Experimental evaluation of the nuclear neutron-proton contact

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Outline

The Contact - Tan's Relations

For a systems of fermions with small interaction range

$$r_0 \ll n_\sigma^{-1/3}, \lambda_T, V^{1/3}$$

Tan relations connects the contact C with:

1. Tail of momentum distribution $|a_{scat}|^{-1} \ll k \ll r_0^{-1}$

$$\boxed{n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}}$$

2. The system's energy

$$\boxed{T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi m a_{scat}} C}$$

3. Adiabatic relation

$$\boxed{\frac{dE}{d1/a_{scat}} = -\frac{\hbar^2}{4\pi m} C}$$

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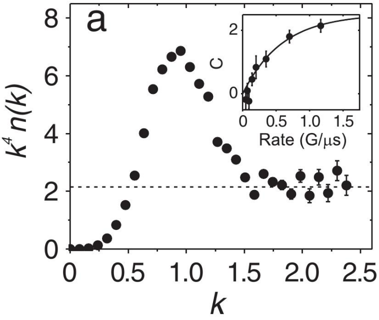
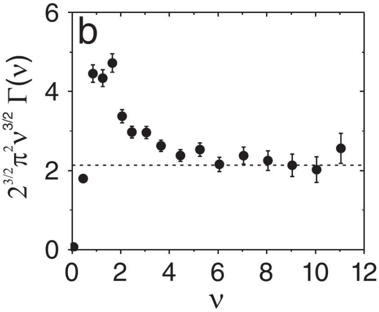
The Contact - Experimental Results

Momentum Distribution



RF line shape



`Verification of Universal Relations in a Strongly Interacting Fermi Gas J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)`

The short range factorization

[Tan, Braatan & Platter, Werner & Castin,...]

- A low density system $k_F r_0 \ll 1$
- The interaction is characterized solely by the scattering length $a_{scat} \gg r_0$.
- The interaction is represented through the boundary condition

$$\left[\partial \log r_{ij}\Psi / \partial r_{ij}\right]_{r_{ij}=0} = -1/a_{scat}$$

- Thus, when two particles approach each other

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a_{scat}) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

- The contact C represents the probability of finding an interacting pair within the system

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k\neq i,j} d\boldsymbol{r}_k \, d\boldsymbol{R}_{ij} \, A_{ij}^{\dagger}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \cdot A_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

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Scales

- The pion dominates the long range NN interaction $\mu^{-1} = \hbar/m_\pi c \approx 1.4$ fm
- The NN scattering lengths $a_t = 5.4$ fm $a_s \approx 20$ fm thus $\mu|a_{scat}| \geq 3.8$
- The nuclear radius is $R \approx 1.2A^{1/3}$ fm
- The interparticle distance $d \approx 2.4$ fm thus $\mu d \approx 1.7$

Few assumptions

- The scattering length dominates the NN interaction
- $\mu d \gg 1$ and $\mu|a_{scat}| \gg 1$

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The Nuclear Contact(s)

- In nuclear physics we have **4** possible one-body states

$$\{p\uparrow,\ p\downarrow,\ n\uparrow,\ n\downarrow\}$$

- Therefore there are **6** possible short range correlated pairs

$$P = \{p\uparrow p\downarrow, n\uparrow n\downarrow, p\uparrow n\downarrow, n\uparrow p\downarrow, p\uparrow n\uparrow, p\downarrow n\downarrow\}$$

- For each pair we define an independent contact

$$C_P \equiv 16\pi^2 \sum_{ij=P} \langle A_{ij} | A_{ij} \rangle \quad ; \; C = \sum_P C_P$$

- Assuming spin symmetry we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
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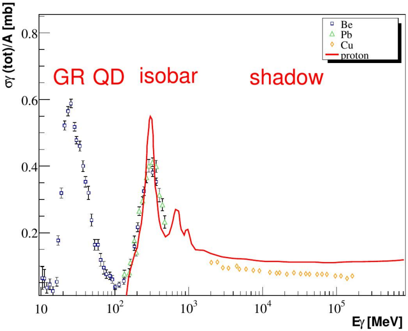
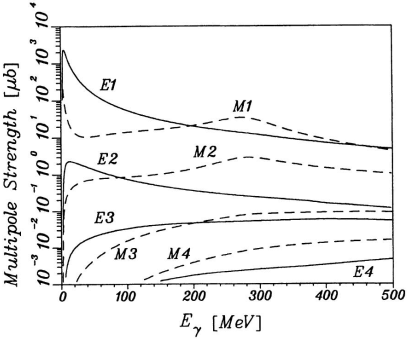
Photoabsorption of Nuclei (I)

Typical nuclear cross-section



R. Al Jebali, PhD Thesis, U. Glasgow (2013)

The Deuteron cross-section



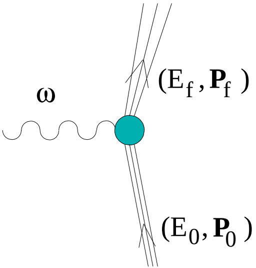
H. Arenhovel, and M. Sanzone, Few-Body Syst. (1991).

Photoabsorption of Nuclei (II)

At low photon energy the response function is dominated by the dipole response

$$\sigma(\omega) = 4\pi^2 \alpha \omega R^{E1}(\omega)$$

$$R^{E1}(\omega) = \sum_f \left| \langle \Psi_f | \epsilon \cdot \hat{D} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$\hat{D} = \sum_i q_i r_i = \sum_i \frac{1+\tau_i^3}{2} r_i$$

Via Siegert theorem MEC are implicitly included in the dipole response

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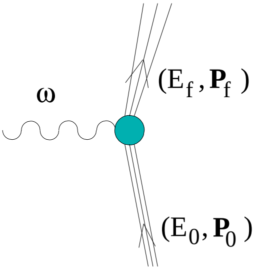

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The Quasi-Deuteron picture

J. S. Levinger
"The high energy nuclear photoeffect", Phys. Rev. **84**, 43 (1951).

- The photon carries **energy** but (almost) **no momentum**
- It is captured by a single proton.
- The proton is ejected without any FSI.
- Momentum conservation $\Rightarrow$ A nucleon with opposite momentum must be ejected $\boldsymbol{k} \approx -\boldsymbol{k}_p$.
- Due to the dipole $E1$ dominance this partner must be a neutron.
- For small photon wavelength the process depends on a universal short range pn wave-function.
- The resulting cross-section is given by

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- L is known as the Levinger Constant

The Quasi-Deuteron in the zero range model

The initial wave function Ψ_0

When a nucleon-proton pair are close together Ψ_0 is factorized into

$$\Psi_0(\mathbf{r}_1,...,\mathbf{r}_A) = \sum_P \left(\frac{1}{r_{pn}} - \frac{1}{a_P}\right) A_P\left(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n}\right) + O(r_{pn}) \qquad r_{pn} \longrightarrow 0$$

where

$$A_P(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n}) = \sum_{J_{A-2}} \left[\chi_P \otimes A_P^{J_{A-2}}(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n})\right]^{J_0 M_0}$$

The pair is coupled into a spin state χ_P with total spin $S_P = 0, 1$.

$$\chi_P = \left[|s_p\rangle \otimes |s_n\rangle\right]^{S_P M_P}$$

The angular momentum of the $A-2$ spectators $J_{A-2} = |J_0 - S|, J_0, J_0 + S$

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$$\boxed{\Psi_f^P(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_A) = \mathcal{N}_P \hat{\mathcal{A}} \left\{ \frac{1}{\sqrt{\Omega}} e^{-i\mathbf{k}\cdot\mathbf{r}_{pn}} A_P(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j \neq p,n}) \right\}}$$

$$\hat{\mathcal{A}} = (1 - \sum_{p' \neq p} (p, p'))(1 - \sum_{n' \neq n} (n, n'))$$

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$$\mathcal{N}_P = \frac{1}{\sqrt{NZ}} \frac{1}{\sqrt{\langle A_P | A_P \rangle}}.$$

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The transition matrix element

Considering now the transition matrix element

$$\langle \Psi_f^P | \boldsymbol{\epsilon}\cdot\hat{\mathbf{D}}|\Psi_0\rangle \quad = \quad NZ\mathcal{N}_P \int \prod_k d\boldsymbol{r}_k \frac{1}{\sqrt{\Omega}} e^{i\mathbf{k}\cdot\mathbf{r}_{pn}} A^\dagger_{S,pn}\, \boldsymbol{\epsilon}\cdot\hat{\mathbf{D}}\Psi_0$$

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$$\langle \chi_P | \boldsymbol{\epsilon}\cdot\hat{\mathbf{D}}_{pn} | \chi_{P'}\rangle = \delta_{PP'}$$

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The Quasi-Deuteron in the zero range model

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The Deuteron in the zero range model

The photoabsorption cross-section

The D's ground state wave-function

$$\psi_{d,0}(\mathbf{r}_{pn}) = \frac{1}{\sqrt{2\pi a_t}} \frac{e^{-r_{pn}/a_t}}{r_{pn}} \xrightarrow[r_{pn}\to 0]{} \frac{1}{\sqrt{2\pi a_t}} \left(\frac{1}{r_{pn}} - \frac{1}{a_t} \right)$$

Neglecting FSI and CM recoil, the final state wave function is

$$\psi_{d,f} = \frac{1}{\sqrt{\Omega}} e^{-i\mathbf{k}\cdot\mathbf{r}_{pn}}$$

The main contribution to the transition matrix-element emerge from $r_{pn} \approx 0$, Hence,

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$(1/r_{pn} - 1/a_P) \approx 1/r_{pn}$ therefore

$$\langle \Psi_f^P | \epsilon \cdot \hat{D} | \Psi_0 \rangle \approx \sqrt{\frac{C_P a_t}{8\pi}} \langle \psi_{d,f} | \epsilon \cdot \hat{D} | \psi_{d,0} \rangle$$

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The Cross-Section

Summing up, the cross-section of any nucleus is proportional to the dueteron cross-section $\sigma_d(\omega)$

$$\boxed{\sigma_A(\omega) = \frac{a_t}{8\pi}(C_s + C_t)\sigma_d(\omega)}$$

Comparing this to Levinger formula

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

The Levinger constant L is a close relative of the nuclear contacts,

$$\boxed{L = \frac{a_t}{8\pi}\frac{A}{NZ}(C_s + C_t)}$$

Define the average proton-neutron contact

$$\bar{C}_{pn} = \frac{1}{2}(C_s + C_t)$$

Then

$$\boxed{\bar{C}_{pn} = \frac{4\pi}{a_t}\frac{NZ}{A}L}$$

Few Comments

Cons

- Nuclei are NOT a dilute Fermi liquid
- The effective range is larger than the interparticle distance
- The simple dipole operator is NOT valid at $E_\gamma \approx 100\ \mathrm{MeV}$

Pros

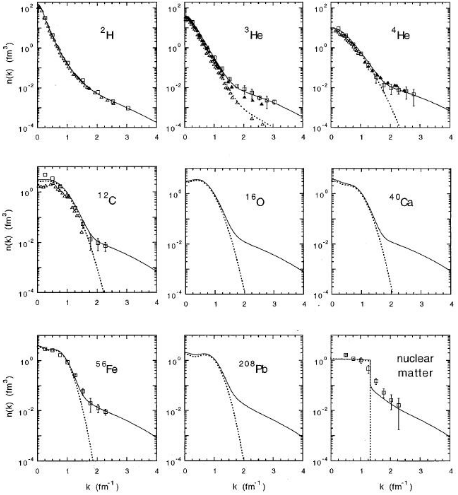
- The Quasi-Deuteron model holds as long as the short range correlations are universal (in the limited nuclear sense)

$$\Psi_0 = \sum_P \varphi_P(\boldsymbol{r}_{pn}) A_P\left(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n}\right) + O(r_{pn}) \qquad r_{pn} \longrightarrow 0$$

- The result doesn't really depends on the details of the E1 operator as long it is spin independent
- For spherical $J_0 = 0$ nuclei even this is not important.
- The two contacts can be separated through spin correlation experiments $\gamma + {}^A X \longrightarrow {}^{A-2}Y + p\uparrow n\uparrow$ where

$$\sigma_A^{\uparrow\uparrow}(\omega) = \frac{1}{3}\frac{a_t}{8\pi} C_t \sigma_d(\omega)$$

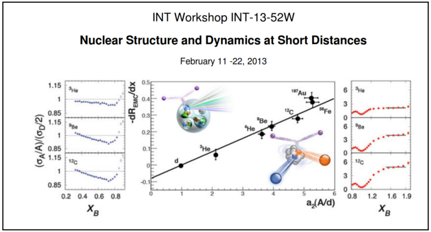
Another Comment - Nuclear Short Range Correlations



C. Ciofi degli Atti, and S. Simula, PRC 53, 1689 (1996)

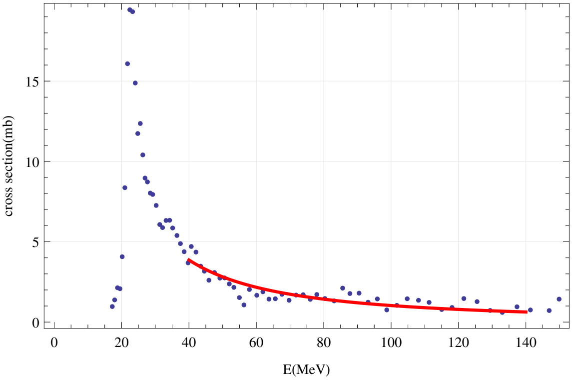
Short range correlations and their universal nature is an intensive line of research in NP

[Ciofi degli Atti, Frankfurt, Strikman, Sargasian, Piasetzky,...]



Experimental Results - fitting the Levinger Constant

The ^{12}C photoabsorption cross-section



Points - data

`Ahrens 1985, http://cdfe.sinp.msu.ru/saladin/gdrmain.html`

Line - the QD model $L = 5.8$

The Levinger Constant

The Levinger constant across the nuclear chart

In his original paper Levinger has estimated

$$\boxed{L = 6.4}$$

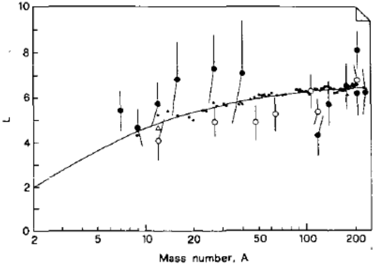
More recent evaluations estimate

$$\boxed{L = 6.8 - 11.2A^{-2/3} + 5.7A^{-4/3}}$$

In view of the available data we can conclude that

$$\boxed{L = 5.50 \pm 0.21}$$

O. A. P. Tavares and M. L. Terranova, J. Phys. G **18**, 521 (1992).



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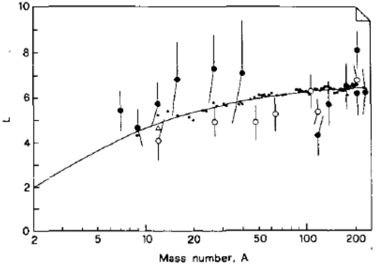
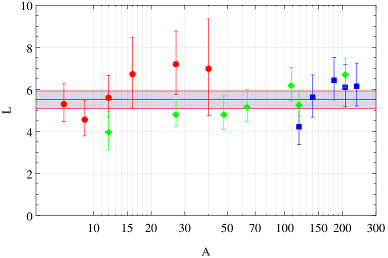




The nuclear contact

Symmetric Nuclei

- The average pn contact

$$\bar{C}_{pn} = \frac{4\pi}{a_t}\frac{NZ}{A}L$$

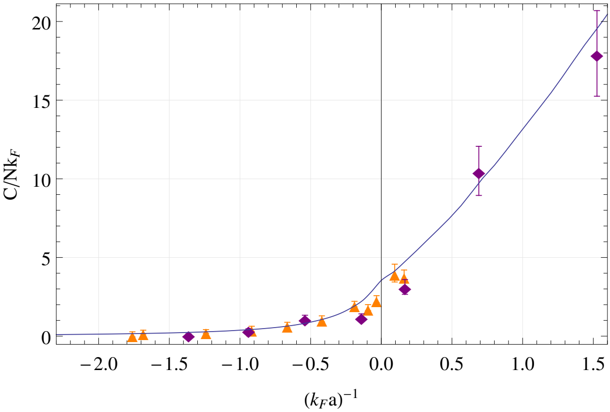
- $N = Z = A/2$
- Normalize by the Fermi monemtum

$$\boxed{\frac{\bar{C}_{pn}}{k_F A} = \frac{\pi}{k_F a_t}(5.50 \pm 0.21)}$$

- $1/k_F a_t \approx 0.15$
- Therefore

$$\boxed{\bar{C}_{pn}/k_F A \approx 2.55 \pm 0.10}$$

So, what about universality?



Atomic data - J. T. Stewart, et al., PRL 104, 235301 (2010)
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Nuclear data - The main source of the orizontal error bar is the range (a_s, a_t)

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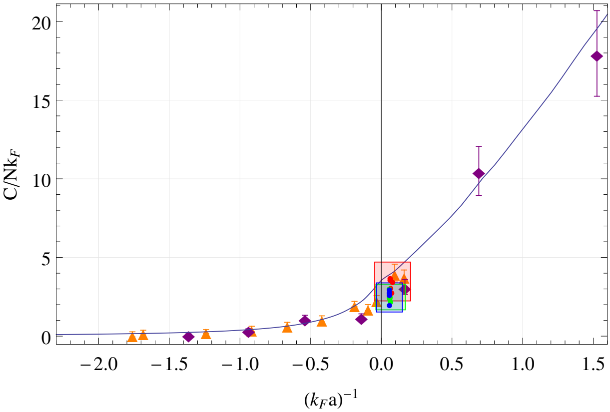

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Summary and Conclusions

- We have rederived the Quasi-Deuteron model utilizing the zero-range model.
- Doing so we have shown that the Levinger constant and the nuclear contacts are close relatives.
- Using previous evaluations of Levinger's constant we have deduced the pn contact in nuclei.
- $\bar{C}_{pn}$ seems to be constant throughout the nuclear chart.
- Its value stands in line with the universal curve measured in ultracold atomic systems.
- Our results can probably be generalized to electron scattering.

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