Effect of quark masses in gluon fusion processes: a theoretical review

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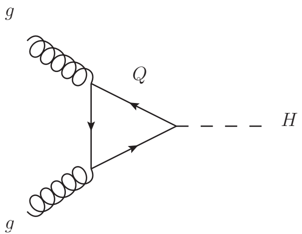
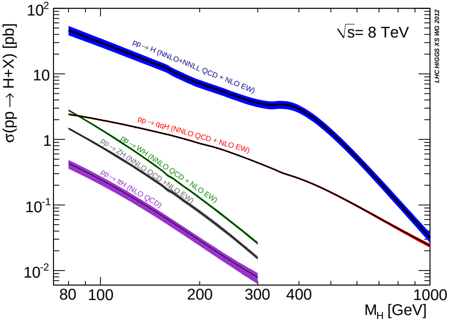
With contributions from: G. Degrassi, S. Frixione, M. Grazzini, P. Nason, C. Oleari, P. Slavich and A. Vicini.

Talk structure

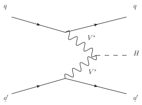
Production channels at the LHC

Gluon fusion

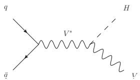




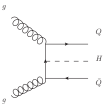
Vector Boson Fusion (VBF)



Higgs Strahlung



Quark associated production

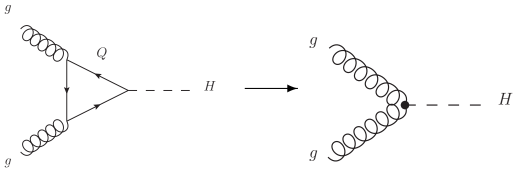


Heavy Quark Effective Field Theory (HQEFT)

In the limit $m_{top} \to \infty$ we can construct an effective Lagrangian for the interaction of the Higgs boson with the gluons

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} \frac{H}{v} (1 + \Delta) \, \mathrm{Tr} \left[G^a_{\mu\nu} G^a_{\mu\nu} \right]$$

In this theory the heavy quark loop shrinks to a point vertex, simplifying the calculations

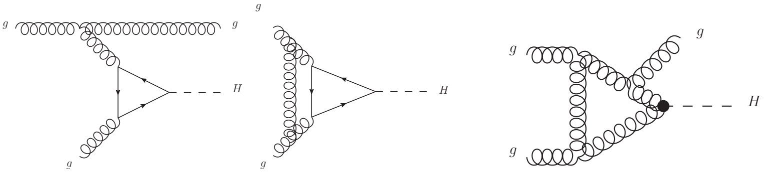


Validity conditions

- $m_H < 2m_{top}$
- Kinematic variables, as p_T^H, less than m_{top}
- No strongly coupled light particles running in the loop (e.g. bottom quark in the MSSM for large $\tan\beta$)

Theoretical results (1)

- LO-QCD: Georgi Glashow Machacek Nanopoulous (1978)
- NLO-QCD - HQEFT : Dawson (1991), Djouadi Graudenz Spira Zerwas (1992)
- NLO-QCD - exact: Spira Djouadi Graudenz Zerwas (1995), Aglietti Bonciani Degrassi Vicini (2007), Anastasiou Beerli Bucherer Daleo Kunstz (2007)
- NNLO-QCD - HQEFT : Anastasiou, Melnikov (2002), Harlander Kilgore (2002), Ravindran Smith Van Neerven (2003)

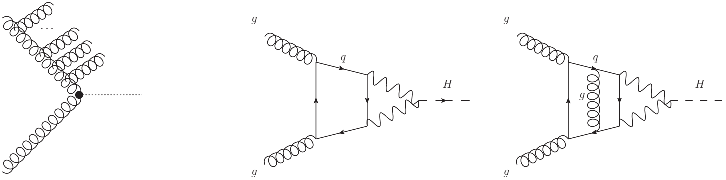


(a) NLO real full (b) NLO virt full (c) NNLO real HQEFT

Examples of Feynman diagrams contributing to the process at various orders

Theoretical results (2)

- NNLO-QCD + finite top mass effects: Marzani Ball Del Duca Forte Vicini (2008), Harlander Ozeren (2009), Pak Rogal Steinhauser (2009), Harlander Mantler Marzani Ozeren (2009)
- NNLO-QCD + soft gluon resummation NNLL-QCD: Catani De Florian Grazzini Nason (2003), Mach Vogt (2005), Idilbi Ji Yuan (2006), Ravindran Smith Van Neerven (2007)
- NLO-EW : Djouadi Gambino (2004), Aglietti Bonciani Degrassi Vicini (2004), Degrassi Maltoni (2004), Actis Passarino Sturm Uccirati (2008)
- mixed NLO EWxQCD: Anastasiou Boughezal Petriello (2009)



(a) Soft-gluon emission (b) NLO EW (c) NLO EWxQCD

Examples of Feynman diagrams contributing to the process at various orders

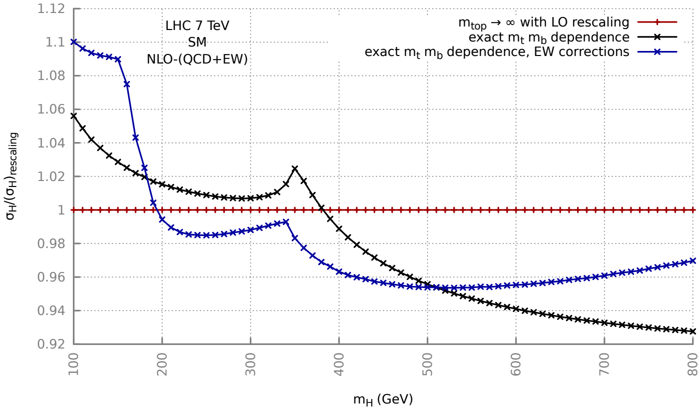
Available codes

Most important programs

- HIGLU, Fehip - NLO full theory
- ggh@nnlo, HNNLO - NNLO-QCD HQEFT
- iHixs - NNLO-QCD HQEFT, NLO-EW,NLO-EW-QCD
- Pythia/Herwig - PS LO HQEFT
- HqT - (NNLO+NNLL) - QCD HQEFT
- HRES - MC NNLO+NNLL QCD (full theory@NLO)
- MC@NLO/POWHEG - MC NLO + PS full theory
- HIGGSNNLO - MC NNLO + PS HQEFT

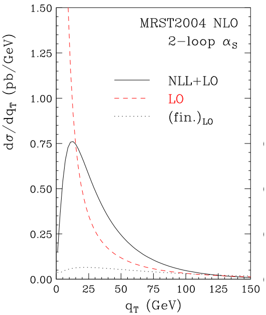
Total cross section

- The total inclusive cross section in the exact theory is well approximated by the result in the *HQEFT* for $m_H < 2m_{top}$.
- The total inclusive cross section obtained in the HQEFT and then rescaled by a factor $k = \frac{LO_{full}}{LO_{HQEFT}}$ gives a good approximation over all the mass range.
- Mass effects studied at NNLO only for the top contribution.

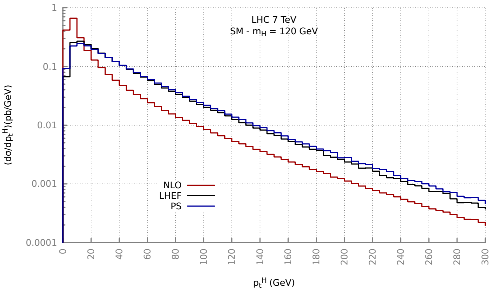


p_T^H distribution

- The Higgs acquires a transverse momentum due to the recoil against QCD radiation.
- At fixed order, the p_T^H distribution diverges in the limit $p_T^H \to 0$.
- The physical behaviour is restored by resumming the divergent log $\left(\frac{p_T^H}{m_H}\right)$ terms, either analytically or numerically (i.e. through a Parton Shower).
- problem: match the resummed and fixed order calculation.



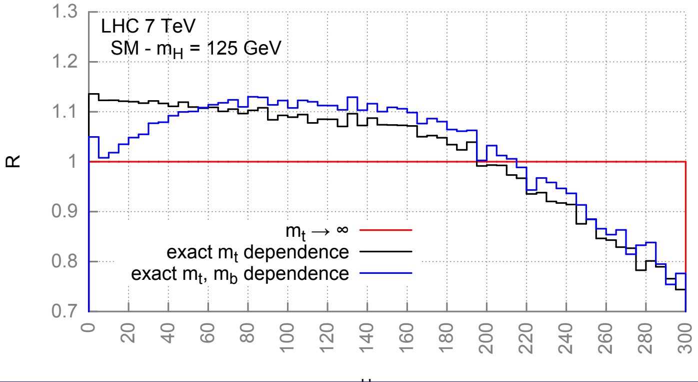
(a) HqT (Bozzi Catani De Florian Grazzini,arXiv:hep-ph/0508068)



(b) POWHEG (EAB,Degrassi,Slavich,Vicini, arXiv:hep-ph/1111.2854)

Mass effects in p_T^H distribution

- The emitted parton can resolve the internal structure of the quark loop if the p_T^H is large enough (mass effects start at $p_T^H > 150$ GeV for the top loop and $p_T^H > 10$ GeV for the bottom one).
- Unique way to include mass effects at fixed order (NLO) – good agreement also numerically between different codes.
- Could be used to characterize Higgs couplings to quarks (e.g MSSM – see backup slides, YR2 and YR3 and arXiv:hep-ph/1111.2854).



Matching of p_T resummation with fixed order calculations

Several different way to perform the matching of $\log\left(\frac{p_T^H}{m_H}\right)$ resummation to NLO calculation.

Analytic approach

- Bozzi Catani De Florian Grazzini (arXiv:hep-ph/0508068).
- Mantler Wiesemann (arXiv:hep-ph/1210.8263).
- Grazzini Sargsyan (arXiv:1306.4581).
- The effect of the resummation (the separation between the "soft" and the "hard" part) is controlled by the so-called resummation scale Q.

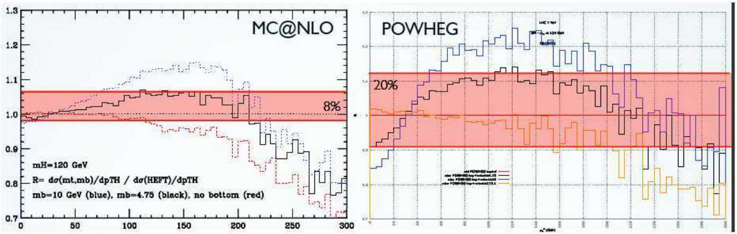
Numeric approaches (NLO+PS)

- Implementation in MC@NLO (Frixione).
- Implementation in POWHEG-BOX (Alioli Nason Oleari, EAB Degrassi Slavich Vicini).
- The effect of the resummation are controlled by parameters that are analogous to Q in HRES. They are the shower scale for MC@NLO and hfact (h) for POWHEG.
- Note: Mass effects are treated differently in the two frameworks due to the different matching procedure (see structure of the POWHEG Sudakov form factor, backup slides).

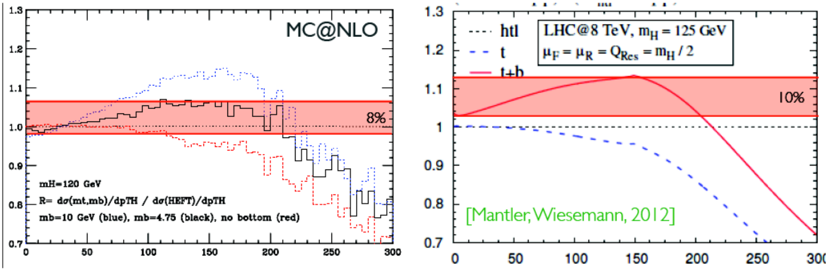
Results comparison - Naïve single scale approach

- First implementation in POWHEG in 2011 (EAB Degrassi Slavich Vicini).
- Followed in 2012 by MC@NLO (Frixione) and Mantler-Wiesemann analytic calculation.

Keeping a single, high, resummation scale of the order of $m_H/2$ gives different results in the various approaches for the top+bottom effects (red band).



MC@NLO (left), POWHEG (right) Frixione, ggF meeting on Higgs pT 20/12/12



MC@NLO (left), Mantler-Wiesemann (right) Frixione, ggF meeting on Higgs pT 20/12/12

Recent developments

- Gained awareness that this is a multi-scale problem and requires an ad-hoc treatment.
- Proposed solution (Grazzini,Vicini): treat separately the top and bottom contribution.

We can always rewrite the full amplitude as

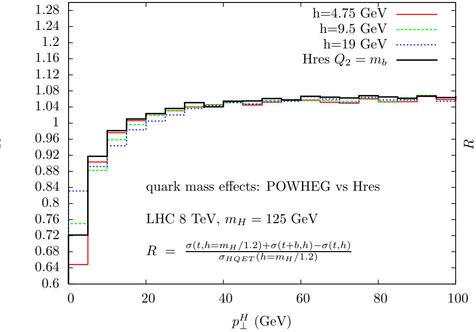
$$|\mathcal{M}(t+b)|^2 = |\mathcal{M}(t)|^2 + \left[|\mathcal{M}(t+b)|^2 - |\mathcal{M}(t)|^2\right]$$

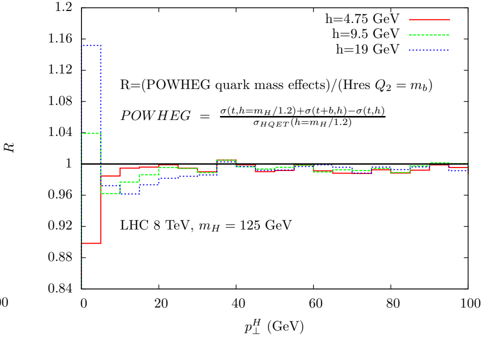
and consider the bottom contribution as a correction to the top one. On the same line we can rewrite the cross section as

$$\begin{aligned}\sigma(t+b) =& \sigma(t, h/Q_1 = \frac{m_H}{1.2}) + \\ & + \left[\sigma(t+b, h/Q_2 = m_b) - \sigma(t, h/Q_2 = m_b)\right]\end{aligned}$$

POWHEG results

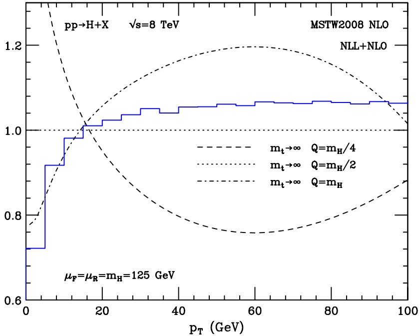
With this new prescription much better agreement between the different approaches.

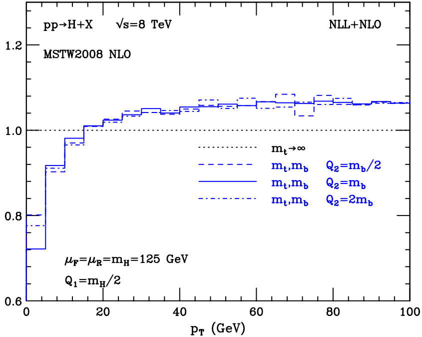




POWHEG (Vicini, ggF meeting on Higgs pT 23/07/13)

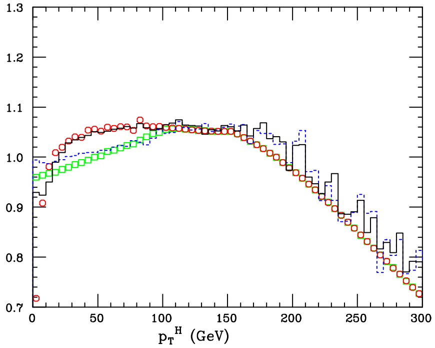
HRES results





HRES(Grazzini, ggF meeting on Higgs pT 23/07/13)

MC@NLO vs HRES



histograms: MC@NLO | symbols: HRes

solid and circles: $Q_2 = \mathcal{O}(m_b)$ | dashed and boxes: $Q_2 = \mathcal{O}(m_H)$

MC@NLO (Frixione, ggF meeting on Higgs pT 23/07/13)

Conclusions and comments

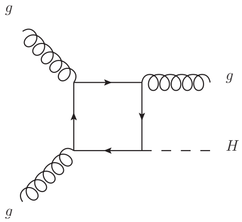
- Complete treatment of the quark masses in gluon processes can give rise to substantial effect in physical observables, as is the case for the p_T^H distribution.
- The Higgs p_T distribution is a multi-scale problem and requires an ad-hoc treatment.
- The Higgs transverse momentum is an important observable for Higgs characterization at the LHC.
- Recently the effect of quark masses was also studied in the Jet Veto observable (Banfi, Monni, Zanderighi - arXiv:1308.4634), finding a much smaller effect.
- Full treatment of quark masses is necessary in theories where the light quark is strongly coupled to the Higgs boson.

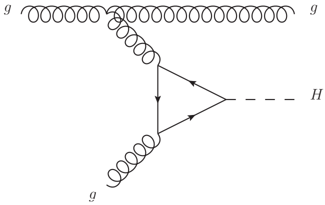
Backup slides

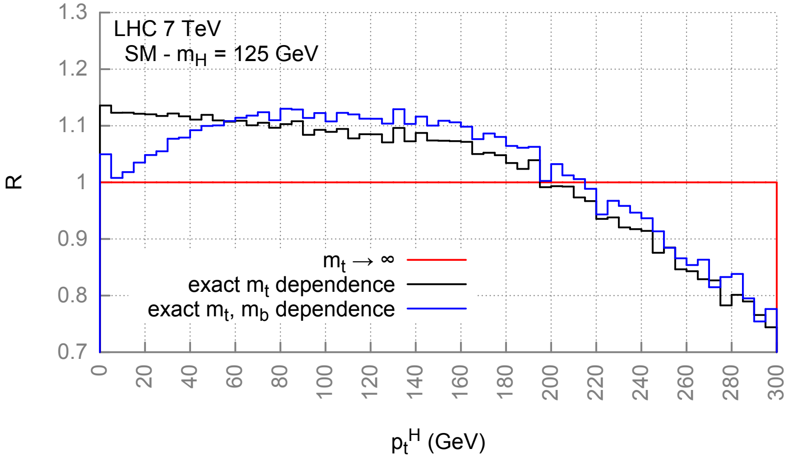
Characterization of mass effects at NLO(1)

- According to the value of the p_T^H the emitted parton can or can not resolve the inner structure of the loop.
- At low p_T^H the leading contributions come from radiation emitted from the incoming partons.

- Triangle diagrams have threshold at $s = 4m_q^2$.
- Box diagrams have enhanced contribution for $p_T^H \approx m_q$.
- Top effect for $p_T^H > 150$ GeV; bottom effect for as low as 10 GeV!







Mass effects in the Higgs p_T^H spectrum at NLO

Characterization of mass effects at NLO(2)

- Due to the different magnitude of the Yukawa couplings, the the bottom diagrams have a suppression factor of $\frac{m_b}{m_t} \approx 0.028$.
- The full amplitude is given by

$$|\mathcal{M}(gg \to gH)|^2 = |\mathcal{M}_t + \mathcal{M}_b|^2 = |\mathcal{M}_t|^2 + 2\mathcal{R}(\mathcal{M}_t\mathcal{M}_b^\dagger) + |\mathcal{M}_b|^2$$

- The bottom effects in the SM are due to the interference term.
- If the theory has enhanced bottom couplings (or more strongly coupled light particles, in the loop) the situation becomes more complex with different terms becoming relevant at different scales.

Parton shower basics

- QCD emissions are enhanced in the collinear limit.
- The parton shower dresses the event with multiple parton emission in this limit.
- In the collinear limit the cross section factorizes

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \rightarrow |\mathcal{M}_n|^2 d\Phi_n \frac{\alpha_s}{2\pi}\frac{dt}{t}P_{ab}(z)dz\frac{d\Phi}{2\pi}$$

- Multiple emission can be described by iterating this formula.
- The showering process stops when the virtuality of the last parton emitted is of the order of Λ_{QCD} and the hadronization regime starts.
- The parton shower effectively implements the resummation of the the leading log (LL) $\log(\frac{p_T^H}{m_H})$. It is also preserves the LO cross-section (unitarity).

Matching between PS and NLO calculation

The problem

- Matching of a NLO-QCD Monte Carlo (MC) event generator and Parton showers (PS) to achieve a better description of experimental data.
- Since a PS includes the Leading Log (LL) terms, which in part are also present in the NLO calculation, it is necessary to develop a strategy to avoid double counting.

The solution

- Two possible approaches: MC@NLO and POWHEG.

POWHEG

P.O.W.H.E.G = POsitive Weight Hardest Emission Generator

The POWHEG solution

- POWHEG generates the hardest emission.
- The interface with the PS requires a p_T ordered (or a p_T vetoed) shower.
- Independent from the specific PS implementation.
- Generates events with positive weight.

POWHEG: the generation of the events

- The POWHEG formula for the generation of the event is:

$$d\sigma = \bar{B}(\bar{\mathbf{\Phi}}_1)d\bar{\mathbf{\Phi}}_1 \left\{ \Delta(\bar{\mathbf{\Phi}}_1, p_T^{\text{min}}) + \Delta(\bar{\mathbf{\Phi}}_1, p_T) \frac{R(\bar{\mathbf{\Phi}}_1, \Phi_{\text{rad}})}{B(\bar{\mathbf{\Phi}}_1)} d\Phi_{\text{rad}} \right\} + \sum_q R_{q\bar{q}}(\bar{\mathbf{\Phi}}_1, \Phi_{\text{rad}}) d\Phi_{\text{rad}} d\bar{\mathbf{\Phi}}_1$$

$$\bar{B}(\bar{\mathbf{\Phi}}_1) = B_{gg}(\bar{\mathbf{\Phi}}_1) + V_{gg}(\bar{\mathbf{\Phi}}_1) + \int d\Phi_{\text{rad}} \left\{ \hat{R}_{gg}(\bar{\mathbf{\Phi}}_1, \Phi_{\text{rad}}) + \sum_q \hat{R}_{qg}(\bar{\mathbf{\Phi}}_1, \Phi_{\text{rad}}) + \sum_q \hat{R}_{gq}(\bar{\mathbf{\Phi}}_1, \Phi_{\text{rad}}) \right\} + c.r.$$

$$\Delta(\bar{\mathbf{\Phi}}_{\mathbf{1}}, p_T) = \exp\left\{ - \int d\Phi_{\text{rad}} \frac{R(\bar{\mathbf{\Phi}}_{\mathbf{1}}, \Phi_{\text{rad}})}{B(\mathbf{\Phi}_1)} \theta(k_T - p_T) \right\}$$

- NLO normalization.
- Sudakov form factor with full matrix elements.

POWHEG: the generation of the events

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- NLO normalization.
- Sudakov form factor with full matrix elements.

Comparison between POWHEG and MC@NLO

$$d\sigma = d\Phi_B \overline{B}^s(\Phi_b)\left[\Delta^s(p_\perp^{\min}) + d\Phi_{R|B}\frac{R^s(\Phi_R)}{B(\Phi_B)}\Delta^s(p_T(\Phi))\right] + d\Phi_R R^f(\Phi_R)$$

$$\overline{B}^s = B(\Phi_b) + \left[V(\Phi_b) + \int d\Phi_{R|B}\hat{R}(\Phi_{R|B})\right]$$

$$\Delta(\bar{\mathbf{\Phi}}_1, p_T) = \exp\left\{-\int d\Phi_{\rm rad}\frac{R^s(\bar{\mathbf{\Phi}}_1, \Phi_{\rm rad})}{B(\mathbf{\Phi}_1)}\theta(k_T - p_T)\right\}$$

MC@NLO

$$R^s \propto \frac{\alpha_s}{t}P_{ij}(z)B(\Phi_B) \quad , \quad R^f = R - R^s$$

- The Sudakov form factor is the one from the P.S., i.e. it uses the collinear splitting function in the exponent.
- The full matrix element appears only in the regular contribution.

POWHEG

$$R^s = \frac{h^2}{h^2 + p_T^2}R \quad , \quad R^f = \frac{p_T^2}{h^2 + p_T^2}R$$

- hfact controls high order effects
- At low p_T R goes into collinear factorization and the Sudakov regains the splitting function in the exponent.

The two approaches differs by higher order terms.

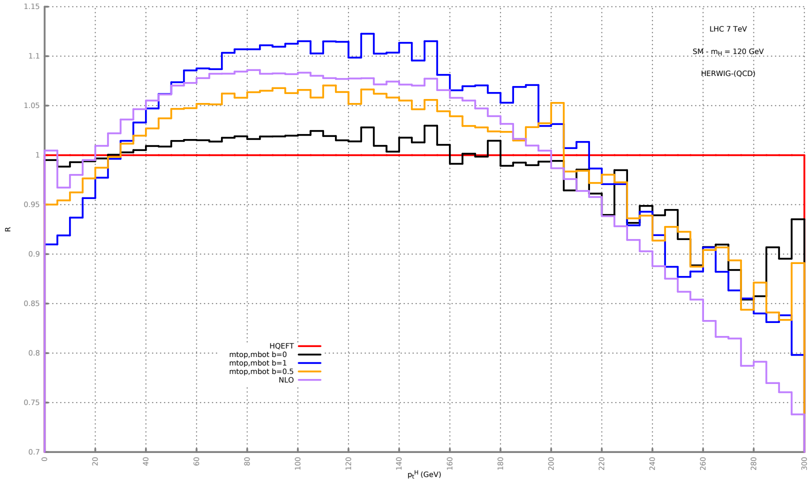
Parametrization of the higher order contribution in POWHEG (preliminary)(1)

$$d\sigma = d\Phi_B \overline{B}^s(\Phi_b)\left[\Delta^s(p_\perp^{\min}) + d\Phi_{R|B}\frac{R^s(\Phi_R)}{B(\Phi_B)}\Delta^s(p_T(\Phi))\right] + d\Phi_R R^f(\Phi_R)$$

$$R^s = \overline{R} + b(R - \overline{R})\ ,\ R^f = (1-b)(R - \overline{R}) + R_{q\bar{q}}\ ,\ \overline{R} = R_{HQEFT}\frac{B_{t+b}}{B_{HQEFT}}$$

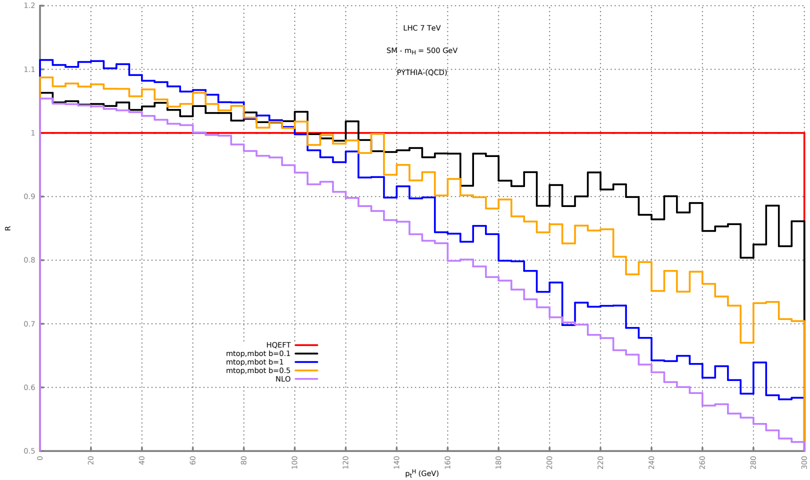
- We consider the mass effect as a small correction over the result obtained in the HQEFT.
- The parameter b specify how this correction is included, either in the R^s or in R^f.
- $b = 0$ is similar to MC@NLO, i.e. mass effects are only in the regular terms. $b = 1$ is the standard POWHEG.
- This parametrization could help in determining the uncertainty band on the p_T^H due to the quark mass effects.

Parametrization of the higher order contribution in POWHEG (preliminary)(2)



Uncertainty band for $m_H = 125$

Parametrization of the higher order contribution in POWHEG (preliminary)(3)



Uncertainty band for $m_H = 150$

Resummation and matching of $\log(\frac{p_T^H}{m_H})$ in HqT

Bozzi Catani De Florian Grazzini (arXiv:hep-ph/0508068)

$$\frac{d\hat{\sigma}^{(\text{res.})}_{F\,ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) = \frac{M^2}{\hat{s}} \int_0^\infty db\, \frac{b}{2} J_0(bq_T)\, \mathcal{W}^F_{ab}(b, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\mathcal{W}^F_{ab,N}(b, M; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) \equiv \int_0^1 dz\, z^{N-1}\, \mathcal{W}^F_{ab}(b, M, \hat{s} = M^2/z; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\mathcal{W}^F_N(b, M; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}^F_N\left(M, \alpha_s(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2\right)$$
$$\times \exp\{\mathcal{G}_N(\alpha_s(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2)\}$$

- The b-space factorization of the cross section for multiple emission is defined at a given scale Q, the so-called "resummation scale".
- The Q scale is unphysical and the complete result does not depend on it. However, at fixed order in perturbation theory, a residual dependence on it is left.
- The choice of the scale Q effectively determines the range where the resummation is effective.

Quark mass effects in HRES (Grazzini Sargsyan arXiv:1306.4581)

$$\mathcal{W}^{(N_1,N_2)} \longrightarrow \mathcal{W}_{\mathrm{top}}^{(N_1,N_2)} + \mathcal{W}_{\mathrm{bot}}^{(N_1,N_2)}$$

where

$$\mathcal{W}_{\mathrm{top}}^{(N_1,N_2)}(b) = \sigma_{LO}(m_t)\mathcal{H}^{(N_1,N_2)}(m_H^2/Q_1^2;m_t)\ \exp\{\mathcal{G}^{(N_1,N_2)}(\widetilde{L}_{Q_1};m_H^2/Q_1^2)\}$$

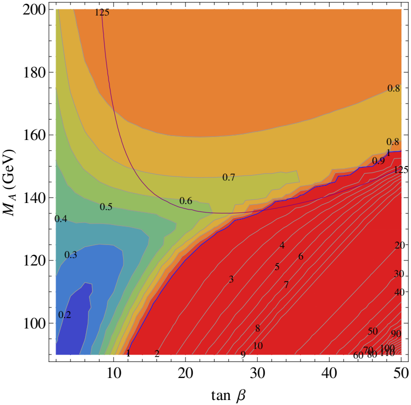
$$\mathcal{W}_{\mathrm{bot}}^{(N_1,N_2)}(b) = \left[\sigma_{LO}(m_t,m_b)\mathcal{H}^{(N_1,N_2)}(m_H^2/Q_2^2;m_t,m_b) - \sigma_{LO}(m_t)\mathcal{H}^{(N_1,N_2)}(m_H^2/Q_2^2;m_t)\right] \times \exp\{\mathcal{G}^{(N_1,N_2)}(\widetilde{L}_{Q_2};m_H^2/Q_2^2)\}\,,$$

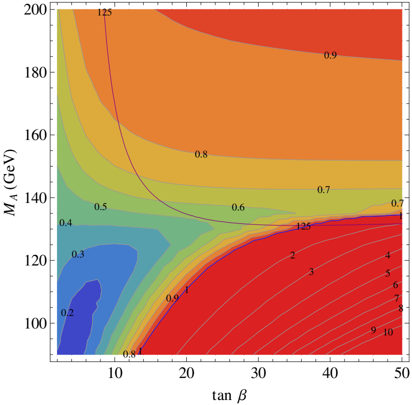
- Use different scale for the top and the bottom contribution.
- The scale used for the bottom contribution is much smaller to take into account the breaking of the factorization at low scale.

Total cross section in the MSSM

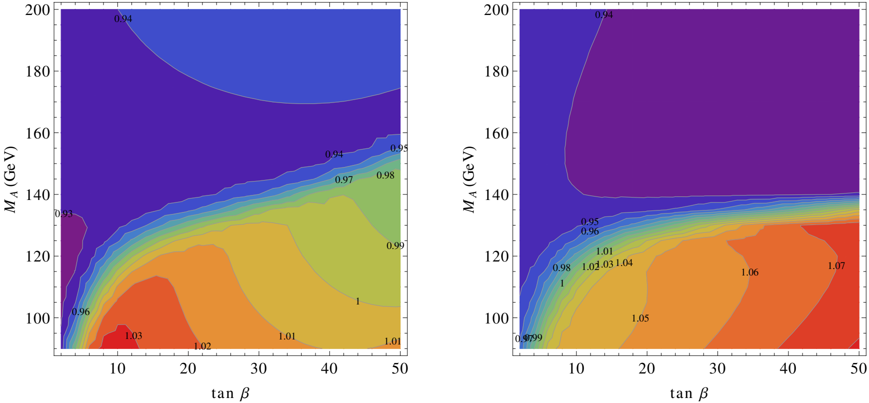
- The bottom contribution could be boosted (i.e. $\tan\beta$ enhancement).
- Exact treatment of mass effects could not be neglected.
- If the squarks are light enough, they can not be neglected.

MSSM vs SM



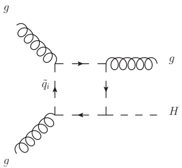


MSSM vs MSSM-with-only-quarks



Ratio of the MSSM complete cross section over the MSSM one with only quarks included.

Higgs p_T^H distribution in the MSSM



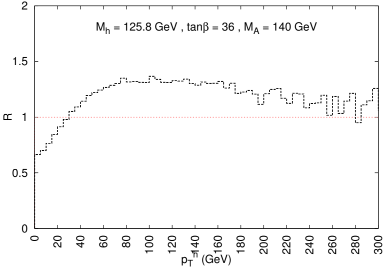
In the MSSM the matrix element squared for the $gg \to gH$ channel is given by

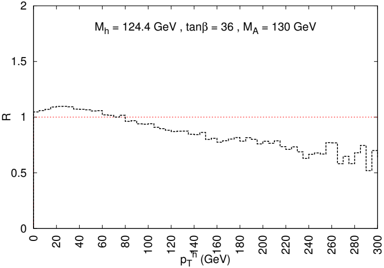
$$\mathcal{M}(gg \to gH)|^2 = |\mathcal{M}_t + \mathcal{M}_b + \mathcal{M}_{\tilde{q}}|^2 =$$

$$= |\mathcal{M}_t|^2 + 2\mathrm{Re}(\mathcal{M}_t \mathcal{M}_b^\dagger) + |\mathcal{M}_b|^2 + 2\mathrm{Re}(\mathcal{M}_t \mathcal{M}_{\tilde{q}}^\dagger) + 2\mathrm{Re}(\mathcal{M}_b \mathcal{M}_{\tilde{q}}^\dagger) + |\mathcal{M}_{\tilde{q}}|^2$$

- The bottom contribution could be boosted (i.e. $\tan\beta$ enhancement).
- This in turn could make the contribution of the interference between the bottom and the squarks relevant.
- Exact treatment of mass effects could not be neglected.

The p_T^H shape could be used to discriminate between the SM and the MSSM.





Ratio of the shapes of the p_T^H distributions in the MSSM over the one computed in the MSSM without squarks.
See also the YR2 and YR3 for a discussion of the MSSM.

Mass effects on total cross section

k-factor (mh=125)	Value
LO(mt,mb)/LO(HQEFT)	0.949
NLO(mt,mb)/NLO(HQEFT)	0.988
NLO(mt)/NLO(HQEFT)	1.061
LO(mt)/LO(HQEFT)	1.066

From Grazzini, Sargsyan (arXiv:1306.4581).