Tunneling of atoms, nuclei and molecules

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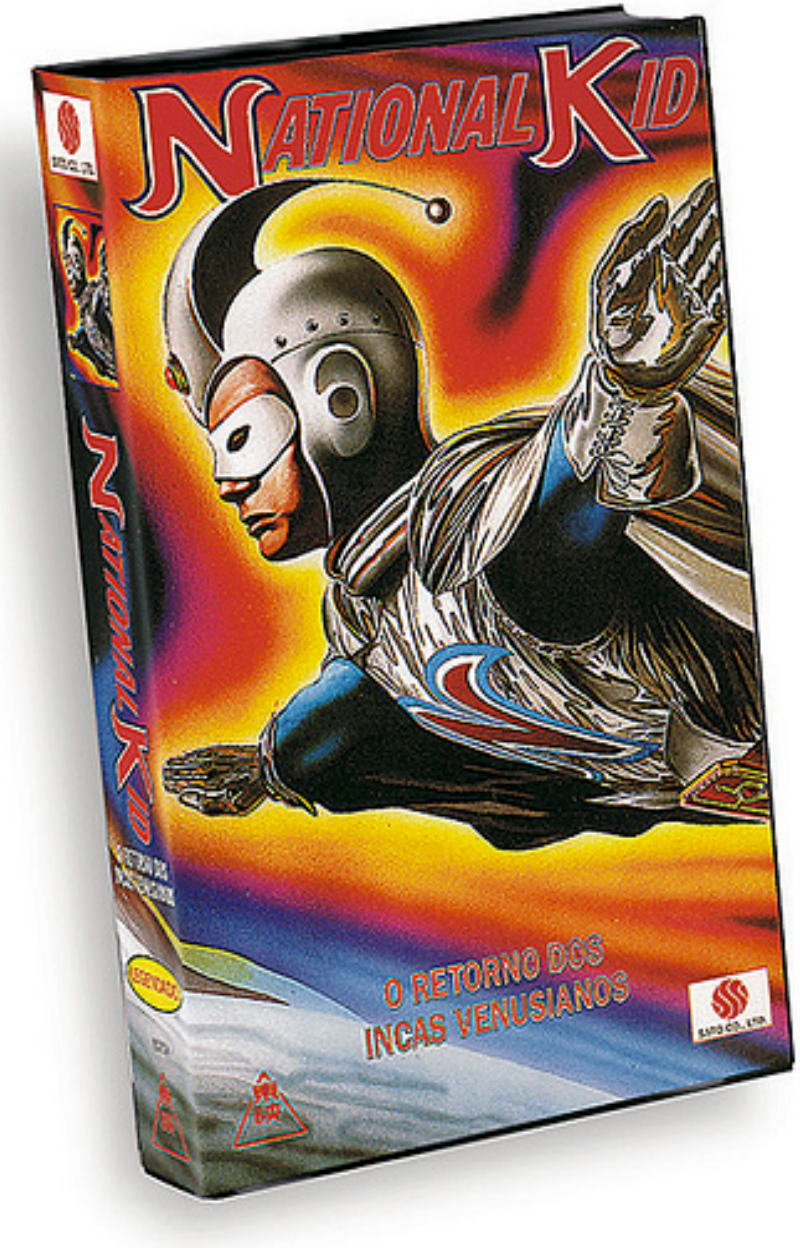
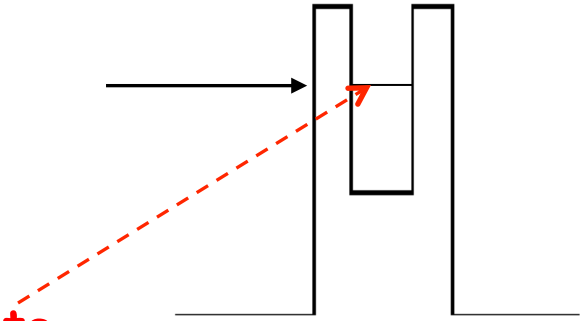


Int. Workshop on Critical Stability, Santos, October 13, 2014

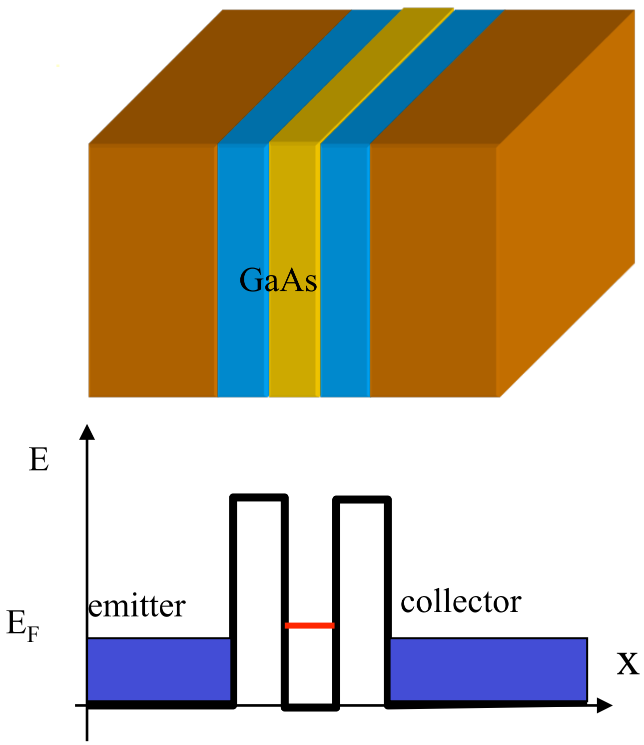
Resonant tunneling

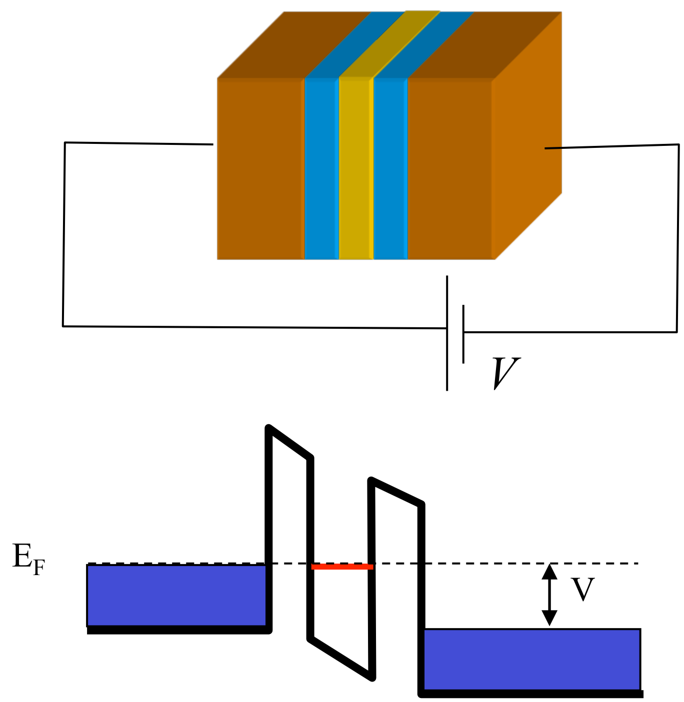
(point particle + double-hump barrier)



Well-known. Used in device technology.

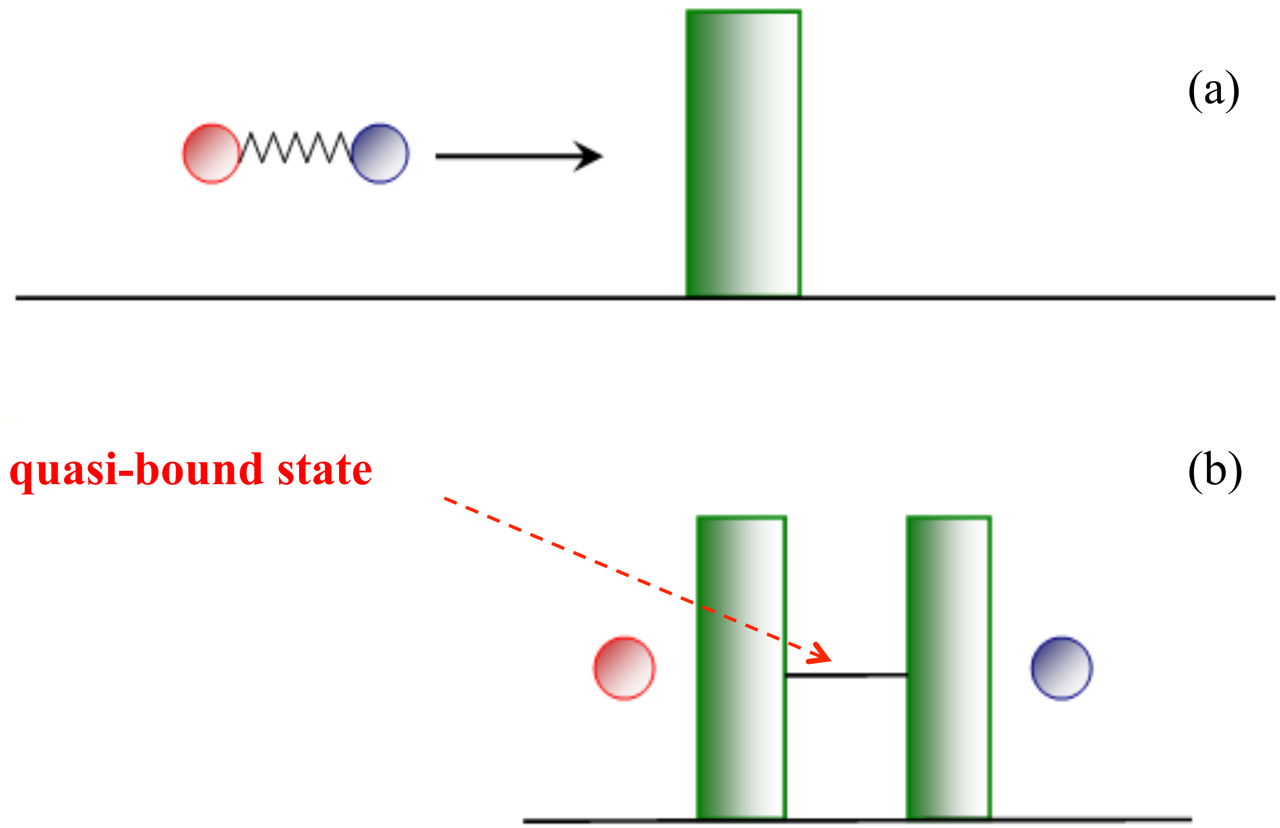
E.g. Resonant Diode Tunneling device.





Resonant tunneling (with single barrier)

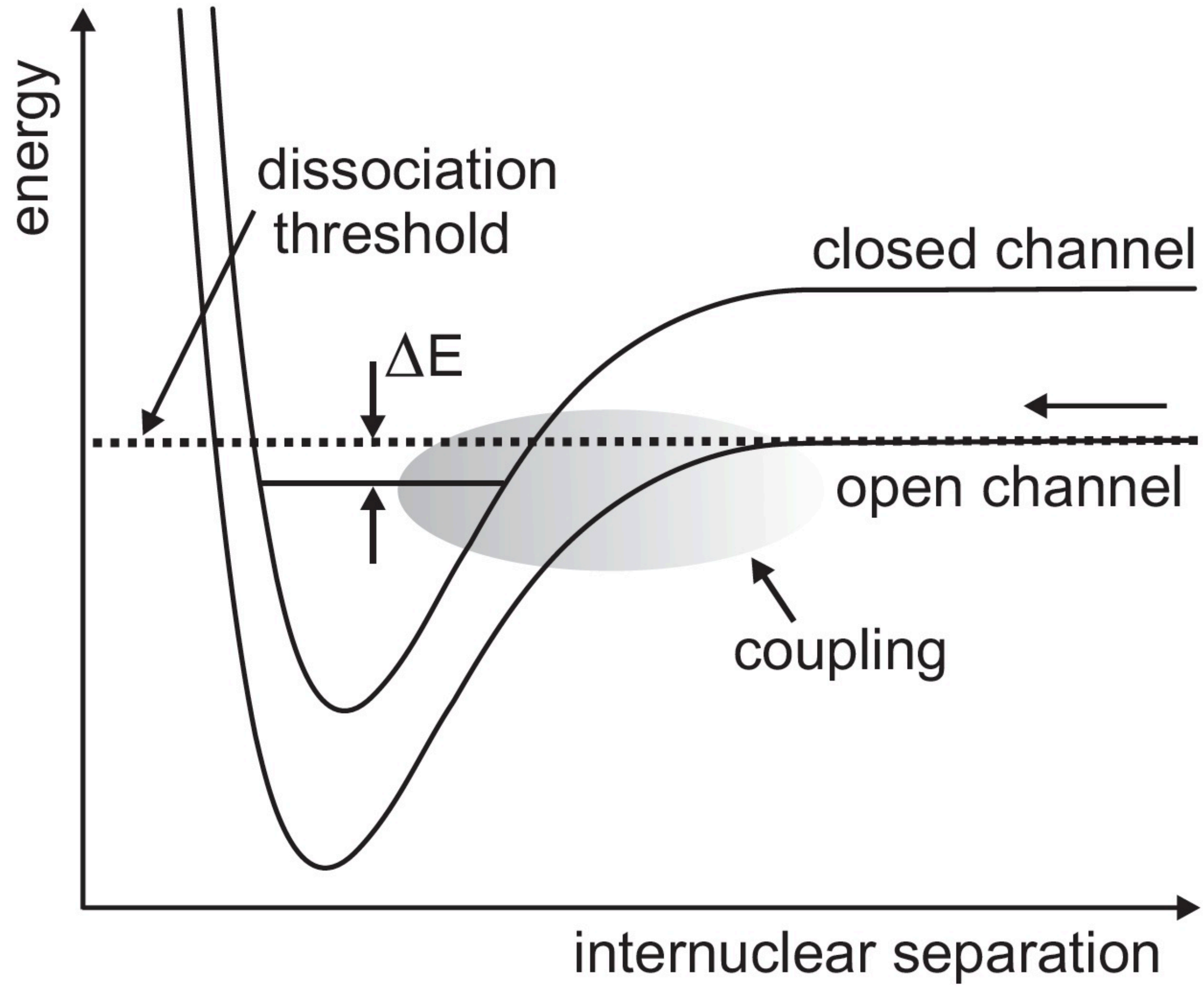
(composite particle + single-step barrier)



Poorly known. Occurs in atomic, molecular and nuclear systems.

E.g. fusion of loosely-bound nuclei.

Feshbach Resonances

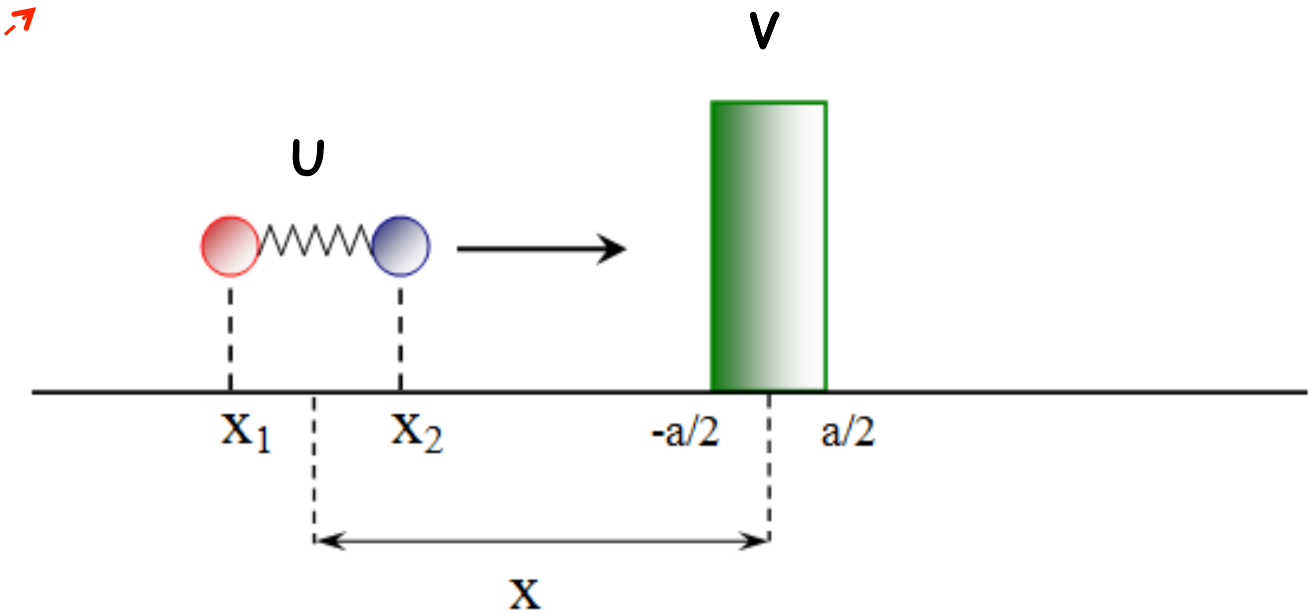


Step-barrier V + square-well U

CB, Flambaum, Zelevinsky, JPG 34, 1 (2007)

barrier

interaction between particles



Schroedinger equation

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + V(x_1) + V(x_2) + U\left(\left|x_1 - x_2\right|\right)$$

Step barrier

$$V(x) = \begin{cases} V_0, & -a/2 \le x \le a/2 \\ 0, & \text{otherwise} \end{cases}$$

Square-well

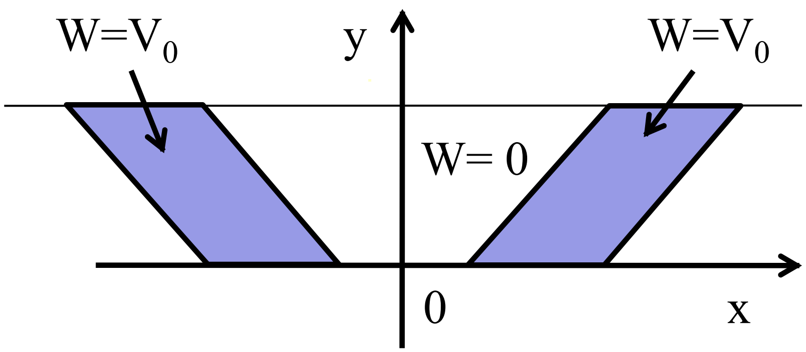
$$U(x) = \begin{cases} 0, & -d/2 \le x \le d/2 \\ \infty, & \text{otherwise} \end{cases}$$

change of variables

$$x = \left(\frac{x_1 + x_2}{2}\right), \qquad y = x_1 - x_2 + \frac{d}{2}$$

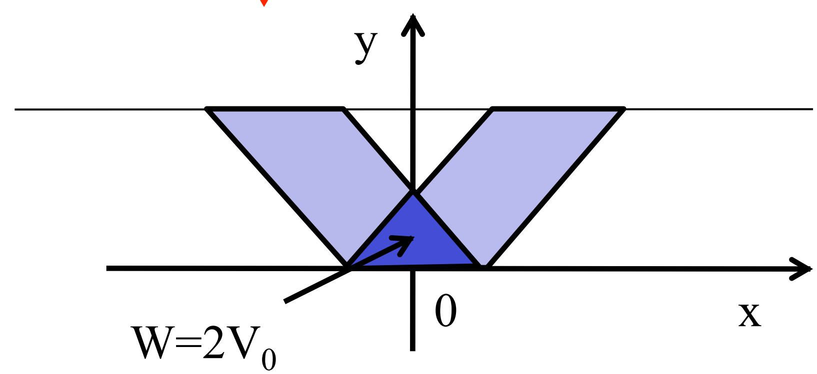
c.m. relative

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + U(y - \frac{d}{2}) + W(x,y)\right]\Psi(x,y) = E\Psi(x,y)$$



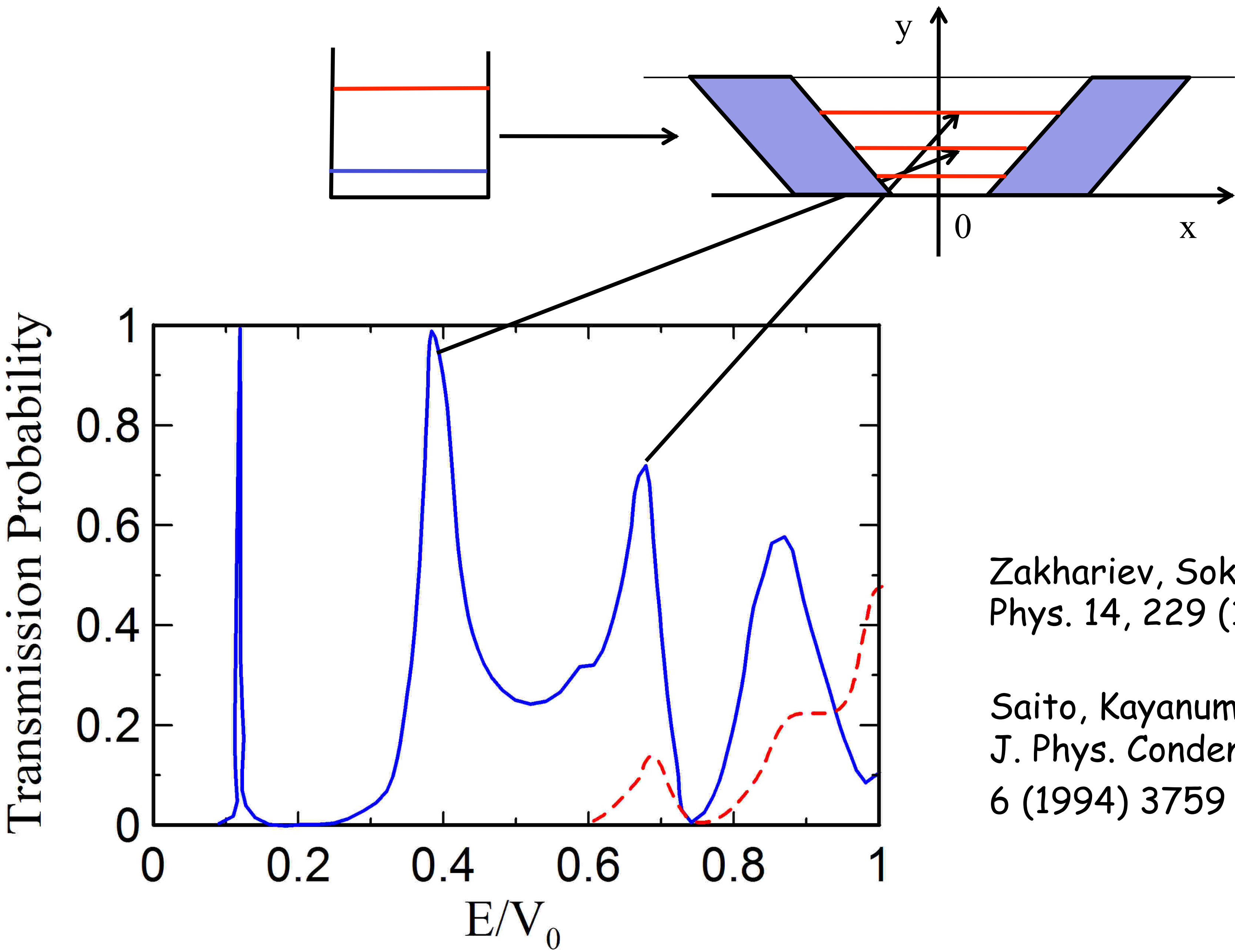
(a) d/2 > a

barrier for c.m. motion



(b) d/2 < a

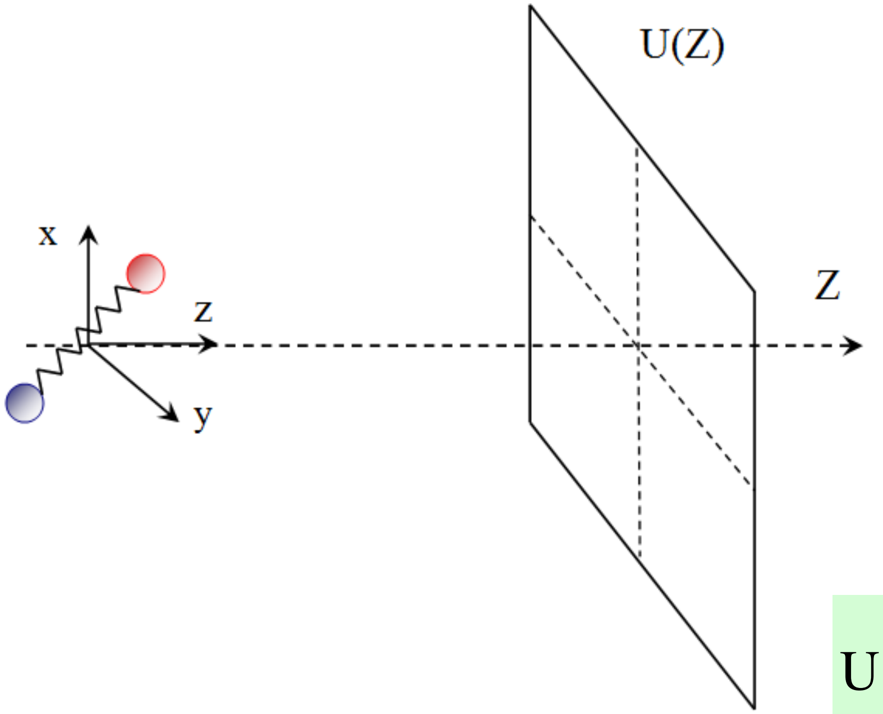
Problem equivalent to a single particle (c.m. coordinate x) tunneling through 2 barriers.



Zakhariev, Sokolov, Ann. d. Phys. 14, 229 (1964)

Saito, Kayanuma, J. Phys. Condens. Matter 6 (1994) 3759

Tunneling of Molecules



$$\mathcal{H} = \frac{\mathbf{P}^2}{4m} + \frac{\mathbf{p}^2}{m} + U\left(Z - \frac{z}{2}\right) + U\left(Z + \frac{z}{2}\right) + V(r)$$

basis

$$\left[\frac{\mathbf{p}^2}{m} + V(r)\right]\phi_n(\mathbf{r}) = \varepsilon_n \phi_n(\mathbf{r})$$

expansion

$$\Psi(Z,\mathbf{r}) = \sum_{n=0}^{\infty} \psi(Z)\phi_n(\mathbf{r})$$

effective potential

$$U_{nm}(Z) = \frac{4m}{\hbar^2}\int \left[U\left(Z + \frac{z}{2}\right) + U\left(Z - \frac{z}{2}\right)\right]\phi_n^*(\mathbf{r})\phi_m(\mathbf{r})d\mathbf{r}$$

effective equation for cm motion

$$\left(\frac{d^2}{dZ^2} + k_n^2\right)\psi_n(Z) - \sum_m U_{nm}(Z)\psi_m(Z) = 0$$

$$k_n^2 = \frac{4m}{\hbar^2}(E - \varepsilon_n)$$

$$\psi_{nl}(Z) = e^{ik_n Z}\delta_{nl} + \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{ik_n(Z-Z')}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

general solution

$$R_{nl}(Z) = \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{ik_n Z'}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

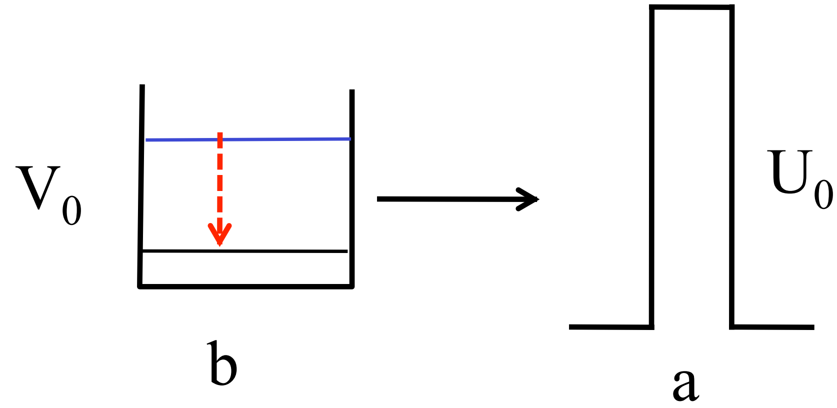
$$T_{nl}(Z) = \delta_{nl} + \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{-ik_n Z'}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

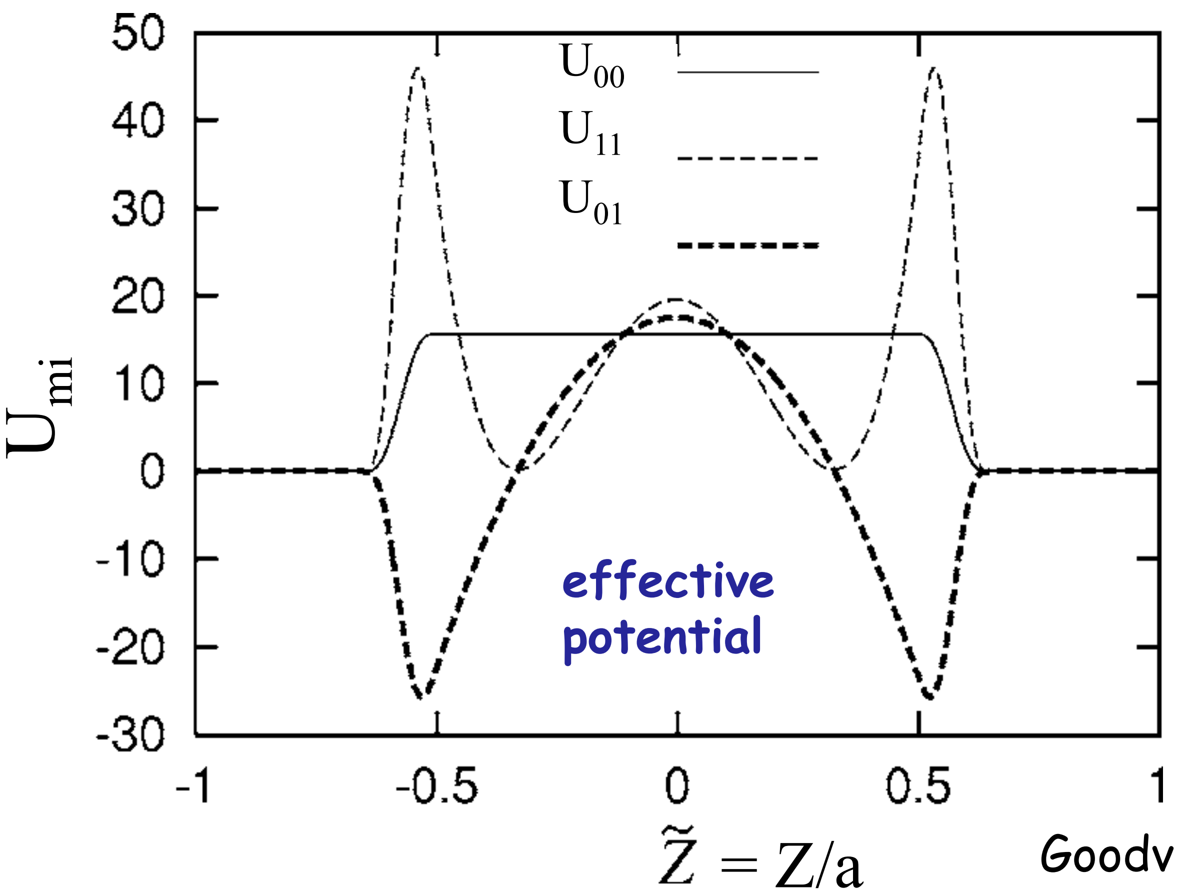
Reflection and transmission probabilities

$$R_1 = \sum_{n=0}^{\infty} \frac{k_n}{k_1} |R_{nl}|^2, \quad T_1 = \sum_{n=0}^{\infty} \frac{k_n}{k_1} |T_{nl}|^2$$

Transition probability

$$P_{n\to 1} = \frac{k_n}{k_1}\left(|R_{nl}|^2 + |T_{nl}|^2\right)$$





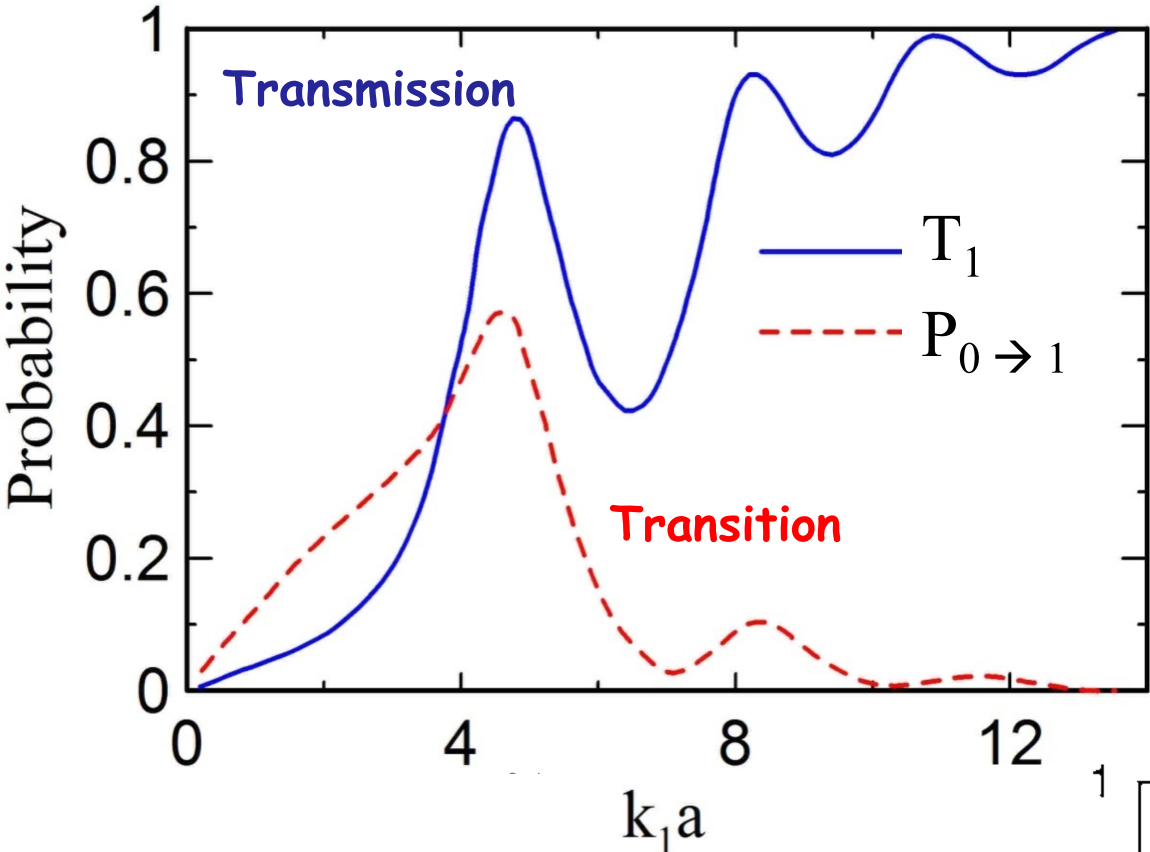
Step barrier parameters

$$\frac{U_0 a}{\hbar c} = 10, \qquad \sqrt{\frac{mU_0}{\hbar^2}}a = 6$$

Square-well particle-particle interaction

$$\frac{V_0 b}{\hbar c} = 2, \qquad \sqrt{\frac{mV_0}{\hbar^2}}b = 4$$

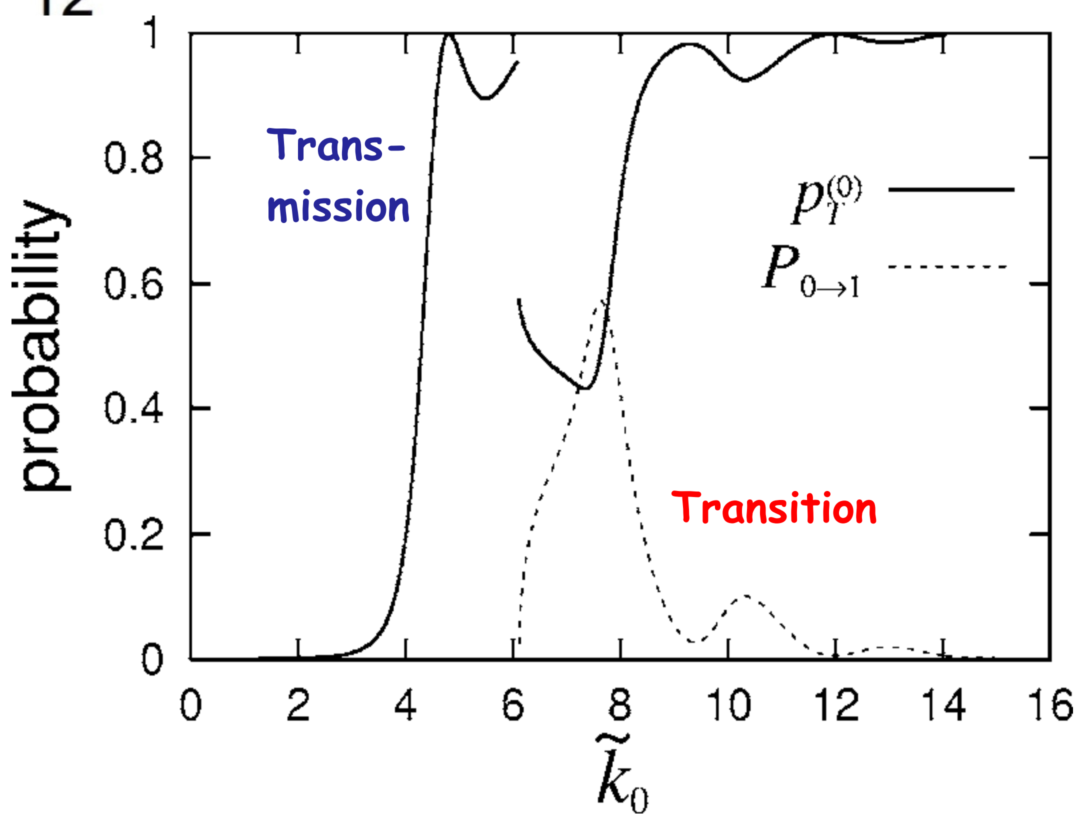
Goodvin, Shegelski, PRA 72, 042713 (2005)



Goodvin, Shegelski,
PRA 72, 042713 (2005)

By moving to its ground state the excited molecule increases its chance of transmission.

The opening of the higher channels decreases the probability of transmission.



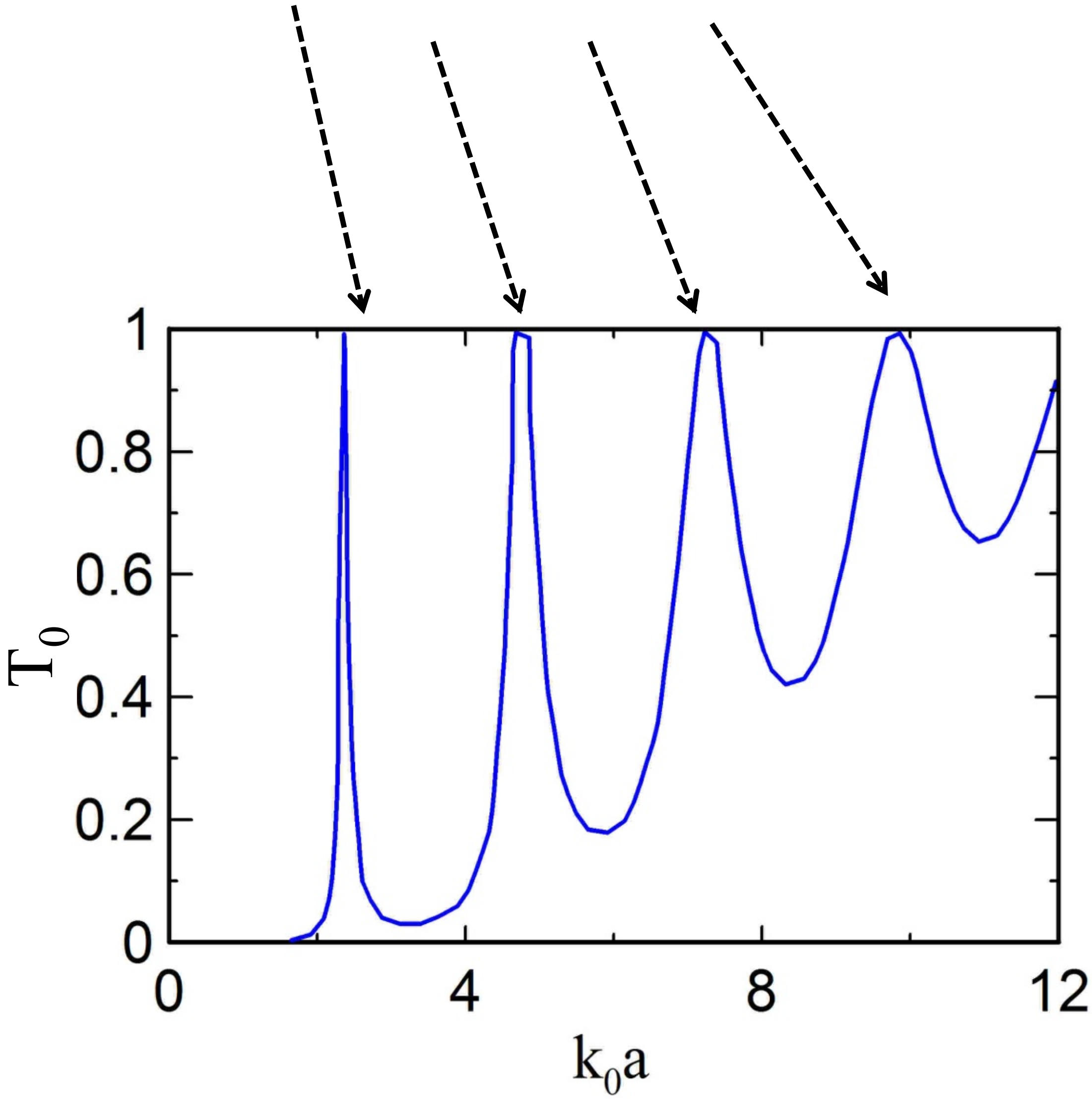
Tunneling of a (two-particle) nucleus

$$k_0 + \frac{2mU_0 a}{\hbar^2}\tan\left(k_0\sqrt{\langle r^2\rangle}\right) = 0$$

valid for

delta barrier

$$U_0 a \rightarrow \delta(Z)$$



For d + d → ^{4}He

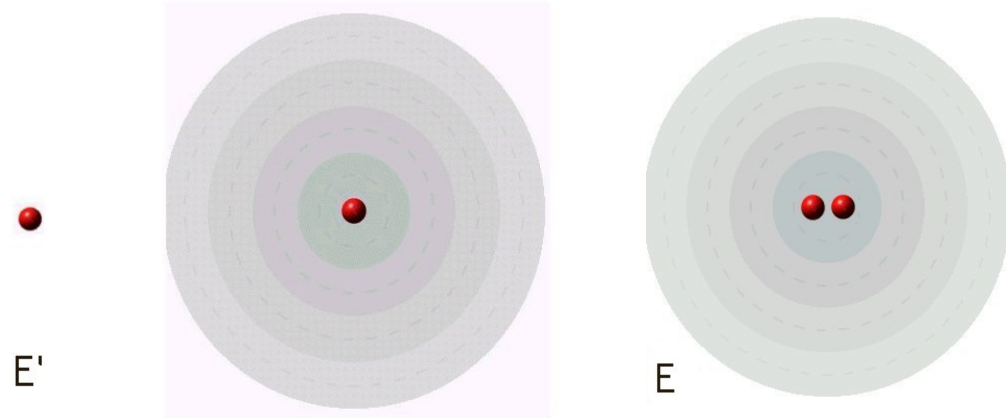
With a and U_0 simulating Coulomb barrier

For large Z's, A's

→ Many resonances possible, if <r^2> large (loosely-bound)

Problem: strong interactions are too strong!

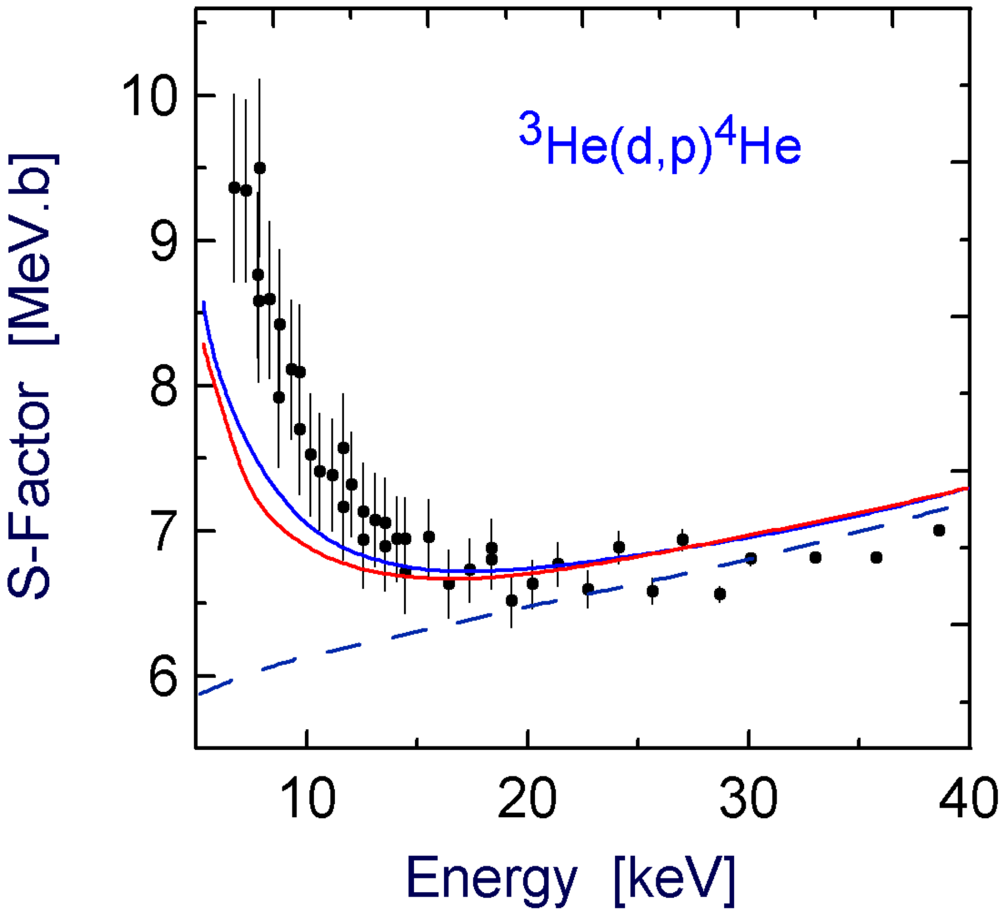
Electron screening in fusion reactions



Adiabatic model: $\Delta E = E' - E$

Electron screening enhancement

$$\sigma_{lab}^{fusion} \sim \sigma_{bare}(E + \Delta E)$$
$$\sim \exp\left[\pi\eta(E)\frac{\Delta E}{E}\right]\sigma_{bare}(E)$$



S_{bare}

Dynamic

Adiabatic

Rolfs, 1995 Reaction	ΔE [eV] experiment	ΔE [eV] adiabatic limit
$d(^3He, p)^4He$	180 ± 30	119
$^6Li(p, \alpha)^3He$	470 ± 150	186
$^6Li(d, \alpha)^4He$	380 ± 250	186
$^7Li(p, \alpha)^4He$	300 ± 280	186
$^{11}B(p, \alpha)2^4He$	620 ± 65	348

Small effects

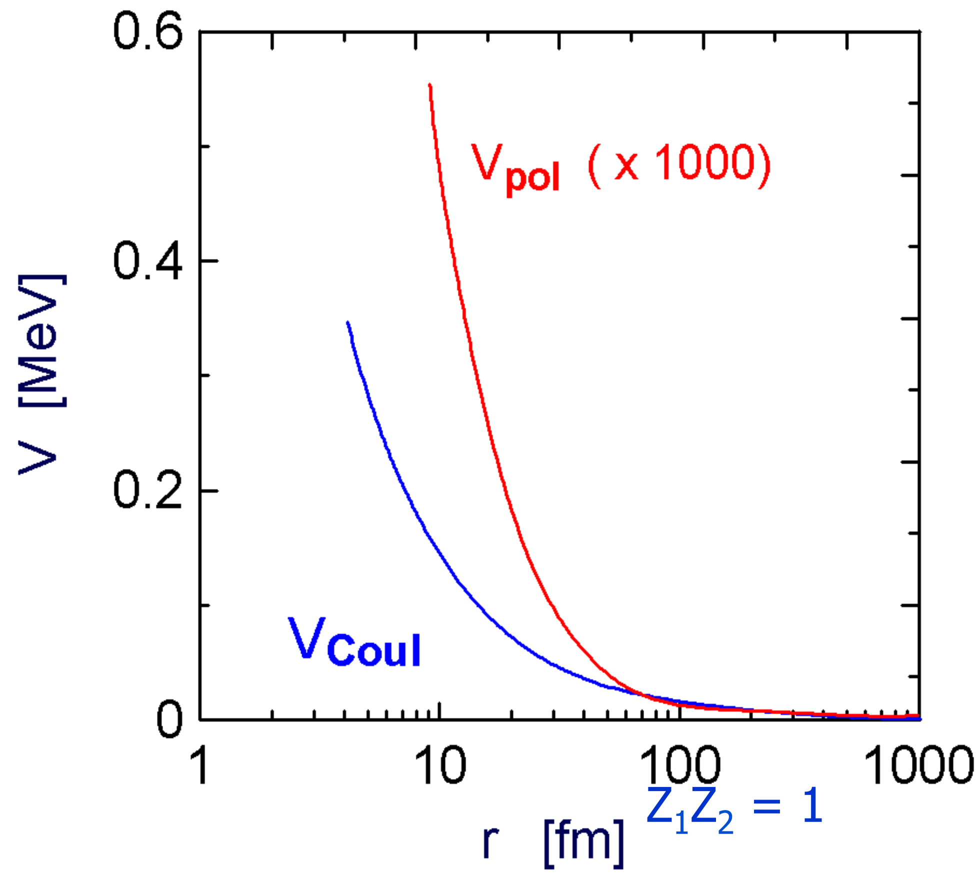
- Thermal motion, lattice vibrations, beam energy spread
- Nuclear breakup channels (in weakly-bound nuclei)
- Dynamics of tunneling

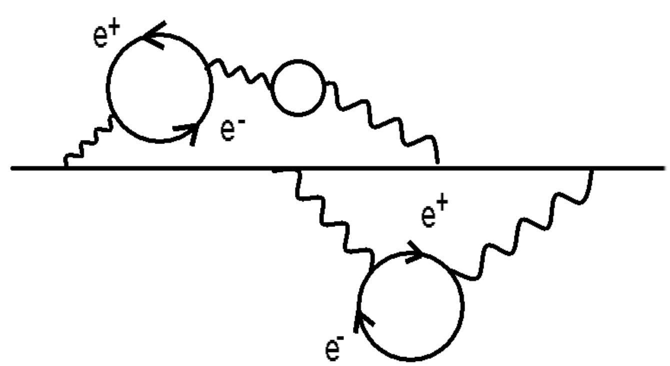
Balantekin, CB, Hussein, NPA 627 (1997)324

Corrections	
Vaccuum Polarization	$\sim 1\%$
Relativity	10^{-3}
Bremsstrahlung	10^{-3}
Atomic porarization	10^{-5}
Nuclear polarization	$< 10^{-10}$

all ≤ 1%

Not a solution! (we need ~ 100%)

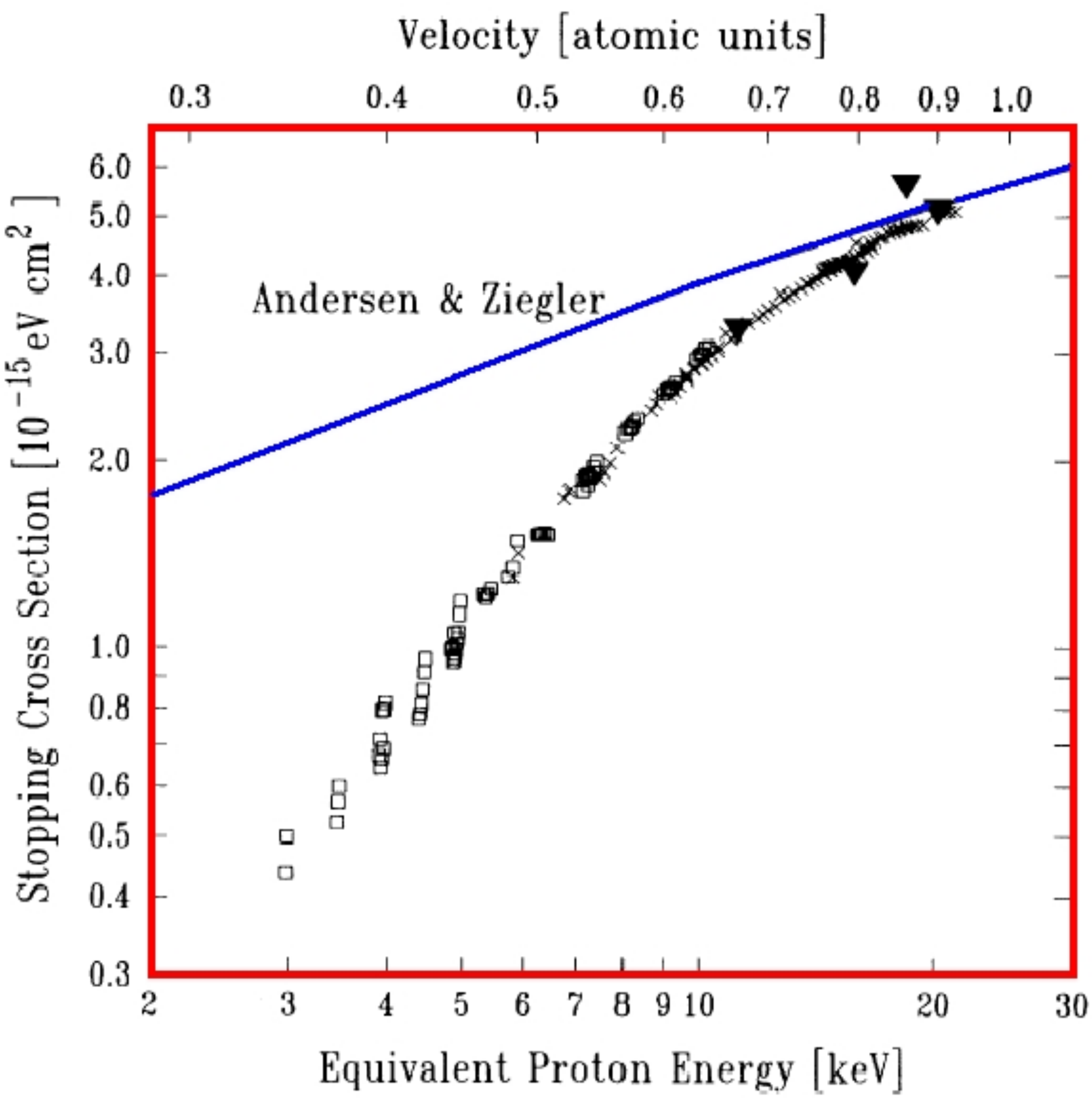


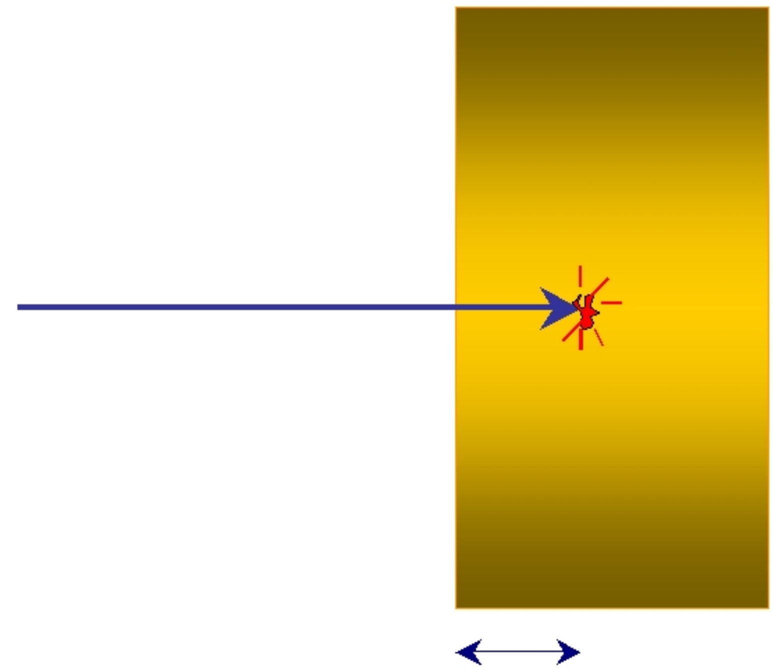


Vacuum polarization

Wrong extrapolation of stopping power

Bang, PRC 53 (1996) R18
Langanke, PLB 369 (1996) 211





$$S_p = -\frac{dE}{dx}$$

Data has to be corrected for stopping power:

$$E' = E - S_p.\Delta x$$

Very few data on stopping at ultra-low energies:

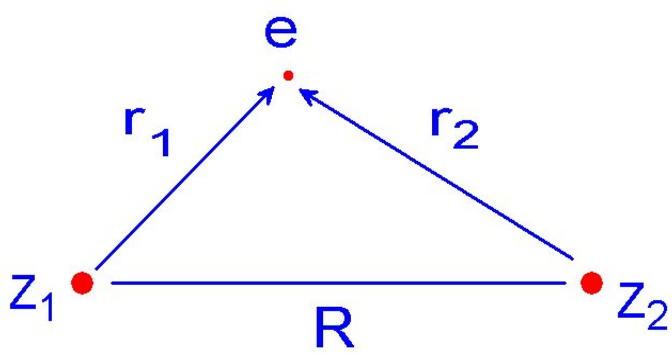
H + He

Golser, Semrad, PRL 14 (1991) 1831

Mainly charge-exchange

Simplest test

CB, de Paula, PRC 62, 045802 (2000)
PLB 585, 35 (2004)

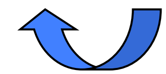


Elliptic coordinates

Charge exchange (pickup) **p + H**

P + D

Projectile slows down
to carry electron with



e^-

change of variables

$$\xi = \frac{r_1 + r_2}{R}; \qquad \eta = \frac{r_1 - r_2}{R}; \qquad \phi$$

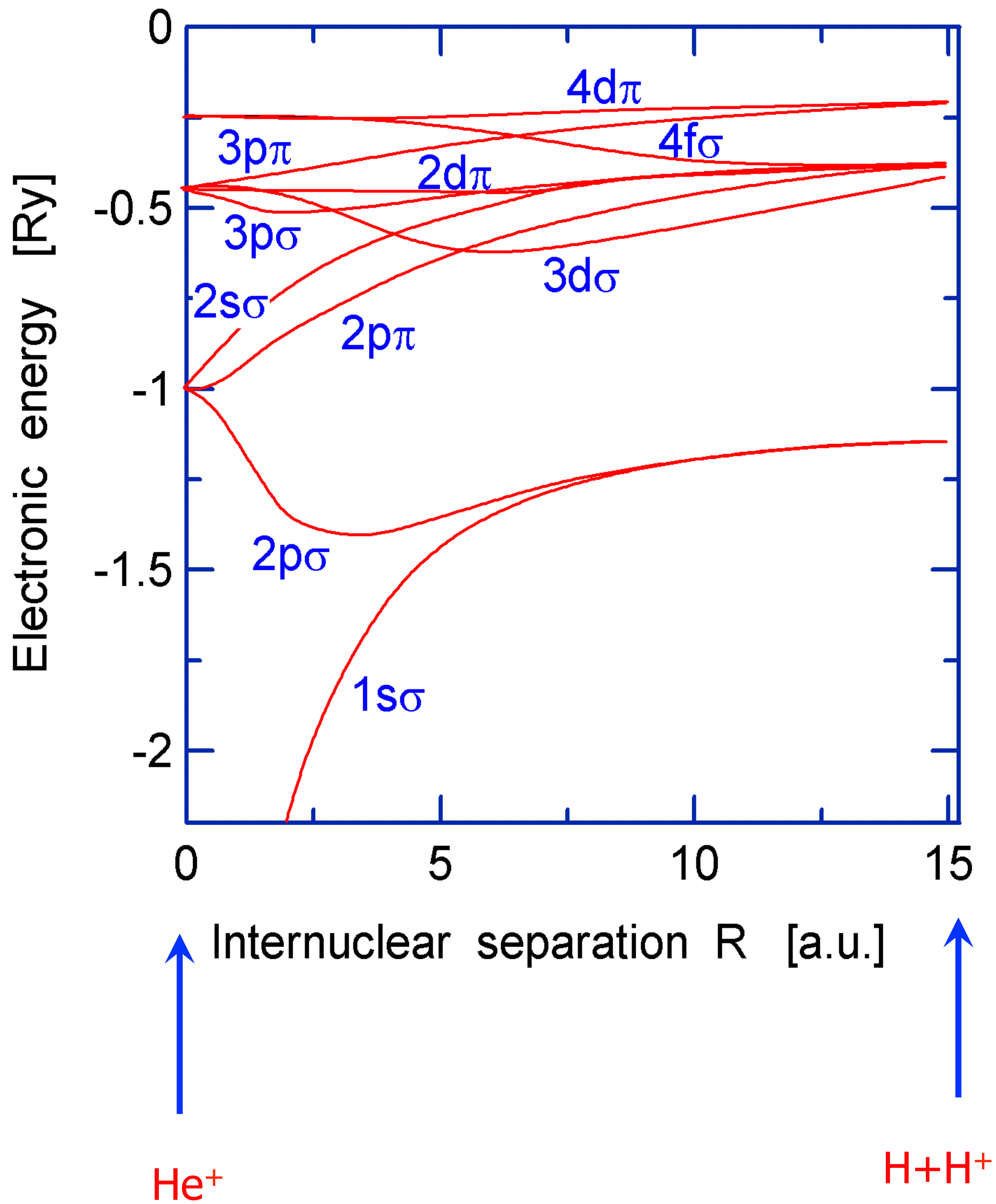
$$\Psi = F(\xi) G(\eta) e^{im\phi}$$

Two-center wfs (molecular orbitals):

$$\frac{d}{d\xi}\left[\left(\xi^2 - 1\right)\frac{dF}{d\xi}\right] + \left[\frac{R^2\xi^2}{2}E + 2R\xi - \frac{m^2}{\xi^2 - 1}\right]F(\xi) = 0$$

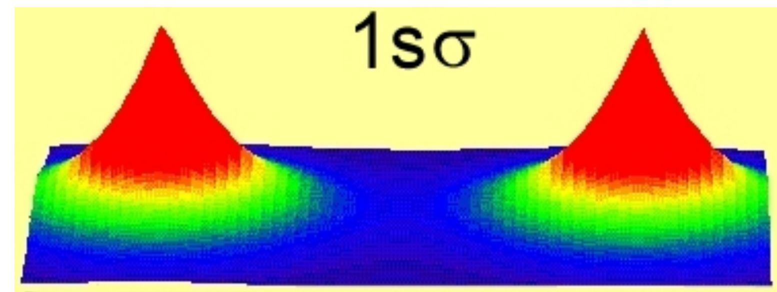
$$\frac{d}{d\eta}\left[\left(1 - \eta^2\right)\frac{dG}{d\eta}\right] - \left[\frac{R^2\xi^2}{2}E + 2R\xi + \frac{m^2}{\eta^2 - 1}\right]G(\eta) = 0$$

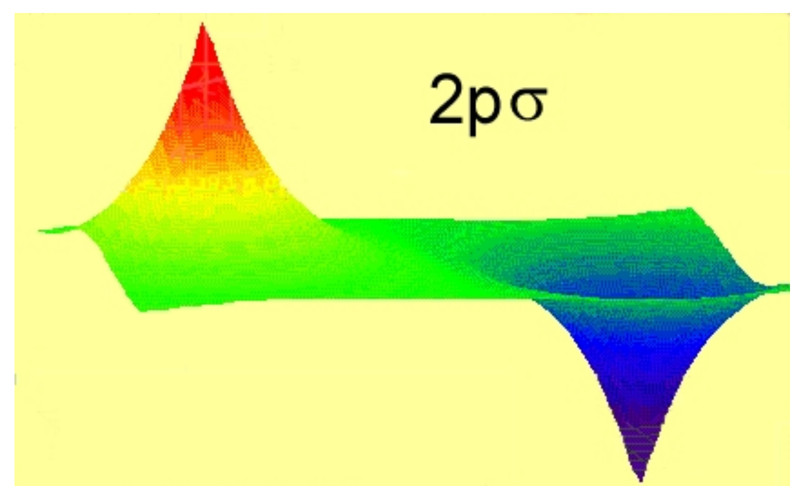
Expansion basis: molecular orbitals for p+H



$$l_z\Phi_s = \pm\lambda\Phi_s$$

Value of λ	0	1	2	3
Code letter	σ	π	δ	$\phi, \cdots$



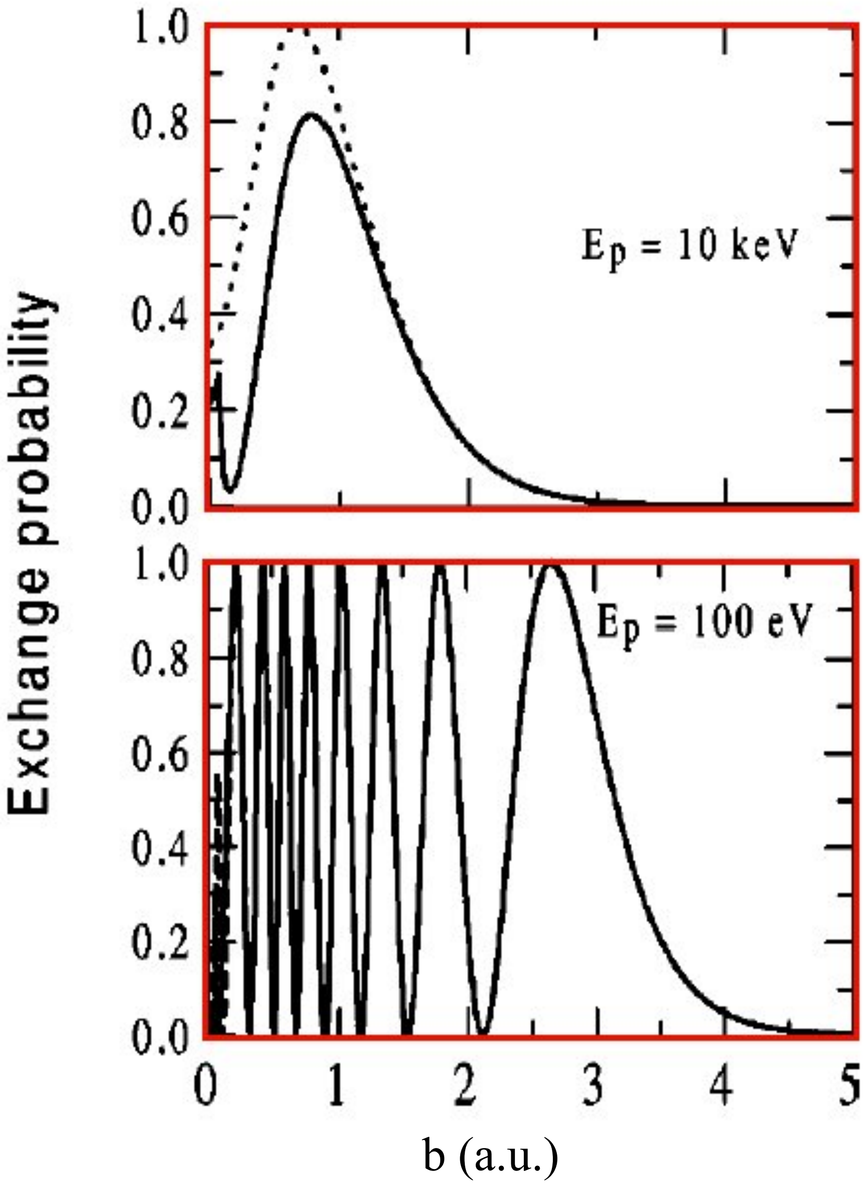


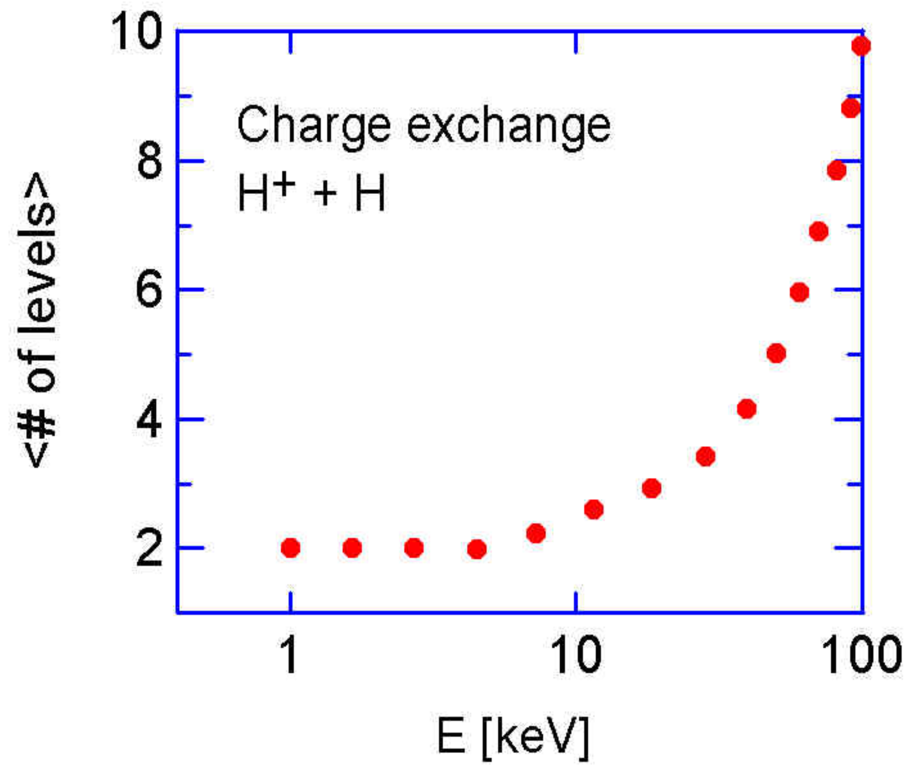
Coupled-channels calculation

$$i\hbar \frac{d}{dt} a_m(t) = E_m(t) a_m(t) - i\hbar \sum_n a_n(t) \left\langle m \left| \frac{d}{dt} \right| n \right\rangle$$

$$\left\langle m \left| \frac{d}{dt} \right| n \right\rangle = \frac{\left\langle m \left| dV_p/dt \right| n \right\rangle}{E_n(t) - E_m(t)},$$

(Hellman, Feynmann relation)





For E_p < 10 keV, only 1sσ and 2pσ

2-level problem - **resonant exchange**

$$P_{exch} \approx \frac{1}{2} + \frac{1}{2} \cos\left\{ \frac{1}{\hbar} \int_{-\infty}^{\infty} \left[E_{2p}(t) - E_{1\sigma}(t) \right] dt \right\}$$

H^+ + He collisions (two-active electrons)

Slater-type orbitals

$$\phi = N r^{n-1} e^{-\xi r} Y_{lm}(\theta,\phi)$$

Two-center basis for two-electrons
Hartree-Fock equations

$$\Phi_i = \sum_{i=1}^{n} \left[c_{ji}^{A} \phi_i^{A} + c_{ji}^{B} \phi_i^{B} \right]$$

→ **F . C = S . C . E**

$$F_{\mu\nu} = H_{\mu\nu} + \sum_{\lambda\rho} P_{\lambda\rho} \left[(\mu\nu \mid \lambda\rho) - \frac{1}{2} (\mu\rho \mid \lambda\nu) \right]$$

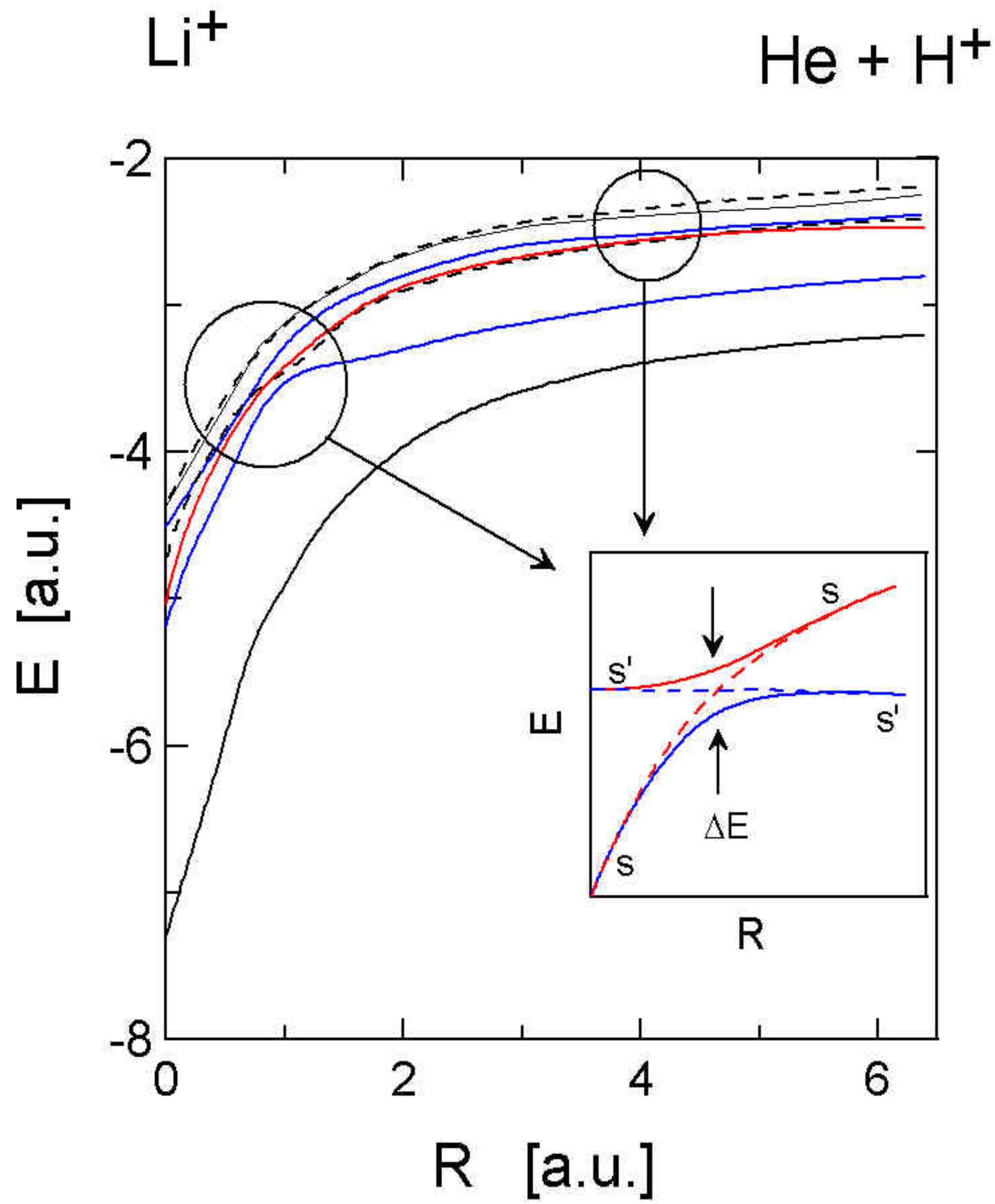
$$H_{\mu\nu} = \iint \phi_{\mu}^{*}(1) \left[-\frac{1}{2}\nabla_1^2 - \sum_{A} \frac{1}{r_{1A}} \right] \phi_{\nu}^{*}(1)\, d\tau_1 , \qquad P_{\lambda\rho} = 2 \sum_{i=1}^{occ} c_{\lambda i} c_{\rho i}$$

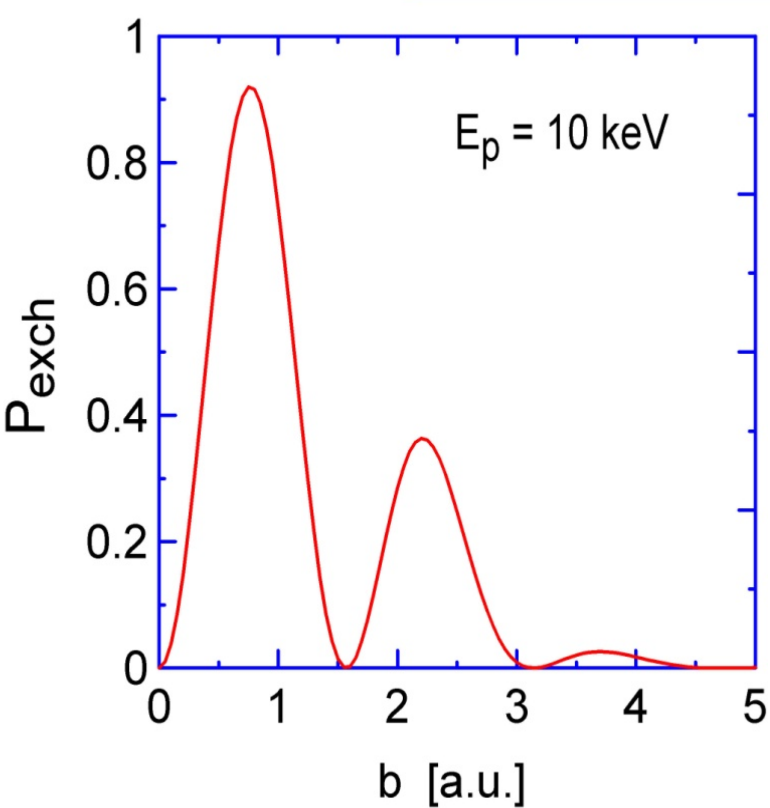
$$(\mu\nu \mid \lambda\rho) = \iint \phi_{\mu}(1)\, \phi_{\nu}(1) \frac{1}{r_{12}} \phi_{\lambda}(2)\, \phi_{\rho}(2)\, d\tau_1 d\tau_2 , \quad S_{\mu\nu} = \int \phi_{\mu}(1)\, \phi_{\nu}(1)\, d\tau_1$$

→ t. d. coupled-channels equations

Damping of resonant exchange H(1s) ⇔ He(1s2s)

Separated atom	United atom
$H^+ + He(1s^2)$	0Σ
$H(1s) + He^+(1s)$	1Σ
$H^+(1s) + He(1s2s)$	2Σ
$H(n=2) + He^+(1s)$	1Π
$H(n=2) + He^+(1s)$	3Σ
$H(n=2) + He^+(1s)$	4Σ
$H^+ + He(1s1p)$	5Σ
$H^+ + He(1s1p)$	2Π





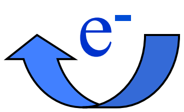
Landau-Zener effect + dissipation

$$P = e^{-\Gamma \Delta t_{coll}} \cos^2\left[\frac{H_{12}\,a}{2v}\right]$$

Stopping power at very low energies

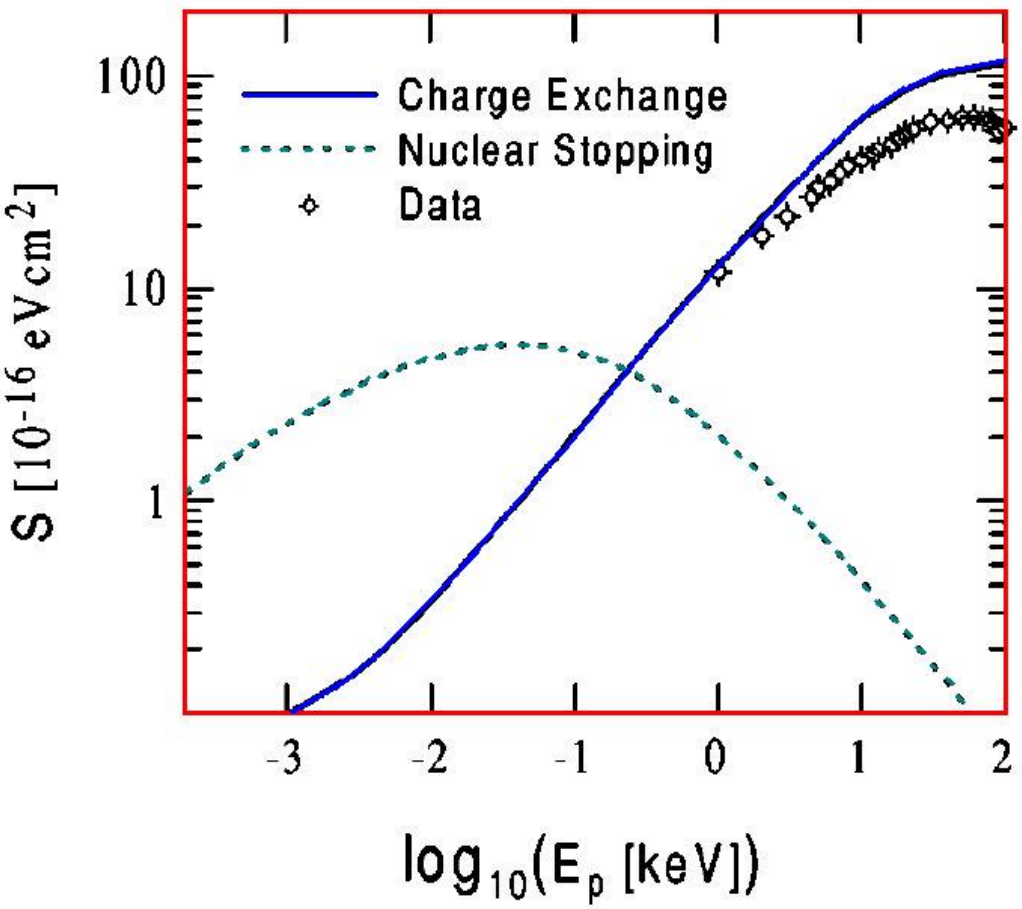
p + H

P + D



CB, PLB 585, 35 (2004)

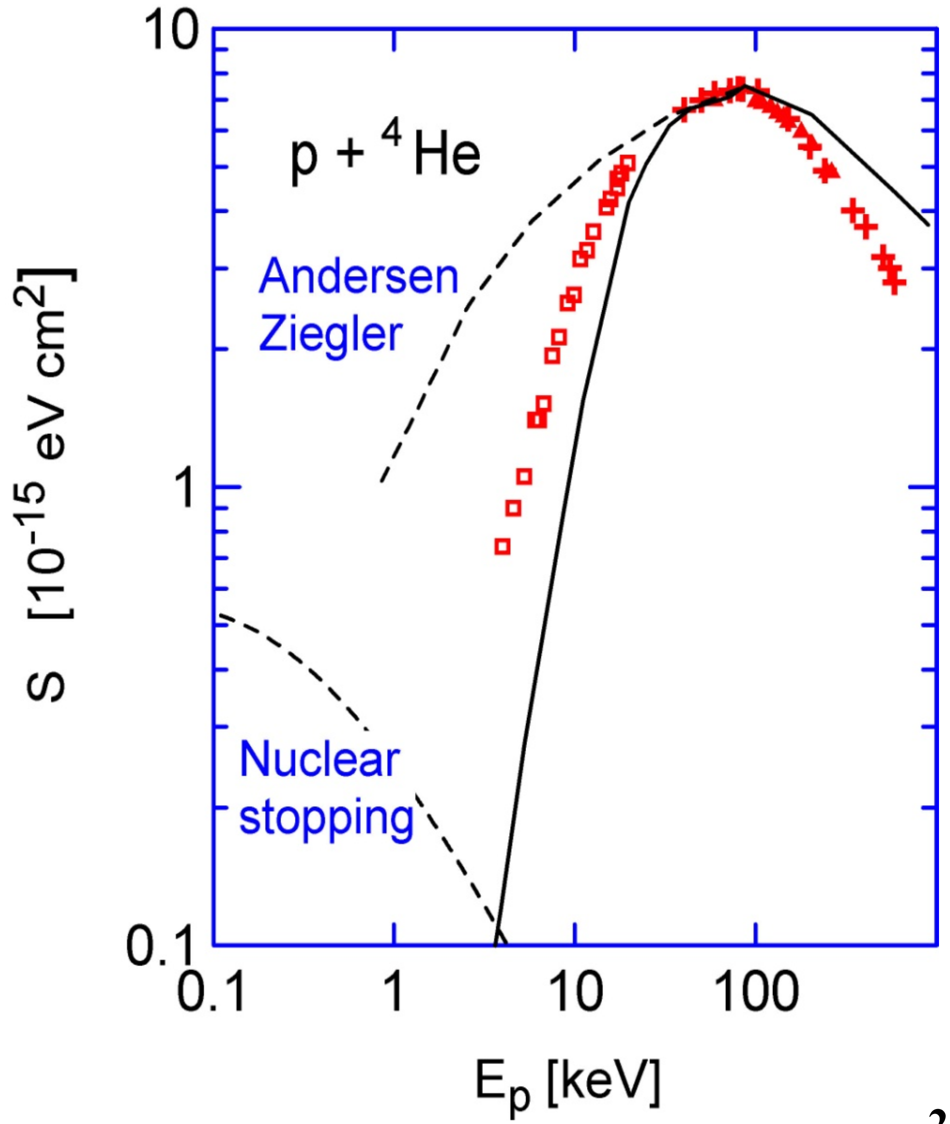
CB, de Paula, PRC 62, 045802 (2000)



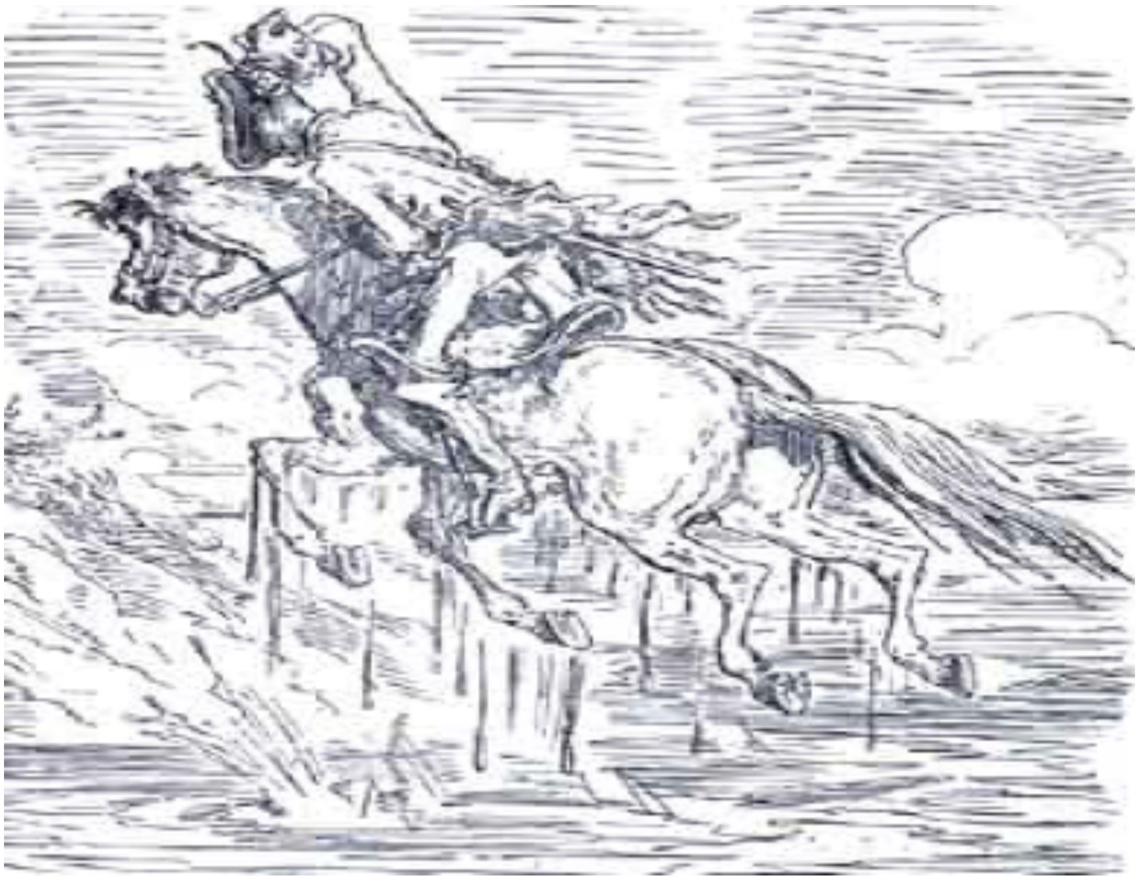
Threshold effect

$$E_P \geq \frac{\mu^2}{4M_P m_e} \Delta E \geq 8 \quad keV$$

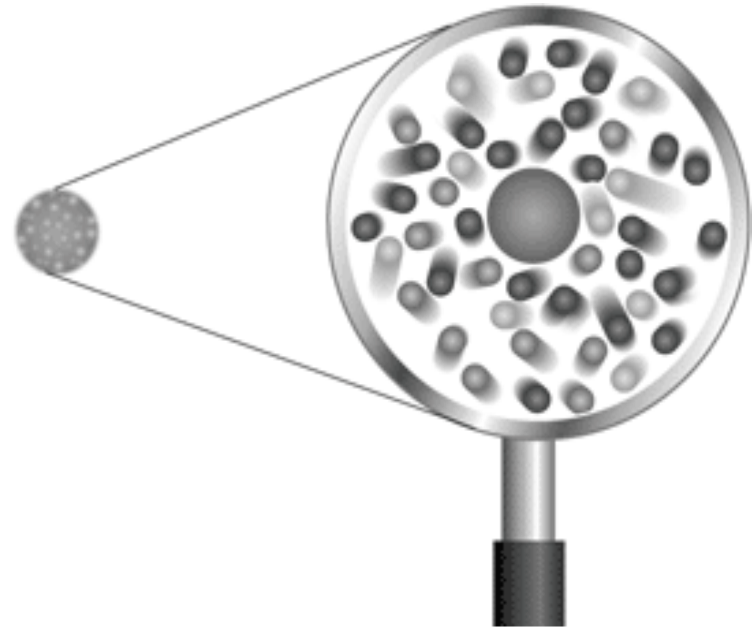
He: $1s^2 \rightarrow 1s2s$: 19.8 eV



Virtual particles enhance tunneling



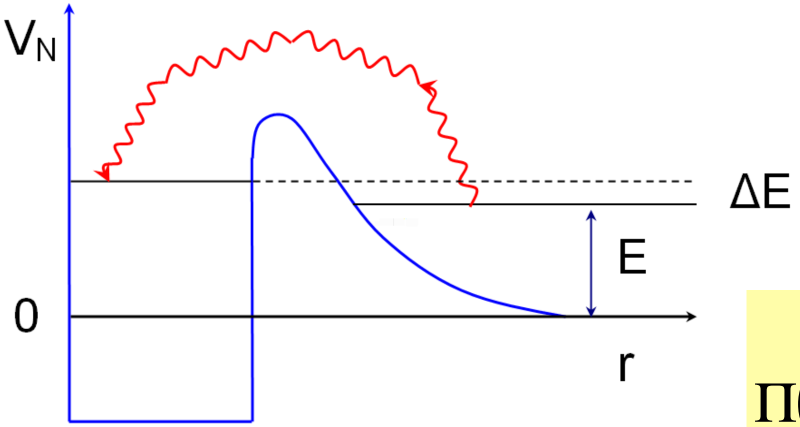
Baron Muenchhausen escaping from a swamp by pulling himself up by his own hair.
G.A. Buerger (1786).



QFT: a "physical" particle consists of a "naked" particle "dressed" in a cloud of short-lived "virtual" particles.

Quantum Muenchhausen

Flambaum , Zelevinsky, PRL 83, 3108 (1999)



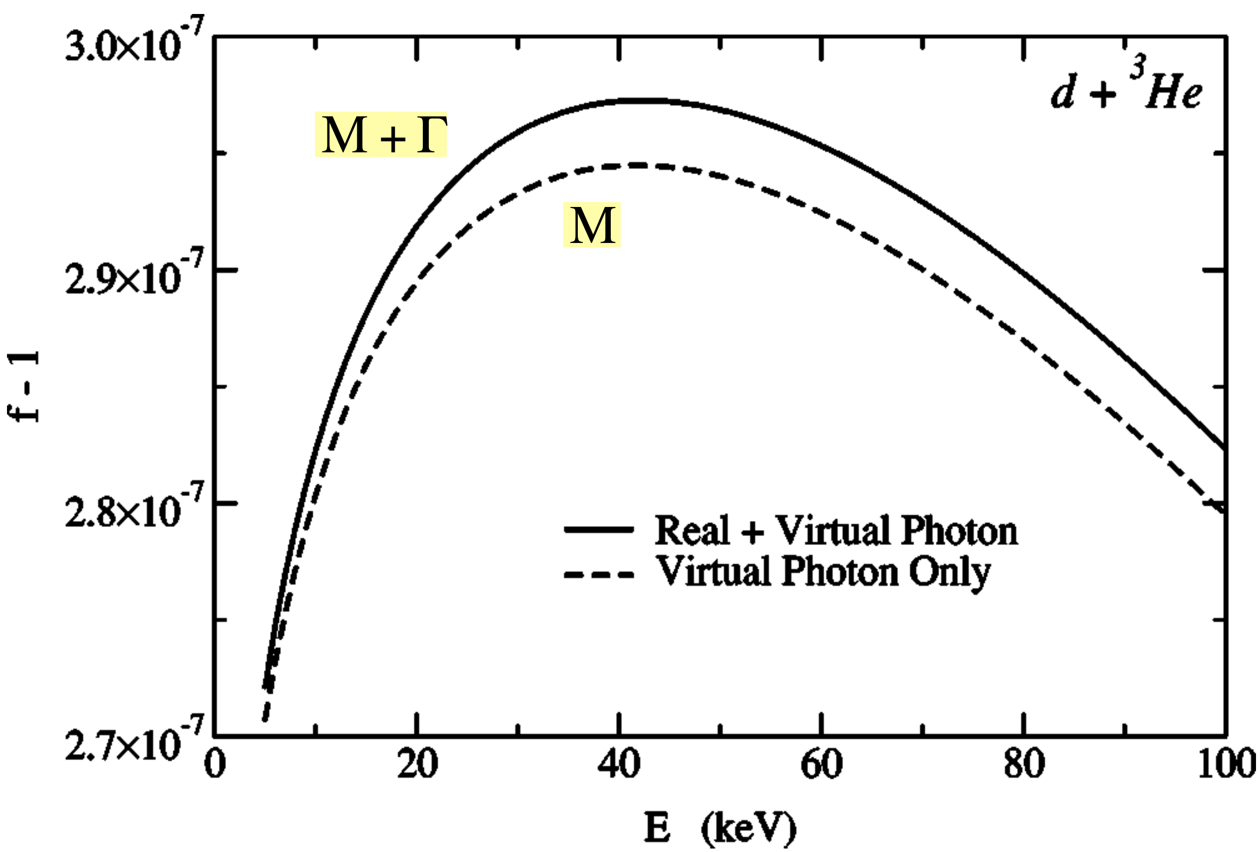
Non-relativistic reduction:

$$H\Psi(\mathbf{r}) + \int \Pi(\mathbf{r},\mathbf{r}';E)\Psi(\mathbf{r}')d^3r = E\Psi(\mathbf{r})$$

Self-energy:

$$\Pi(\mathbf{r},\mathbf{r}';E) = \sum_{\mathbf{k},\lambda}|g_{\mathbf{k}}|^2 \sum_n \frac{\langle \mathbf{r}|(\hat{\mathbf{p}}\cdot \mathbf{e}_{\mathbf{k}\lambda})e^{i\mathbf{k}\cdot\mathbf{r}}|n\rangle\langle n|(\hat{\mathbf{p}}\cdot \mathbf{e}^*_{\mathbf{k}\lambda})e^{-i\mathbf{k}\cdot\mathbf{r}}|\mathbf{r}'\rangle}{E - E_n - \omega_k - i0}$$

$$= M(\mathbf{r},\mathbf{r}';E) + \Gamma(\mathbf{r},\mathbf{r}';E)$$



M = self-energy, mass renormalization, virtual photons

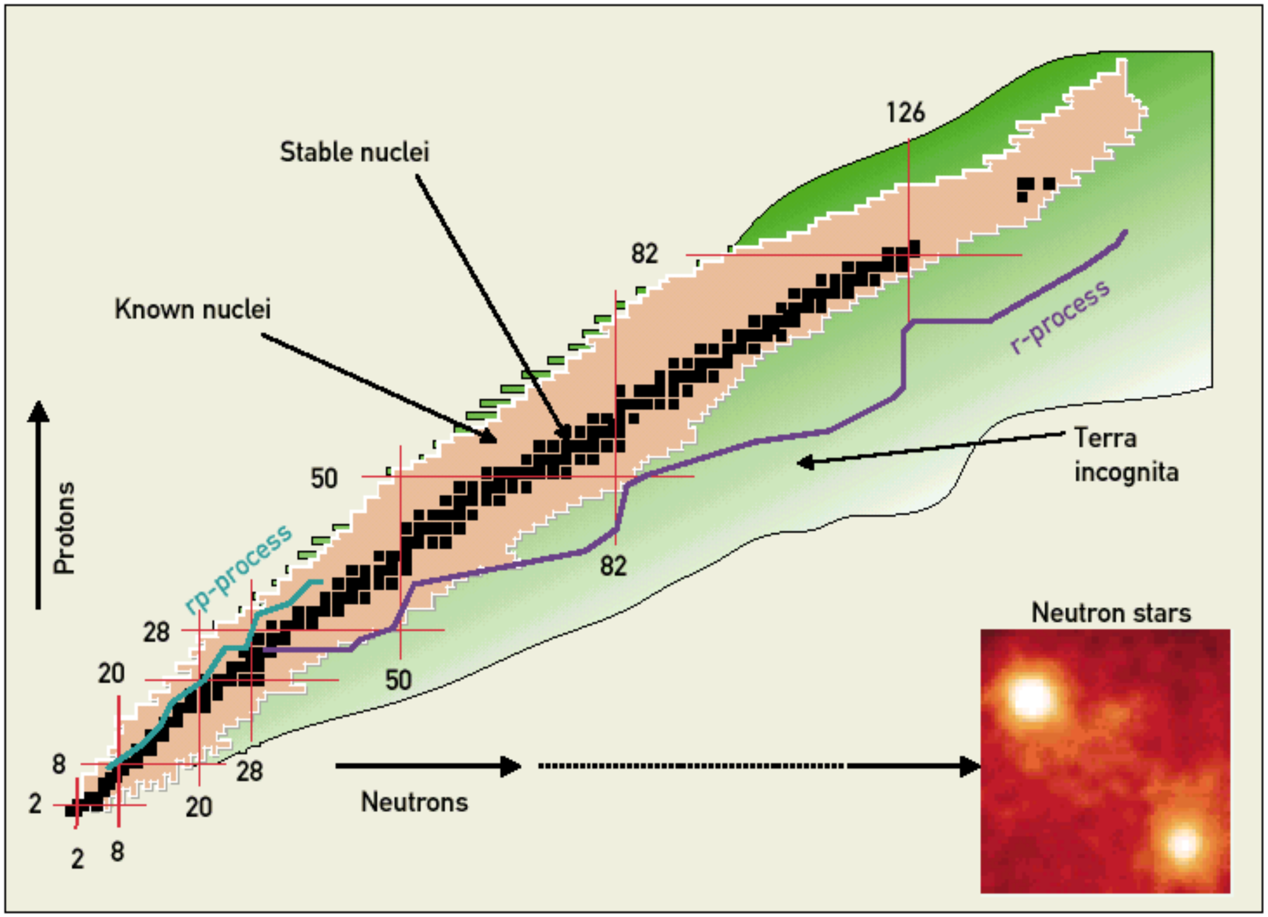
Γ = decay width, real photons

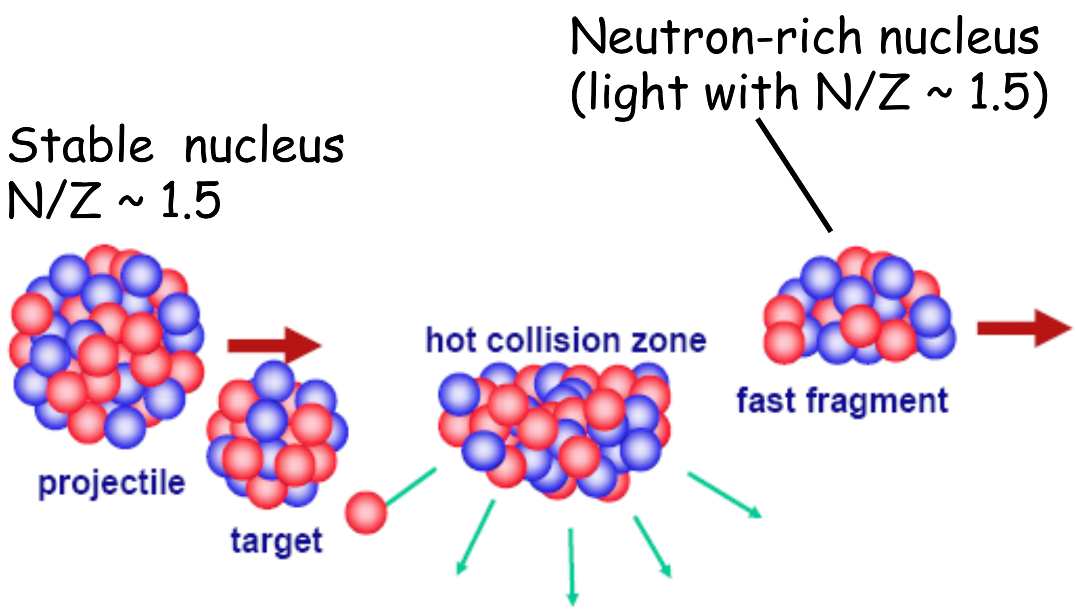
Hagino, Balantekin
PRC 66, 055801 (2002)

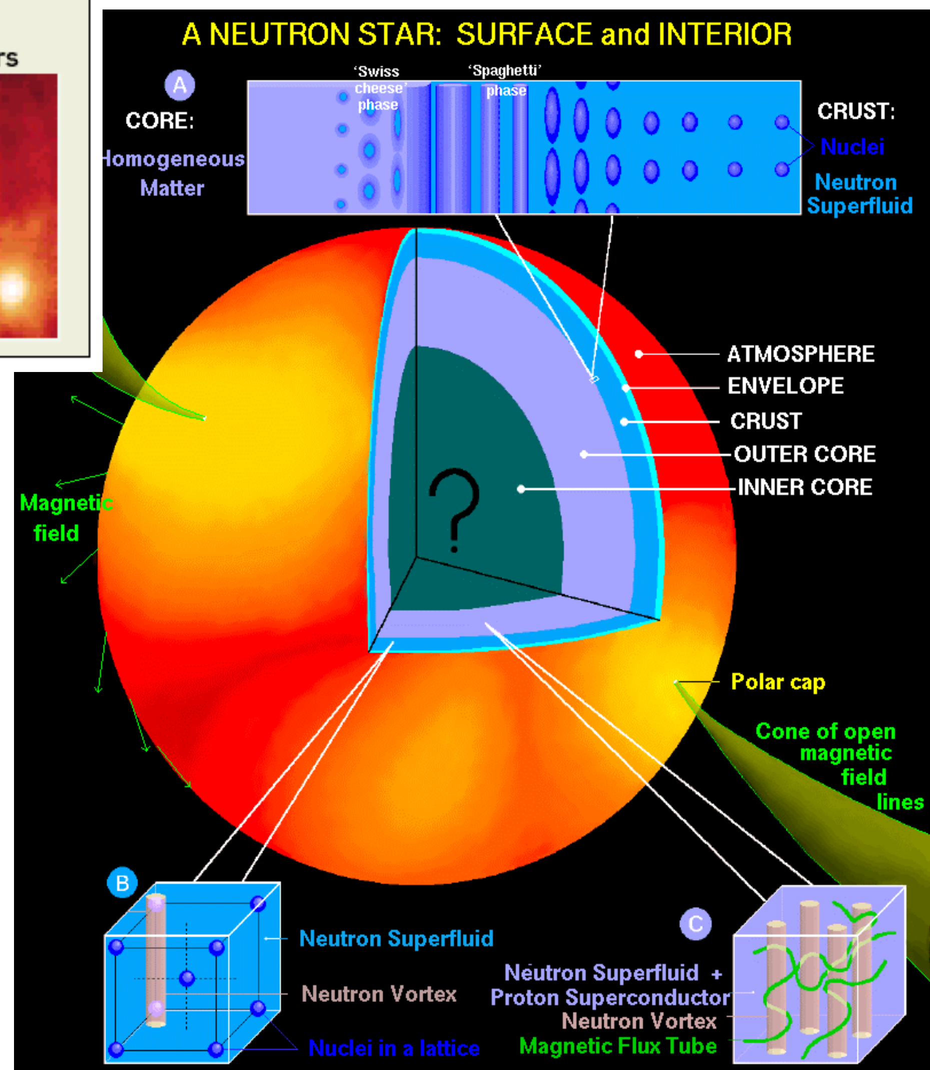
Small effect for tunneling of stable nuclei.

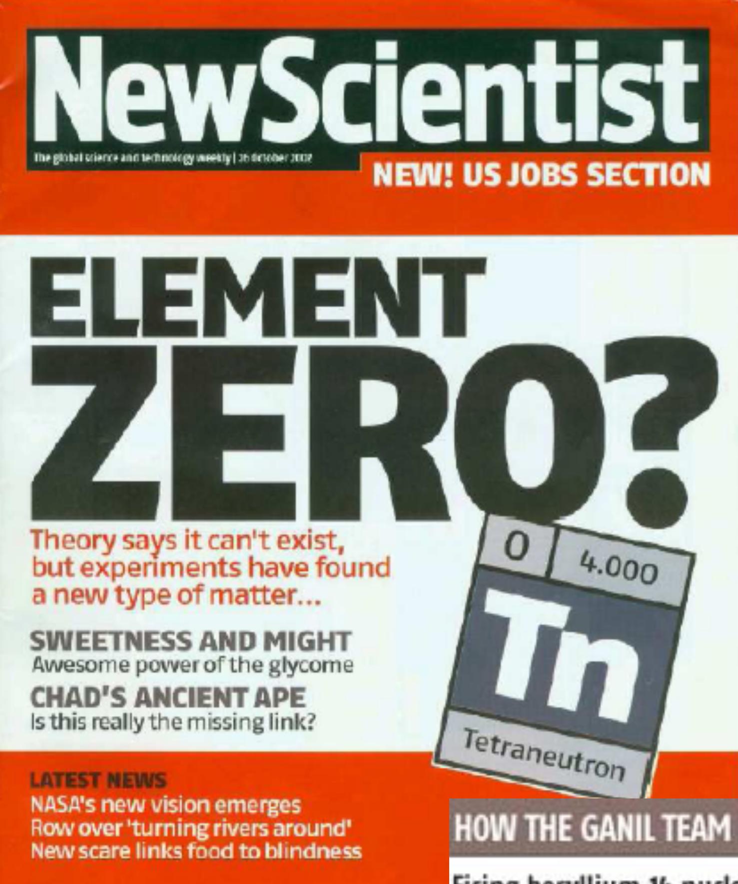
Effect for loosely-bound nuclei (and molecules) unexplored.

Neutron stars and neutron-rich nuclei





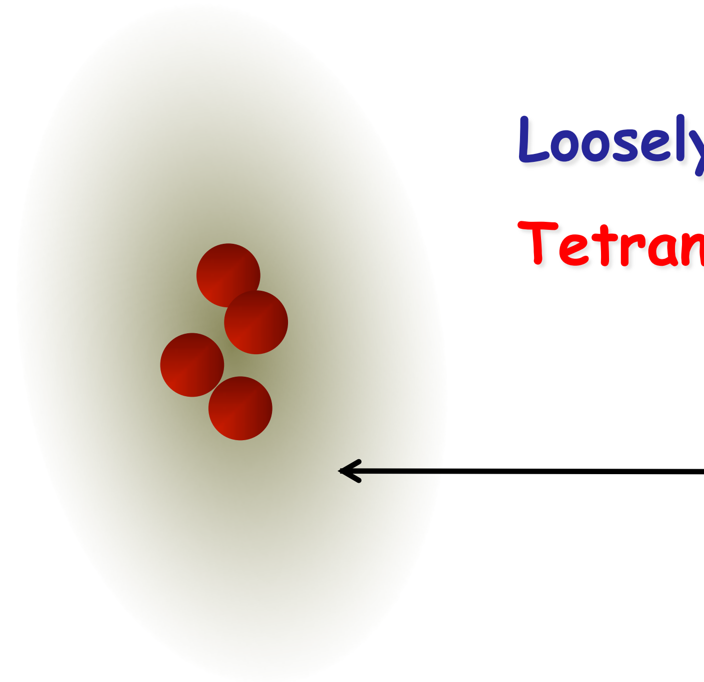




10/26/2002

Loosely-bound nuclei

Tetraneutron?

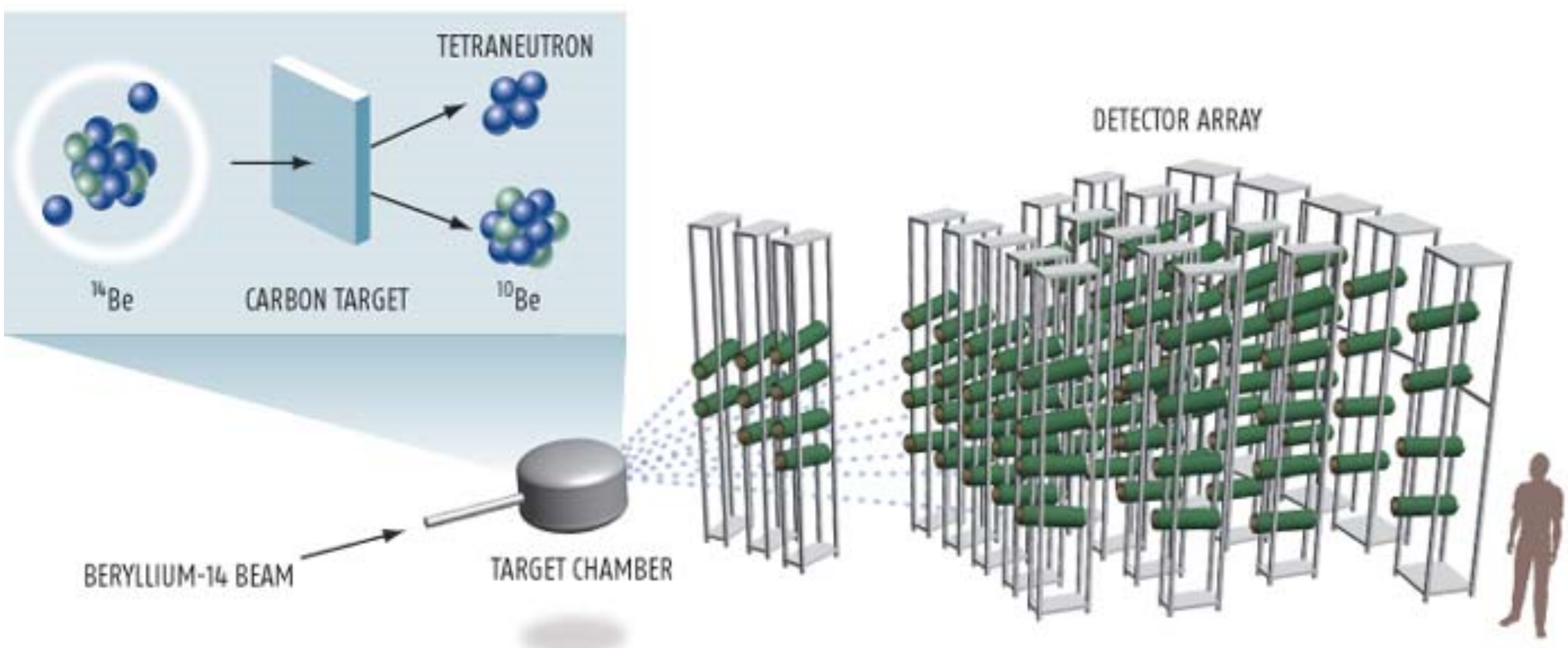


halo

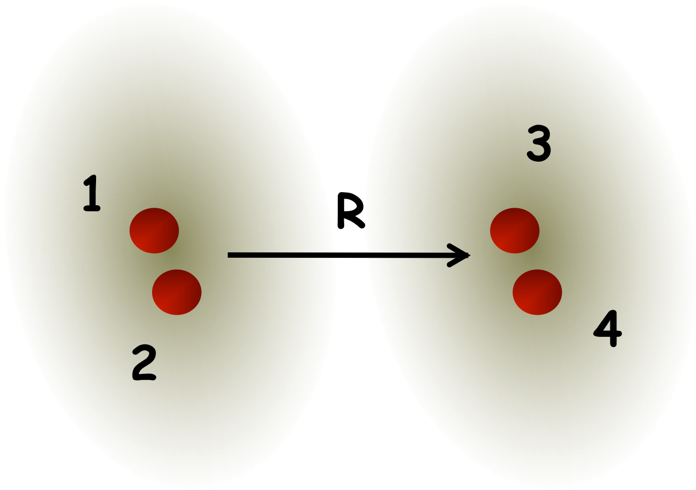
Marques et al, PRC 65, 044006 (2002)

HOW THE GANIL TEAM CREATED AND DETECTED TETRANEUTRONS

Firing beryllium-14 nuclei at a carbon target produces a spray of nuclear fragments that fly into over 100 separate detectors. When this nuclear debris hits a detector, it's energy is transformed into a flash of light



Tetraneutron as a dineutron-dineutron molecule



CB, Zelevinsky, J. Phys. G 29 (2003) 2431

Antissimetrization (Pauli-principle)

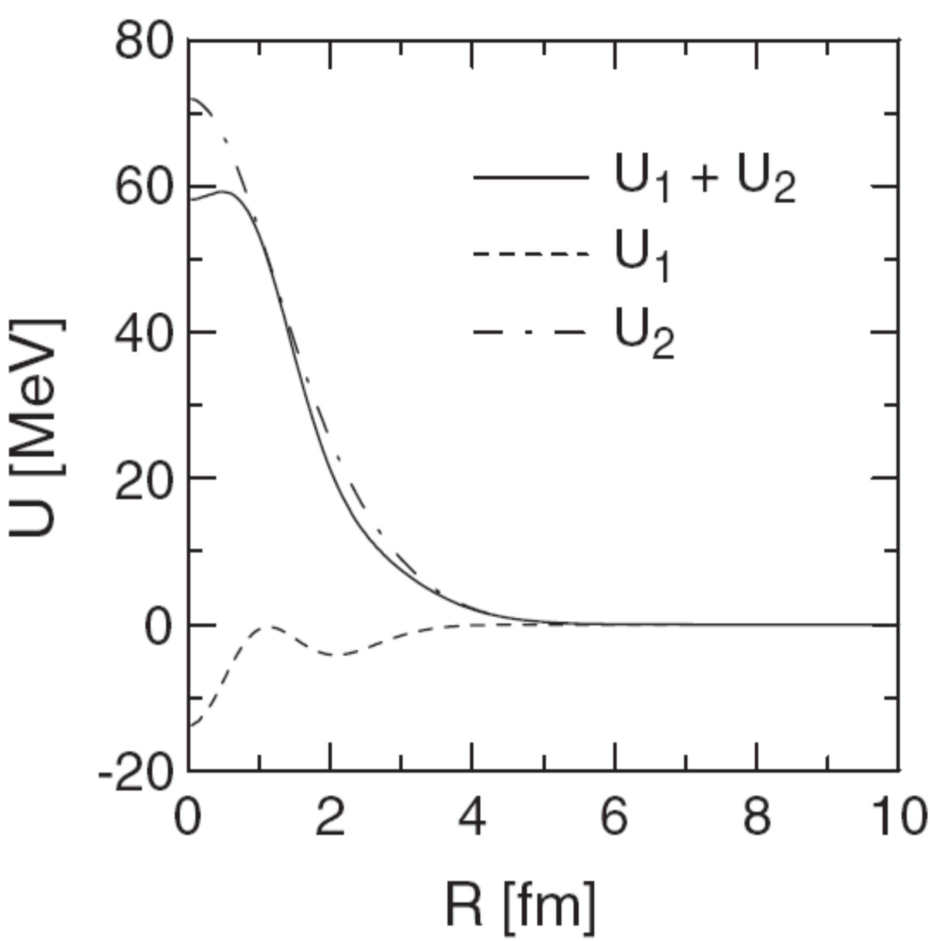
$$\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \mathrm{A}\left\{\psi(\mathbf{R})\phi_a(\mathbf{r}_1,\mathbf{r}_2)\phi_b(\mathbf{r}_3,\mathbf{r}_4)\right\}$$

With realistic NN potentials v_{ij}

$$\mathrm{H} = \mathrm{T}_R + \mathrm{T}_a + \mathrm{T}_b + \sum_{i<j} v_{ij}(\mathbf{r}_{ij})$$

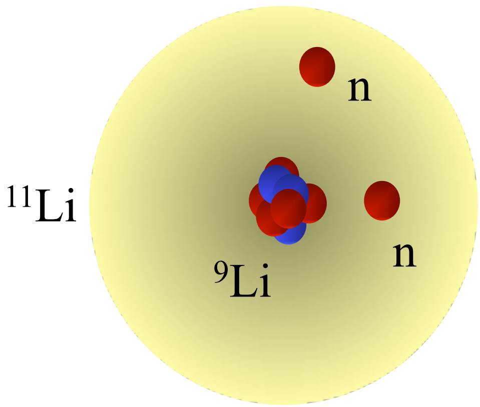
Effective wave equation

$$\left[\frac{\mathbf{P}^2}{2m_N} + U_1(\mathbf{R}) + U_2(\mathbf{R})\right]\psi(\mathbf{R}) = E\psi(\mathbf{R})$$

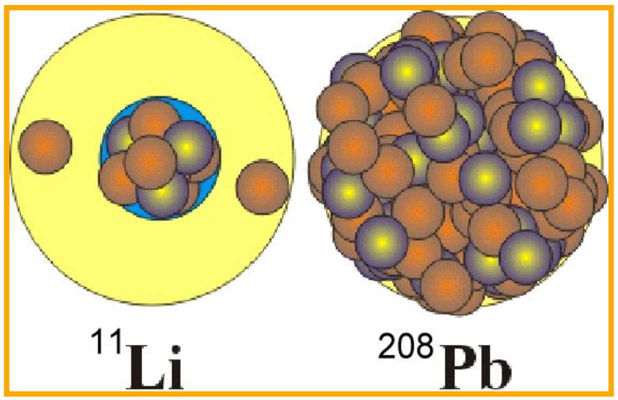


- Effective potential repulsive
- **No margin for pocket or state**
- **no tetraneutron in singlet, or triplet state!**
- Confirmed by Pieper, PRL 90 252501 (2003)

Fusion of halo nuclei



Loosely bound nuclei are like
Rydberg states in atoms



CB, Balantekin,
PLB 314, 275 (1993)

Model Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V_A(r_{1A}) + V_B(r_{1B}) + V_A(r_{2A}) + V_B(r_{2B})$$

Landau approximation

$$\Psi_\pm \cong N\left[\Psi_A(r_{1A}) \pm c_\pm \Psi_B(r_{1B})\right]$$

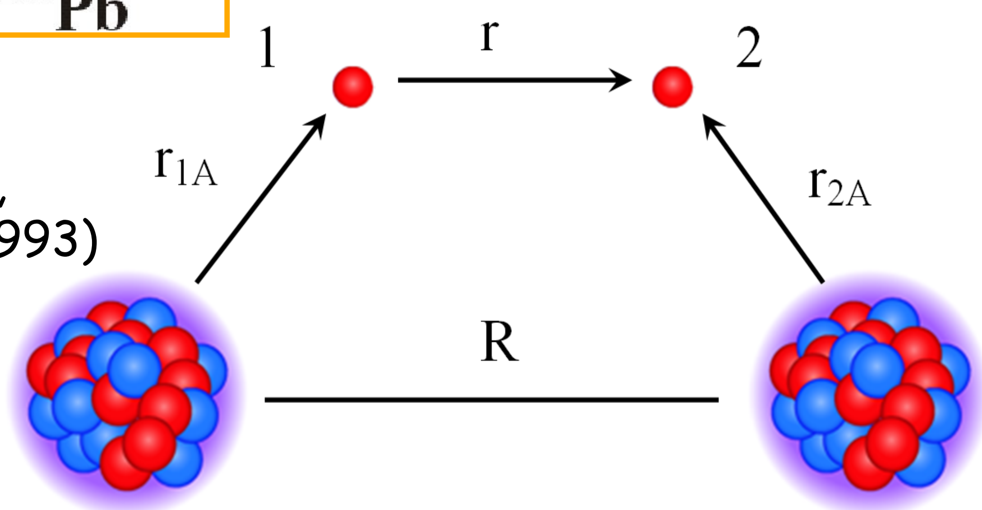
Covalent bond

$$E(R) = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{S_{2n}\left(1+O^2\right)+2OJ+2I}{1+O^2}$$

$$I = \left\langle \Psi_A \left\| V_B(r_{1B}) \right\| \Psi_A \right\rangle$$

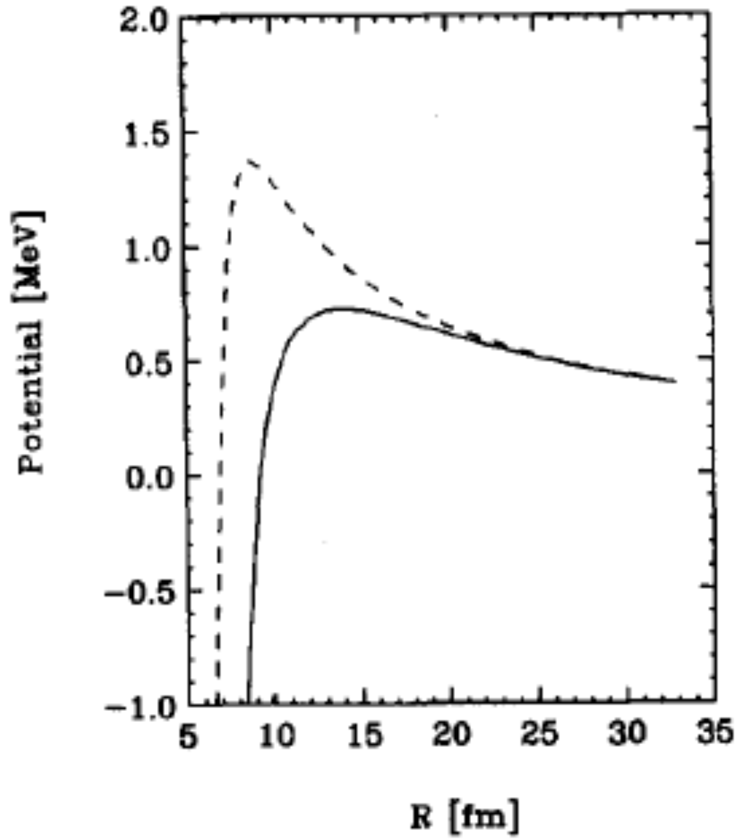
$$L = \left\langle \Psi_A \left\| V_B(r_{1B}) \right\| \Psi_B \right\rangle$$

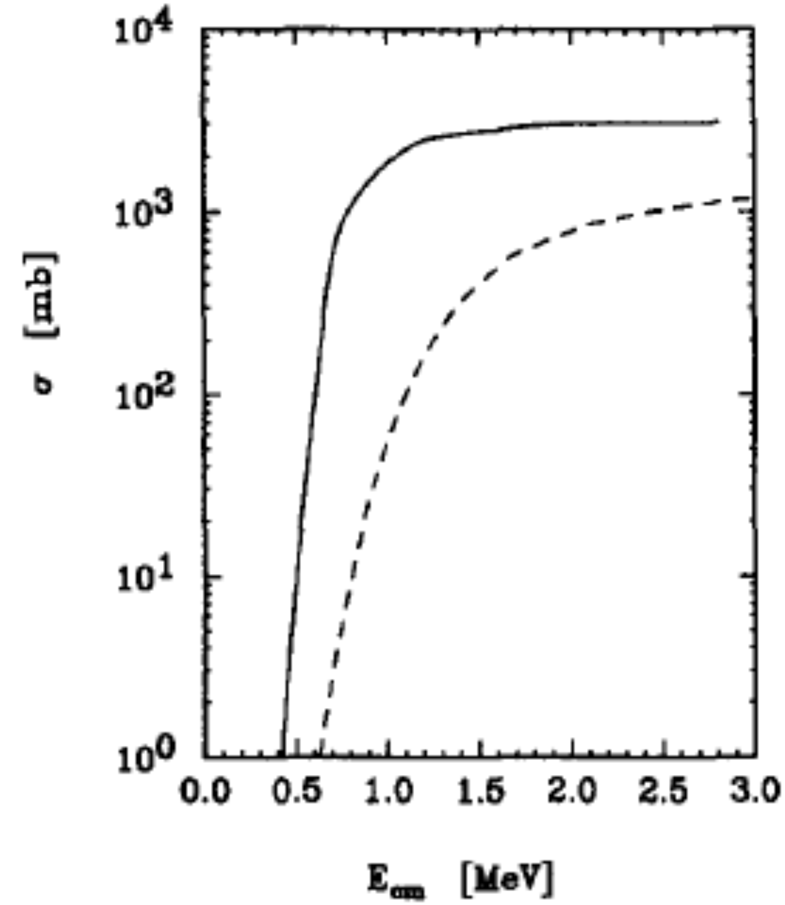
$$O = \left\langle \Psi_A \,|\, \Psi_B \right\rangle$$



$^{9}Li + ^{11}Li$

CB, Balantekin, PLB 314, 275 (1993)





Effective covalent potential

$$V(R) = E(R) - S_{2n}$$

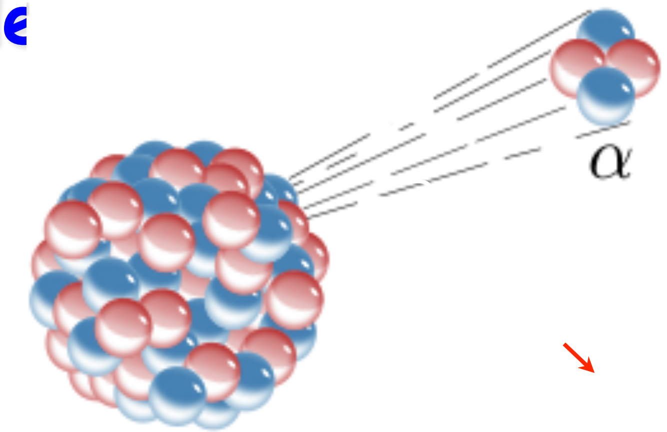
Fusion cross section

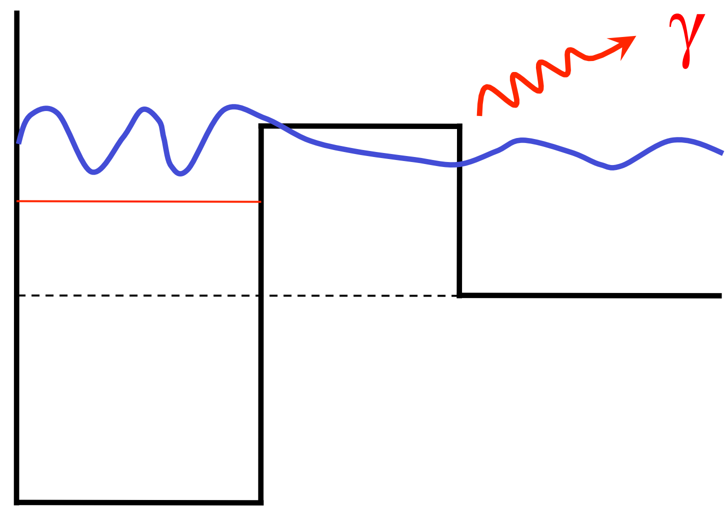
$$\sigma(E_{cm}) = \frac{\pi}{k^2}\sum_l (2l+1) T_l(E_{cm})$$

More on fusion with weakly-bound nuclei :

Canto, Gomes, Donangelo, Hussein, Phys. Rep. 424, 1 (2005)

Bremsstrahlung in α-de

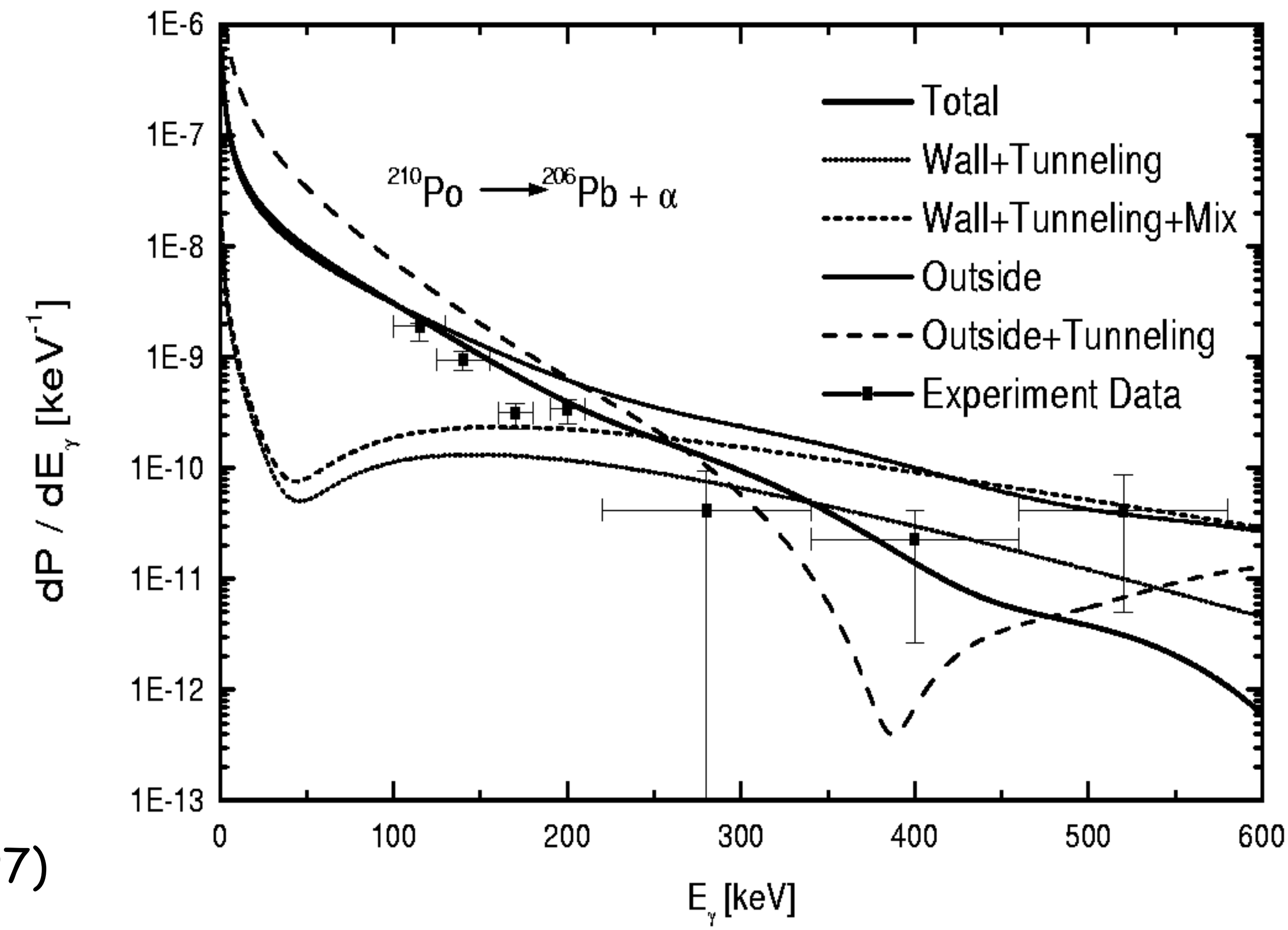




Interference between photons emitted within and outside the barrier

↓

tunneling time



Data:
Kasagi, PRL 79, 371 (1997)

Time-dependent analysis

Power emitted:

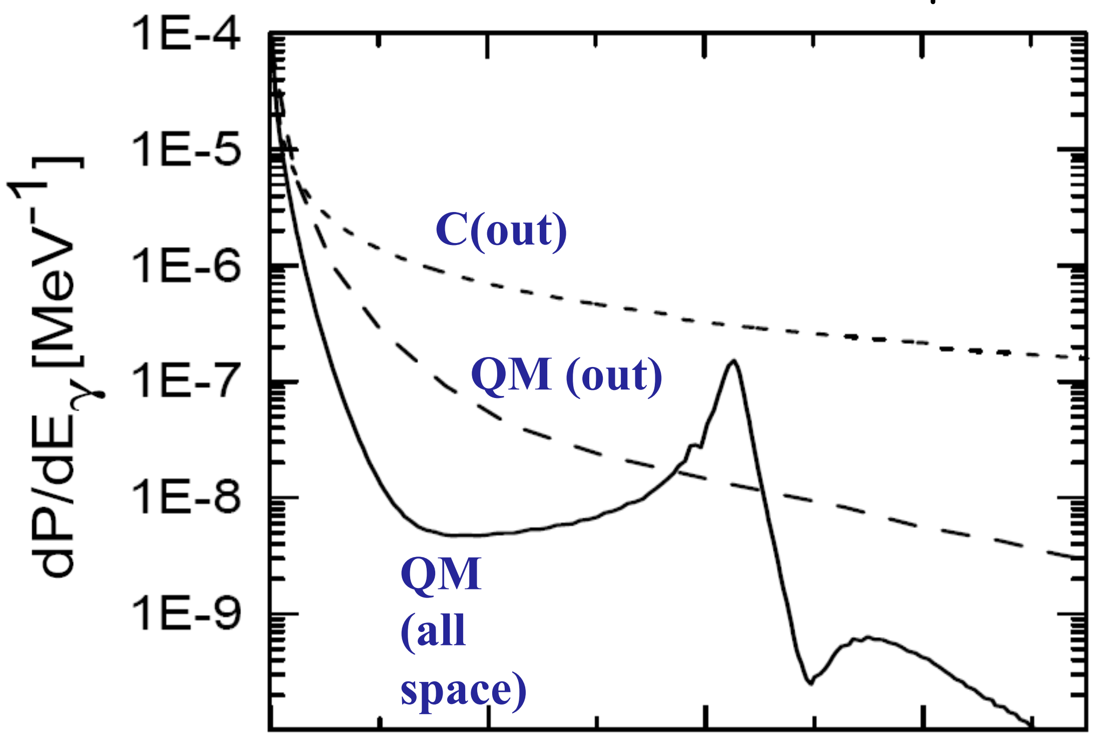
$$\frac{dP}{dE_\gamma} = \frac{1}{E_\gamma}\frac{dE(\omega)}{dE_\gamma}$$

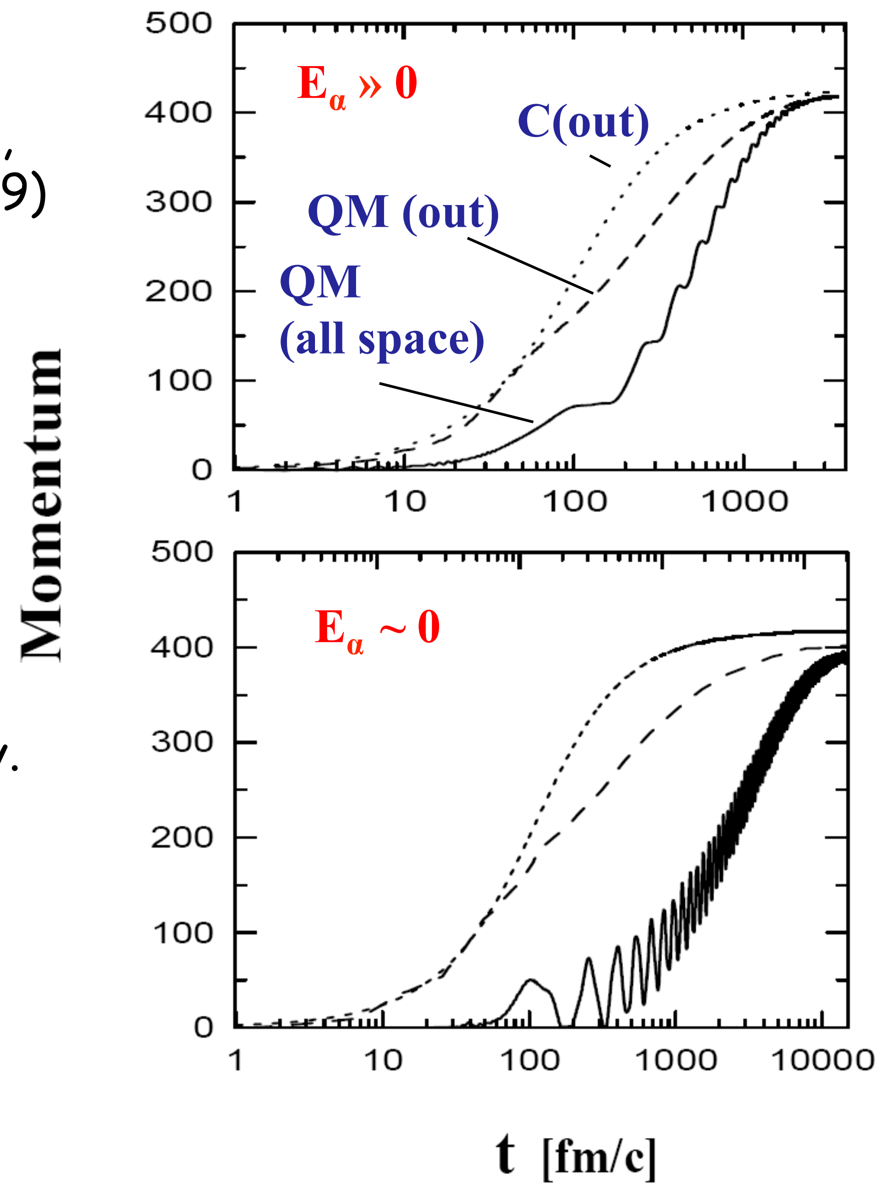
CB, de Paula, Zelevinsky,
PRC 60, 031602(R) (1999)

Solving for t.d. w.f. → power emitted:

$$dE(\omega) = \frac{8\pi\omega^2}{3m^2c^3} Z^2e^2 \left| \int \Psi(\mathbf{r},t)\left[-i\hbar\nabla\Psi(\mathbf{r},t)\right] d^3r \right|^2$$

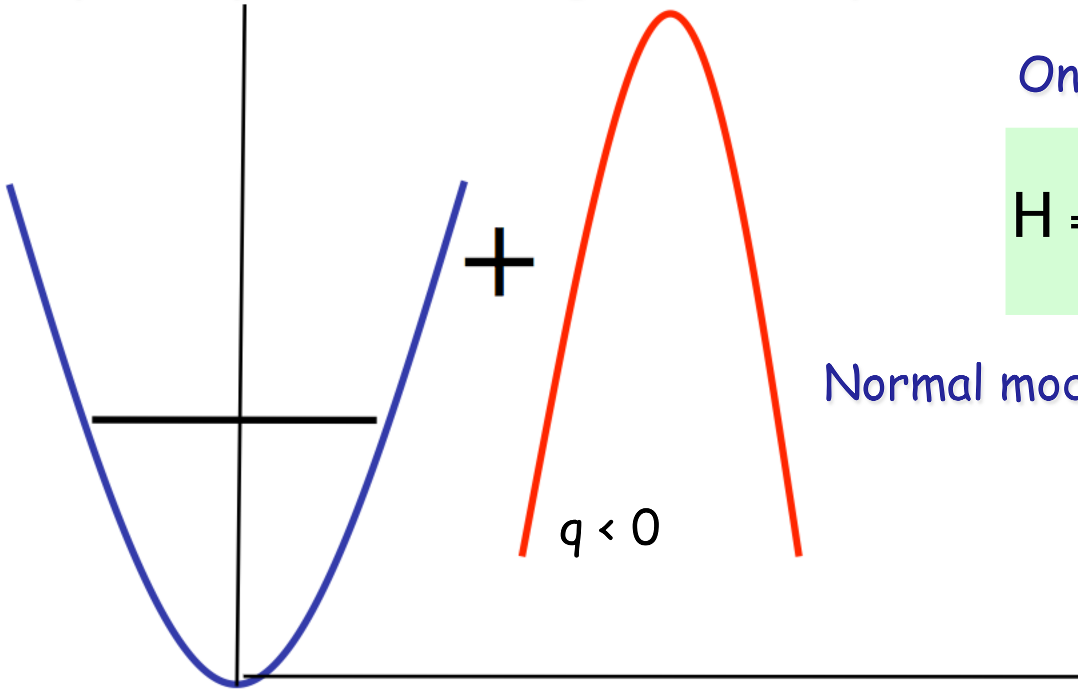
Not well understood. Not verified experimentally.





Composite particles seeing different potentials

Zelevinsky, Flambaum, JPG 34, 355 (2005)



Only one particle sees barrier:

$$H = \frac{p_x^2}{2m_x} + \frac{p_y^2}{2m_y} + \frac{1}{2}k(x-y)^2 + \frac{1}{2}qx^2$$

Normal modes: $m_x \to 0: \quad \omega_+^2 = \frac{k}{\mu} + \frac{q}{m_x}, \quad \omega_-^2 \to 0$

$m_y \to 0: \quad \omega_+^2 = \frac{k}{\mu}, \quad \omega_-^2 \to \frac{q}{m_x}$

Equal masses and k >> q: $|\omega_-| = \omega_0\left(1 + \frac{|q|}{8k}\right), \qquad \omega_0^2 = \frac{|q|}{2m}$

Tunneling probability: $P = \exp\left(\frac{2\pi|E|}{\hbar|\omega_-|}\right) = \exp\left(-\frac{2\pi|E|}{\hbar\omega_0}\right) \times \exp\left(\frac{\pi|E|m\omega_0}{2\hbar k}\right)$

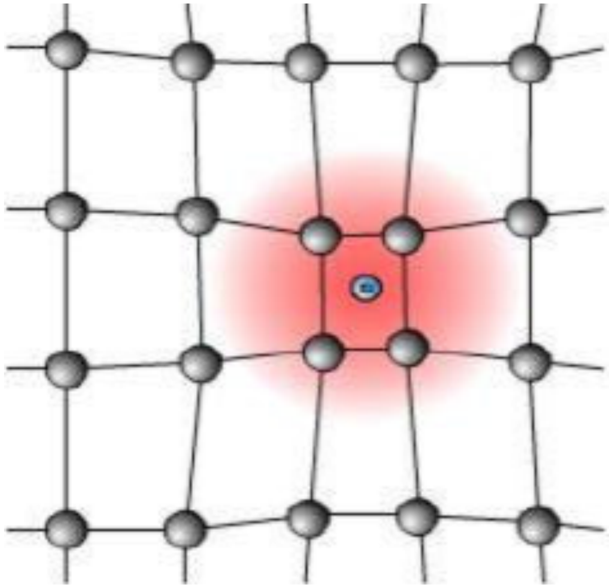
$m_x/m_y << 1: \quad |\omega_-| \approx \sqrt{\frac{|q|}{m_x+m_y}}\left(1 + \frac{|q|}{8k}\frac{m_y}{m_x}\right)$

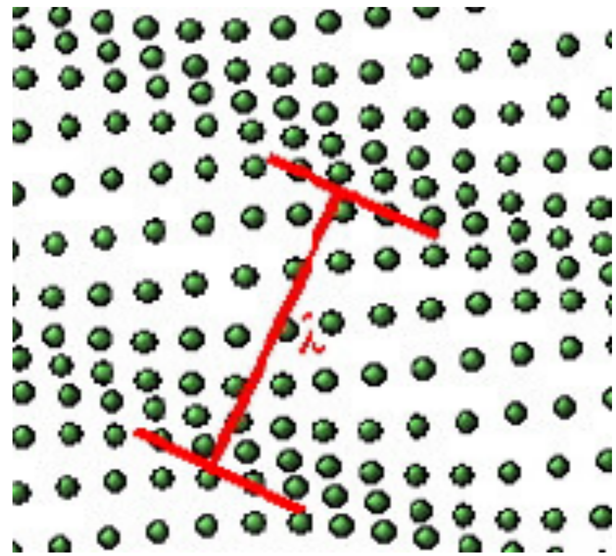
enhancement

Finite size always enhance tunneling probability

Tunneling of Cooper pairs

Zelevinsky, Flambaum, JPG 34, 355 (2005)





Both particles see barrier:

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}k(x-y)^2 + \frac{1}{2}q(x^2+y^2)$$

Normal modes:

$$\omega_+^2 = \frac{2k+q}{m}, \quad \omega_-^2 = \frac{q}{m}$$

Does not depend on k → no finite size effect

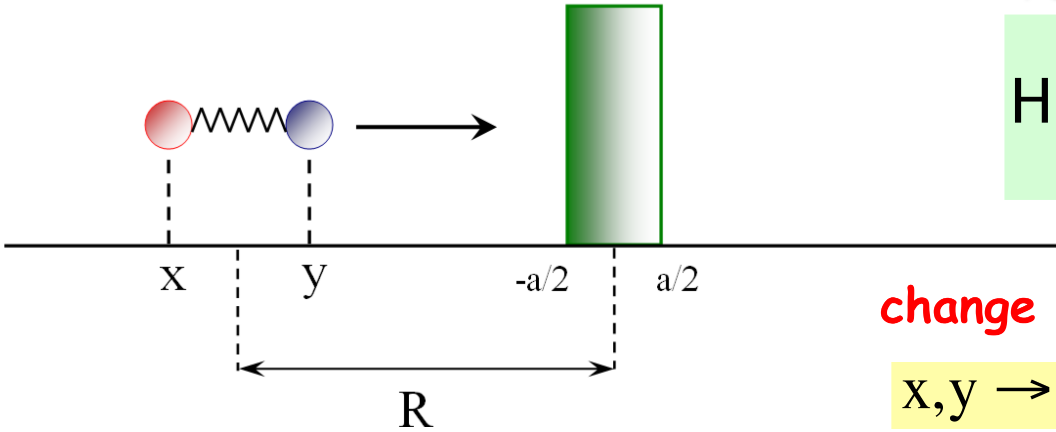
Energy transfer from internal to c.m. motion

$$E = E(\infty) + \frac{\hbar\sqrt{2k/m}}{2} - \frac{\hbar\sqrt{(2k+q)/m}}{2}$$

Adiabatic approx. not valid for tunneling through a Josephson junction (more complicated)

Composite particle fusion enhancement

CB, Flambaum, Zelevinsky, JPG 34, 1 (2007)



Particles see different barriers:

$$H = -\frac{p_x^2}{2m_x} + \frac{p_y^2}{2m_y} + V(x-y) + U(x) + U(y)$$

change of variables

$$x, y \rightarrow r, R$$

Adiabatic approximation

$$\Psi(r,R) = \psi(R)\phi(r,R)$$

$$\psi(R) = u(R)\exp\left[-\int^{R} \alpha(R')dR'\right]$$

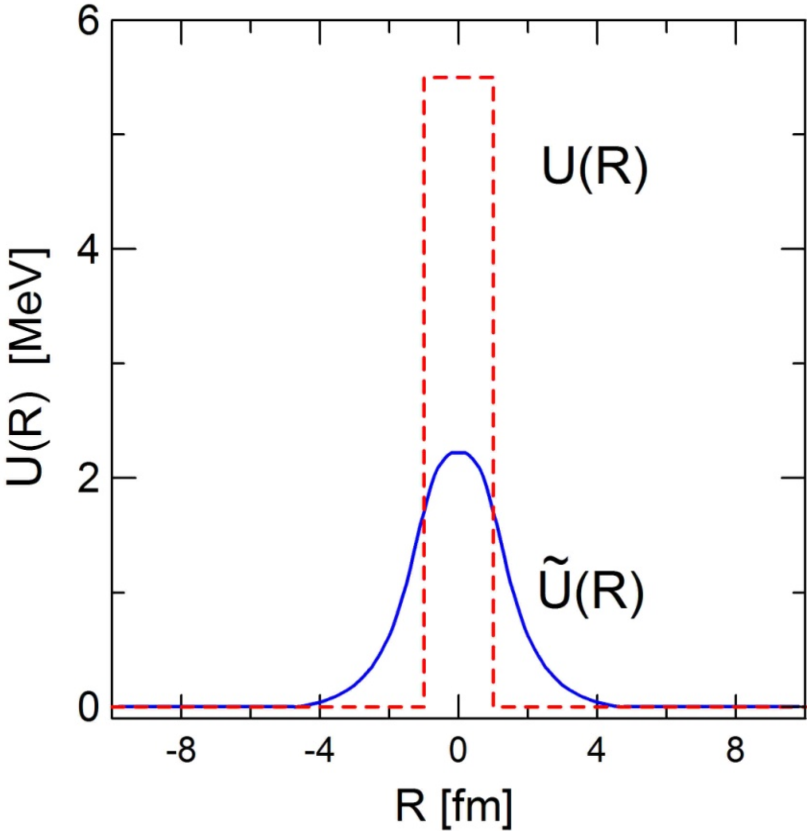
ϕ normalized

$$\alpha(R) = \left\langle \phi \middle| \frac{\partial \phi}{\partial R} \right\rangle$$

$$u''(R) + \frac{2M}{\hbar^2}\left[E - \tilde{U}(R)\right]u(R) = 0$$

$$\tilde{U}(R) = \varepsilon(R) - E_0 + \frac{\hbar^2}{2M}\left[\alpha^2(R) + \alpha'(R) + \beta(R)\right]$$

$$\beta(R) = \left\langle \phi \middle| \frac{\partial^2 \phi}{\partial R^2} \right\rangle$$

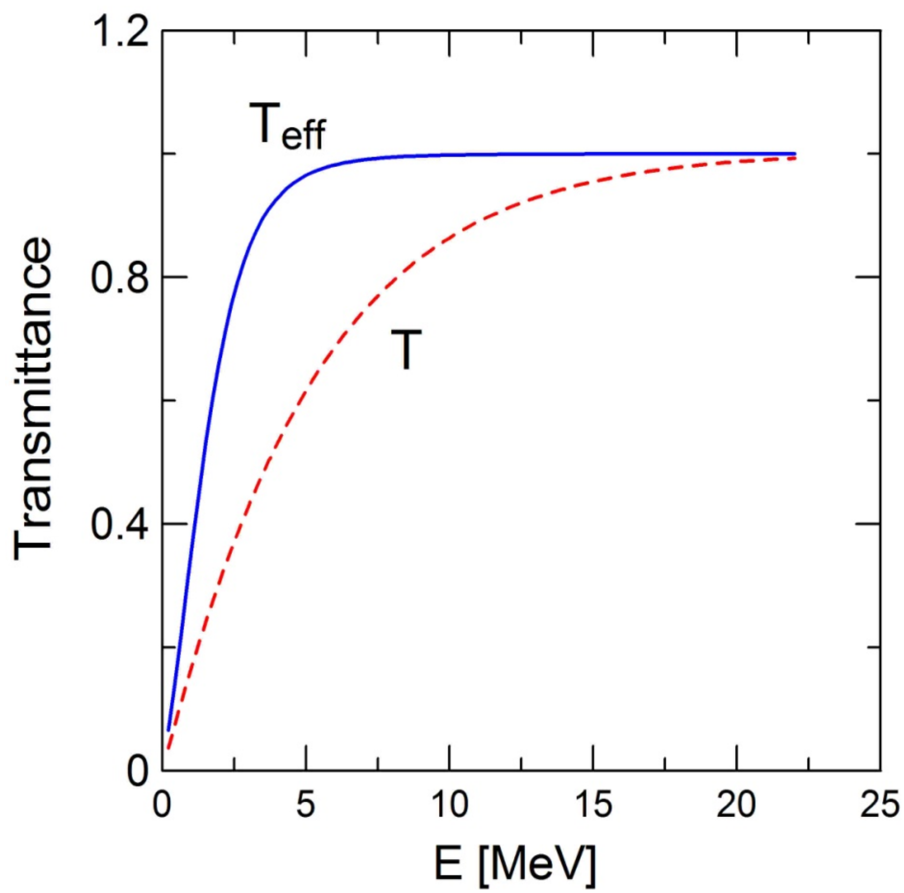


Deuteron tunneling through barrier step.

$$E_0 = -2.225 \text{ MeV}$$

$$r_0 = 2 \text{ fm}$$

CB, Flambaum, Zelevinsky, JPG 34, 1 (2007)

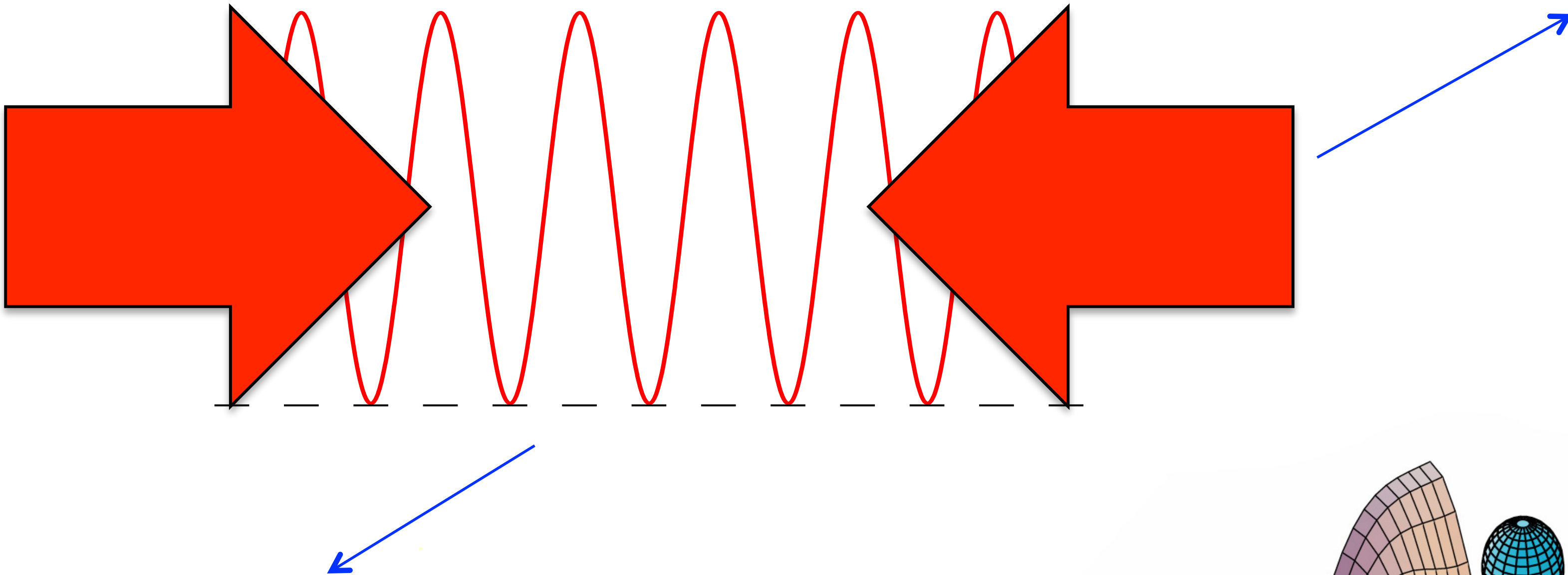


Features probably seen in experiments.

But not disentangled from uncertainties in potentials, polarization effects, etc.

Optical Lattices

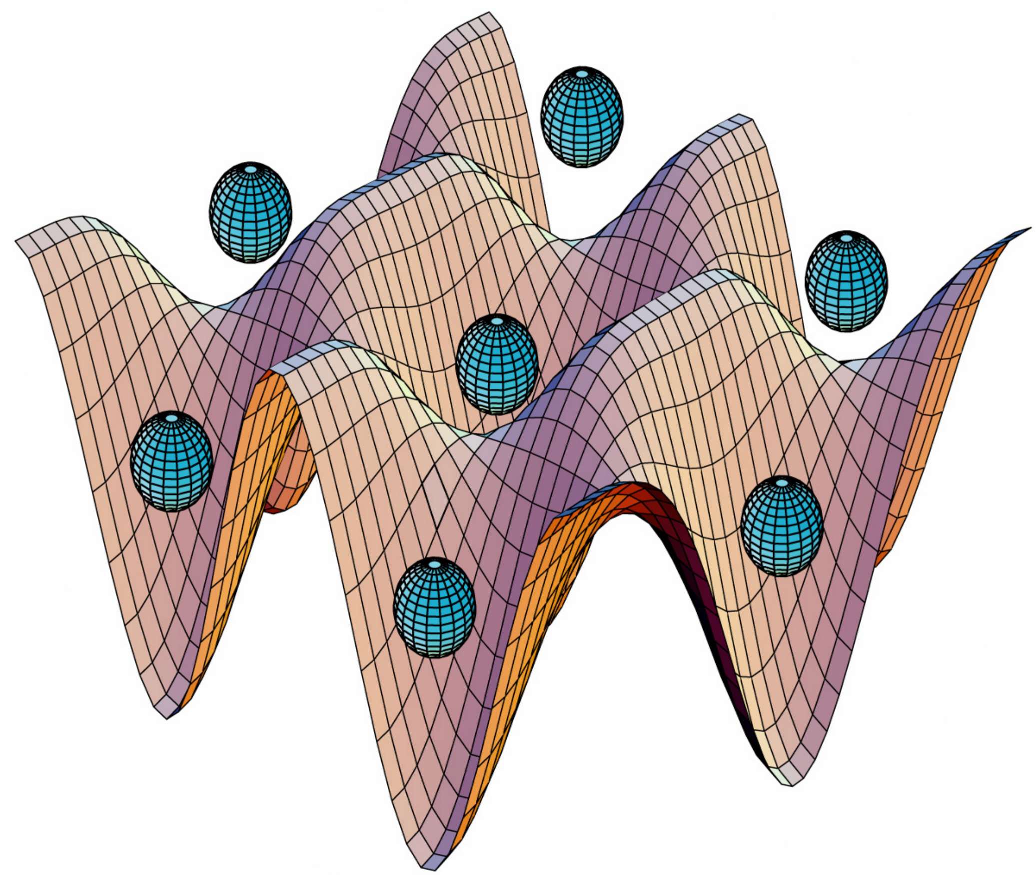
Laser



1D

Interference

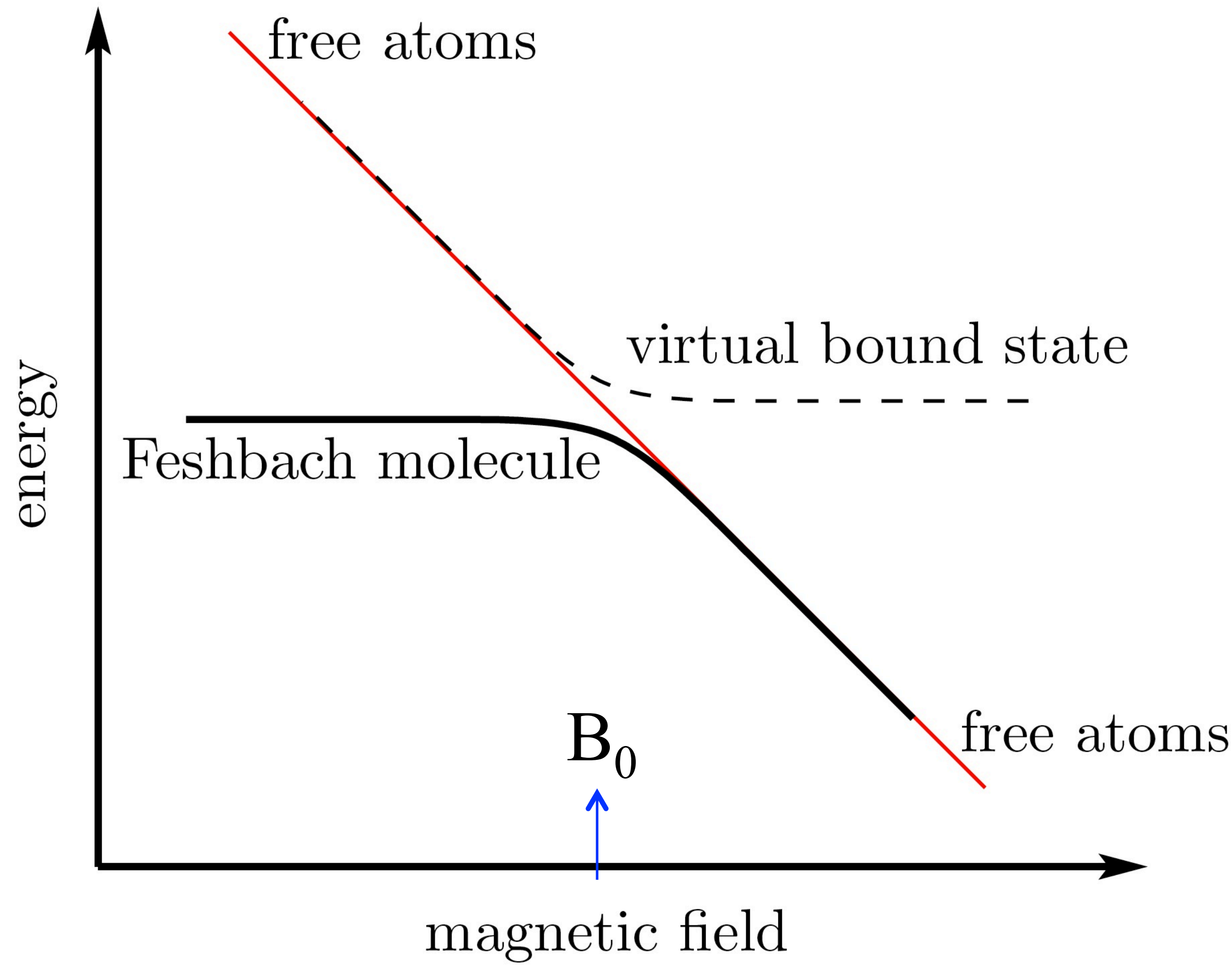
2D



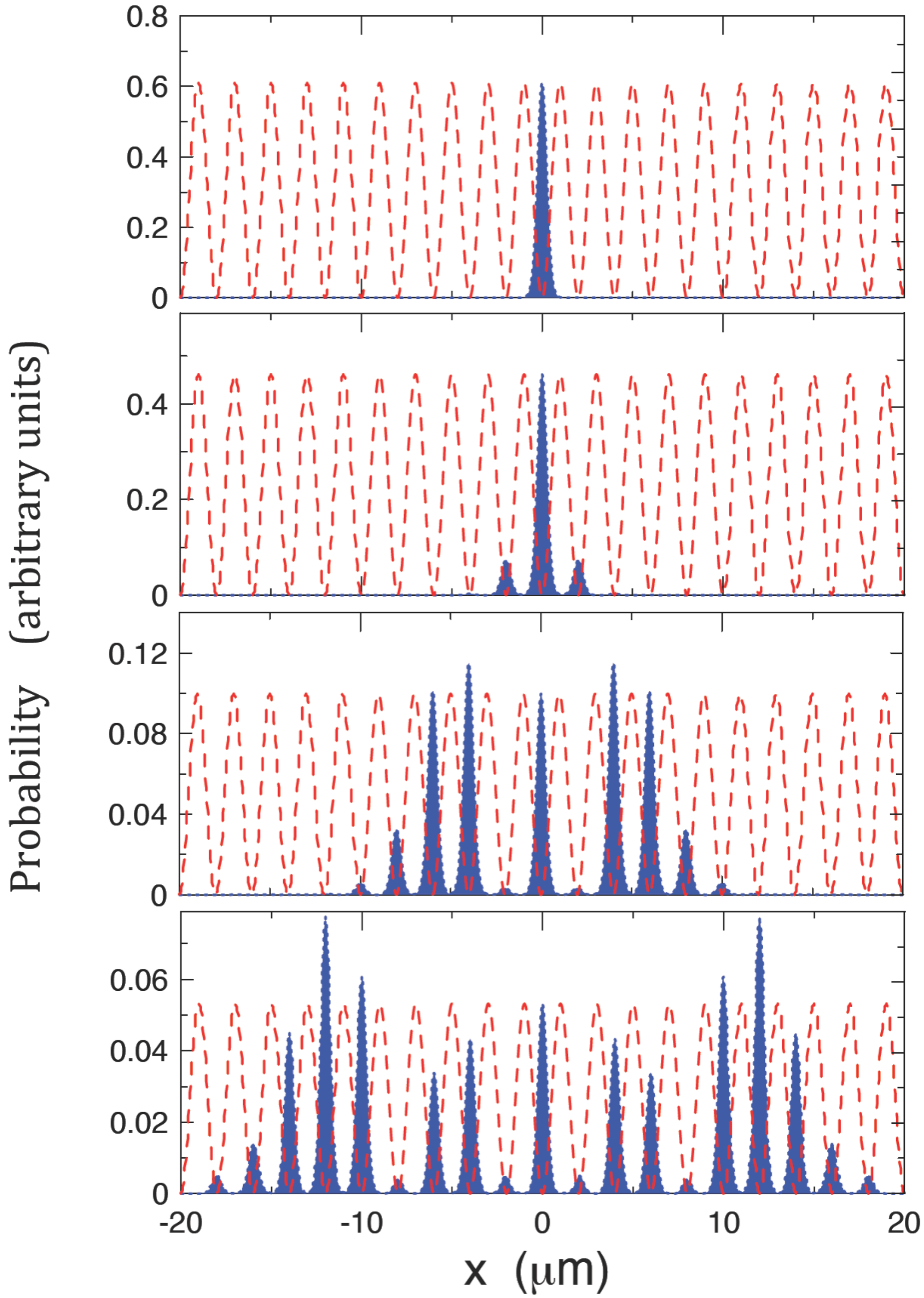
Feshbach Resonances

$$a(B) = a_{B=0}\left[1 - \frac{\Delta}{(B - B_0)}\right]$$

Δ = B-field width at the resonance



Diffusion of Strongly Bound Molecules

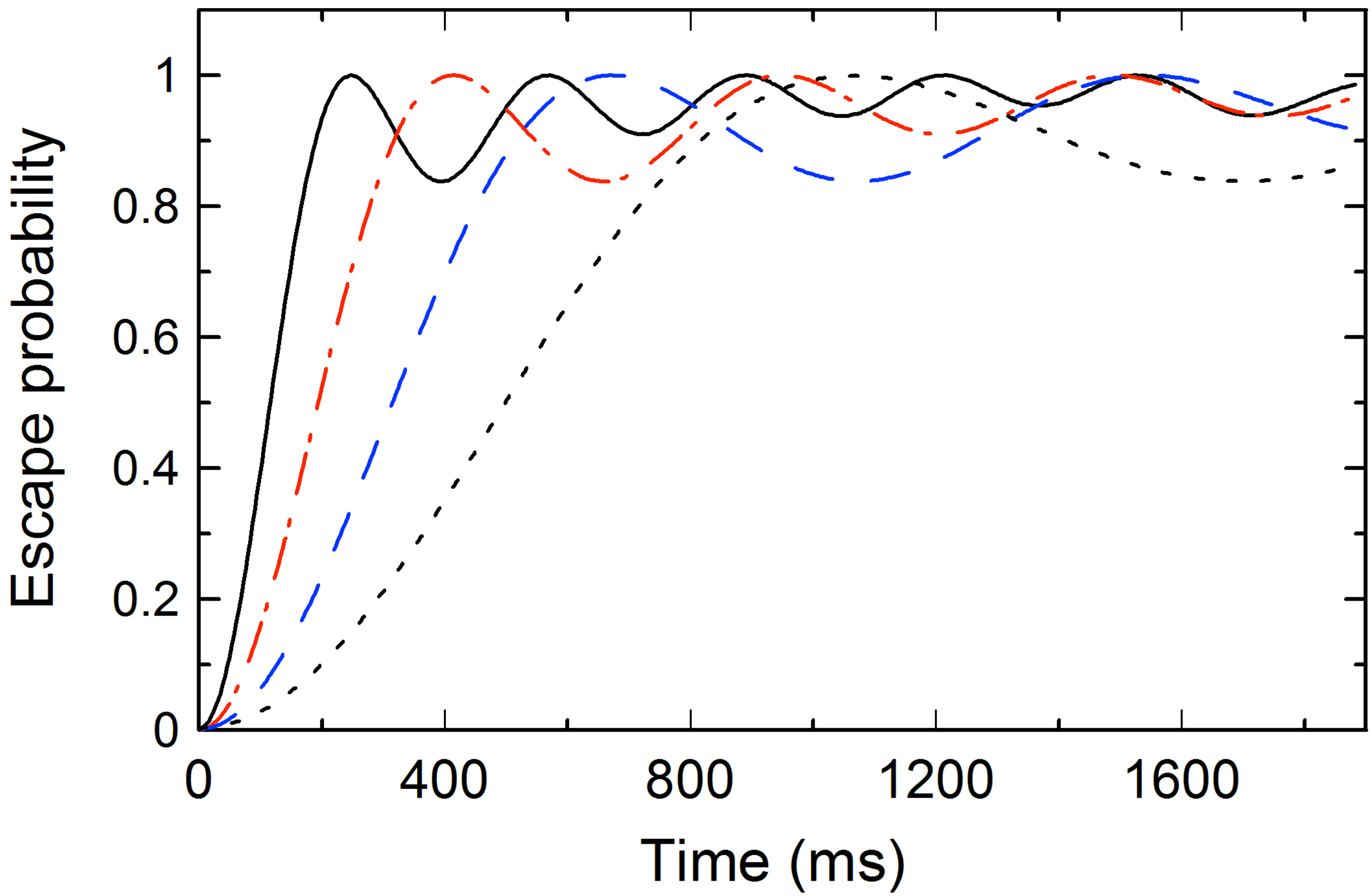


Probability distribution of strongly bound (E_b=10 kHz) rubidium molecules in optical lattice with size D = 2 μm.

From top to bottom the time is t = 0, t = 10 ms, t = 25 ms and t = 100 ms.

T. Bailey, CB, E. Timmermans, PR A 85, 033627 (2012)

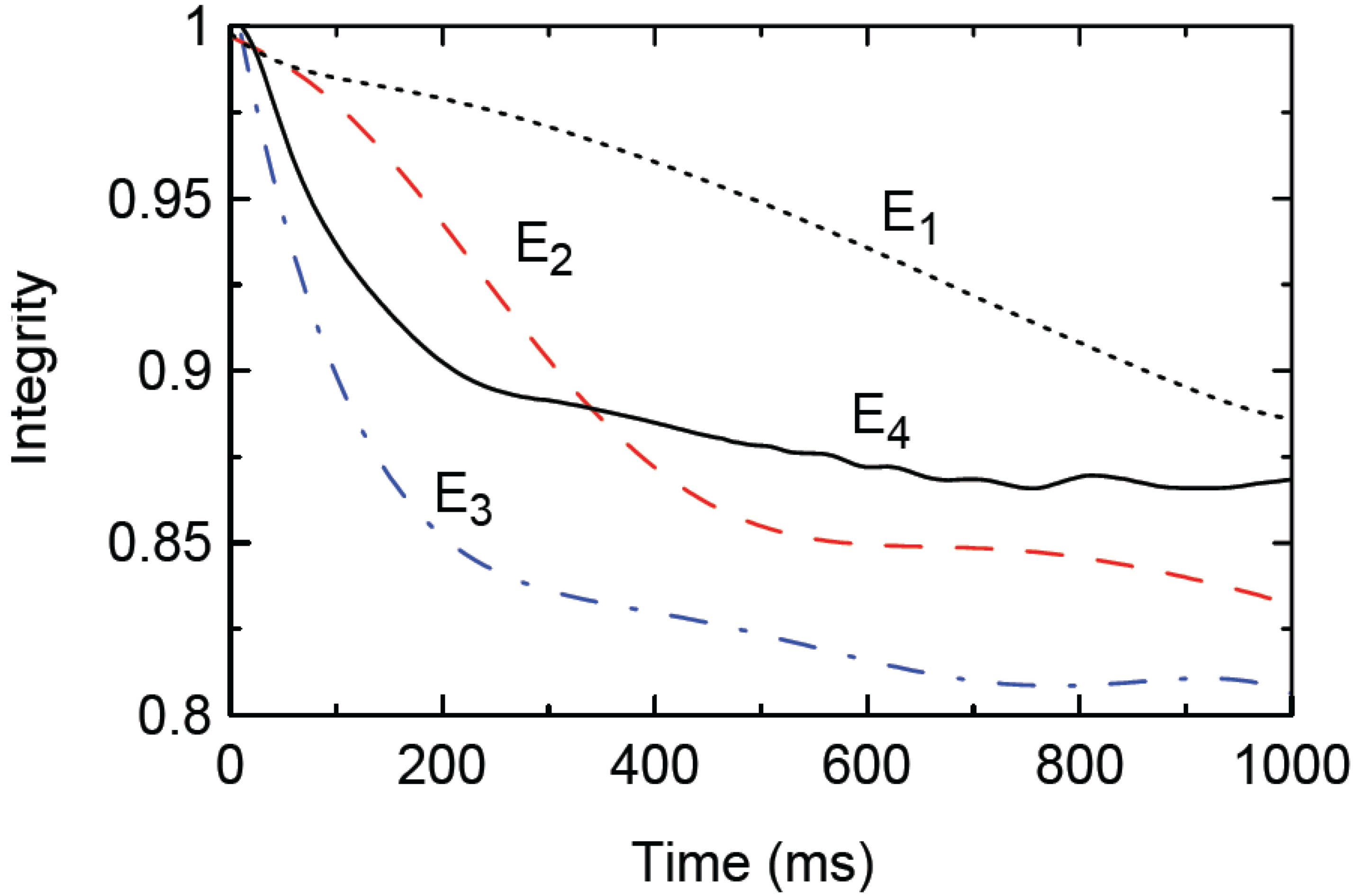
Diffusion of Strongly Bound Molecules



Escape probability of strongly bound rubidium molecules in optical lattice with D=2 μm, for increasing potential barrier heights.

Barrier heights increase by 1.5 from the solid to dashed-dotted, from dashed-dotted to long-dashed, and from long-dashed to dotted curve.

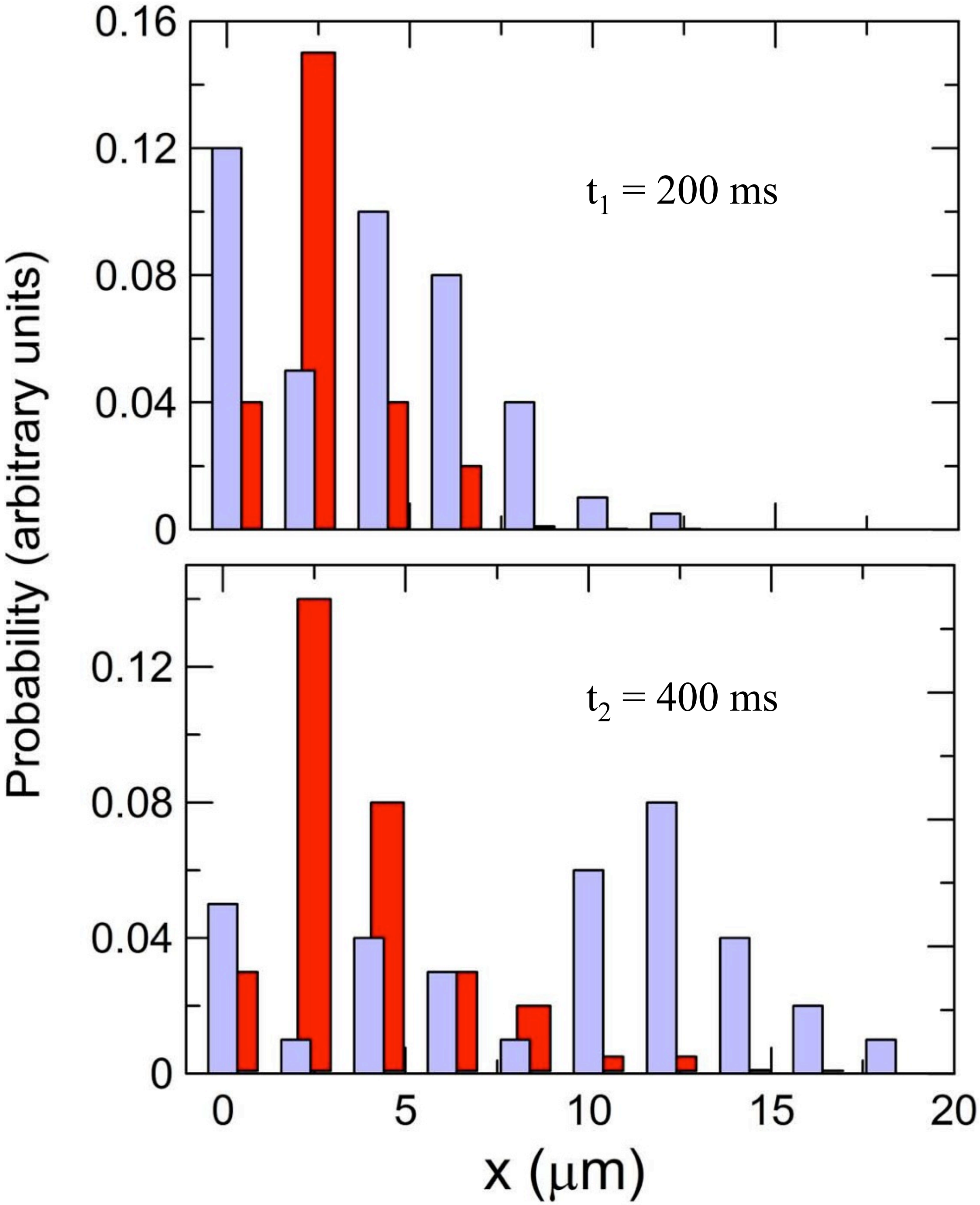
Diffusion of Loosely Bound Molecules



Integrity probability of loosely bound rubidium molecules in an optical lattice D = 2 μm, and potential barrier height V_0=5 kHz.

The binding energies were parametrized in terms of the barrier height, with $E_4=V_0/20$, $E_3=V_0/5$, $E_2=V_0/2$, and $E_1=V_0/1.2$.

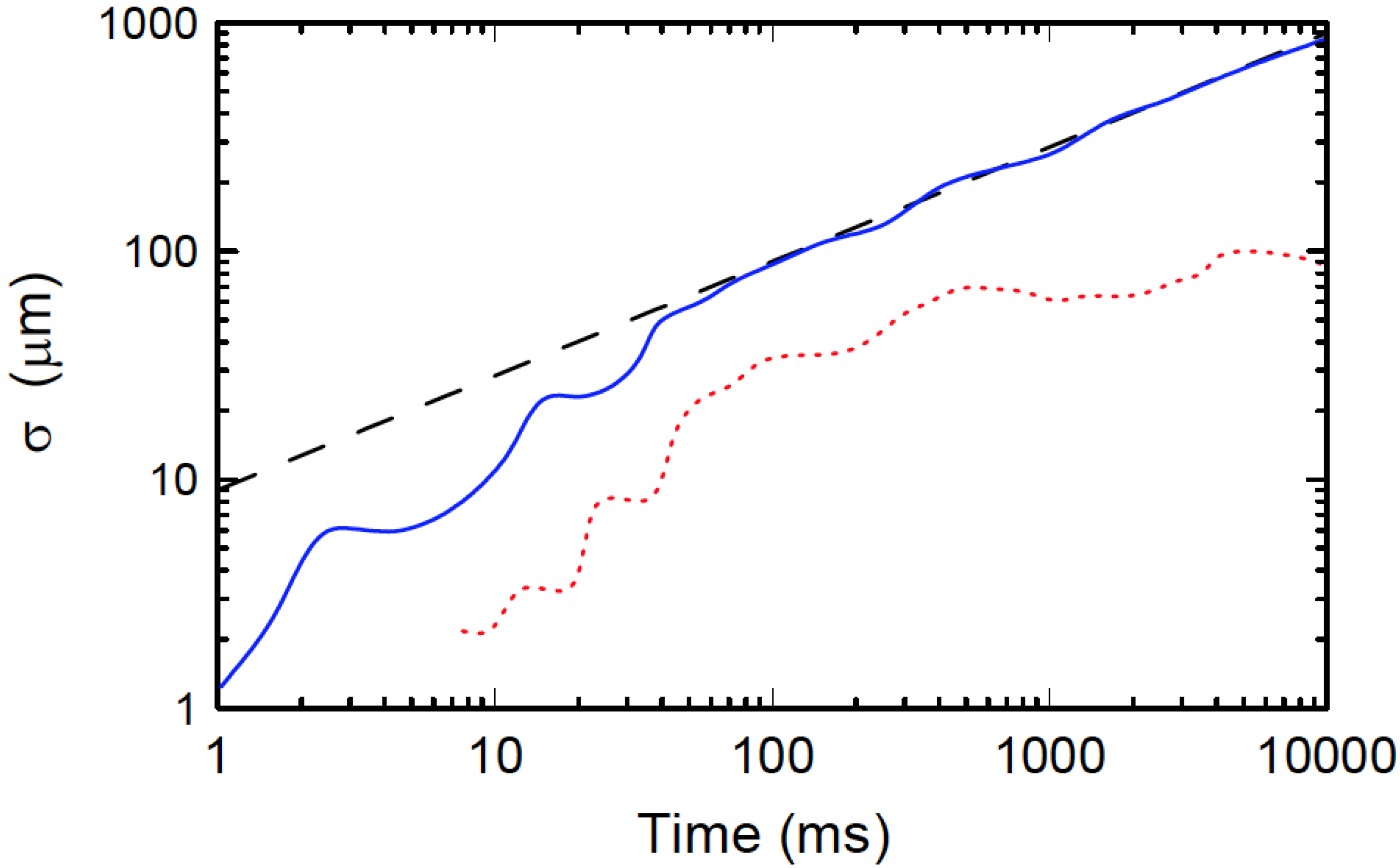
Diffusion of Loosely Bound Molecules



Blue histograms are the relative probability of finding a molecule in its ground state at a given position along the lattice.

Red histograms give the relative probability of finding individual atoms after the dissociation.

Diffusion of Loosely Bound Molecules



Spreading position width of bound molecules, $\sigma_M(t)$, shown by solid line and of dissociated atoms, $\sigma_A(t)$, shown by dotted line. The dashed curve is a fit to the asymptotic time dependence $\sigma_M(t) \sim t^{1/2}$.

No clear asymptotic dependence is found for the dissociated atoms.

$$D = 0.156 \frac{\hbar^2}{mb\sigma_0^2}$$

Compared to Einstein diffusion coefficient

$$D = \frac{kT}{b}$$

$$\Rightarrow \quad T \sim \frac{\hbar^2}{m\sigma_0^2}$$

Conclusions:

"Although a [large] number of theoretical works have studied tunneling phenomena in various situations, quantum tunneling of a *composite particle, in which the particle itself has an* internal structure, has yet to be clarified."

Saito and Kayanuma, J. Phys.: Condens. Matter 6 (1994) 3759

- ***20* years after:** *still remains open to imagination and creativity.*