

GDR – PH – QCD

Workshop on Radiative Corrections in Annihilation and Scattering Experiments

Simulations with MC generators for e^+e^- (X) processes

for dummies...

- Simulations with EKHARA^{*} will be presented
- Trick&tips on the implementation of EKHARA in the BES III code
- Examples from B factories: $e^+e^- \rightarrow e^+e^-X$
- PANDA: my first simulations for $p\bar{p} \rightarrow Y \rightarrow e^+e^-$
- Remarks

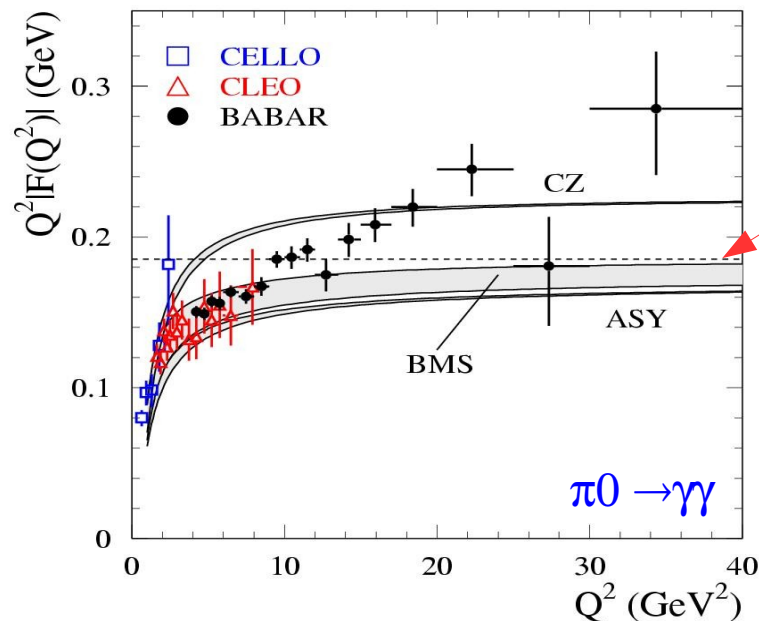
* The work with the MC generator EKHARA was performed at the J.G. University of Mainz in 2011/2012

- EKHARA is a MC generator written for specific processes $e^+e^- \rightarrow e^+e^-X$,
 $X = \pi^0, \eta, \eta', \pi^+\pi^-$
- It is published and available online: prac.us.edu.pl/~ekhara/
Phys.Rev. D85 (2012) 094010, H.Czyz, S. Ivashyn
- It makes use of the double-octet model
- Fit makes use of the results from “real” data
- Very good simulations for projects willing to study form factors, cross section, in $e^+e^- \rightarrow e^+e^-X$ via $\gamma\gamma$ interactions.

π^0 transition form factor

- The process under study here is: $e^+e^- \rightarrow e^+e^-\pi^0$ via $\gamma\gamma$ interactions

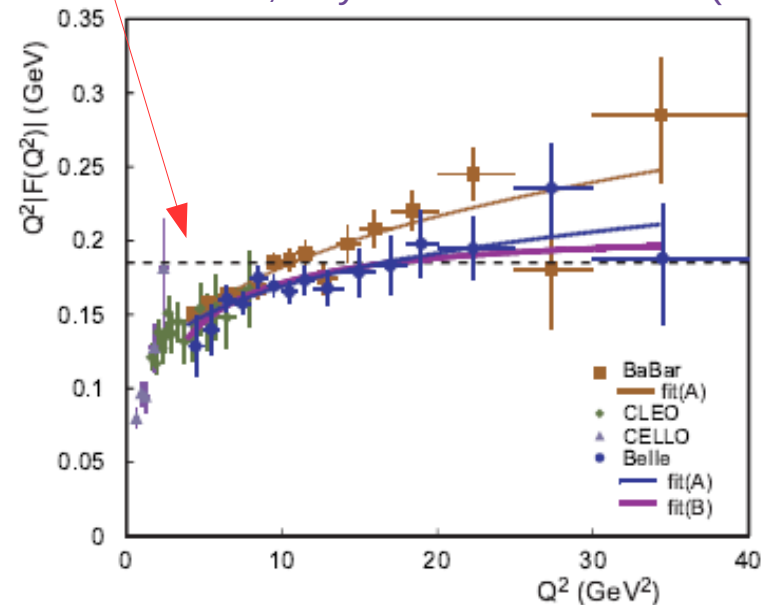
BABAR, PhysRevD.80.052002 (2009)



...measurements from different experiments are not consistent

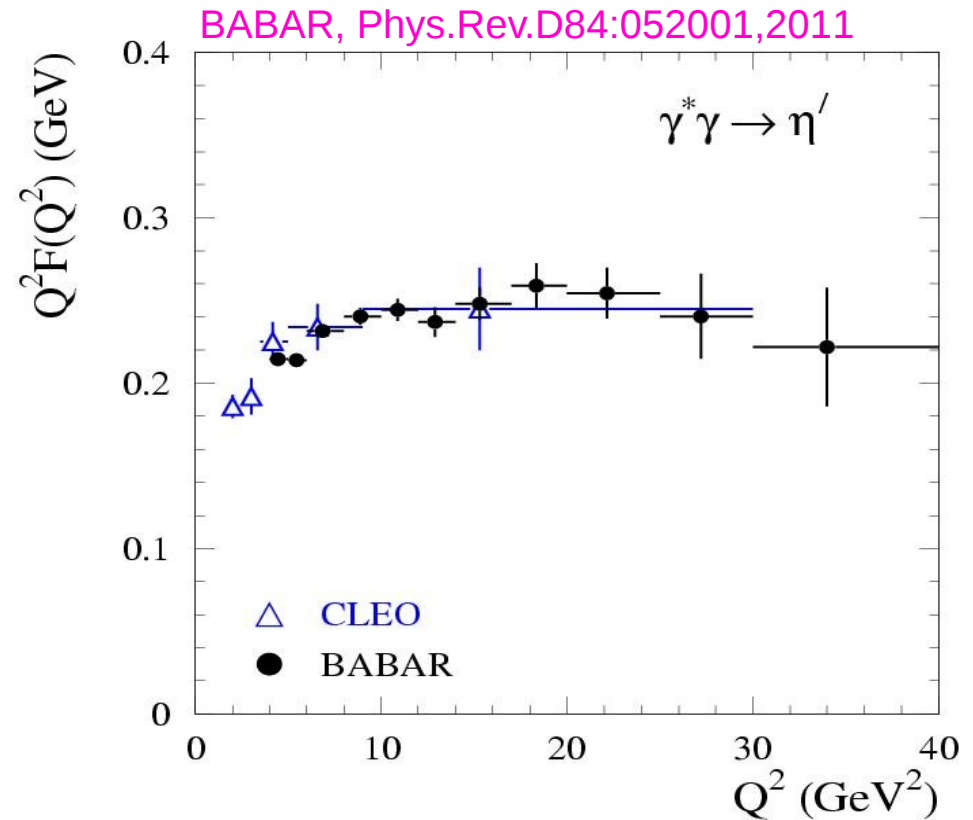
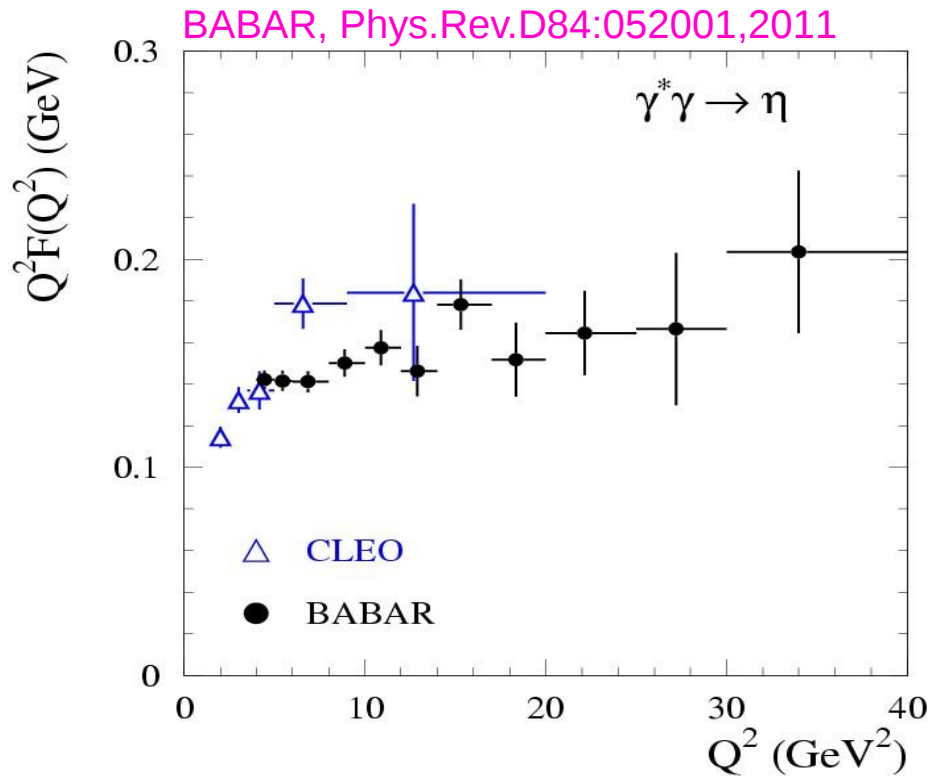
Theoretically an asymptotic limit is expected (Brodsky-Lepage-Mackenzie); but...

BELLE, Phys.Rev.D.86.092007 (2012)



η, η' transition form factor

$Q^2 \cdot F(Q^2)$ tends asymptotically to a limit



PUZZLING, compared to the case with π^0 !

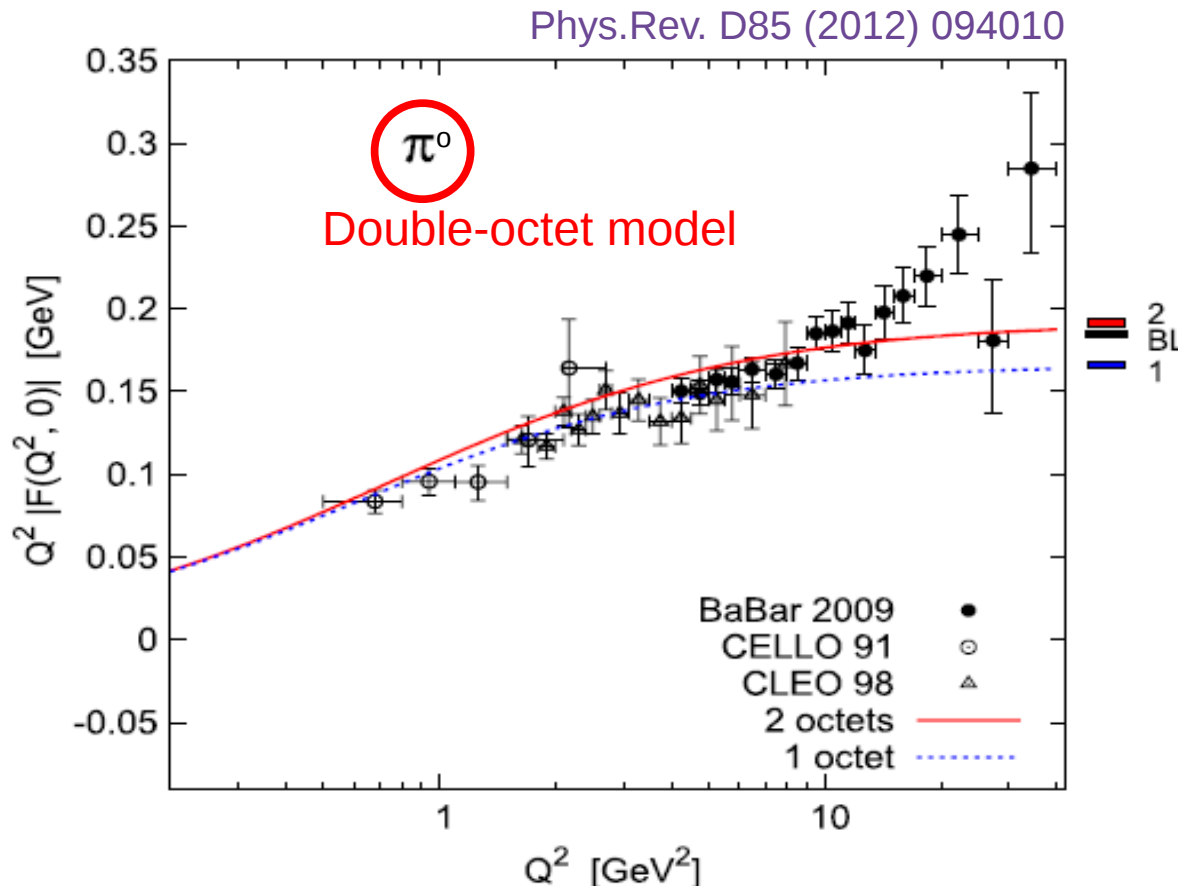


FIG. 2 (color online). Transition form factor $\gamma^* \gamma \pi^0$ compared to the data. The Brodsky-Lepage [30] high- Q^2 limit (BL) is shown as a bold solid straight line at $2 \times f_\pi = 2 \times 0.0924$ GeV. The high- Q^2 limit in our 1-octet ansatz and 2-octet ansatz are marked as (1) and (2), respectively.

- Still not a lot of information at very low Q^2 , from experiments

- Need to know the distribution of $F(Q^2) \cdot Q^2$ vs Q^2 in order to give important contribution to understanding LBL hadronic corrections to $(g-2)_\mu$

- Theoretically, the most interesting range for the LBL hadronic corrections to $(g-2)_\mu$ is:
 $Q^2 \in [0 \div 2] \text{ GeV}^2$

- Double-octet model makes prediction for $F(q_1^2, q_2^2)$ with one lepton tagged, or both leptons tagged, or none of them. Tagging one of the 2 leptons makes the calculation of $F(q_1^2, q_2^2)$ easier, as it depends only on the transfer momentum of the **tagged lepton** (e.g. $q_1=Q$, so $q_2=0$). There are no free parameters in $F(Q^2, 0)$ in this model, except only one (the coupling constant $Hv1$)

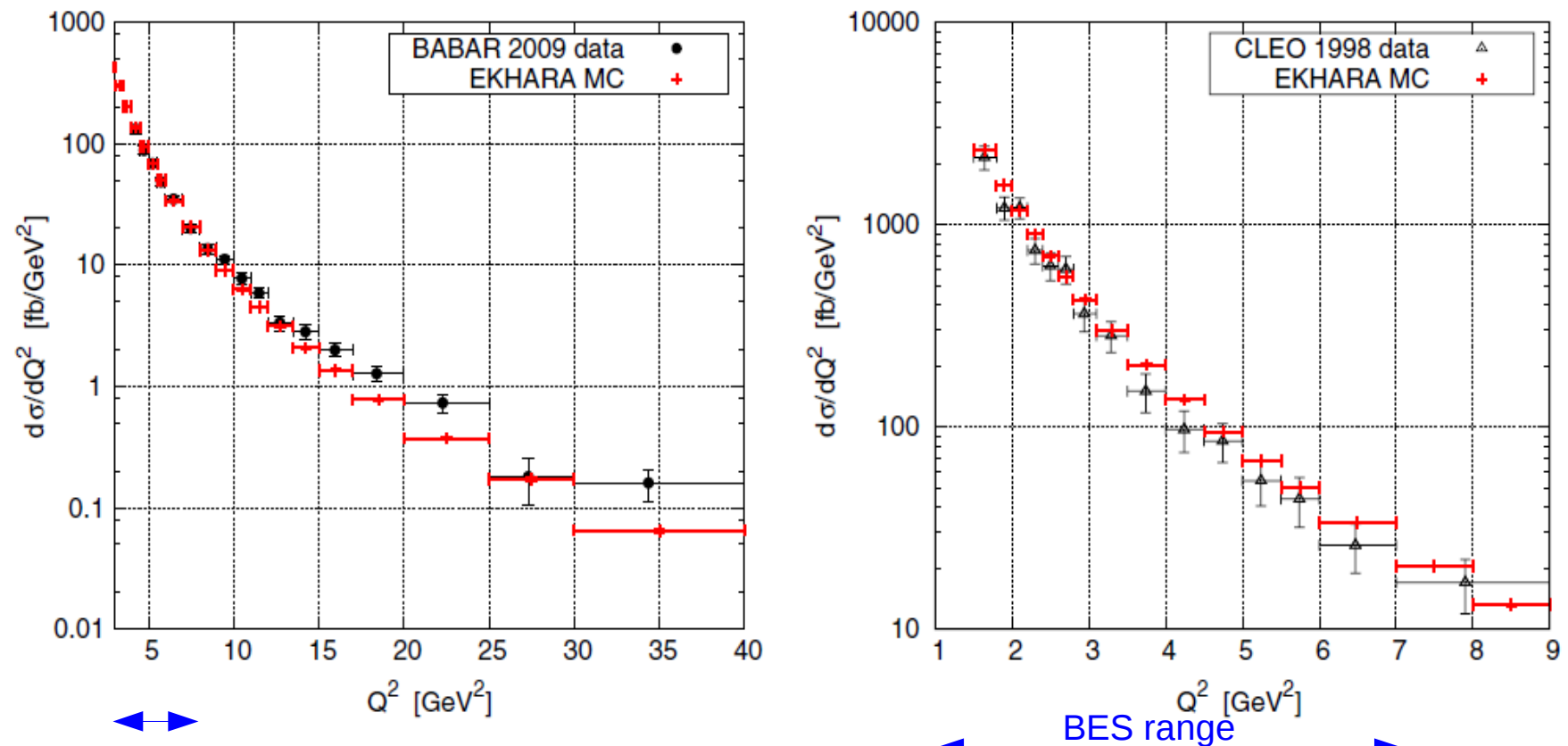
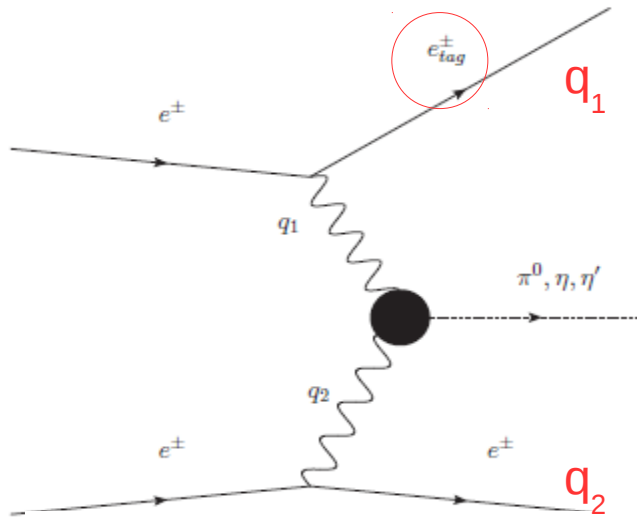


FIG. 5 (color online). The cross section $d\sigma/dQ^2$ for the process $e^+e^- \rightarrow e^+e^-\pi^0$ compared to BABAR [1] (left) and CLEO [48] (right).

- Double-octet model is used in these simulations

Some calculations as exercise:



$$\mathcal{M}[\gamma^*(q_1, \nu) \gamma^*(q_2, \beta) \rightarrow \mathcal{P}] = e^2 \epsilon_{\mu\nu\alpha\beta} q_1^\mu q_2^\alpha F_{\gamma^*\gamma^*\mathcal{P}}(t_1, t_2)$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor

$$F_{\gamma^*\gamma^*\mathcal{P}}(t_1, t_2) = F_{\gamma^*\gamma^*\mathcal{P}}(t_2, t_1) \quad \text{Bose symmetry of photons}$$

$$(q_1^2 = t_1, q_2^2 = t_2)$$

$$F_{\gamma^*\gamma^*\pi^0}(t_1, t_2) = -\frac{N_c}{12\pi^2 f_\pi} + \sum_{i=1}^n \frac{4\sqrt{2}h_{V_i}f_{V_i}}{3f_\pi} t_1 \left(D_{\rho_i}(t_1) + D_{\omega_i}(t_1) \right) \\ + \sum_{i=1}^n \frac{4\sqrt{2}h_{V_i}f_{V_i}}{3f_\pi} t_2 \left(D_{\rho_i}(t_2) + D_{\omega_i}(t_2) \right) - \sum_{i=1}^n \frac{4\sigma_{V_i}f_{V_i}^2}{3f_\pi} t_2 t_1 \left(D_{\rho_i}(t_2)D_{\omega_i}(t_1) + D_{\rho_i}(t_1)D_{\omega_i}(t_2) \right)$$

where n = number of vector meson resonance octets

f_π = pion decay constant

N_c = number of quark color

f_{vi} = coupling for vectors representation of the spin-1 fields for a fixed octet

$D_V(Q^2) = [Q^2 - M_V^2 + i\sqrt{Q^2}\Gamma_{tot,V}(Q^2)]^{-1}$ is the vector meson propagator

Phys.Rev. D85 (2012) 094010

...let's continue our calculation!

$$\lim_{t_1 \rightarrow -\infty} F_{\gamma^* \gamma^* \mathcal{P}}(t_1, t_2) \Big|_{t_2 = \text{const}} = 0$$

In our case $t_2 = 0$. It implies:

$$\sqrt{2} h_{V_i} f_{V_i} - \sigma_{V_i} f_{V_i}^2 = 0, \quad i = 1, \dots, n$$

$$-\frac{N_c}{4\pi^2} + 8\sqrt{2} \sum_{i=1}^n h_{V_i} f_{V_i} = 0 \quad \Rightarrow \quad f_{V_1} h_{V_1} = \frac{3}{32\pi^2 \sqrt{2}}$$

In the double-octet model used in these simulations, we have:

$$f_{V_1} = 0.20173(86) \quad \text{derived from:} \quad \Gamma(\rho \rightarrow ee) = \frac{e^4 M_\rho f_{V_1}^2}{12\pi}$$

Then it is possible to calculate: $h_{V_1} = 0.03121(14)$

$$h_{V_2} f_{V_2} = \frac{3}{32\pi^2 \sqrt{2}} - h_{V_1} f_{V_1} = 0.42(5) \times 10^{-3}$$

$$h_{V_3} f_{V_3} = \frac{3}{32\pi^2 \sqrt{2}} - h_{V_1} f_{V_1} - h_{V_2} f_{V_2} \approx 7.45 \times 10^{-3}$$

Phys.Rev. D85 (2012) 094010

For convenience, the authors of EKHARA define the slope of the transition form factor as:

$$a_P \equiv \frac{1}{F_{\gamma^* \gamma^* P}(0, 0)} \left. \frac{dF_{\gamma^* \gamma^* P}(t, 0)}{dx} \right|_{t=0}$$

P = pseudoscalar meson
 $t = -Q^2$
 $x \equiv t/m_P^2$

$$a_\pi = \frac{16\sqrt{2}\pi^2 m_\pi^2}{N_C} \sum_{i=1}^n h_{V_i} f_{V_i} \left(\frac{1}{M_{\rho_i}^2} + \frac{1}{M_{\omega_i}^2} \right)$$

Coefficients evaluated in 2-octet model

TABLE III. Model prediction for the slope parameters a_P and two most recent experimental values. The “2 octets” column is calculated with the parameter values given by our global fit. The first error in experimental value is due to statistics and the second one is systematics.

	1 octet	2 octets	Experiments	
a_π	0.03003(1)	0.02870(9)	0.026(24) (48)	0.025(14) (26)
a_η	0.546(9)	0.521(2)	0.576(105) (39)	0.585(18) (13)
$a_{\eta'}$	1.384(3)	1.323(4)

SINDRUM1

NA62

- In the source code **flags** are setup
- You can change those from a text file, depending on your analysis:
 1. **Tagging angle**
 2. **Energy in c.m.**
 3. **Min, max energy**
 4. **Amplitude model (5 models are available)**
 5. **Which final state ($\pi^0, \eta, \eta', \pi^+ \pi^-$)**
 6. **Matrix (t, s, t+s)**
- The source code is written in fortran, with quadrupole precision variables
In order to run it, you need to install intel-fortran, which is not free for academic purposes

A free demo is available for a maximum period of 30 days.

<http://software.intel.com/en-us/intel-compilers>

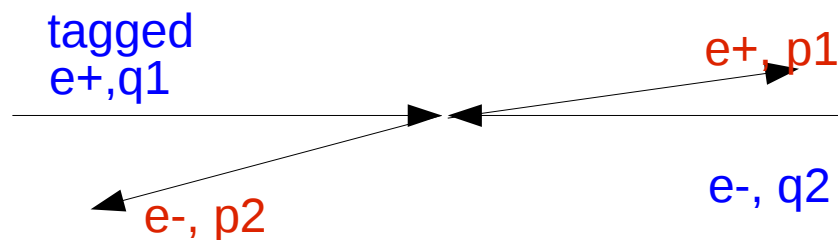
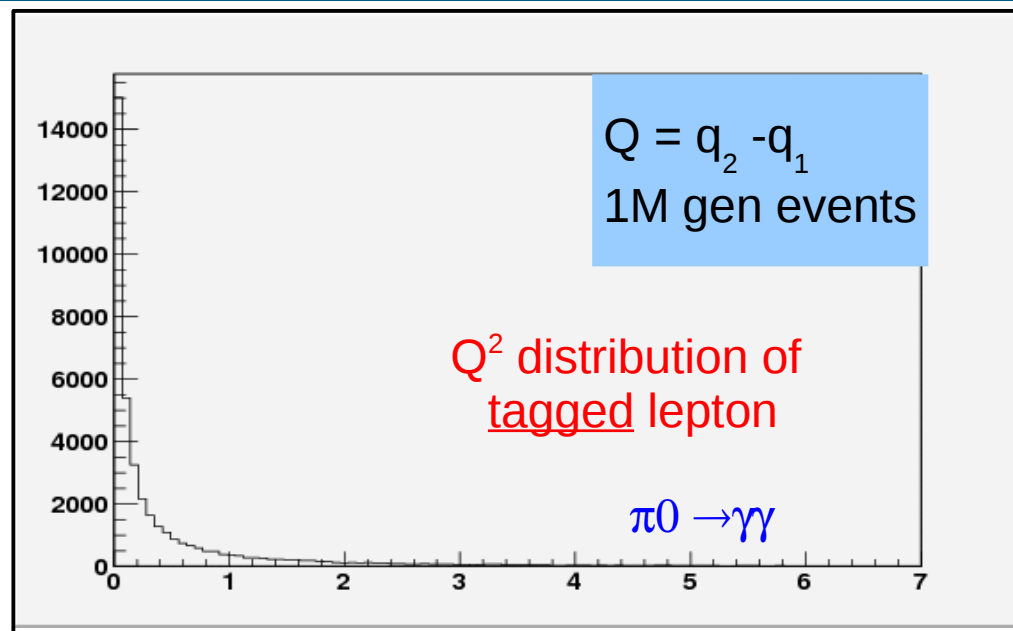
- Example of the EKHARA performance:
energy c.m. flag is set up to 3.77 GeV (Ψ'')

EKHARA simulation	$e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\pi^0$ (nb)	$e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\eta$ (nb)	$e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\eta'$ (nb)
Non tagged	$(832.2 \pm 2.9) \times 10^{-3}$	$(297.2 \pm 1.0) \times 10^{-3}$	$(212.2 \pm 1.1) \times 10^{-3}$
→ Tagged e^+ $21.6 < \theta < 158.4$	$(6.672 \pm 0.059) \times 10^{-3}$	$(5.240 \pm 0.019) \times 10^{-3}$	$(6.776 \pm 0.039) \times 10^{-3}$
Double tagging	$(2.020 \pm 0.014) \times 10^{-4}$	$(1.451 \pm 0.010) \times 10^{-4}$	$(3.613 \pm 0.025) \times 10^{-4}$

No detector acceptance
No radiative corrections are included

- Small cross section
- A drastic cross section reduction simulated when tagging one lepton
This number is even smaller if 2 leptons are both tagged:
measurement much cleaner, but high statistics needed in such a case

Some examples: Generated momentum transfer at $E_{cm} = 3.77 \text{ GeV}$

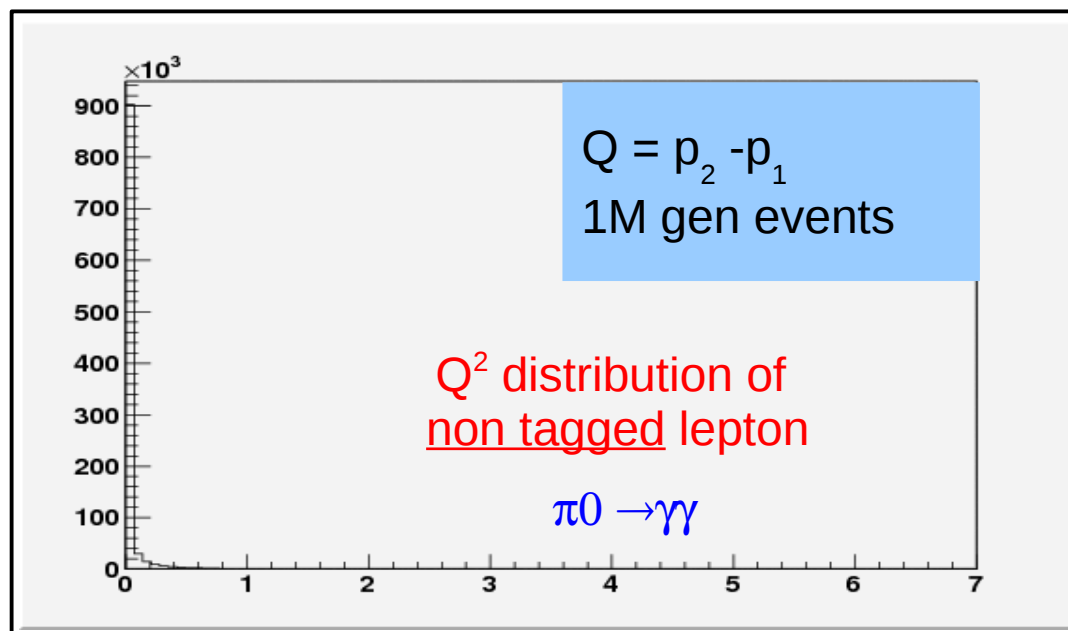


- The lepton (e^+) is tagged at large angle
- Scattering of e^- at small angle \Rightarrow identification of non-tagged lepton
- Non-tagged particle is not reconstructed: we use the missing momentum of the event

- BaBar could not check very low Q^2 values due to the trigger
- Simulations show that in BESIII it is possible
- $Q^2 \in [0 \div 2] \text{ GeV}^2$ is theoretically the best range to test hadronic LBL correction to $(g-2)_\mu$

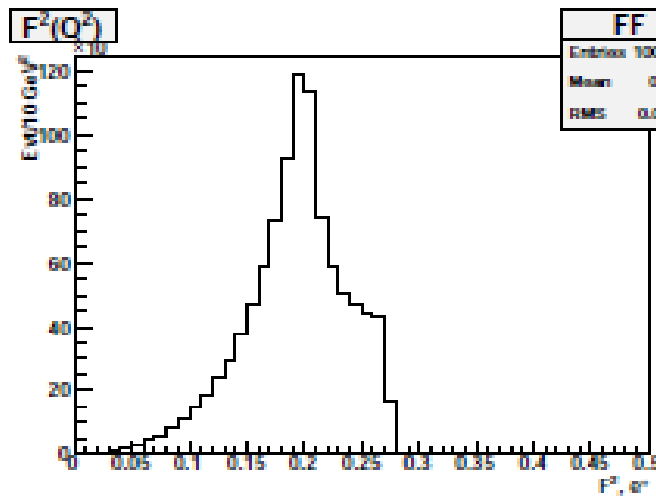


INPUT TO THE THEORY!

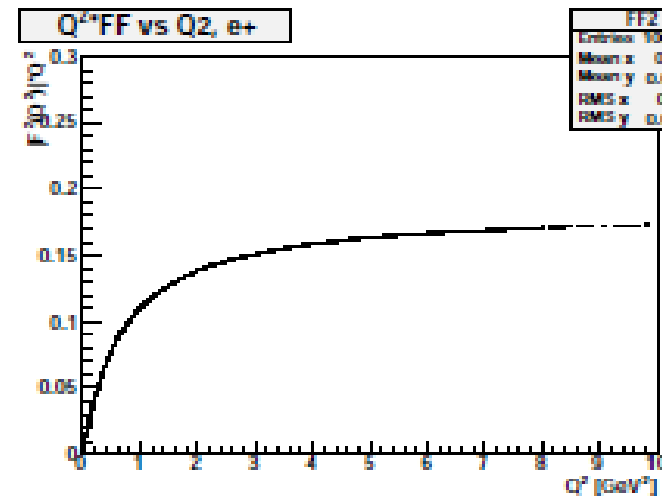


Some examples: FF generated at $E=3.77$ GeV

Tagged
lepton

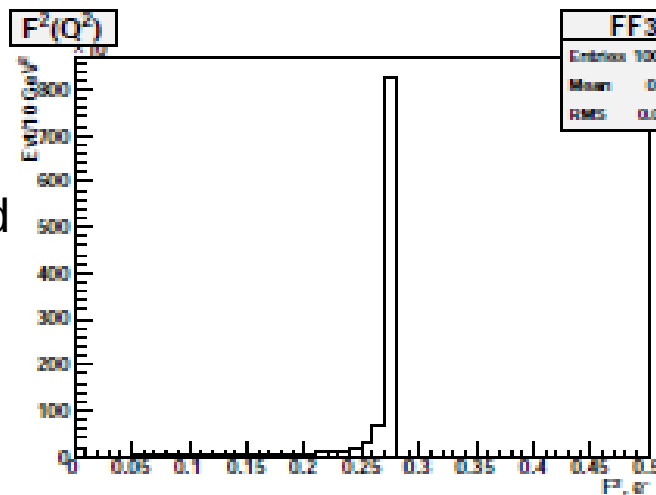


FF
Entries: 1000000
Mean: 0.1922
RMS: 0.04472

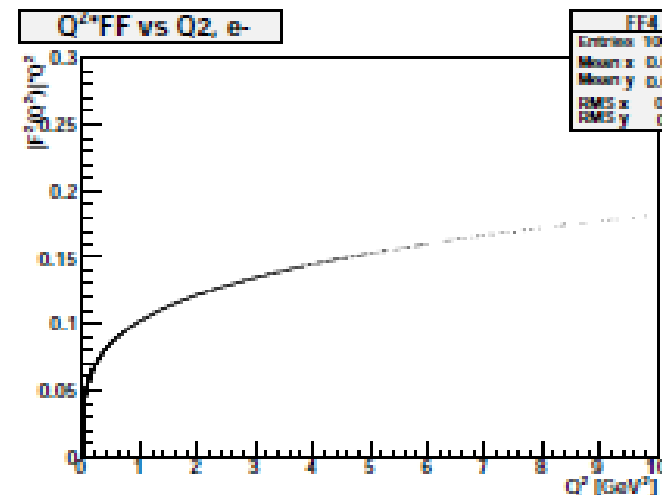


FF2
Entries: 1000000
Mean x: 0.3671
Mean y: 0.05025
RMS x: 0.4092
RMS y: 0.00006

Non-tagged
lepton



FF3
Entries: 1000000
Mean: 0.2694
RMS: 0.02217



FF4
Entries: 1000000
Mean x: 0.00026
Mean y: 0.01649
RMS x: 0.1456
RMS y: 0.0002

No detector acceptance
No radiative corrections are included

Single-tag mode

Test to verify the EKHARA interface

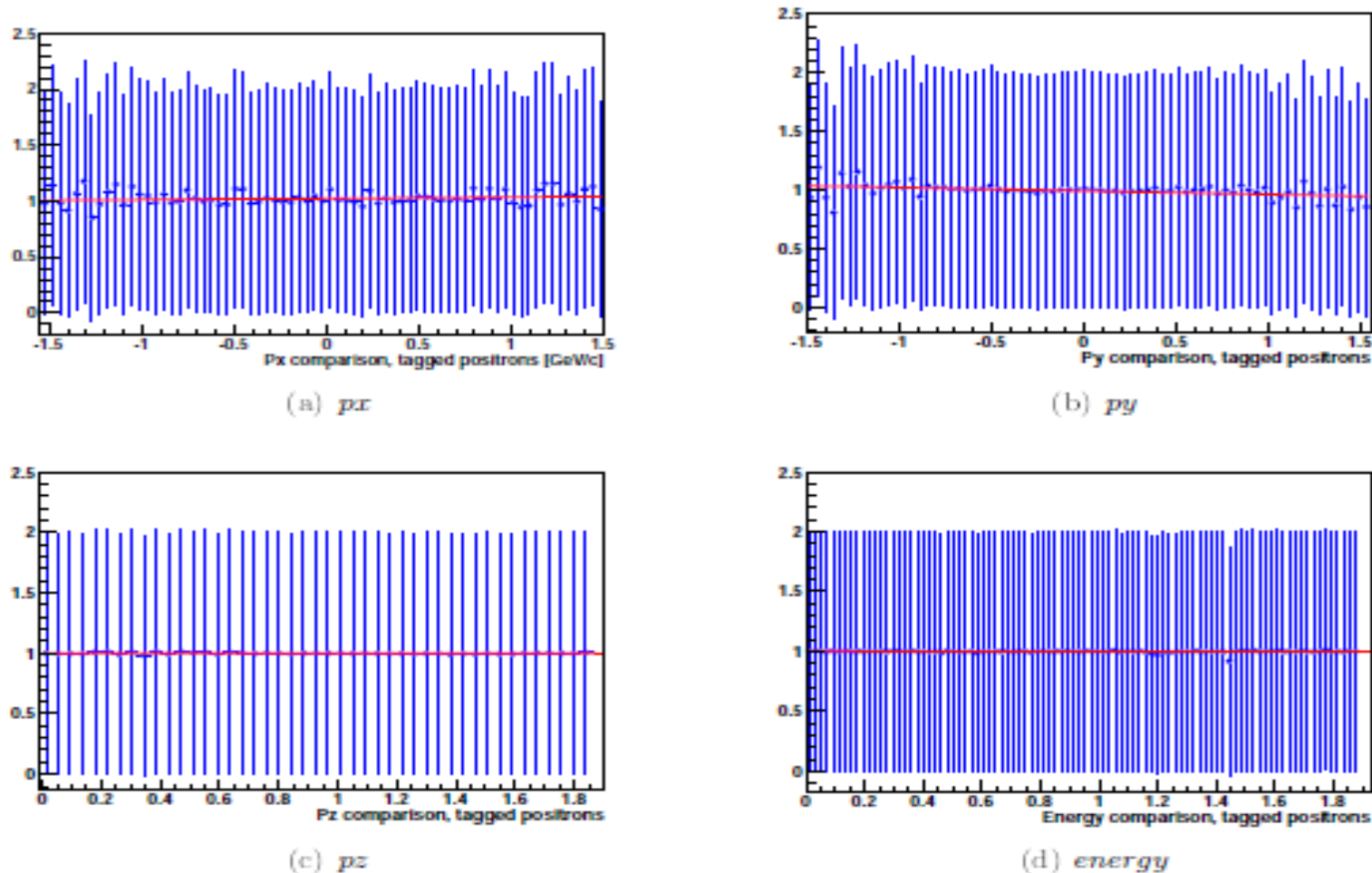
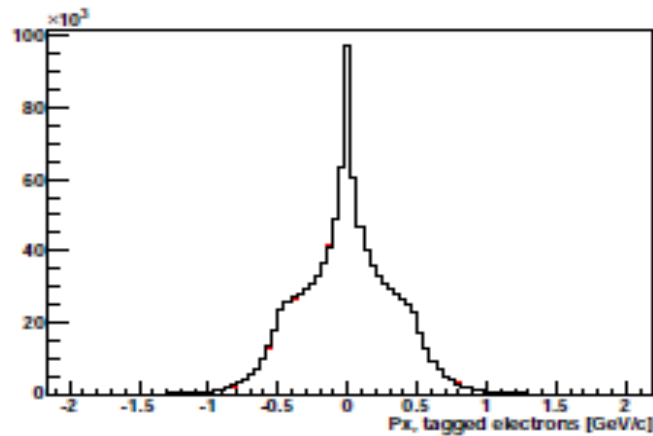


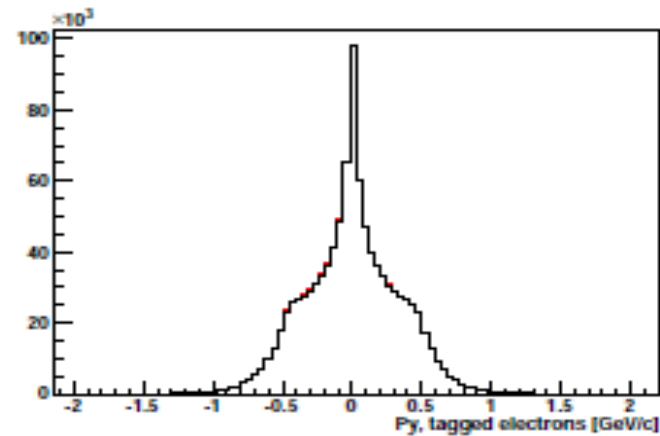
Figure 4: Studies performed with the MC generator EKHARA for the channel $e^+e^- \rightarrow e^+e^-\pi^0$, with tagged electrons: the momentum and energy of the tagged lepton is compared

- EKHARA fortran / EKHARA in BES, true values: distributions must be aligned o 1

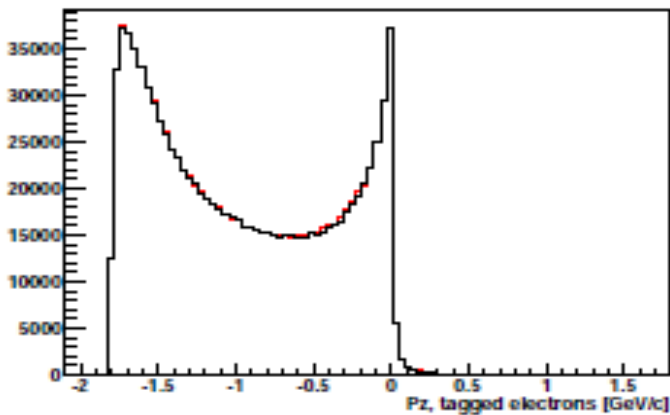
Some example: generated 4-momomentum distributions



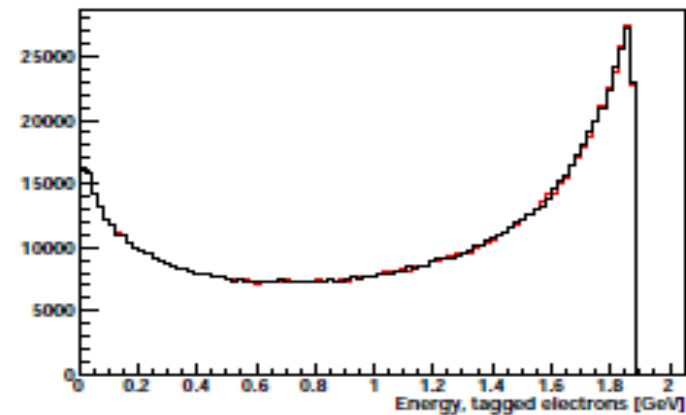
(a) p_x



(b) p_y



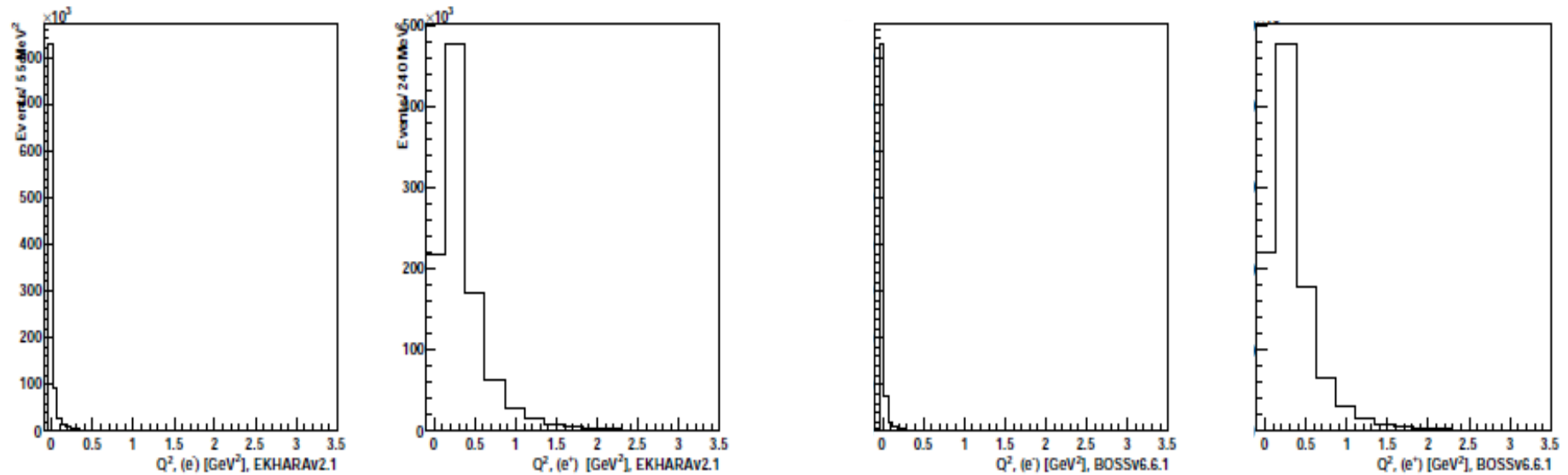
(c) p_z



(d) $energy$

Figure 8: Studies performed with the MC generator EKHARA for the channel $e^+e^- \rightarrow e^+e^-\pi^0$, with tagged electrons: the momentum and energy of the tagged lepton is shown

No detector acceptance
No radiative corrections are included
 Single-tag mode



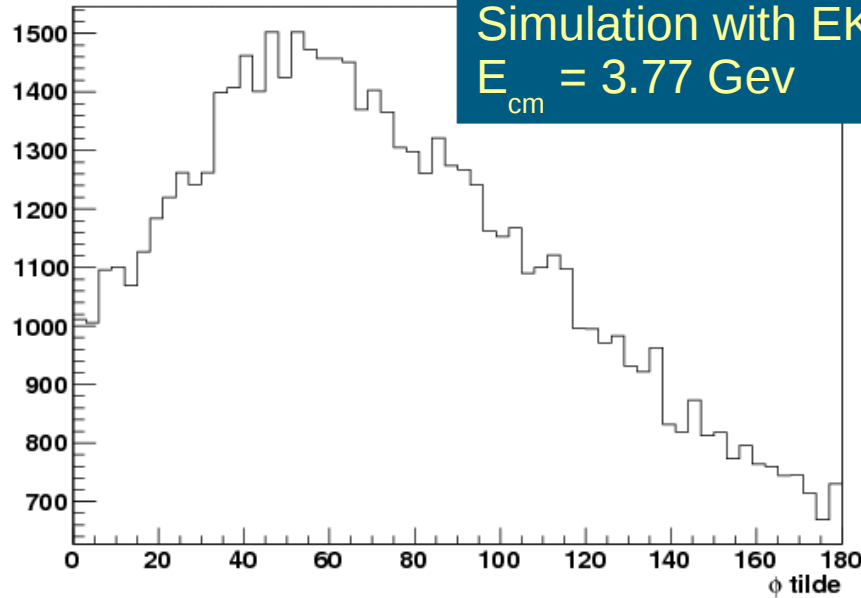
(a) Q^2 distribution

(b) Q^2 distribution

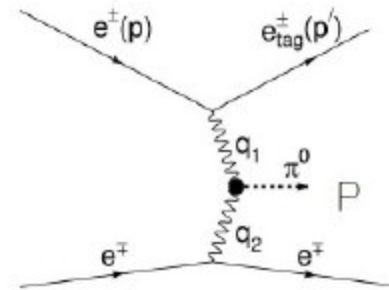
Figure 7: Distribution of the momentum transfer (Q^2) performed with the MC generator EKHARA for the channel $e^+e^- \rightarrow e^+e^-\pi^0$, with tagged positrons, in both cases: a) EKHARA v2.1 standalone code; b) EKHARA v2.1 in BOSSv. 6.6.1, respectively for the non-tagged lepton (left) and tagged lepton (right).

Some example: azimuthal angular correlation

anglephi



No detector acceptance
Simulation with EKHARA
 $E_{\text{cm}} = 3.77 \text{ GeV}$



$$d\sigma = F \{ v_{TT} \sigma_{TT} + v'_{TT} \cos(2\tilde{\phi}) (\sigma_{||} - \sigma_{\perp}) + h_1 h_2 v''_{TT} \frac{1}{2} (\sigma_0 - \sigma_2) + v_{LL} \sigma_{LL} + v_{TL} \sigma_{TL} + v_{LT} \sigma_{LT} + v'_{TL} \cos(\tilde{\phi}) \tau_{TL} + h_1 h_2 v''_{TL} \cos(\tilde{\phi}) \tau_{TL}^a \}$$

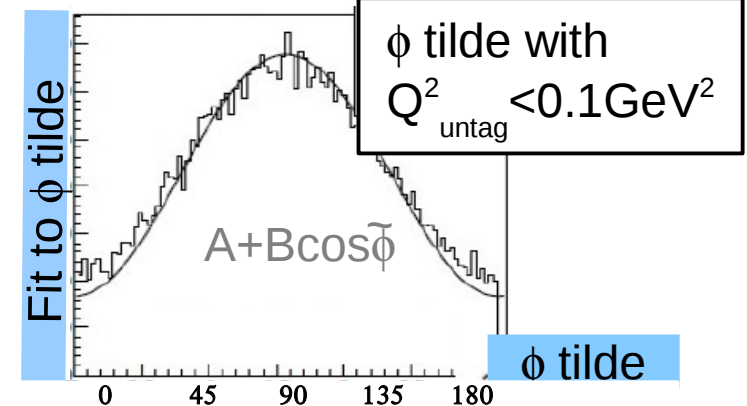
For pseudoscalar mesons, only $\sigma_{\perp} = \sigma_0 = 2\sigma_{TT}$ are different from 0

$$(\cos\phi)_{\text{c.m.}ee} \equiv - \frac{p'_{1\perp} \cdot p'_{2\perp}}{[(p'_{1\perp})^2 (p'_{2\perp})^2]^{1/2}}$$

lepton
frame

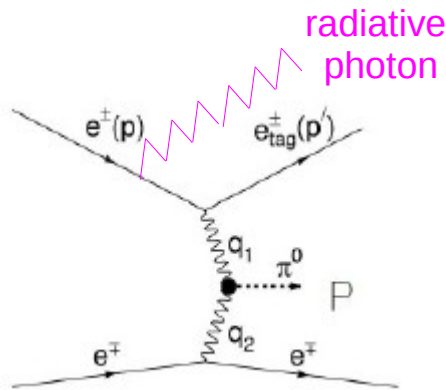
$$\cos\tilde{\phi} \equiv - \frac{\tilde{p}_{1\perp} \cdot \tilde{p}_{2\perp}}{[(\tilde{p}_{1\perp})^2 (\tilde{p}_{2\perp})^2]^{1/2}}$$

$\gamma\gamma$ frame



This analysis can be performed only if the momentum resolution is very good!

- Radiative corrections are not implemented in EKHARA
problems with numerical integration occur
not easy to solve
- A MC generator where the rad. corr. are implemented is RABHAT (old fortran code)
used from the Belle Collaboration for the analysis $e^+e^- \rightarrow e^+e^-\pi^0$
- The main background of this analysis is the VCS: $e^+e^- \rightarrow e^+e^-\gamma$
- A random photon can combine with one photon in $e^+e^-\gamma$ and peak close to the π^0 mass. Or a hard photon can be emitted from the electron and leads difference in Q^2 .
- To reduce these sources of backgrounds it is possible to build *ad-hoc* variables
(lesson learned by BaBar, PRD 80, 052002 (2009)): *the variables r and $\cos\theta_{EP}$*



$$r_\gamma = 2E_\gamma^*/\sqrt{s}$$

$$Q_{\text{meas}}^2 = Q_{\text{true}}^2(1+r_\gamma)$$

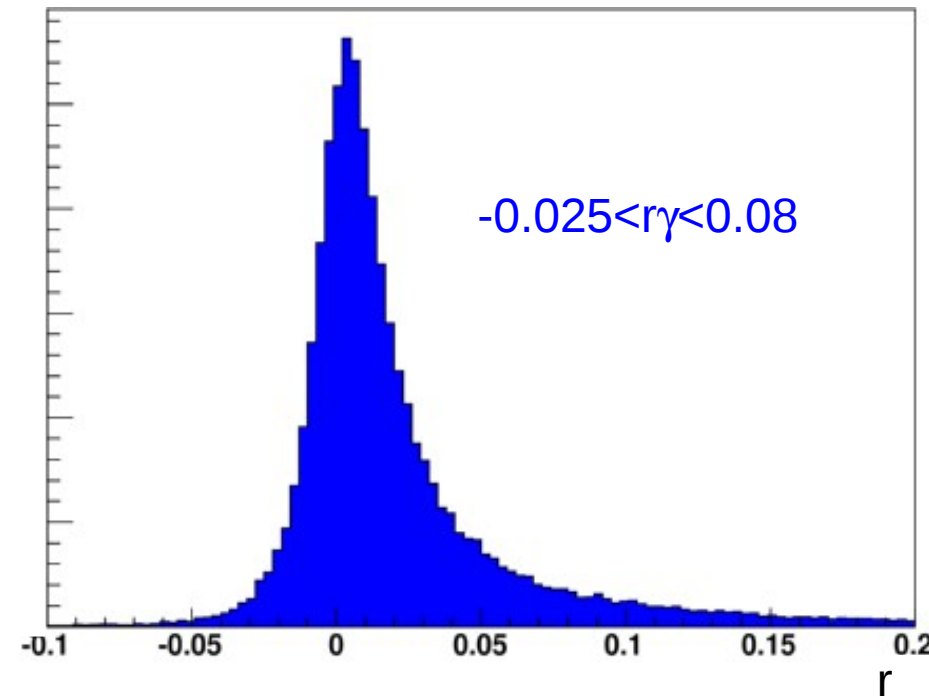
$$r = \frac{\sqrt{s} - E_{e\pi}^* - p_{e\pi}^*}{\sqrt{s}}$$

\sqrt{s} = c.m. energy

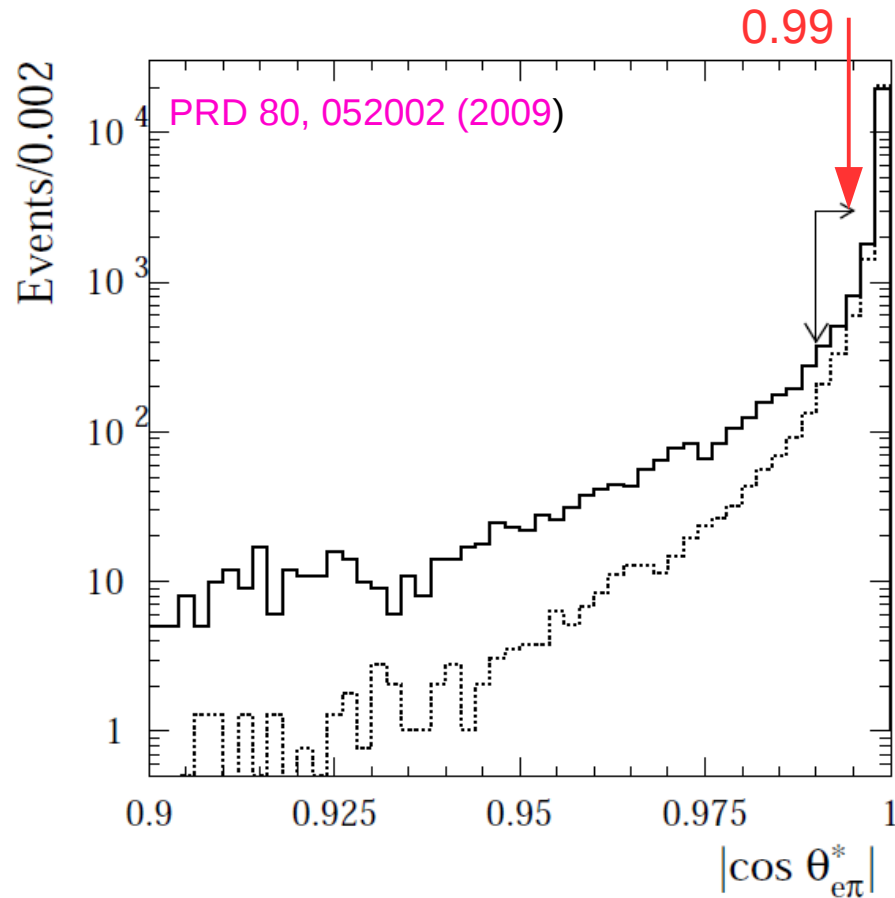
$E_{e\pi}^*$ = c.m. energy in [eP] system

$p_{e\pi}^*$ = magnitude of momentum in [eP] system

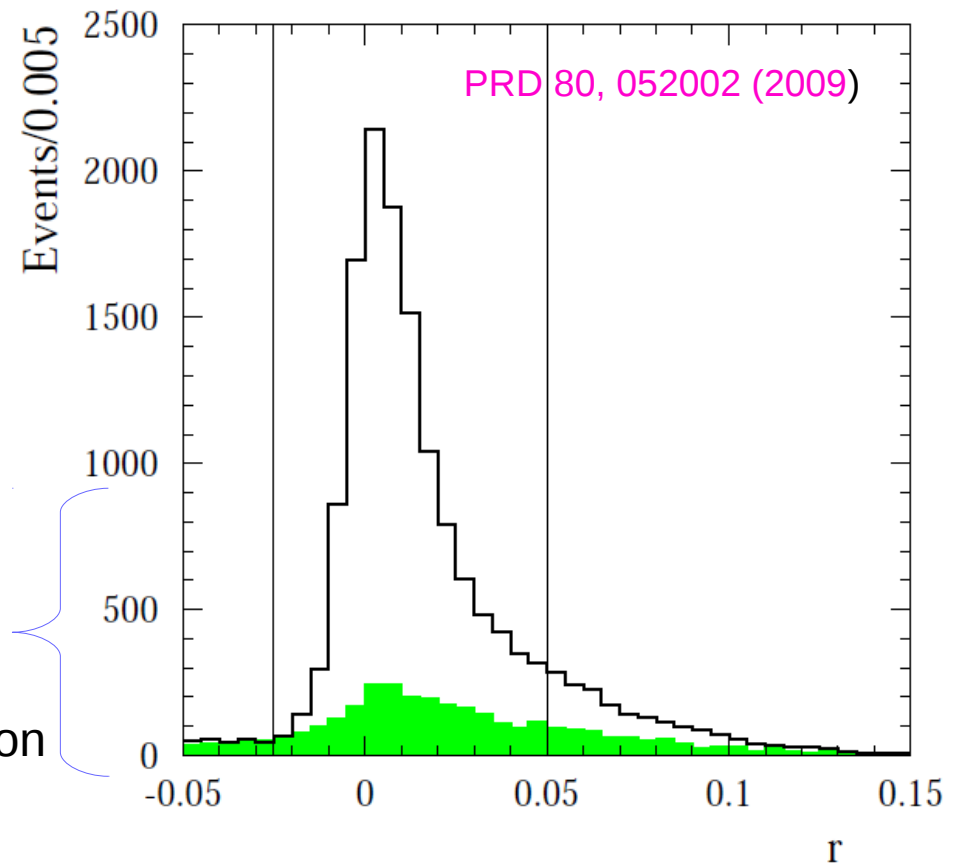
[eP] = system of the reco. particles ($e_{\text{tag}} + P$)



Examples from BaBar



- Scattering at small angle \Rightarrow identification of the non-tagged particle using the missing momentum of e_{TAG} and π^0
- Combinatorial + VCS background rejected after this angular selection cut



- Tail on data is due to the radiative photon
- On MC simulations it looks pretty symmetric
- The “ r ” selection cut rejects ISR contamination

Examples from Belle

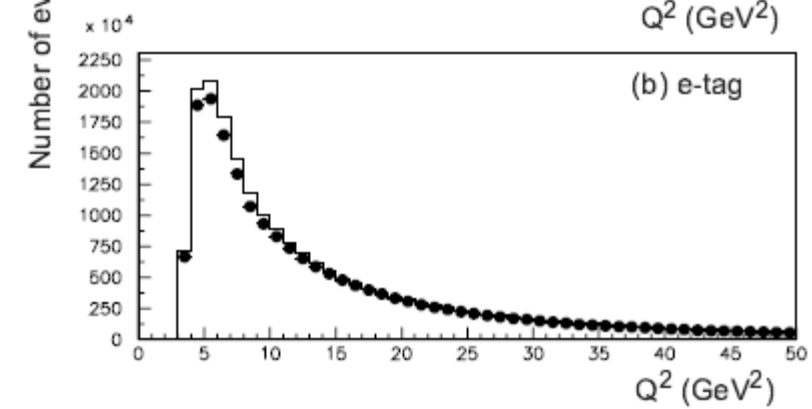
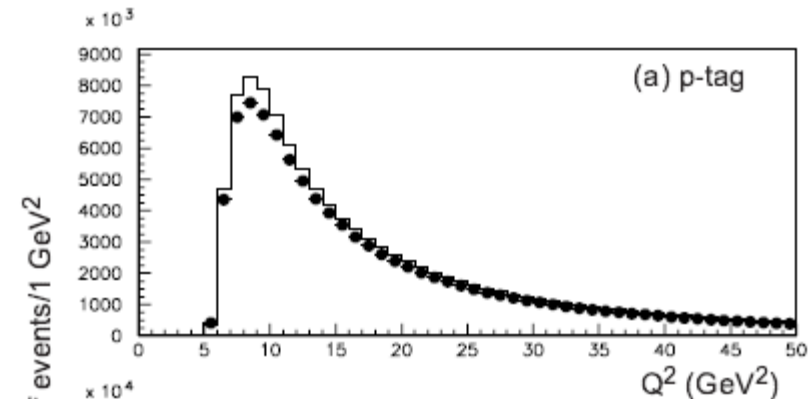
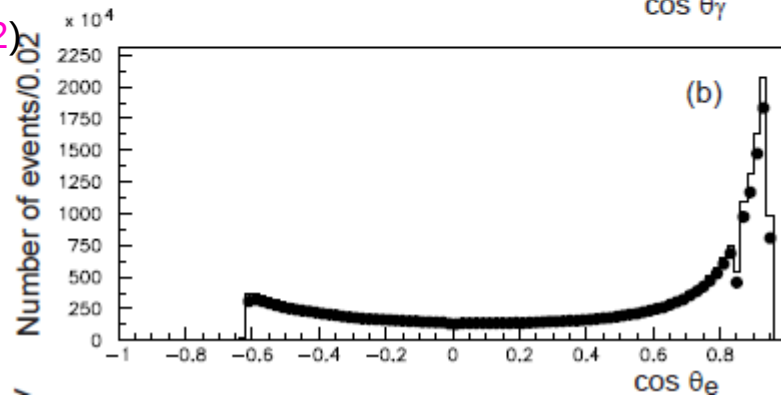
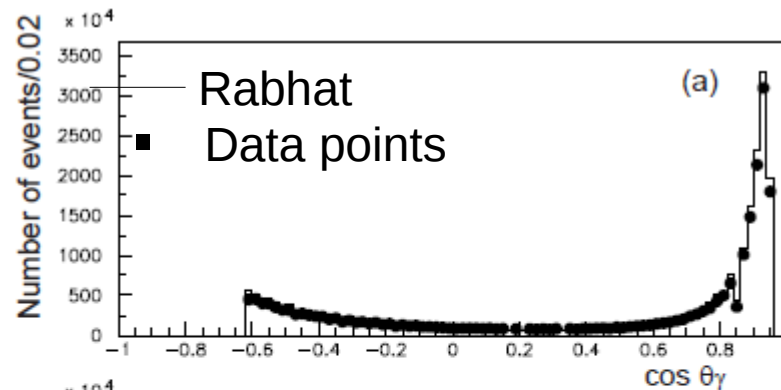
- The Belle Collaboration chose the MC generator Rabhat to simulate events with a virtual Compton configuration

K. Tobimatsu and Y. Shimizu, Comp. Phys. Comm. **55**, 337 (1989).

- Rabhat simulates the radiative Bhabha process with *t-channel* mass singularity
- The singularity occurs in a topology where the virtual photon, which is emitted in an extremely forward angle from one of the incident electron, is hard scattered off the other electron
- **Rabhat** adopts a particular solution for integration variables and numerical integration method

Plots from the
Belle paper:

PRD 86, 092007 (2012)



More on radiative corrections: MC generator *BabaYaga@NLO*

- Approach from Pavia: MC generator Babayaga
- It can generate events for QED processes, mainly for e^+e^- colliders in the Q^2 range 1-10 GeV^2
- It is a general MC generator for flavor factories for the processes: e^+e^- , $\mu^+\mu^-$, $\gamma\gamma$, $\pi^+\pi^-$.
- It is accommodated for LO, NLO and higher orders, which can have important impact on the cross section measurement
- For radiative corrections: QED Parton Shower model

<http://www2.pv.infn.it/~hepcomplex/babayaga.html>

DOCUMENTATION

BabaYaga@NLO [Eur.Phys.J.C71:1680,2011](#)

- This version is a complete rewriting of the generator, which now could also simulate the production of a single dark matter massive photon decaying in e^+e^- , $\mu^+\mu^-$ pairs.
- The code includes also non-log order alpha corrections, which were the main source of theoretical error in the previous releases of the code.
- The theoretical error of the new release is estimated to be around 0.1%, for Standard Model processes.

For any information, please contact the authors (names/e-mails are on the web site)

- Anti-proton against a fix target: process of interest $p\bar{p} \rightarrow e^+e^-$
- Clearly, the calculations are different compared to e^+e^- colliders
- My attention has been to the decay of $Y(4160)$ and $Y(4260)$ to e^+e^-

$$p\bar{p} \rightarrow Y \rightarrow e^+e^-$$

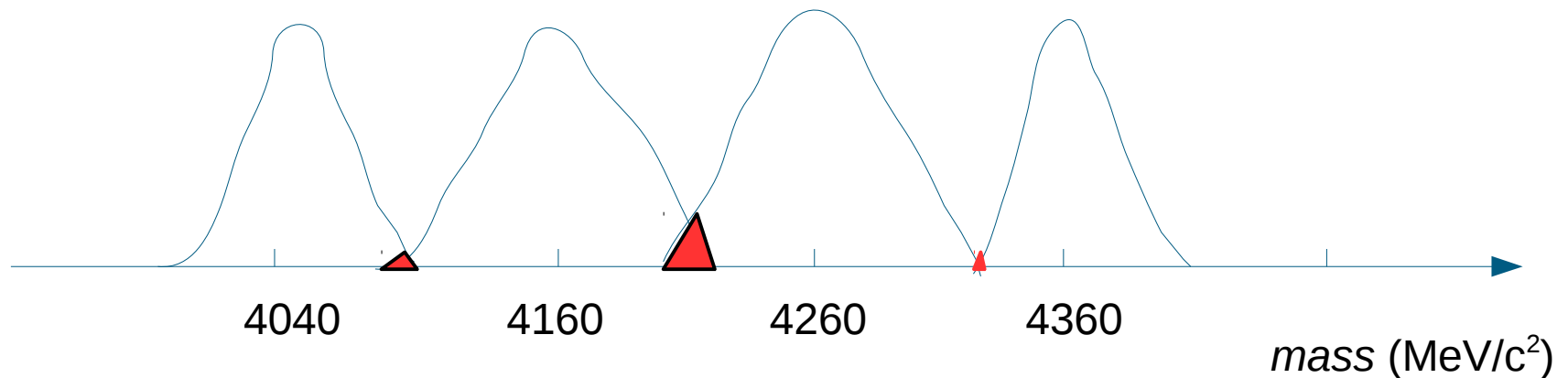
The goal of this study is:

- Study a rare process and determine the BR with precision
- Study interference between the Y states, decaying to a common final state

$$V \rightarrow e^+e^-$$

Vector state	BR($\rightarrow e^+e^-$)	Width (MeV)
$\psi(4040)$	$(1.07 \pm 0.16) \times 10^{-5}$	80 ± 10
$\psi(4160)$	$(8.1 \pm 0.9) \times 10^{-6}$	103 ± 8
$Y(4260)$	—	108 ± 12
$Y(4360)$	—	74 ± 18
$Y(4660)$	—	48 ± 15

- Vectors: $J^{PC} = 1^{--}$
- Expected to decay to e^+e^-
- Only 2 measured: very low BR
- Large width, probably they interfere
- PANDA can do better than its predecessors: $16400 * Y(4260)/\text{day}$
similar $Y(4160)/\text{day}$

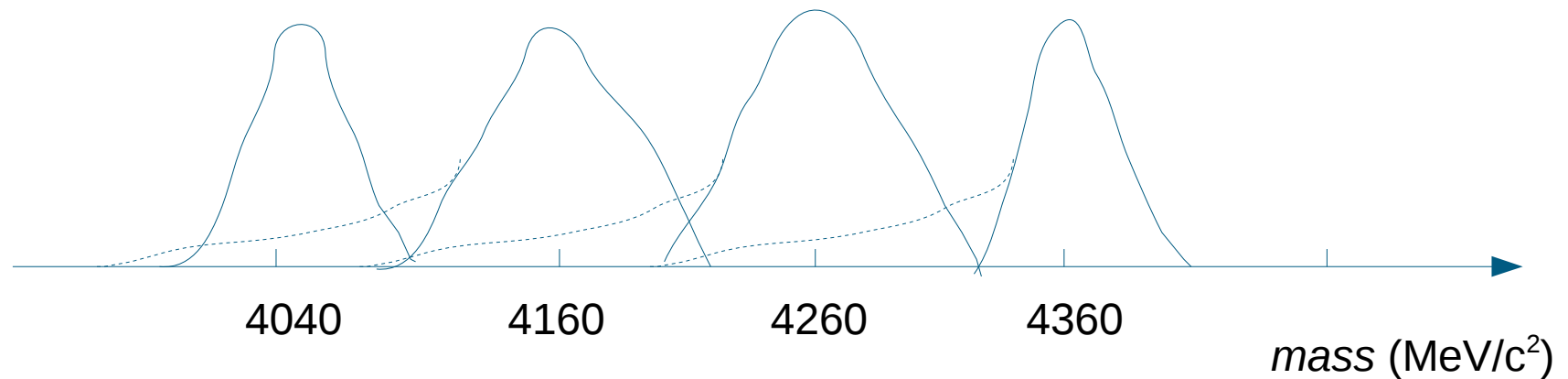


* in high resolution mode

$$V \rightarrow e^+e^-$$

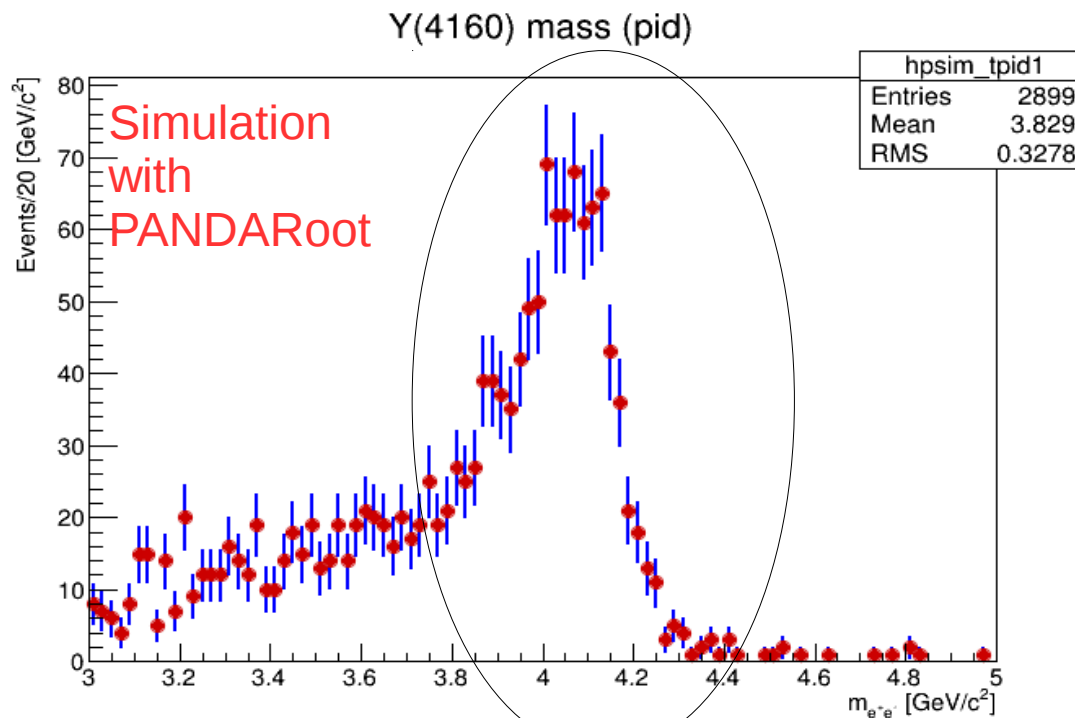
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similar $Y(4160)/\text{day}$



- Due to Bremsstrahlung we expect a long tail on the left of the resonant state

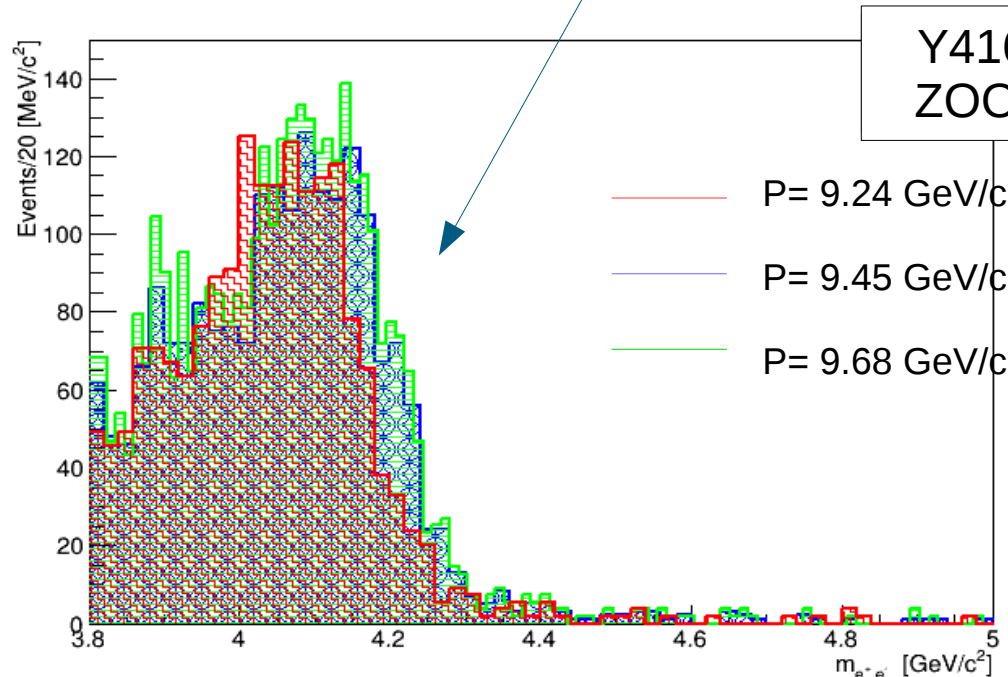
* in high resolution mode



- **PHOTOS** is used in these simulations

- Momentum: 9.24 GeV/c
Threshold of production of Y(4160)
Standalone simulation of Y(4160)

- Momentum: 9.45 GeV/c
It is between the 2 reson. production
Y(4160) and Y(4260)



- Momentum: 9.68 GeV/c
Threshold of production of Y(4260)
Simulation with Y(4160) and Y(4260)

- Very high reconstruction efficiency

- Interference can modify the line shape
A specific model needs to study this effect

- I tried to use several MC generators in processes with $e^+e^-(X)$ in the final state: EKHARA, PHOTOS.
RABHAT: work in progress.

BABAYAGA @ NLO: I have still to try

- All these MC generators are optimized for analysis at e^+e^- colliders
- Implementing EKHARA in a new framework was trial, but feasible: main problems due to the fortran version (ifort) and conversion of quadrupole precision variables to C++
Problems solved in BES using cfortran.h lib, and adding one subroutine to the original source code
- Radiative corrections are still not implemented in EKHARA
- Implementation of MC generators with correction N(N)L0 is tricky due to problems of numerical integration (Rabhat): divergences not easy to cancel
Solution for numerical integration: BASES/SPRING (tested at SLAC, KEK, CERN) <http://www.cpc.cs.qub.ac.uk/>
- When using EKHARA in BES, specific variable were built to reduce VCS...
- Radiative corrections are important even in $p\bar{p}$ processes: what is here the estimation?
- If any algorithm is available, I can give help to introduce it in PANDA and make tests

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Thanks for your attention!