Experimental constraints on the uncoupled Galileon model

A&A 555, A53 (2013), arXiv:1302.2786

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October 29, 2013

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CEA - Saclay

The Galileon cosmology

1 - The Galileon cosmology

Galileon π : a new scalar field to explain accelerated expansion.

Principles : Nicolis, Rattazzi & Trincherini, 2009

The π Lagrangian is constructed so that the equation of motion of π is invariant under Galilean symmetry

$$\pi \mapsto \pi + a + b_\mu x^\mu$$

$\Rightarrow$ only 5 L_i terms are possible $\Rightarrow$ 5 free parameters c_i

- Imposing Galilean symmetry justified by Xdim considerations (DGP).

Action of the galileon (Appleby & Linder 2011) :

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} - L_{SM} - \frac{1}{2}\sum_{i=1}^{5} c_i L_i - L_{couplings}\right)$$

- General Relativity
- Standard matter Lagrangian
- Covariant Galileon Lagrangian from Deffayet et al. 2009
- Optional couplings between matter and the Galileon field

Lagrangians (with $M^3 = M_P H_0^2$) :

$$L_1 = M^3\pi \qquad L_2 = (\nabla_\mu\pi)(\nabla^\mu\pi) \qquad L_3 = (\Box\pi)(\nabla_\mu\pi)(\nabla^\mu\pi)/M^3$$

$$L_4 = (\nabla_\mu\pi)(\nabla^\mu\pi)\left[2(\Box\pi)^2 - 2\pi_{;\mu\nu}\pi^{;\mu\nu} - R\,(\nabla_\mu\pi)(\nabla^\mu\pi)/2\right]/M^6$$

$$L_5 = (\nabla_\mu\pi)(\nabla^\mu\pi)\times$$
$$\left[(\Box\pi)^3 - 3(\Box\pi)\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu}^{;\nu}\pi_{;\nu}^{;\rho}\pi_{;\rho}^{;\mu} - 6\pi_{;\mu}\pi^{;\mu\nu}\pi^{;\rho}\,G_{\nu\rho}\right]/M^9$$

- π coupled to Ricci scalar and Einstein tensor ⇒ modified gravity !
- $L_i \propto \nabla_\mu\pi^i/M^{3(i-2)}$

Remark

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Properties

- Based on a very restrictive symmetry : only 5 c_i free parameters (beside the couplings to matter).
- Can assume $c_1 = 0$ to avoid an explicit cosmological constant
- Close to massive objects : no ghosts, no instabilities, preserves General Relativity thanks to Vainshtein screening effect.

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- Derivation of the two Einstein equations and π EoM, e.g. :

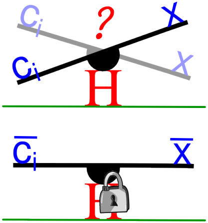
$$\text{(00)-Einstein equation : } \frac{\partial S}{\partial g_{00}} = 0$$

$$\Rightarrow \bar{H}^2 = \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \underbrace{\frac{c_2}{6}\bar{H}^2 x^2 - 2c_3\bar{H}^4x^3 + \frac{15}{2}c_4\bar{H}^6x^4 - 7c_5\bar{H}^8x^5}_{\Omega_\pi = \text{"new" } \Omega_{DE}}$$

$$\bar{H} = H/H_0 \qquad x = \pi'/M_P, \quad ' = d/d\ln a$$

- Two problems :
  1. unknown initial condition for x
  2. degeneracy to break between the c_is and x
- One solution : reparametrize with $x_0 = x(z=0)$:

$$\boxed{\bar{c}_i = c_i x_0^i, \ \bar{x} = x/x_0}$$



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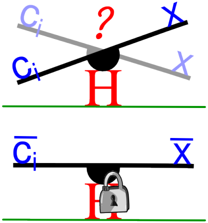


(00)-Einstein, (ij)-Einstein, and π EoM :

$\Rightarrow$ 3 coupled differential equations to solve in $\bar{H}(z)$ and $\bar{x}(z)$

- only use 2 equations, with the 2 known initial conditions :

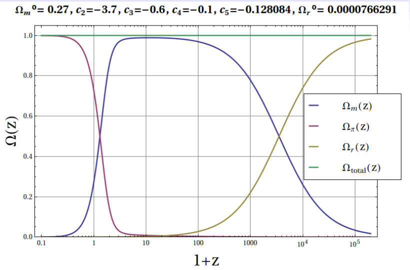
$$\bar{x}(z=0)=1, \quad \bar{H}(z=0)=1$$

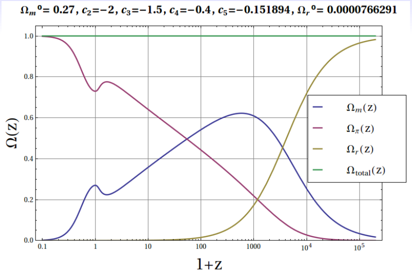
- 1 constraint equation ((00)-Einstein) : used to fix $\bar{c}_5$ given Ω_m^0, Ω_r^0 and the other $\bar{c}_i$'s :

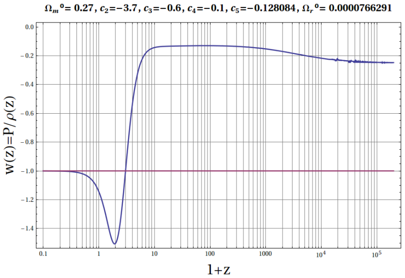
$$\bar{c}_5 = \frac{1}{7}\left(-1+\Omega_m^0+\Omega_r^0+\frac{\bar{c}_2}{6}-2\bar{c}_3+\frac{15}{2}\bar{c}_4\right)$$

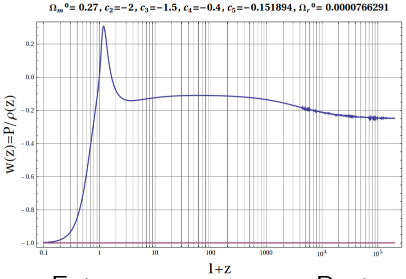
$\Rightarrow$ 5 free parameters to constrain : $\Omega_m^0, \Omega_r^0, \bar{c}_2, \bar{c}_3, \bar{c}_4$

Let's compute some Galileon Universes !









← Future Past → ← Future Past →

Growth of structure in a Galileon Universe :

- linear perturbation of Galileon field $\delta\pi$ and metric :

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- after computation, we obtain a new Poisson equation for Newtonian gravity, with an **effective gravitational coupling** :

$$\nabla^2\psi = 4\pi a^2 G_{\text{eff}}^{(\psi)}(z)\rho_m\delta_m$$

$$G_{\text{eff}}^{(\psi)}(z) = \frac{4(\kappa_3\kappa_6 - \kappa_1^2)}{\kappa_5(\kappa_4\kappa_1 - \kappa_5\kappa_3) - \kappa_4(\kappa_4\kappa_6 - \kappa_5\kappa_1)} G_N$$

and other quantities :

- $\delta\pi$ kinetic normalisation factor
- squared sound speed of propagation for the $\delta\pi$ perturbation $c_s^2(z)$

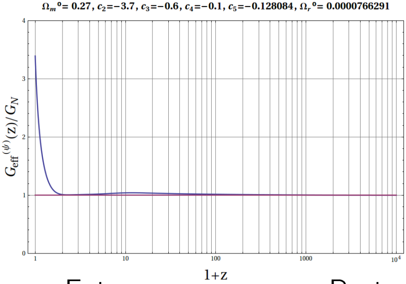
$$\kappa_1 = -6\bar{c}_4\bar{H}^3\bar{x}^3\left(\bar{H}'\bar{x} + \bar{H}\bar{x}' + \frac{\bar{H}\bar{x}}{3}\right) + \bar{c}_5\bar{H}^5\bar{x}^3(12\bar{H}\bar{x}' + 15\bar{H}'\bar{x} + 3\bar{H}\bar{x})$$

$$\kappa_3 = -1 - \frac{\bar{c}_4}{2}\bar{H}^4\bar{x}^4 - 3\bar{c}_5\bar{H}^5\bar{x}^4(\bar{H}'\bar{x} + \bar{H}\bar{x}')$$

$$\kappa_4 = -2 + 3\bar{c}_4\bar{H}^4\bar{x}^4 - 6\bar{c}_5\bar{H}^6\bar{x}^5$$

$$\kappa_5 = 2\bar{c}_3\bar{H}^2\bar{x}^2 - 12\bar{c}_4\bar{H}^4\bar{x}^3 + 15\bar{c}_5\bar{H}^6\bar{x}^5$$

$$\kappa_6 = \frac{\bar{c}_2}{2} - 2\bar{c}_3(\bar{H}^2\bar{x}' + \bar{H}\bar{H}'\bar{x} + 2\bar{H}^2\bar{x}) + \bar{c}_4(12\bar{H}^4\bar{x}\bar{x}' + 18\bar{H}^3\bar{x}^2\bar{H}' + 13\bar{H}^4\bar{x}^2) - \bar{c}_5(18\bar{H}^6\bar{x}^2\bar{x}' + 30\bar{H}^5\bar{x}^3\bar{H}' + 12\bar{H}^6\bar{x}^3).$$



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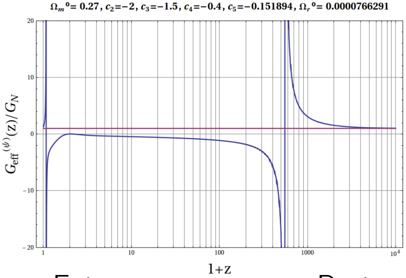
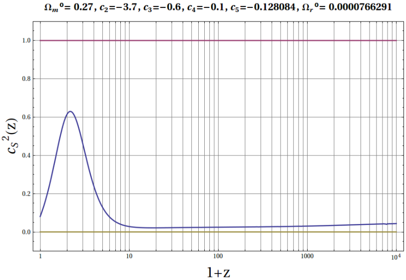
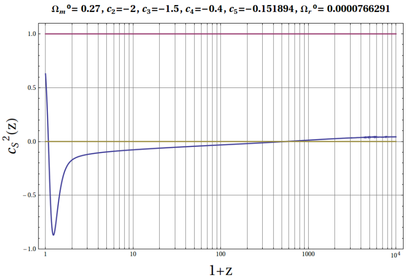


← Future Past →

A good Galileon cosmological scenario must have $\forall z > 0$ good theoretical properties, e.g. positive kinetic term normalization (no-ghost condition) for $\delta\pi$, positive squared sound speed (no instabilities) $c_s^2 > 0$...



$$\forall z > 0,\ c_s^2(z) > 0$$
$\Rightarrow$ good scenario !
$\Rightarrow$ keep it for fitting



$$\exists z > 0,\ c_s^2(z) < 0$$
$\Rightarrow$ bad scenario !
$\Rightarrow$ reject it

Experimental constraints

2 - Experimental constraints

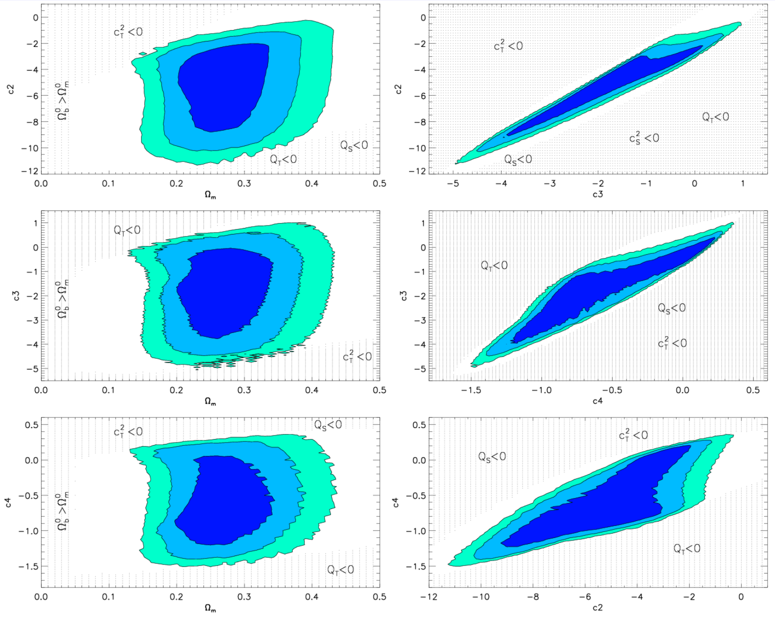
Supernovae

- 472 well measured SNe Ia from the SuperNovae Levagy Survey (SNLS3) data sample with systematics (Conley et al. 2011)
- Prediction of B band magnitude peak for SNe Ia :

$$m_B^{mod} = 5\log_{10}\left[(1+z_{hel})\int_0^{z_{CMB}} \frac{dz}{\bar{H}(z,\text{cosmo})}\right] - \alpha(s-1) + \beta.\mathcal{C} + \mathcal{M}_{\mathcal{B}}$$

  to be compared to measurements.
- α, β and $\mathcal{M}_{\mathcal{B}}$: nuisance parameters fitted with the cosmological parameters.

Figure: **SNLS3** constraints



Cosmological Microwave Background

- No prediction of the full power spectrum
  $\Rightarrow$ use of simplified set of observables : l_a, R, z_*

$$R = \sqrt{\Omega_m^0} \int_0^z \frac{dz'}{\bar{H}(z')}, \quad l_a = (1+z_*) \frac{\pi D_A(z_*)}{r_s(z_*)}$$

$$D_A(z) = \frac{c}{H_0} \frac{1}{1+z} \int_0^z \frac{dz'}{\bar{H}(z')}, \quad r_s(z) = \frac{c}{H_0} \int_0^{\frac{1}{1+z}} da \frac{\bar{c}_{s,m}(a)}{a^2 \bar{H}(a)}$$

- $\bar{c}_{s,m}(a)$ not modified by the Galileon field
- H_0 Gaussian prior from direct measurement of Riess et al. 2011 :

$$h = 0.738 \pm 0.024$$

- Technical details :
  - z_* from fitting formula of Hu & Sugiyama 1996
  - Minimization on h and $\Omega_b^0 h^2$ together with CMB predictions (following prescription of Komatsu et al. 2011).

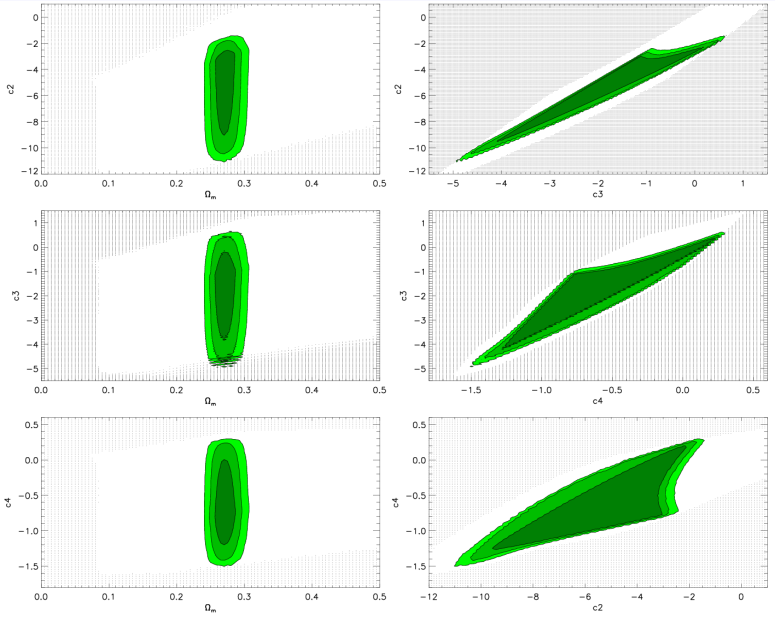
Baryonic Acoustic Oscillations

- 3 BAO $y_s(z)$ measurements coming from 6dF, SDSS-II and BOSS surveys :

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)}\right]^{1/3}, \quad y_s(z) = r_s(z_d)/D_V(z)$$

- Technical details :
  - Minimization on h and $\Omega_b^0 h^2$ together with CMB predictions

Figure: **CMB+BAO+H_0 constraints**



Growth of structure data

- 9 $f\sigma_8(z)$ growth rate and 5 $F(z)$ Alcock-Paczynski parameter measurements from 6dFGRS, 2dFGRS, WiggleZ, SDSS, and BOSS surveys

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}^{(\psi)}(t)\rho_m\delta_m = 0$$

$$D(a) = \delta_m(a)/\delta_m(1), \quad f(a) = \frac{d\ln D(a)}{d\ln a}, \quad F(a) = \frac{1}{c}\frac{D_A(a)H(a)}{a}$$

- Measurements **independent** from any fiducial cosmology and GR requirement
- Technical details :
  - Hypothesis : same σ_8 value at decoupling for the ΛCDM and Galileon models :

$$\sigma_8(a) = \sigma_8(a_{\text{initial}})\frac{D(a)}{D(a_{\text{initial}})}, \quad \sigma_8(a_{\text{initial}}) = \sigma_8^{\text{WMAP7}}(1)\frac{D^{\Lambda\text{CDM}}(a_*)}{D^{\Lambda\text{CDM}}(1)}$$

Figure: **GoS** (filled) and **SN+CMB+BAO+H_0** (dotted) constraints

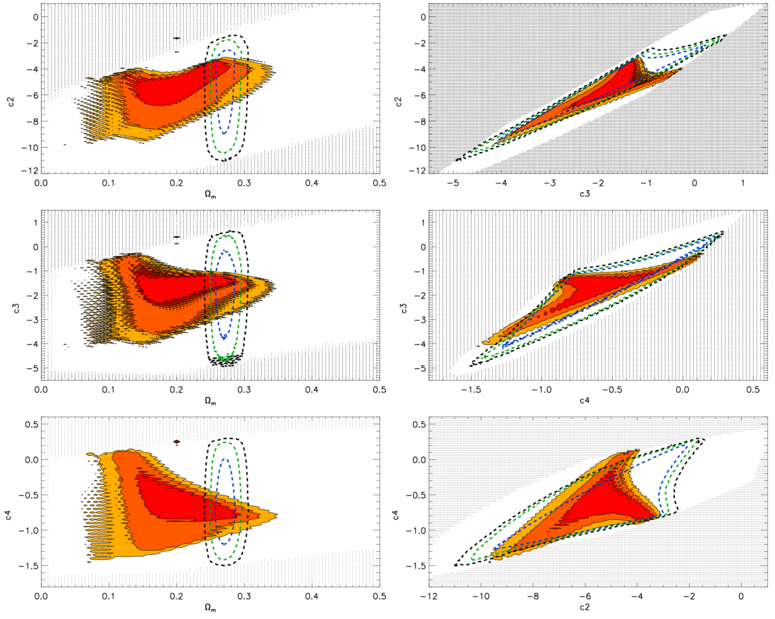


Figure: **all data** constraints

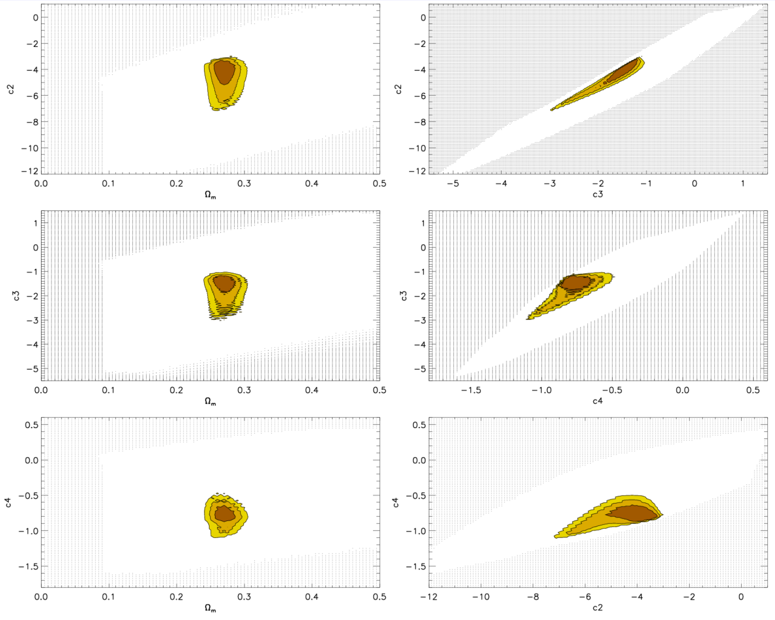
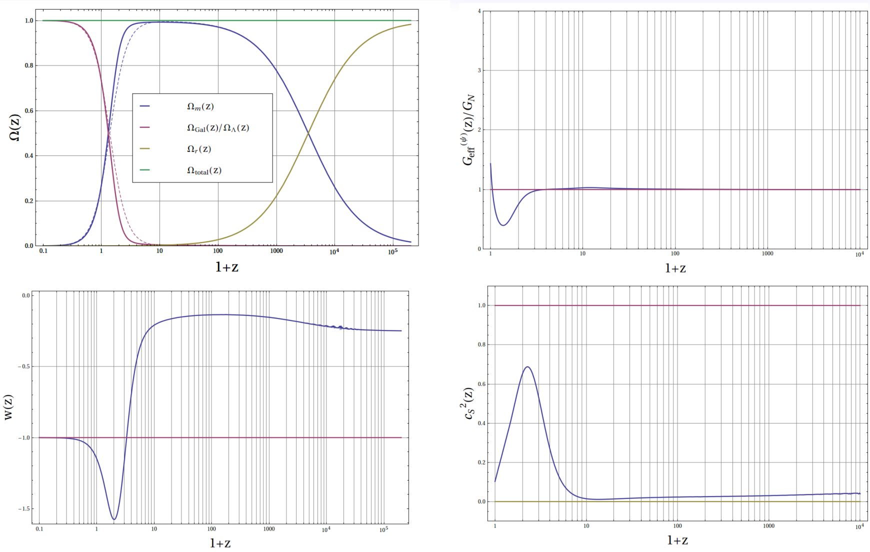


Table: Cosmological constraints on the Galileon model.

Probe	Ω_m^0	$\bar{c}_2$	$\bar{c}_3$	$\bar{c}_4$	χ^2
SNLS3	$0.273^{+0.054}_{-0.042}$	$-5.240^{+1.880}_{-2.802}$	$-1.781^{+1.071}_{-1.426}$	$-0.588^{+0.516}_{-0.348}$	420.1
Growth	$0.200^{+0.047}_{-0.044}$	$-5.430^{+0.850}_{-1.563}$	$-1.757^{+0.365}_{-1.251}$	$-0.635^{+0.272}_{-0.179}$	19.83
BAO+WMAP7+H0	$0.272^{+0.014}_{-0.009}$	$-5.591^{+1.973}_{-2.655}$	$-1.926^{+1.008}_{-1.407}$	$-0.619^{+0.468}_{-0.335}$	2.14
All	$0.271^{+0.013}_{-0.008}$	$-4.352^{+0.518}_{-1.220}$	$-1.597^{+0.203}_{-0.726}$	$-0.771^{+0.098}_{-0.061}$	450.4

- Result of minimization : $h = 0.713$ and $\Omega_b^0 h^2 = 0.0224$ with all data.
- For standard models : $\chi^2_{\Lambda\mathrm{CDM}} = 440.2$ and $\chi^2_{\mathrm{FWCDM}} = 440.2$ (with same program and same data)

Best fit behaviour



Conclusion

3 - Conclusion

- Galileon is a very good candidate to dark energy :
  - good theoretical properties
  - no modification of local gravitation
  - produces accelerated expansion
- **Galileon model in agreement with current data.**
- **Galileon, ΛCDM and FWCDM have equivalent χ^2.**
- Our results were in contradiction with previous works that concluded the Galileon model is ruled out by data (main difference is in the treatment of initial conditions and of the growth of structures)...
- ... but were then confirmed by a paper of Barreira et al., including full CMB power spectrum prediction (arXiv :1302.6241)

Thanks for your attention !

Backup slides

Possible direct couplings to matter

- Linear coupling :

$$L_0 = \frac{c_0}{M_P} \pi T^\mu_\mu$$

- Derivative coupling :

$$L_G = \frac{c_G}{M^3 M_P} T^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$$

$\Rightarrow$ 2 more possible parameters c_0 and c_G

Note

Direct coupling to matter is not mandatory, but has to be weak to preserve solar tests of gravitation.

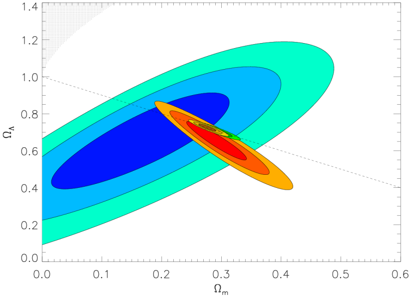


Figure: ΛCDM

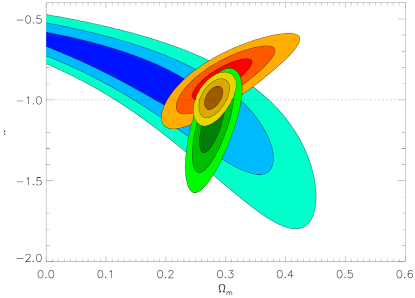


Figure: FWCDM

Blue : SNLS, red : WiggleZ, green : BAO+WMAP7+H0, yellow : all data

(00)-Einstein equation : $\frac{\partial S}{\partial g_{00}} = 0$

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$$x = \pi'/M_P, \quad ' = d/d\ln a, \quad \bar{H} = H/H_0$$

Degeneracy problem !

Equation invariant under a scale factor γ :
$x \mapsto x/\gamma$, $c_2 \mapsto c_2 \times \gamma^2$,
$c_3 \mapsto c_3 \times \gamma^3$, $c_4 \mapsto c_4 \times \gamma^4$, $c_5 \mapsto c_5 \times \gamma^5$!

$\Rightarrow$ *same $\bar{H}(z)$ can be obtain from small x and high c_i's or big x and small c_i's*

$\Rightarrow$ degeneracy to break !

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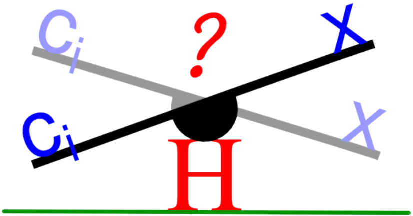
Two solutions :

- x value is known at some point of the history of the Universe : any idea ?
- or we get rid of the degeneracy by a new parametrization



New parametrization

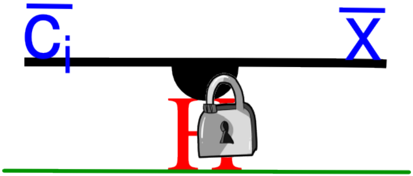
Set $x_0 = x(z=0)$:

$$\boxed{\bar{c}_i = c_i x_0^i,\ \ \bar{x} = x/x_0}$$

$$\bar{H}^2 = \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{c_2}{6}\bar{H}^2 x^2 - 2c_3\bar{H}^4 x^3 + \frac{15}{2}c_4\bar{H}^6 x^4 - 7c_5\bar{H}^8 x^5$$

$$= \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{\bar{c}_2}{6}\bar{H}^2 \bar{x}^2 - 2\bar{c}_3\bar{H}^4 \bar{x}^3 + \frac{15}{2}\bar{c}_4\bar{H}^6 \bar{x}^4 - 7\bar{c}_5\bar{H}^8 \bar{x}^5$$

with $\bar{x}(z=0) = 1 \Rightarrow \bar{x}$ is known at z=0 !



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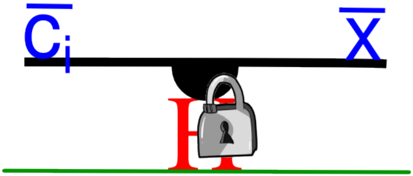
$$\begin{aligned}\bar{H}^2 &= \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{c_2}{6}\bar{H}^2 x^2 - 2c_3\bar{H}^4 x^3 + \frac{15}{2}c_4\bar{H}^6 x^4 - 7c_5\bar{H}^8 x^5 \\ &= \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{\bar{c}_2}{6}\bar{H}^2 \bar{x}^2 - 2\bar{c}_3\bar{H}^4 \bar{x}^3 + \frac{15}{2}\bar{c}_4\bar{H}^6 \bar{x}^4 - 7\bar{c}_5\bar{H}^8 \bar{x}^5\end{aligned}$$

with $\bar{x}(z=0) = 1 \Rightarrow \bar{x}$ is known at z=0 !



$$\text{(00)-Einstein equation : } \frac{\partial S}{\partial g_{00}} = 0$$

$$\Rightarrow \bar{H}^2 = \frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \frac{\bar{c}_2}{6}\bar{H}^2\bar{x}^2 - 2\bar{c}_3\bar{H}^4\bar{x}^3 + \frac{15}{2}\bar{c}_4\bar{H}^6\bar{x}^4 - 7\bar{c}_5\bar{H}^8\bar{x}^5$$

$$\left.\begin{array}{ll} \text{(ij)-Einstein equation : } & \dfrac{\partial S}{\partial g_{ij}} = 0 \\ \pi \text{ equation of motion : } & \dfrac{\partial S}{\partial \pi} = 0 \end{array}\right\} \Rightarrow \left\{ \begin{array}{ll} \bar{H}' & = f(\bar{c}_i, \bar{x}, \bar{H}, \Omega_r^0) \\ \bar{x}' & = g(\bar{c}_i, \bar{x}, \bar{H}, \Omega_r^0) \end{array}\right.$$

- 2 differential equations with 2 known initial conditions :

$$\bar{x}(z=0) = 1, \quad \bar{H}(z=0) = 1$$

- 1 constraint equation ((00)-Einstein) : used to fix $\bar{c}_5$ given Ω_m^0, Ω_r^0 and the other $\bar{c}_i$'s :

$$\bar{c}_5 = \frac{1}{7}(-1 + \Omega_m^0 + \Omega_r^0 + \frac{\bar{c}_2}{6} - 2\bar{c}_3 + \frac{15}{2}\bar{c}_4)$$

Let's compute some Galileon Universe !

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Let's compute some Galileon Universe !

Effective dark energy equation of state

$$\frac{\rho_\pi}{H_0^2 M_P^2} = 6c_0\bar{H}^2x + \frac{c_2}{2}\bar{H}^2x^2 - 6c_3\bar{H}^4x^3 + \frac{45}{2}c_4\bar{H}^6x^4 - 21c_5\bar{H}^8x^5 - 9c_G\bar{H}^4x^2$$

$$\frac{P_\pi}{H_0^2 M_P^2} = -c_0\left[4\bar{H}^2x + 2\bar{H}(\bar{H}x)'\right] + \frac{c_2}{2}\bar{H}^2x^2 + 2c_3\bar{H}^3x^2(\bar{H}x)' - c_4\left[\frac{9}{2}\bar{H}^6x^4 + 12\bar{H}^6x^3x' + 15\bar{H}^5x^4\bar{H}'\right] + 3c_5\bar{H}^7x^4\left(5\bar{H}x' + 7\bar{H}'x + 2\bar{H}x\right) + c_G\left[6\bar{H}^3x^2\bar{H}' + 4\bar{H}^4xx' + 3\bar{H}^4x^2\right]$$

$$w_\pi \equiv P_\pi/\rho_\pi$$

Linear perturbations of Galileon field :

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

$$\text{(00) Einstein} \Rightarrow \frac{1}{2}\kappa_4\bar{\nabla}^2\psi - \kappa_3\bar{\nabla}^2\phi = \kappa_1\bar{\nabla}^2\delta y$$

$$\text{(ij) Einstein} \Rightarrow \kappa_5\bar{\nabla}^2\delta y - \kappa_4\bar{\nabla}^2\phi = \frac{a^2\rho_m}{H_0^2 M_P^2}\delta_m$$

$$\pi \text{ EoM} \Rightarrow \frac{1}{2}\kappa_5\bar{\nabla}^2\psi - \kappa_1\bar{\nabla}^2\phi = \kappa_6\bar{\nabla}^2\delta y$$

$$\text{matter EoS} \Rightarrow \bar{H}^2\delta_m'' + \bar{H}\bar{H}'\delta_m' + 2\bar{H}^2\delta_m' = \frac{1}{a^2}\bar{\nabla}^2\psi$$

where $\delta y = \delta\pi/M_P$, $\bar{\nabla} = \nabla/H_0$, ρ_m matter density, $\delta_m = \delta\rho_m/\rho_m$ contrast matter density and κ_is :

$$\kappa_1 = -6\bar{c}_4\bar{H}^3\bar{x}^3\left(\bar{H}'\bar{x} + \bar{H}\bar{x}' + \frac{\bar{H}\bar{x}}{3}\right)$$
$$+ \bar{c}_5\bar{H}^5\bar{x}^3(12\bar{H}\bar{x}' + 15\bar{H}'\bar{x} + 3\bar{H}\bar{x})$$
$$\kappa_3 = -1 - \frac{\bar{c}_4}{2}\bar{H}^4\bar{x}^4 - 3\bar{c}_5\bar{H}^5\bar{x}^4(\bar{H}'\bar{x} + \bar{H}\bar{x}')$$
$$\kappa_4 = -2 + 3\bar{c}_4\bar{H}^4\bar{x}^4 - 6\bar{c}_5\bar{H}^6\bar{x}^5$$
$$\kappa_5 = 2\bar{c}_3\bar{H}^2\bar{x}^2 - 12\bar{c}_4\bar{H}^4\bar{x}^3 + 15\bar{c}_5\bar{H}^6\bar{x}^5$$
$$\kappa_6 = \frac{\bar{c}_2}{2} - 2\bar{c}_3(\bar{H}^2\bar{x}' + \bar{H}\bar{H}'\bar{x} + 2\bar{H}^2\bar{x})$$
$$+ \bar{c}_4(12\bar{H}^4\bar{x}\bar{x}' + 18\bar{H}^3\bar{x}^2\bar{H}' + 13\bar{H}^4\bar{x}^2)$$
$$- \bar{c}_5(18\bar{H}^6\bar{x}^2\bar{x}' + 30\bar{H}^5\bar{x}^3\bar{H}' + 12\bar{H}^6\bar{x}^3).$$

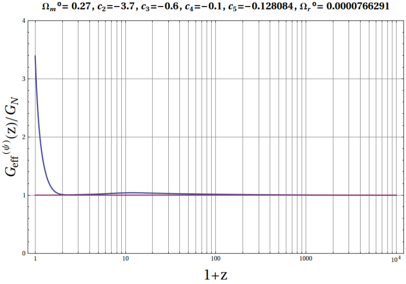
We obtain a new Poisson equation for gravity at sub-horizon scales :

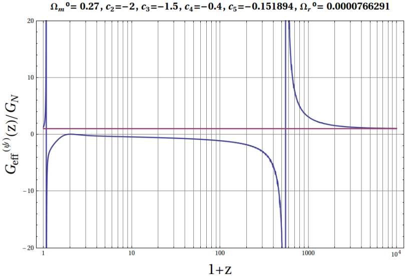
$$\nabla^2\psi = 4\pi a^2 G_{eff}^{(\psi)}(z)\rho_m\delta_m$$

$$G_{\text{eff}}^{(\psi)}(z) = \frac{4(\kappa_3\kappa_6 - \kappa_1^2)}{\kappa_5(\kappa_4\kappa_1 - \kappa_5\kappa_3) - \kappa_4(\kappa_4\kappa_6 - \kappa_5\kappa_1)} G_N$$

and other quantities :

- $\delta\pi$ kinetic normalisation factor
- speed of propagation


$\Omega_m{}^o$= 0.27, c_2=−3.7, c_3=−0.6, c_4=−0.1, c_5=−0.128084, $\Omega_r{}^o$= 0.0000766291
$G_{\rm eff}{}^{(\psi)}(z)/G_N$
1+z


$\Omega_m{}^o$= 0.27, c_2=−2, c_3=−1.5, c_4=−0.4, c_5=−0.151894, $\Omega_r{}^o$= 0.0000766291
$G_{\rm eff}{}^{(\psi)}(z)/G_N$
1+z

A good Galileon cosmological scenario must have $\forall z > 0$:

- for the field perturbations $\delta\pi$:
  1. positive kinetic term normalization (no-ghost condition) : $Q_S^2 > 0$
  2. positive squared sound speed (no instabilities) : $c_s^2 > 0$
- for the metric tensorial perturbations :
  3. positive kinetic term normalization (no-ghost condition) : $Q_T^2 > 0$
  4. positive squared sound speed (no instabilities) : $c_T^2 > 0$

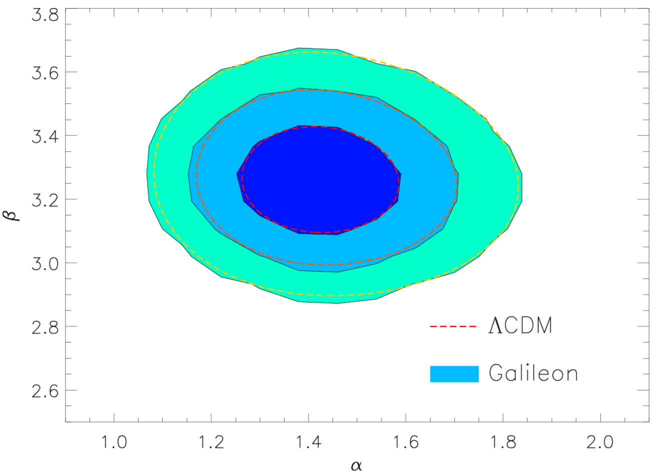


Figure: Confidence contours for the SN nuisance parameters α and β.

Table: Cosmological constraints on the Galileon model from the SNLS3 sample

Method	Ω_m^0	$\bar{c}_2$	$\bar{c}_3$	$\bar{c}_4$	α	β	$\mathcal{M}_B^1$	$\mathcal{M}_B^2$	χ^2
Stat+sys+$\alpha\beta$	$0.273^{+0.057}_{-0.042}$	$-5.235^{+1.875}_{-2.767}$	$-1.779^{+1.073}_{-1.416}$	$-0.587^{+0.515}_{-0.349}$	$1.428^{+0.121}_{-0.098}$	$3.263^{+0.121}_{-0.103}$	23.997	23.950	415.4
Stat+sys	$0.273^{+0.054}_{-0.042}$	$-5.240^{+1.880}_{-2.802}$	$-1.781^{+1.071}_{-1.426}$	$-0.588^{+0.516}_{-0.348}$	1.428	3.263	23.997	23.950	420.1
Stat only	$0.294^{+0.045}_{-0.039}$	$-4.765^{+1.725}_{-2.921}$	$-1.586^{+0.987}_{-1.474}$	$-0.541^{+0.502}_{-0.338}$	1.451	3.165	24.022	23.951	441.8

Table: WMAP7 measurements

l_a	302.09 ± 0.76
R	1.725 ± 0.018
z_*	1091.3 ± 0.91

+ covariances

Table: BAO measurements used in this work.

z	$y_s^{mes}(z)$	Surveys
0.106	0.336 ± 0.015	6dF
0.35	0.1126 ± 0.0022	SDSS-II
0.57	0.0732 ± 0.0012	BOSS

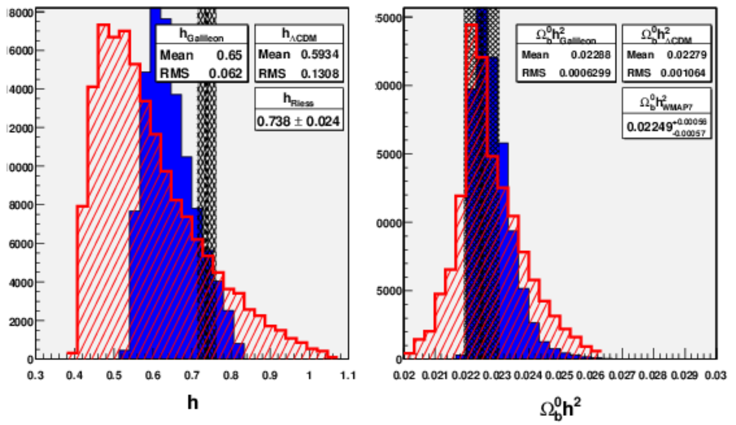
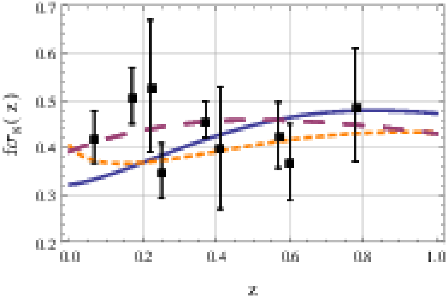


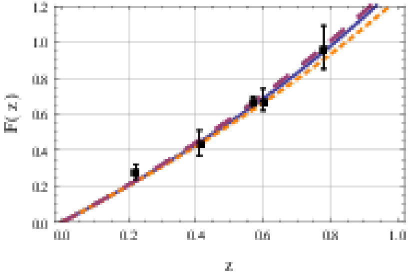
Figure: Minimized values of h and $\Omega_b^0 h^2$ for a large subset of tested scenarios, in ΛCDM (red dashed histogram) and in the Galileon cosmology (blue filled histogram).

Table: Growth data

z	$f\sigma_8(z)$	$F(z)$	r	Survey
0.067	0.423 ± 0.055	-	-	6dFGRS (a)
0.17	0.51 ± 0.06	-	-	2dFGRS (a)
0.22	0.53 ± 0.14	0.28 ± 0.04	0.83	WiggleZ
0.25	0.351 ± 0.058	-	-	SDSS LRG (b)
0.37	0.460 ± 0.038	-	-	SDSS LRG (b)
0.41	0.40 ± 0.13	0.44 ± 0.07	0.94	WiggleZ
0.57	0.430 ± 0.067	0.677 ± 0.042	0.871	BOSS CMASS
0.6	0.37 ± 0.08	0.68 ± 0.06	0.89	WiggleZ
0.78	0.49 ± 0.12	0.49 ± 0.12	0.84	WiggleZ

r is the cross-correlation in $(F, f\sigma_8)$. (a) Alcock-Paczynski effect is negligible at low redshift. (b) Values of $f\sigma_8$ are corrected for the Alcock-Paczynski effect but no $F(z)$ values are provided.

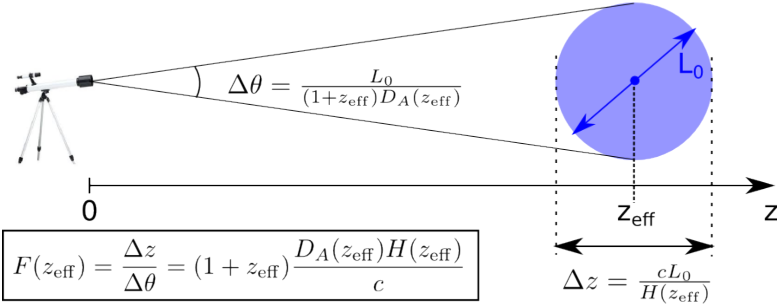




Dashed purple : ΛCDM best fit
Solid blue : Galileon best fit
Dashed orange : Galileon best fit using only growth data

Alcock-Paczynski effect

See Blake et al. 2011 (arXiv :1108.2637)
"The Alcock-Paczynski test (Alcock & Paczynski 1979) is a geometric probe of the cosmological model based on the comparison of the observed tangential and radial dimensions of objects which are assumed to be isotropic in the correct choice of model."



$$F(z_{\rm eff}) = \frac{\Delta z}{\Delta\theta} = (1+z_{\rm eff})\frac{D_A(z_{\rm eff})H(z_{\rm eff})}{c}$$

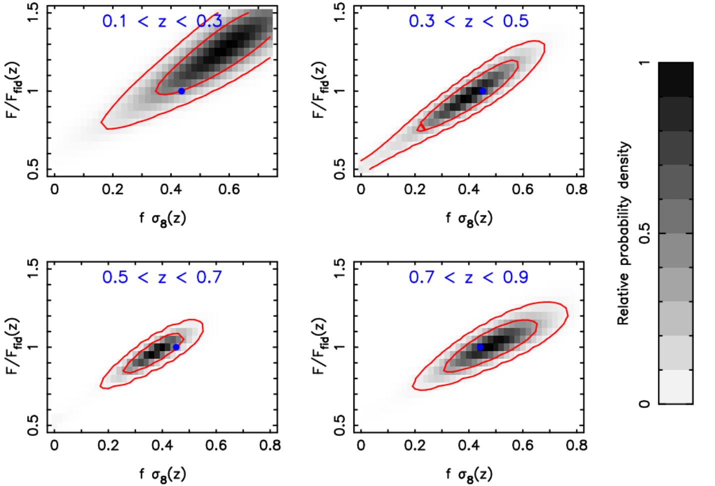


Figure 2. This Figure displays the joint likelihood of the Alcock-Paczynski scale distortion parameter $F(z)$ relative to the fiducial value $F_{\rm fid}$, and the growth rate quantified by $f\,\sigma_8(z)$, obtained from fits to the 2D galaxy power spectra of the WiggleZ Dark Energy Survey in four redshift slices. In order to produce this Figure we marginalized over the linear galaxy bias b^2 and the pairwise velocity dispersion σ_v. There is some degeneracy between F and $f\,\sigma_8$ but their characteristic dependence on the angle to the line-of-sight is sufficiently different that both parameters may be successfully extracted. The probability density is plotted as both greyscale and contours enclosing 68% and 95% of the total likelihood. The solid circles indicate the parameter values in our fiducial cosmological model.

- Finding x_0 thanks to (00) Einstein equation :

$$1-\Omega_m^0-\Omega_r^0-\frac{1}{6}c_2x_0^2+2c_3x_0^3-\frac{15}{2}c_4x_0^4+7c_5x_0^5=0$$

- At most 5 solutions to consider
- To a small x_0 corresponds high c_i's and vice versa.
- Leads to unstable contours.

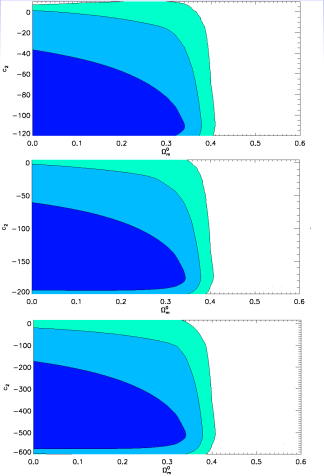


Figure: SNLS3 data.

Protecting local gravity with Vainshtein screening effect

Important remark

If the Galileon is not coupled to matter, there is no effect on local gravitation. Only direct coupling is studied.

Equation of motion :

$$\frac{\delta L}{\delta \pi} = 0 \Leftrightarrow \sum_{i=1}^{5} c_i E_i + \frac{c_0}{M_P} T^\mu_\mu = 0 \quad , \quad E_i = \frac{\delta L_i}{\delta \pi}$$

To study the Galileon effect near a massive object :

- background solution in de Sitter Universe (negligible matter, $\dot{H} = 0$, $w \approx -1$) : π_{dS}
- study of Galileon perturbation with the eom due to the presence of a point-like massive object :

$$\pi \rightarrow \pi_{dS} + \pi$$

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$$\text{eom of the perturbation} \Leftrightarrow \sum_{i=2}^{5} d_i E_i = -\frac{c_0}{M_P} T^{\mu}_{\mu}$$

with d_i linear combinations of the c_i.

Spherical solution around an object of mass m :

$$\sum_{i=2}^{5} d_i E_i = \frac{c_0}{M_P} m\delta(\vec{r})$$

$$\Rightarrow d_2 \left(\frac{1}{r}\frac{d\pi}{dr}\right) + 2\frac{d_3}{M^3}\left(\frac{1}{r}\frac{d\pi}{dr}\right)^2 + 2\frac{d_4}{M^6}\left(\frac{1}{r}\frac{d\pi}{dr}\right)^3 = \frac{c_0}{M_P}\frac{m}{4\pi r^3}$$

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Force created by the perturbation of the Galileon field around m :

$$F_\pi = \frac{1}{M_P}\frac{d\pi}{dr}$$

Newtonian gravitational field created by the mass m :

$$F_N = \frac{m}{M_P^2 r^2}$$

At small distances :

$$2\frac{d_4}{M^6}\left(\frac{1}{r}\frac{d\pi}{dr}\right)^3 = \frac{c_0}{M_P}\frac{m}{4\pi r^3} \Leftrightarrow F_\pi = \left(\frac{c_0 m M^6}{8\pi d_4 M_P}\right)^{1/3}$$

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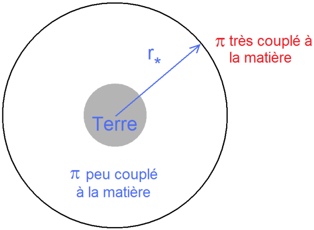
We introduce Vainshtein radius r_v :

$$\frac{F_\pi}{F_N} = \left(\frac{c_0 H_0^4 M_P^4}{8\pi d_4 m^2}\right)^{1/3} r^2 = \left(\frac{r}{r_v}\right)^2 \Rightarrow r_v = \left(\frac{8\pi d_4 m^2}{c_0 H_0^4 M_P^4}\right)^{1/6}$$

So, for distances $r << r_v$, the Galileon field has no effect on gravity.
Numerical application for $d_4 \approx c_0 \approx 1$:

- Sun : $r_v \approx 2650$ pc » solar system
- Earth : $r_v \approx 25$ pc » solar system



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