangle$ Å		11.8246	6.8575	8.5512	7.2220	9.9060
	$\langle R^2\rangle^{1/2}$ Å		13.0404	6.9949	8.7115	7.3383	10.1317
Ar	E_n (K)			0.2823	0.0311	0.0262	0.0010
	$\langle R\rangle$ Å			9.6011	14.8778	16.2743	32.7279
	$\langle R^2\rangle^{1/2}$ Å			9.7904	15.3308	16.7389	35.6998
Kr	E_n (K)				0.0001	0.02957	0.0006
	$\langle R\rangle$ Å				53.3392	16.6931	36.4596
	$\langle R^2\rangle^{1/2}$ Å				62.4463	17.0551	39.1313
Xe	E_n (K)					0.004802	0.009926
	$\langle R\rangle$ Å					24.4651	22.0605
	$\langle R^2\rangle^{1/2}$ Å					25.2103	22.5518
Rn	E_n (K)						0.001673
	$\langle R\rangle$ Å						31.1108
	$\langle R^2\rangle^{1/2}$ Å						32.0870

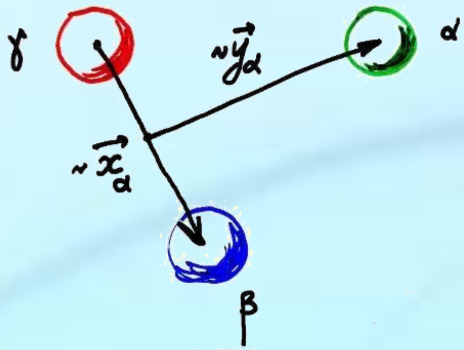
TABLE. Weakest state energy of the heterogeneous rare gases dimers (in K), average distance and mean root square radius (both in Å).

Three-body, theory formalism

In describing the three-body system we use the standard Jacobi coordinates [4] $\boldsymbol{x}_\alpha, \boldsymbol{y}_\alpha$, $\alpha = 1, 2, 3$, expressed in terms of the position vectors of the particles $\boldsymbol{r}_i \in \mathbb{R}^3$ and their masses m_i,

$$\boldsymbol{x}_\alpha = \left[\frac{2m_\beta m_\gamma}{m_\beta + m_\gamma}\right]^{1/2} (\boldsymbol{r}_\beta - \boldsymbol{r}_\gamma)$$

$$\boldsymbol{y}_\alpha = \left[\frac{2m_\alpha(m_\beta + m_\gamma)}{m_\alpha + m_\beta + m_\gamma}\right]^{1/2} \left(\boldsymbol{r}_\alpha - \frac{m_\beta \boldsymbol{r}_\beta + m_\gamma \boldsymbol{r}_\gamma}{m_\beta + m_\gamma}\right)$$



where (α, β, γ) stands for a cyclic permutation of the indices $(1, 2, 3)$. The coordinates $\boldsymbol{x}_\alpha, \boldsymbol{y}_\alpha$ fix the six-dimensional vector $X \equiv (\boldsymbol{x}_\alpha, \boldsymbol{y}_\alpha) \in \mathbb{R}^6$. The vectors $\boldsymbol{x}_\beta, \boldsymbol{y}_\beta$ corresponding to the same point X as the pair $\boldsymbol{x}_\alpha, \boldsymbol{y}_\alpha$ are obtained using the transformations

$$\boldsymbol{x}_\beta = c_{\beta\alpha}\boldsymbol{x}_\alpha + s_{\beta\alpha}\boldsymbol{y}_\alpha \qquad \boldsymbol{y}_\beta = -s_{\beta\alpha}\boldsymbol{x}_\alpha + c_{\beta\alpha}\boldsymbol{y}_\alpha$$

where the coefficients $c_{\beta\alpha}$ and $s_{\beta\alpha}$ fulfil the conditions $-1 < c_{\beta\alpha} < +1$ and $s_{\beta\alpha}^2 = 1 - c_{\beta\alpha}^2$ with $c_{\alpha\beta} = c_{\beta\alpha}$, $s_{\alpha\beta} = -s_{\beta\alpha}$, $\beta \neq \alpha$ and depend only on the particle masses [4]. For equal masses $c_{\beta\alpha} = -\frac{1}{2}$.

[4] - L.D.Faddeev,S.P.Merkuriev, 1993, *Quantum scattering theory for several particles*

When the total angular momentum L of the system is fixed, the three-body dynamics is constrained onto three-dimensional internal space [5], which can be parametrized by coordinates

$$x_\alpha = |\boldsymbol{x}_\alpha|,\; y_\alpha = |\boldsymbol{y}_\alpha|,\, z_\alpha = \cos\theta_\alpha = (\hat{\boldsymbol{x}}_\alpha, \hat{\boldsymbol{y}}_\alpha)$$

For zero angular momentum the Faddeev equations in internal space are given by the set of three coupled three-dimensional equations

$$(H_0 + V_\alpha - E)F_\alpha(x_\alpha, y_\alpha, z_\alpha) = -V_\alpha \sum_{\beta\neq\alpha} F_\beta(x_\beta, y_\beta, z_\beta)$$

$$x_\beta = \sqrt{c_{\beta\alpha}^2 x_\alpha{}^2 + s_{\beta\alpha}^2 y_\alpha{}^2 + 2c_{\beta\alpha}s_{\beta\alpha}x_\alpha y_\alpha z_\alpha}$$

$$y_\beta = \sqrt{s_{\beta\alpha}^2 x_\alpha{}^2 + c_{\beta\alpha}^2 y_\alpha{}^2 - 2c_{\beta\alpha}s_{\beta\alpha}x_\alpha y_\alpha z_\alpha}$$

$$H_0 = -\frac{\partial^2}{\partial x_\alpha{}^2} - \frac{\partial^2}{\partial y_\alpha{}^2} - \left(\frac{1}{x_\alpha{}^2} + \frac{1}{y_\alpha{}^2}\right)\frac{\partial}{\partial z_\alpha}(1 - z_\alpha)^{1/2}\frac{\partial}{\partial z_\alpha}$$

$$x_\beta y_\beta z_\beta = \sqrt{(c_{\beta\alpha}^2 - s_{\beta\alpha}^2)x_\alpha y_\alpha z_\alpha - c_{\beta\alpha}s_{\beta\alpha}(x_\alpha{}^2 - y_\alpha{}^2)}$$

or in hyperspherical coordinates

$$\rho = \sqrt{x_\alpha^2 + y_\alpha^2},\, \tan\vartheta_\alpha = y_\alpha / x_\alpha,\, \eta_\alpha = (\hat{\boldsymbol{x}}_\alpha, \hat{\boldsymbol{y}}_\alpha)$$

$$(H_0 + V_\alpha - E)\Phi_\alpha(\rho, \vartheta_\alpha, \eta_\alpha) = -V_\alpha \sum_{\beta\neq\alpha} \Phi_\beta(\rho, \vartheta_\beta, \eta_\beta)$$

$$\Phi(x, y, \eta) \underset{\rho\to\infty}{=} \psi_d(x)\exp(ipy)a_0(\eta; E) + \frac{\exp(i\sqrt{E}\rho)}{\rho^{1/2}} A(\vartheta, \eta; E)$$

[5] - V.V.Kostrykin,A.A.Kvitsinsky,S.P.Merkuriev, Few-Body Syst. 6 (1989) 97

For computational purposes, one can reduce the dimension by expanding the Faddeev components into an auxiliary basis, at the expense of dealing with an infinite number of partial equations. Expanding the function F_α in a series of bispherical harmonics

$$F_\alpha(x, y, \theta) = \sum_{l,\lambda} \frac{\Phi^{(\alpha)}_{l\lambda\alpha}(x, y)}{xy} \,|\, l\lambda 0 >$$

One can obtain the partial equation

$$(H_0 + V_\alpha - E)\Phi^{(\alpha)}_{l\lambda}(x_\alpha, y_\alpha) = -V_\alpha \sum_{\beta\neq\alpha} \sum_{l'\lambda'} \int_{-1}^{1} d\eta \, h^{(\alpha\beta)}_{l\lambda l'\lambda'}(x_\alpha, y_\alpha, \eta)\Phi^{(\beta)}_{l'\lambda'}(x_\beta, y_\beta)$$

$$H_0 = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + \frac{l(l+1)}{x^2} + \frac{\lambda(\lambda+1)}{y^2}$$

$$x_\beta = \sqrt{c^2_{\beta\alpha} x_\alpha{}^2 + s^2_{\beta\alpha} y_\alpha{}^2 + 2c_{\beta\alpha} s_{\beta\alpha} x_\alpha y_\alpha \eta}$$

$$y_\beta = \sqrt{s^2_{\beta\alpha} x_\alpha{}^2 + c^2_{\beta\alpha} y_\alpha{}^2 - 2c_{\beta\alpha} s_{\beta\alpha} x_\alpha y_\alpha \eta}$$

Three-body, theory formalism

where

$$
\begin{aligned}
& h^{L}_{(\alpha;l\lambda L)(\beta;l'\lambda' L)}(x,y,\eta) \\
&= \frac{xy}{x_{\beta\alpha}(\eta)y_{\beta\alpha}(\eta)}(-1)^{l+L}\frac{(2\lambda+1)(2l+1)}{2^{\lambda+l}}\left[(2\lambda)!(2l)!(2\lambda'+1)(2l'+1)\right]^{1/2} \\
&\quad \times \sum_{k=0}^{k_{max}}(-1)^{k}(2k+1)P_{k}(\eta)\sum_{\substack{\lambda_1+\lambda_2=\lambda,\\ l_1+l_2=l}}\frac{y^{\lambda_1+l_1}x^{\lambda_2+l_2}}{[y_{\beta\alpha}(\eta)]^{\lambda}[x_{\beta\alpha}(\eta)]^{l}}(-1)^{\lambda_1}\mathrm{c}_{\beta\alpha}{}^{\lambda_1+l_2}\mathrm{s}_{\beta\alpha}{}^{\lambda_2+l_1} \\
&\quad \times\left[(2\lambda_1)!(2l_1)!(2\lambda_2)!(2l_2)!\right]^{-1/2}\sum_{\lambda'' l''}(2\lambda''+1)(2l''+1)\begin{pmatrix}\lambda_1 & l_1 & \lambda''\\ 0 & 0 & 0\end{pmatrix} \qquad (9) \\
&\quad \times\begin{pmatrix}\lambda_2 & l_2 & l''\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}k & \lambda'' & \lambda'\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}k & l'' & l'\\ 0 & 0 & 0\end{pmatrix} \\
&\quad \times\begin{Bmatrix}l' & \lambda' & L\\ \lambda'' & l'' & k\end{Bmatrix}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ l_1 & l_2 & l\\ \lambda'' & l'' & L\end{Bmatrix},
\end{aligned}
$$

$$k_{max} = \frac{1}{2}(l+\lambda+l'+\lambda').$$

$P_k(\eta) \Longleftrightarrow$ the Legendre polynomial of order k

$$
\begin{aligned}
x_{\beta\alpha}(\eta) &= \sqrt{\mathrm{c}^2_{\beta\alpha}x^2+2\mathrm{c}_{\beta\alpha}\mathrm{s}_{\beta\alpha}xy\eta+\mathrm{s}^2_{\beta\alpha}y} \\
y_{\beta\alpha}(\eta) &= \sqrt{\mathrm{s}^2_{\beta\alpha}x-2\mathrm{c}_{\beta\alpha}\mathrm{s}_{\beta\alpha}xy\eta+\mathrm{c}^2_{\beta\alpha}y}
\end{aligned}
$$

$^4He_2$ - ^4He

Boundary conditions

The asymptotic condition for the partial-wave Faddeev components of the $(2+1 \rightarrow 2+1\,;\, 1+1+1)$ scattering wave function reads, (as $\rho \rightarrow \infty$ and/or $y \rightarrow \infty$)

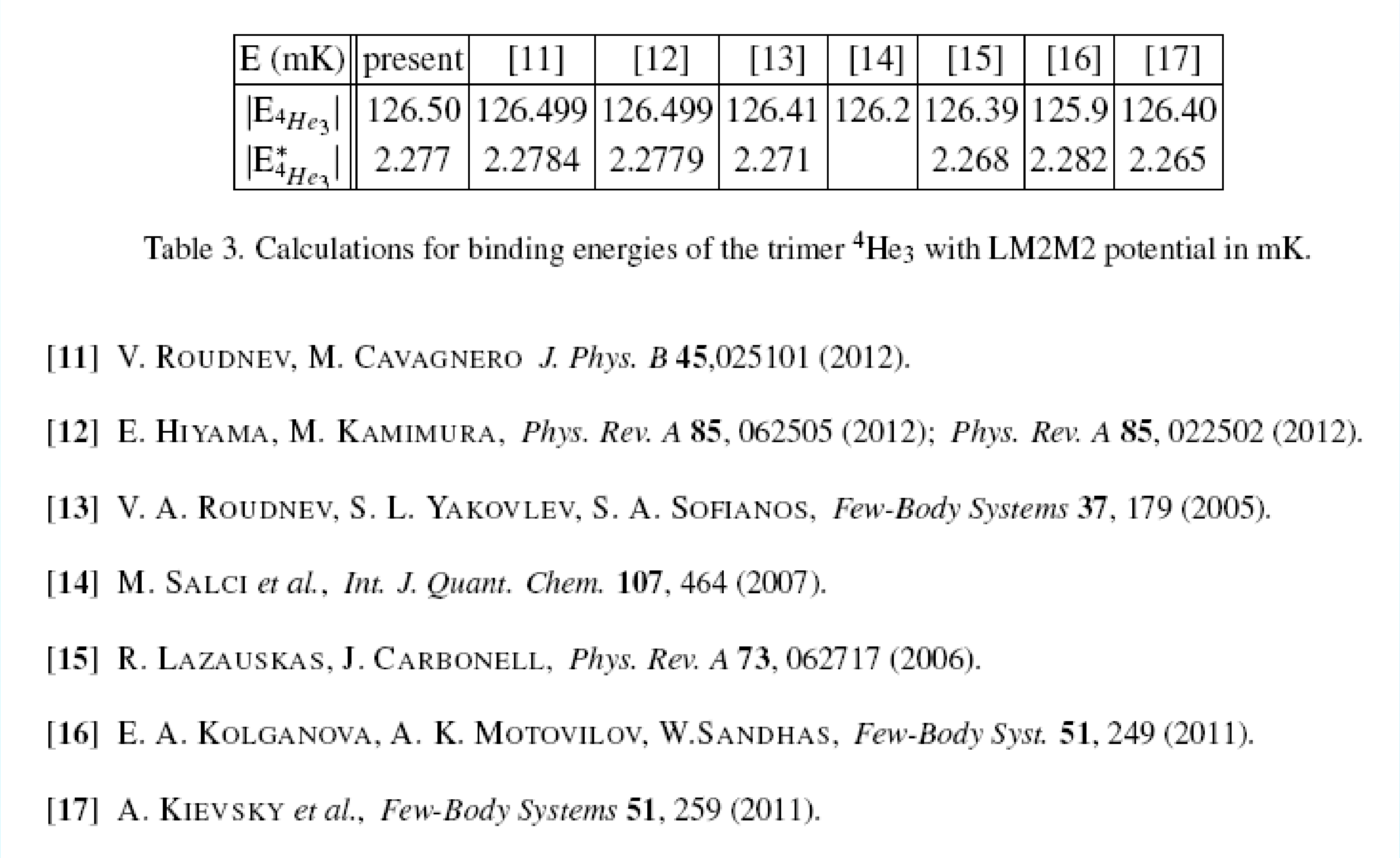
$$\Phi_l(x,y;p) = \delta_{l0}\psi_d(x)\{\sin(py) + \exp(ipy)[a_0(p) + o(y^{-1/2})]\}$$
$$+\frac{\exp(i\sqrt{E}\rho)}{\sqrt{\rho}}[A_l(\theta) + o(\rho^{-1/2})].$$

Here ψ_d is the dimer wave function, E stands for the scattering energy given by $E = \varepsilon_d + p^2$ with ε_d the dimer energy, and p is the relative momentum conjugate to the variable y. The coefficient $a_0(p)$ is nothing but the elastic scattering amplitude, while the functions $A_l(\theta)$ provides us, at $E > 0$, with the corresponding partial-wave Faddeev breakup amplitudes. The scattering length is given by

$$l_{sc} = -\frac{\sqrt{3}}{2}\lim_{p\rightarrow 0}\frac{a_0(p)}{p}$$

E.K, A.Motovilov,S.Sofianos
J.Phys.B **31**, 1279 (1998)

$^4He_3$



E (mK)	present	[11]	[12]	[13]	[14]	[15]	[16]	[17]
$\lvert E_{^4He_3} \rvert$	126.50	126.499	126.499	126.41	126.2	126.39	125.9	126.40
$\lvert E^*_{^4He_3} \rvert$	2.277	2.2784	2.2779	2.271		2.268	2.282	2.265

Table 3. Calculations for binding energies of the trimer $^4He_3$ with LM2M2 potential in mK.

[11] V. Roudnev, M. Cavagnero *J. Phys. B* **45**,025101 (2012).

[12] E. Hiyama, M. Kamimura, *Phys. Rev. A* **85**, 062505 (2012); *Phys. Rev. A* **85**, 022502 (2012).

[13] V. A. Roudnev, S. L. Yakovlev, S. A. Sofianos, *Few-Body Systems* **37**, 179 (2005).

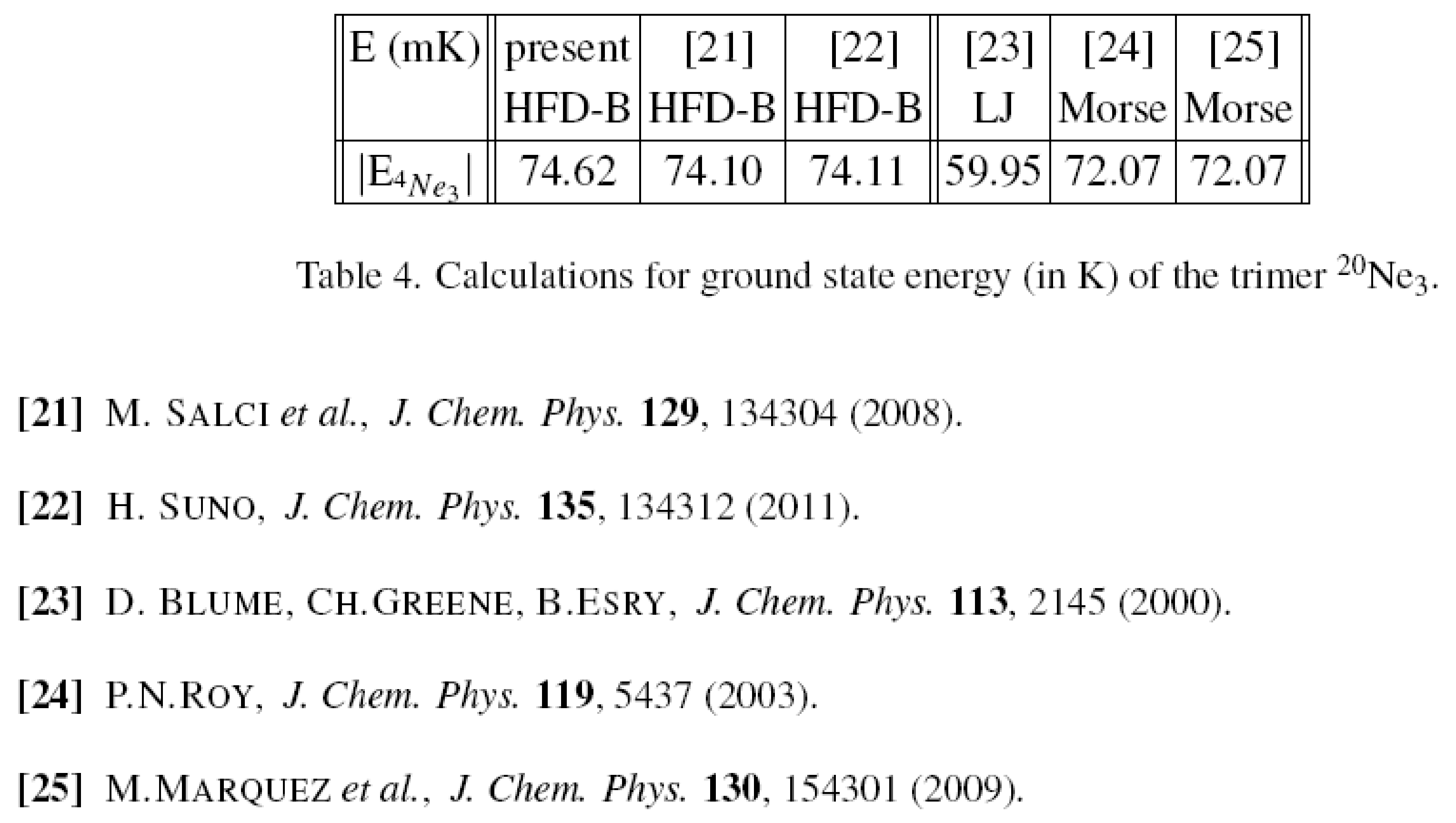
[14] M. Salci *et al.*, *Int. J. Quant. Chem.* **107**, 464 (2007).

[15] R. Lazauskas, J. Carbonell, *Phys. Rev. A* **73**, 062717 (2006).

[16] E. A. Kolganova, A. K. Motovilov, W.Sandhas, *Few-Body Syst.* **51**, 249 (2011).

[17] A. Kievsky *et al.*, *Few-Body Systems* **51**, 259 (2011).

$^4Ne_3$



E (mK)	present HFD-B	[21] HFD-B	[22] HFD-B	[23] LJ	[24] Morse	[25] Morse
$\lvert E_{^4Ne_3} \rvert$	74.62	74.10	74.11	59.95	72.07	72.07

Table 4. Calculations for ground state energy (in K) of the trimer $^{20}Ne_3$.

[21] M. SALCI *et al.*, *J. Chem. Phys.* **129**, 134304 (2008).

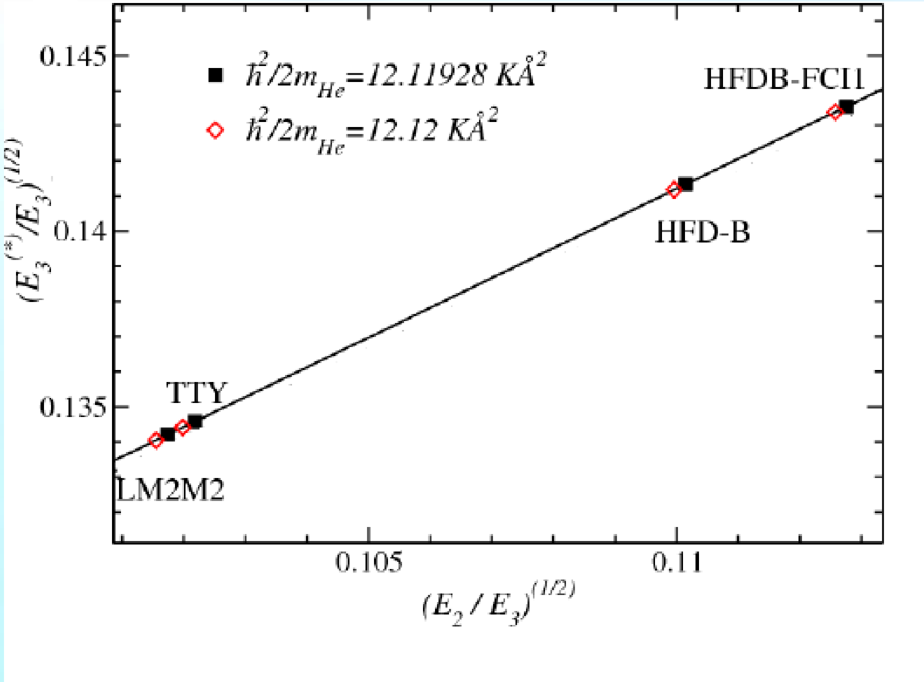
[22] H. SUNO, *J. Chem. Phys.* **135**, 134312 (2011).

[23] D. BLUME, CH.GREENE, B.ESRY, *J. Chem. Phys.* **113**, 2145 (2000).

[24] P.N.ROY, *J. Chem. Phys.* **119**, 5437 (2003).

[25] M.MARQUEZ *et al.*, *J. Chem. Phys.* **130**, 154301 (2009).

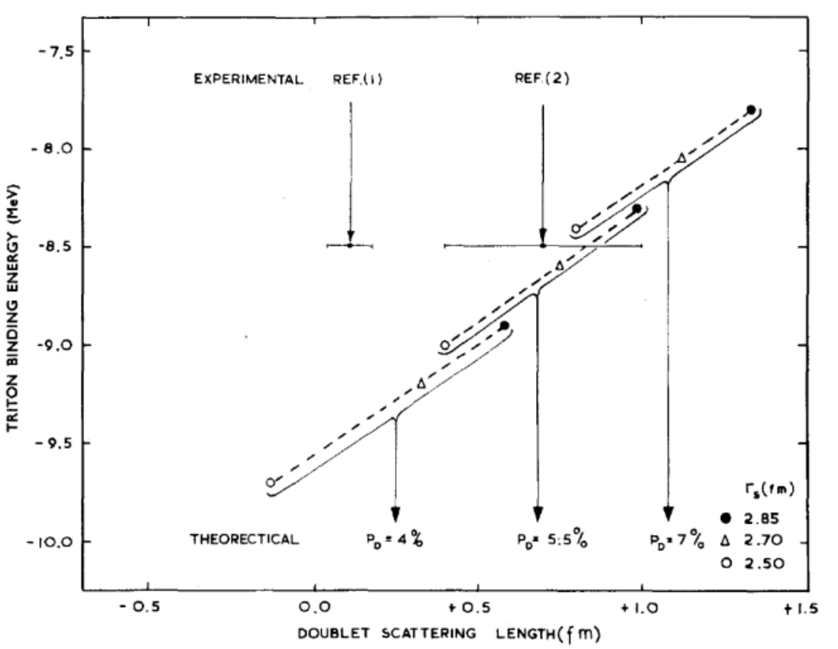
	*	[22]
E_0 (He_3) HFD-B	133.0 mK	130.57 mK

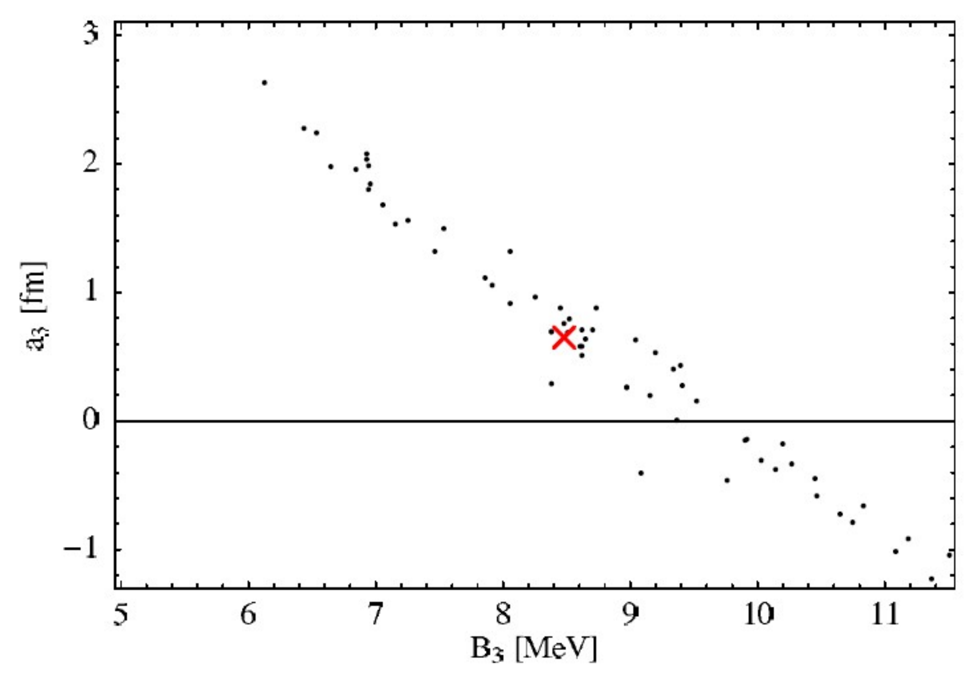


V. Roudnev and M. Cavagnero //*J. Phys. B* **45,** 025101 (2012)

^{3}H - system

Three-body, theory





Phillips line from the original paper, showing the unexpected linear correlation

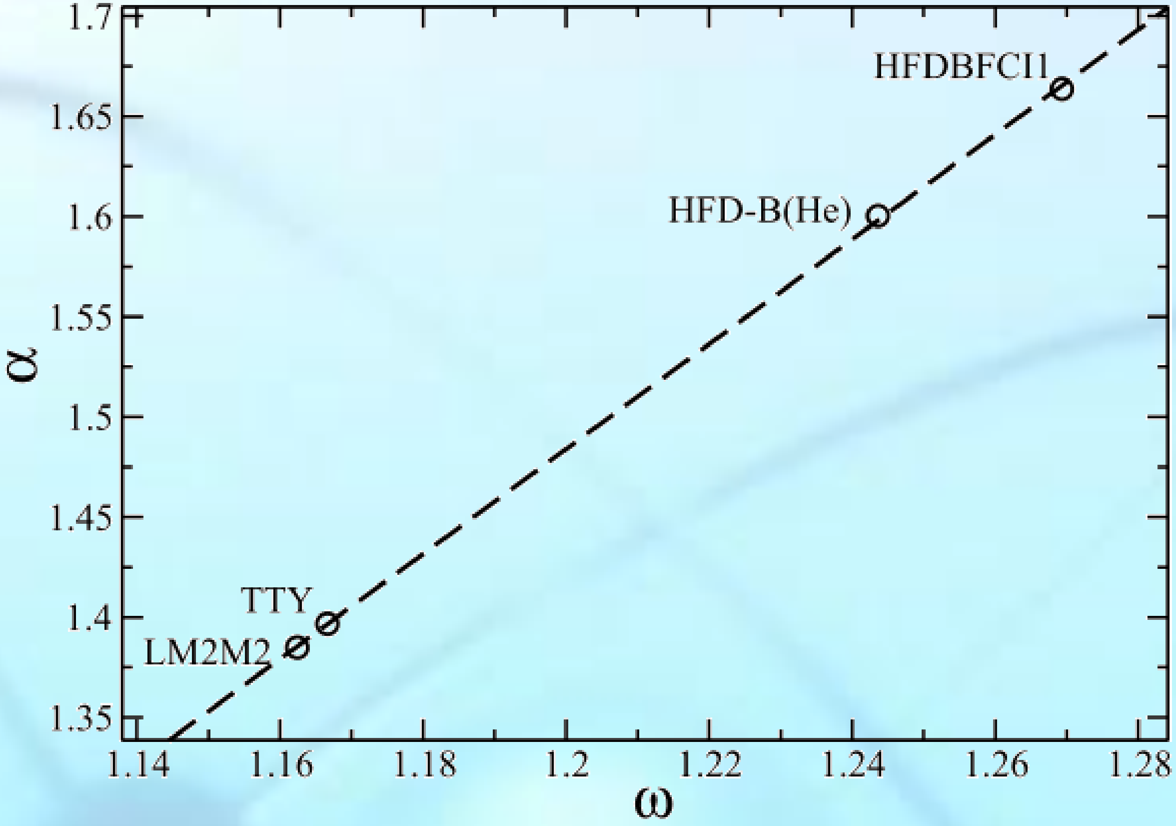
A.C. Phillips Nucl. Phys A **107**, 209 (1968)

$^4He_2$ - ^4He

Three-body, theory

$$E_2 - E_3 \approx 1/(2m_{12}a_3^2)$$

V.Efimov, E.G.Tkachenko,
Phys.Lett. B **157**, 108 (1985)



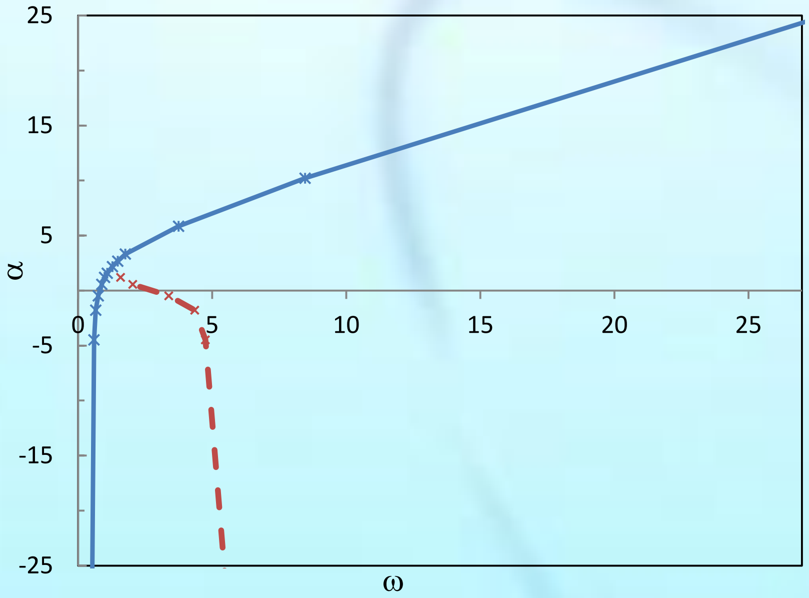
$$\alpha \equiv a_3\sqrt{-\mu E_2} \propto 1/\sqrt{E_3/E_2 - 1} \equiv \omega$$

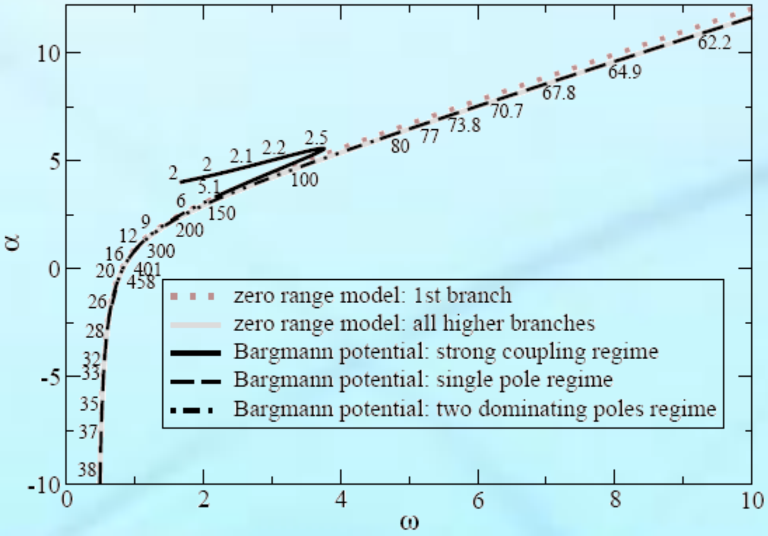
V.Roudnev, M.Cavagnero
Phys.Rev.Lett. **108**, 110402 (2012)

$^{4}He_{2}$ - ^{4}He

Three-body, theory

$$V(x) = \lambda\, V_{HFD-B}(x)$$



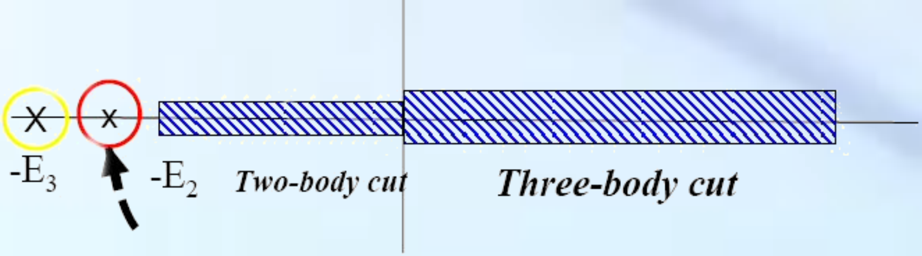


$$\alpha \equiv a_3\sqrt{-\mu E_2} \propto 1/\sqrt{E_3/E_2-1} \equiv \omega$$

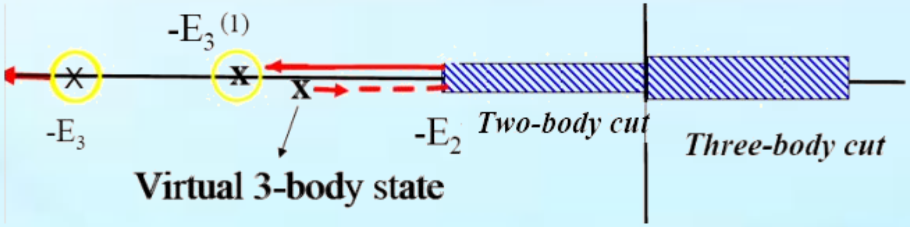
E.A.K, *Few-Body Syst.* **55**, 957 (2014)

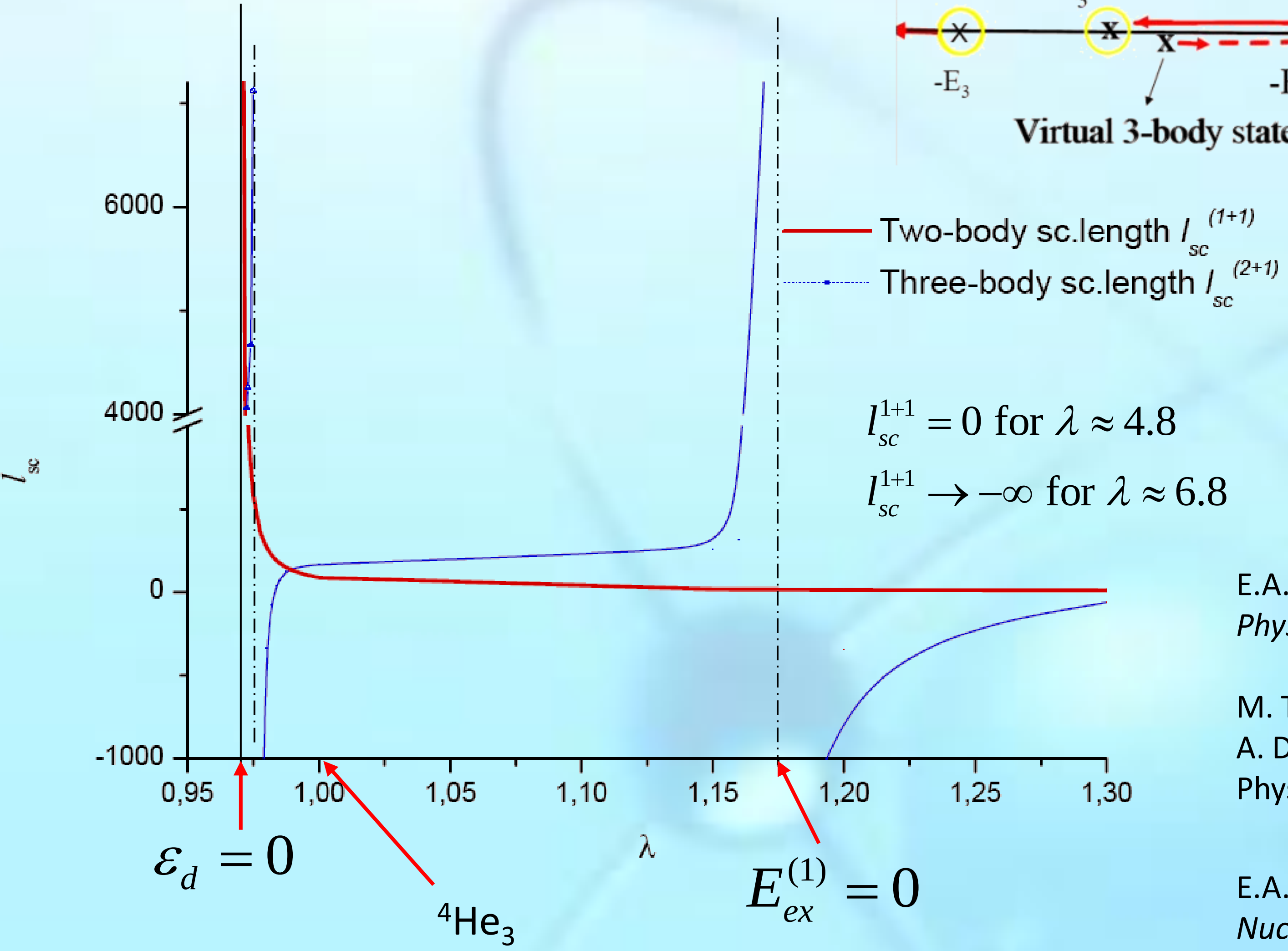
V.Roudnev, M.Cavagnero
Phys.Rev.Lett. **108**, 110402 (2012)

$^4He_2$ - ^4He



$$V(x) = \lambda V_{HFD-B}(x)$$





$$l_{sc}^{1+1} = 0 \text{ for } \lambda \approx 4.8$$

$$l_{sc}^{1+1} \rightarrow -\infty \text{ for } \lambda \approx 6.8$$

E.A.K, A.Motovilov,
Phys. At. Nucl. **62**, 1179 (1999)

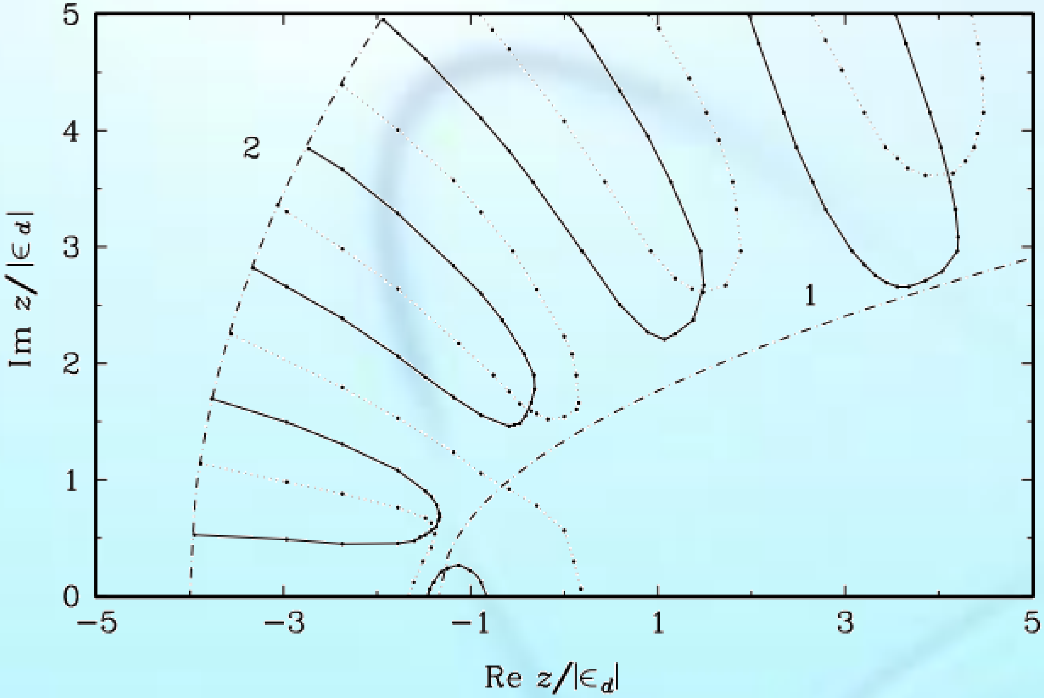
M. T. Yamashita, T. Frederico,
A. Delfino, L. Tomio ,
Phys. Rev. A 66, 052702 (2002)

E.A.K, A.Motovilov, W.Sandhas
Nucl.Phys. A **790**, 752 (2007)

$^{4}He_{2}$ - ^{4}He

Three-body, theory

S-matrix



$$\lambda = 1$$

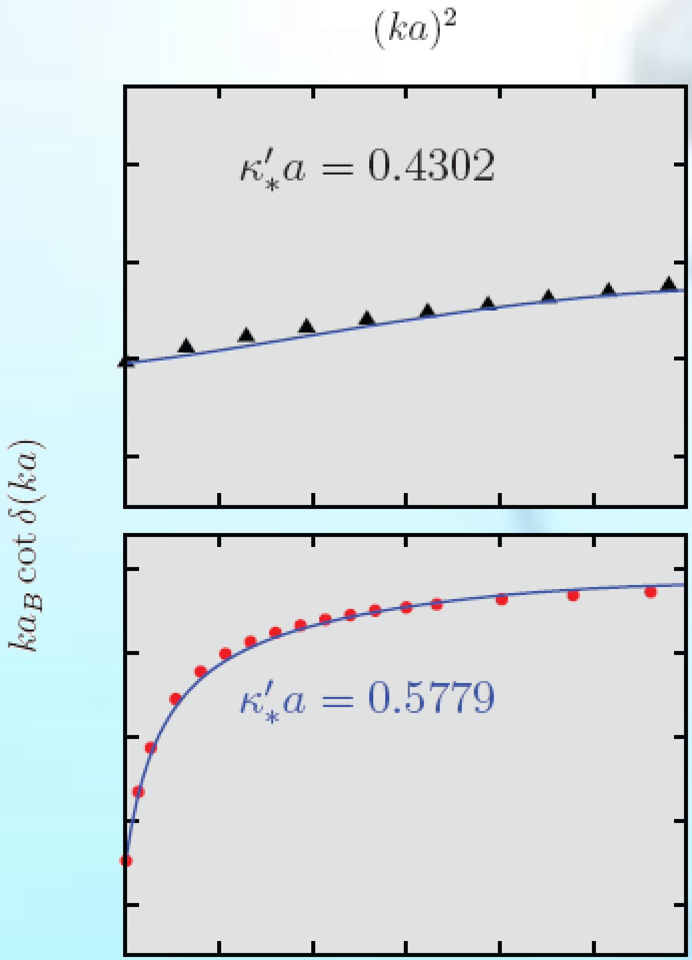
A.K.Motovilov, *Math.Nachrichten* **187**,147(1997)

E.A.K, A.K.Motovilov *Phys.At.Nucl.* **60**, 235 (1997)

E.A.K., A.K.Motovilov,Y.K.Ho *Nucl.Phys.A* **684**, 623 (2001)

Fig. 1. Root locus curves of the real and imaginary parts of the scattering matrix $\mathrm{S}_0(z)$ in case of helium trimer. The solid lines correspond to $\mathrm{Re}\,\mathrm{S}_0(z) = 0$, while the tiny dashed lines, to $\mathrm{Im}\,\mathrm{S}_0(z) = 0$. The Numbers 1, 2 denote the boundaries of the domains $\Pi^{(S)}$ and $\Pi^{(\Psi)}$, respectively. Complex roots of the function $\mathrm{S}_0(z)$ are represented by the crossing points of the curves $\mathrm{Re}\,\mathrm{S}_0(z) = 0$ and $\mathrm{Im}\,\mathrm{S}_0(z) = 0$ and are located at $(-2.34 + \mathrm{i}\,0.96)$ mK, $(-0.59 + \mathrm{i}\,2.67)$ mK, $(2.51 + \mathrm{i}\,4.34)$ mK and $(6.92 + \mathrm{i}\,6.10)$ mK.

$$V(x) = \lambda V_c(x)$$



^{4}He – ^{4}He$_2$ scattering

A.Kievsky, M.Gattobigio,
Phys. Rev. A **87**, 052719 (2013)

the same parametrization describes
a very different system: nucleon-deuteron
scattering below the deuteron breakup
Threshold and elastic atom-dimer scattering
below the dimer breakup threshold.

n – d scattering
in the doublet channel

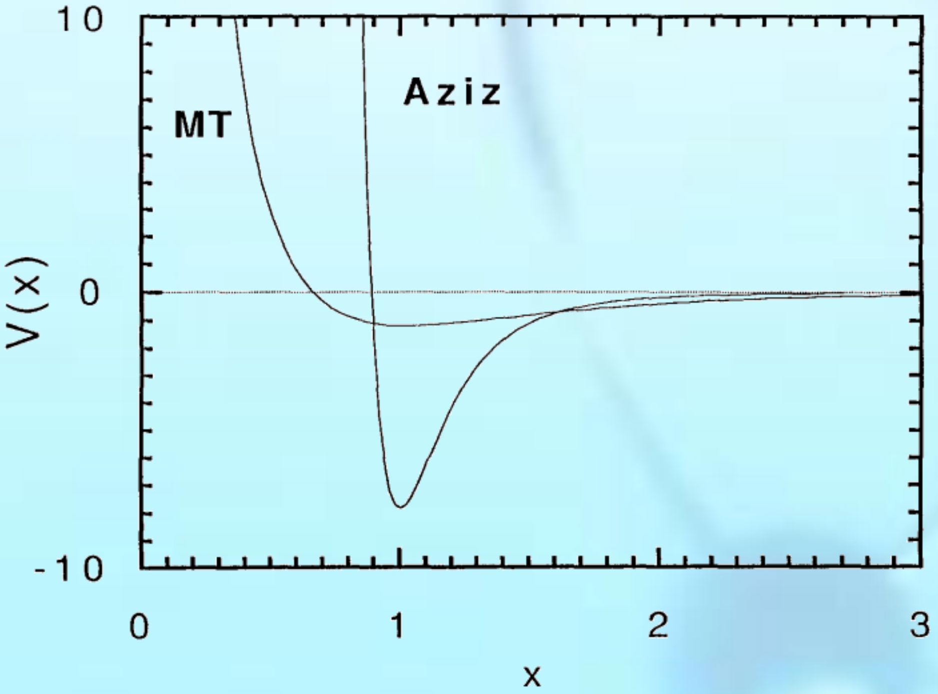
$$ka_B \cot\delta = c_1(ka) + c_2(ka)\cot[s_0 \ln(\kappa'_* a) + \phi(ka)] \,.$$

^{3}H - system

MT I-III

E_d = - 2.224 MeV

Virtual state of (nnp) -2.69 MeV (0.47 MeV)

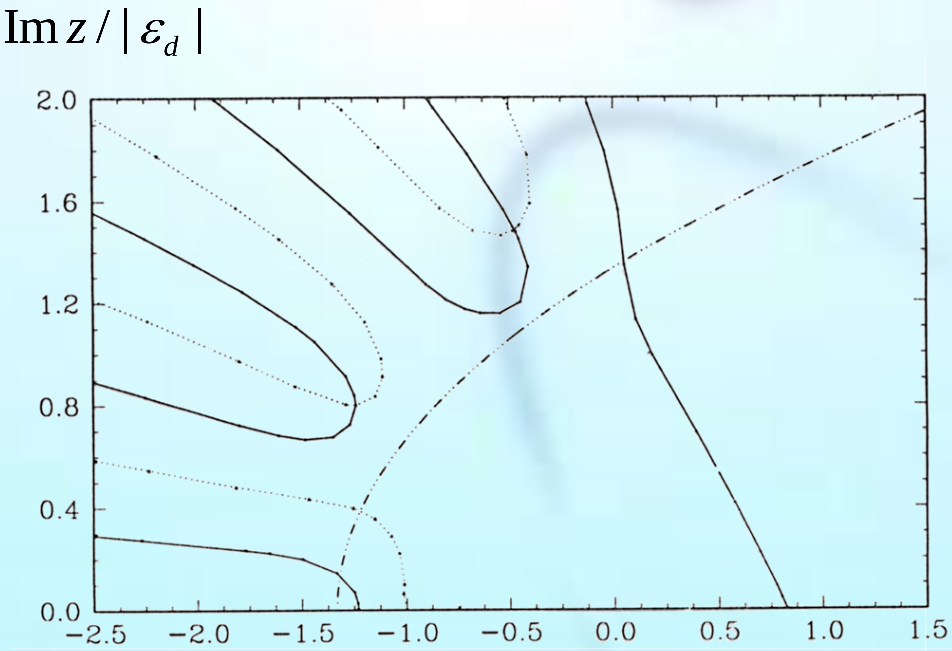


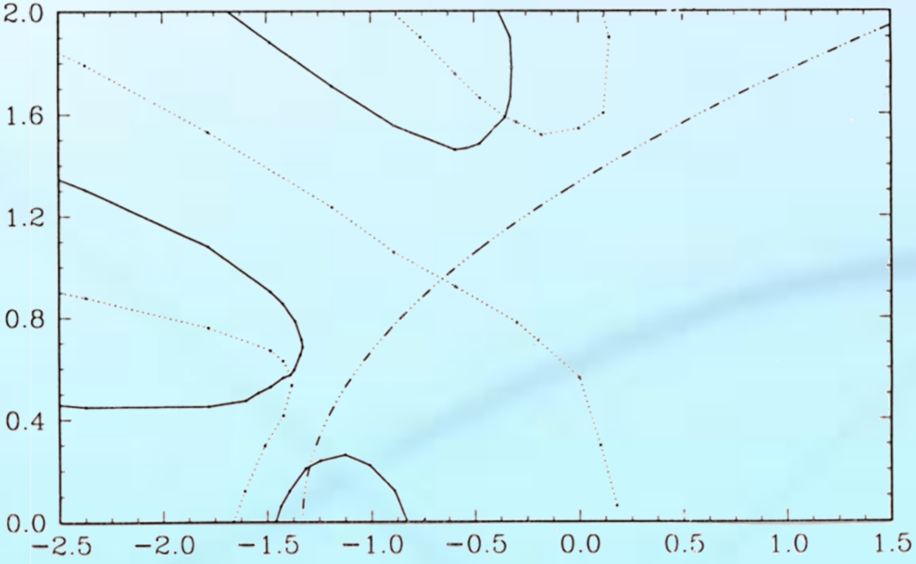
A=3
Nucl. Phys. A **848**, 1 (2010) 0.48 MeV
Orlov, Nikitina *Phys.At.Nucl.* 69 (2006)

Fig. 1. The dimensionless MT I-III $^3S_1$ nuclear and the HFD-B atomic potentials. The length unit for each potential is the position of its minimum (L). The potentials are made dimensionless according to the Schrödinger equation, i.e. $v = L^2 V/(\hbar^2/m)$

Carbonell, Gignoux, Merkuriev *FBS* **15**, 15 (1993)

Root locus curve of scattering matrix





$\mathrm{Re}\, z / |\varepsilon_d|$

^{3}H

^{4}He$_3$

S-matrix root lines in nnp system

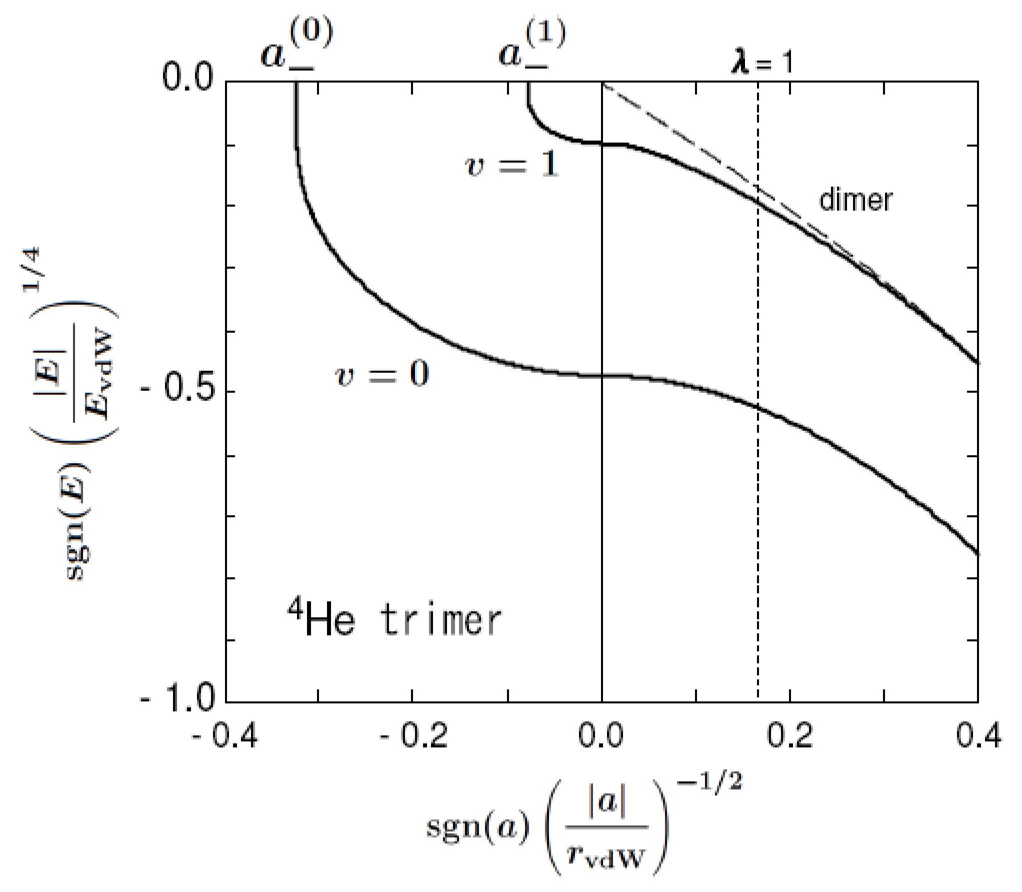
S-matrix root lines in ^{4}He$_3$ system

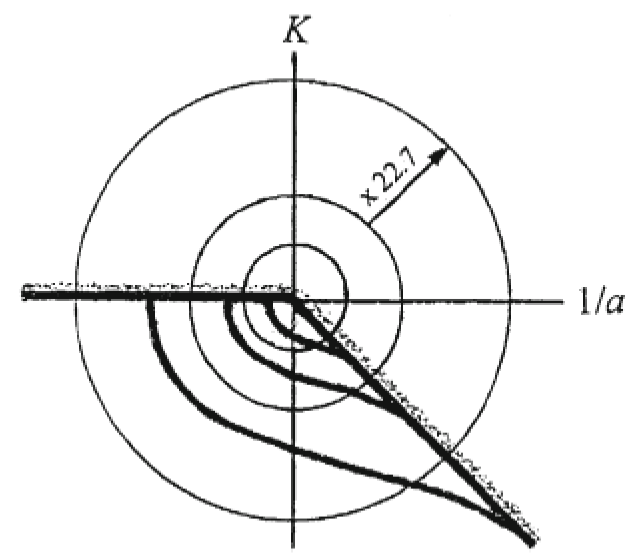
Solid line - Re(S)=0, tiny dashed line – Im(S)=0

E.A.K, *Few-Body Syst.* **55**, 957 (2014)

Three-body parameter

E.Hiyama, M.Kamimura, arxiv: 1409.2501



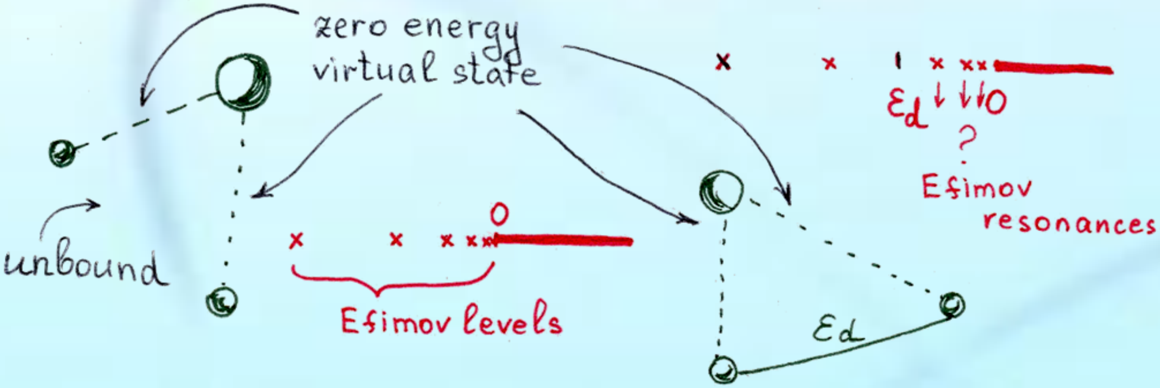


$$\lim_{n\to\infty}\frac{E_{n+1}}{E_n} = \exp(-2\pi/\omega_0) \approx \frac{1}{515.035},$$

$$N_{\mathrm{Efi}} \simeq \frac{\omega_0}{\pi}\ln\left|\frac{\ell_{\mathrm{sc}}^{(2)}}{r_0}\right|.$$

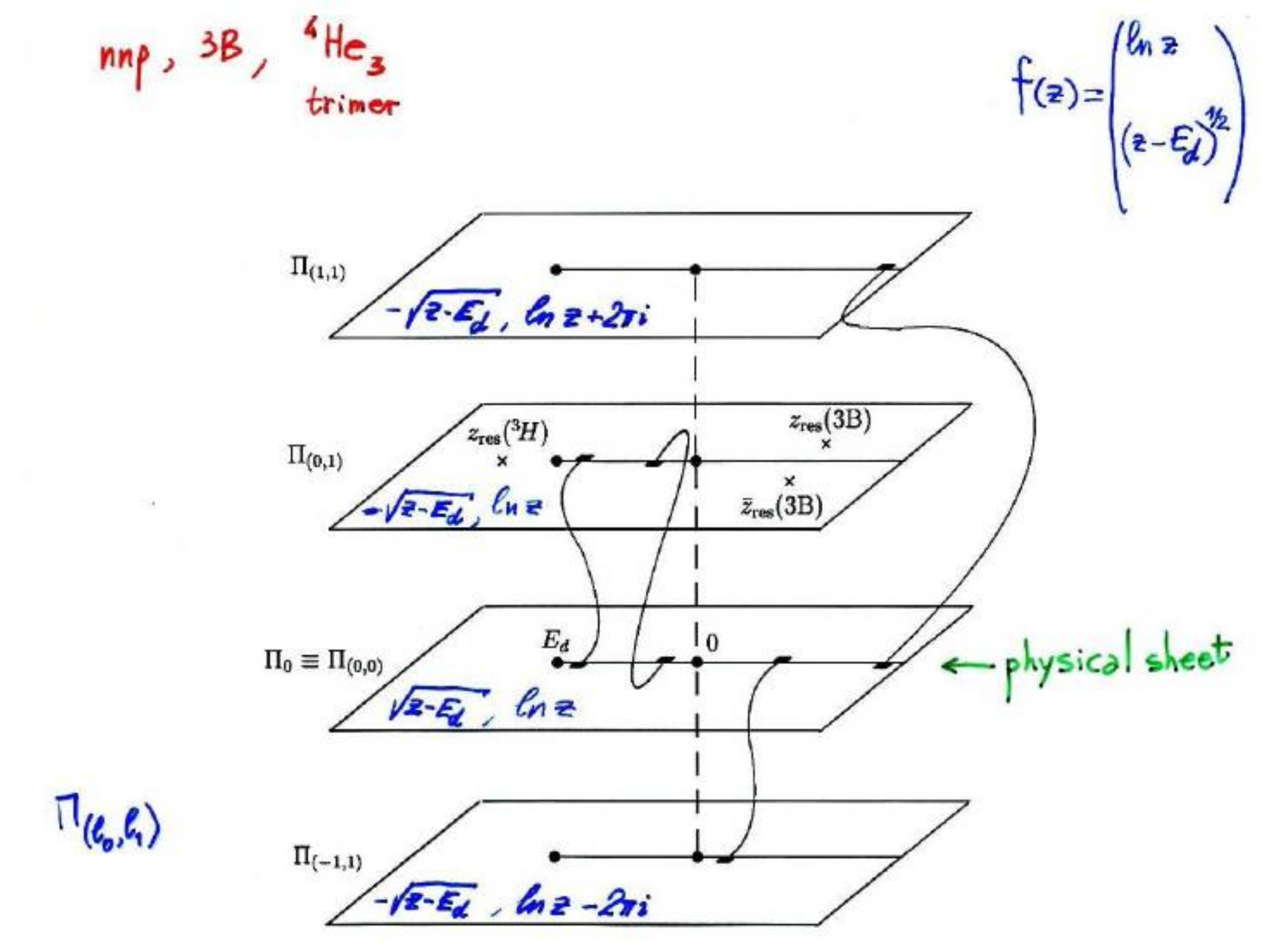
$$\left[T + \sum_{1=i<j}^{A} \lambda V_2(r_{ij}) - E_A\right]\Psi_A = 0,$$

V.Efimov, *Few-Body Syst.* **51**, 79 (2011)


zero energy
virtual state
unbound
0
Efimov levels
εd ↓ ↓↓0
?
Efimov
resonances
εd

BACK UP SLIDES

$^4He_3$ and nnp systems



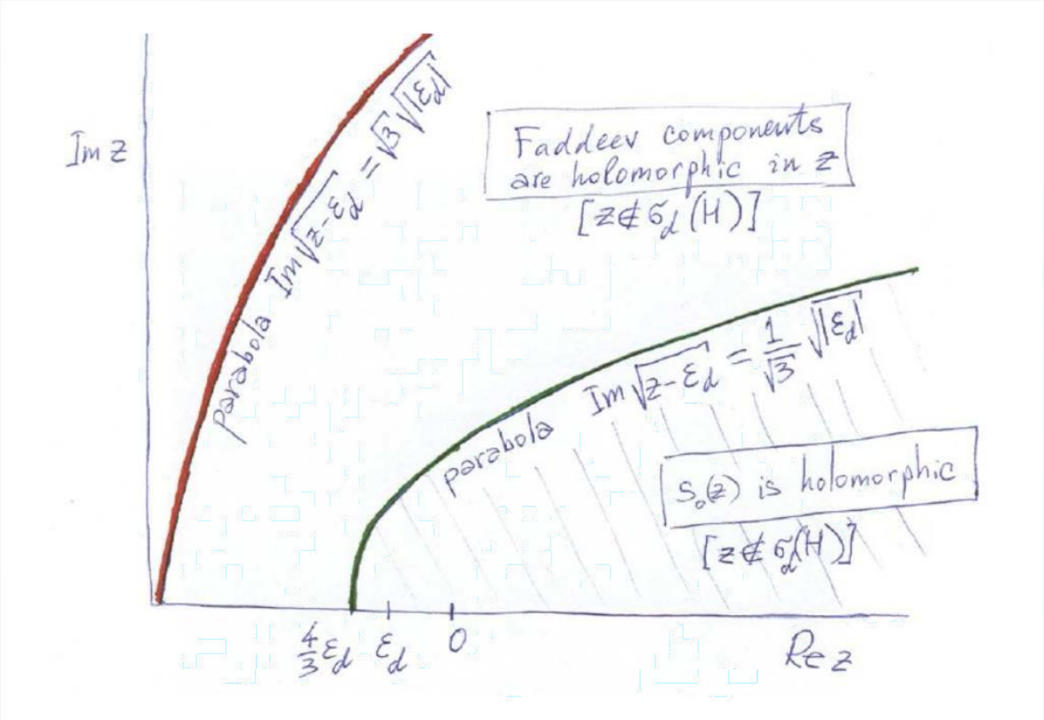
$^4He_3$

Three-body, theory
resonances

$$E \longrightarrow z$$

$$a_0(E) \longrightarrow a_0(z)$$

$$S_0(z) = 1 + 2ia_0(z)$$



LJ potential model [1]

Ne_2 (K)	Ar_2 (K)	Kr_2 (K)	Xe_2 (K)
20,178	101,454	150,716	210,674
3,859	71,019	126,280	188,599
0,5981	47,139	104,534	168,059
	29,157	85,329	148,985
	16,334	68,552	131,357
	7,892	54,038	115,107
	2,985	41,689	100,213
	0,693	31,308	86,630
		22,780	74,270
		15,924	63,111
		10,578	53,085
		6,576	44,171
		3,723	36,279
		1,837	29,366
		0,722	23,364
		0,230	18,229
			13,916
			10,315
			7,380
			5,068

Ne_2 (K)	Ar_2 (K)	Kr_2 (K)	Xe_2 (K)
20,179	101,361	150,719	210,665
3,861	70,799	126,294	188,596
0,0429	46,863	104,542	168,051
	28,871	85,339	148,986
	16,084	68,554	131,354
	7,704	54,050	115,120
	2,865	41,685	100,226
	0,6821	31,313	86,626
	0,025	22,780	74,277
		15,925	63,114
		10,580	53,099
		6,573	44,173
		3,721	36,281
		1,836	29,368
		0,725	23,374
		0,1859	18,242
		0,014	13,909
			10,313
			7,389
			5,072
			3,295
			1,988
			1,080
			0,501
			0,178
			0,036
			6,360E-04

[1] Leither D.M., Doll J.D., Whitnell R.M. // J. Chem. Phys. 1991. Vol. 94. P. 6644.