

e^+e^- , Tau and Hadronic Vacuum Polarization



Project activity report 2013-2014



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Outline

□ Update of ALEPH spectral functions

- Its applications in precision QCD studies and a_μ calculation

□ Summary of e^+e^- results from Babar

- $e^+e^- \rightarrow \mu^+ \mu^-, \pi^+ \pi^-, K^+ K^-$ with ISR method

Update of the ALEPH non-strange spectral functions from hadronic τ decays

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Some of the related earlier publications:

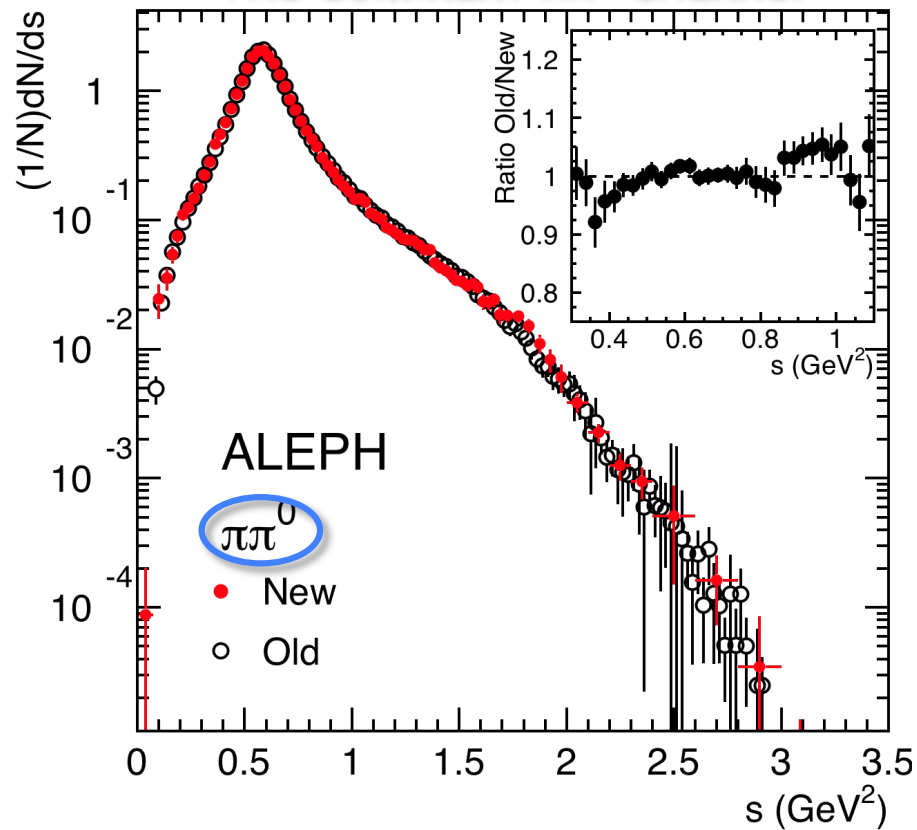
- ALEPH Collaboration, Phys. Rep. 421 (2005) 191, hep-ex/0506072
- M. Davier et al., Eur. Phys. J. C 56 (2008) 305, 0803.0979
- M. Davier et al., Eur. Phys. J. C 66 (2010) 127, 0906.5443

The Main Content of the Paper

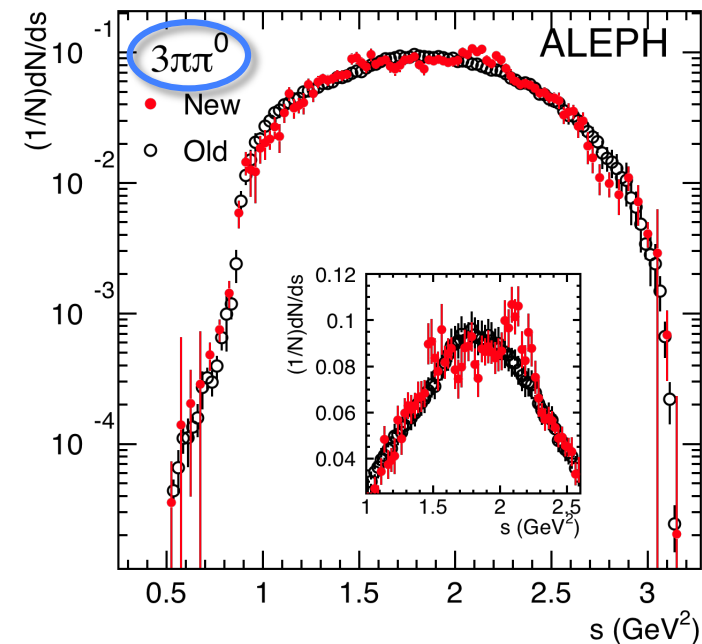
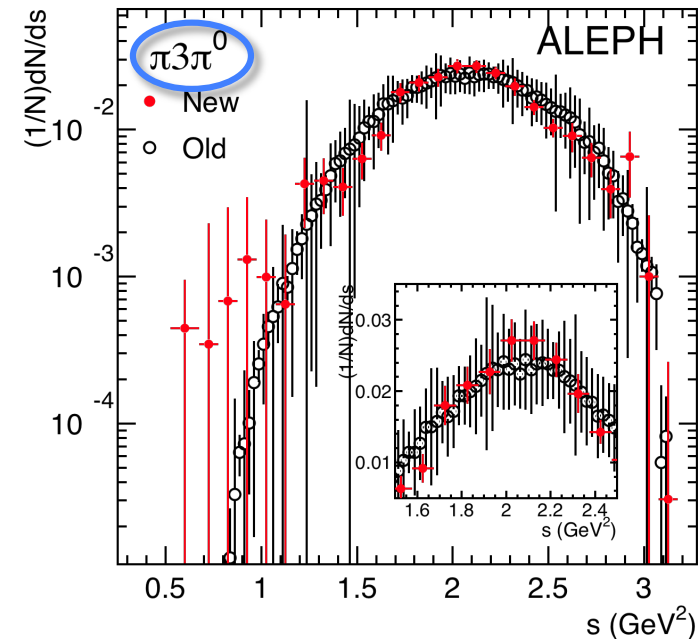
- ❑ Raw data (invariant mass distributions) unchanged
5 channels: $\pi\pi^0$, $\pi 2\pi^0$, 3π , $\pi 3\pi^0$, $3\pi\pi^0$
- ❑ Main improvement
New & more robust unfolding method applied
Fixing a problem in the statistical covariance matrix
- ❑ Calibration and resolution related systematic uncertainties modified
Based on specific studies performed
- ❑ Update results compared with previous one
Find good agreement

Comparison of New and Old Unfolded Spectra

The dominant $\pi\pi^0$ channel



→ Good agreement observed



Applications of the Tau Hadronic Decay Data

Rich and sometime unique testing ground for the SM

Example applications in this publications:

1) Determination of strong coupling constant α_s

Spectral functions (SFs) $v, a, v+a$ are used

2) LO hadronic contribution to muon magnetic anomaly $a_\mu = (g-2)/2$

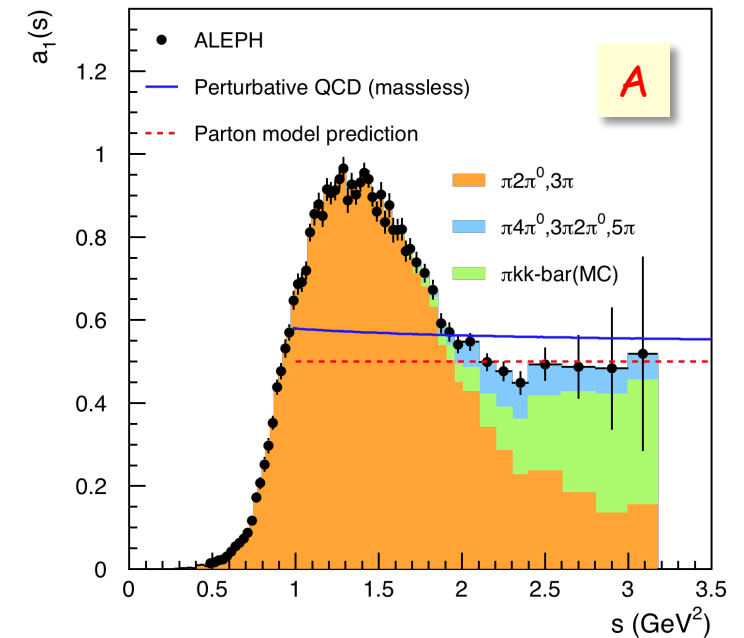
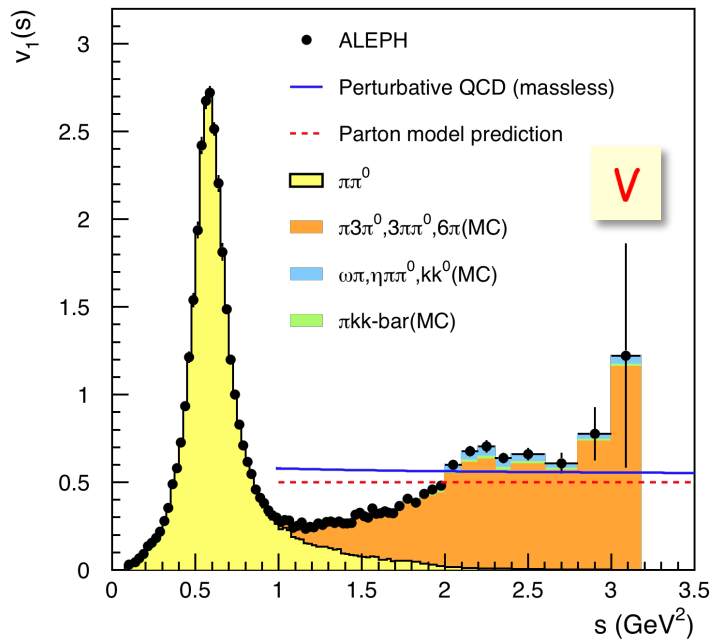
$\pi\pi^0, 3\pi\pi^0, \pi 3\pi^0$ channels are used

3) Line shape fit to $\pi\pi^0$ mass spectrum

Connection between tau mass spectrum and spectral functions (SFs)

$$v[\tau^- \rightarrow \pi^- \pi^0 \nu_\tau] \propto \underbrace{\frac{\text{BR}[\tau^- \rightarrow \pi^- \pi^0 \nu_\tau]}{\text{BR}[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]}}_{\text{branching fractions}} \underbrace{\frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds}}_{\text{mass spectrum}} \underbrace{\frac{m_\tau^2}{(1-s/m_\tau^2)^2 (1+s/m_\tau^2)}}_{\text{kinematic factor (PS)}}$$

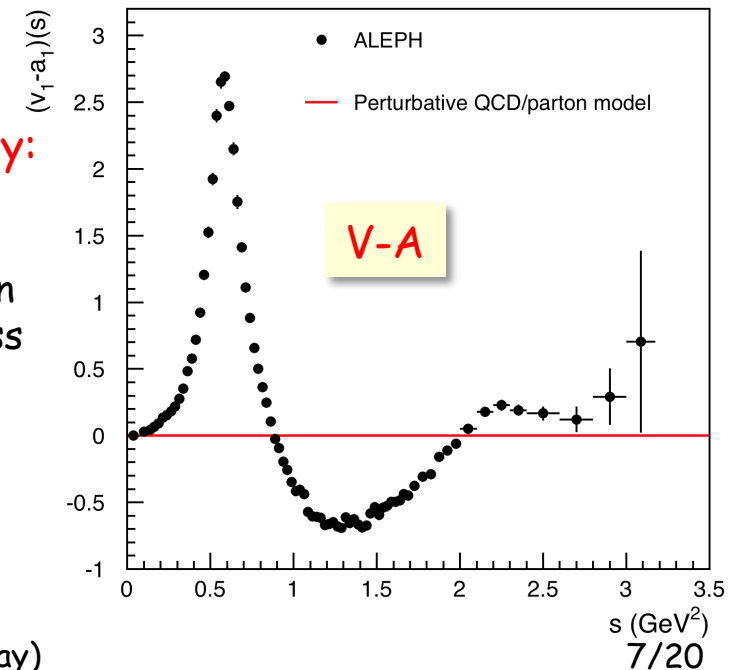
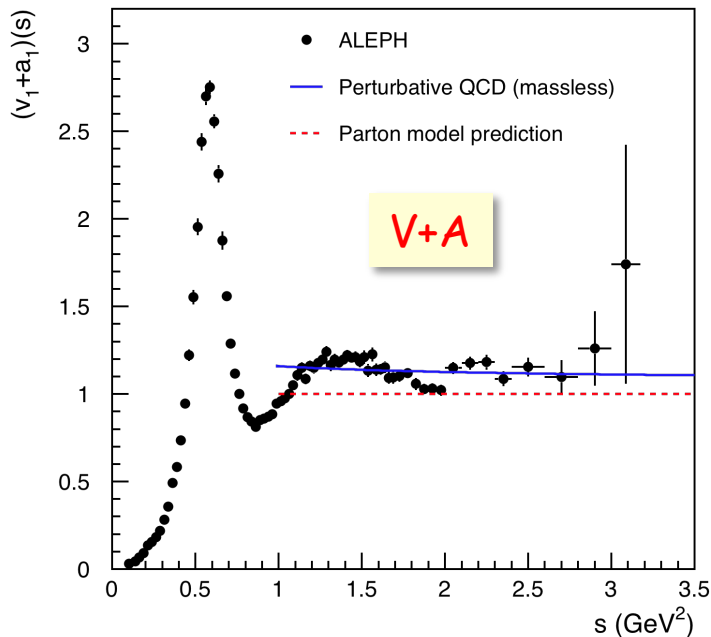
Spectral Functions



The difference between parton model and pert. QCD predictions is due to expansion in α_s

→ α_s determination

For V & A, data at high masses not yet reach asymptotic limit



V+A: quark-hadron duality:

- large oscillating resonance structure at low mass
- stabilizing to q contribution from pert QCD at high mass

(1) α_s Determination

- On experimental side:

- Decay rate
- Spectral moments

$$R_{\tau,V+A} = R_{\tau} - R_{\tau,S} = B_{\text{had}}/B_e = (1 - B_e - B_{\mu})/B_e$$

$$R_{\tau,V/A}^{k\ell} = \int_0^{m_{\tau}^2} ds \left(1 - \frac{s}{m_{\tau}^2}\right)^k \left(\frac{s}{m_{\tau}^2}\right)^{\ell} \frac{dR_{\tau,V/A}}{ds}$$

In reality, we use $D_{\tau,V/A}^{k\ell} \equiv \frac{R_{\tau,V/A}^{k\ell}}{R_{\tau,V/A}}$

- On theoretical side:

$$R_{\tau,V/A}^{k\ell} = \frac{3}{2} |V_{ud}|^2 S_{\text{EW}} \left(1 + \delta^{(0,k\ell)} + \delta'_{\text{EW}} + \delta_{ud,V/A}^{(2-\text{mass},k\ell)} + \underbrace{\sum_{D=4,6,\dots} \delta_{ud,V/A}^{(D,k\ell)}}_{\text{Nonpert. contribution}} \right)$$

$\sim f(\alpha_s, \delta^4, \delta^6, \delta^8)$

$$R_{\tau,V/A}^{00} = R_{\tau,V/A}$$

Nonpert. contribution

The dominant pert. part δ^0 known to α_s^4
 Several expansion methods exist, e.g.
CIPT: Contour-Improved fixed-order Pert. Theory
FOPT: Fixed-Order Pert. Theory

- A fit thus determines $\alpha_s, \delta^4, \delta^6, \delta^8$ and can check the dominance of δ^0 term

(1) Fit Results with CIPT

Parameter	Vector (V)	Axial-Vector (A)	V + A
$\alpha_s(m_\tau^2)$	$0.346 \pm 0.007 \pm 0.008$	$0.335 \pm 0.008 \pm 0.009$	$0.341 \pm 0.005 \pm 0.006$
δ^4	$(1.0 \pm 1.6) \cdot 10^{-4}$	$(-6.3 \pm 0.1) \cdot 10^{-3}$	$(-3.1 \pm 0.1) \cdot 10^{-3}$
δ^6	$(2.8 \pm 0.2) \cdot 10^{-2}$	$(-3.7 \pm 0.2) \cdot 10^{-2}$	$(-4.6 \pm 1.5) \cdot 10^{-3}$
δ^8	$(-8.2 \pm 0.5) \cdot 10^{-3}$	$(10.9 \pm 0.5) \cdot 10^{-3}$	$(1.3 \pm 0.3) \cdot 10^{-3}$
Total NP	$(2.0 \pm 0.3) \cdot 10^{-2}$	$(-3.2 \pm 0.2) \cdot 10^{-2}$	$(-6.4 \pm 1.3) \cdot 10^{-3}$

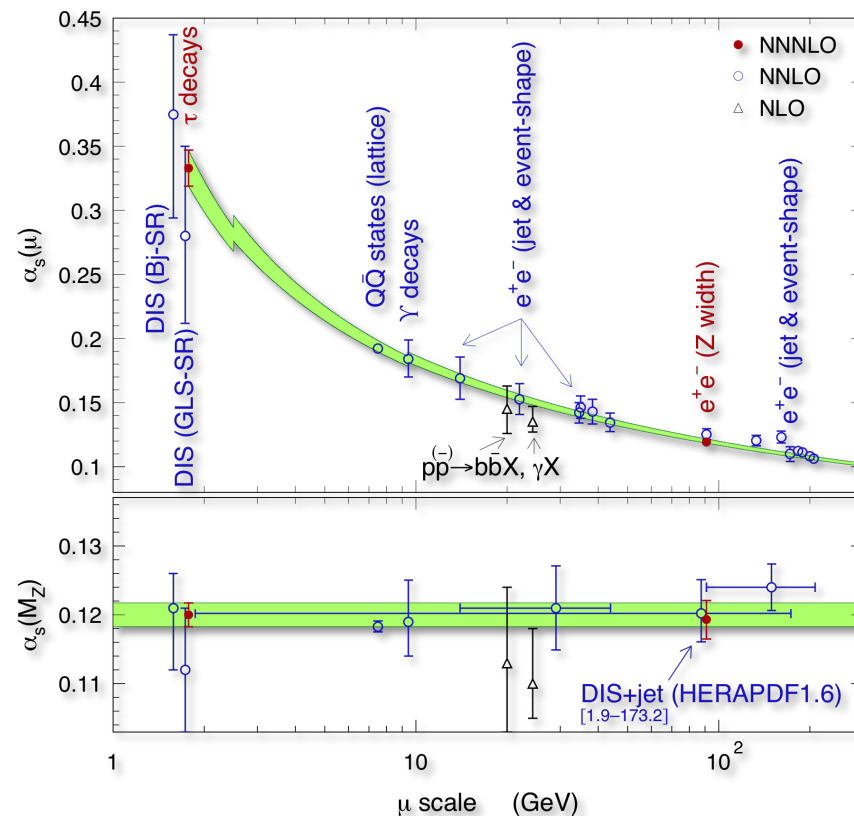
was 0.344

- Remarkable feature of approximate cancellation of δ^6 , δ^8 & total NP in V+A
- The small nonpert. contribution in V+A
 $\rightarrow \alpha_s(m_\tau^2)$ in V+A expected to be more robust than V or A.
- Similar result with FOPT, combine CIPT & FOPT

$$\alpha_s(m_\tau^2) = 0.332 \pm 0.005_{\text{exp}} \pm 0.011_{\text{theo}}$$

- Extrapolate to Z mass

$$\begin{aligned} \alpha_s(M_Z^2) &= 0.1199 \pm 0.0006_{\text{exp}} \pm 0.0012_{\text{theo}} \pm 0.0005_{\text{evol}} \\ &= 0.1199 \pm 0.0015_{\text{tot}}, \end{aligned}$$



(2) Application to a_μ

All numbers shown in 10^{-10}

$$a_\mu^{\text{SM}} \equiv \left(\frac{g-2}{2} \right)_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had,LO}} + a_\mu^{\text{had,NLO}} + a_\mu^{\text{weak}}$$

$\sigma^{\text{Exp}} = 6.3$

$\sigma_{\text{QED}}^{\text{SM}} \approx 0.02$

$\sigma_{\text{had,LO}}^{\text{SM}} \approx 4$
Dominant error

$\sigma_{\text{had,NLO}}^{\text{SM}} \approx \sigma_{\text{had,LBLS}}^{\text{SM}} \approx 3$

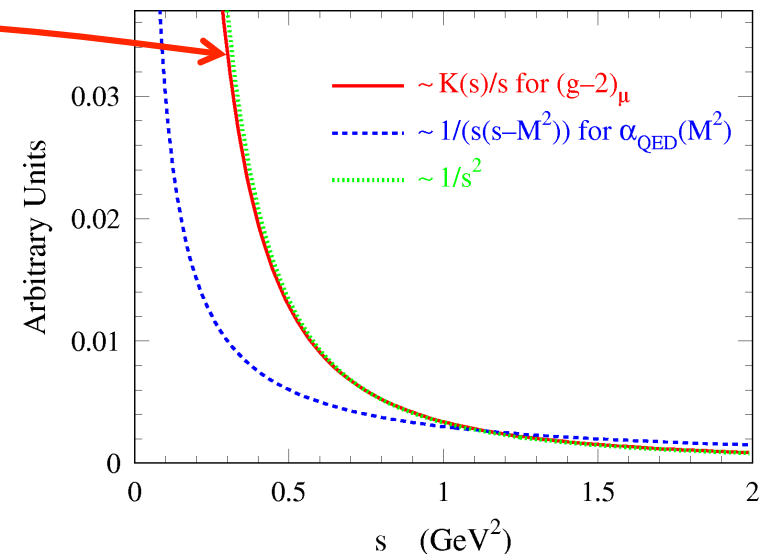
$\sigma_{\text{weak}}^{\text{SM}} \approx 0.2$

← ← ← ←

LO hadronic contribution could not predict from 1st principle but can be rigorously calculated using ee annihilation data via Dispersion Relation

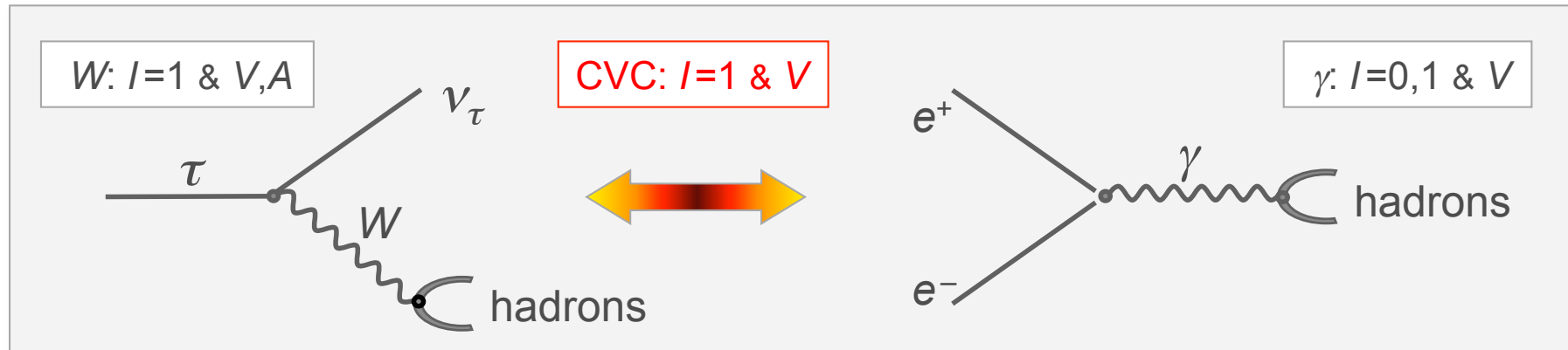
$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \left(\frac{K(s)}{s} \right) R(s)$$

$$R(s) = \frac{\sigma_0(e^+e^- \rightarrow \text{hadrons}(\gamma))}{\sigma_{pt}(e^+e^- \rightarrow \mu^+\mu^-)}$$



(2) Connection Between e^+e^- and tau

R. Alemany, M. Davier, A. Hoecker, Eur. Phys. J. C 2, 123 (1998)



Hadronic physics factorizes in **Spectral Functions** :

Isospin symmetry connects $I=1$ e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}[e^+e^- \rightarrow \pi^+\pi^-] = \frac{4\pi\alpha^2}{s} v[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]$$

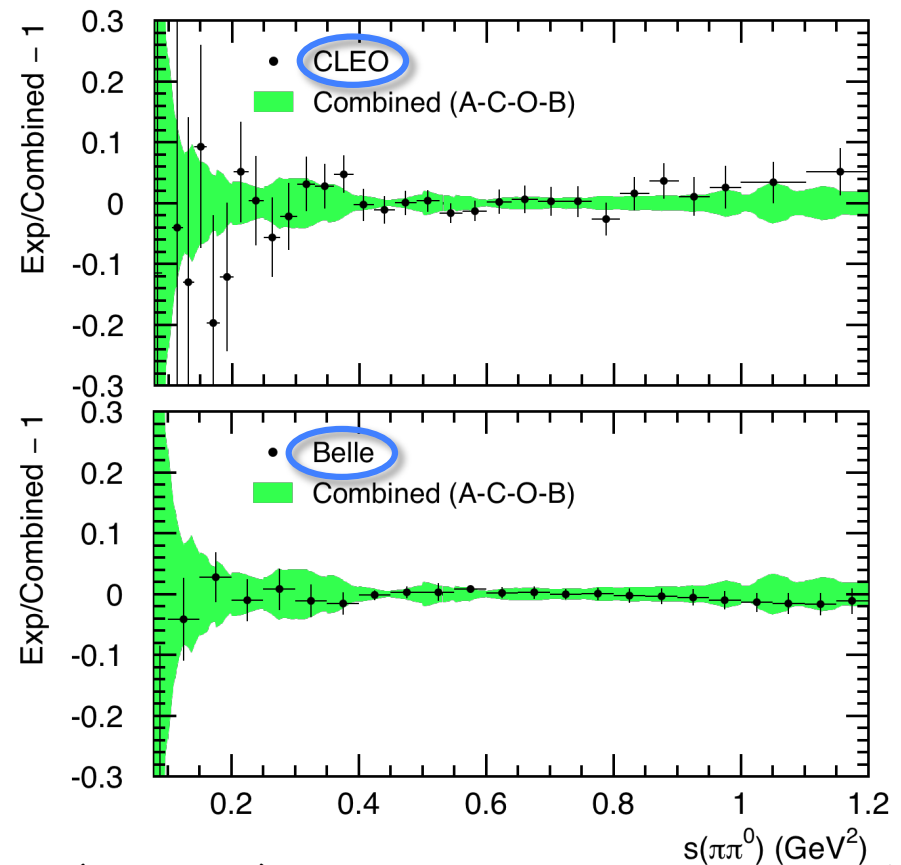
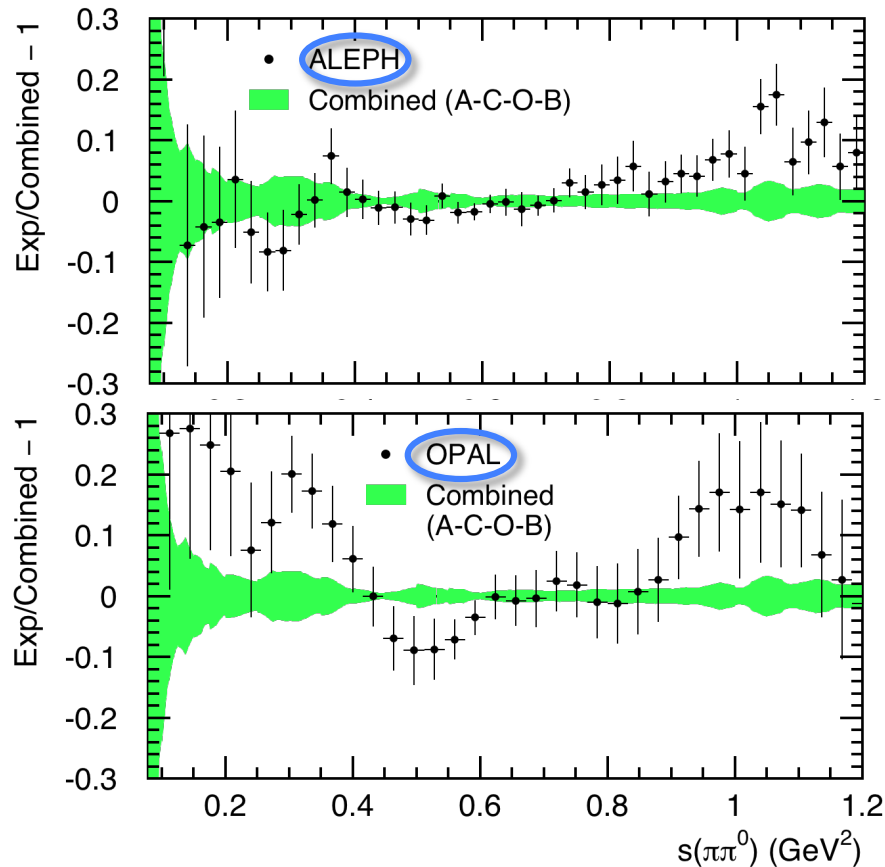
fundamental
ingredient relating
long distance
(resonances) to
short distance
description (QCD)

All isospin breaking effects were studied and taken into account in our early paper

Eur. Phys. J. C66 (2010) 127, arXiv:0106.5443

(2) The Dominant $\pi\pi^0$ Channel

Experiment	$a_\mu^{\text{had,LO}}[\pi\pi, \tau] (10^{-10})$	
	$2m_{\pi^\pm} - 0.36 \text{ GeV}$	$0.36 - 1.8 \text{ GeV}$
ALEPH	was 9.46 $9.80 \pm 0.40 \pm 0.05 \pm 0.07$	was 499.2 $501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	was 9.76 $9.82 \pm 0.13 \pm 0.04 \pm 0.07$	was 505.5 $506.4 \pm 1.9 \pm 2.2 \pm 1.9$



(2) The tau-based a_μ Results & Status

Including contributions from 4π channels

$$2\pi 2\pi^0: \quad 14.7 \pm 0.28_{\text{exp}} \pm 1.01_{\text{B}} \pm 0.40_{\text{IB}} \quad \text{was } 14.89$$

$$4\pi: \quad 7.07 \pm 0.41_{\text{exp}} \pm 0.48_{\text{B}} \pm 0.35_{\text{IB}} \quad \text{was } 6.31$$

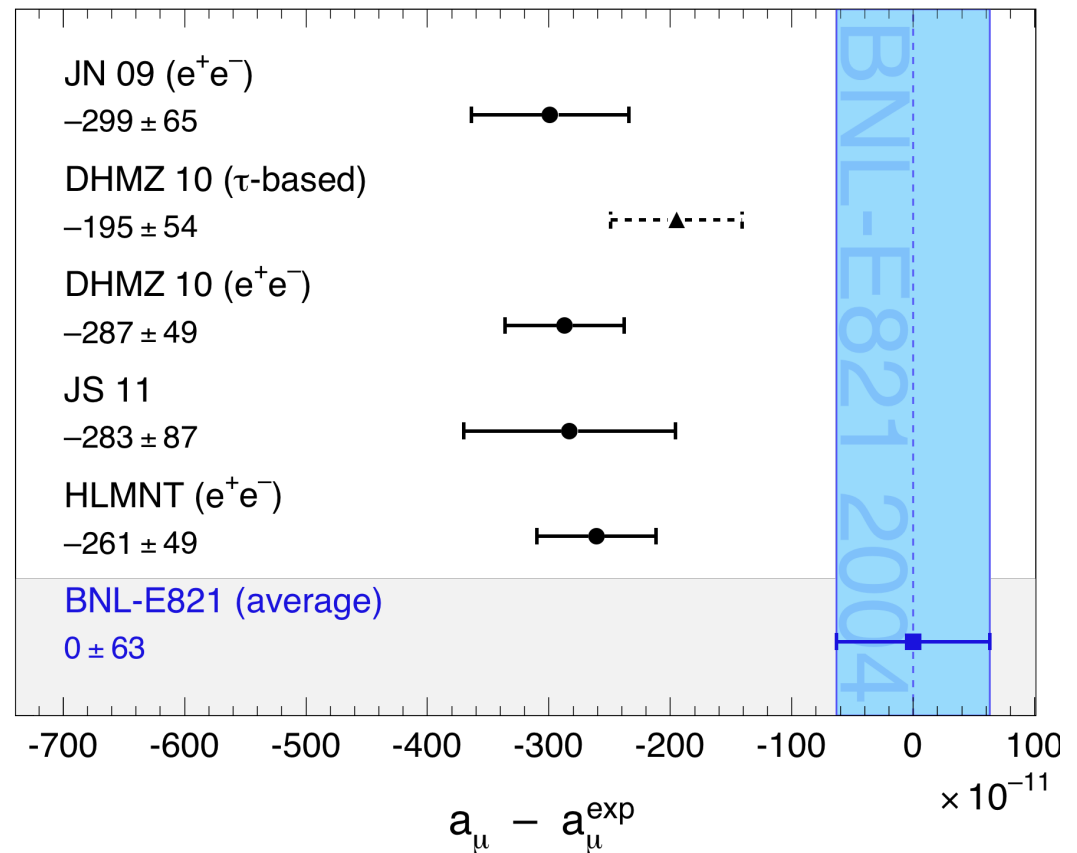
one gets total tau-based LO hadronic contributions:

$$537.9 \pm 3.1_{\text{exp+B}} \pm 2.0_{\text{IB}}$$

was 536.4

The difference between tau
and e^+e^- based predictions
changed from 1.8σ to 2.2σ

Discrepancy between tau/ e^+e^- -
based predictions and direct
measurement remains
(version of Photon'13)



(3) Comparison of Line Shape Fit to $\pi\pi^0$

Use Gounaris-Sakurai parameterization (Phys. Rev. Lett. 21 (1968) 244)

$$F_{\pi}^{I=1,0}(s) = \frac{\text{BW}_{\rho(770)}(s) \times \left(1 + \alpha \frac{s}{m_{\omega(783)}^2} \text{BW}_{\omega(783)}(s)\right) + \beta \text{BW}_{\rho(1450)}(s) + \gamma \text{BW}_{\rho(1700)}(s)}{1 + \beta + \gamma}$$

with 7 free parameters

Parameter	ALEPH 2005	This analysis
$m_{\rho^\pm(770)}$ (MeV)	775.5 ± 0.7	775.5 ± 1.1
$\Gamma_{\rho^\pm(770)}$ (MeV)	149.0 ± 1.2	151.4 ± 1.9
β	0.120 ± 0.008	0.120 ± 0.016
ϕ_β (degrees)	153 ± 7	177 ± 17
$m_{\rho^\pm(1450)}$ (MeV)	1328 ± 15	1404 ± 29
$\Gamma_{\rho(1450)}$ (MeV)	468 ± 41	474 ± 84
γ	0.023 ± 0.008	0.012 ± 0.022
$m_{\rho^\pm(1700)}$ (MeV) [fixed]	1713	1713
$\Gamma_{\rho(1700)}$ (MeV) [fixed]	235	235
χ^2/DF	119/110	50.4/69

→ Good agreement between new and old fit results observed

The difference is mainly due to the new calibration & resolution

Outline

- Update of ALEPH spectral functions
 - Its applications in precision QCD studies and a_μ calculation

- Summary of e^+e^- results from Babar
 - $e^+e^- \rightarrow \mu^+ \mu^-, \pi^+ \pi^-, K^+K^-$ with ISR method

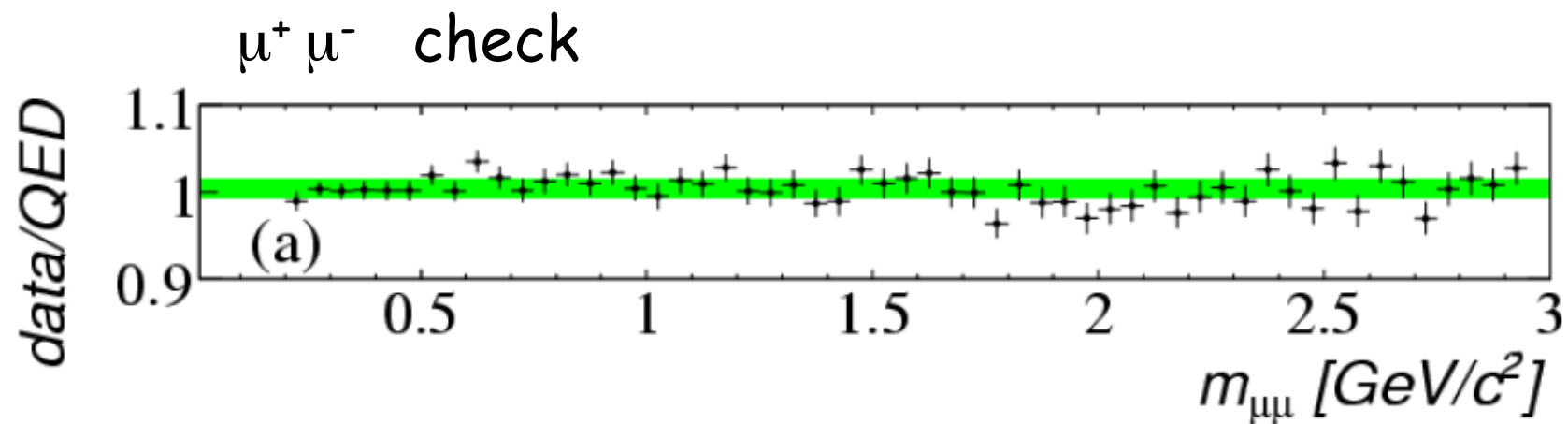
Babar: $e^+e^- \rightarrow \mu^+\mu^-$, $\pi^+\pi^-$, K^+K^- with ISR Method

M. Davier, B. Malaescu, Wang Wenfeng (LAL), Wang Liangliang (IHEP)

$e^+e^- \rightarrow \mu^+\mu^- \gamma_{\text{ISR}}$, $\pi^+\pi^- \gamma_{\text{ISR}}$, $K^+K^- \gamma_{\text{ISR}}$ measured simultaneously

Extensive program of precision measurements to improve the accuracy of hadronic vacuum polarization contribution to $g-2$ and $\alpha(M_Z)$

Papers published 2009-2013



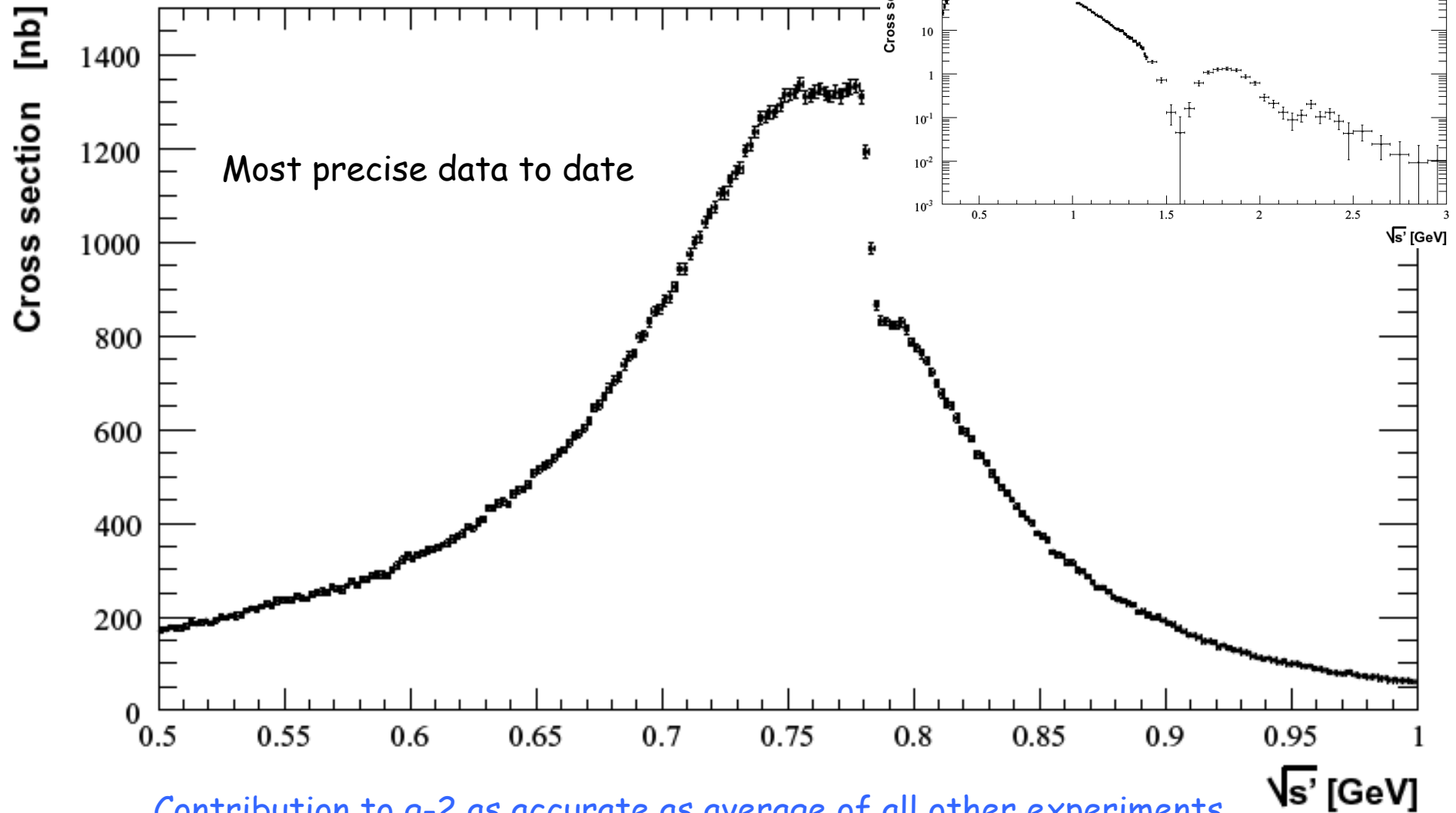
$$\frac{\sigma_{\mu\mu\gamma(\gamma)}^{\text{data}}}{\sigma_{\mu\mu\gamma(\gamma)}^{\text{NLO QED}}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4) 10^{-3}$$

PRL 2009, PRD 2012
Thesis Wang Liangliang

Babar: $e^+e^- \rightarrow \pi^+\pi^-$

PRL 2009, PRD 2012

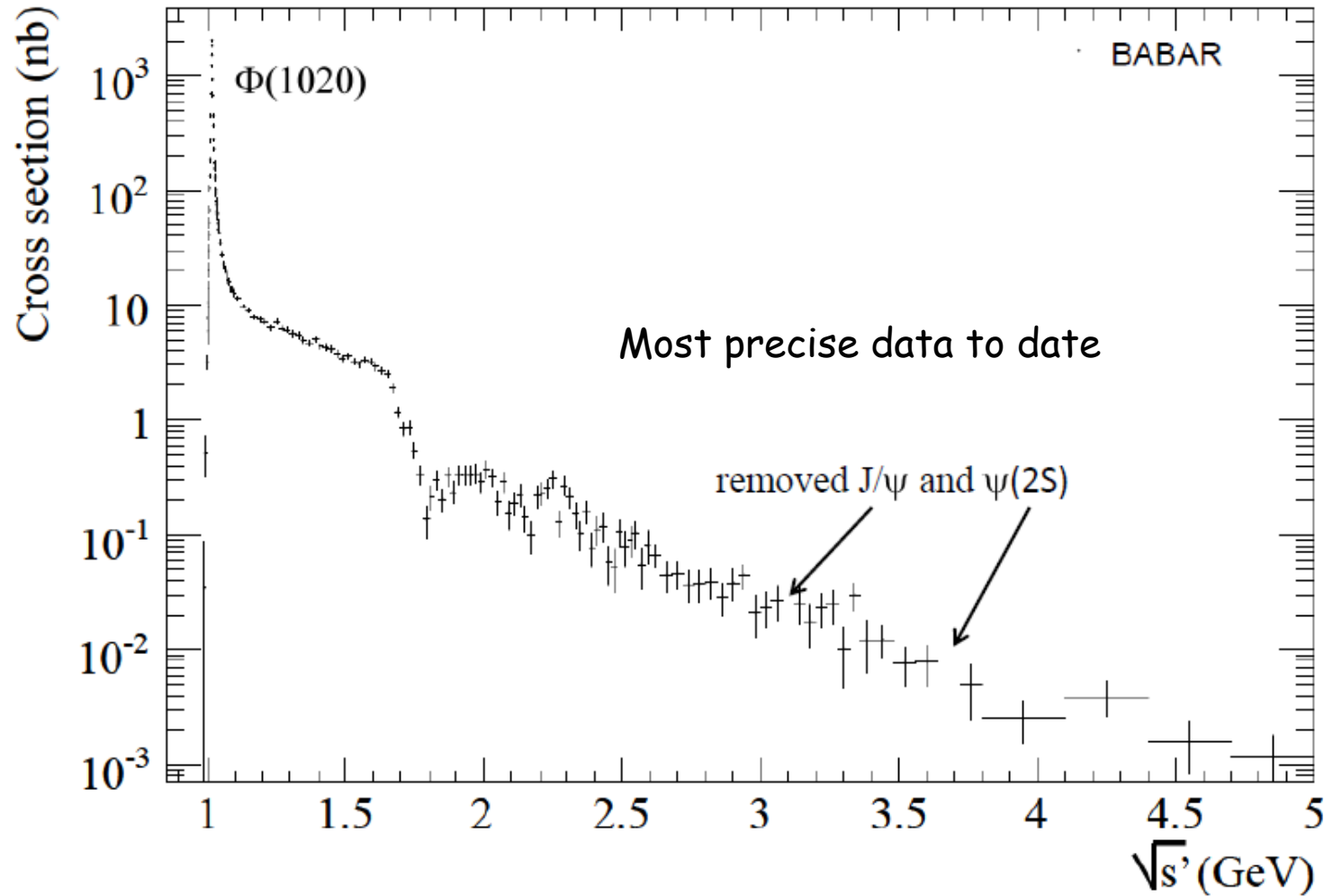
thesis Wang Liangliang + Wang Wenfeng



Contribution to $g-2$ as accurate as average of all other experiments

Babar: $e^+e^- \rightarrow K^+K^-$

PRD 2013, Thesis B. Malaescu + Wang Liangliang



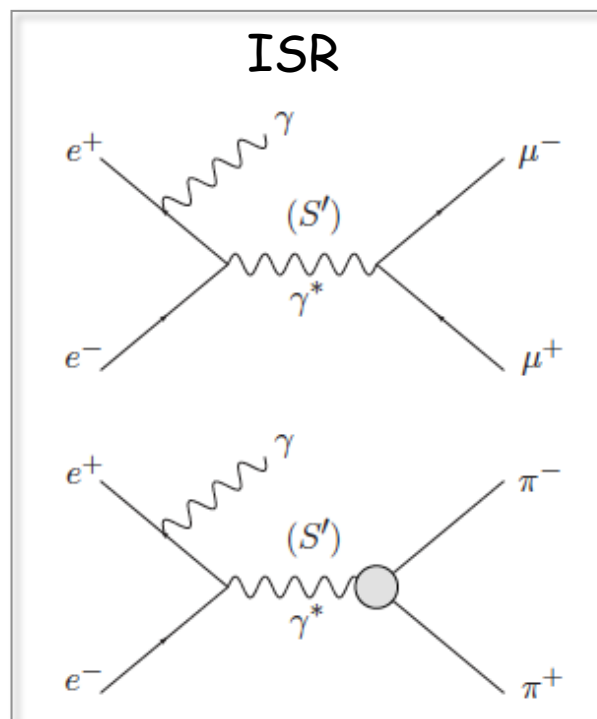
Contribution to $g-2$ more accurate than average of all other experiments

Babar: ISR-FSR Interference

M. Davier, Wang Liangliang 2014

$$x = 2E_{\gamma}^*/\sqrt{s}$$

$$s' = s(1 - x)$$



LO FSR was assumed to be negligible for $\pi\pi$
 $s \sim (10.6 \text{ GeV})^2$

How to check?

Charge conjugation $C_{XX} = -1$ for ISR, $+1$ for FSR

- ISR-FSR interference changes sign when X^+ and X^- are interchanged
- measure the **charge asymmetry**

First determination, analysis completed, under review in BABAR

Summary

- Update of ALEPH spectral functions recently published in EPJC
 - Use a new unfolding method and fixed 2 minor technical problems
 - Main results are in good agreement with the previous one
- Active collaboration on Babar data analyses/publications continues
- Perspective for a_μ is good
 - New e^+e^- measurements expected from KLOE2, VEPP-2000)
 - New recent calculation had, NNLO $(1.24 \pm 0.01) \cdot 10^{-10}$ (1403.6400)
 - Expect a factor of 4 error reduction in direct measurements from Fermilab & J-PARC
 - We will continue to be the leading actor on the subject

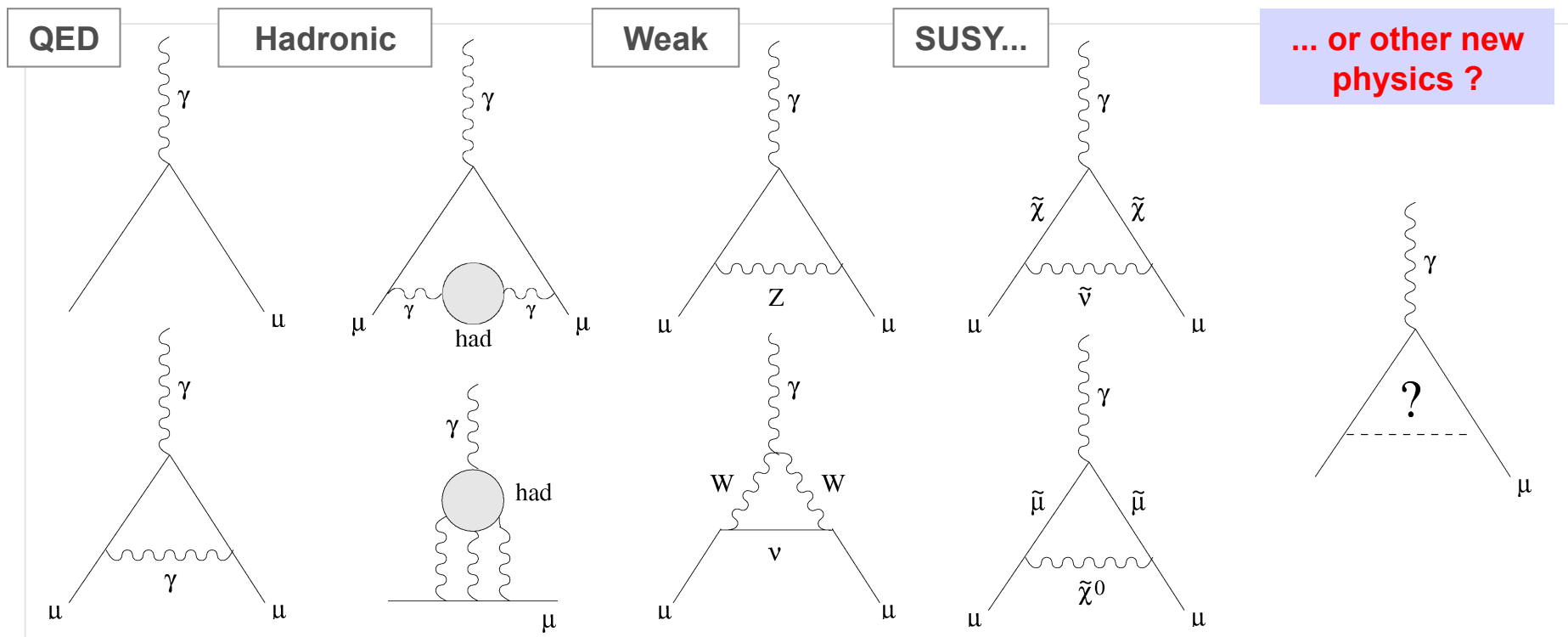
Muon Magnetic Moment Anomaly

$$\vec{\mu} = g \frac{\pm e}{2m} \vec{s} \quad g = 2 + \dots \quad \rightarrow \text{Magnetic Moment anomaly: } a_l = \frac{g - 2}{2}$$

a_e is better measured but a_μ is more sensitive to new physics effects by $(m_\mu/m_e)^2 \sim 43000$

$$a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{non-SM}},$$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{Weak}}$$



SM Predictions: $a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{Weak}}$

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$

Leading-Order Higher-Order Light-By-Light

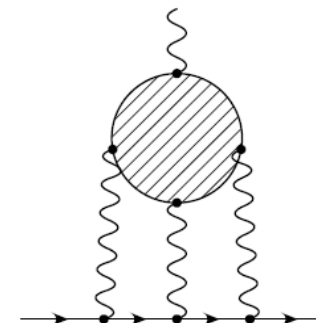
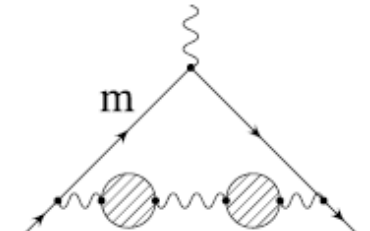
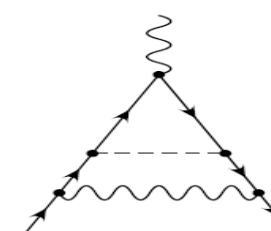
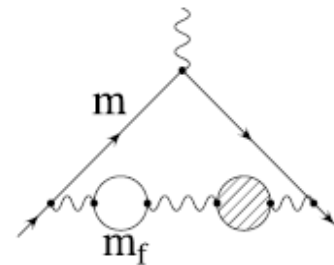
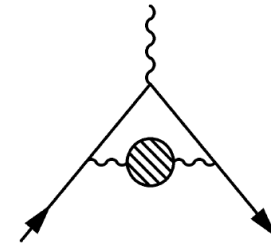
$$a_\mu^{\text{had,LO}} \sim (700 \pm 5) \times 10^{-10}$$

→ dominant uncertainty
(both e^+e^- and τ based)

$$a_\mu^{\text{had,HO}} = (-9.8 \pm 0.1) \times 10^{-10}$$

$$a_\mu^{\text{had,LBL}} \sim (10.5 \pm 2.6) \times 10^{-10}$$

→ 2nd leading uncertainty



Comparing Measurements with Predictions

Measurement (BNL-E821)

PRD73(06)072003,
hep-ex/0602035

$$11\,659\,208.9 \pm 5.4_{\text{stat}} \pm 3.3_{\text{syst}} [10^{-10}]$$

SM predictions:

QED

$$11\,658\,471.809 \pm 0.014_{\text{5th order}} \pm 0.008_{\delta\alpha} [10^{-10}]$$

Improved (Kinoshita et al.)

HAD

- LO

$$\text{DHMZ10 } e^+e^-: 692.3 \pm 4.2 \pm 0.2_{\psi} \pm 0.3_{\text{QCD}} [10^{-10}]$$

$$\text{HLMNT11 } e^+e^-: 694.9 \pm 3.7 \pm 2.1_{\text{rad}} [10^{-10}]$$

$$\text{DHMZ10 } \tau: 701.5 \pm 4.2 \pm 0.3_{\text{rad}} \pm 1.9_{\text{SU}(2)} [10^{-10}]$$

- HO

$$-9.8 \pm 0.1 [10^{-10}]$$

- LBL

$$10.5 \pm 2.6 [10^{-10}]$$

Weak

$$15.4 \pm 0.2 [10^{-10}]$$

New Unfolding Method

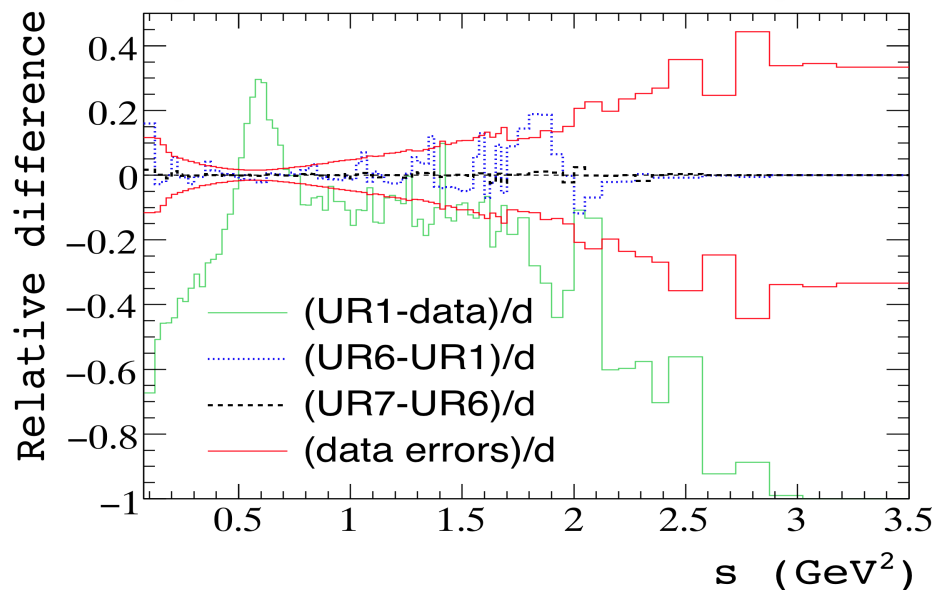
- New: Iterative, Dynamically Stabilized (IDS) method
- Old: Singular Values Decomposition (SVD) method

0907.3791, 1106.3107

Hep-ph/9509307

Both methods were developed by members of our collaboration and widely used in different analyses

Relative correction to the measured spectrum in each iteration step



A data-driven closure test

