LIGHT NUCLEI IN A QUIRKY WORLD

U. van Kolck

Institut de Physique Nucléaire d'Orsay and University of Arizona

Supported by CNRS and US DOE

Outline

- QCD at Low Energies and the Lattice
- Pionless Effective Field Theory
- EFT for Lattice Nuclei
- Outlook(/Extrapolation in Pion Mass)
- Conclusion

Goal

Derivation of nuclear physics consistent with Standard Model (SM) of particle physics

- correct symmetries
- systematic

Why?

- Nucleus as the simplest complex system: quarks and gluons interacting strongly, yet exhibiting many regularities
  - QCD at large distances an unsolved part of the SM
  - tools for non-perturbative quantum (field) theories, *e.g.* cold atoms
- Nucleus as a laboratory: properties of the SM and beyond
  - nuclear matrix elements for symmetry tests
  - reaction rates for nucleosynthesis
  - equation of state for stellar structure
  - variation of parameters for cosmology
  - …

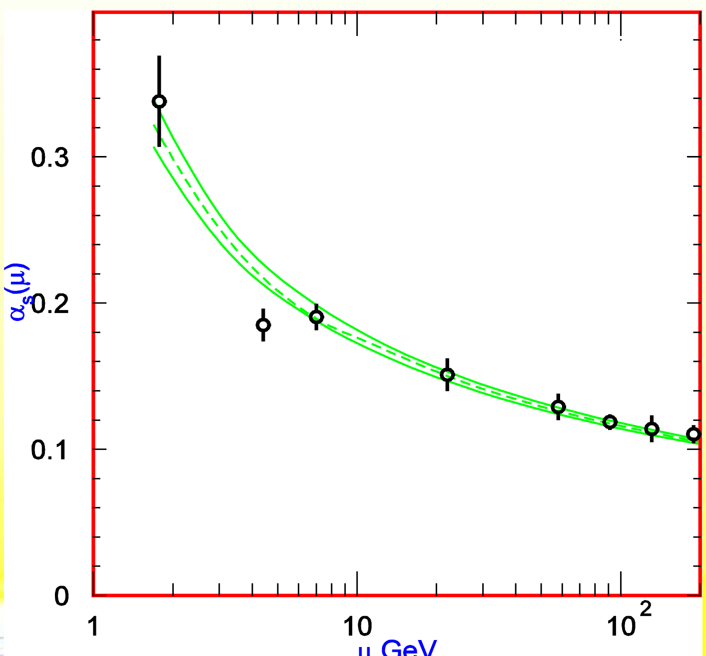
QCD

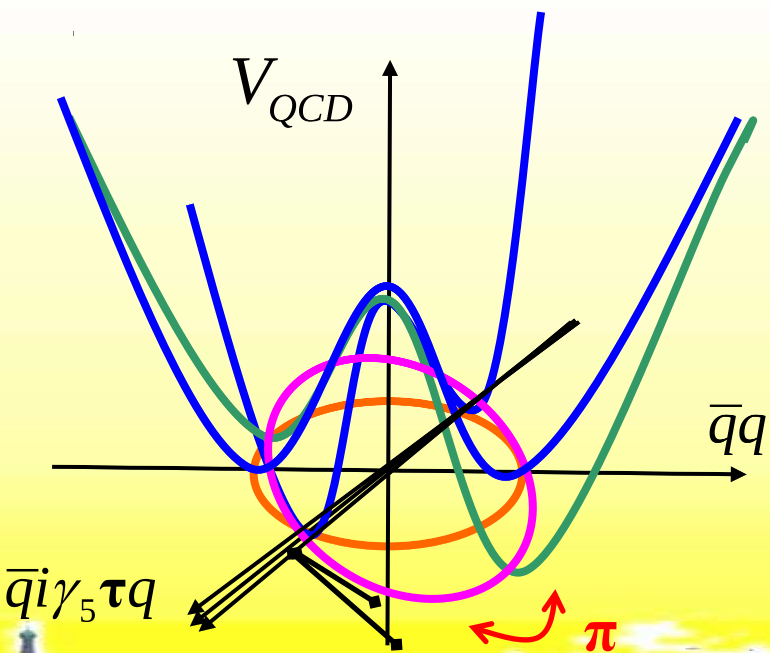
d.o.f.s quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ gluons: G^a_μ (photon: A_μ)

symmetries $SO(3,1)$ global, $SU_c(3)$ gauge ($+U_{em}(1)$ gauge)

$$\mathcal{L}_{QCD} = \bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu}G_{\mu\nu} \quad + \bar{m}\,\bar{q}\left(1-\varepsilon\tau_3\right)q \quad + \ldots$$

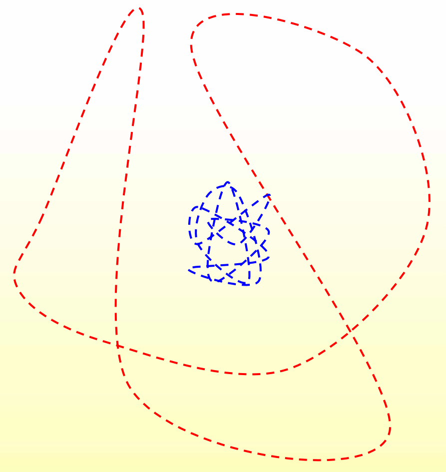
Basic mass scales $M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots \sim 1\,\mathrm{GeV}$ $m_\pi \sim \sqrt{\bar{m}M_{QCD}} \simeq 140\,\mathrm{MeV}$





$$f_\pi \sim M_{QCD}/4\pi + \mathcal{O}(\bar{m}) \simeq 100\,\mathrm{MeV}$$

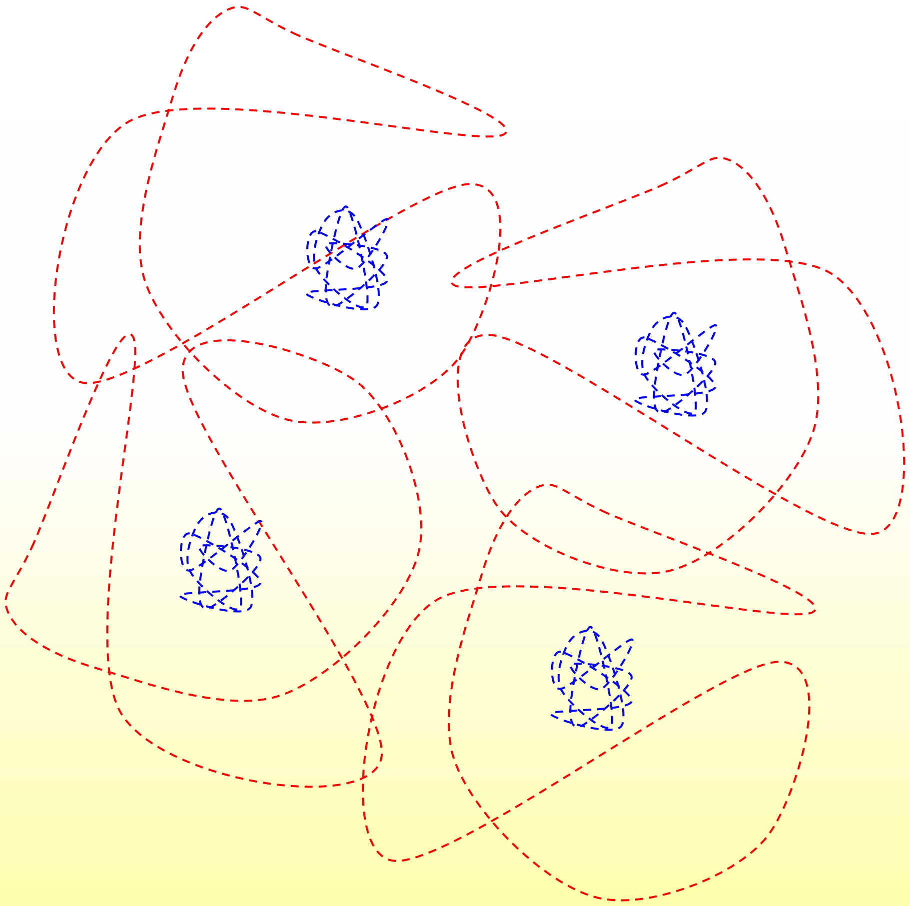
nucleon



$1/M_{QCD} \approx 0.3 \text{ fm}$

$1/m_{\pi} \cong 1.4 \text{ fm}$

nucleus



$1/M_{QCD} \approx 0.3$ fm

$1/m_\pi \cong 1.4$ fm

$R \sim \rho\left(m_\pi / f_\pi\right) A^{1/3}/m_\pi \approx 1.2\ A^{1/3}$fm

How? Lattice QCD + Effective Field Theory

$$\left\{ \begin{array}{l} T = T^{(\infty)}(Q \sim m \ll M) \propto \sum_{\nu=\nu_{\min}}^{\infty} \left[\frac{Q}{M}\right]^{\nu} \sum_i \tilde{c}_{\nu,i}(\Lambda)\, F_{\nu,i}\left(\frac{Q}{m};\frac{Q}{\Lambda}\right) \\ \frac{\partial T}{\partial \Lambda} = 0 \end{array} \right.$$

light scales (Q, m) — hard scales (M)

"power counting" — counting index (ν)

"low-energy constants" ($\tilde{c}_{\nu,i}(\Lambda)$)

non-analytic, from loops ($F_{\nu,i}$)

arbitrary regulator (Λ)

For $Q \sim m$, truncate …

$$T = T^{(\bar{\nu})}\left[1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right]$$

controlled

$\Rightarrow$ … consistently with RG invariance

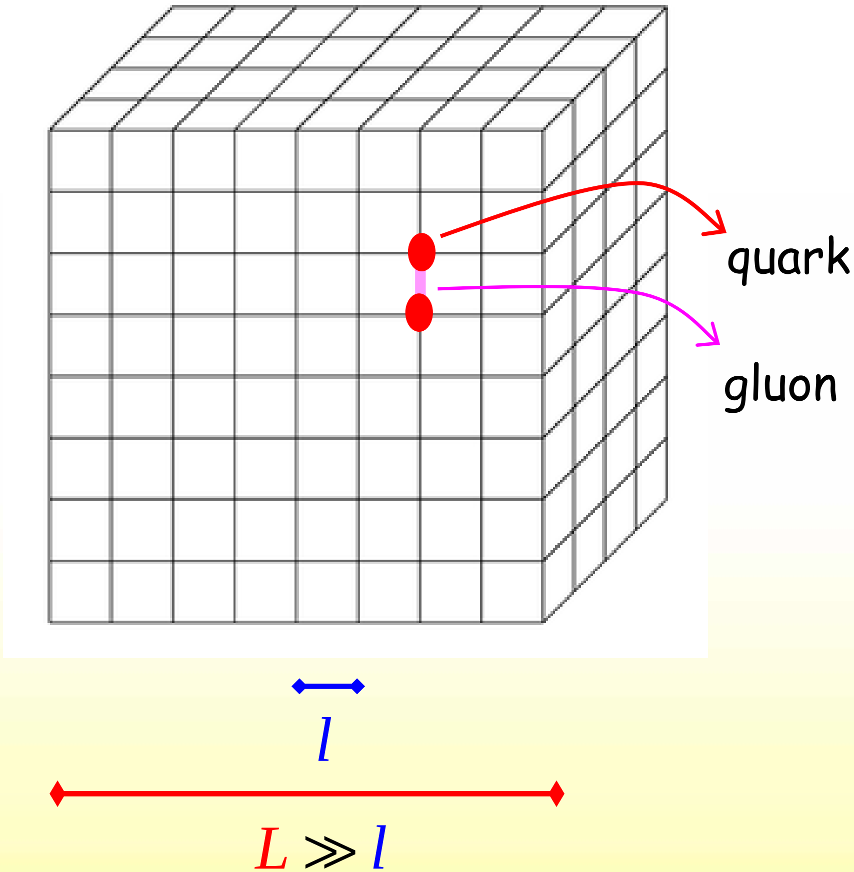
$$\frac{\Lambda}{T^{(\bar{\nu})}} \frac{\partial T^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1$$

model independent

If so:
- to minimize cutoff errors, want $\Lambda \gtrsim M$
- realistic full error estimate comes from variation $\Lambda \in [M, \infty)$

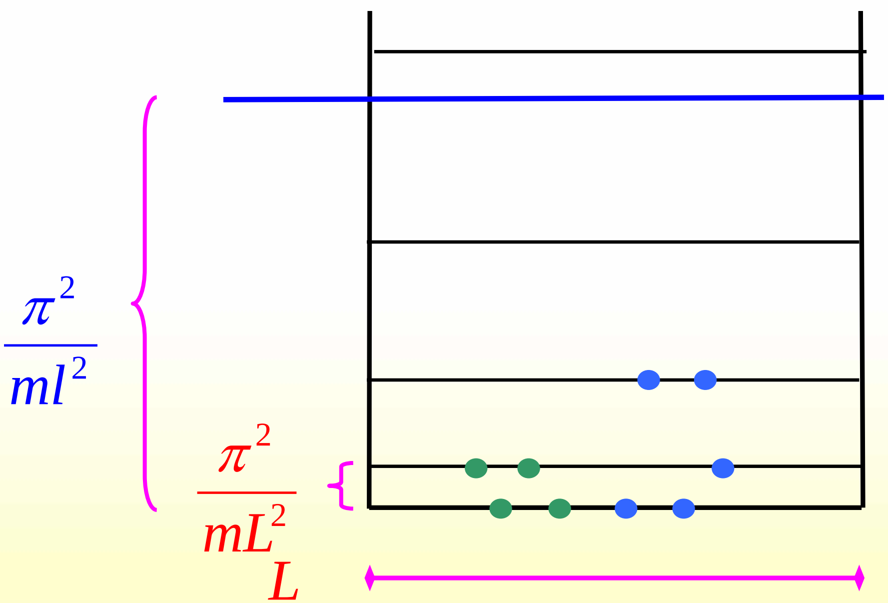
match amplitudes
- lattice QCD
- *most general* hadronic Hamiltonian with QCD symmetries

Lattice QCD



path integral solved with Monte Carlo methods, typically for unrealistically large quark masses

lattice "model space"



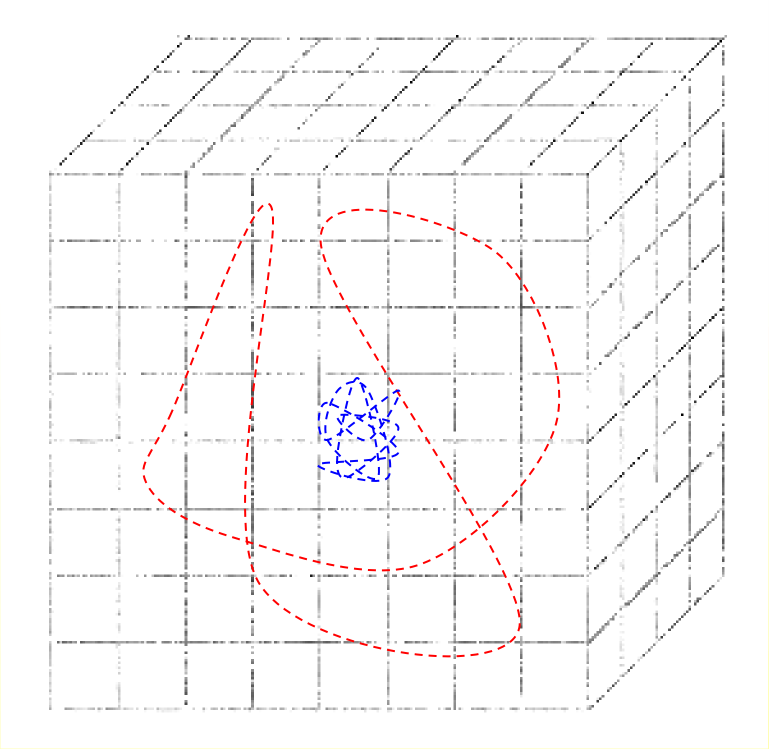
$$\cot\delta(E) = \frac{4}{\sqrt{mEL}}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - mEL^2} - \frac{L}{l}\right]$$

Lüscher '91

nucleon

$l \ll 1/M_{QCD}$

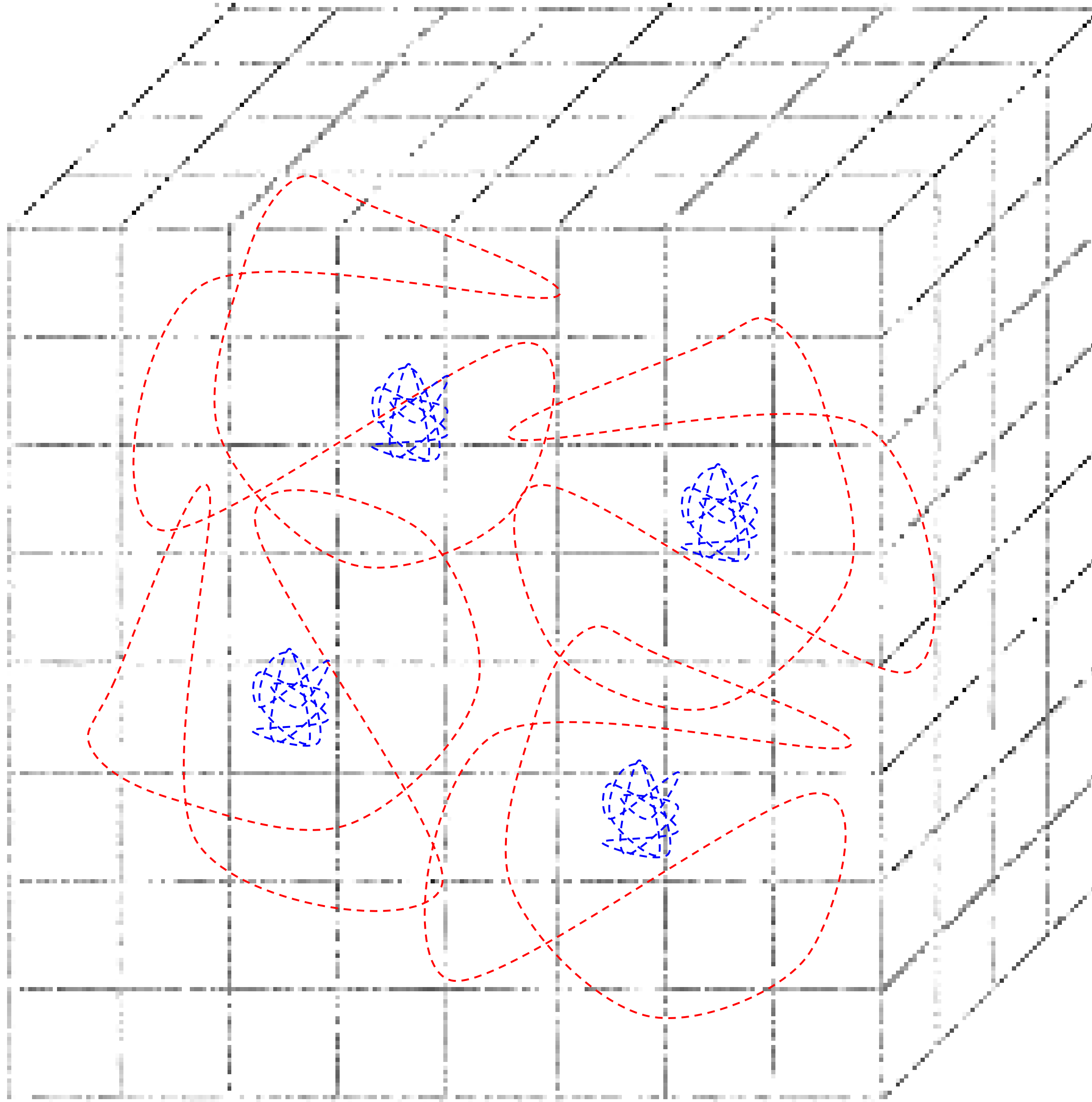
$1/M_{QCD} \approx 0.3\ \mathrm{fm}$



$1/m_\pi \cong 1.4\ \mathrm{fm}$

$L \gg 1/m_\pi$

nucleus



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$L \gg \rho(m_\pi / f_\pi)\, A^{1/3}/m_\pi$

two-step strategy

II) solve EFT for

II) fit LECs for

$A \leq 3, 4$

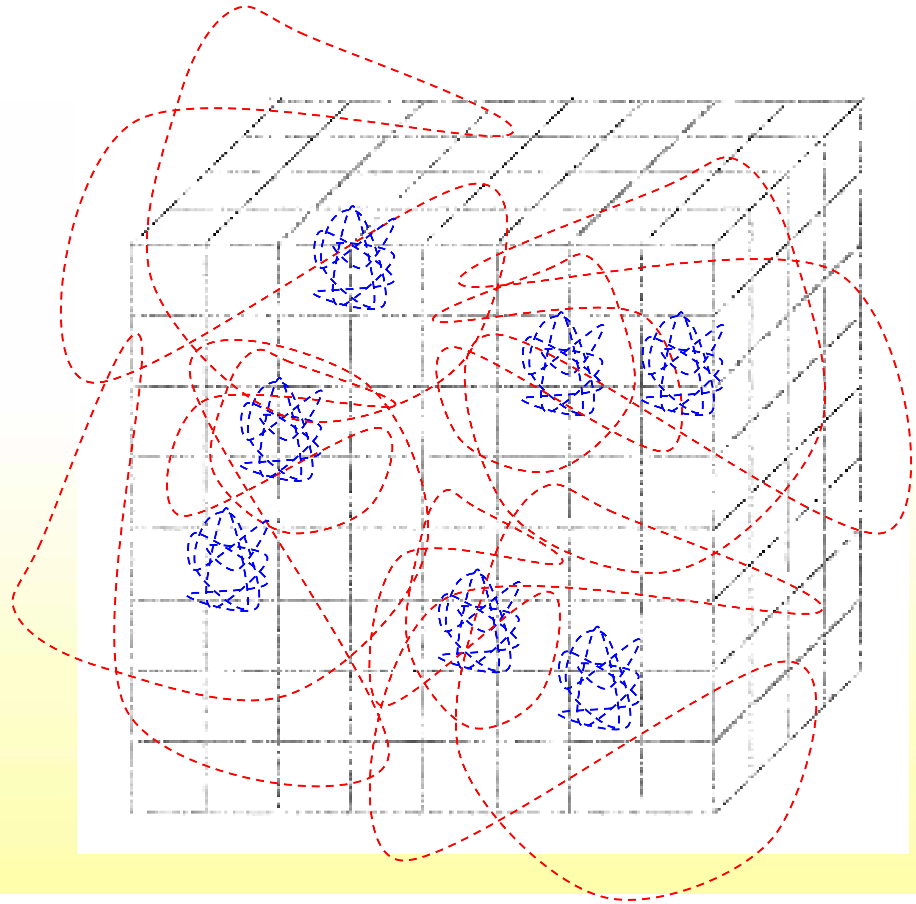
any A

$m_\pi \geq M_\pi \sim 300, 400$ MeV

any m_π

$l \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3$ fm



$1/m_\pi$ $1/M_\pi$

$\rho\left(M_\pi/f_\pi\right) a^{1/3}/M_\pi$

$\rho\left(m_\pi/f_\pi\right) A^{1/3}/m_\pi$

$L \gg \rho\left(M_\pi/f_\pi\right) a^{1/3}/M_\pi$

Experimental and LQCD data

m_π	140	510	805
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[5] Yamazaki *et al.* '12
[6] Beane *et al.* '12

Beane *et al.* '13

$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \quad , \quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad , \quad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

Scales (MeV)

m_N	940	1320	1630
$\sqrt{2m_N(m_\Delta - m_N)}$	750	890	765
m_π	140	500	800
$\sqrt{2m_N B/A}\,(A=2\mapsto 4)$	$45\mapsto 115$	$130\mapsto 170$	$185\mapsto 300$

⇓

$$\aleph \equiv \sqrt{m_N B} \ll M \equiv m_\pi \lesssim M_{QCD}$$

Pionless EFT $Q \sim \aleph \ll M$

- degrees of freedom: nucleons
- symmetries: Lorentz, P̸, T̸

$$\mathcal{L}_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N - \frac{C_0}{2} N^{+}N\, N^{+}N - \frac{D_0}{6} N^{+}N\, N^{+}N\, N^{+}N$$

$$+ N^{+}\frac{\nabla^4}{8m_N^3}N - \frac{C_2}{4} N^{+}N\, \nabla^2 N^{+}N + \ldots$$

(omitting spin, isospin)

- expansion in: $\frac{Q}{M} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$

Universality: first orders apply also to neutral atoms

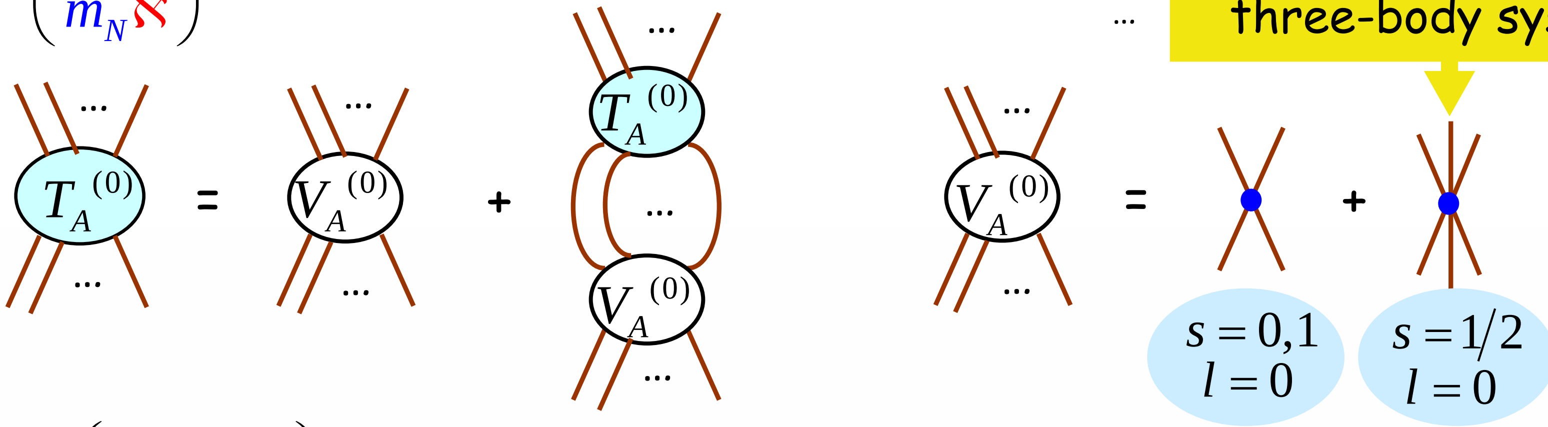
$m_\pi \to 1/l_{vdW}$ *where* $V(r) = -\frac{l_{vdW}^4}{2m_{at}r^6} + \ldots$

Bedaque, Hammer + v.K. '99 '00
Bedaque, Braaten + Hammer '01
...

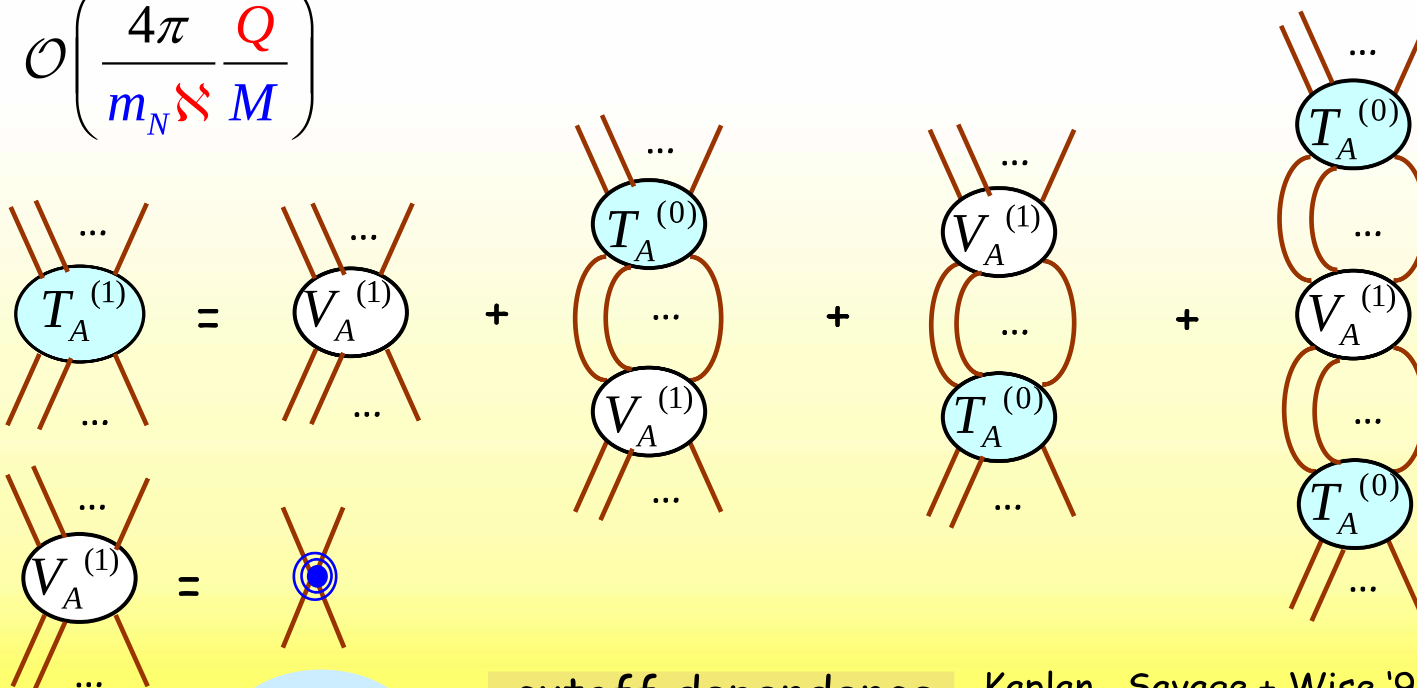
LO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\right)$

Bedaque + v.K. '97
...

needed to renormalize three-body system



NLO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\frac{Q}{M}\right)$



$s = 0,1$
$l = 0$

cutoff dependence of LO interactions

Kaplan, Savage + Wise '98
v.K. '98
...

etc.

v.K. '97
Kaplan, Savage +Wise '98
...

$A=2$ in each S-wave channel with shallow b.s.

$$\frac{m_N\Lambda}{4\pi}C_0(\Lambda) = -\left(\# - \frac{4\pi}{m_N\Lambda C_0^{(R)}}\right)^{-1} + \dots$$

regularization-dependent numbers

$$\frac{4\pi\Lambda}{m_N}\frac{C_2(\Lambda)}{C_0^2(\Lambda)} = \# + \frac{4\pi\Lambda}{m_N}\frac{C_2^{(R)}}{C_0^{(R)2}} + \dots$$

etc.

RG non-trivial fixed point!

$$C_0^{(R)} \sim \frac{4\pi}{m_N\aleph}$$

$$C_2^{(R)} \sim \frac{4\pi}{m_N M\aleph^2}$$

renormalized LECs

LO NLO N2LO

$$T_2(k) = \frac{4\pi}{m_N}\left(-\frac{4\pi}{m_N C_0^{(R)}} - ik\right)^{-1}\left[1 - \left(-\frac{4\pi}{m_N C_0^{(R)}} - ik\right)^{-1}\frac{16\pi C_2^{(R)}}{m_N C_0^{(R)2}}\frac{k^2}{2} + \mathcal{O}\left(\frac{\aleph^2}{M^2}\right)\right]$$

scattering length $= \frac{1}{a_2} \sim \aleph$

$= r_2 \sim \frac{1}{M}$ effective range

b.s. at $Q \sim \aleph$

effective-range expansion

Bedaque, Hammer + v.K. '99 '00
...

$A = 3$ { bosons; fermions with more than two states

$$T^{(0)}_{2+1}(\Lambda \gg p \gg \aleph; D_0 = 0) \approx A \cos\left(s_0 \ln\frac{p}{\Lambda} + \delta\right) \quad \Rightarrow \quad \frac{\Lambda}{T^{(0)}_{2+1}} \frac{\partial T^{(0)}_{2+1}}{\partial \Lambda}(p \sim \aleph; D_0 = 0) \sim 1$$

approximate scale invariance

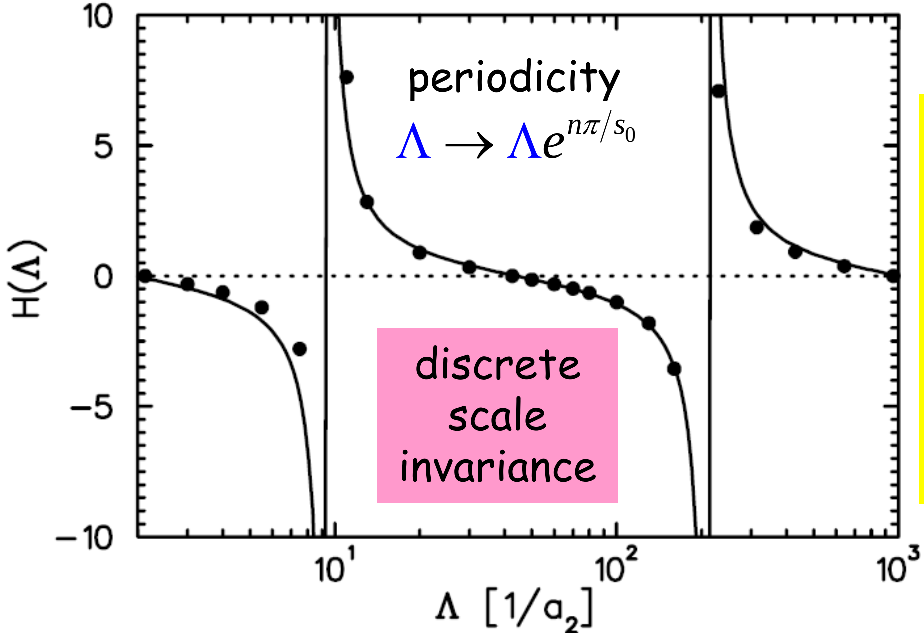
$s_0 = 1.0064\ldots$

unless $D_0^{(R)} \sim \dfrac{(4\pi)^2}{m_N \aleph^4}$ LO

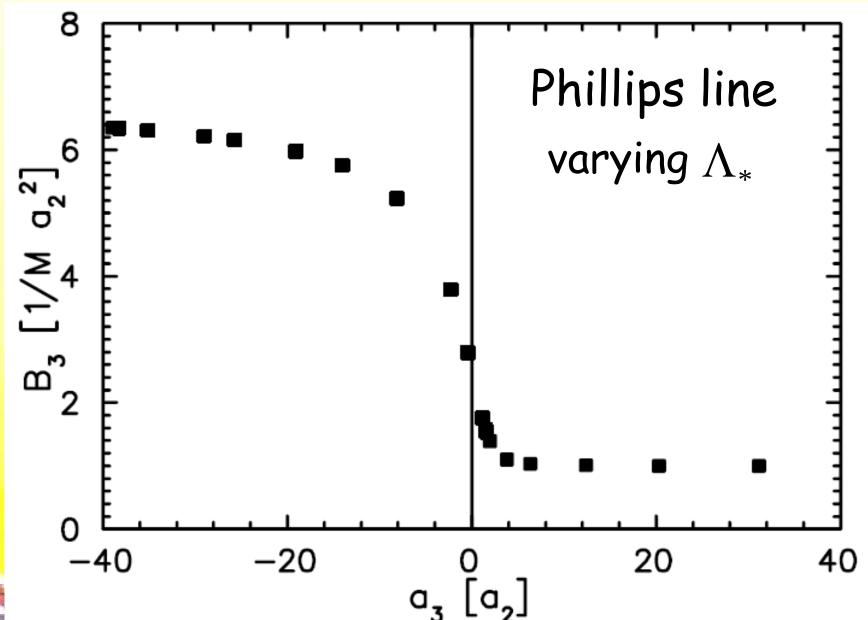
dimensionful parameter (dimensional transmutation)

not *just* the effective-range expansion

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m_N C_0^2(\Lambda)} \simeq \frac{\sin\left(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)\right)}$$



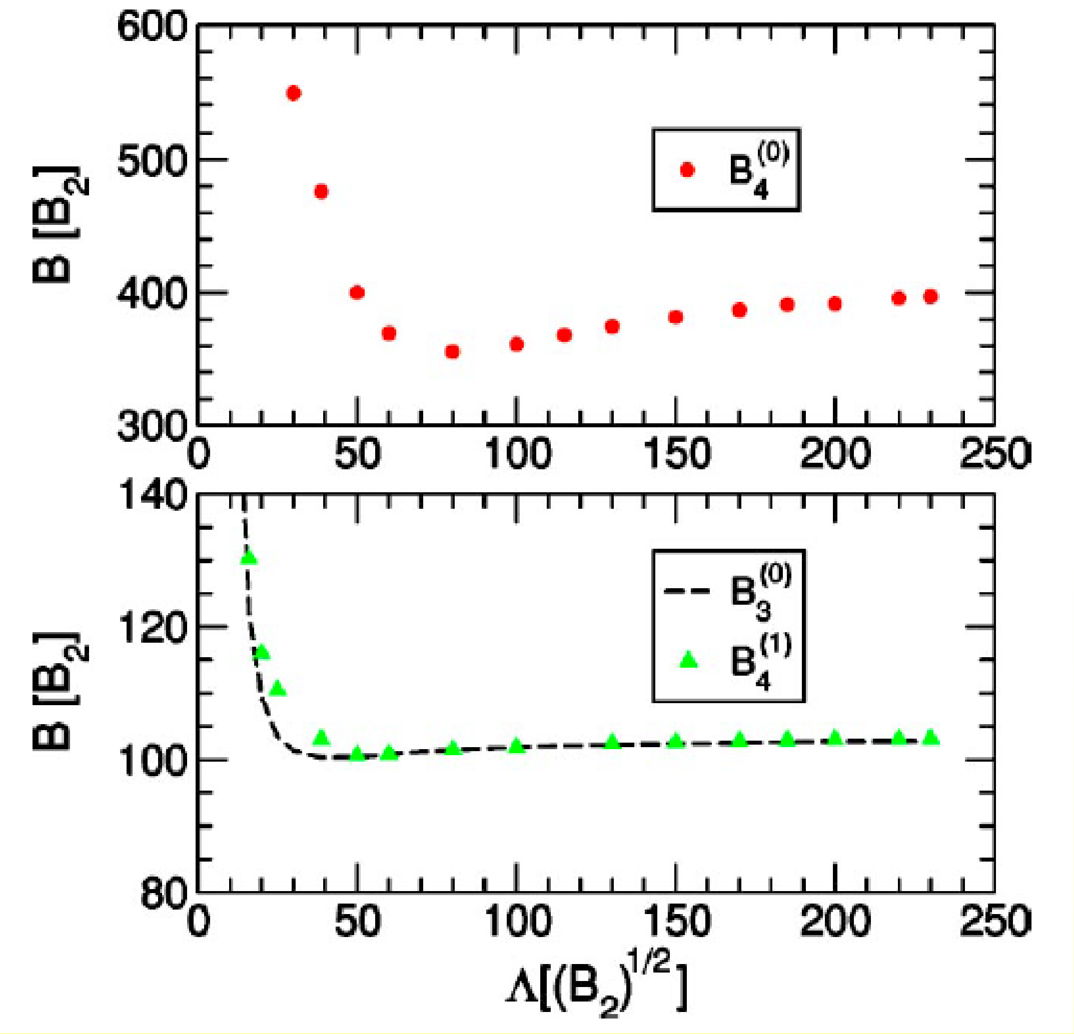
RG limit cycle!



Platter, Hammer + Meissner '04

....

$A = 4$



No 4-body force at LO!?

Experimental and LQCD data

LO pionless fit:

m_N, C_{01}, C_{10}, D_1

Stetcu, Barrett + v.K. '06

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Beane *et al.* '13

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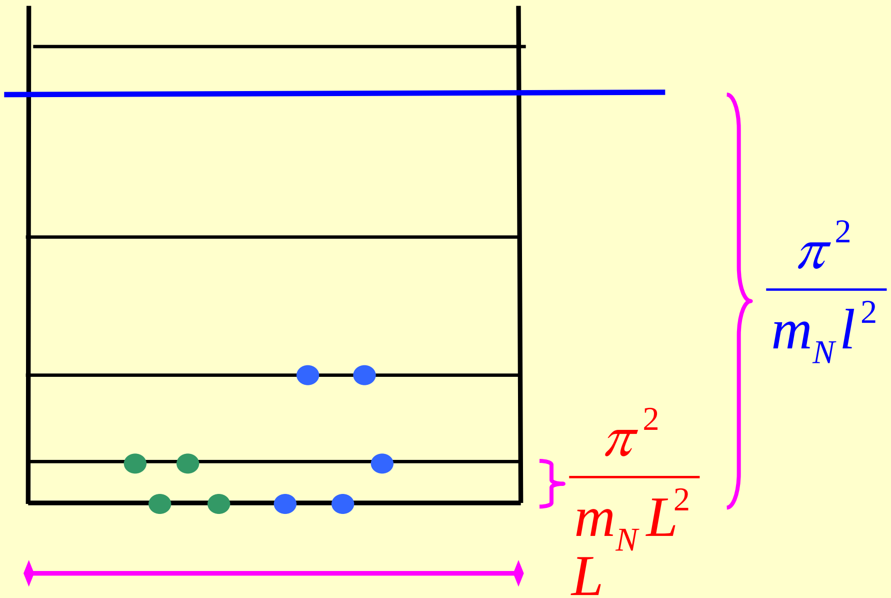
$A \gtrsim 4$ As A grows, given computational power limits number of accessible one-nucleon states ⟹ IR cutoff

Lattice Box



Mueller *et al.* '99

nuclear matter

Lee *et al.* '05

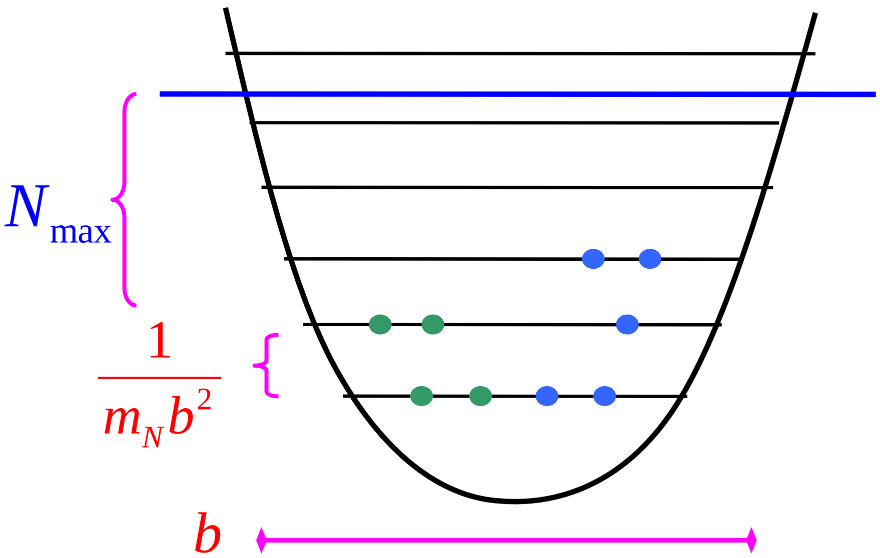
few nucleons

...

$$\cot\delta(E) = \frac{4}{\sqrt{m_N E L}}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - m_N E L^2} - \frac{L}{l}\right]$$

Lüscher '91

Harmonic Oscillator "No-Core Shell Model"



Stetcu *et al.* '06

finite nuclei

...

$$\cot\delta(E) = -\frac{2}{\sqrt{m_N E b}} \frac{\Gamma\left(\frac{3}{4} - \frac{m_N E b^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{m_N E b^2}{2}\right)}$$

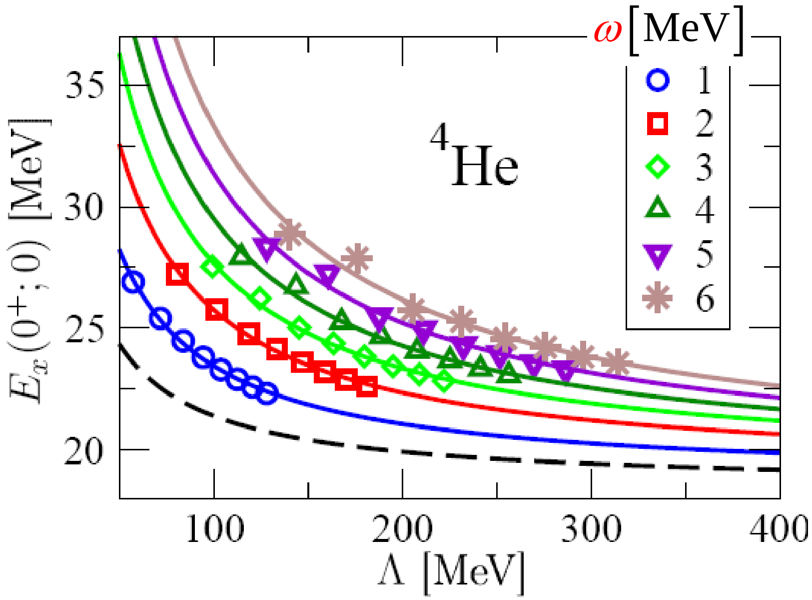
Busch *et al.* '99

...

Stetcu, Barrett + v.K. '06

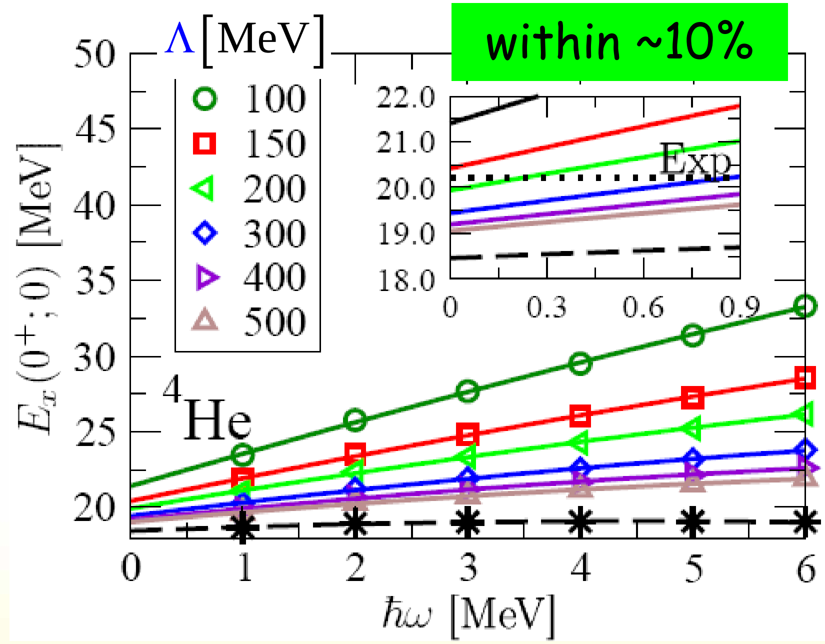
Pionless EFT: LO

(parameters fitted to d, t, α ground-state binding energies)

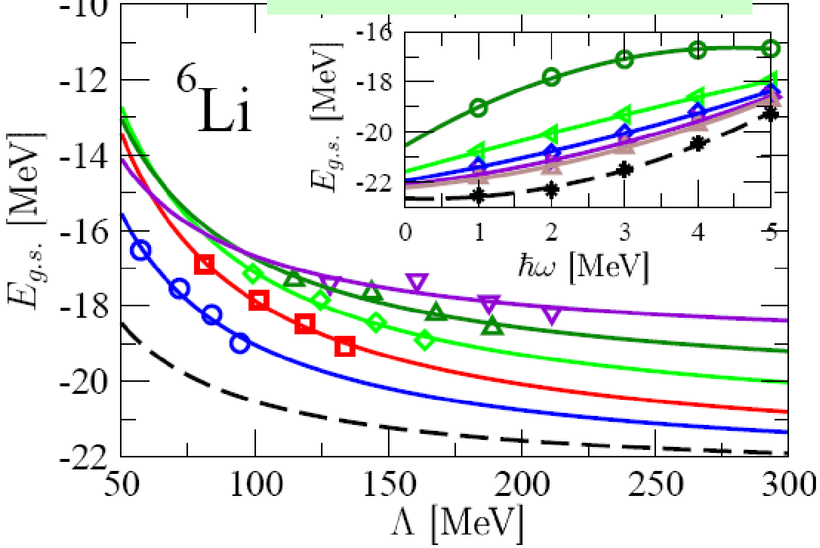


$N_{max} \leq 16$

within ~10%



works within ~30%

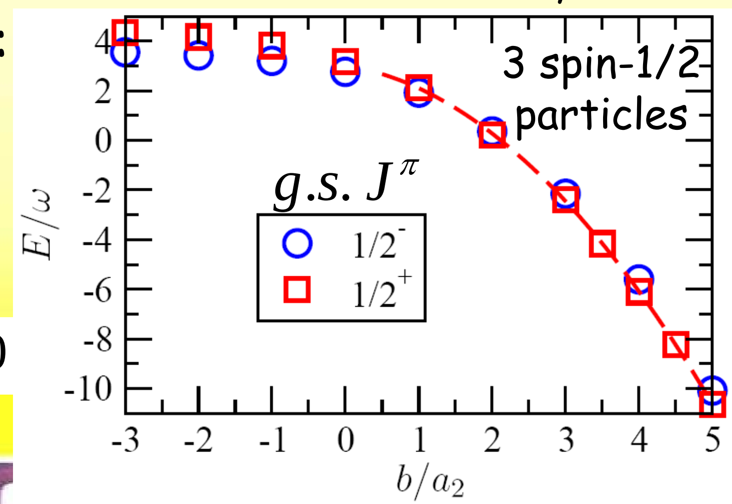


$N_{max} \leq 8$

$N_{max} \lesssim 30$

Stetcu, Barrett, Vary + v.K. '08

Bonus:



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[6] Beane *et al.* '12

* (p; nn; D; ^{3}H/^{3}He)

LO pionless fit:

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Barnea, Contessi, Gazit, Pederiva + v.K. '13

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Ab initio methods employed

- Effective-Interaction Hyperspherical Harmonics (EIHH) Barnea *et al.* '00' 01
  - ✓ hyperspherical coordinates: hyperradius + 3*A*-4 hyperangles
  - ✓ model space: hyperangular momentum $K \leq K_{max}$
  - ✓ wavefunction: expanded in antisymmetrized spin/isospin states
  - ✓ effective interaction: Lee-Suzuki projection to subspace "in medium"
  - ✓ extrapolation: $K_{max} \to \infty$

- Auxiliary-Field Diffusion Monte Carlo (AFDMC) Schmidt + Fantoni '99
  - ✓ integral equation for evolution of wavefunction in imaginary time τ in terms of Green's function (diffusion)
  - ✓ two- and more-body operators linearized by auxiliary fields (Hubbard-Stratonovich transformation)
  - ✓ trial wavefunction probed stochastically with weight given by the Green's function
  - ✓ lowest-energy state with symmetry of trial wavefunction projected onto as $\tau \to \infty$

Barnea, Contessi, Gazit, Pederiva + v.K. '13

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$
$$+\frac{1}{4}\sum_{i<j}\left[\left(3C_{10}(\Lambda)+C_{01}(\Lambda)\right)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4}$$
$$+\sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

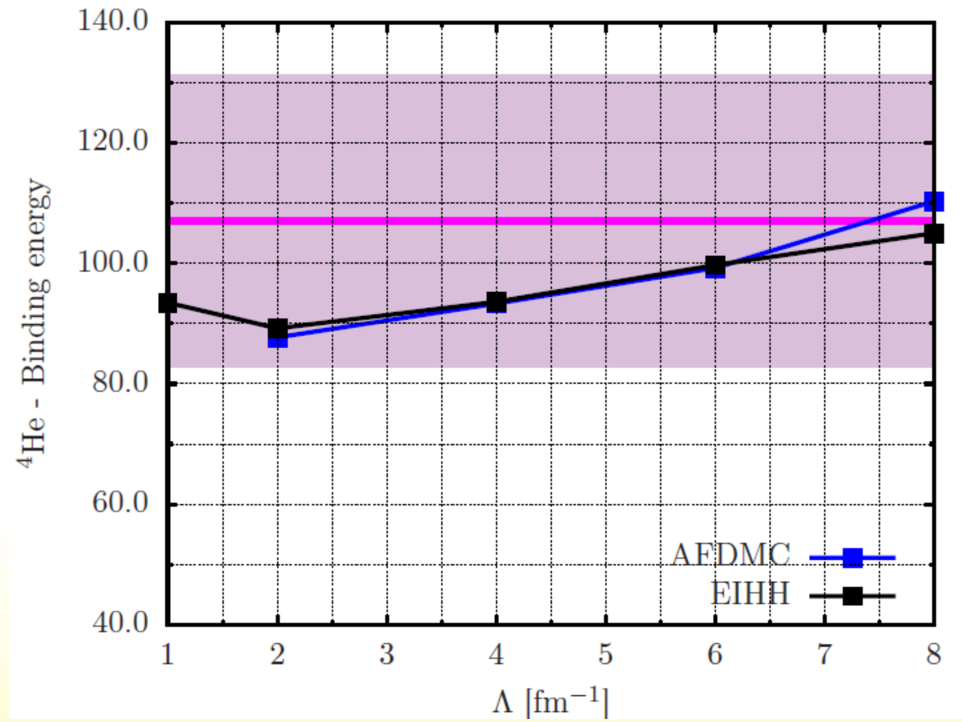
TABLE III. The LO LECs [GeV] for lattice nuclei at m_π = 805 MeV, as a function of the momentum cutoff Λ [fm^{-1}].

Λ	$C_{1,0}$	$C_{0,1}$	D_1
2	−0.1480	−0.1382	−0.07515
4	−0.4046	−0.3885	−0.3902
6	−0.7892	−0.7668	−1.147
8	−1.302	−1.273	−2.648

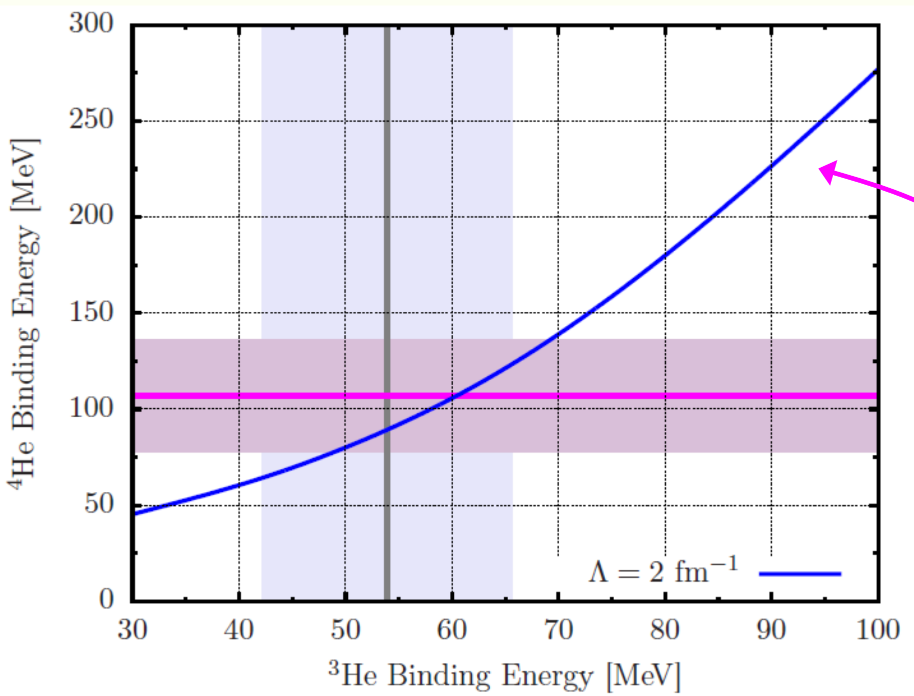
Barnea, Contessi, Gazit, Pederiva + v.K. '13

$$a^{(^3S_1)} = (1.2 \pm 0.5)\ \text{fm}$$

cutoff variation 2 to 14 fm^{-1}



Tjon line



varying D_1

at fixed C_{01}, C_{10}

Barnea, Contessi, Gazit, Pederiva + v.K. '13

- no excited states for A =2,3,4
- no ^{3}n droplet

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^{5}He	27.50			98 ± 39
^{5}Li	26.61			98 ± 39
^{6}Li	32.00			122 ± 50

predictions (^{4}He, ^{5}He, ^{5}Li, ^{6}Li)

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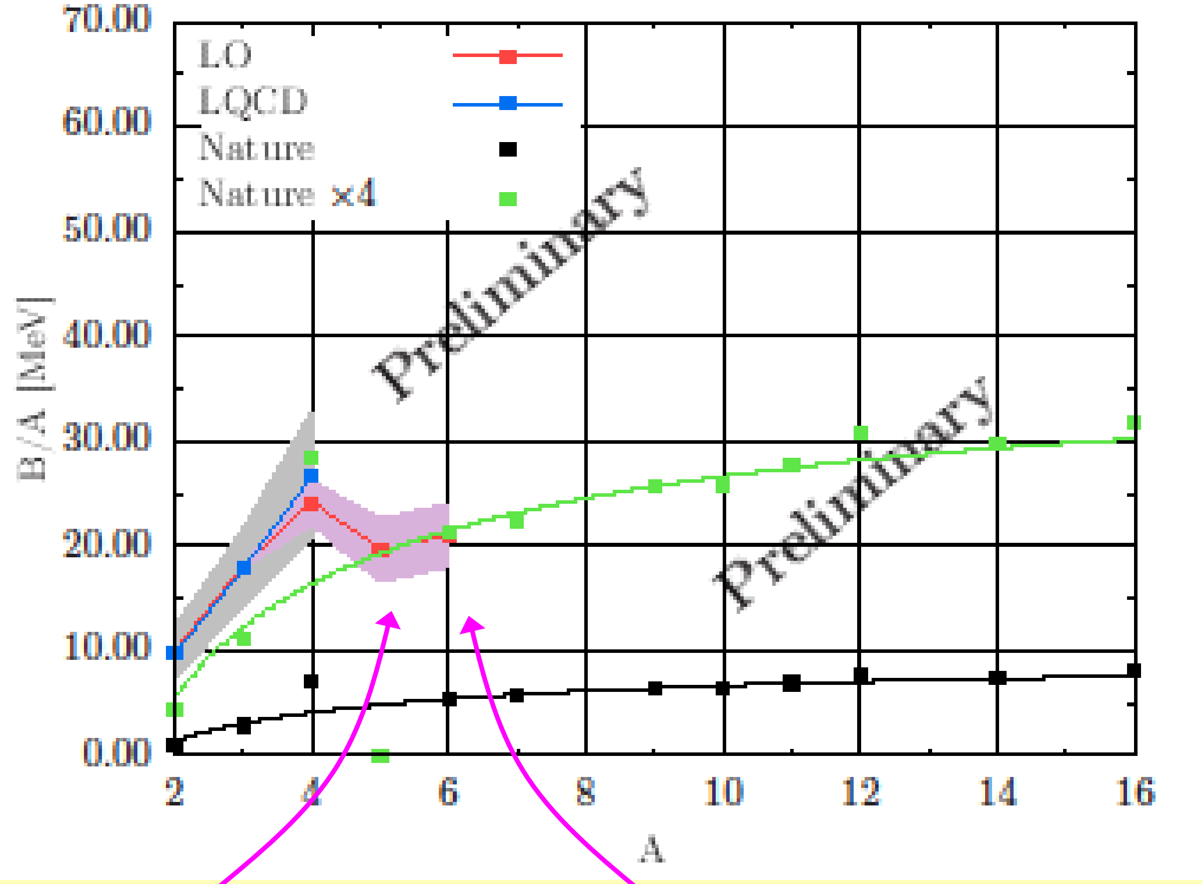
[6] Beane *et al.* '12
[This work] Barnea *et al.* '13

Barnea, Contessi, Gazit, Pederiva + v.K. '13



$$B_5 \approx B_4$$

A=5 gap persists!?

$$\frac{B_6}{6} \approx \frac{B_4}{4}$$

nuclear saturation survives!?

What next?

- NLO, larger cutoff at m_π = 805 MeV
- LO at m_π = 510 MeV
- larger A with AFDMC
- hypernuclei
- chiral EFT at lower pion masses when available
- …

Conclusion

- EFT is constrained *only* by symmetries and thus can be matched onto lattice QCD

- EFT allows controlled extrapolations of lattice results in nucleon number (and pion mass)

- First, proof-of-principle calculation carried out at $m_\pi \approx 800$ MeV with pionless EFT

- World at large pion mass *might* be just a denser version of ours