Modified Gravity and its Tests

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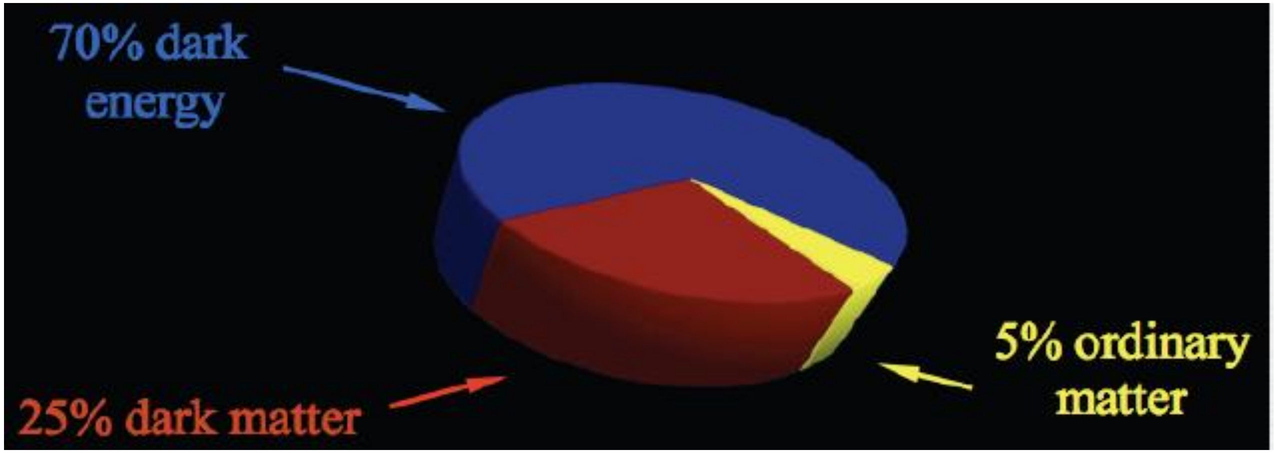
The Big Puzzle



Evidence: The Hubble Diagram

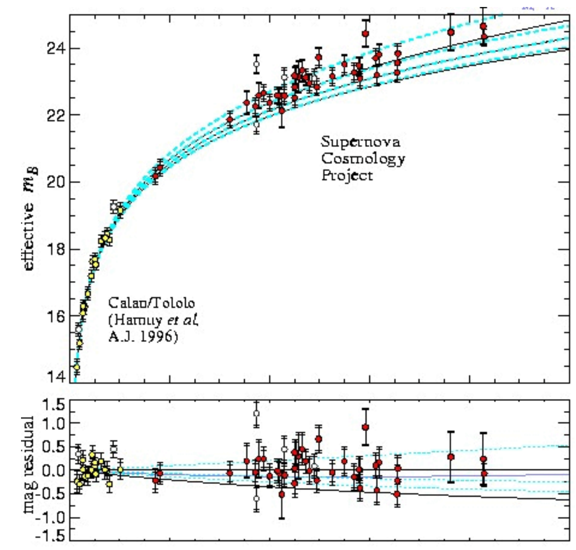
The explosion of high red-shift SN Ia (standard candles):

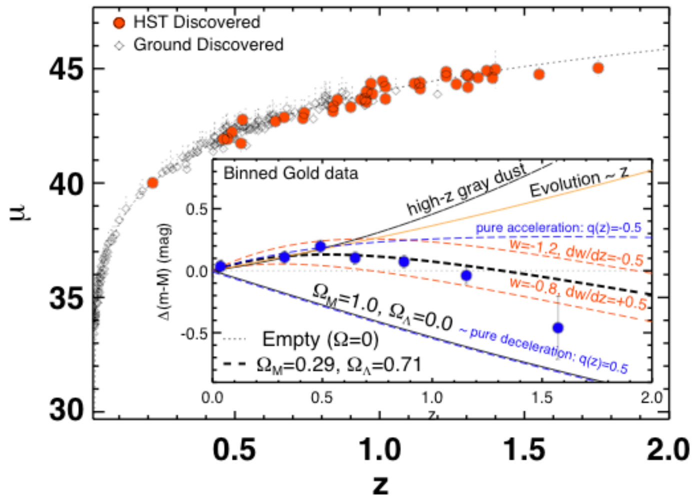
$$q_0 \equiv -\frac{a_0\ddot{a}_0}{(\dot{a}_0)^2} \simeq -0.67 \pm 0.25$$

Within General Relativity, link to matter and dark energy

$$q_0 = -\Omega_\Lambda + \frac{1}{2}\Omega_m \sim -0.67$$

Dark Energy must exist!





The Cosmic Microwave Background

Fluctuations of the CMB temperature across the sky lead to acoustic peaks and troughs, snapshot of the plasma oscillations at the last scattering

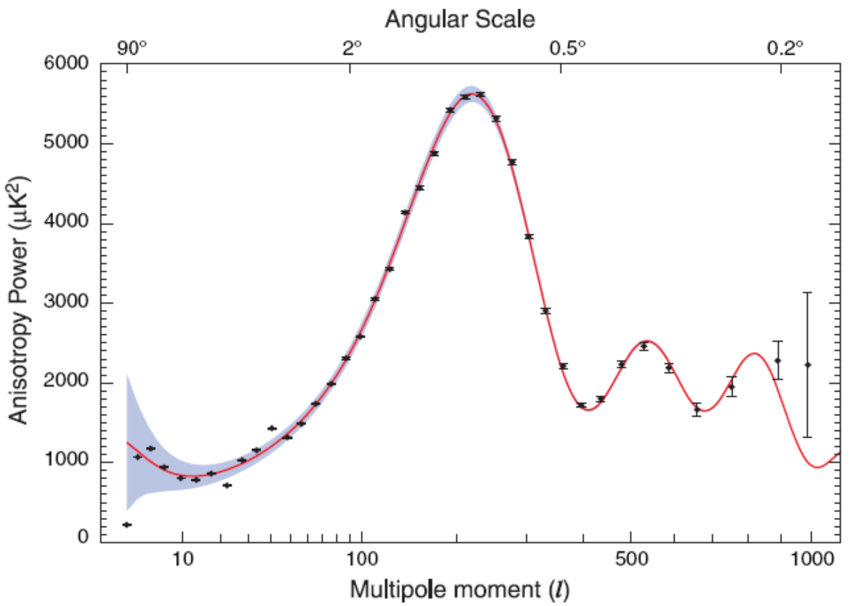
The position of the first peak:

$$l_1 \approx \frac{220}{\sqrt{\Omega_\Lambda + \Omega_m}}$$

The universe is spatially flat

$$\Omega_\Lambda + \Omega_m = 1$$

$$\Omega_\Lambda = \frac{2}{3}\left(\frac{1}{2} - q_0\right) \sim 0.78$$

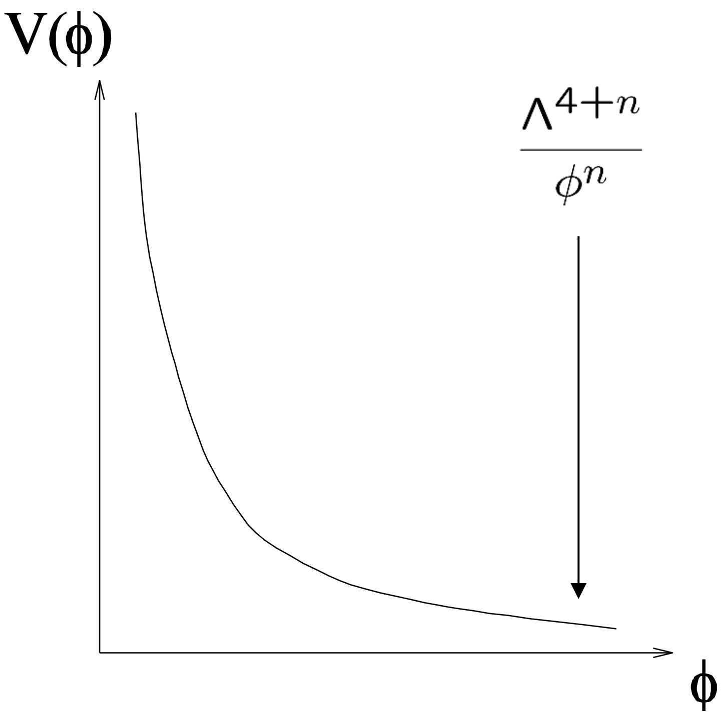


WMAP data

Dark Energy ?

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$$

A scalar field is a good candidate as its energy density can be dominated by its potential implying that its pressure is almost opposite to ρ



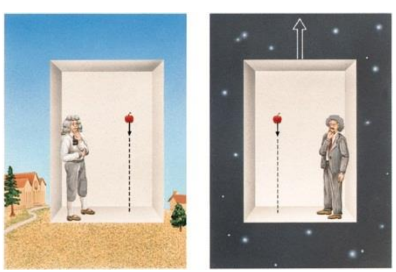
A field rolling down a runaway potential, reaching large values now?

It could also be that what we interpret as acceleration is in fact a manifestation of something more subtle:

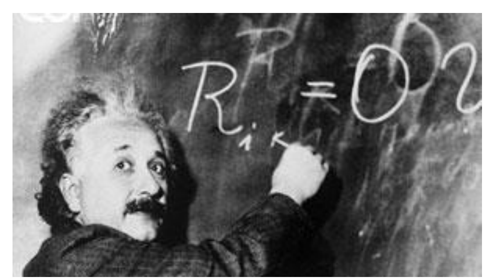
A modification of the laws of gravity on large scales

Gravity is described by the general theory of relativity (Einstein 1915) and encompasses several aspects which needs to be carefully understood when one tries to “modify gravity”.

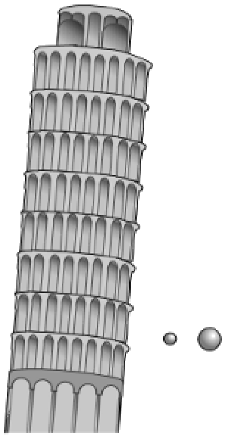
The equivalence principle



The Einstein equation of General Relativity



The equivalence principle is at the heart of general relativity. There are several versions which must be distinguished:



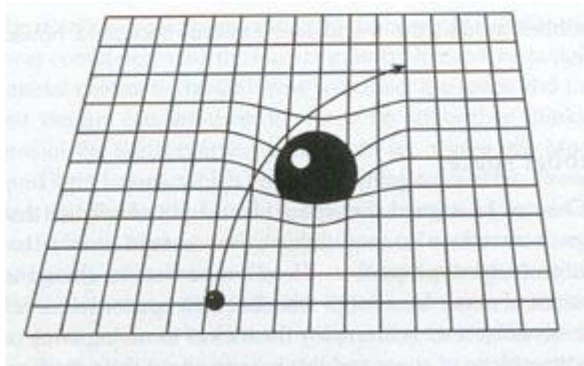
The weak equivalence principle (or universality of free fall) states that to massive tests bodies fall in the same way independently of their composition.

$$m_{\text{inert}} a = m_{\text{grav}} g$$

The equality of the inertial and gravitational masses for test bodies has been tested at the level of thirteen decimal places!

$$m_{\text{grav}} = m_{\text{inert}}$$

General relativity satisfies a stronger version of the equivalence principle.



General relativity is a metric theory relating energy and geometry of the Universe:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

Ricci tensor

Energy momentum tensor

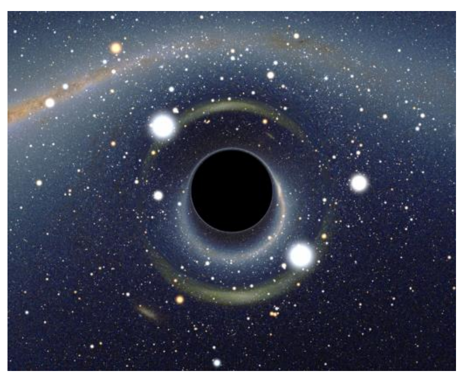
General relativity even satisfies a strong equivalence principle

The weak equivalence principle is valid even for bodies with self-gravity.

Result of any local experiment independent of the velocity of free falling frame.

Result of any experiment independent of where and when performed.

This is violated in the theories which modify gravity on large scales.



Modifying Gravity

Mass term for a graviton

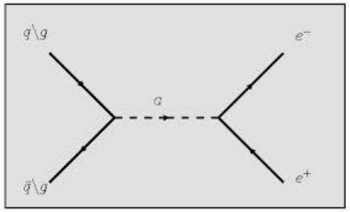
The simplest modification is massive gravity (Pauli-Fierz):

$$\delta \mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations. In the presence of matter, the graviton wave function takes the form:

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \to h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not gives GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal.



Pauli- Fierz massive gravity has a ghost in curved backgrounds (cosmology).

An infinite class of modified gravity models can be considered:

$$S_{\mathrm{MG}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

These Lagrangian field theories fall within the category of higher derivative theories

Ostrosgradski's theorem states that these theories are *generically* plagued with ghosts. Quantum mechanically, this implies an explosive behaviour with particles popping out of the vacuum continuously. In particular an excess in the gamma ray background.

A large class is ghost-free though, the f(R) models:

$$S_{\mathrm{MG}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R)$$

f(R) is totally equivalent to an effective field theory with gravity and scalars!

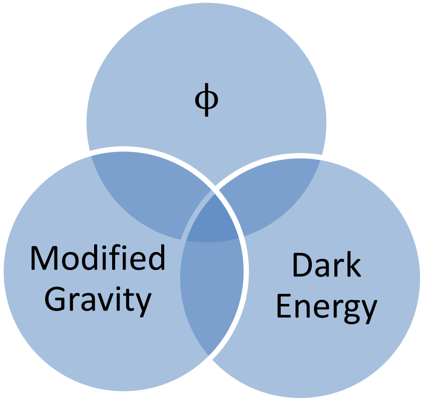
$$S=\int d^4x\sqrt{-g}(\frac{1}{16\pi G_N}R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\rm Pl}}g_{\mu\nu}))$$

Crucial coupling between matter and the scalar field

The potential V is directly related to f(R)

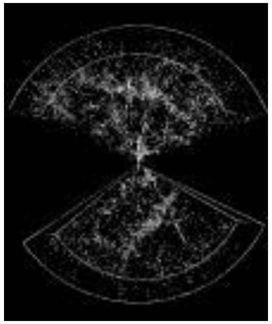
$$V(\phi)=m_{\rm Pl}^2\frac{Rf'-f}{2f'^2},\ \ f'=e^{-2\phi/\sqrt{6}m_{\rm Pl}}$$

The acceleration of the Universe could be due to either:



In both cases, current models use scalar fields. In modified gravity models, this is due to the scalar polarisation of a massive graviton (5=2+2+1). In dark energy, it is by analogy with inflation.

The fact that the scalar field acts on cosmological scales implies that its mass must be large compared to solar system scales.



$$\beta = \frac{1}{\sqrt{6}}$$ F(R) gravity

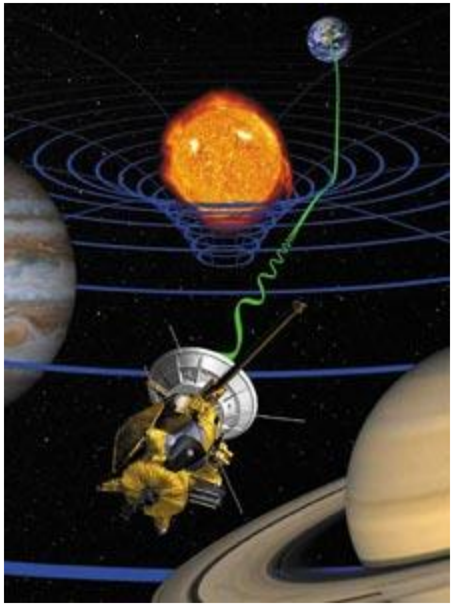
Deviations from Newton's law are parametrised by:

$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

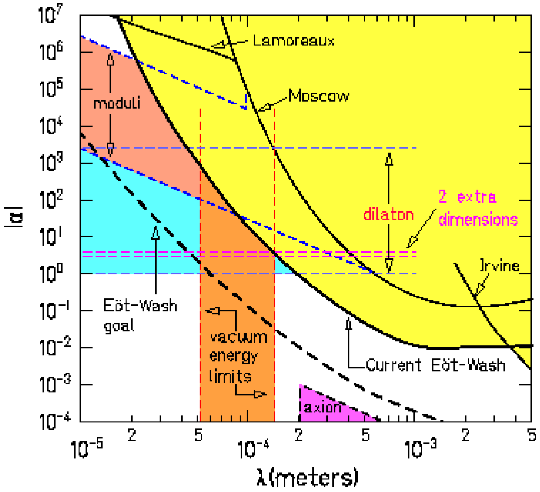
For large range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 4 \cdot 10^{-5}$$

The effect of a long range scalar field must be screened to comply with this bound and preserve effects on cosmological scales.



Bertotti et al. (2004)



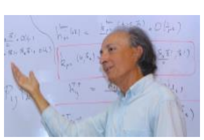
Around a background configuration and in the presence of matter, the Lagrangian can be linearised and the main screening mechanisms can be schematically distinguished :

$$\mathcal{L} \supset -\frac{Z(\phi_0)}{2}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T \ ,$$

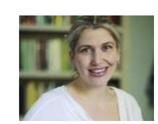
The chameleon mechanism makes the range become smaller in a dense environment by increasing m

The Damour-Polyakov mechanism reduces β in a dense environment

The Vainshtein mechanism reduces the coupling in a dense environment by increasing Z





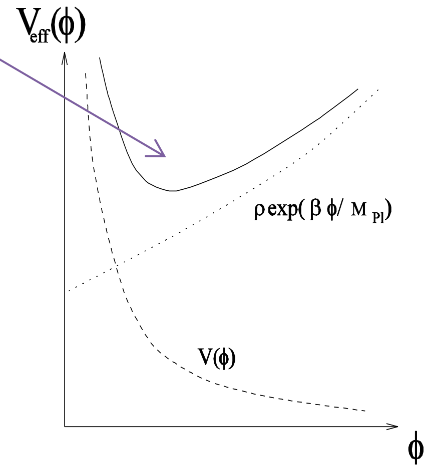


The effect of the environment (excluding Vainshtein)

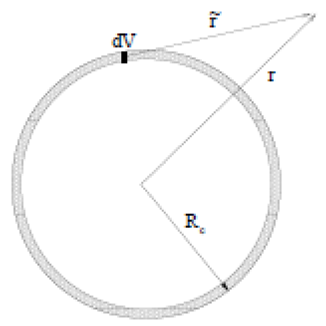
When coupled to matter, scalar fields have a matter dependent effective potential

$$V_{eff}(\phi) = V(\phi) + \rho_m (A(\phi) - 1)$$

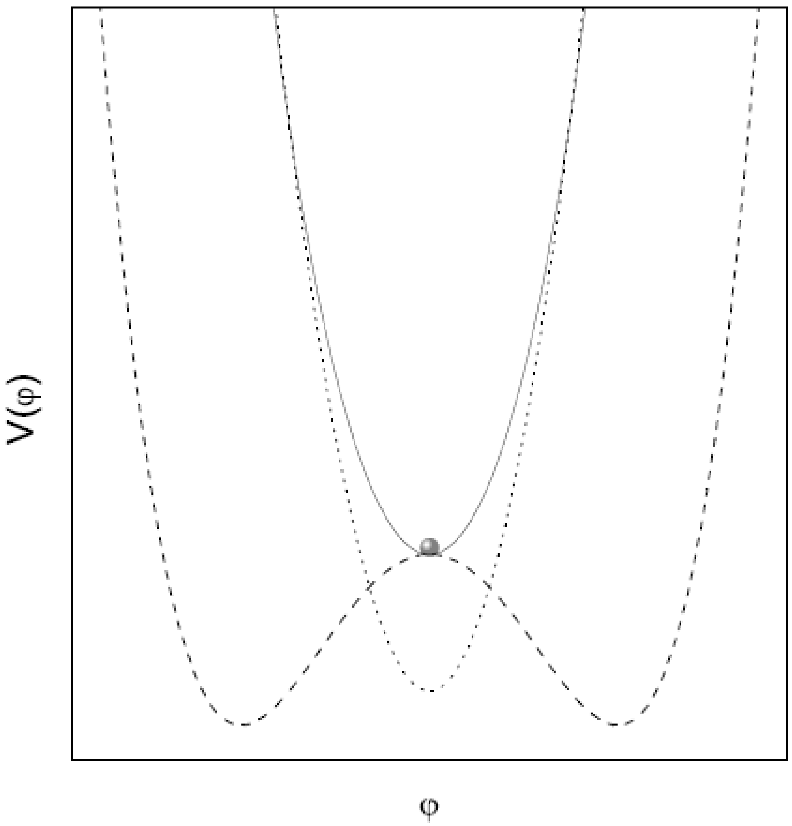
Environment dependent minimum

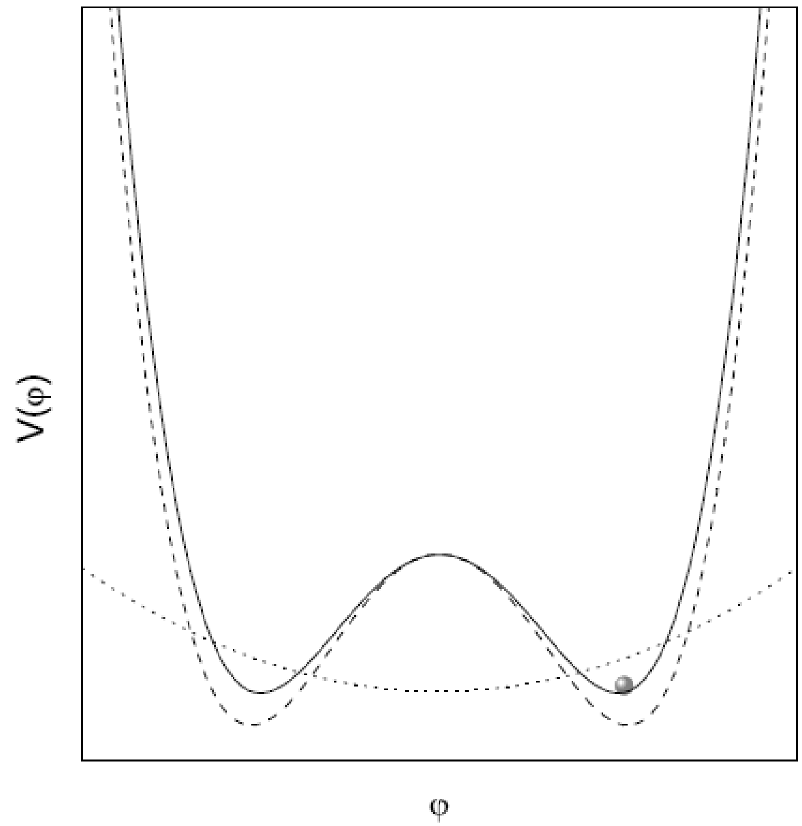


Chameleon



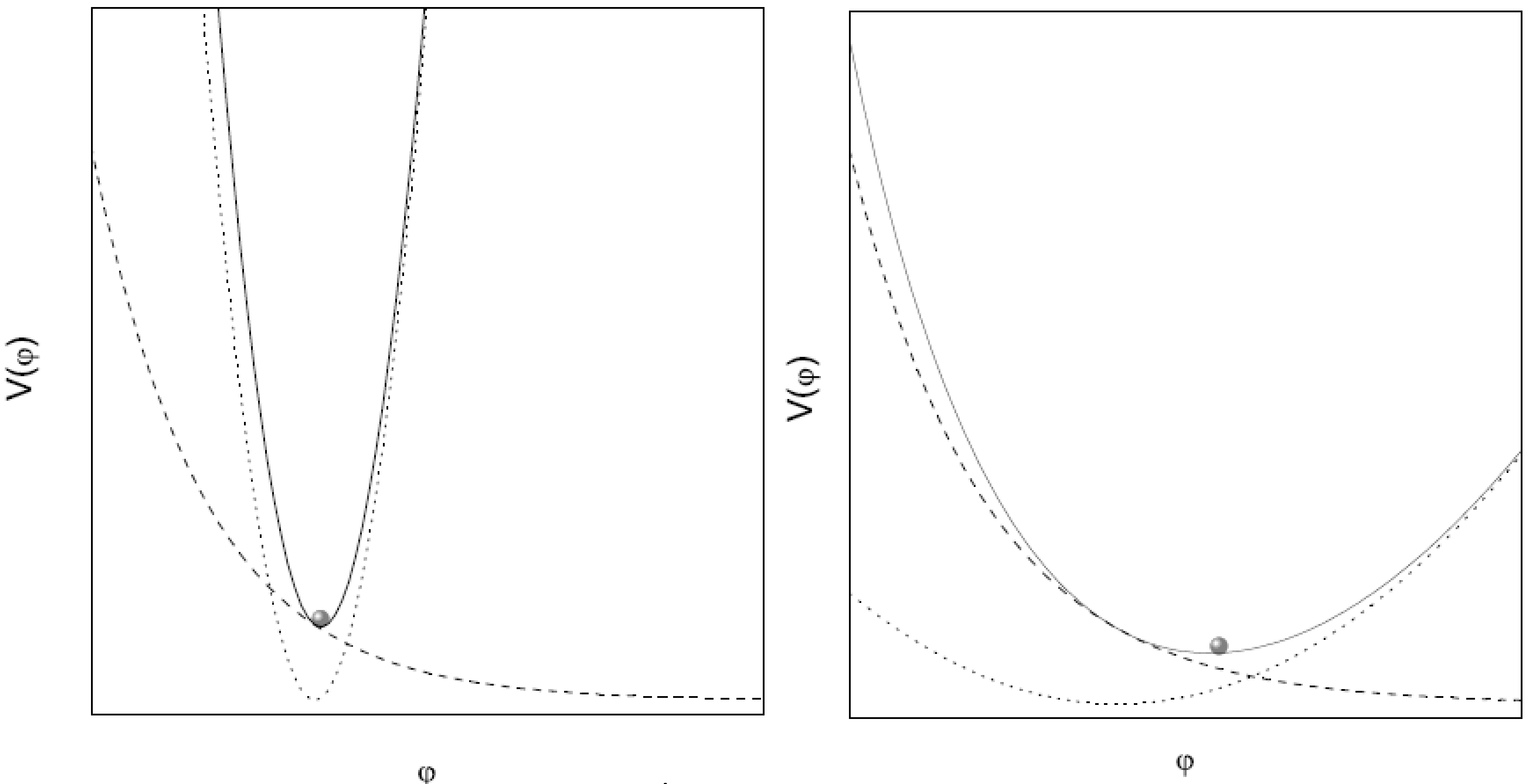
The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.





Symmetron

$$V(\phi) = V_0 - \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4, \;\; A(\phi) = 1 + \frac{A_2}{2m_{\mathrm{Pl}}^2}\phi^2$$

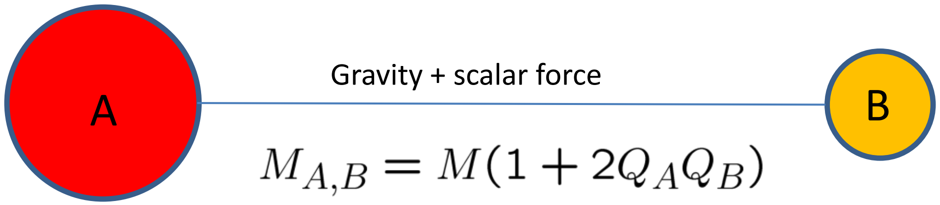


Dilaton

$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}^2}(\phi - \phi_\star)^2$$

Due to the scalar interaction, within the Compton wavelength of the scalar field, the inertial and gravitational masses differ for screened objects:

VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE



$$M_{A,B} = M(1 + 2Q_A Q_B)$$

$$Q_A \leq \beta_\infty$$

Screening criterion

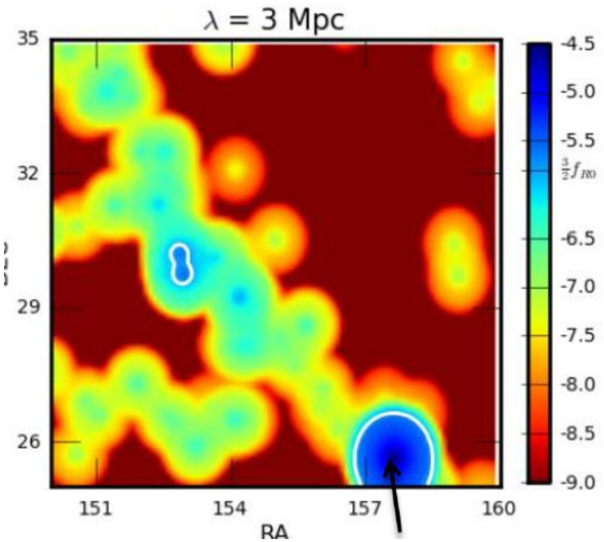
$$Q_A = \frac{\phi_\infty}{2m_{\rm Pl}\Phi_A}$$

Value of the field far away

Newtonian potential at the surface of the body.

Massive bodies with differ scalar charges fall differently.
Hence a violation of the strong equivalence principle.

Astrophysical tests: try to see if screened and unscreened objects fall at different rates, for instance the gas and stars in the halo of an unscreened galaxy.



screened

$\xi = 10^{-6}$

SDSS catalogue, within 200 Mpc, scalar range 3 Mpc, some halos are screened, others are not.

$\Phi_N \sim 10^{-7}$

No effects on the luminosity of stars or on the trajectories measured so far: bound on the range of the scalar interaction.

All these models can be entirely characterised by 2 time dependent functions. The non-linear potential and coupling of the model can be reconstructed using:

m(a) cosmological mass
β(a) cosmological coupling

$$\phi(a) = \phi_i + \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta(a)\rho(a)}{am^2(a)}$$
$$V(a) = V_i - \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta^2(a)\rho^2(a)}{am^2(a)}$$

Works for chamelons, dilatons, symmetrons etc…

Screening in the Milky Way:

$$\frac{m_0}{H_0} \geq 10^3$$

Models with cosmological effects on scales smaller than 1 Mpc.

Equation of state equal to -1 at the 1 per million level!

The cosmological background evolves like in the concordance model. The main difference coming from the modification of gravity arises at the perturbation level where the Cold Dark Matter density contrast evolves like:

$$\delta_c''+\mathcal{H}\delta_c'-\frac{3}{2}\mathcal{H}^2\frac{\rho_c}{\rho_c+\rho_b+\rho_\gamma+\rho_\phi}\left(1+\frac{2\beta^2(a)}{1+\frac{m^2a^2}{k^2}}\right)\delta_c=0$$

Modified gravity

The new factor in the bracket is due to a modification of gravity depending on the comoving scale k.

The growth of structures depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length (matter era)

$$\delta \sim a$$

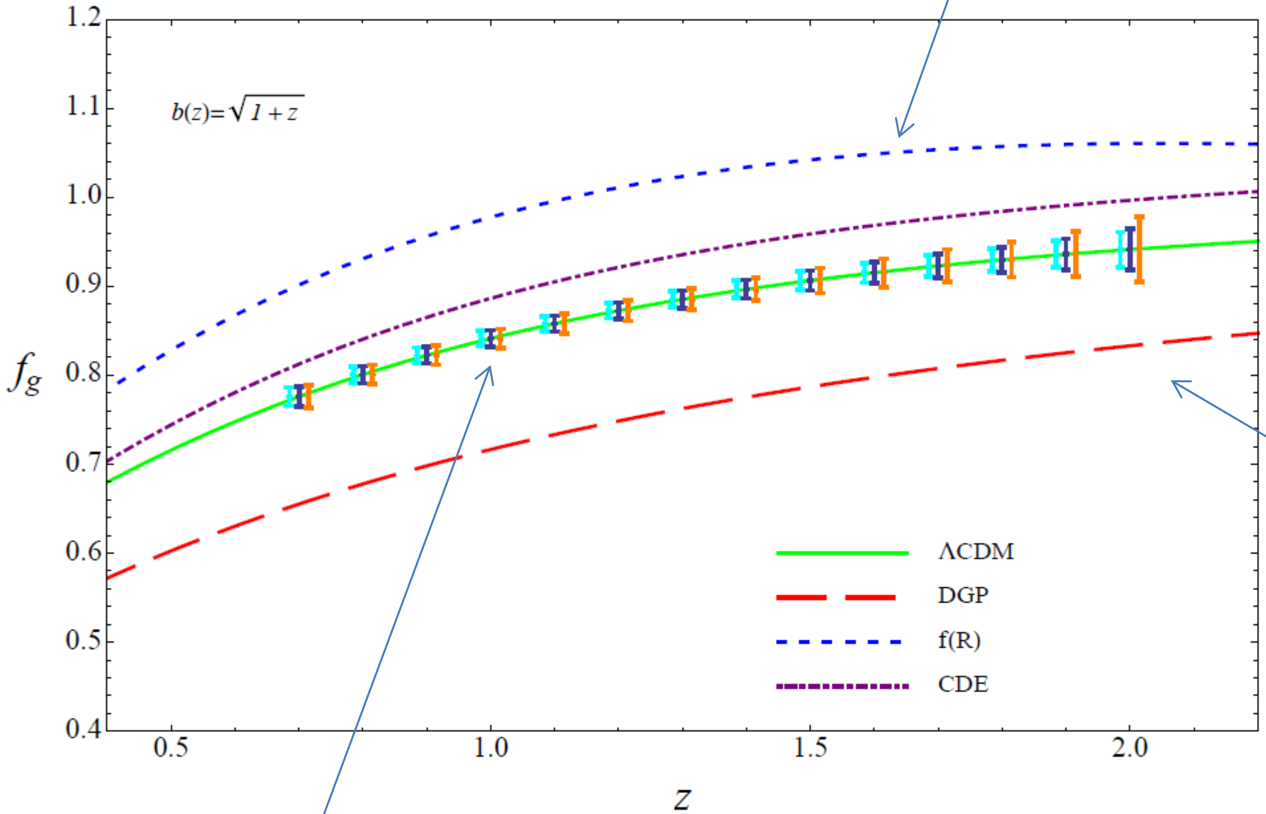
Gravity is modified inside the Compton length with MORE growth (matter era):

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

EUCLID forecast: growth rate at the percent level

more growth

$$f_g(z) = \frac{d \ln \delta}{d \ln a}$$



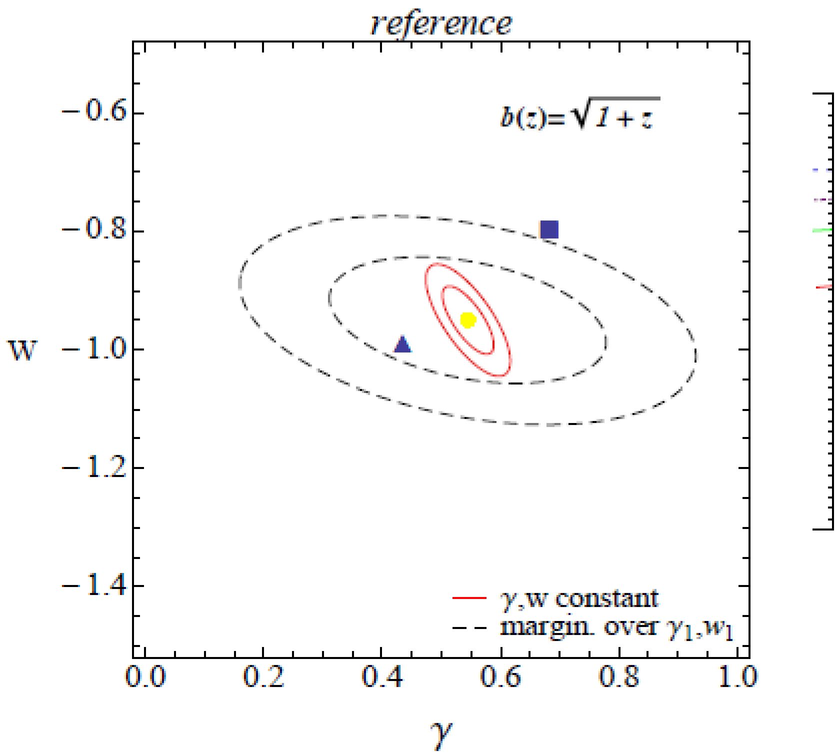
ΛCDM

Less growth

Forecast in the linear regime

EUCLID Forecast

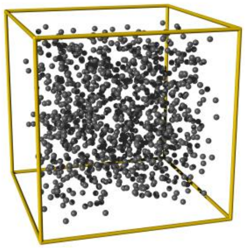
Equation of state



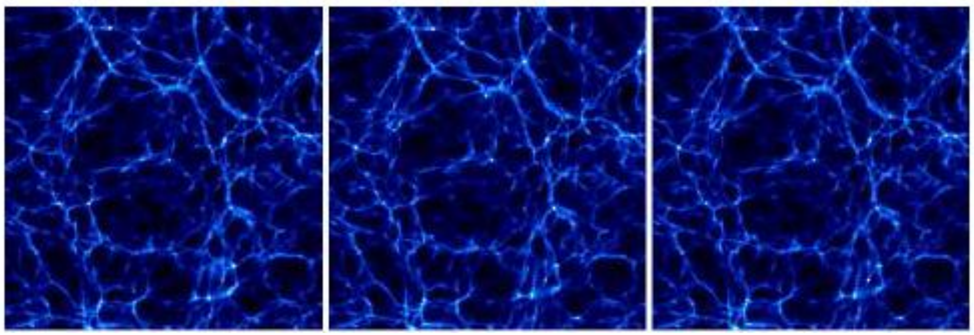
Growth index

$$f_g(z) = \Omega_m^{\gamma}$$

Properties around 1Mpc and below necessitates large scale N-body simulations



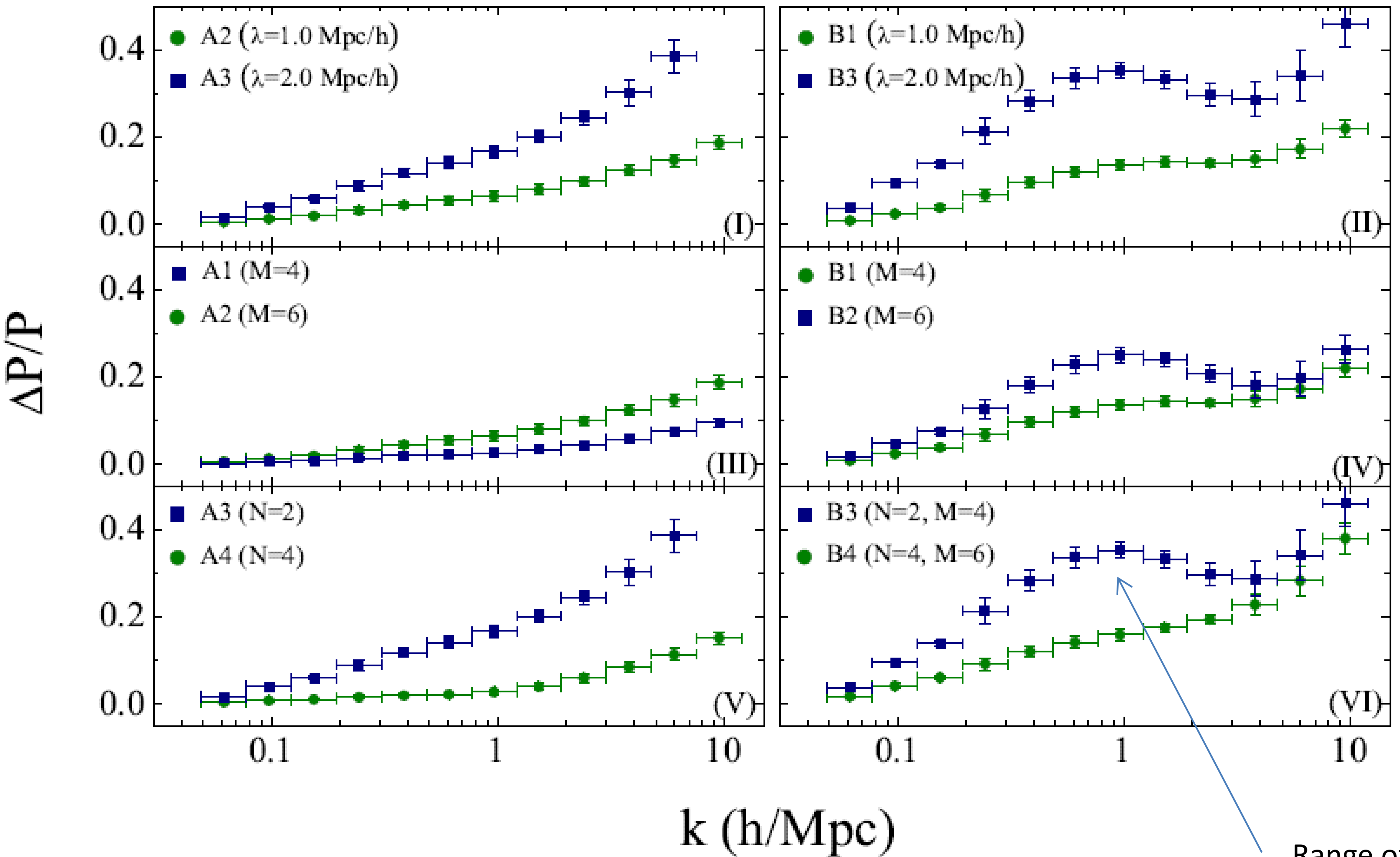
N-body simulations with around 10 million particles in a box up to 1 Gpc



Λ-CDM Linear Non-linear

Koyama, Li and Zhao (2013)

Large scale structures in the non-linear regime determined by m(a) and β(a). Non-linear effects simulated with ECOSMOG. Deviation of the power spectrum of the density spectrum from GR (Fourier space). Deviations in the quasi-linear and non-linear regimes.



Range of the scalar field.

Power Spectrum deviation (symmetron)

Deviations in the quasi-linear regime can be tackled in perturbation theory by going to higher order than linear perturbation theory and using partial resummation techniques. The non linearities are important to go beyond large scales to smaller scales where modified gravity effects appear.

First let us consider the Klein-Gordon equation in the sub-horizon limit and in the quasi-static approximation:

$$n \geq 1: \quad \beta_n(\overline{\varphi}) = M_{\rm Pl}^n \frac{{\rm d}^n A}{{\rm d}\varphi^n}(\overline{\varphi}),$$

$$\left(\frac{\nabla^2}{a^2} - m^2\right) \cdot \delta\varphi = \frac{\beta\,\delta\rho}{c^2 M_{\rm Pl}} + \frac{\beta_2\,\delta\rho}{c^2 M_{\rm Pl}^2}\delta\varphi + \sum_{n=2}^{\infty}\left(\frac{\kappa_{n+1}}{M_{\rm Pl}^{n-1}} + \frac{\beta_{n+1}\,\delta\rho}{c^2 M_{\rm Pl}^{n+1}}\right)\frac{(\delta\varphi)^n}{n!}.$$

$$n \geq 2: \quad \kappa_n(\overline{\varphi}, \overline{\rho}) = \frac{M_{\rm Pl}^{n-2}}{c^2}\frac{\partial^n V_{\rm eff}}{\partial\varphi^n}$$

One can expand the solution in a perturbation series:

$$\delta\tilde{\varphi}(\mathbf{k}) = \sum_{n=1}^{\infty}\int {\rm d}\mathbf{k}_1..{\rm d}\mathbf{k}_n\; \delta_D(\mathbf{k}_1 + .. + \mathbf{k}_n - \mathbf{k}) \times h_n(\mathbf{k}_1, .., \mathbf{k}_n)\,\delta\tilde{\rho}(\mathbf{k}_1)\ldots\delta\tilde{\rho}(\mathbf{k}_n),$$

Particles evolves according to the generalised potential and its expansion:

$$\Psi = \Psi_{\rm N} + \Psi_{\rm A},$$

$$\frac{1}{a^2}\nabla^2\Psi_{\rm N} = 4\pi\mathcal{G}\,\delta\rho, \qquad \Psi_{\rm A} = c^2(A - \bar{A}),$$

$$\tilde{\Psi}(\mathbf{k}) = \sum_{n=1}^{\infty}\int {\rm d}\mathbf{k}_1..{\rm d}\mathbf{k}_n\; \delta_D(\mathbf{k}_1 + .. + \mathbf{k}_n - \mathbf{k}) \times H_n(\mathbf{k}_1, .., \mathbf{k}_n)\,\delta\tilde{\rho}(\mathbf{k}_1)\ldots\delta\tilde{\rho}(\mathbf{k}_n).$$

The scalar field modifies the dynamics of the Cold Dark Matter particles treated as a fluid:

$$\begin{aligned} \frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] &= 0, \\ \frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} &= -\nabla \cdot \Psi, \end{aligned}$$

This can be simplified by introducing a vector and matrix notations:

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \equiv \begin{pmatrix} \delta \\ -(\nabla\cdot\mathbf{v})/\dot{a} \end{pmatrix},$$

$$\begin{aligned} \mathcal{O}(x,x') &= \delta_D(\eta'-\eta)\,\delta_D(\mathbf{k}'-\mathbf{k}) \\ &\times \begin{pmatrix} \frac{\partial}{\partial\eta} & -1 \\ -\frac{3}{2}\Omega_{\rm m}(\eta)(1+\epsilon(k,\eta)) & \frac{\partial}{\partial\eta}+\frac{1-3w\Omega_{\rm de}(\eta)}{2} \end{pmatrix} \end{aligned}$$

$$1+\epsilon(k,\eta) = -2M_{\rm Pl}^2 a^{-2} k^2\, H_1(k,\eta).$$

$$\mathcal{O}(x,x')\cdot\tilde{\psi}(x') = \sum_{n=2}^{\infty} K_n^s(x;x_1,..,x_n)\cdot\tilde{\psi}(x_1)\ldots\tilde{\psi}(x_n),$$

The kernels are defined by:

$$\begin{aligned} K_n^s(x;x_1,..,x_n) &= \delta_D(\eta_1-\eta)..\delta_D(\eta_n-\eta) \\ &\times \delta_D(\mathbf{k}_1+..+\mathbf{k}_n-\mathbf{k})\ \gamma^s_{i;i_1,..,i_n}(\mathbf{k}_1,..,\mathbf{k}_n;\eta). \end{aligned}$$

The kernels of order 2 comprise the usual dynamics in the absence of modified gravity:

$$\gamma^s_{2;2,2}(\mathbf{k}_1,\mathbf{k}_2) = \hat{\beta}(\mathbf{k}_1,\mathbf{k}_2), \qquad \gamma^s_{1;1,2}(\mathbf{k}_1,\mathbf{k}_2) = \frac{\hat{\alpha}(\mathbf{k}_2,\mathbf{k}_1)}{2}, \quad \gamma^s_{1;2,1}(\mathbf{k}_1,\mathbf{k}_2) = \frac{\hat{\alpha}(\mathbf{k}_1,\mathbf{k}_2)}{2},$$

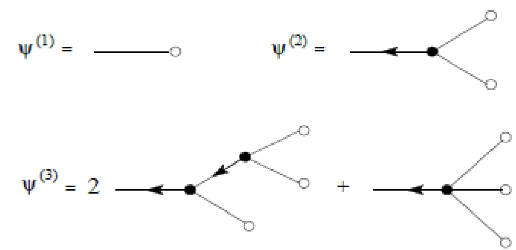
$$\hat{\alpha}(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{k}_1}{k_1^2}, \hat{\beta}(\mathbf{k}_1,\mathbf{k}_2) = \frac{|\mathbf{k}_1+\mathbf{k}_2|^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{2k_1^2k_2^2}.$$

New vertices appear from the second order onwards:

$$
\begin{aligned}
n \geq 2: \quad \gamma^{s}_{2;1,..,1}(\mathbf{k}_1, .., \mathbf{k}_n; \eta) \;&=\; -\frac{k^2}{a^2 H^2}\,(3\Omega_{\rm m} H^2 M_{\rm Pl}^2)^n \\
&\times \frac{1}{n!} \sum_{\rm perm.} H_n(\mathbf{k}_1, .., \mathbf{k}_n; \eta), \quad (68)
\end{aligned}
$$

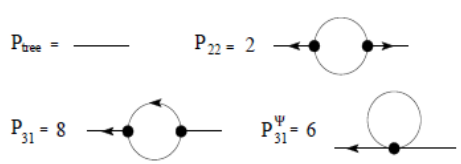
The solution can be expanded in the linear solution and the corresponding diagrammatic expansion:

$$\psi(x) = \sum_{n=1}^{\infty} \psi^{(n)}(x), \quad \text{with} \quad \psi^{(n)} \propto \psi_L^n.$$



White dot: linear solution
Black arrow: retarded propagator
Black dot: vertices

One can then get the one loop power spectrum:

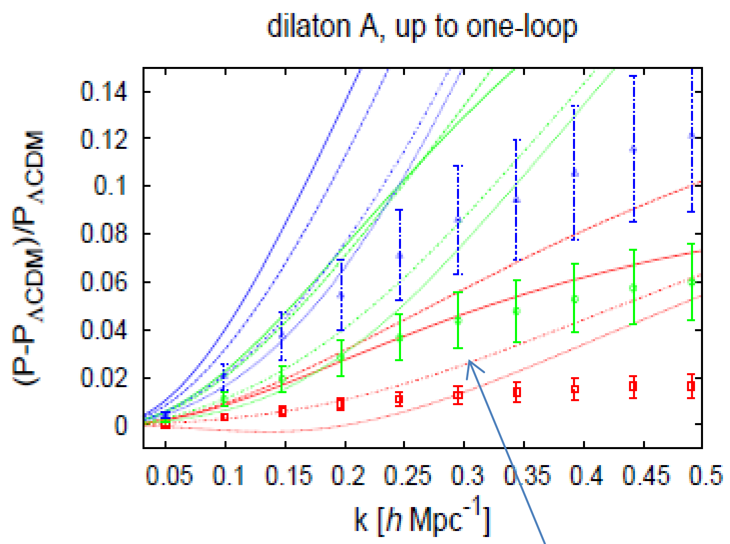


$$m(a) = m_0 \, a^{-r},$$

As an example, we focus on the dilaton models:

$$\beta(a) = \beta_0 \, e^{-s(a^{2r-3}-1)/(3-2r)}, \quad \text{with} \quad s = \frac{9A_2\Omega_{\mathrm{m}0}H_0^2}{c^2 m_0^2}.$$

model name	$m_0[h/\mathrm{Mpc}]$	r	β_0	s
A1	0.334	1	0.5	0.6
A2	0.334	1	0.5	0.24
A3	0.334	1	0.5	0.12
B1	0.334	1	0.25	0.24
B3	0.334	1	0.75	0.24
B4	0.334	1	1	0.24



Good approximation up quasi-linear scales.

To go further into the non-linear, need to study the spherical collapse of a shell of mass M under the influence of both gravity and the scalar field:

$$\ddot{r} = -\frac{\partial \Psi}{\partial r} = -\frac{\partial \Psi_{\mathrm{N}}}{\partial r} - \frac{\partial \Psi_{\mathrm{A}}}{\partial r}, \qquad y(t) = \frac{r(t)}{a(t)q} \quad \text{with} \quad q = \left(\frac{3M}{4\pi\overline{\rho}_0}\right)^{1/3}, \quad y(t=0) = 1,$$

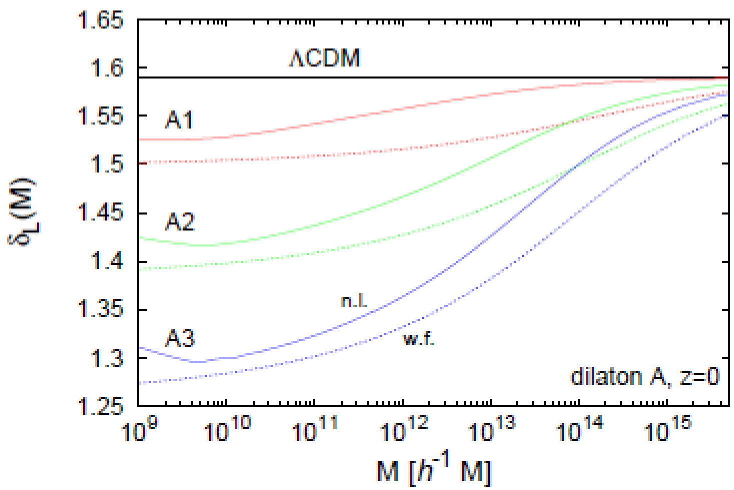
It is governed by the dynamics of the shell:

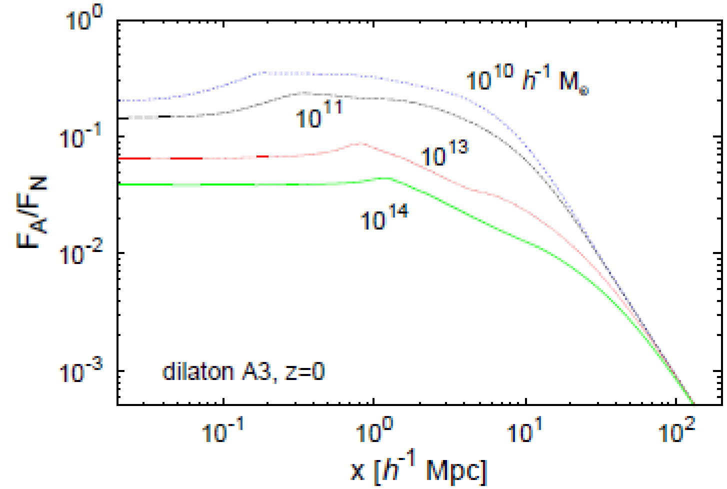
$$\frac{\mathrm{d}^2 y_M}{\mathrm{d}\eta^2} + \frac{1-3w\Omega_{\mathrm{de}}}{2}\frac{\mathrm{d}y_M}{\mathrm{d}\eta} + \frac{\Omega_{\mathrm{m}}}{2}(y_M^{-3}-1)y_M = \frac{-9\Omega_{\mathrm{m}} a \beta_\alpha^2 y_M}{m_\alpha^2 \alpha^4 x_M}\frac{\partial \alpha}{\partial x},$$

Coupled to the Klein-Gordon equation in spherical coordinates:

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2} + \frac{2}{x}\frac{\mathrm{d}\alpha}{\mathrm{d}x} + \left[\frac{\mathrm{d}\ln\beta_\alpha}{\mathrm{d}\alpha} - 2\frac{\mathrm{d}\ln m_\alpha}{\mathrm{d}\alpha} - \frac{4}{\alpha}\right]\left(\frac{\mathrm{d}\alpha}{\mathrm{d}x}\right)^2 = \frac{m_\alpha^2\alpha^4}{3a}\left[1+\delta-\frac{a^3}{\alpha^3}\right]. \qquad \alpha = a(\varphi),$$

Because of the extra force, the linear contrast required to reach a density contrast of 200 (clusters) is lower.





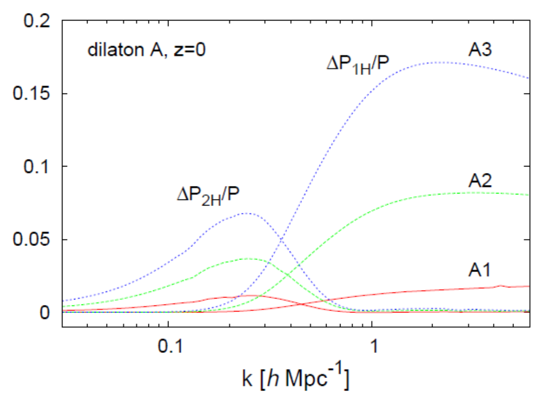
For the interesting range of cluster masses, the decrease of the critical density contrast is significant. Moreover, the extra force on a cluster is reduced for large ones. This plays a crucial role in the distribution function of clusters n(M).

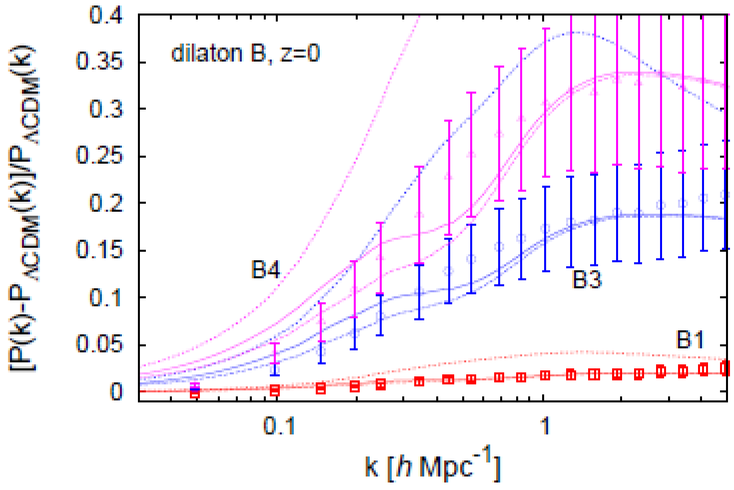
$$n(M)\frac{\mathrm{d}M}{M} = \frac{\overline{\rho}}{M} f(\nu)\frac{\mathrm{d}\nu}{\nu}, \quad \text{with} \quad \nu = \frac{\delta_L(M)}{\sigma(M)}.$$

On small scales, the power spectrum is dominated by the "1 halo" term which corresponds to the contribution of particles belonging to the same cluster:

$$P_{1\mathrm{H}}(k) = \int_0^{\infty} \frac{\mathrm{d}\nu}{\nu} f(\nu) \frac{M}{\overline{\rho}(2\pi)^3} \left(\tilde{u}_M(k) - \tilde{W}(kq_M) \right)^2 ,$$

There is also a "2 halo" term which extends the perturbative results:





Good agreement with numerical simulations, but much easier to compute!

Conclusions

Effects of modified gravity can be modelled out with the dynamics of a scalar field having a long range in the cosmological vacuum.

On smaller scales, the scalar field tends to be screened in the presence of matter

Still there are interesting effects on large scales in the quasi to non-linear regime of structure formation.

Requires either numerical simulations or resummation techniques to study deviations from General Relativity.

Many models, too many! Hopefully observations will soon appear.