Low-energy reactions involving halo nuclei: a microscopic version of CDCC

P. Descouvemont
Université Libre de Bruxelles, Belgium

In collaboration with
- M.S. Hussein (USP)
- E.C. Pinilla (ULB)
- J. Grineviciute (ULB)

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   P.D., M. Hussein, Phys. Rev. Lett. 111 (2013) 082701
7. Application to $^{17}F+^{208}Pb$
   J. Griveniciute, P.D., Phys. Rev. C 90 (2014) 034616
8. Application to $^{8}B+^{208}Pb$, $^{8}B+^{58}Ni$
   Very preliminary!
9. Conclusion

1. Introduction

Main goal in nuclear physics: understanding the structure of exotic nuclei

Available data:

- Elastic (inelastic) scattering
- Breakup
- Fusion
- Etc

Role of the theory: how to interpret these scattering data?

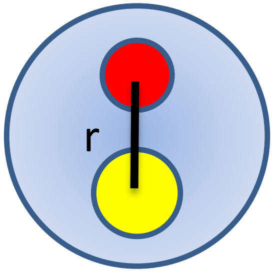
Combination of two ingredients

1. **Description of the scattering process**
   - Low energies (around the Coulomb barrier): optical model, **CDCC**
   - High energies (typically ~50-100 MeV/u): eikonal (+variants)
2. **Description of the projectile**
   - 2-body, 3-body
   - **Microscopic**

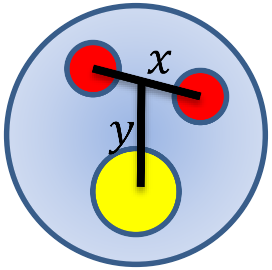
Spectroscopy of light nuclei

2. Spectroscopy of light nuclei – theoretical models

Non-microscopic models: 2-body or 3-body



Ex: $^{11}Be = ^{10}Be + n$
$^{7}Li = \alpha + t$
$^{15}C = ^{14}C + n$



Ex: $^{6}He = \alpha + n + n$
$^{9}Be = \alpha + \alpha + n$
$^{12}C = \alpha + \alpha + \alpha$

- A substructure is assumed (2-body, 3-body,etc.)
- Internal structure of the constituents (clusters) is neglected
- The constituents interact by a nucleus-nucleus potential
  2 body: $H_0(r) = T_r + V_{12}(r)$
  3 body $H_0(x,y) = T_x + T_y + V_{12}(x) + V_{13}(x,y) + V_{23}(x,y)$
- Pauli principle approximated (appropriate choice of the nucleus-nucleus potential)

2. Spectroscopy of light nuclei – theoretical models

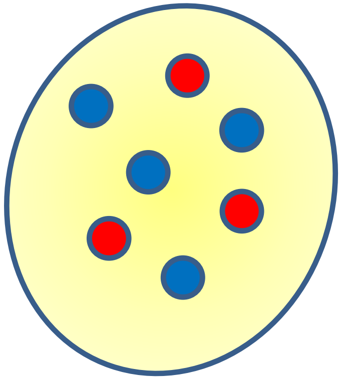
Microscopic models

- Pauli principle taken into account
- Depend on a nucleon-nucleon (NN) interaction → more predictive power

$$H_0(r_1, \dots r_A) = \sum_i T_i + \sum_{ij} V_{ij}$$

- Two approaches

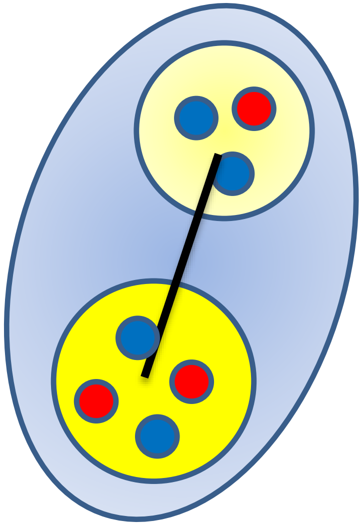
« Ab initio » (No-cluster approximation)



- Try to find an exact solution of the (A-body) Schrödinger equation
- Use realistic NN interactions (fitted on NN properties)
- Variants: VMC, NCSM, FMD, AMD, etc.
- In general:
  - $A \leq 12$
  - Scattering states difficult/impossible to obtain
  - Not well adapted to halo structure, resonant states

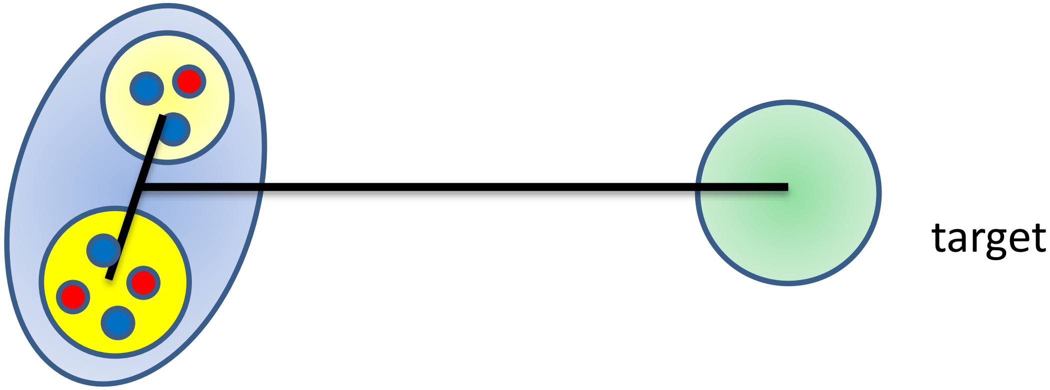
2. Spectroscopy of light nuclei – theoretical models

Cluster approximation



- Wave function defined by
$\Psi = \mathcal{A}\Phi_1\Phi_2 g(r)$ (Φ_1, Φ_2=internal wave functions (shell-model))
=Resonating Group Method (RGM)
- Effective NN interactions (Minnesota, Volkov)
- Extensions to 3 clusters, 4 clusters, etc.
- Core excitations can be easily included
- Scattering states possible
- Calculations easier than in ab initio theories
→Many applications (up to Ne isotopes) in spectroscopy and scattering

➔ Next step: using microscopic wave functions in collisions



Application to nucleus-nucleus scattering

3. Application to nucleus-nucleus scattering

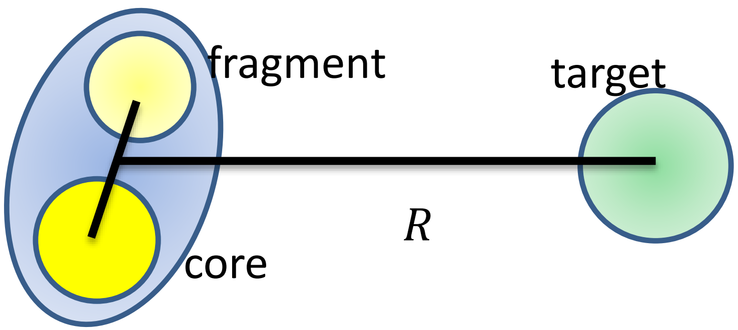
The CDCC method (Continuum Discretized Coupled Channel)

- Structure of the target is neglected
- Valid at low energies (partial-wave expansion the total wave function)
- Introduced to describe deuteron induced reactions (deuteron weakly bound)

G. Rawitscher, Phys. Rev. C 9 (1974) 2210

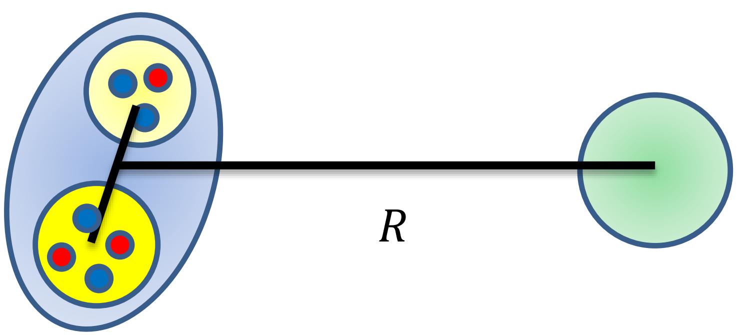
N. Austern et al., Phys. Rep. 154 (1987) 126

Standard CDCC (2-body projectile)



$$H = H_0(r) - \frac{\hbar^2}{2\mu}\Delta_R + V_{ct}\left(-\frac{A_f}{A}r + R\right) + V_{ft}\left(\frac{A_c}{A}r + R\right)$$

Microscopic CDCC (2-cluster projectile)

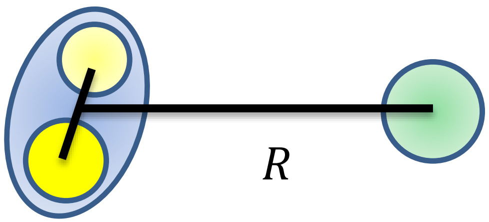


$$H = H_0(r_1 \dots r_A) - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$$

3. Application to nucleus-nucleus scattering

Comparison between both variants

Non-microscopic CDCC



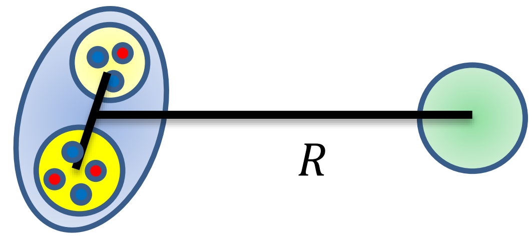
- $H = H_0(r) - \frac{\hbar^2}{2\mu}\Delta_R + V_{ct}\left(-\frac{A_f}{A}r + R\right) + V_{ft}\left(\frac{A_c}{A}r + R\right)$

- Depends on **nucleus-target** interactions between the core/fragment and the target

- **Approximate wave functions** of the projectile

- Core excitations **difficult** (definition of the potentials?)

Microscopic CDCC



- $H = H_0(r_1 \dots r_A) - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$

- Depends on a **nucleon-target** interactions (in general well known) Different for neutron-target and proton-target

- **Accurate wave functions** of the projectile

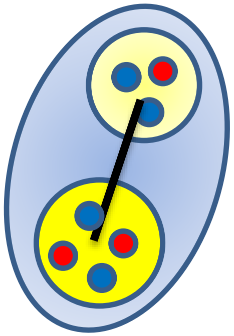
- Core excitations « **automatic** »

Outline of microscopic CDCC

4. Outline of microscopic CDCC

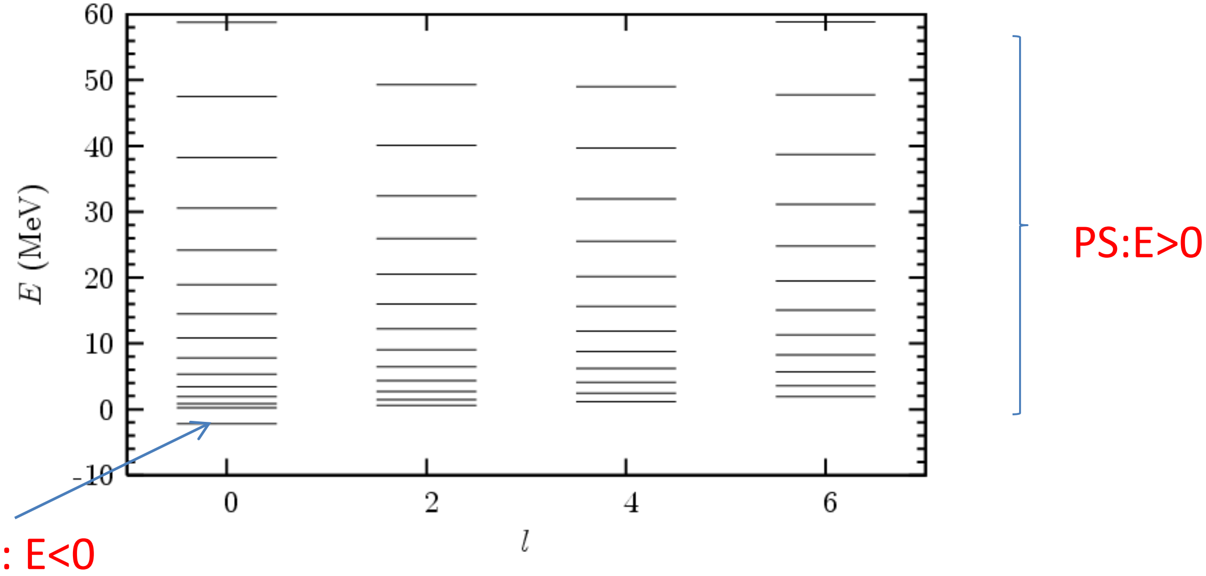
First step: wave functions of the projectile

Solve $H_0\Phi_k^J = E_k\Phi_k^J$ for several J: ground-state but also additional J values



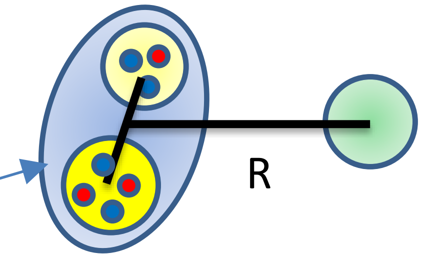
with $E_k < 0$: physical states
$E_k > 0$: pseudostates (approximation of the continuum)
$\Phi_k^J = \mathcal{A}\Phi_1\Phi_2 g_k^J(r)$: combination of Slater determinants (RGM)

Example: deuteron d=p+n



4. Outline of microscopic CDCC

Second step: wave function for projectile + target



$$H = H_0 - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$$

Expansion over projectile states: Microscopic wf

$$\Psi^{JM\pi}(\boldsymbol{r_i}, \boldsymbol{R}) = \sum_{jLk} \left[\Phi_k^j(\boldsymbol{r_1}, \boldsymbol{r_2}, \dots, \boldsymbol{r_A}) \otimes Y_L(\Omega_R)\right]^{JM} \chi_{jkL}^{J\pi}(R)$$

We define $c = (j, k, L)$, j, k: projectile state (bound states + continuum states)

L: relative angular momentum between projectile and target

→ Set of coupled equations

$$\left(-\frac{\hbar^2}{2\mu}\Delta_R + E_c \; - E\right)\chi_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R)\, \chi_{c'}^{J\pi}(R) = 0$$

- Common to all CDCC variants
- Solved with: Numerov algorithm / **R-matrix method**
- From asymptotic behaviour of $\chi_c^{J\pi}(R)$: → scattering matrix → cross sections

4. Outline of microscopic CDCC

Coupling potentials $V_{k,k'}^{jm,j'm'}(\boldsymbol{R}) =< \Phi_k^{jm}(\boldsymbol{r_i}) \left| \sum_i v(\boldsymbol{r_i} - \boldsymbol{R}) \right| \Phi_{k'}^{j'm'}(\boldsymbol{r_i}) >$

- $\Phi_k^{jm}(\boldsymbol{r_i})$=combination of Slater determinants
- $v(\boldsymbol{r_i} - \boldsymbol{R})$ =nucleon-target interaction (including Coulomb)

→ Standard one-body matrix element (such as kinetic energy, rms radius…)
→ Must be expanded in multipoles:

$$V_{k,k'}^{jm,j'm'}(\boldsymbol{R}) = \sum_{\lambda} < jm\, \lambda m'-m | j'm' > V_{k,k'}^{j,j'}(R) Y_{\lambda}^{m'-m}(\Omega_R)$$

4. Outline of microscopic CDCC

→ Two calculation methods:

1) Brink's formula for Slater determinants

One-body matrix elements (kinetic energy, rms radius, densities, etc.)

- Matrix elements between individual orbitals φ_i : $M_{ij} =< \varphi_i | v(\boldsymbol{r}-\boldsymbol{R}) | \varphi_j >$
- Overlap matrix $B_{ij} =< \varphi_i | \varphi_j >$
- Angular momentum projection

2) Folding procedure

$$V_{k,k'}^{jm,j'm'}(\boldsymbol{R}) =< \Phi_k^{jm} \left| \sum_i v(\boldsymbol{r_i}-\boldsymbol{R}) \right| \Phi_{k'}^{j'm'} >$$

$$= \int d\boldsymbol{S}\, v(\boldsymbol{S}-\boldsymbol{R}) < \Phi_k^{jm} \left| \sum_i \delta(\boldsymbol{r}_i-\boldsymbol{S}) \right| \Phi_{k'}^{j'm'} >$$

$$= \int d\boldsymbol{S}\, v(\boldsymbol{S}-\boldsymbol{R}) \rho_{kk'}^{jm,j'm'}(\boldsymbol{S})$$

With $\rho_{kk'}^{jm,j'm'}(\boldsymbol{S}) =< \Phi_k^{jm} \left| \sum_i \delta(\boldsymbol{r}_i-\boldsymbol{S}) \right| \Phi_{k'}^{j'm'} >$ =nuclear densities

expanded in multipoles as $\rho_{kk'}^{jm,j'm'}(\boldsymbol{S}) = \sum_\lambda < jm\, \lambda m'-m | j'm' > \rho_{k,k'}^{jj'\lambda}(S)\, Y_\lambda^{m'-m}(\Omega_S)$

→ Test of the calculation

→ 2nd method more efficient since changing the potential is a minor work

4. Outline of microscopic CDCC

In practice:

- folding potentials are computed with Fourier transforms

$$\tilde{V}(q) = \tilde{v}(q)\tilde{\rho}(q)$$

- $V_{cc'}^{J\pi}(R)$ obtained from $V_{k,k'}^{jm,j'm'}(\boldsymbol{R})$ with additional angular momenum couplings

The R-matrix method

5. The R-matrix method

Scattering matrices, cross sections

Asymptotic form of the relative wave functions $\chi_c^{J\pi}(R) \to I_c(kR)\delta_{c\omega} - O_c(kR)U_{c\omega}^{J\pi}$

- $I_c(kR)$, $O_c(kR)$=incoming/outgoing Coulomb functions
- ω=entrance channel
- $U_{c\omega}^{J\pi}$=scattering matrix → cross sections: elastic, inelastic, transfer, etc.

R-matrix theory: based on 2 regions (channel radius a)

Lane and Thomas, Rev. Mod. Phys. 30 (1958) 257

P.D. and D. Baye, Rep. Prog. Phys. 73 (2010) 036301

Internal region: $R \le a$ | $R = a$ | External region: $R \ge a$ | R

Full Hamiltonian

$\chi_c^{J\pi}(R)$ expanded over a basis (N functions)

$$\chi_{c,int}^{J\pi}(\boldsymbol{R}) = \sum_{i=1}^{N} c_i \phi_i(R)$$

Only Coulomb

$$V_{cc'}^{J\pi}(R) = \frac{Z_p Z_t e^2}{R}\delta_{cc'}$$

$\chi_c^{J\pi}(R)$ has its asymptotic form

$$\chi_{c,ext}^{J\pi}(\boldsymbol{R}) = I_c(kR)\delta_{c\omega} - O_c(kR)U_{c\omega}^{J\pi}$$

matching at *R=a* provides: scattering matrices $\boldsymbol{U}^{J\pi}$ → cross sections

5. The R-matrix method

Matrix elements

$$C^{J\pi}_{cn,c'n'} = \langle \varphi_n \mid (T_c + \mathcal{L}_c + E_c - E)\delta_{cc'} + V^{J\pi}_{cc'} \mid \varphi_{n'} \rangle_{\text{int}}$$

- In general: integral over *R* (from *R=0* to *R=a*)
  Lagrange mesh: value of the potential at the mesh points ➔ very fast
- From matrix ***C*** → R matrix

$$R^{J\pi}_{c,c'} = \sum_{n,n'} (\boldsymbol{C}^{J\pi})^{-1}_{cn,c'n'} \varphi_n(a)\, \varphi_{n'}(a)$$

→ Collision matrix ***U*** → phase shifts, cross sections

- Typical sizes: ~100-200 channels c, N~30-40
- Test: collision matrix ***U*** does not depend on *N, a*
- Choice of *a*: compromize (*a* too small: R-matrix not valid, *a* too large: *N* large)

- Gauss approximation: $\int_0^a g(x)dx \approx \sum_{k=1}^{N} \lambda_k g(x_k)$
  - N= order of the Gauss approximation
  - x_k=roots of an orthogonal polynomial, λ_k=weights
  - If interval [0,*a*]: Legendre polynomials
    [0,∞]: Laguerre polynomials

- Lagrange functions for [0,1]: $f_i(x) \sim PN(2x-1)/(x-xi)$
  - x_i are roots of $P_N(2xi-1)=0$
  - with the Lagrange property: $f_i(x_j) = \lambda_i^{-1/2}\,\delta_{ij}$

- Matrix elements with Lagrange functions: Gauss approximation is used

$$< f_i|f_j >= \int f_i(x) f_j(x) dx \;\approx \sum_{k=1}^{N} \lambda_k f_i(x_k) f_j(x_k) \approx \delta_{ij}$$

$< f_i|T|f_j >$ analytical

$$< f_i|V|f_j >= \int f_i(x) V(x) f_j(x) dx \;\approx V(x_i)\delta_{ij}$$

⇒ no integral needed very simple!

Computer time: 2 main parts

- Matrix elements: very fast with Lagrange functions
- Inversion of (complex) matrix C → R-matrix (long times for large matrices)

For reactions involving halo nuclei:

- Long range of the potentials (Coulomb)

$$\frac{Z_1 Z_t e^2}{\boldsymbol{R}+\frac{A_2}{A_p}\boldsymbol{r}} + \frac{Z_2 Z_t e^2}{\boldsymbol{R}-\frac{A_1}{A_p}\boldsymbol{r}} = \sum_\lambda V_\lambda(r,R) P_\lambda(\cos\theta_{Rr})$$

$$V_{cc'}(R) \approx \frac{Z_p Z_t e^2}{R} + \frac{Z_t Q_p}{R^3} + \dots$$

Can be large (large quadrupole moments of PS)

- Radius *a* must be large
- Many basis functions (*N* large)

→

- Distorted Coulomb functions (FRESCO)
- Propagation techniques in the R-matrix (well known in atomic physics)

    Ref.: Baluja et al. Comp. Phys. Comm. 27 (1982) 299

    Well adapted to Lagrange-mesh calculations

Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$ at low energies

P.D., M. Hussein, Phys. Rev. Lett. 111 (2013) 082701

6. Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$

- Data: E_{lab}=27 to 60 MeV (Coulomb barrier ~ 35 MeV)

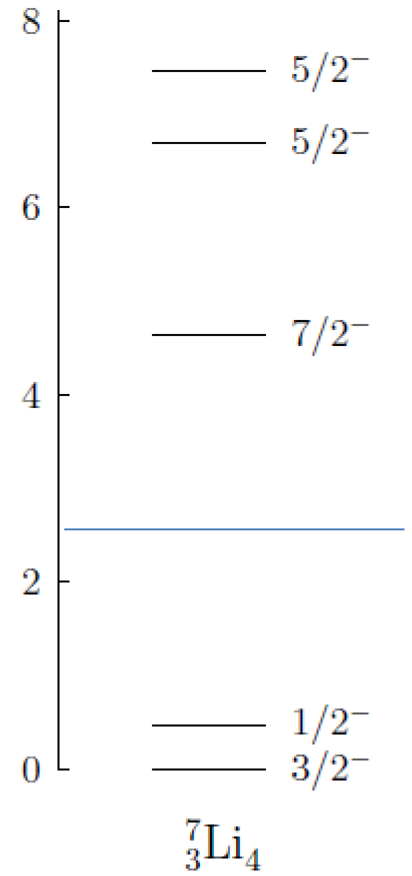
- **Non-microscopic calculation** at 27 MeV:
  - Parkar et al, PRC78 (2008) 021601
  - uses $\alpha - {}^{208}\mathrm{Pb}$ and $t - {}^{208}\mathrm{Pb}$ potentials renormalized by 0.6!

- **Microscopic calculation**
  - $^{7}\mathrm{Li}$ wave functions: include gs,$1/2^-$,$7/2^-$,$5/2^-$ and pseudostates (E>0)
    Nucleon-nucleon potential: Minnesota interaction
    Reproduces ^{7}Li/^{7}Be, α+^{3}He scattering, ^{3}He(α,γ)^{7}Be cross section

  - Q(3/2-)=−37.0 e.mb (GCM), -40.6 ± 0.8 e.mb (exp.)
    B(E2, $3/2^- \rightarrow 1/2^-$)=7.5 e^2fm^4 (GCM), 7.3 ± 0.5 e^2fm^4 (exp)

  - $n - {}^{208}\mathrm{Pb}$ potential:
    local potential of Koning-Delaroche (Nucl. Phys. A 713 (2003) 231)
  - $p - {}^{208}\mathrm{Pb}$ potential:
    only Coulomb (E_p=27/7 ~4 MeV, Coulomb barrier ~12 MeV)

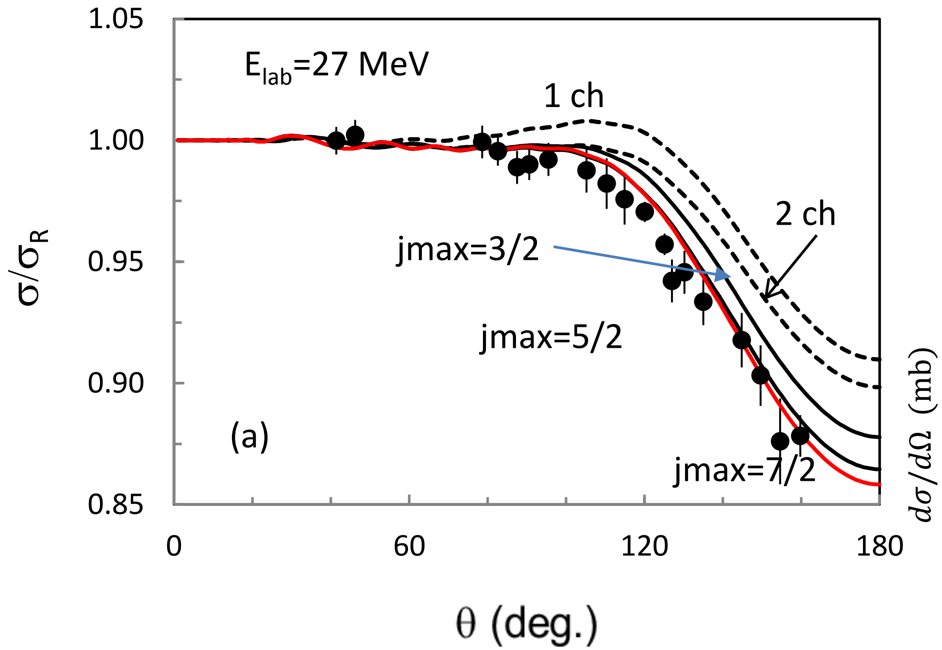
→ NO PARAMETER



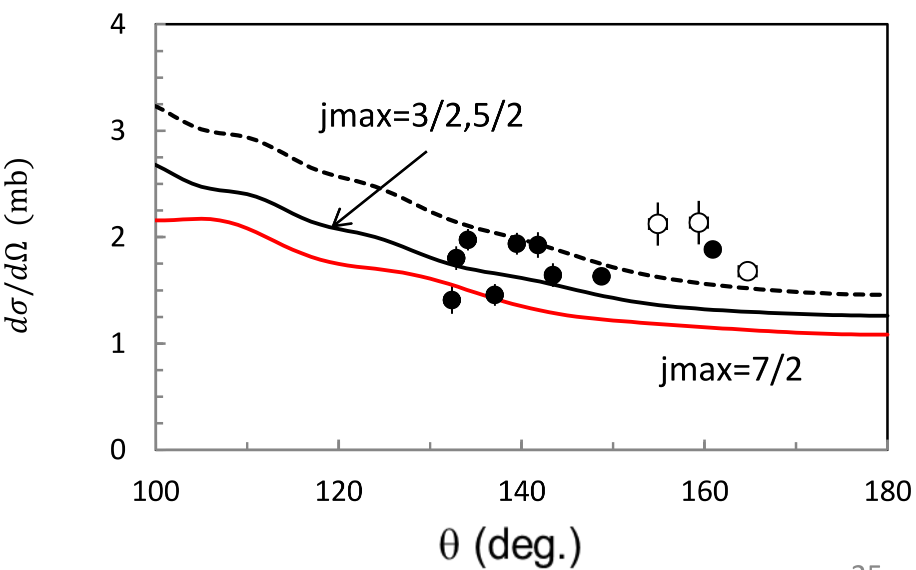
6. Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$

Convergence test: single-channel: $^{7}\mathrm{Li}(3/2^{-}) + {}^{208}\mathrm{Pb}$
two channels: $^{7}\mathrm{Li}(3/2^{-}, 1/2^{-}) + {}^{208}\mathrm{Pb}$
multichannel: $^{7}\mathrm{Li}(3/2^{-}, 1/2^{-}, \ldots) + {}^{208}\mathrm{Pb}$
pseudostates up to 20 MeV

Elastic scattering at Elab=27 MeV

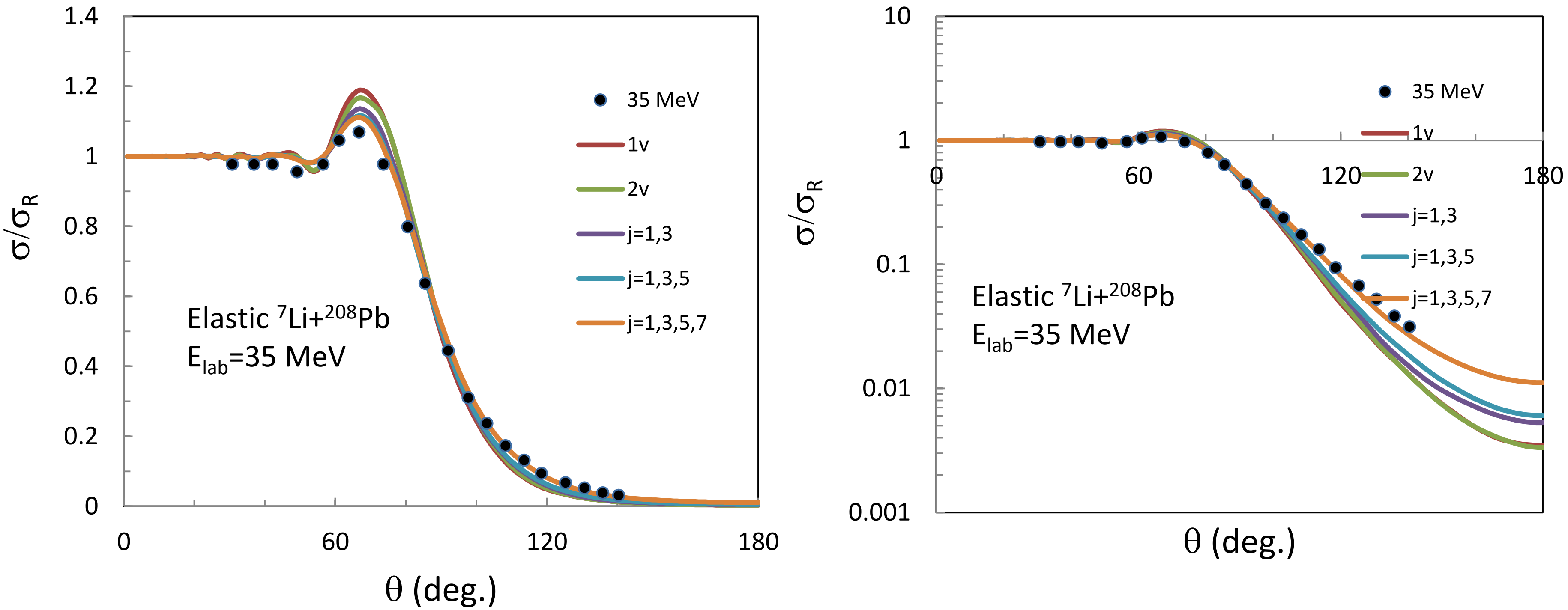


Inelastic scattering



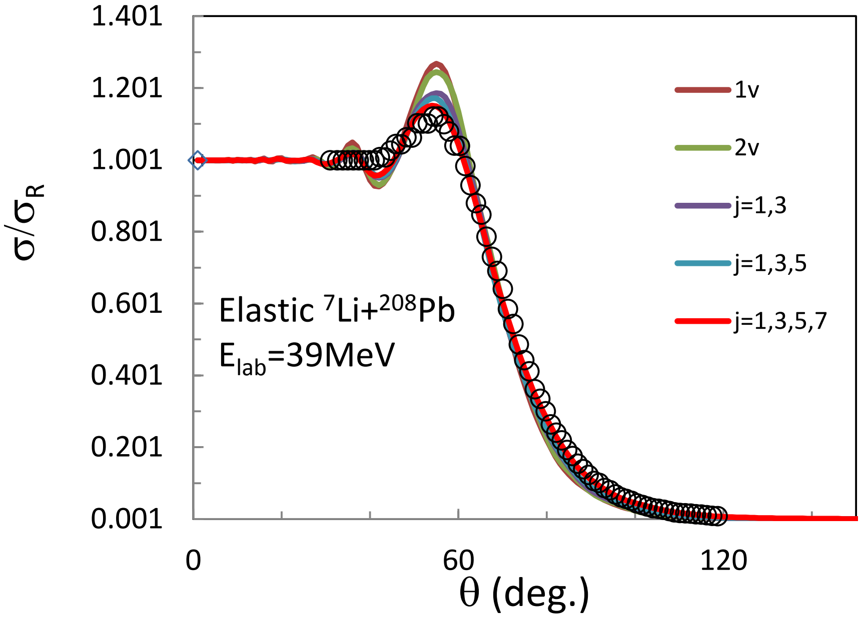
6. Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$

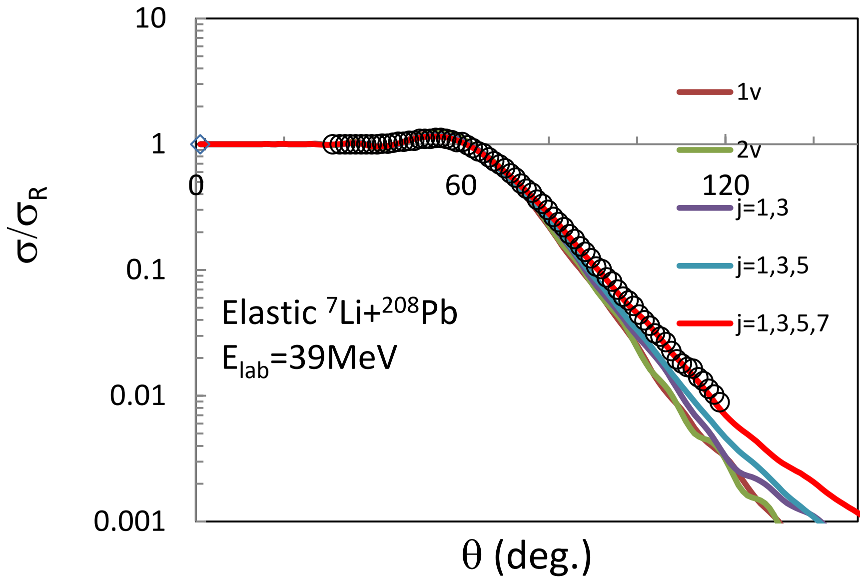
Elab=35 MeV



6. Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$

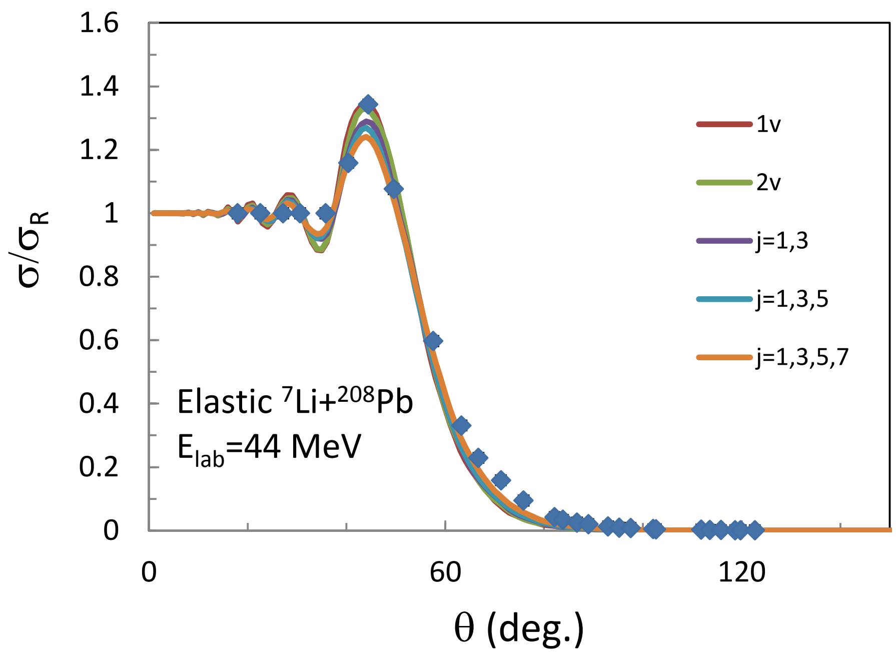
Elab=39 MeV

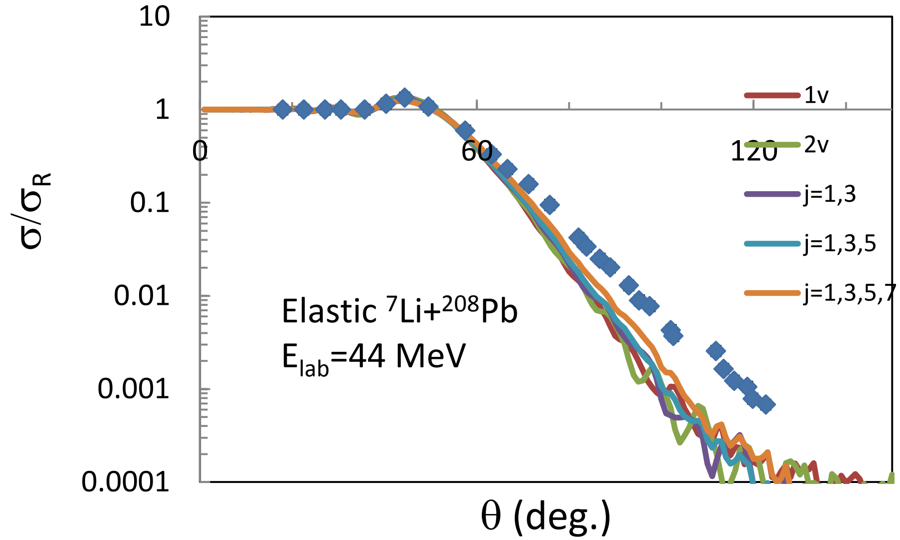




6. Application to $^{7}\mathrm{Li} + {}^{208}\mathrm{Pb}$

Elab=44 MeV





Underestimation at large angles and high energies

N. Timofeuyk and R. Johnson (Phys. Rev. Lett. **110**, 2013, 112501)

- suggest that the nucleon energy in A(d,p) reaction mush be larger than Ed/2
- →Similar effect here?

Role of target excitations?

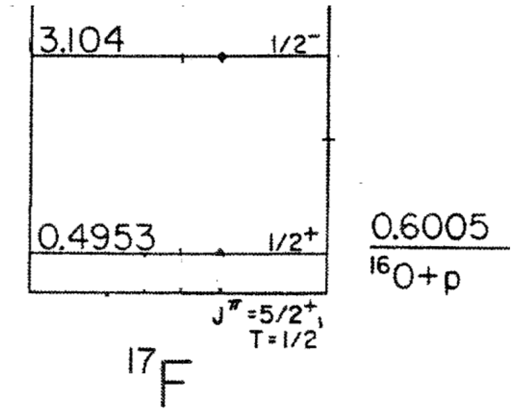
Application to $^{17}\mathrm{F} + {}^{208}\mathrm{Pb}$

J. Gineviciute, P.D., Phys. Rev. C **90**, 034616 (2014)

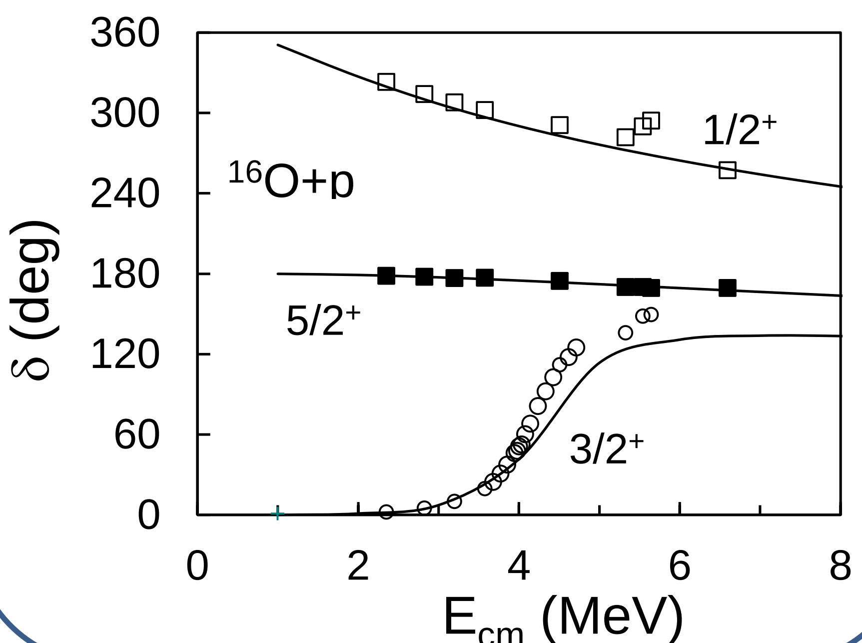
8. Application to $^{17}\mathrm{F} + {}^{208}\mathrm{Pb}$

^{17}F described by ^{16}O+p

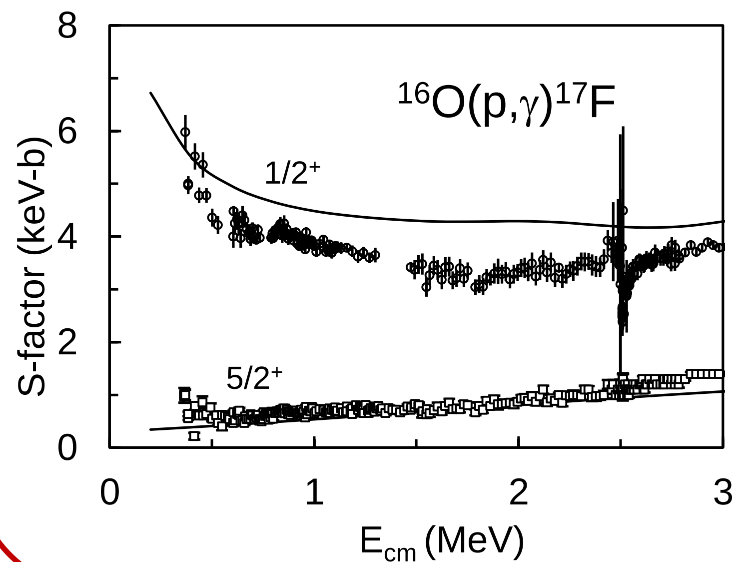
- NN interaction: Minnesota + spin-orbit
- Q(5/2$^+$)=-7.3 e.fm2 (exp: -10±2 e.fm^2)
- B(E2)=19.1 W.u., exp=25.0±0.5 W.u.
- Ref. D. Baye, P.D., M. Hesse, PRC 58 (1998) 545



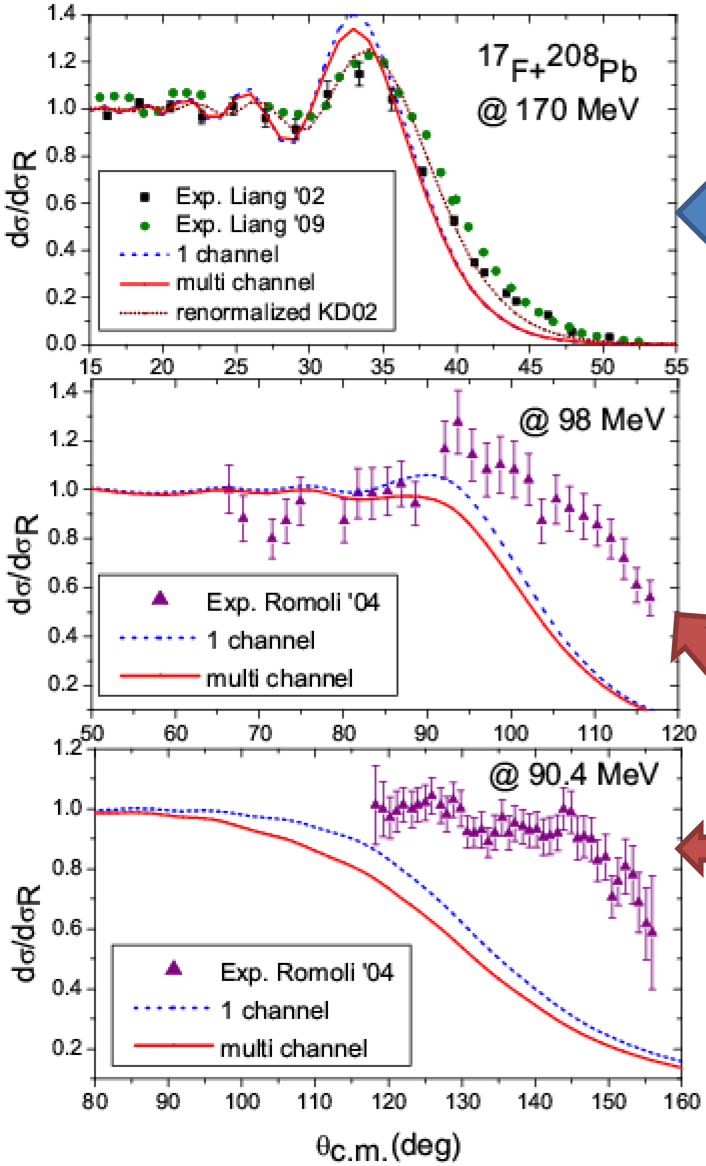
^{16}O+p phase shifts



^{16}O(p,γ)^{17}F radiative capture



8. Application to $^{17}F + ^{208}Pb$



- Data from Liang et al., PLB681 (2009) 22; PRC65 (2002) 051603
- Renormalization: real part x0.65
- Weak effect of breakup channels
- Similar to Y. Kucuk, A. Moro, PRC86 (2012) 034601

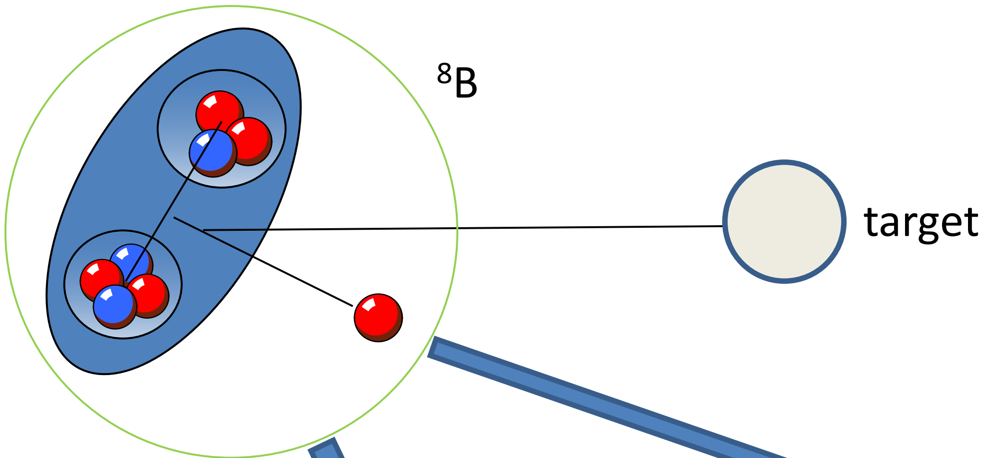
- Data from M. Romoli et al. PRC69 (2004) 064614
- Poor agreement with experiment!

Application to $^{8}\mathrm{B}$+nucleus scattering

Very preliminary!!

9. Application to ^{8}B + nucleus

The model can be extended to 3-cluster projectiles: many data with ^{6}He, ^{9}Be, ^{8}B, ^{8}Li



Same theory

J(^{8}B)=0,1,2,3,4 (Gs=2+)

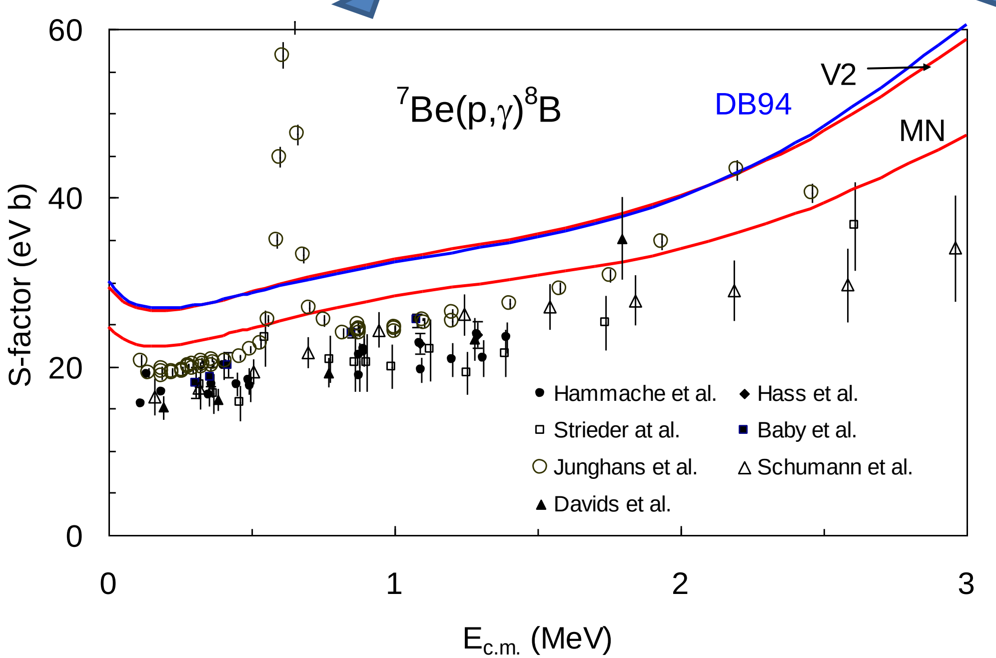
l(^{7}Be)=0,1,2,3 (gs=3/2-)

L(p-^{7}Be):0 to 7

→ many channels for ^{8}B

Calculations much longer

- 2 generator coordinates
- Angular-momentum projection



	experiment	theory
μ (2^{+}) (μ_N)	1.03	1.52
Q(2^{+}) (e.fm^{2})	6.83 ± 0.21	6.0
B(M1,1^{+}→2^{+}) (W.u.)	5.1 ± 2.5	3.8

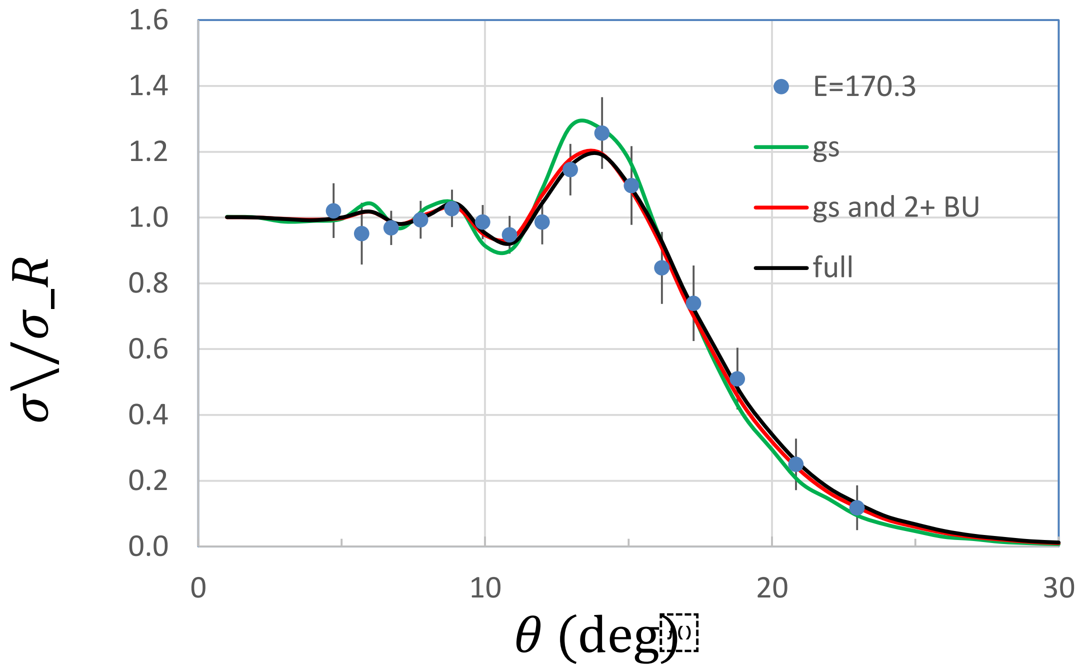
P.D., Phys. Rev. C70, 065802 (2004)

9. Application to $^{8}B + ^{208}Pb$

$^{8}B+^{208}Pb$ at Elab=170.3 MeV

Data from Y. Yang et al., Phys. Rev. C **87**, 044613

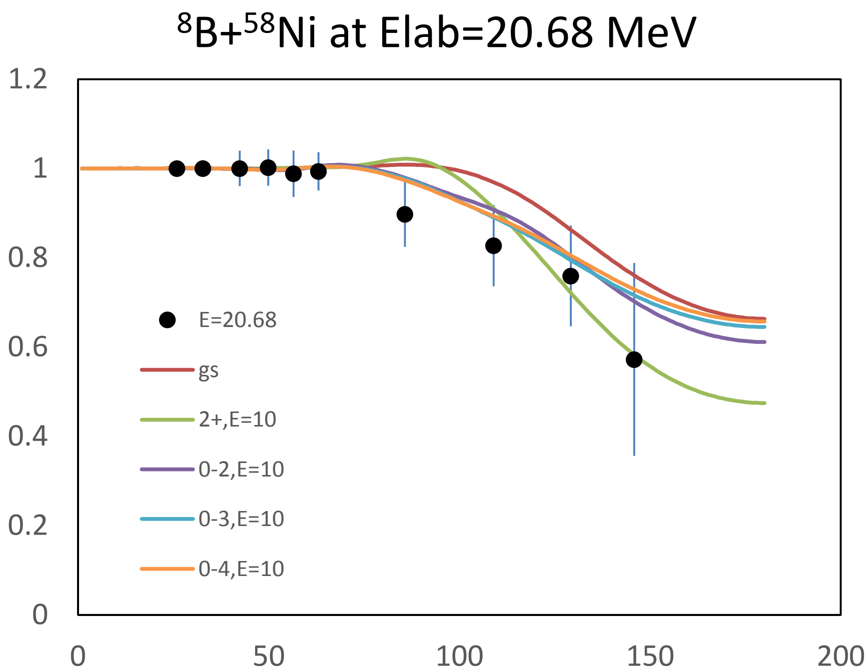
Calculation with the KD nucleon-^{208}Pb potential

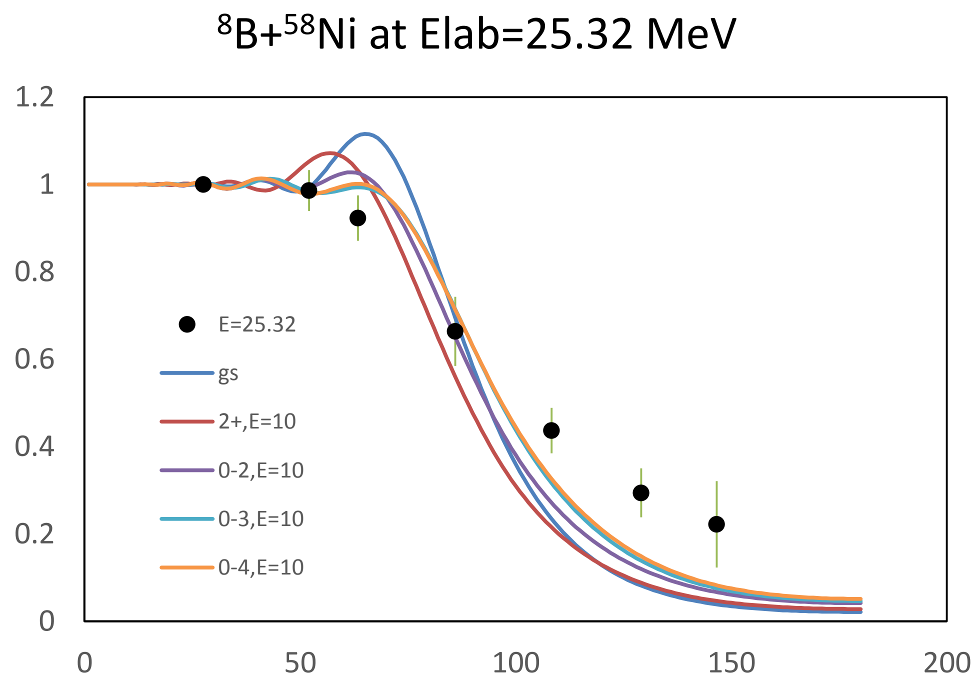


Weak influence of breakup channels (high energy)

9. Application to $^{8}\mathrm{B} + {}^{58}\mathrm{Ni}$

$^{8}B+^{58}Ni$ around the Coulomb barrier
Data from E. Aguilera et al., Phys. Rev. C 79, 021601(R) (2009)
Calculation with the KD nucleon-^{58}Ni potential





10. Conclusion

Microscopic CDCC

- Combination of CDCC and microscopic cluster model for the projectile
- Excited states of the projectile are included
- Continuum simulated by pseudostates (bins are possible)
- Only a nucleon-target is necessary
- Open issues
  - Target excitations
  - Around the Coulomb barrier, weak constraint for the p-target potential (Elab/A much smaller than the CB → Rutherford cross section)
  - Multichannel description of the projectile:
    - Elastic, inelastic OK
    - Breakup: pseudostates contain a mixing of all channels → PS method cannot be applied
- Future works: core excitations (^{11}Be), fusion, etc.