Mass-imbalanced 3B systems in 2D: bound states and the one-body density

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Outline

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Objective/Motivation

Theoretical

- Obtaining universal properties of weakly binding systems in 2D;
    - Universal properties?

Experimental

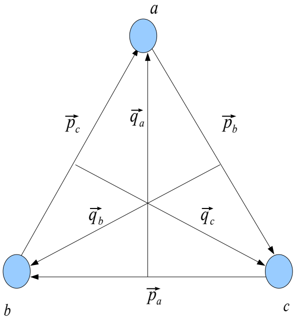
Ultra-cold quantum atomic gases[1]:

- Evidence of Efimov states;
- Tuneability of interaction strength: Feshbach resonances;
- Quasi-2D samples of ^{133}Cs, ^{23}Na,^{87}Rb, ^{40}K e ^{6}Li.

[1] references in F. Bellotti *et al.*, J. Phys. B: At. Mol. Opt. Phys. **44**, 205302 (2011)

Momentum space / $s-$wave zero-range interaction / $L_z = 0$

Jacobi momenta



Wave function

$$\Psi\left(\mathbf{q}_\alpha, \mathbf{p}_\alpha\right) = \frac{f_\alpha\left(q_\alpha\right) + f_\beta\left(q_\beta\right) + f_\gamma\left(q_\gamma\right)}{-E_3 + \frac{q_\alpha^2}{2m_{\beta\gamma,\alpha}} + \frac{p_\alpha^2}{2m_{\beta\gamma}}}.$$

- $(\alpha, \beta, \gamma) \leftrightarrow (a, b, c)$;

$$f_\alpha\left(q\right) = \tau_{\beta\gamma}^{-1}(E_3, q, E_{\beta\gamma}) \int_0^\infty dk\, k \left[K_{\alpha\beta}(E_3, q, k)\, f_\beta(k) + K_{\alpha\gamma}(E_3, q, k) f_\gamma(k)\right]$$

$$f_\beta\left(q\right) = \tau_{\alpha\gamma}^{-1}(E_3, q, E_{\alpha\gamma}) \int_0^\infty dk\, k \left[K_{\beta\alpha}(E_3, q, k)\, f_\alpha(k) + K_{\beta\gamma}(E_3, q, k) f_\gamma(k)\right]$$

$$f_\gamma\left(q\right) = \tau_{\alpha\beta}^{-1}(E_3, q, E_{\alpha\beta}) \int_0^\infty dk\, k \left[K_{\gamma\alpha}(E_3, q, k)\, f_\alpha(k) + K_{\gamma\beta}(E_3, q, k) f_\beta(k)\right]$$

Difference between 2D and 3D worlds in 3BBS

Centrifugal barrier

- **3D**: Zero or Repulsive.
- **2D**: Attractive for $L_z = 0$.

Influence

- **3D**: Finite amount of attraction to produce bound states.
- **2D**: An infinitesimal attraction will produce a bound state. [2]

Three identical bosons

- **3D**: Infinitely many bound states $\rightarrow$ Efimov effect. [3]
- **2D**: At most two universal bound states. [4]
  - $E_3^0 = 16.52 E_2$;
  - $E_3^1 = 1.27 E_2$.

[2] E. Nielsen *et al.*, Physics Reports **347**, 373 (2001)
[3] V. Efimov, Sov. J. Nucl. Phys. **12**, 589 (1970)
[4] J. A. Tjon, Phys. Lett. B **56**, 217 (1975)

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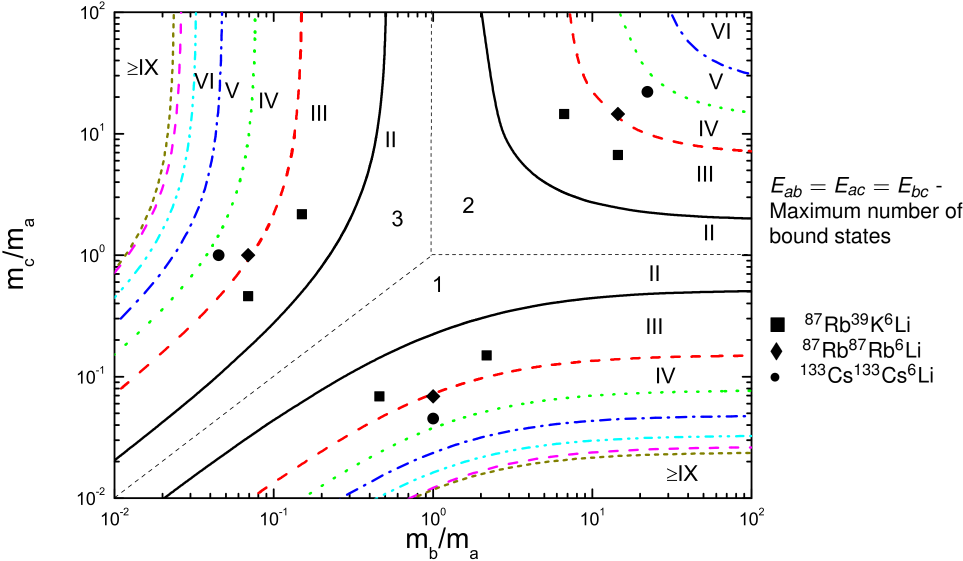
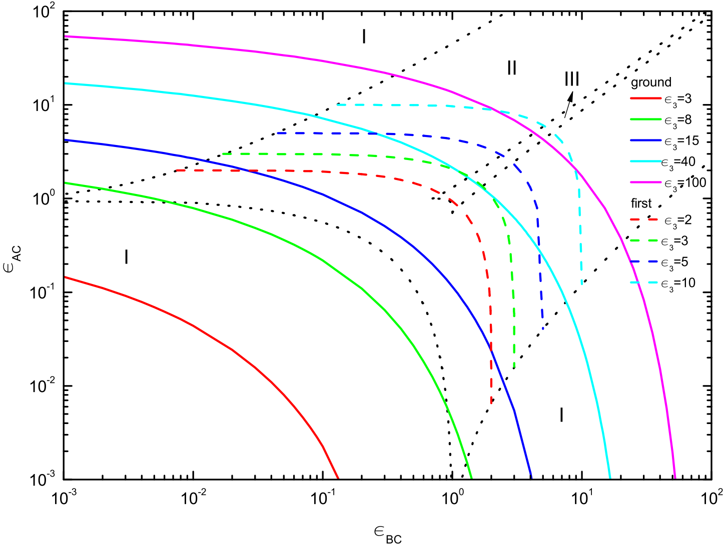
Mass diagram for the occurrence of *II*, *III*, *IV*, ... bound states with $E_{ab} = E_{ac} = E_{bc}$



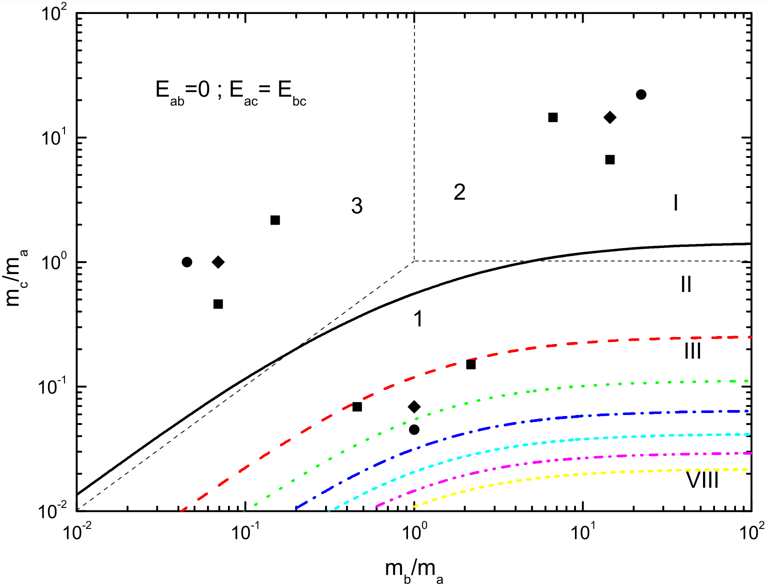
- F. F. Bellotti *et al.*, Phys. Rev. A **85**, 025601 (2012)

Varying the energies for $^{87}Rb^{39}K^{6}Li$ system



- F. F. Bellotti *et al.*, Phys. Rev. A **85**, 025601 (2012)

Heavy-heavy-light system → BO approximation



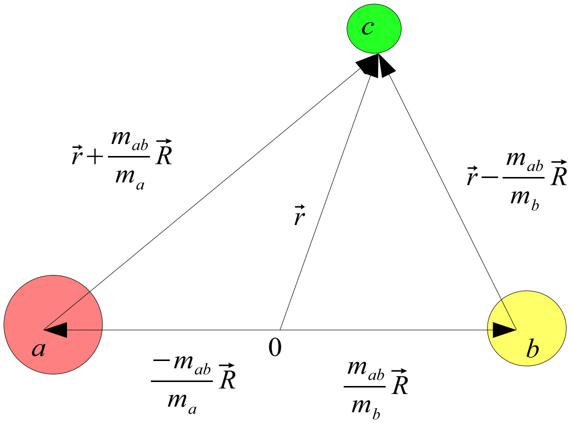
2D: *ab* system non-interacting → $E_{ab} = 0$.

■ $^{87}Rb^{39}K^{6}Li$
♦ $^{87}Rb^{87}Rb^{6}Li$
● $^{133}Cs^{133}Cs^{6}Li$

- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Outline

System of coordinates



- 3D: A. C. Fonseca *et al.*, Nucl. Phys. A **320**, 273 (1979)
- 2D: T. K. Lim and B. Shimer, Z. Phys. A **297**, 185 (1980)

Effective potential - $\epsilon(R)$

Light-particle equation

$$\left[-\frac{\hbar^2\nabla_r^2}{2m_{ab,c}} + v_a\left(\mathbf{r} - \frac{m_{ab}}{m_b}\mathbf{R}\right) + v_b\left(\mathbf{r} + \frac{m_{ab}}{m_a}\mathbf{R}\right)\right]\psi(\mathbf{r},\mathbf{R}) = \epsilon(R)\psi(\mathbf{r},\mathbf{R})$$

Heavy-heavy particles equation

$$\left(-\frac{\hbar^2\nabla_R^2}{2m_{ab}} + v_c(\mathbf{R}) + \epsilon(R)\right)\phi(\mathbf{R}) = E\phi(\mathbf{R})$$

Effective potential

$$\ln\frac{|\epsilon(R)|}{|E_2|} = 2K_0\left(\sqrt{\frac{2m_{ab,c}|\epsilon(R)|}{\hbar^2}}R\right)$$

- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Analytic approach to the effective potential

Small distance

$$\frac{|\epsilon_{asymp}(R)|}{|E_2|} \to \frac{2e^{-\gamma}}{s(R)} \left(1 - \frac{e^{-\gamma}}{2} s(R) \left[(1-\gamma) - \frac{1}{2} \ln\left(\frac{e^{-\gamma}}{2} s(R)\right)\right]\right)^{-1} , \\ \to -\frac{2e^{-\gamma}}{s(R)} .$$

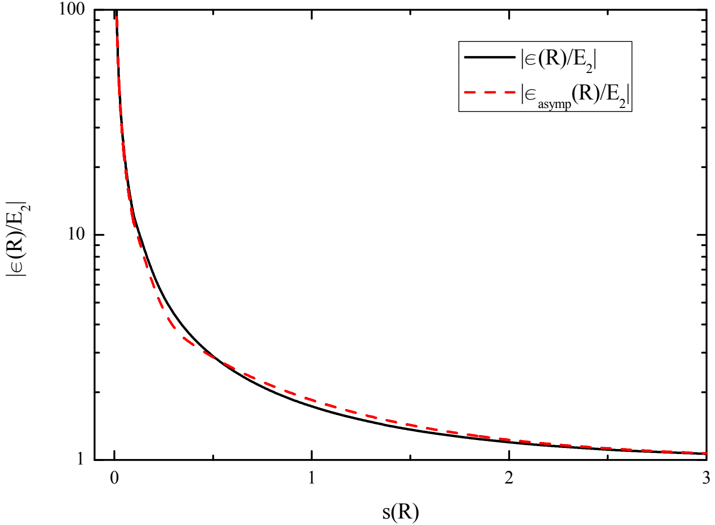
Large distance

$$\frac{|\epsilon_{asymp}(R)|}{|E_2|} \to 1 + \frac{2K_0\left(s(R)\right)}{1 + s(R)\, K_1\left(s(R)\right)} , \\ \to 1 + \sqrt{2\pi} \frac{e^{-s(R)}}{\sqrt{s(R)}} .$$

$s(R) = \sqrt{\frac{2m_{ab,c}|E_2|}{\hbar^2}} R$, $m_c \to 0 \Rightarrow 2m_{ab,c} \to \frac{4m}{m+2} \to 0$, $m = \frac{m_c}{m_a}$.

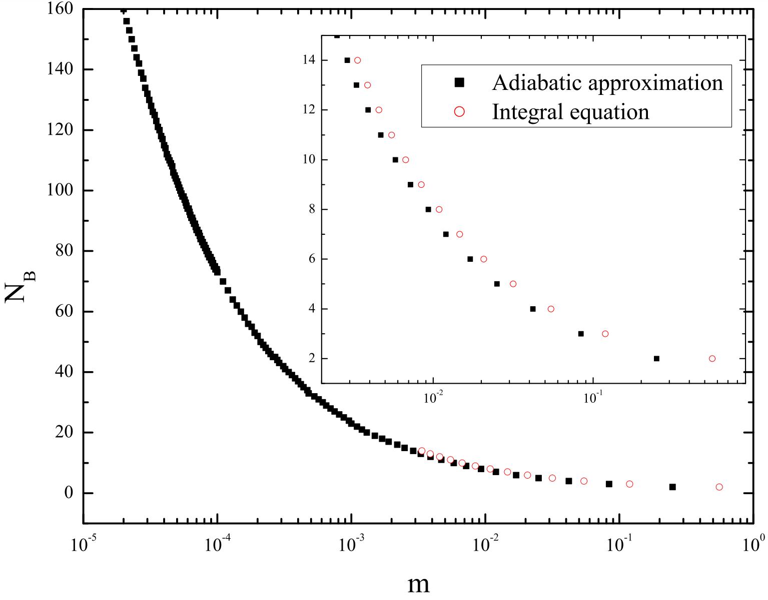
- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

Validity of the analytic approach to the effective potential



- F. F. Bellotti *et al.*, Few-body system **55**, 847 (2014)

Adiabatic approximation and semi-classical estimative to N_B



- Fit: $N_B \approx \frac{0.731}{\sqrt{m}}$;
- JWKB: $N_B = \frac{0.733}{\sqrt{m}}$.

- F. F. Bellotti *et al.*, J. Phys. B **46**, 055301 (2013)

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Asymptotic behavior of the spectator function

$$f_\alpha(q) = \left[2m_{\beta\gamma}\ln\left(\sqrt{\frac{\frac{q^2}{2m_{\beta\gamma,\alpha}}+E_3}{E_{\beta\gamma}}}\right)\right]^{-1}\times$$

$$\int k\,dk\left(\frac{f_\beta(k)}{\sqrt{\left(-E_3+\frac{q^2}{2m_{\alpha\gamma}}+\frac{k^2}{2m_{\beta\gamma}}\right)^2+\frac{k^2q^2}{m_\gamma^2}}}+\frac{f_\gamma(k)}{\sqrt{\left(-E_3+\frac{q^2}{2m_{\alpha\beta}}+\frac{k^2}{2m_{\beta\gamma}}\right)^2+\frac{k^2q^2}{m_\beta^2}}}\right).$$

$$\vdots$$

$$\vdots$$

Asymptotic behavior of the spectator function

$$f_\alpha\left(q\right) = \left[2m_{\beta\gamma}\ln\left(\sqrt{\frac{\frac{q^2}{2m_{\beta\gamma,\alpha}}+E_3}{E_{\beta\gamma}}}\right)\right]^{-1} \times$$

$$\int k\,dk\left(\frac{f_\beta\left(k\right)}{\sqrt{\left(-E_3+\frac{q^2}{2m_{\alpha\gamma}}+\frac{k^2}{2m_{\beta\gamma}}\right)^2+\frac{k^2q^2}{m_\gamma^2}}}+\frac{f_\gamma\left(k\right)}{\sqrt{\left(-E_3+\frac{q^2}{2m_{\alpha\beta}}+\frac{k^2}{2m_{\beta\gamma}}\right)^2+\frac{k^2q^2}{m_\beta^2}}}\right).$$

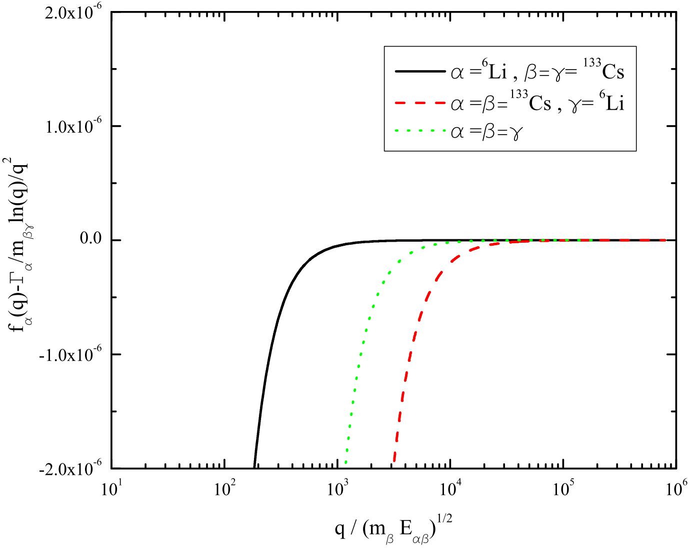
$$\vdots$$

$$\vdots$$

$$\lim_{q\to\infty} f_\alpha(q) \to \frac{\Gamma}{m_{\beta\gamma}}\frac{\ln q}{q^2}\,.$$

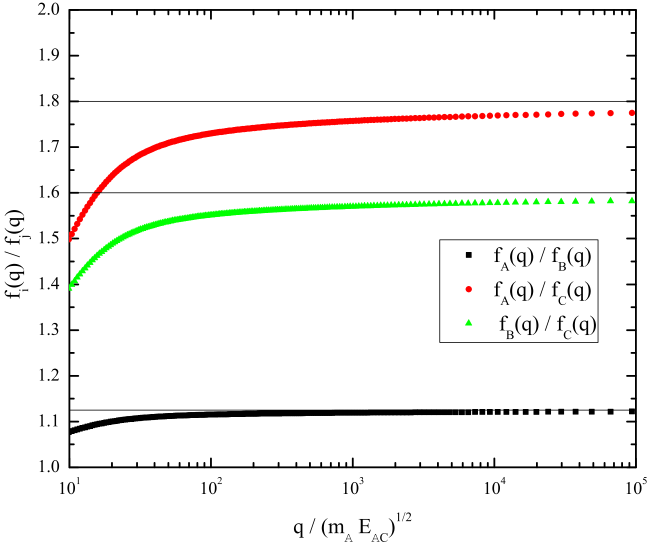
- F. F. Bellotti *et al.*, Phys. Rev. A **87**, 013610 (2013) (Identical bosons)
- F. F. Bellotti *et al.*, New Journal of Physics **16**, 013048 (2014)

Validity of the asymptotic behavior of the spectator function



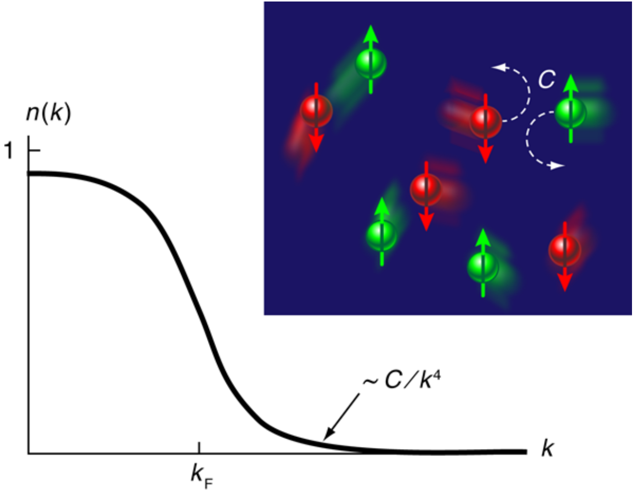
Notice the scale on $y-$axis!

Mass-coefficients in the asymptotic expression: $\frac{f_\alpha(q)}{f_\beta(q)} = \frac{m_{\alpha\gamma}}{m_{\beta\gamma}}$



Outline

Tan's contact parameter [5]



$$\frac{dE}{da^{-1}} = -\frac{\hbar^2 V}{4\pi m} C$$

$$2\pi \frac{dE}{d[-1/(k_F a)]} = C$$

$$E - 2V = -\frac{C}{4\pi k_F a}$$

- J. T. Stewart *et al.*, Phys. Rev. Letters **104**, 235301 (2010);
- D. E. Sheehy, Physics **3**, 48 (2010). Illustration: Alan Stonebraker.

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[5] S. Tan, Annals of Physics **323(12)**, 2952 (2008)

The scenario

- one-body density: $n(q_\alpha) = \int d^2p_\alpha |\Psi(\mathbf{q}_\alpha, \mathbf{p}_\alpha)|^2$;
- $\lim_{q\to\infty} n(q) \to \frac{C_2}{q^4} + C_3 F(q)$;
- few-body $\leftrightarrow$ many-body: C_2, C_3 (only for bosons) [6];
- $F(q) \leftrightarrow f(q)$;
  - 3D[7]: $f(q) \propto \frac{\sin(s_0 \ln(q/q^*))}{q^2}$;
  - 2D[8] [9]: $f(q) \propto \frac{\ln(q)}{q^2}$;
- $n_{2D}(q) \to \frac{1}{q^4} C_2 + \frac{\ln^3(q)}{q^6} C_3$.

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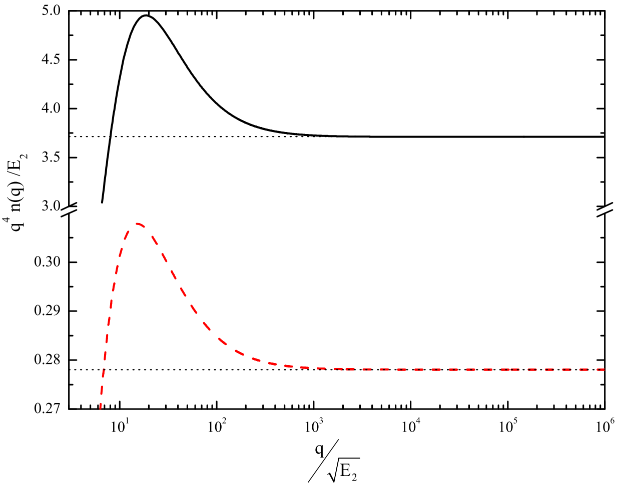
[6] R. J. Wild *et al.*, Phys. Rev. Letters **108**, 145305 (2012)
[7] G. S. Danilov, Zh. Eksp. Teor. Fiz **40**, 698 (1961)
[8] F. F. Bellotti *et al.*, Phys. Rev. A **87**, 013610 (2013)
[9] F. F. Bellotti *et al.*, New Journal of Physics **16**, 013048 (2014)

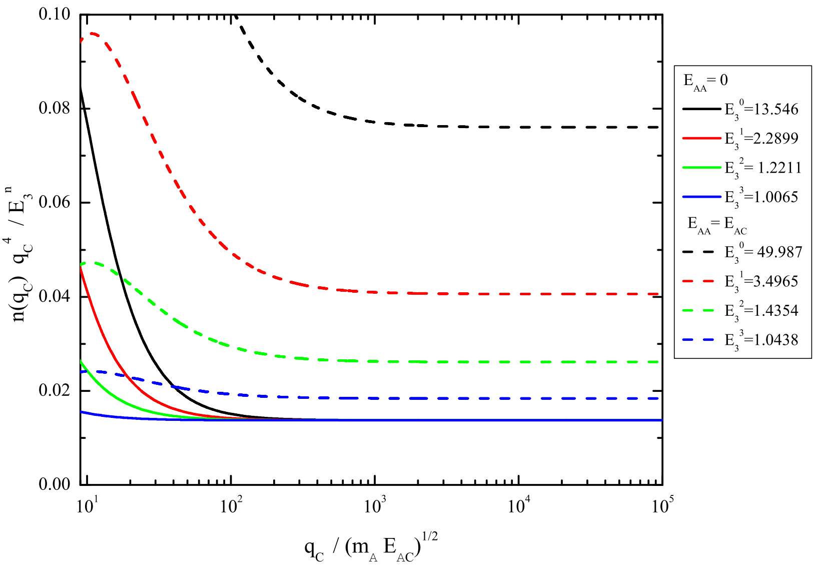
Large-momentum expansion: Leading order for three identical bosons



$$\frac{C_2}{E_3} = 0.2220 \pm 0.0025\ .$$

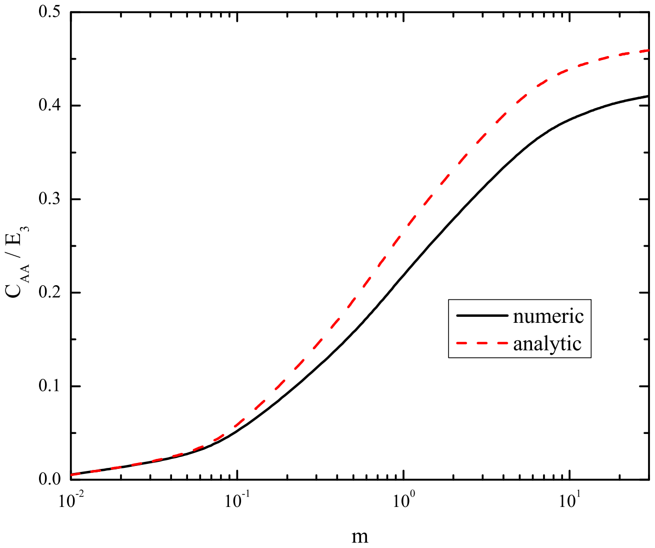
- F. F. Bellotti *et al.*, Phys. Rev. A **87**, 013610 (2013)
- agrees with: F. Werner and Y. Castin, Phys. Rev. A **86** 053633 (2012)

Large-momentum expansion: Leading order for a ^{133}Cs-^{133}Cs-^{6}Li system



- F. F. Bellotti *et al.*, New Journal of Physics **16**, 013048 (2014)

Large-momentum expansion: Analytical expression



- $\frac{C_{aa}}{E_3} = 16\pi \frac{m^2}{(1+m)(2+m)} f_a^2(0) \left(1 + \frac{2}{\ln(E_3)} + \frac{2}{\ln^2(E_3)}\right)$

Outline

Summary

- Number of available bound states in 2D systems is mass-dependent;
  - $E_{ab} = E_{ac} = E_{bc}$ gives the maximum number of bound states;
- heavy-heavy-light system with zero-range interactions
  - Adiabatic (effective) potential;
  - Asymptotic form $\rightarrow$ Analytic approach;
  - Rich energy spectrum in 2D;
- Large momentum behavior of the spectator function in 2D: form and coefficient;
- Universal two-body contact parameter in 2D;
- Analytical estimate of the two-body contact parameter in the ground state.

Outlook

- Background Fermi sea;
- Range correction;
- Changing dimensionality;
- 4-Body problem in 2D.

The end

Advisors

- Tobias Frederico - ITA/Brazil
- Aksel S. Jensen - AU/Denmark

Collaborators (Co-Advisors)

- Marcelo T. Yamashita - IFT-UNESP/Brazil
- Nikolaj T. Zinner - AU/Denmark
- Dmitri V. Fedorov - AU/Denmark

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Thank you for your attention!