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# Horizons & Spheres: The Superluminal Expansion

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# Outline

- I) Concepts to understand by the end of this talk
- II) Hubble Law, Hubble Sphere and Event Horizon
- III) Mind-blowing animations
- IV) Concepts not to understand (i.e. my real work)



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## I) Concepts to understand by the end of this talk

- Expansion of the Universe / Current accelerated expansion  
(Hubble law) (Dark Energy)
- Some galaxies recede from us at superluminal speed...?!  
(yes, they do. They totally do)  
But no problem with Special Relativity.



## I) Concepts to understand by the end of this talk

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(Hubble law) (Dark Energy)
- Some galaxies recede from us at superluminal speed...?!  
(yes, they do. They totally do)  
But no problem with Special Relativity.
- It is doable to present 238 slides in 20 minutes.



## I) Concepts to understand by the end of this talk

- Hubble Sphere & Event Horizon
- Let us consider a galaxy beyond our Hubble Sphere, but within our Event Horizon. Therefore it is receding from us ..... than the speed of light.



## I) Concepts to understand by the end of this talk

- Hubble Sphere & Event Horizon
- Let us consider a galaxy beyond our Hubble Sphere, but within our Event Horizon. Therefore it is receding from us FASTER than the speed of light. However, we will be able to see it as it is right now, in the future!



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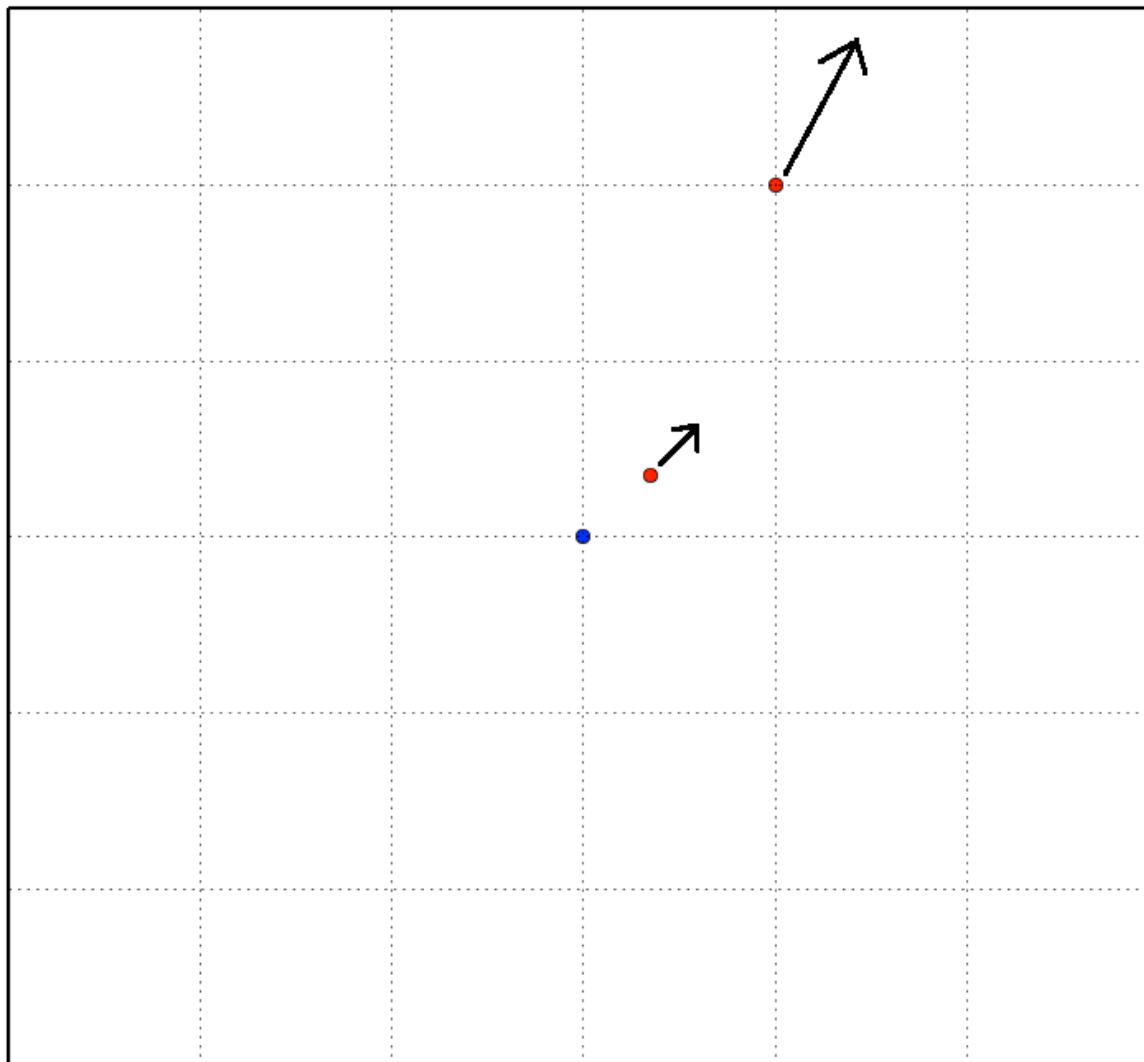
## II) Hubble Law, Hubble Sphere and Event Horizon

- Hubble Law:

$$v_{rec} = H.d$$



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## II) Hubble Law, Hubble Sphere and Event Horizon

- Hubble Law:

$$v_{rec} = H.d$$

- This is the law for expansion (Edwin Hubble, 1929). Not to be confused with current phase of accelerated expansion (Ries/Perlmutter & al., 1998).
- The more we increase the distance, the greater the recession velocity. If we go far enough, we can reach the speed of light!
- In fact, nothing prevents us from going further still, and having recession velocities greater than c.



## II) Hubble Law, Hubble Sphere and Event Horizon

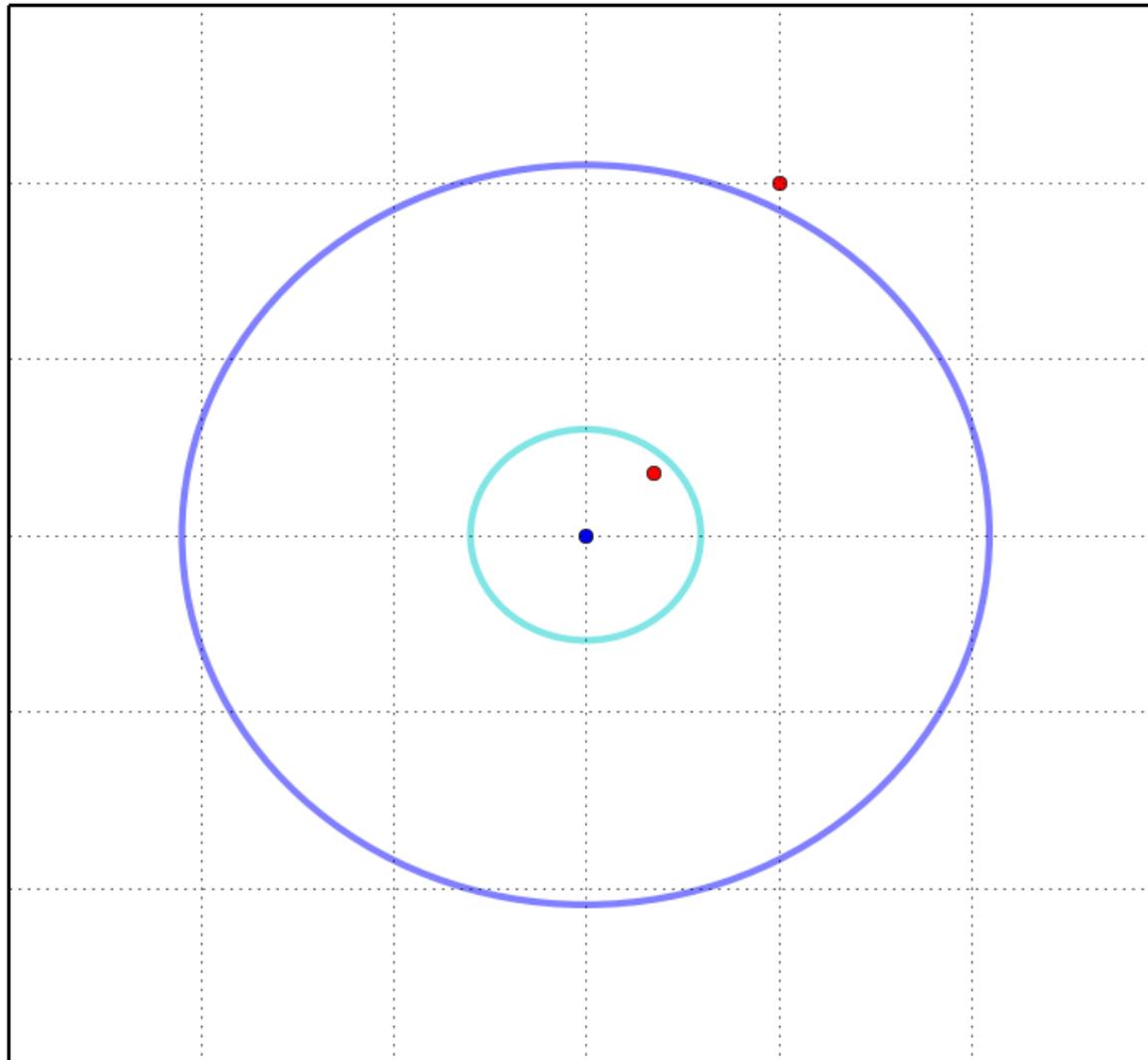
- Hubble Sphere:

$$v_{rec} = H \cdot d = c \Rightarrow d = \frac{c}{H} = R_H$$

- Divides the Universe into regions with **subluminal** expansion (inside the sphere) and regions with **superluminal** expansion (outside).
- The Event Horizon is the sphere that divides all the comoving objects into two families: those that we will one day be able to see as they are right now, and those that we will **NEVER** be able to see.



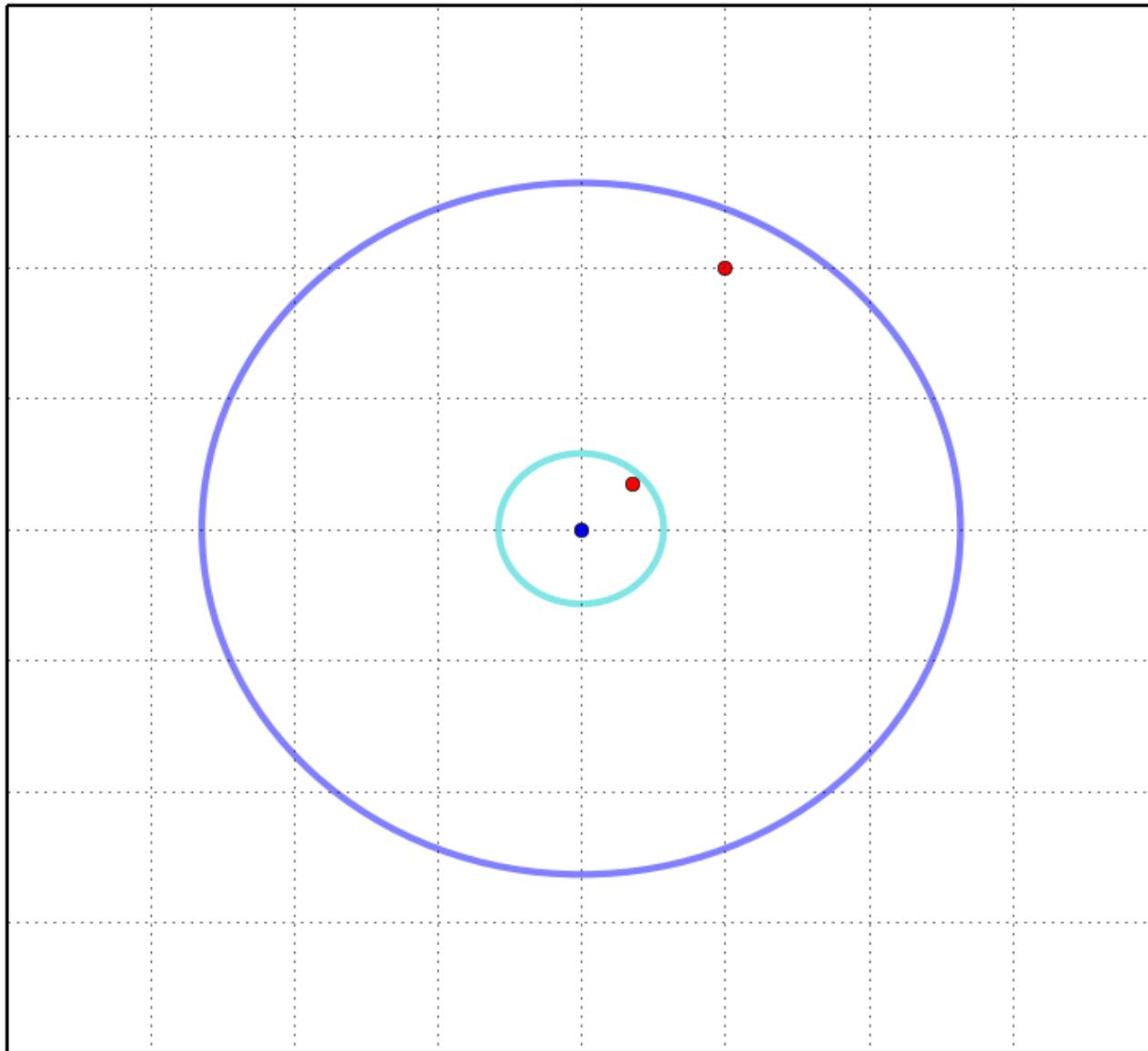
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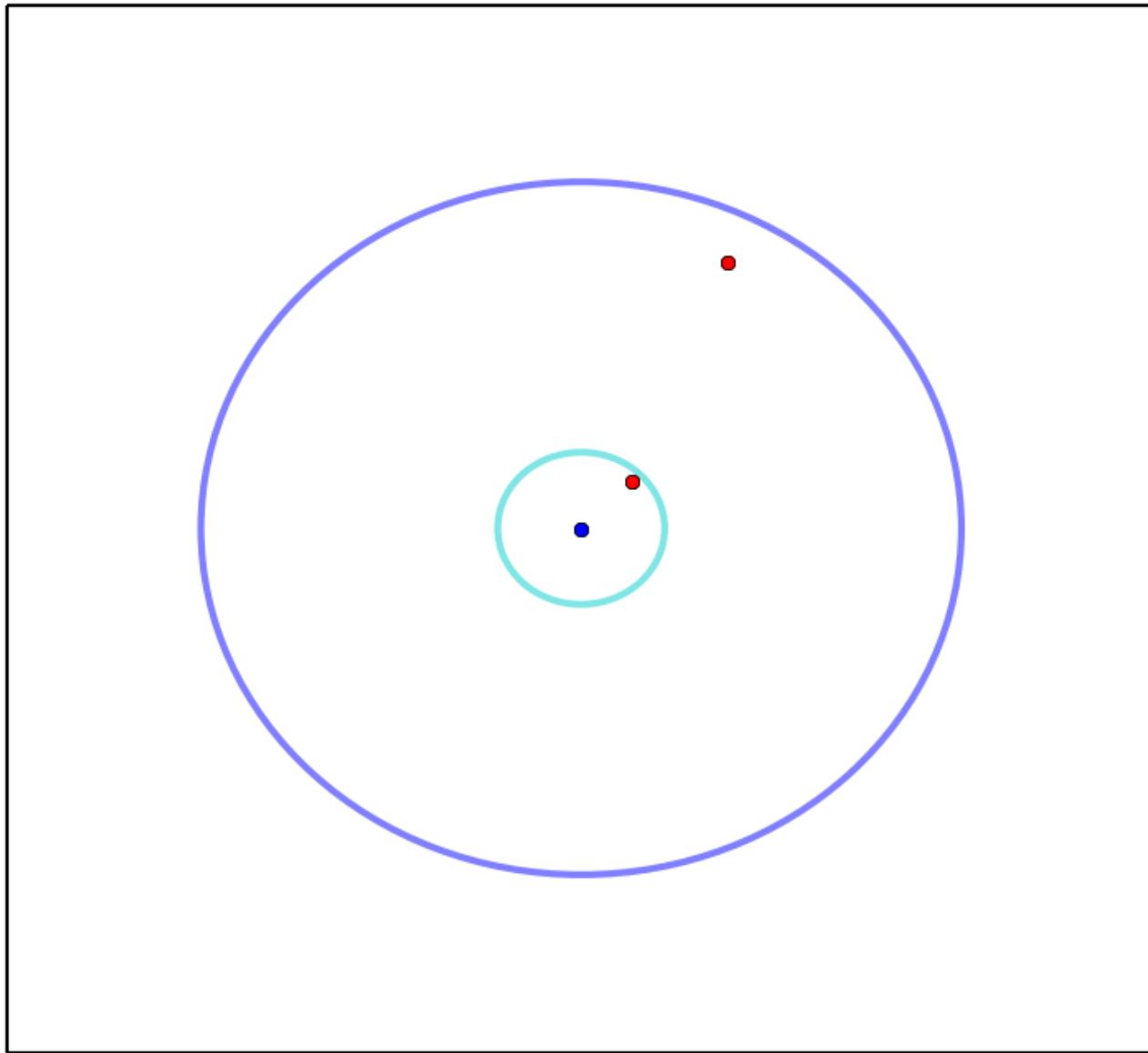


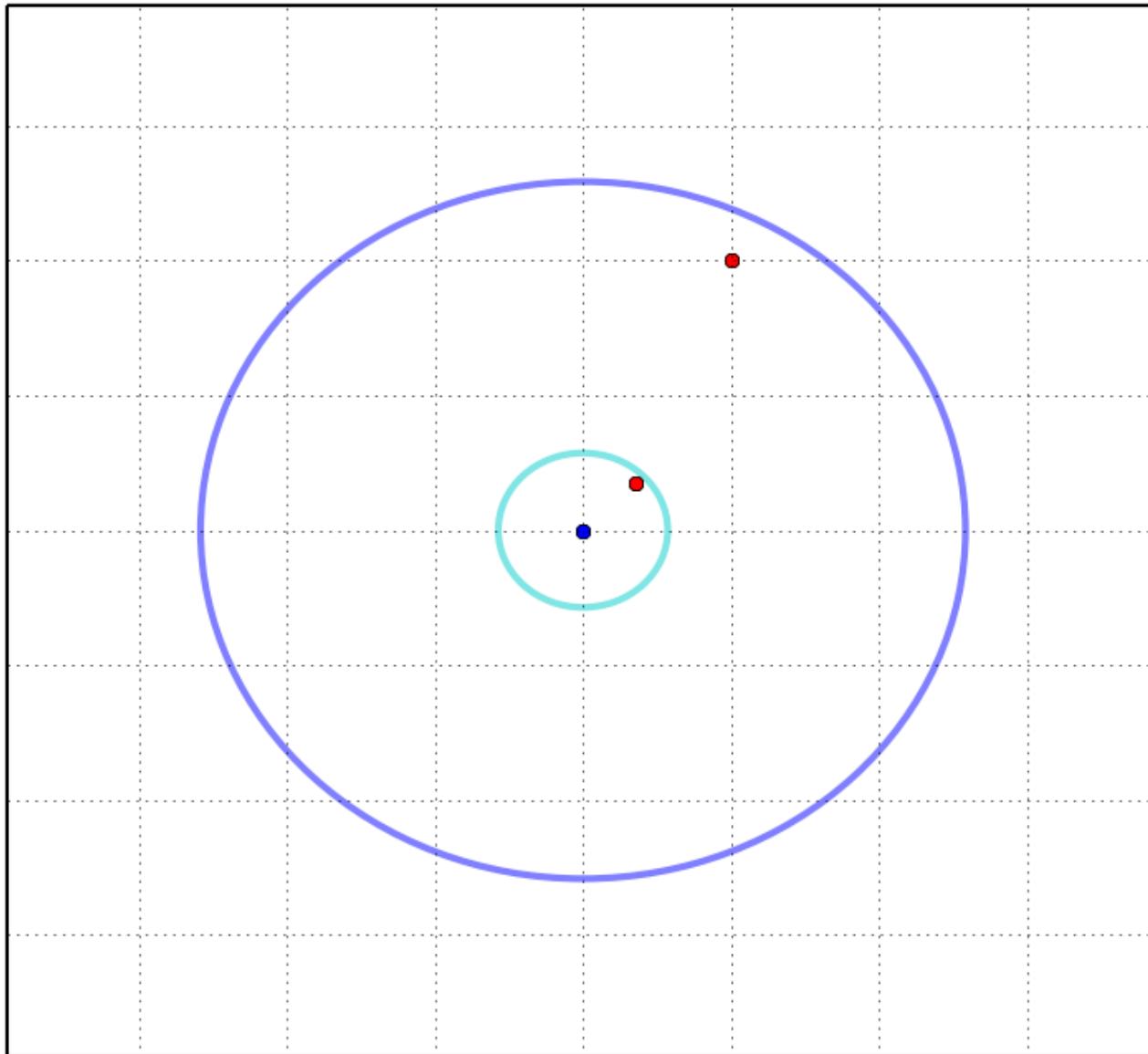
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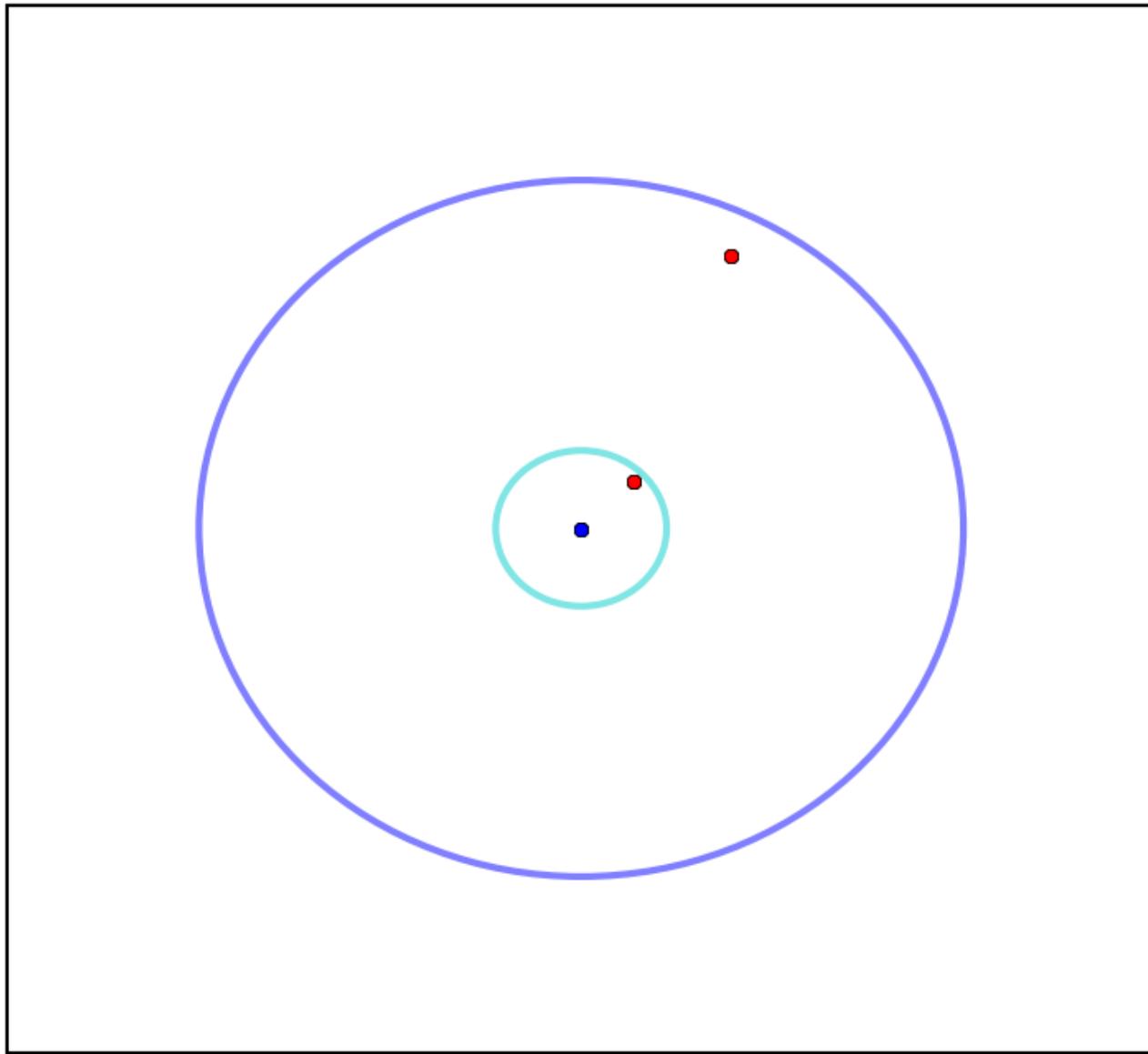


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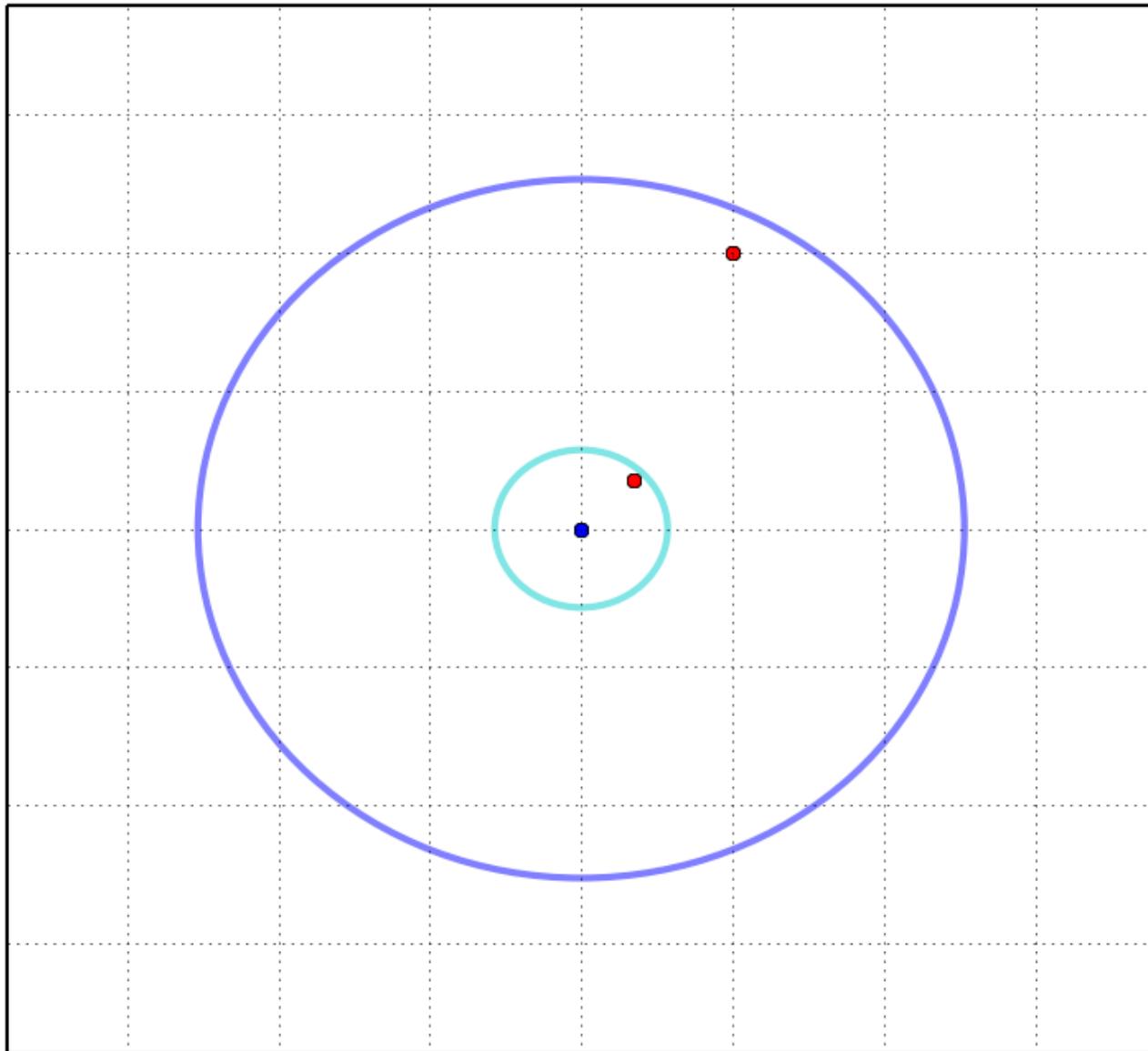




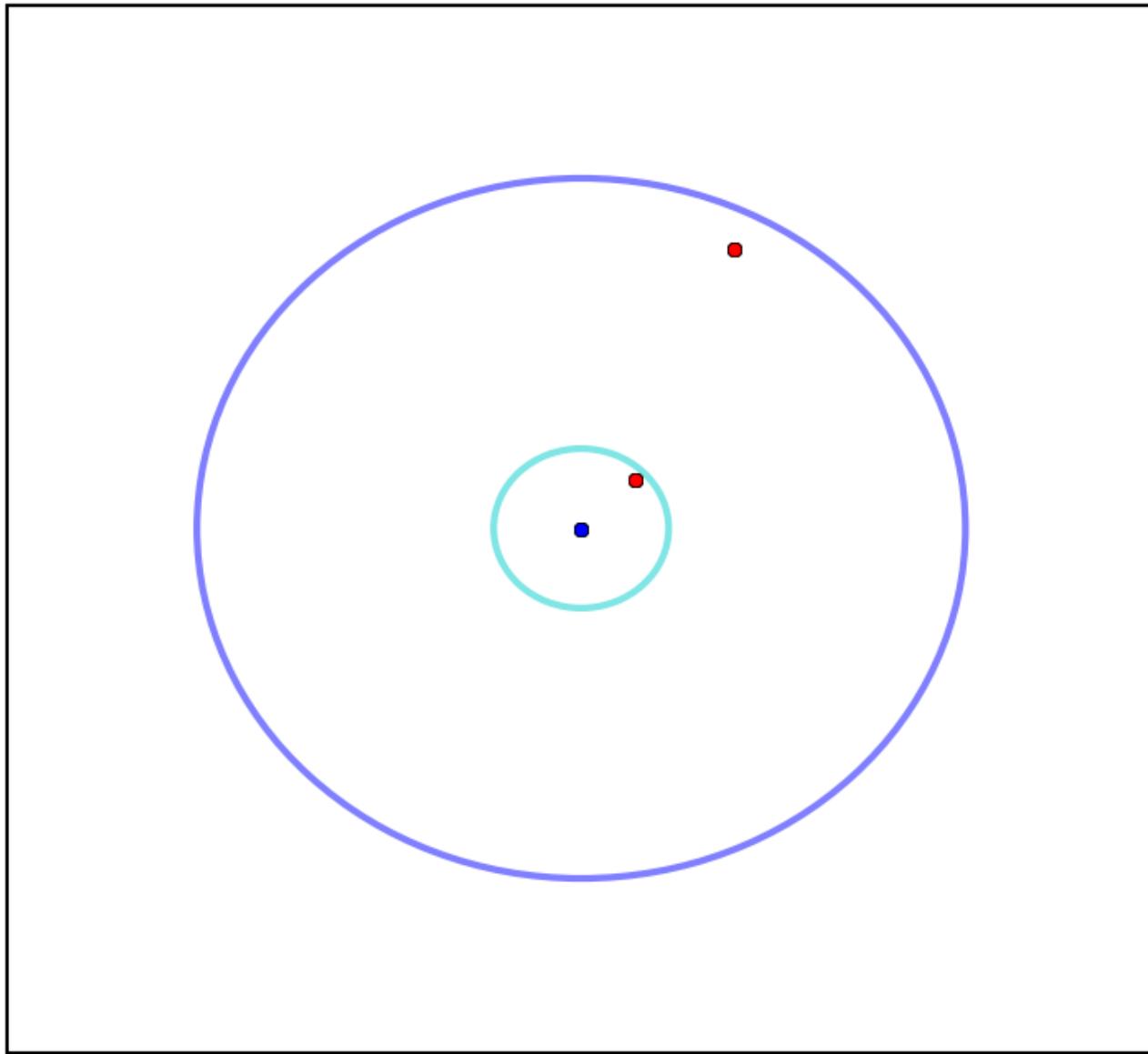
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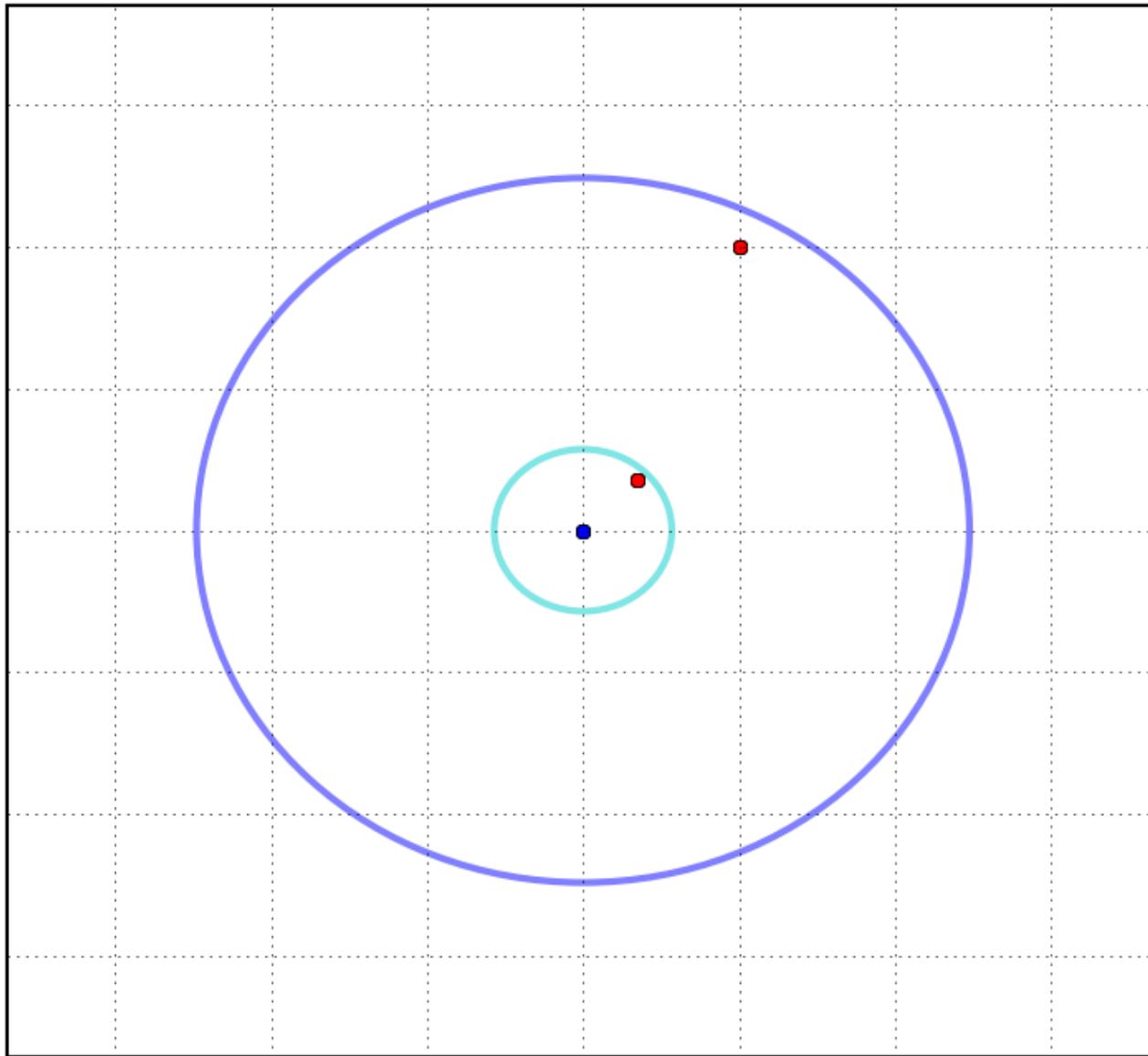


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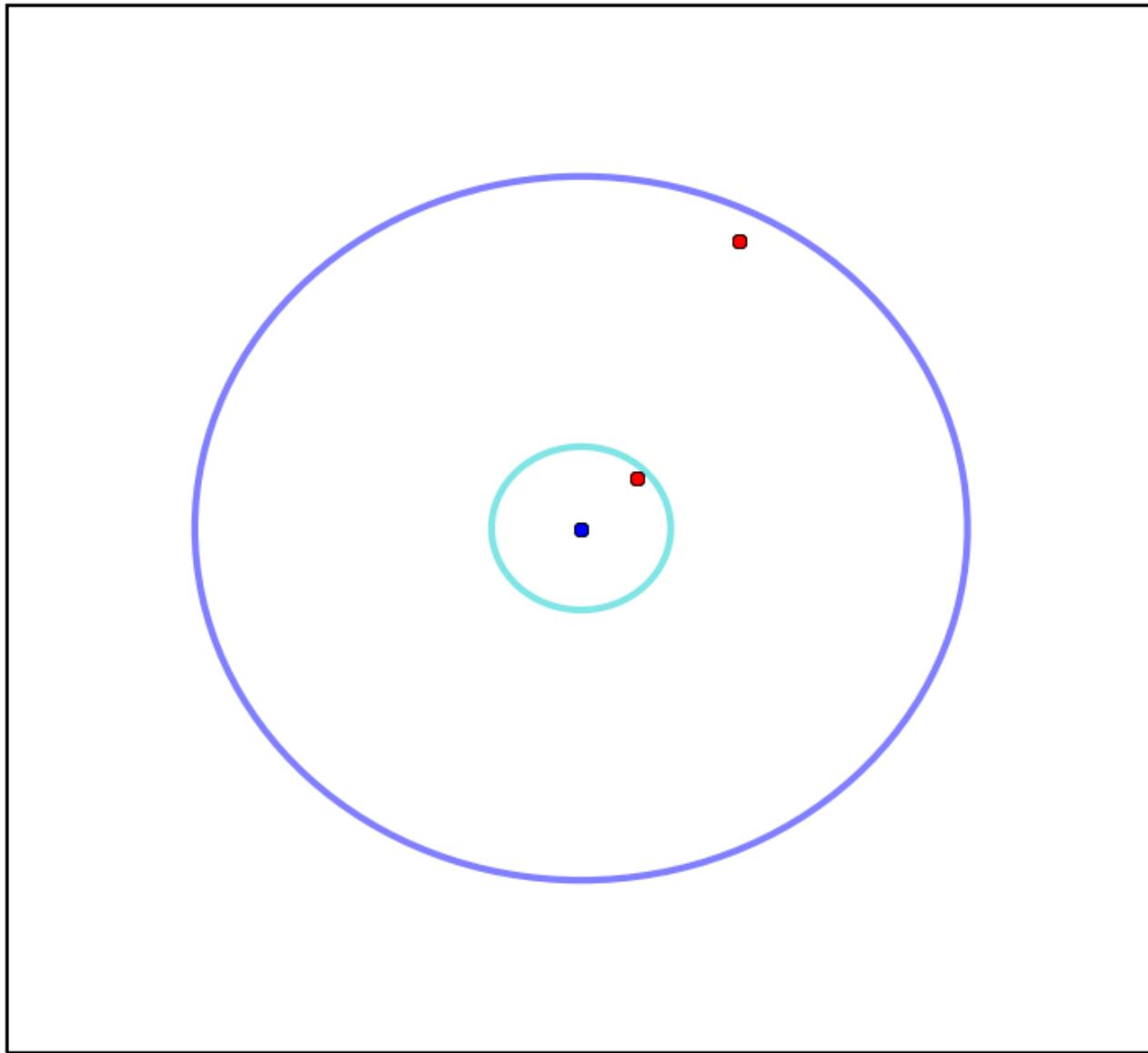


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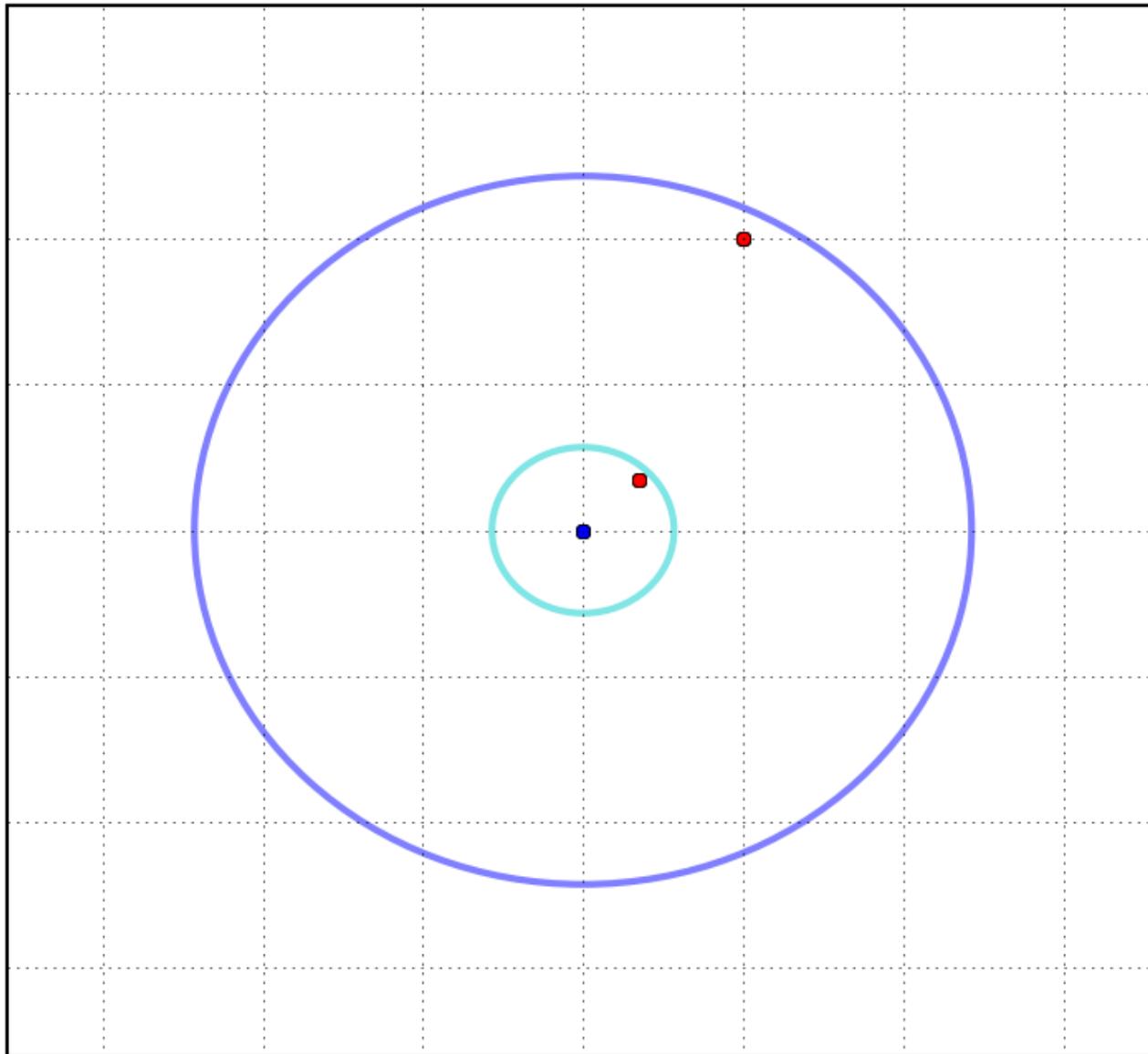




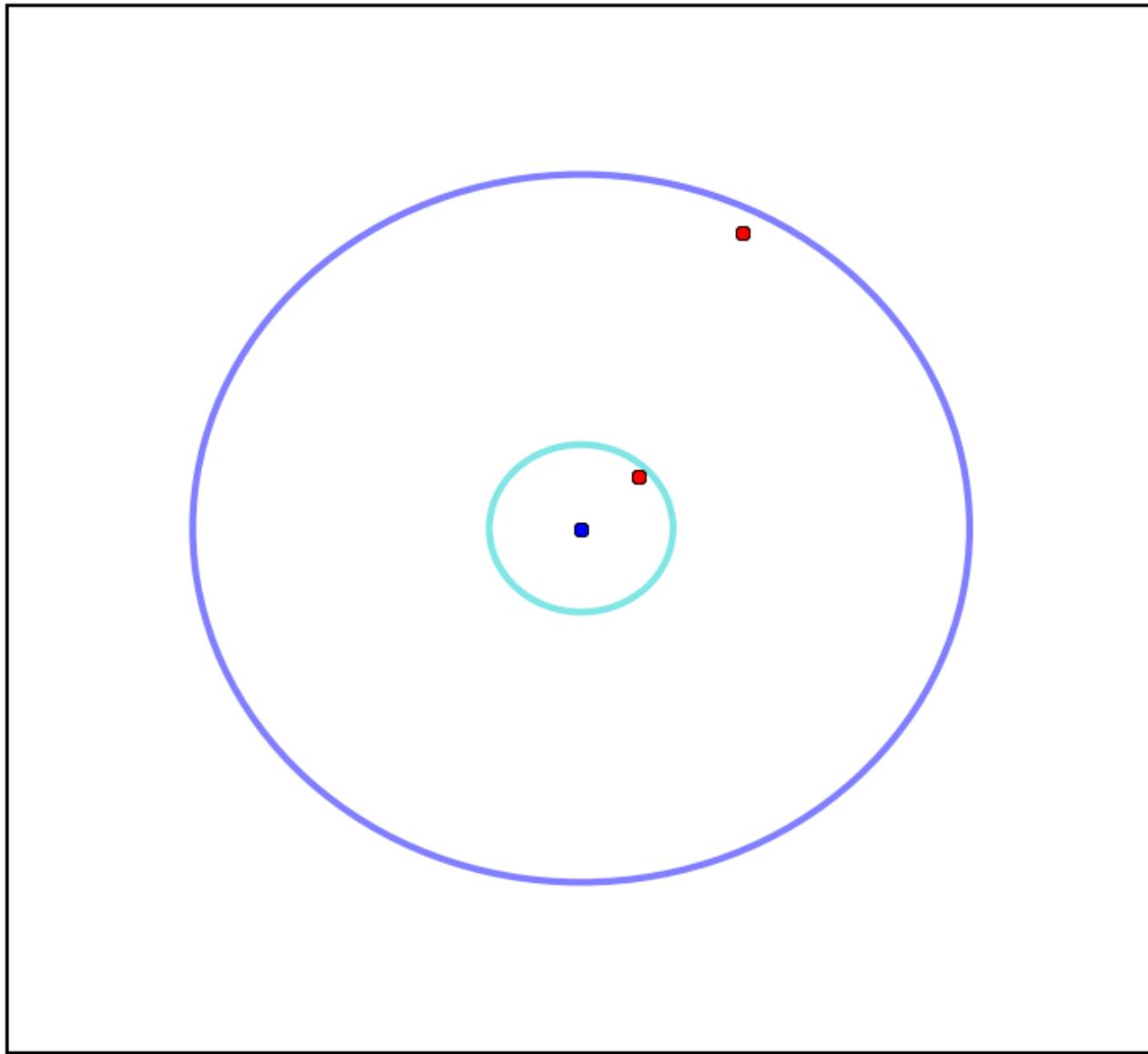
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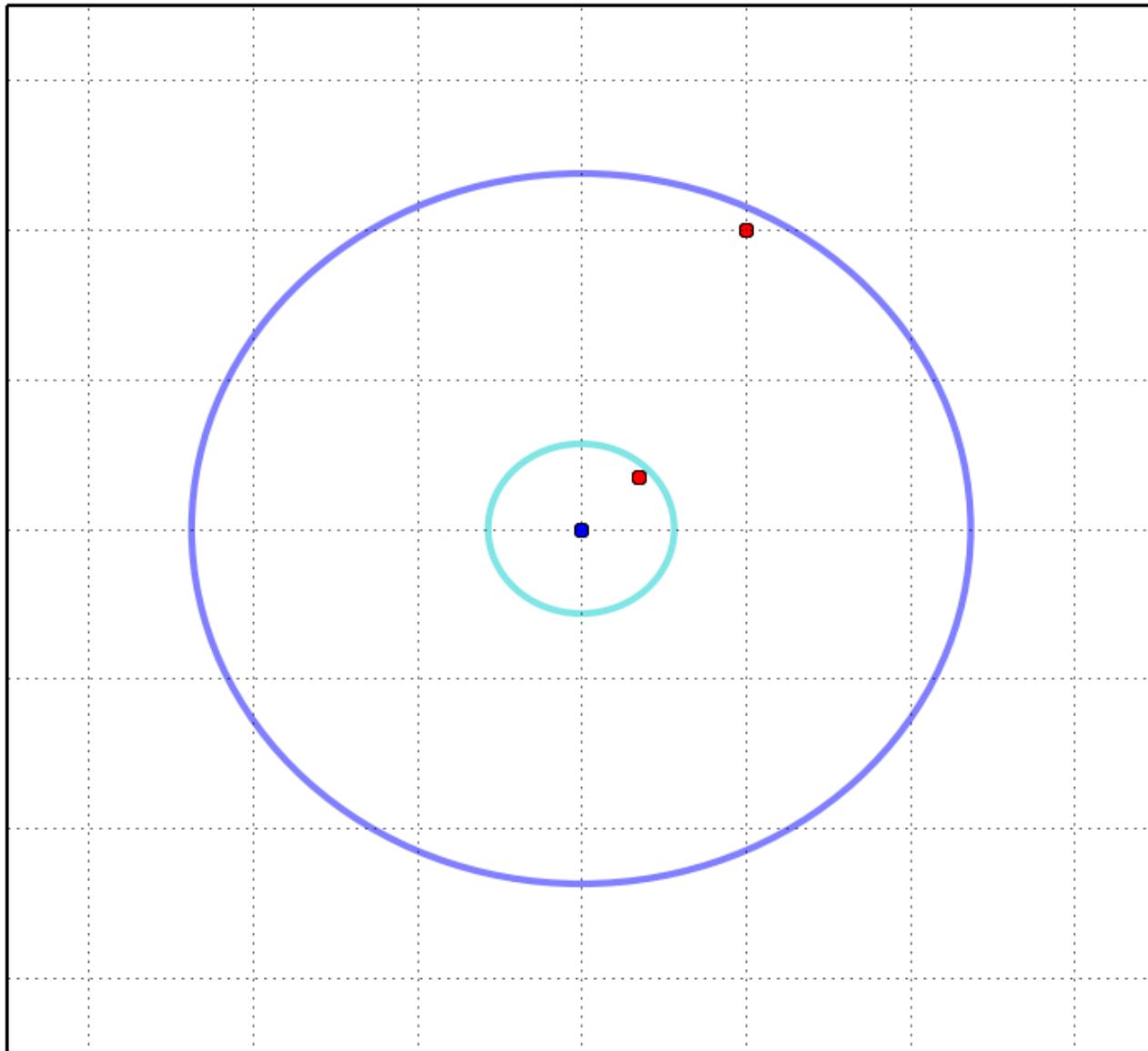


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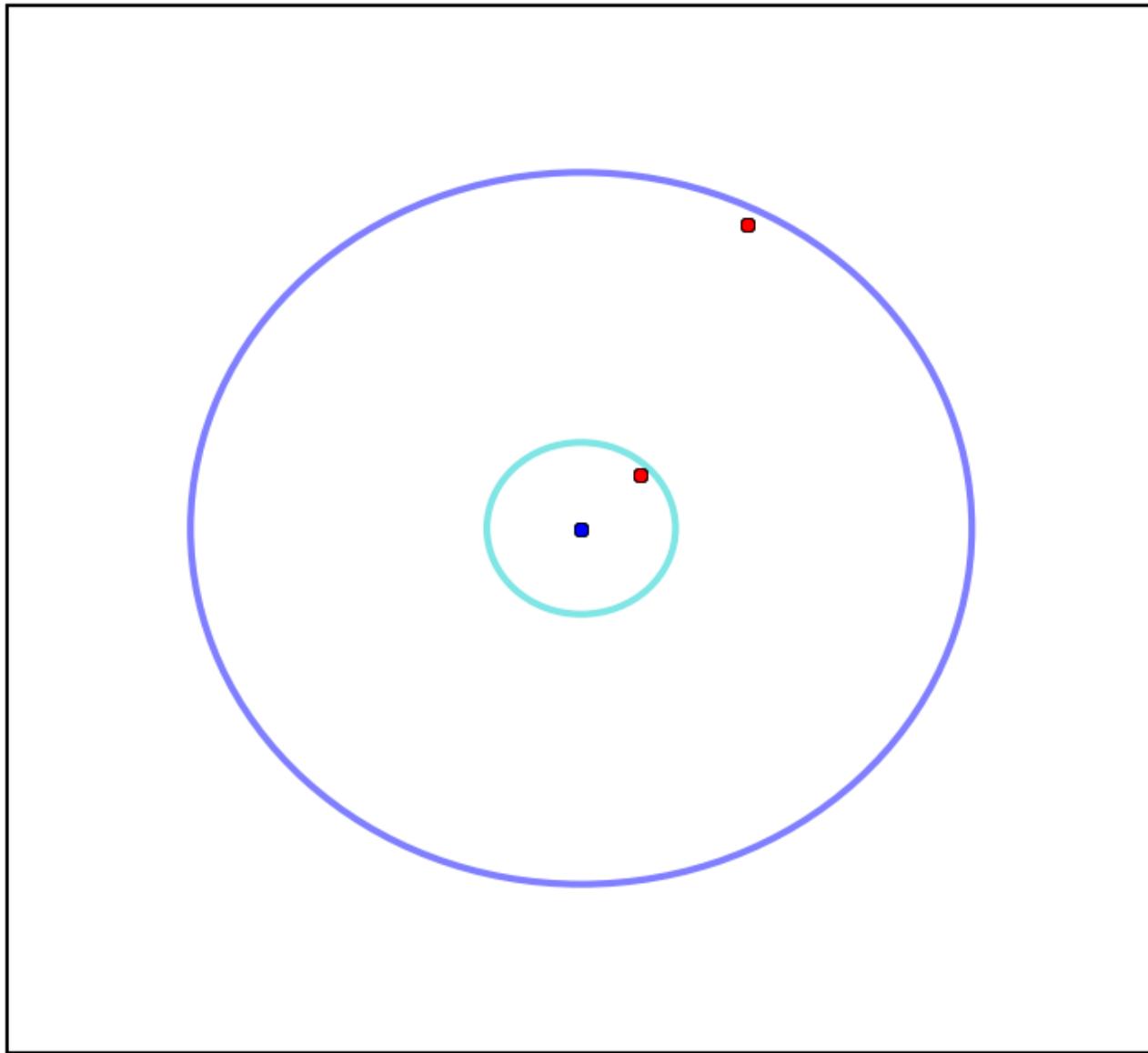


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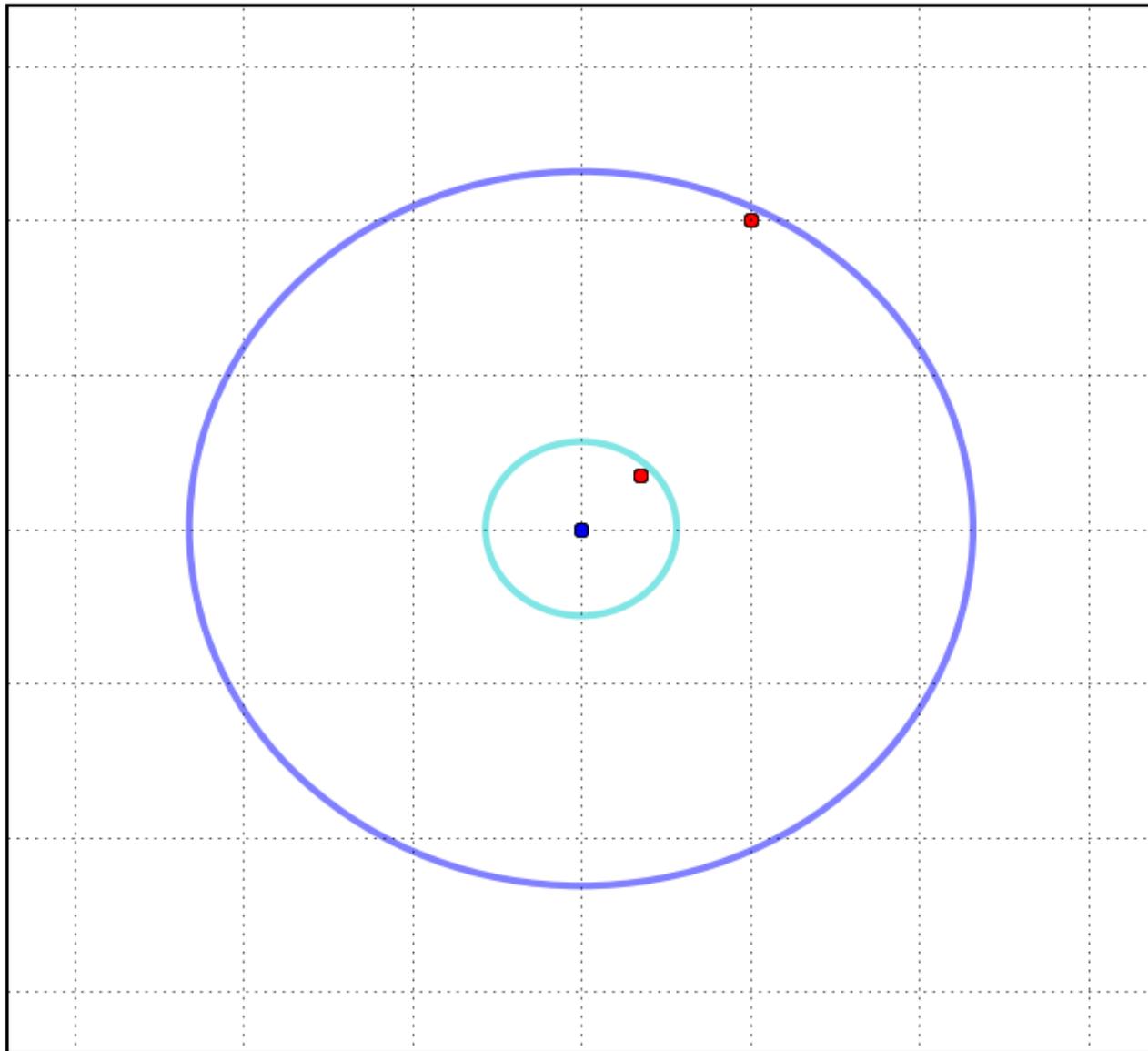




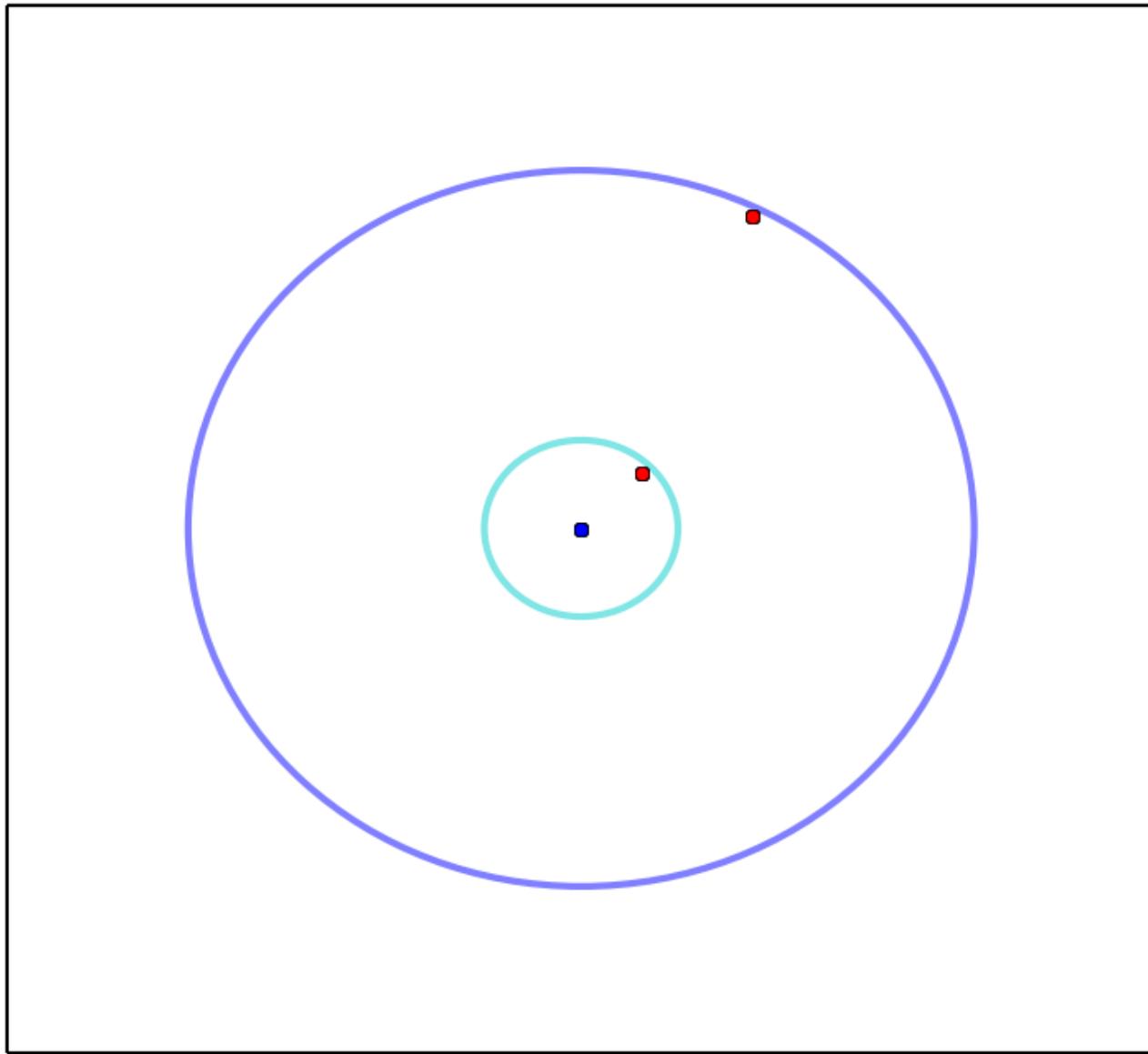
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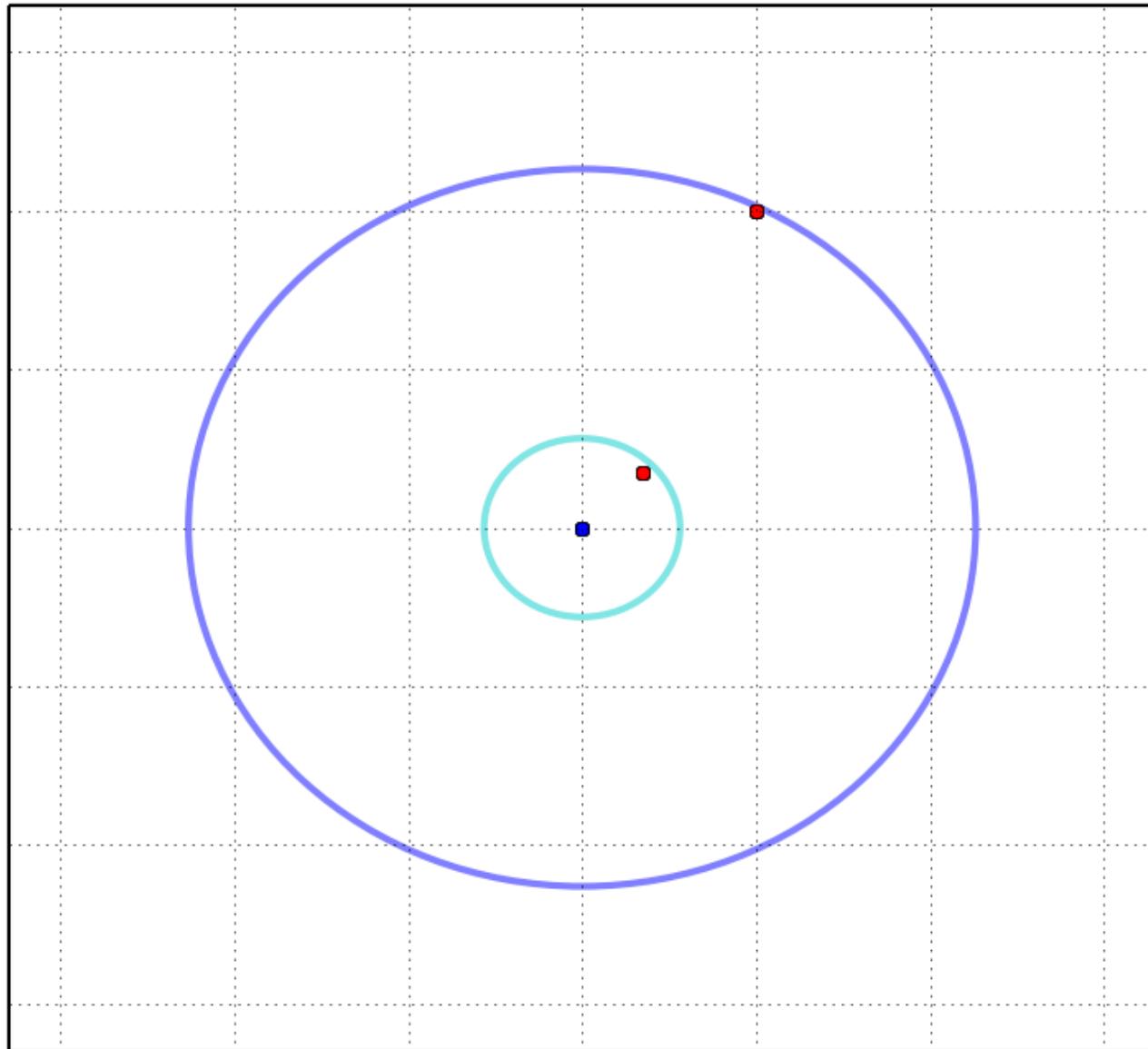
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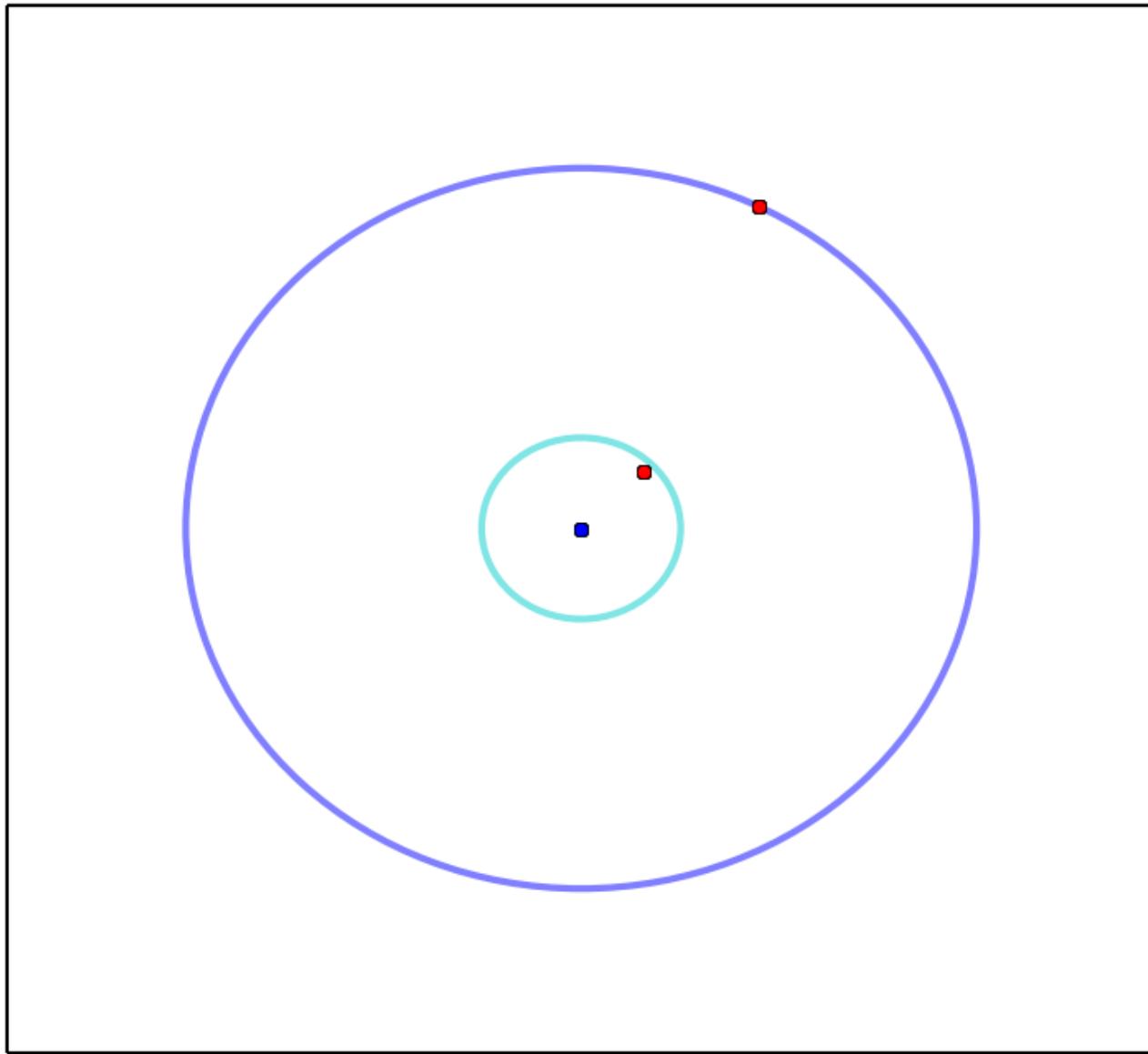
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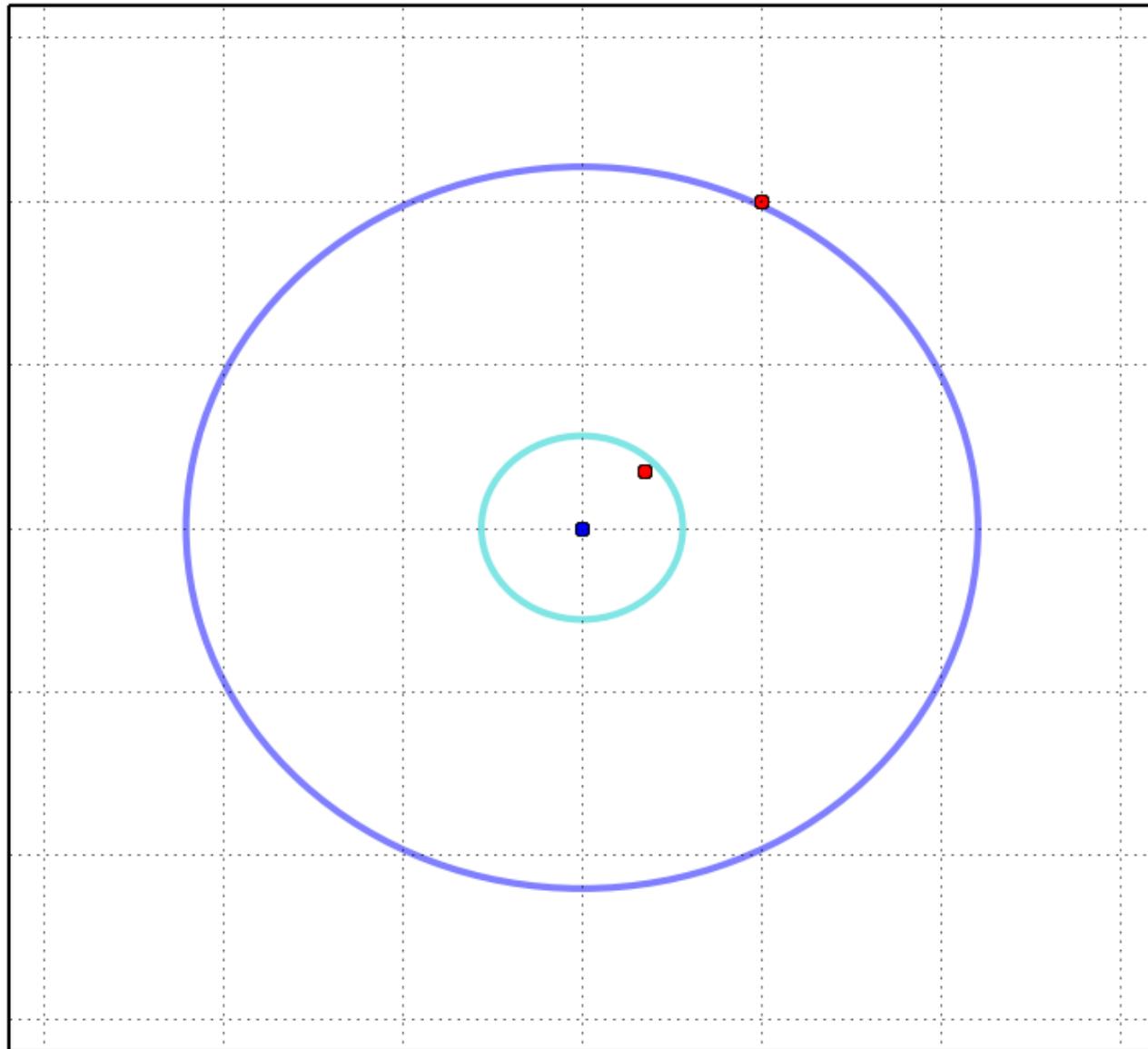
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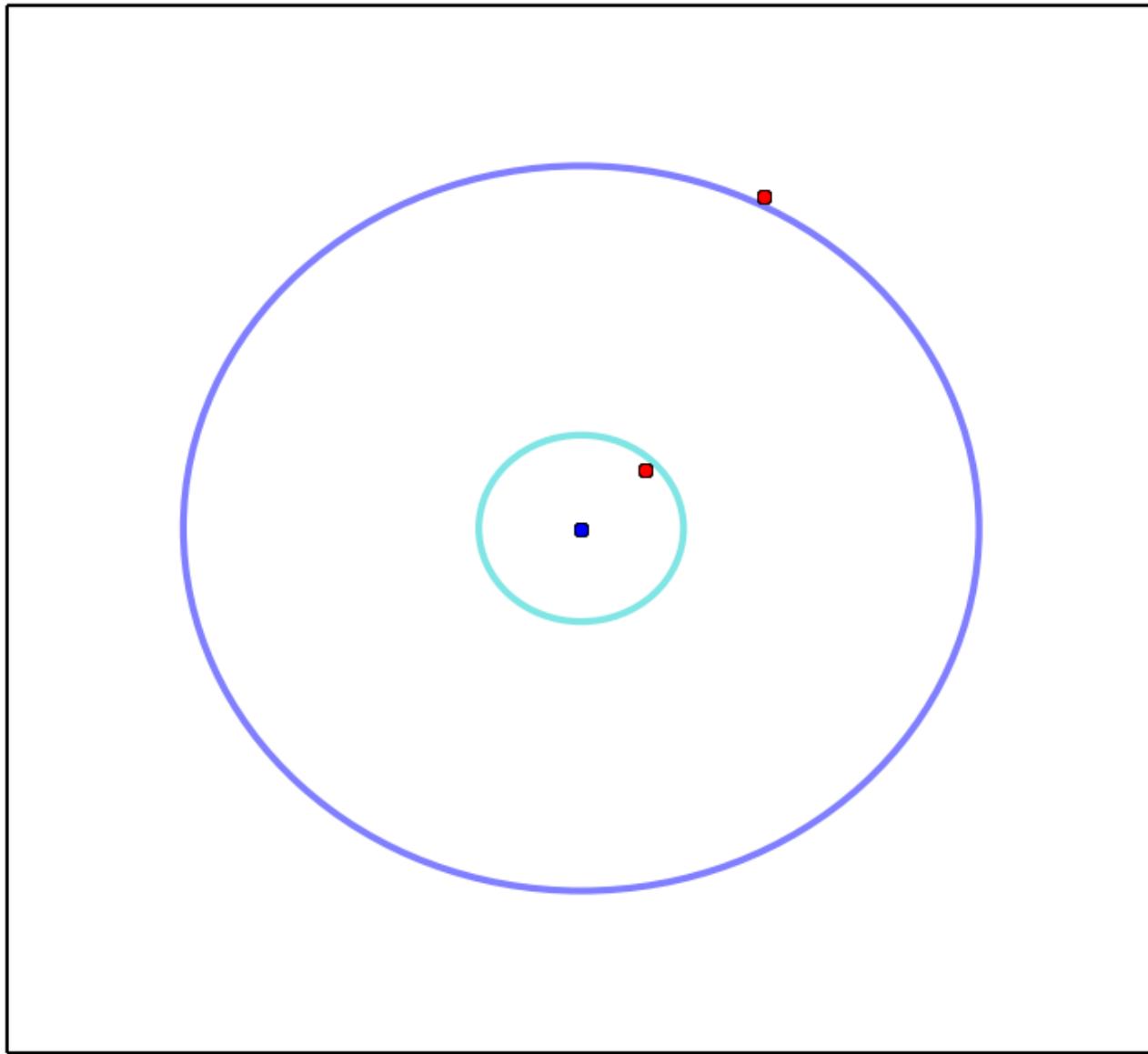
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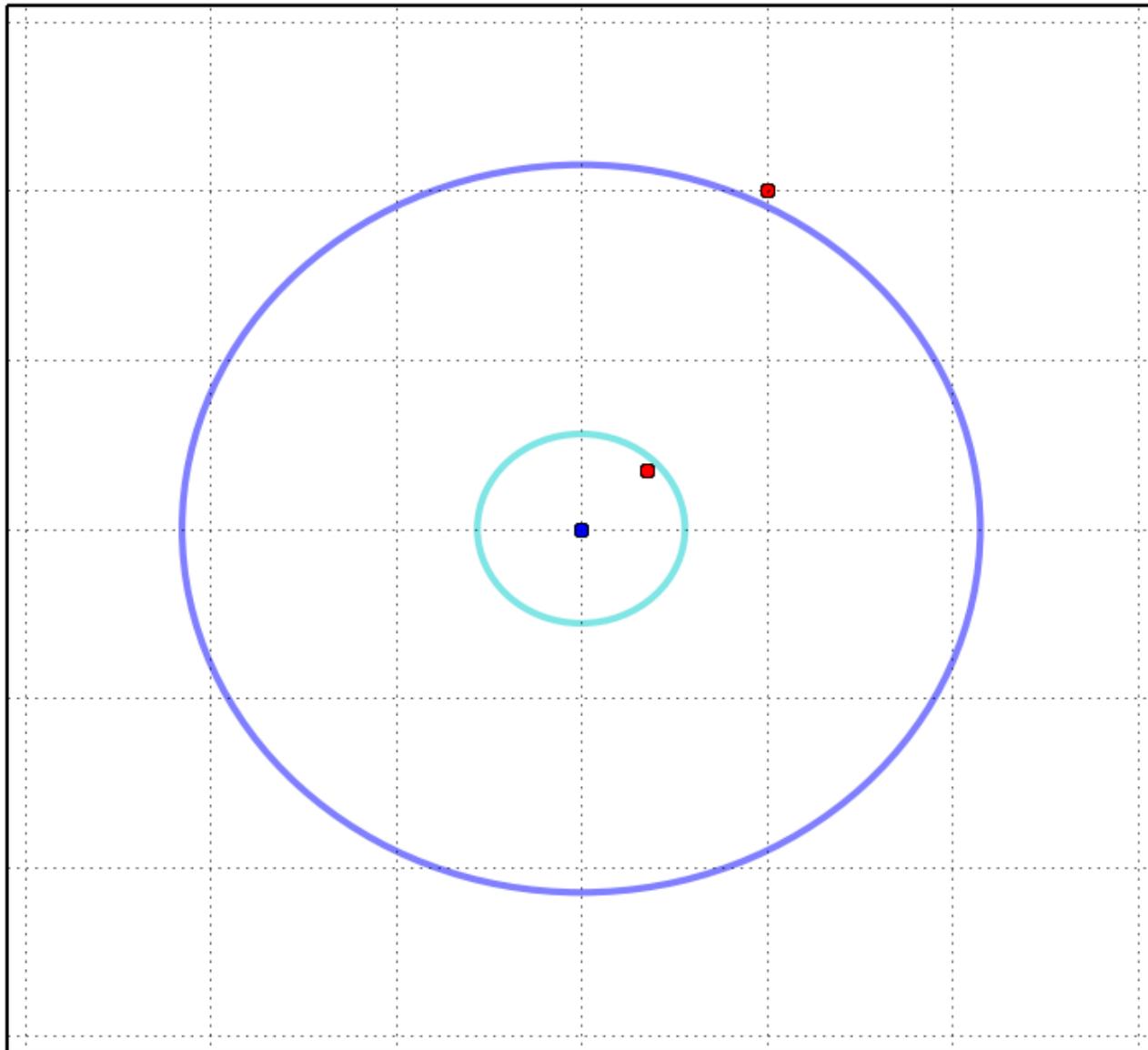
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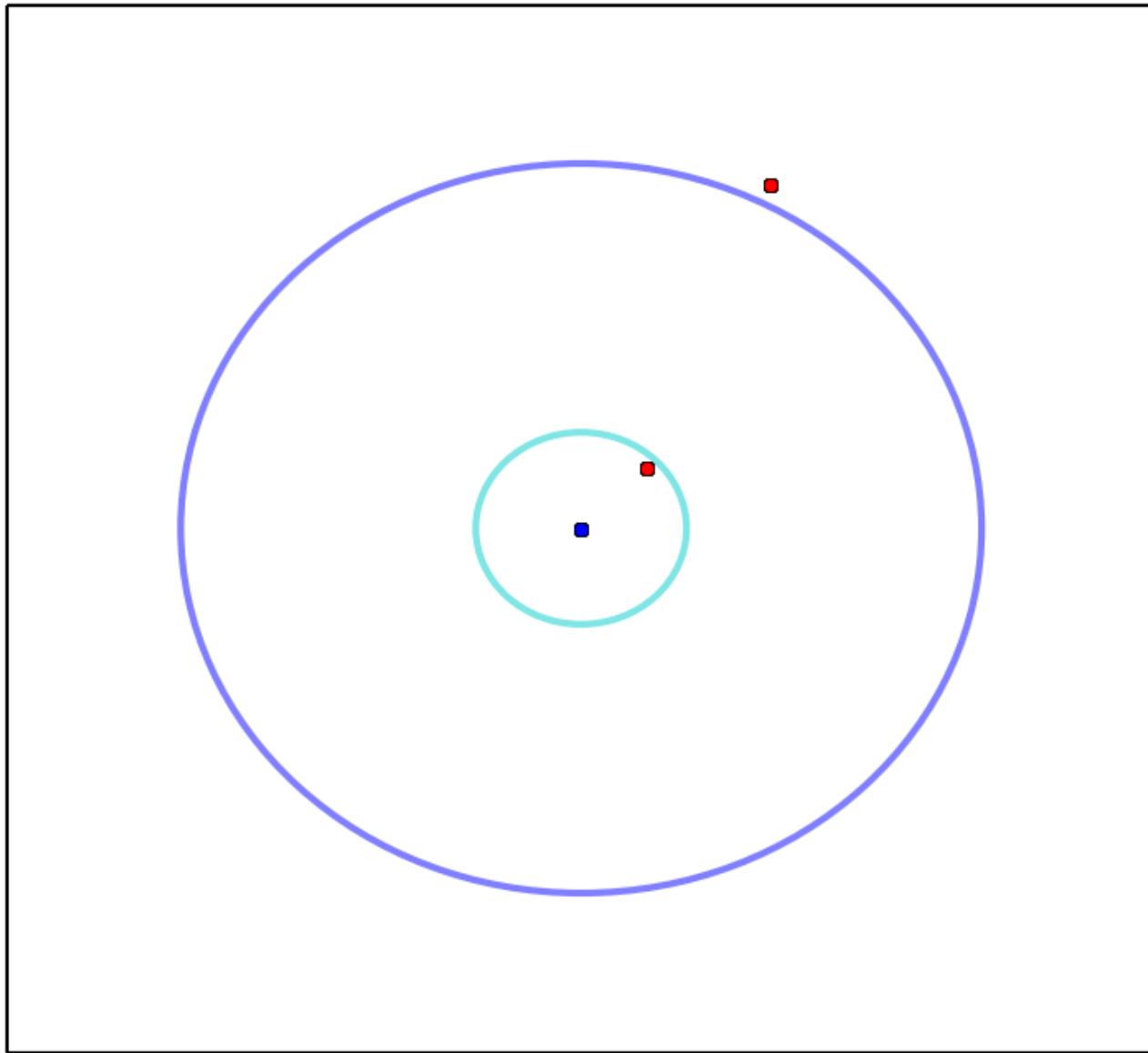


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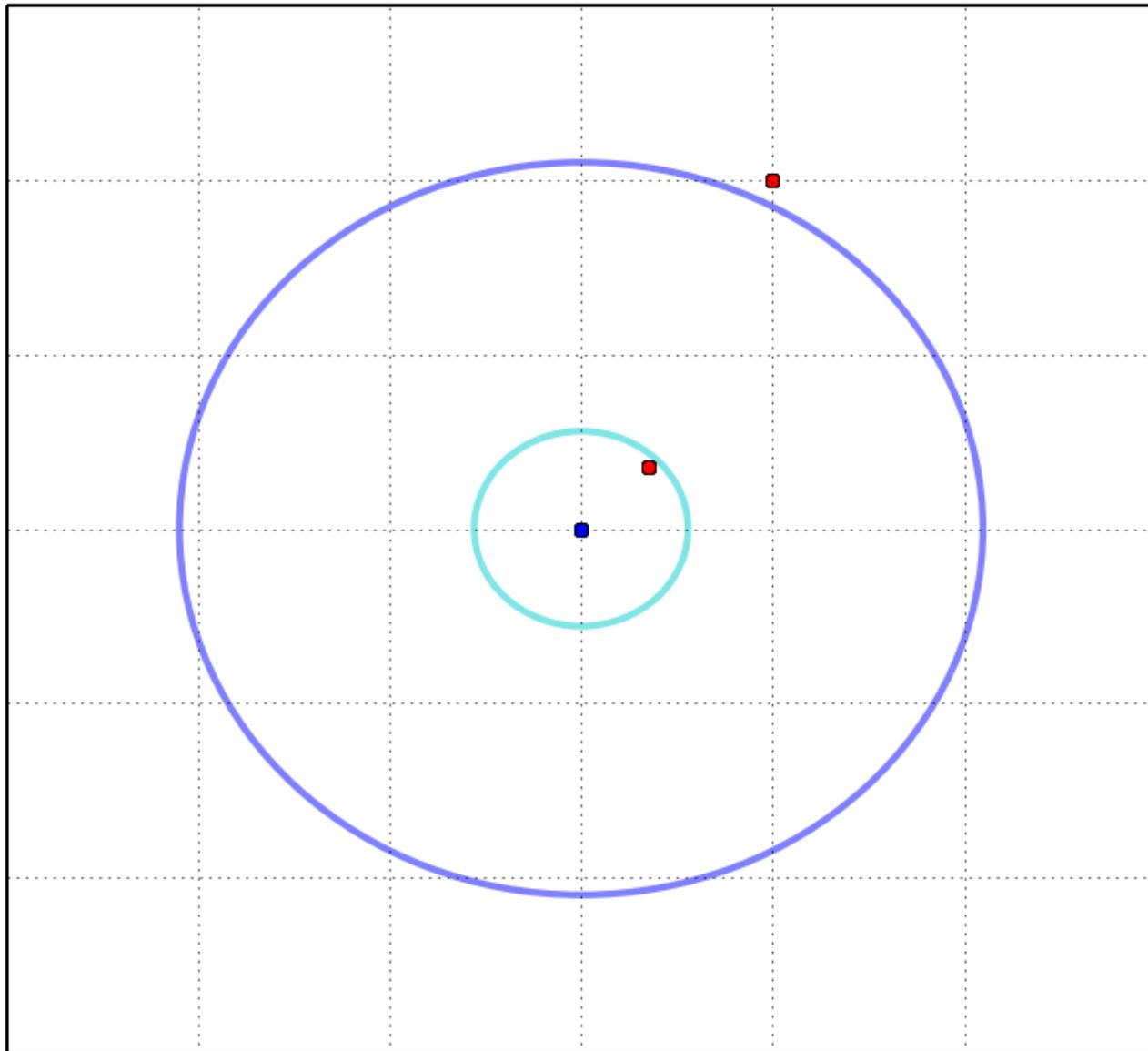


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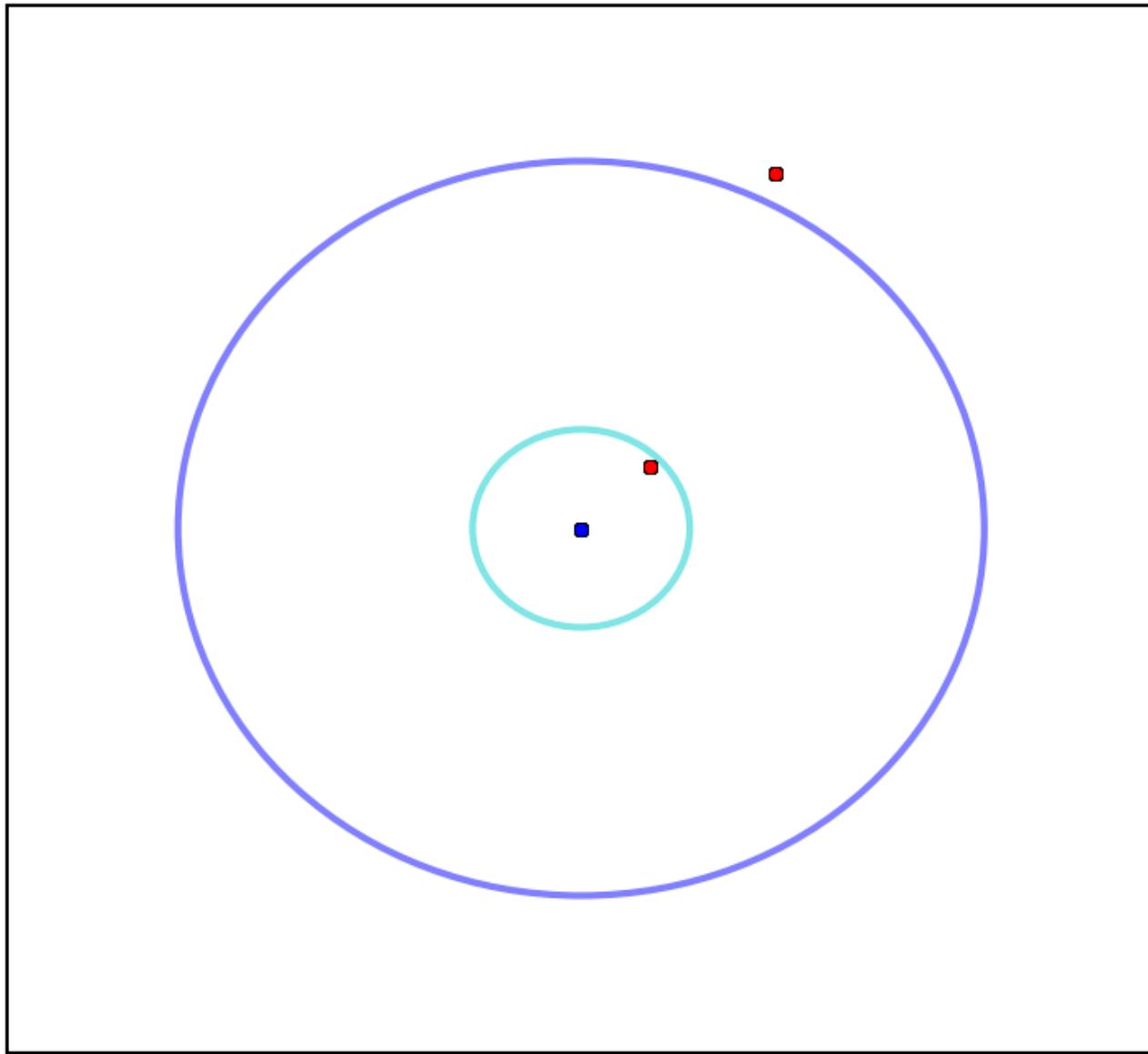


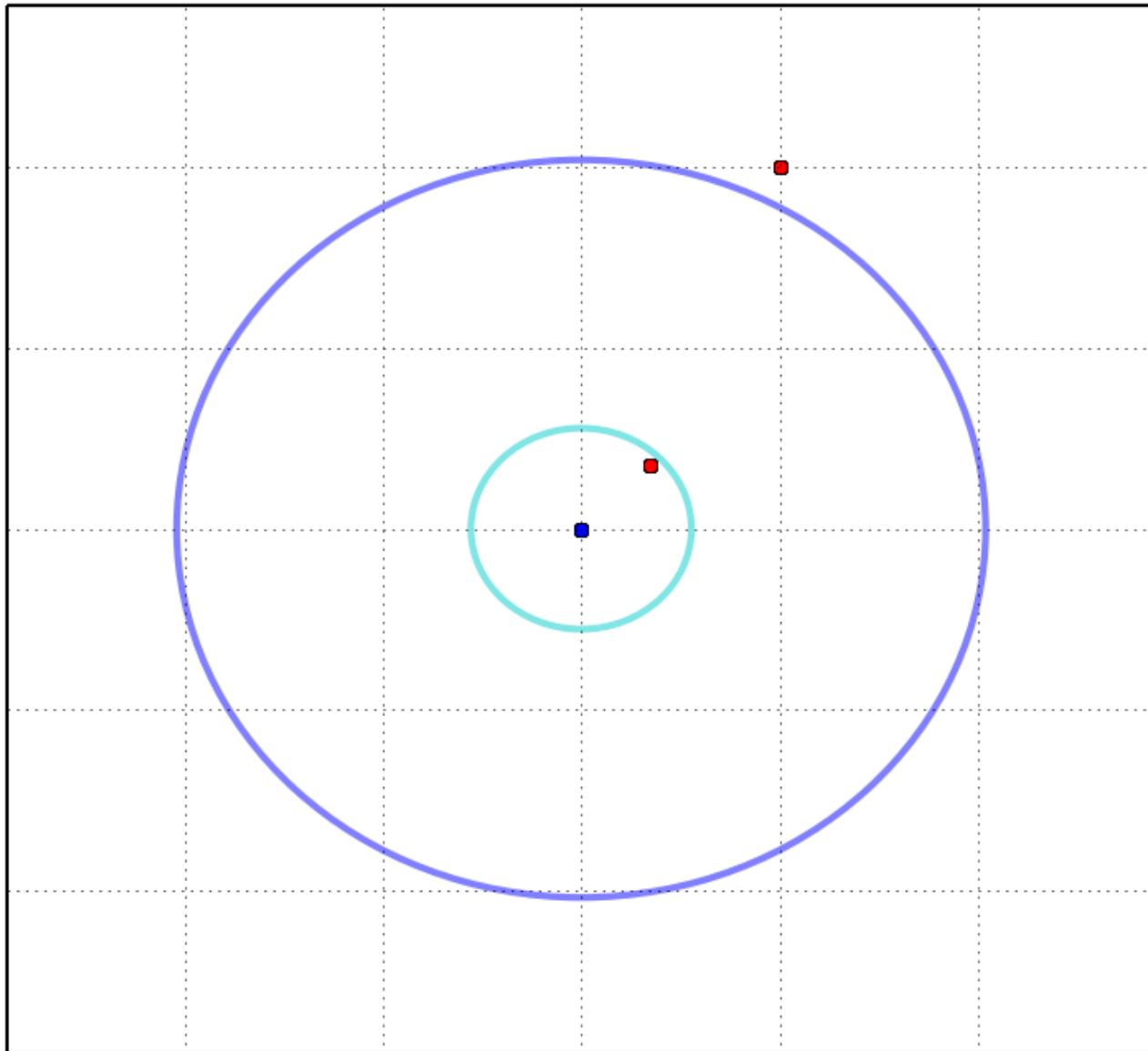


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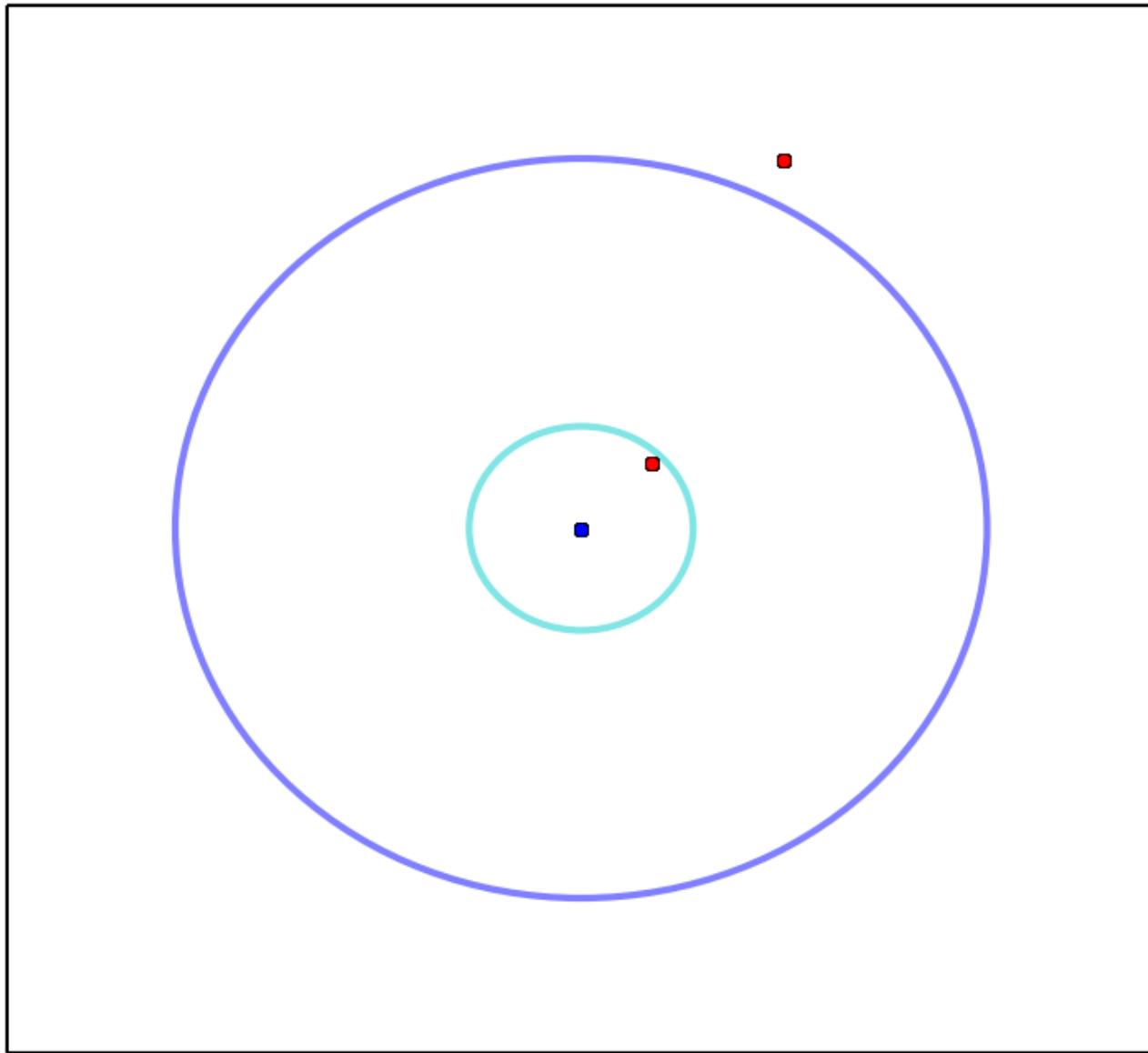


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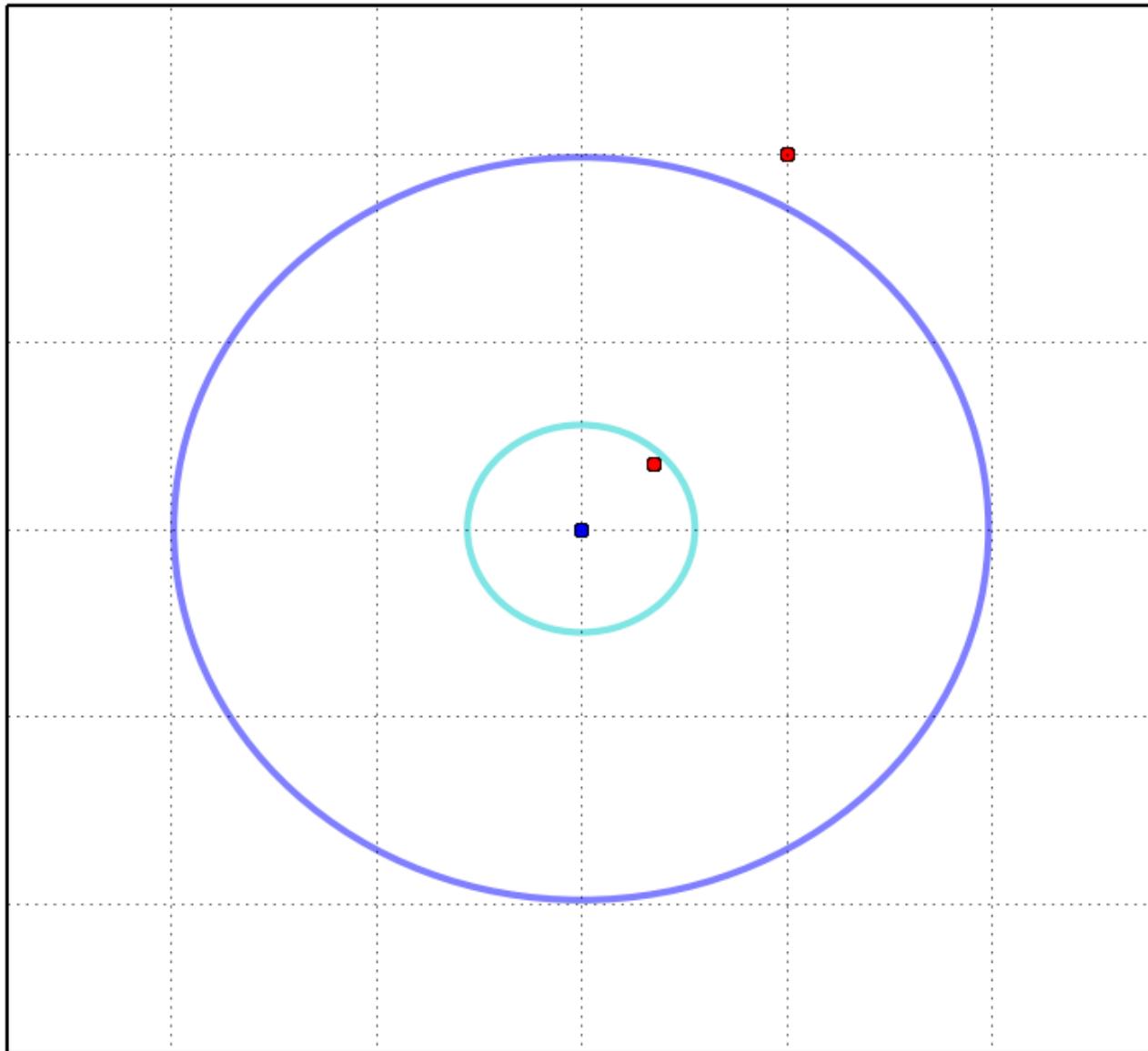




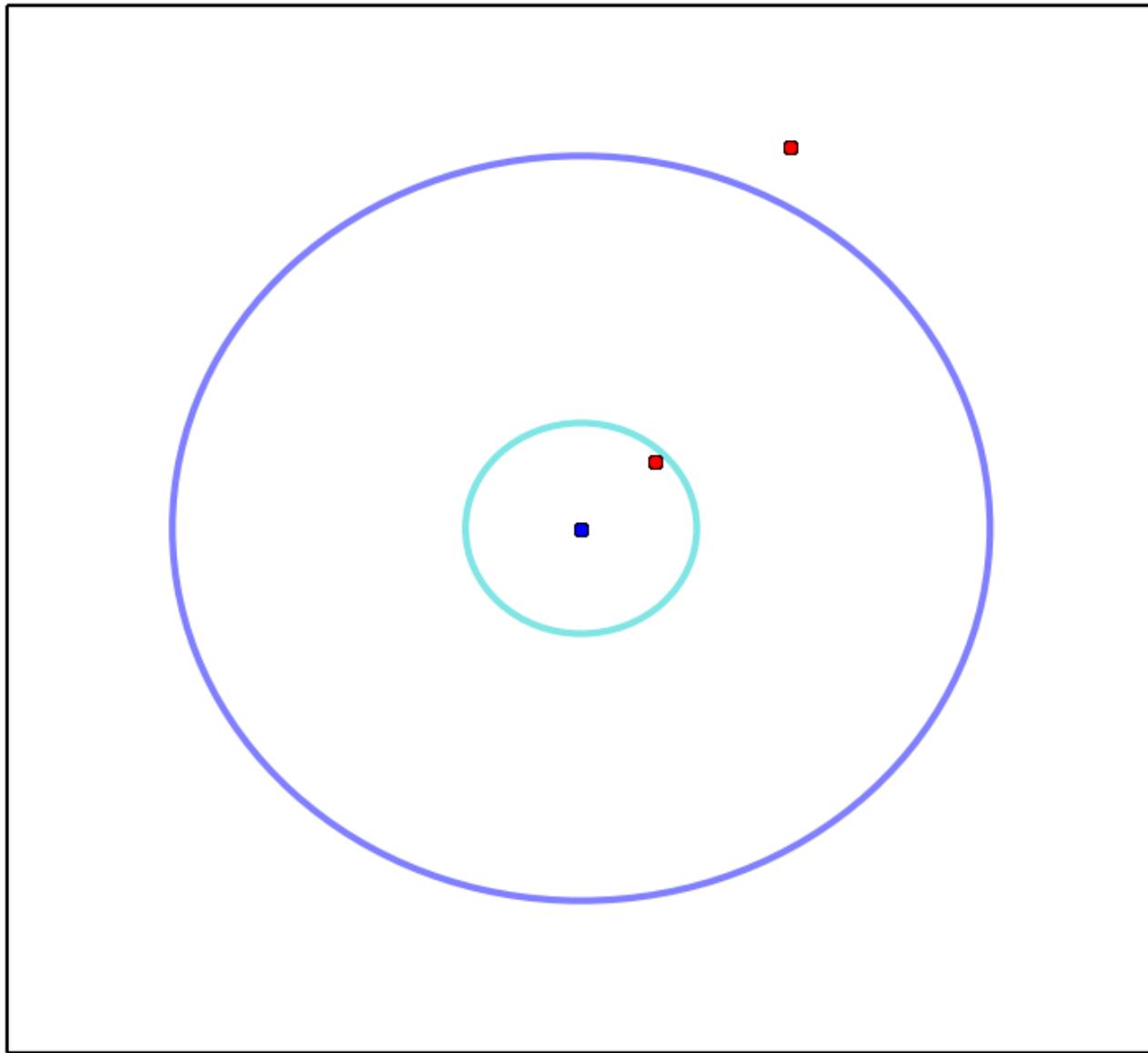
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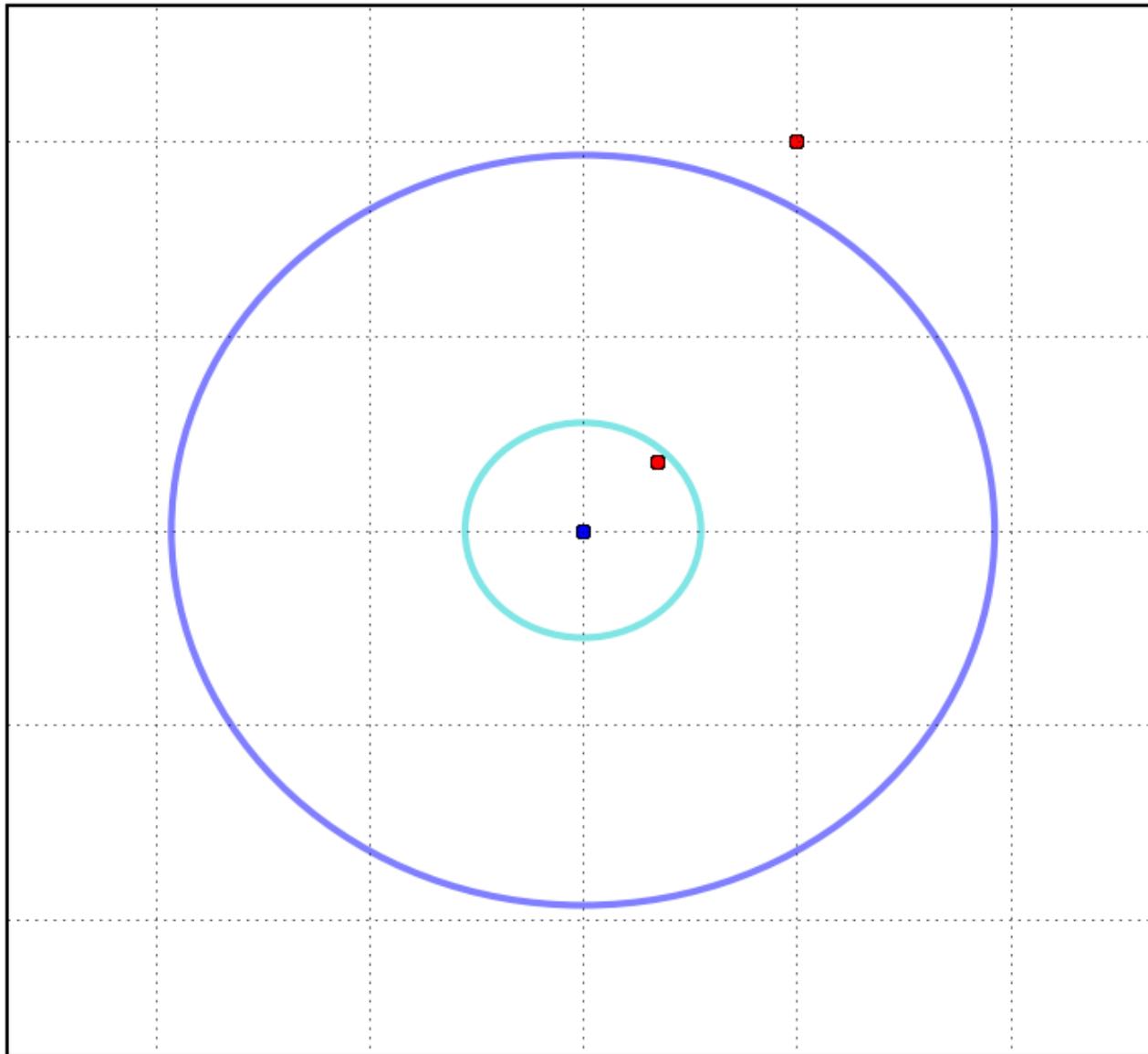


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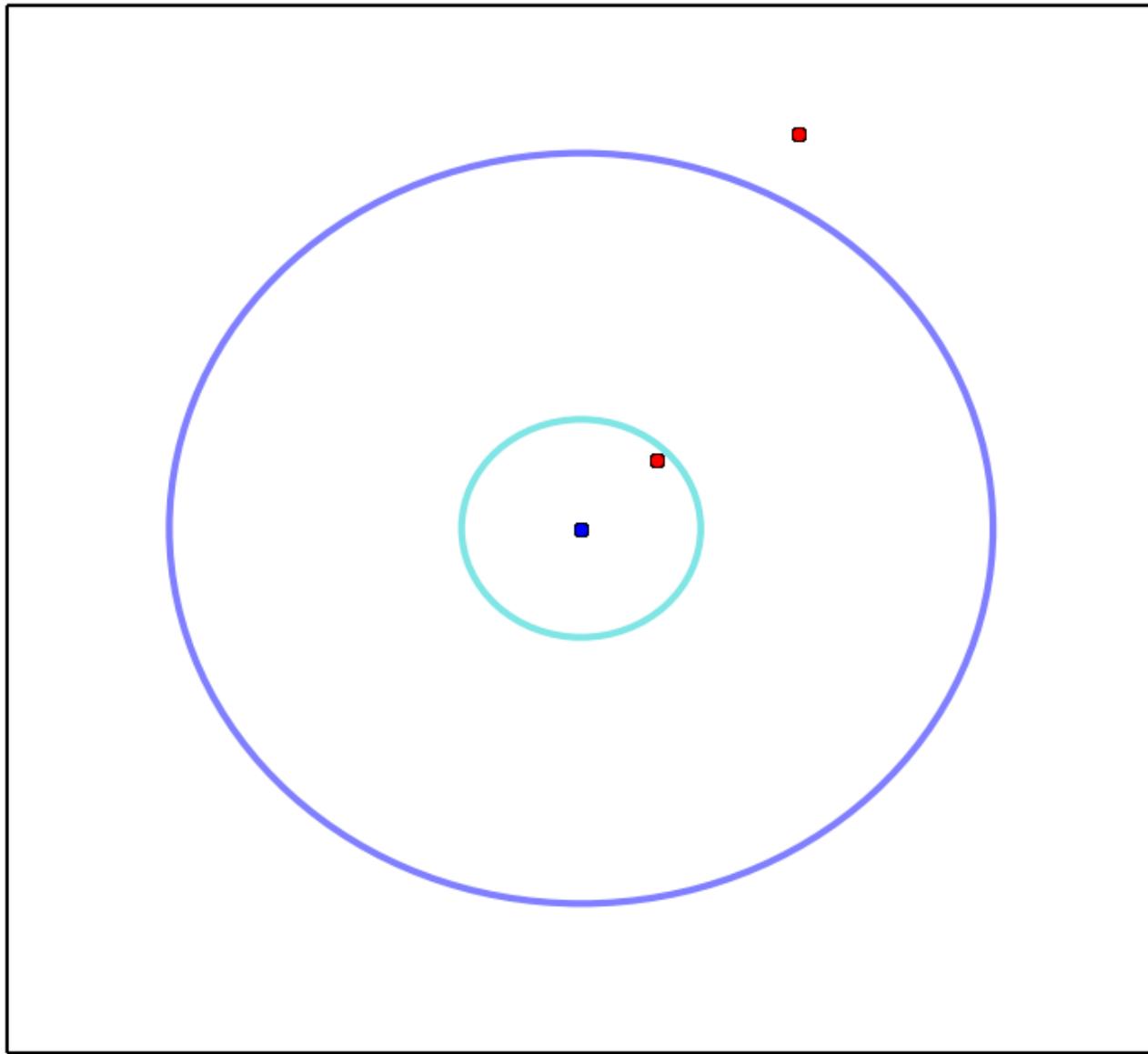


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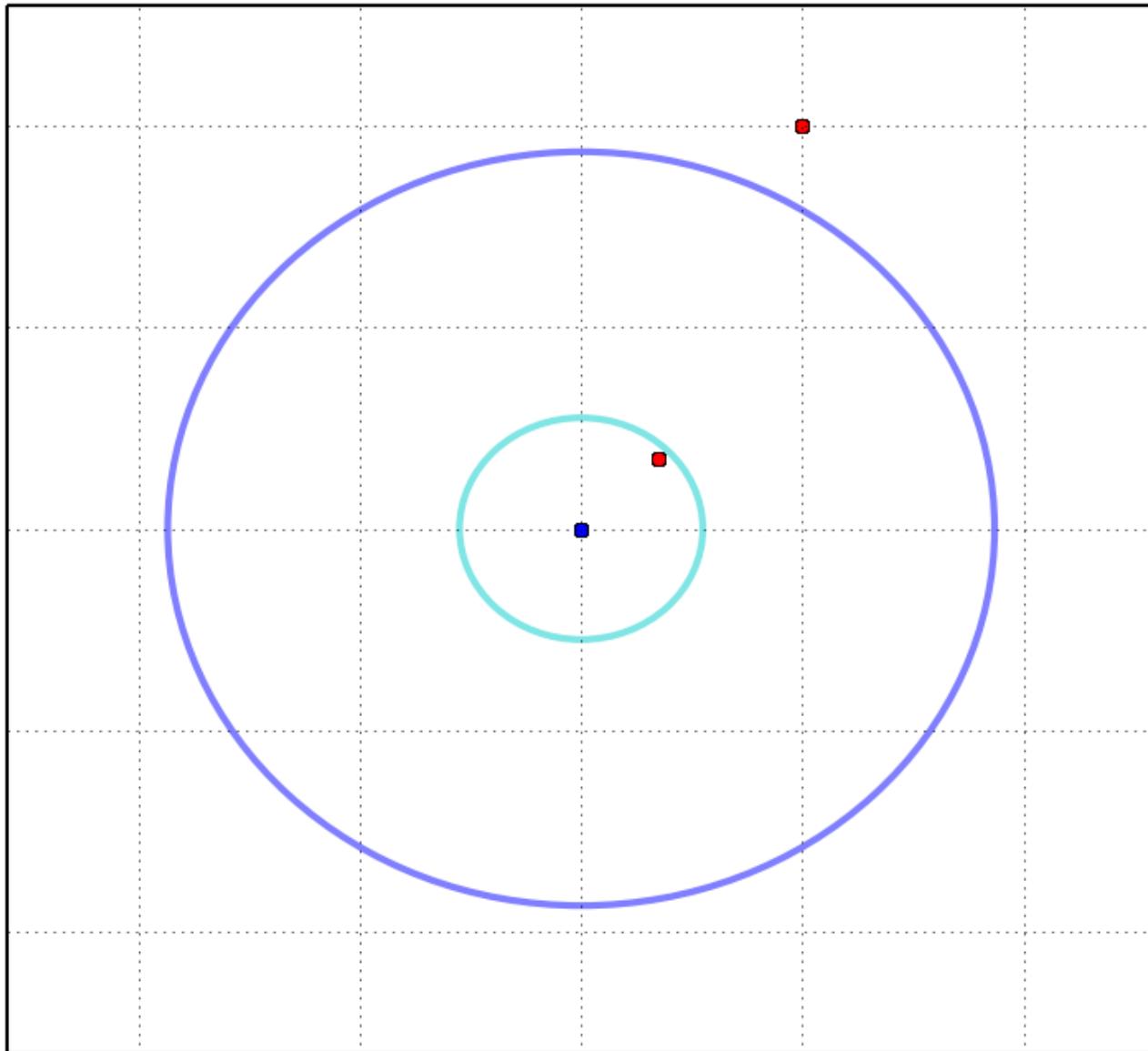




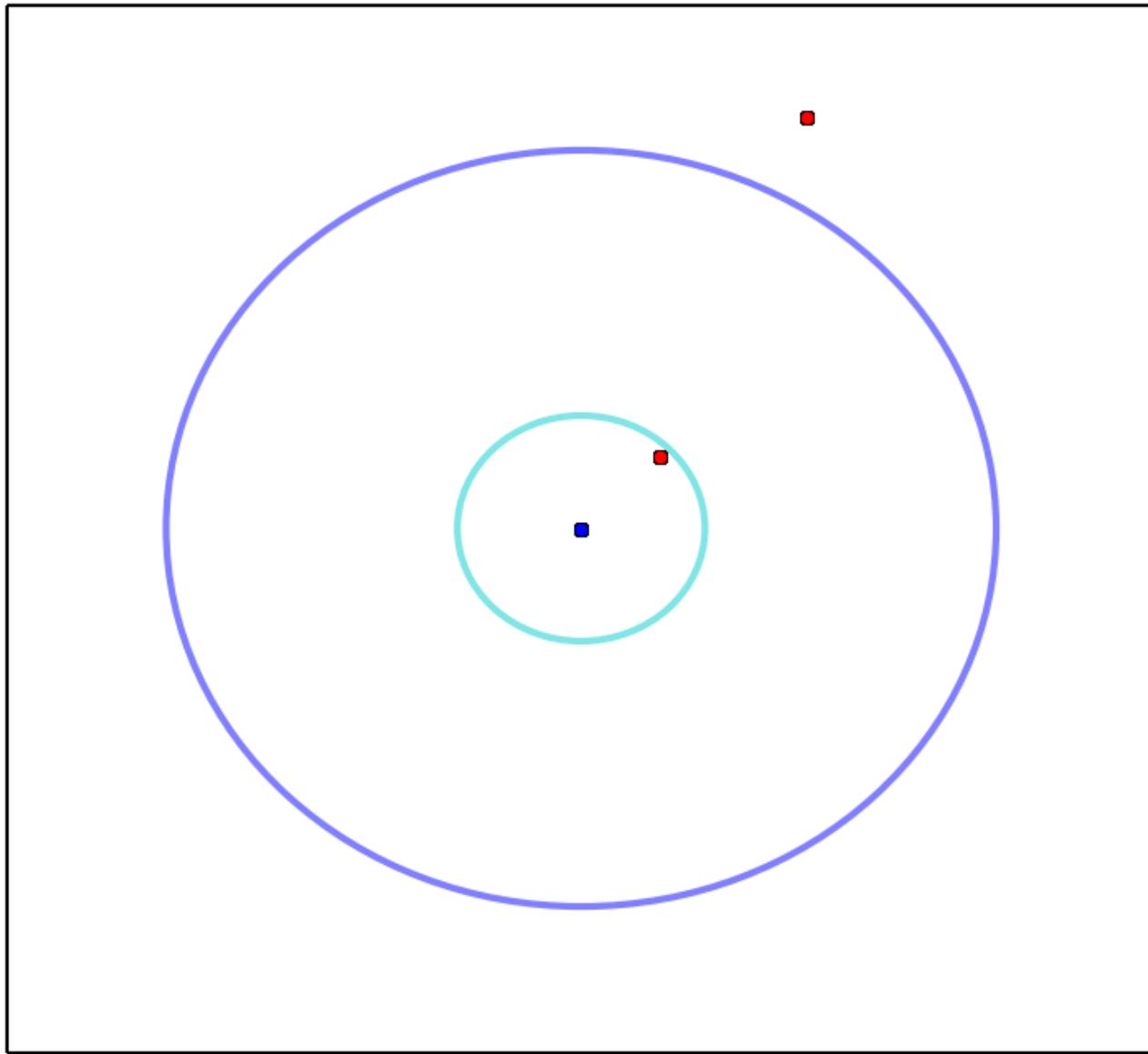
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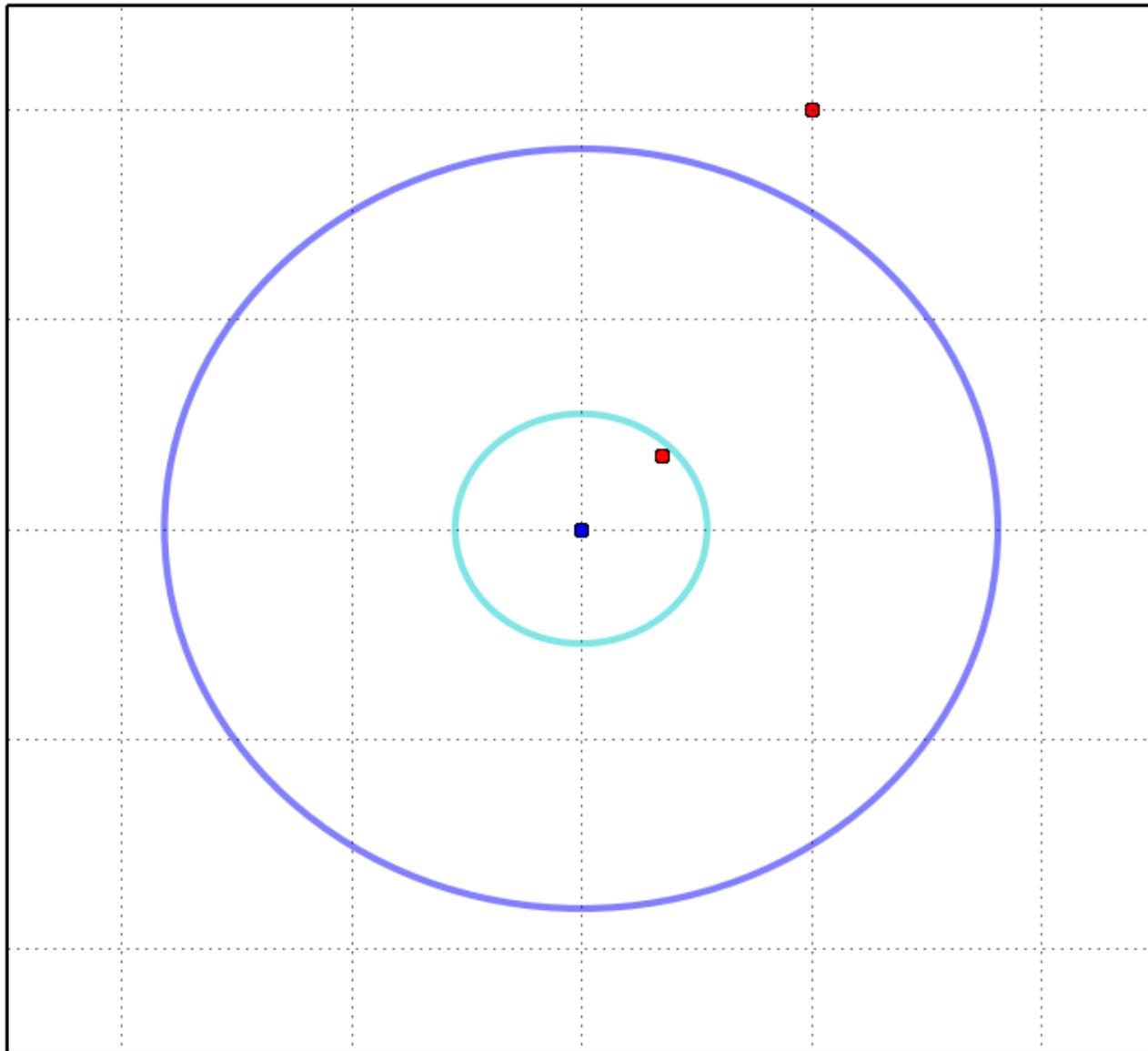
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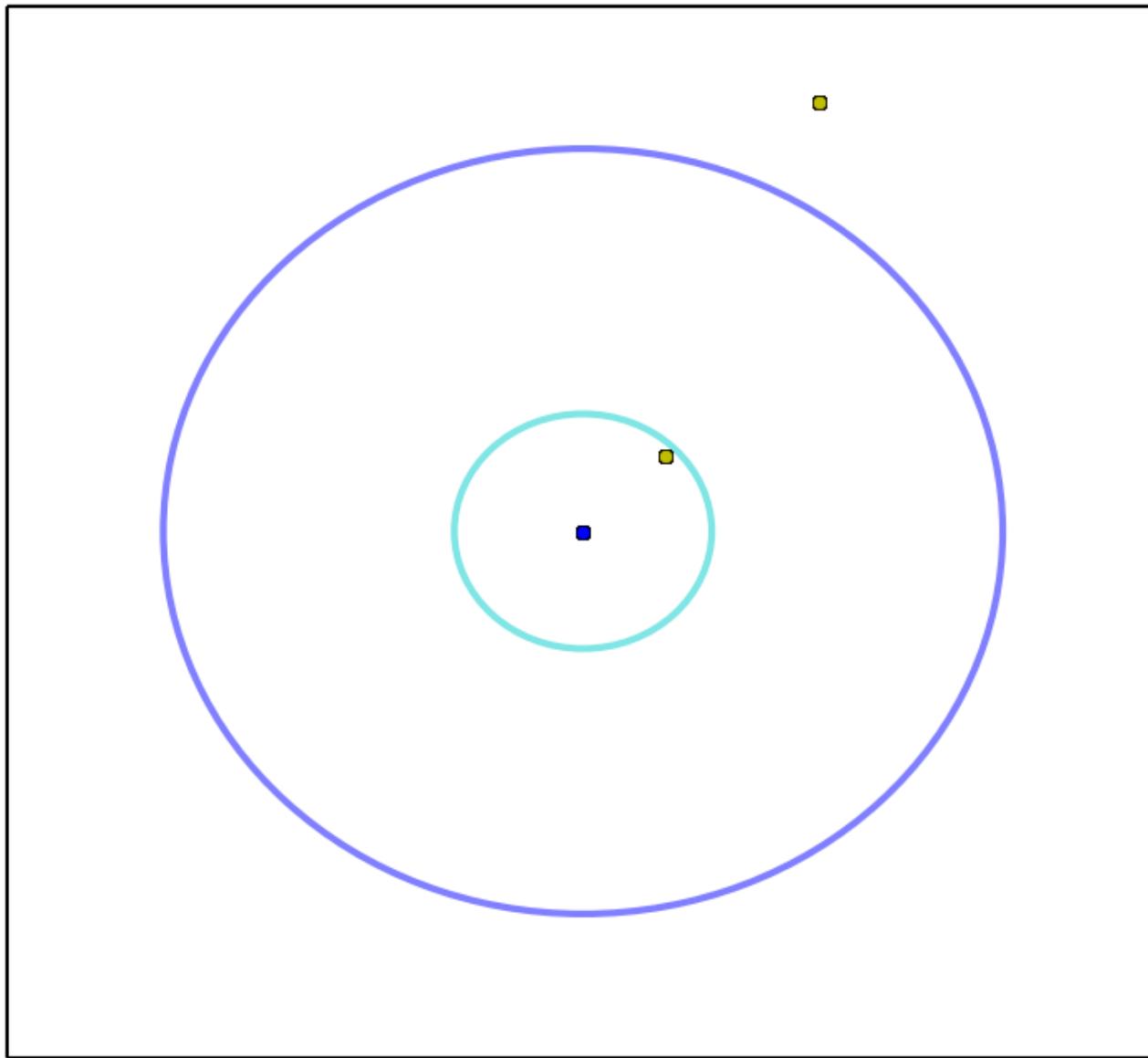
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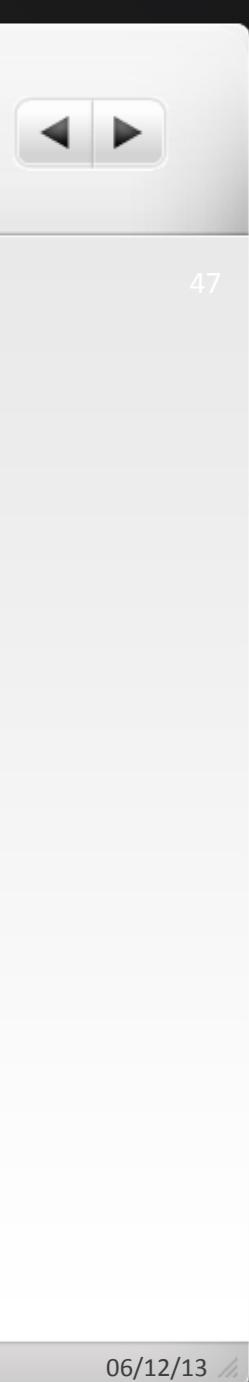
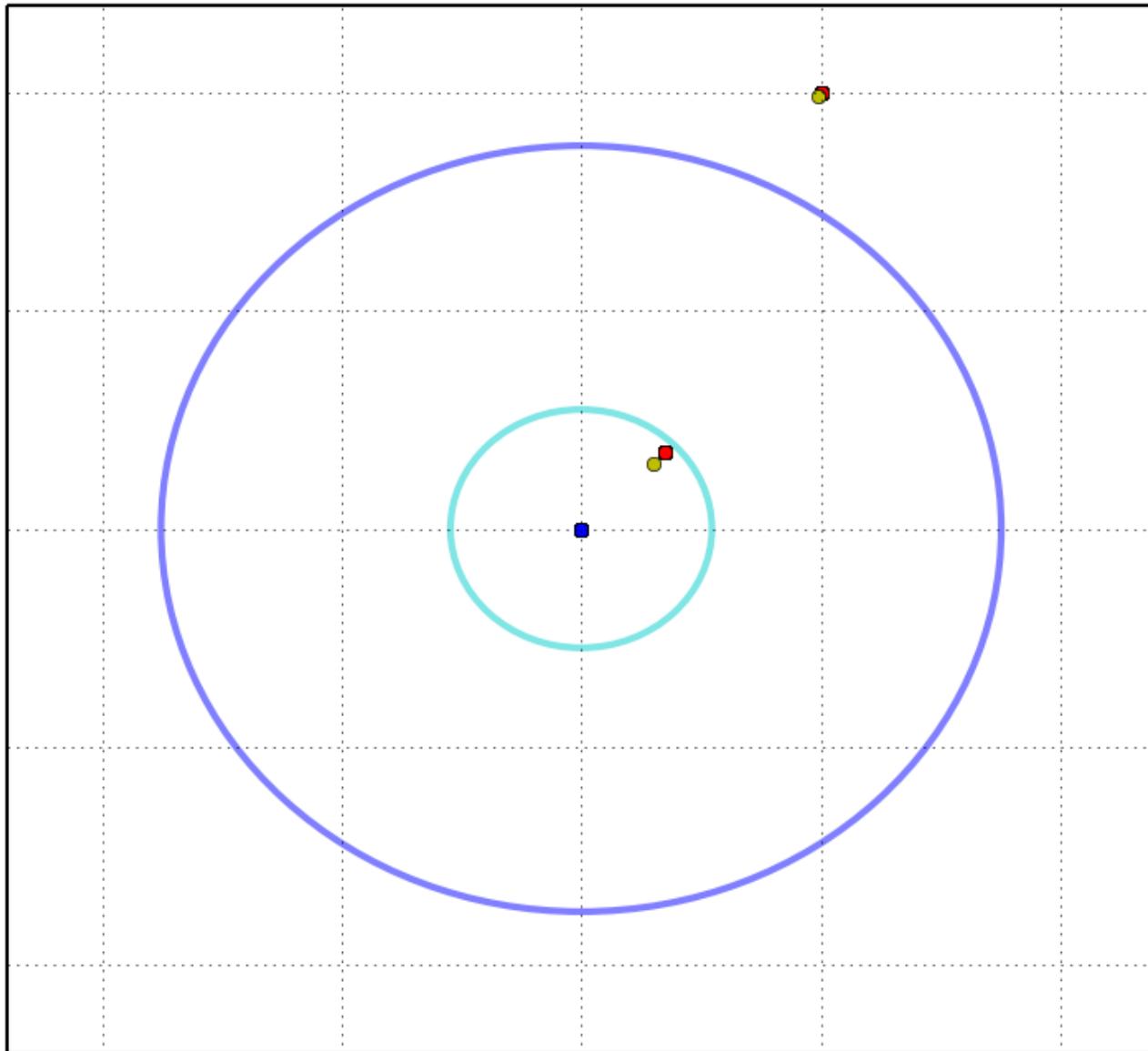


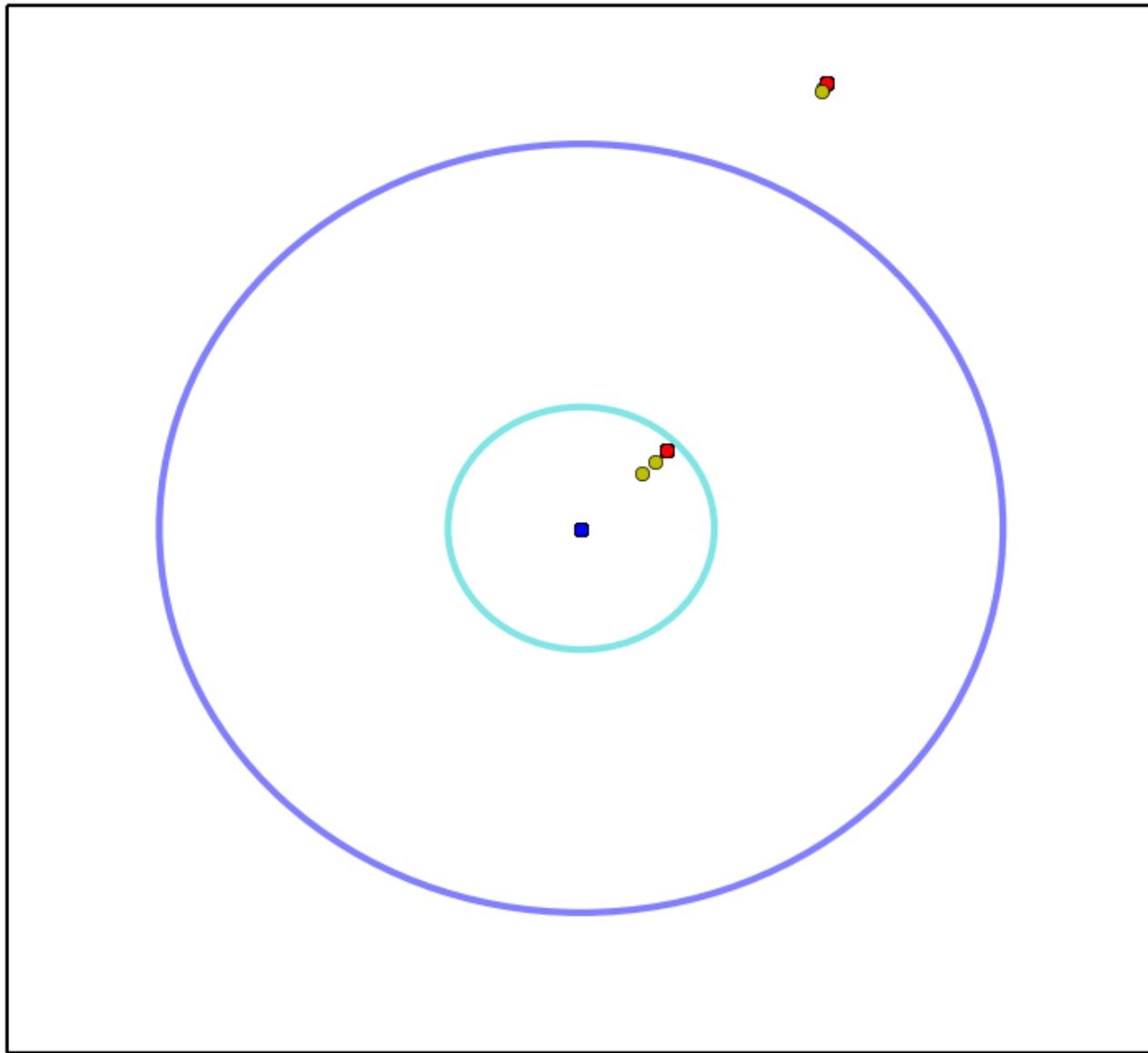
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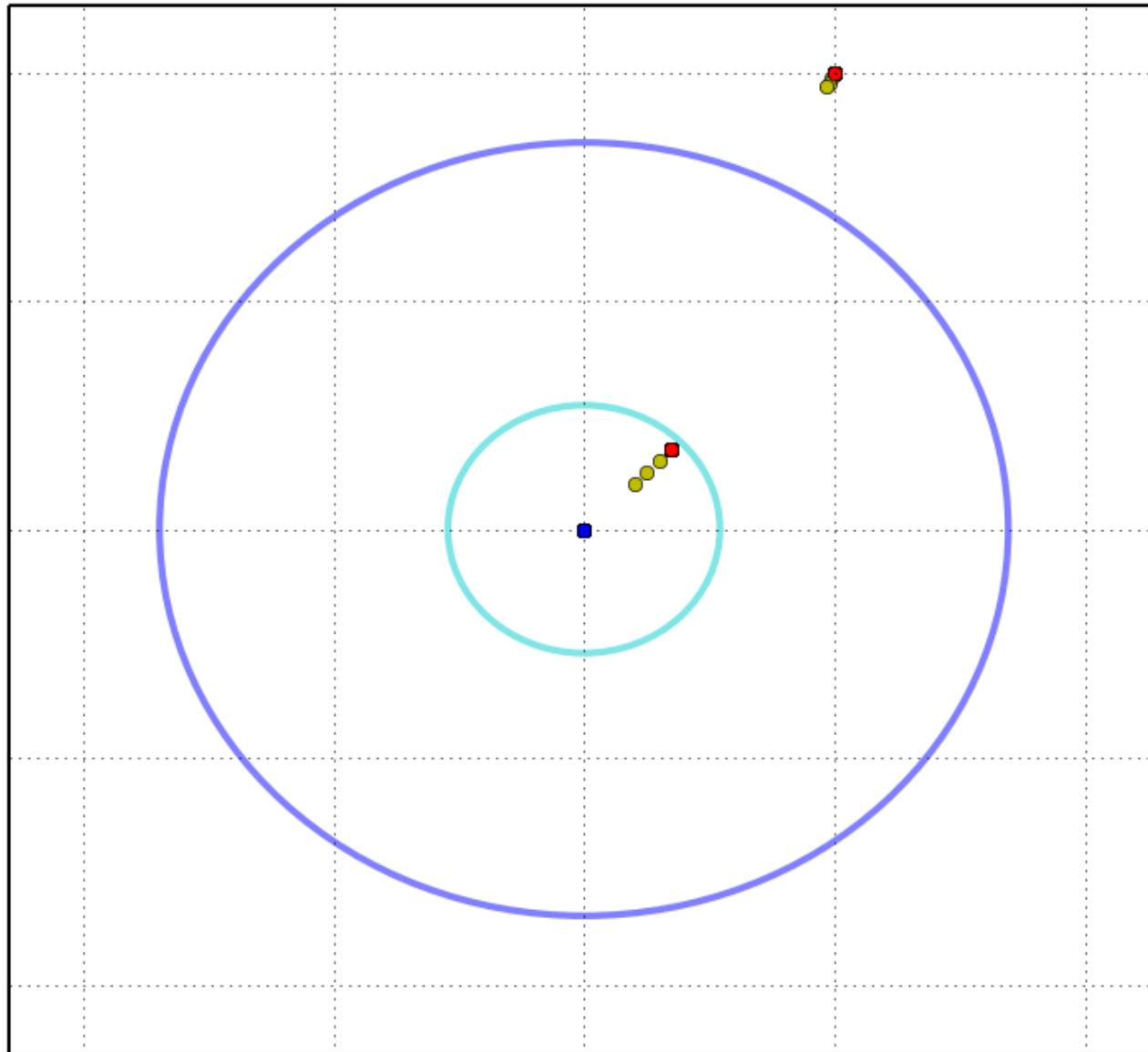
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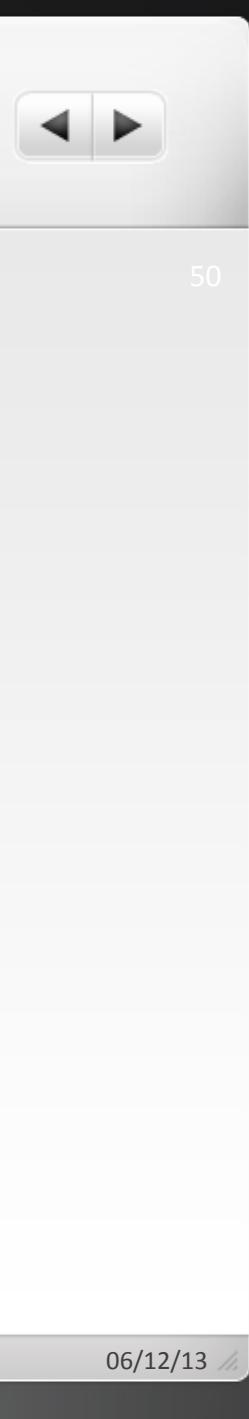
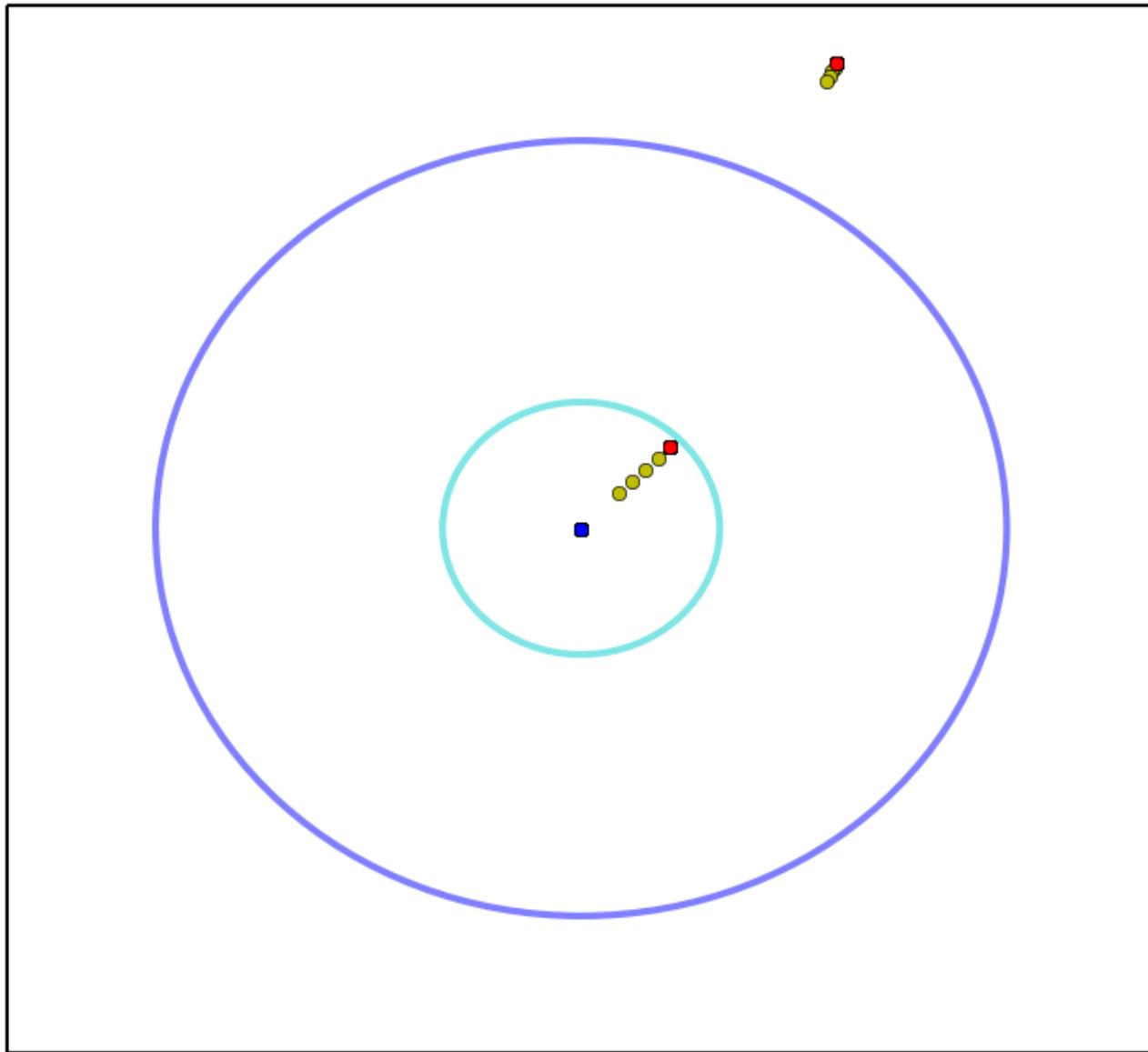




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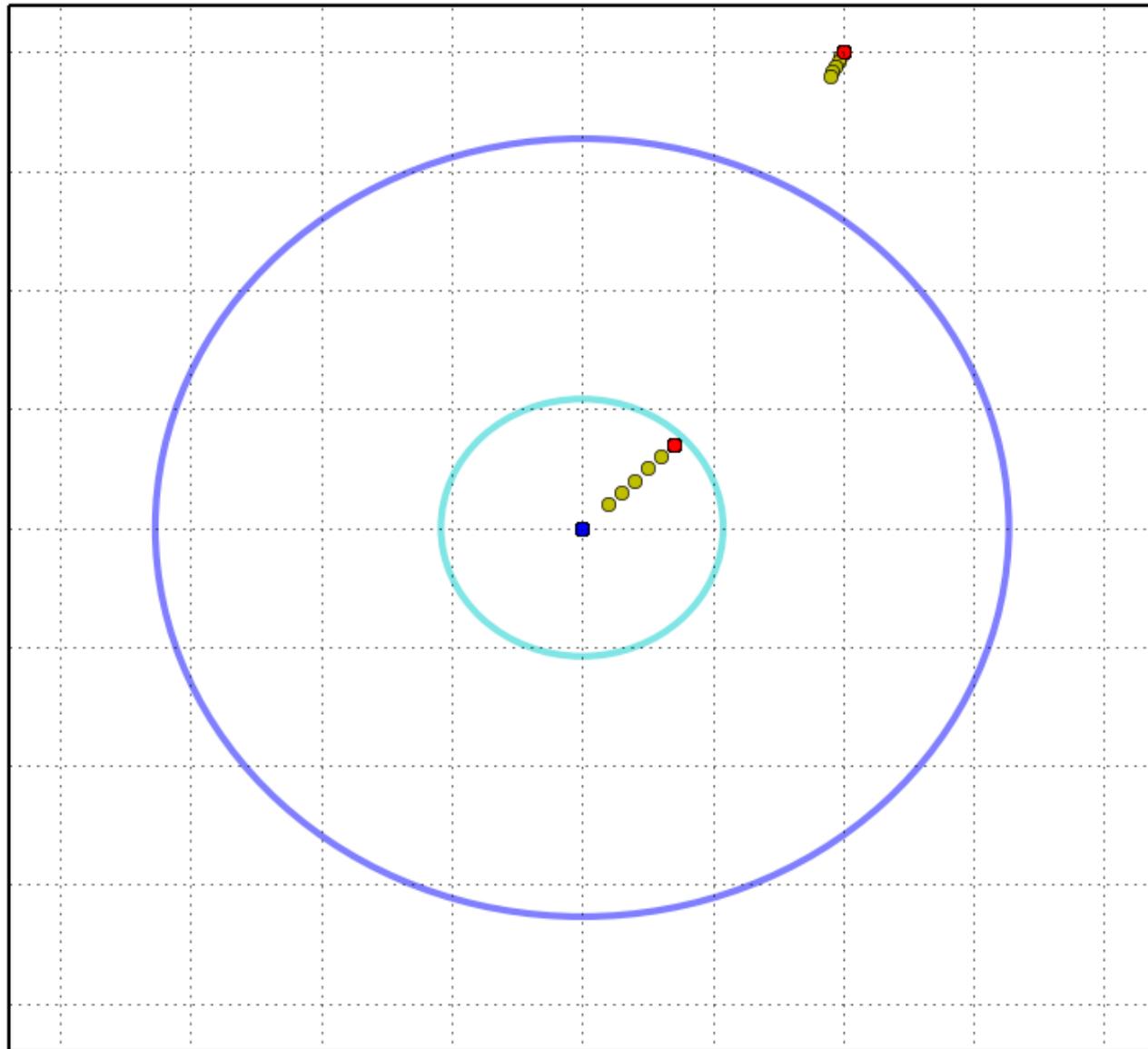
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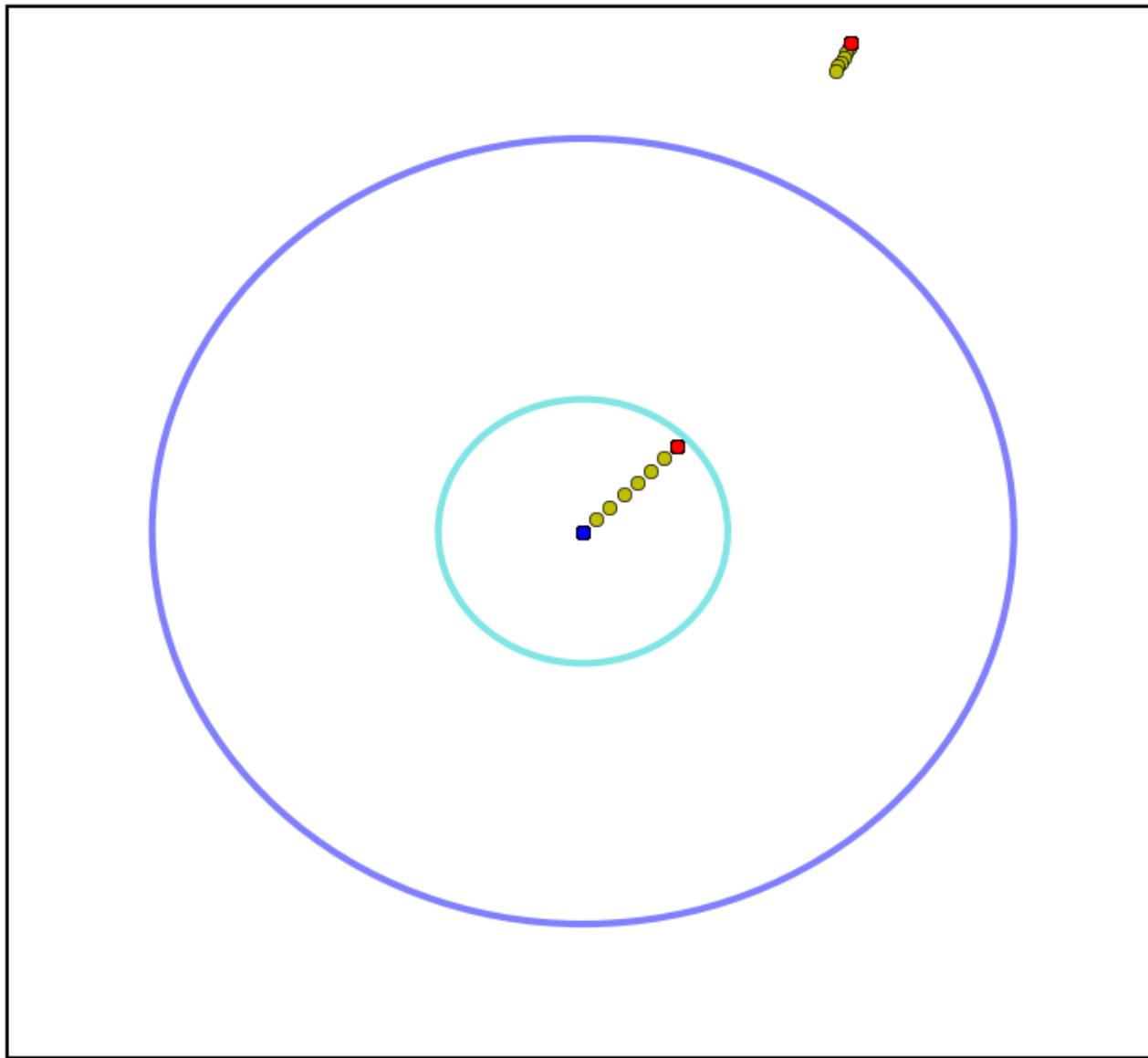


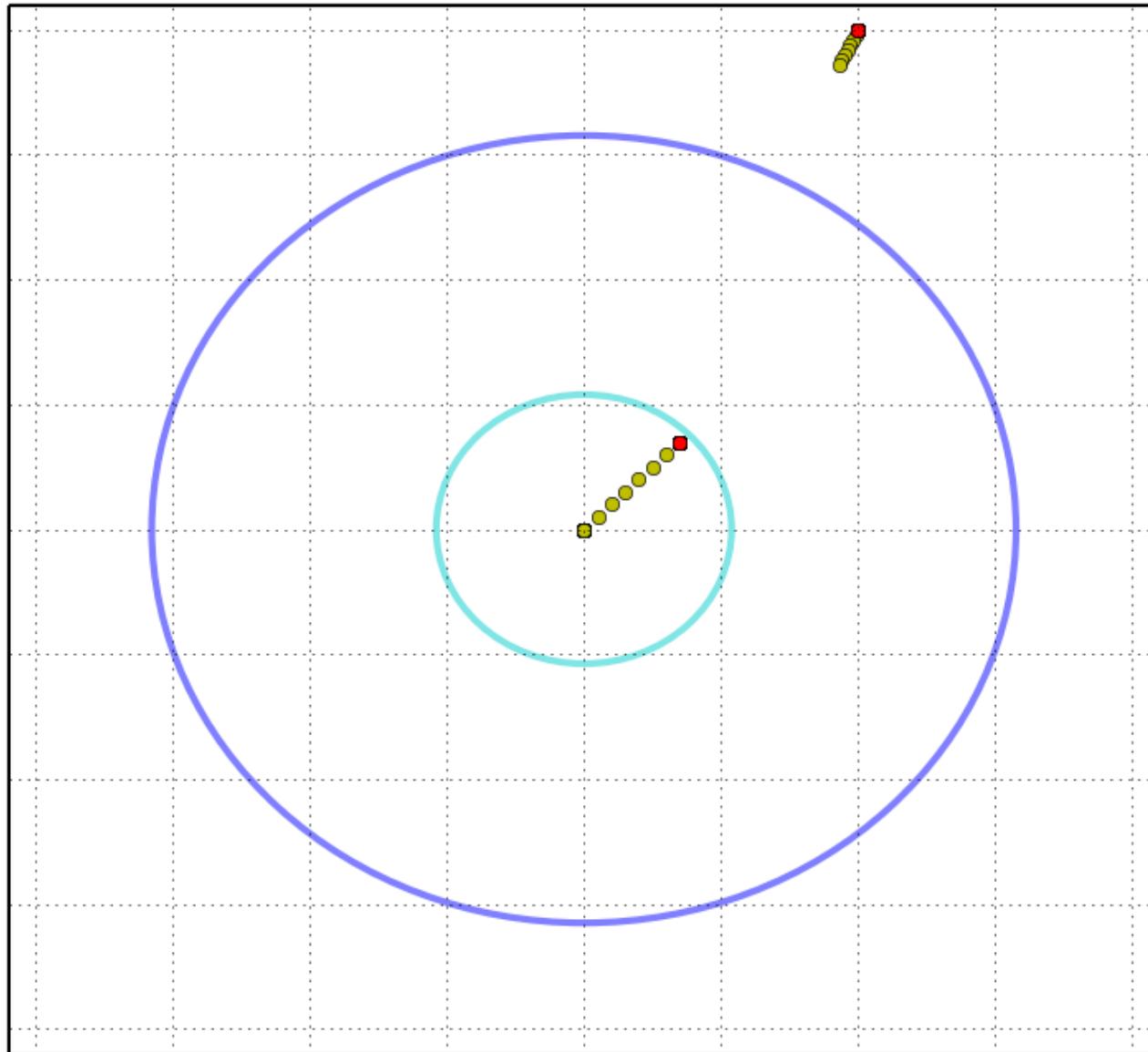
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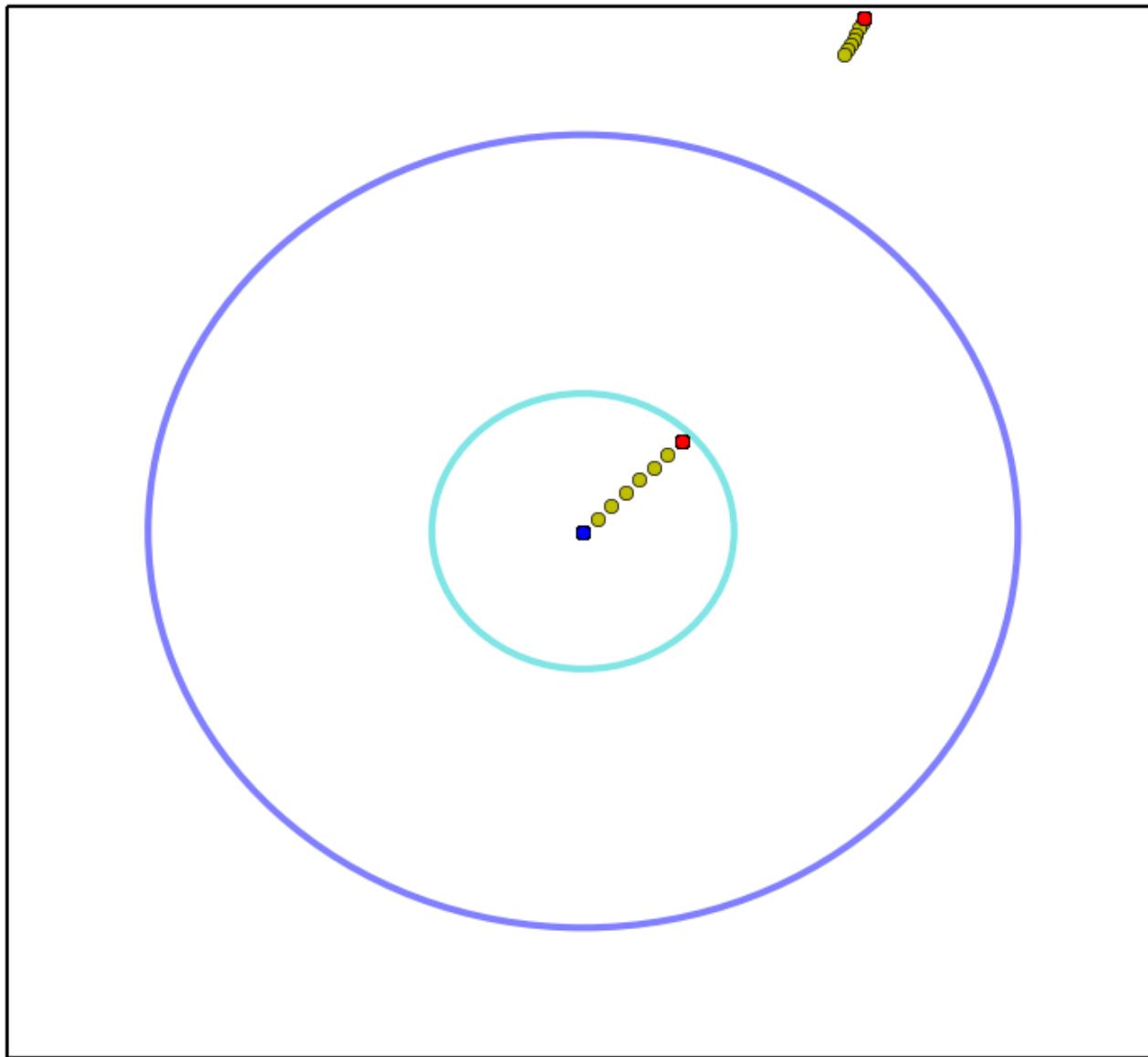


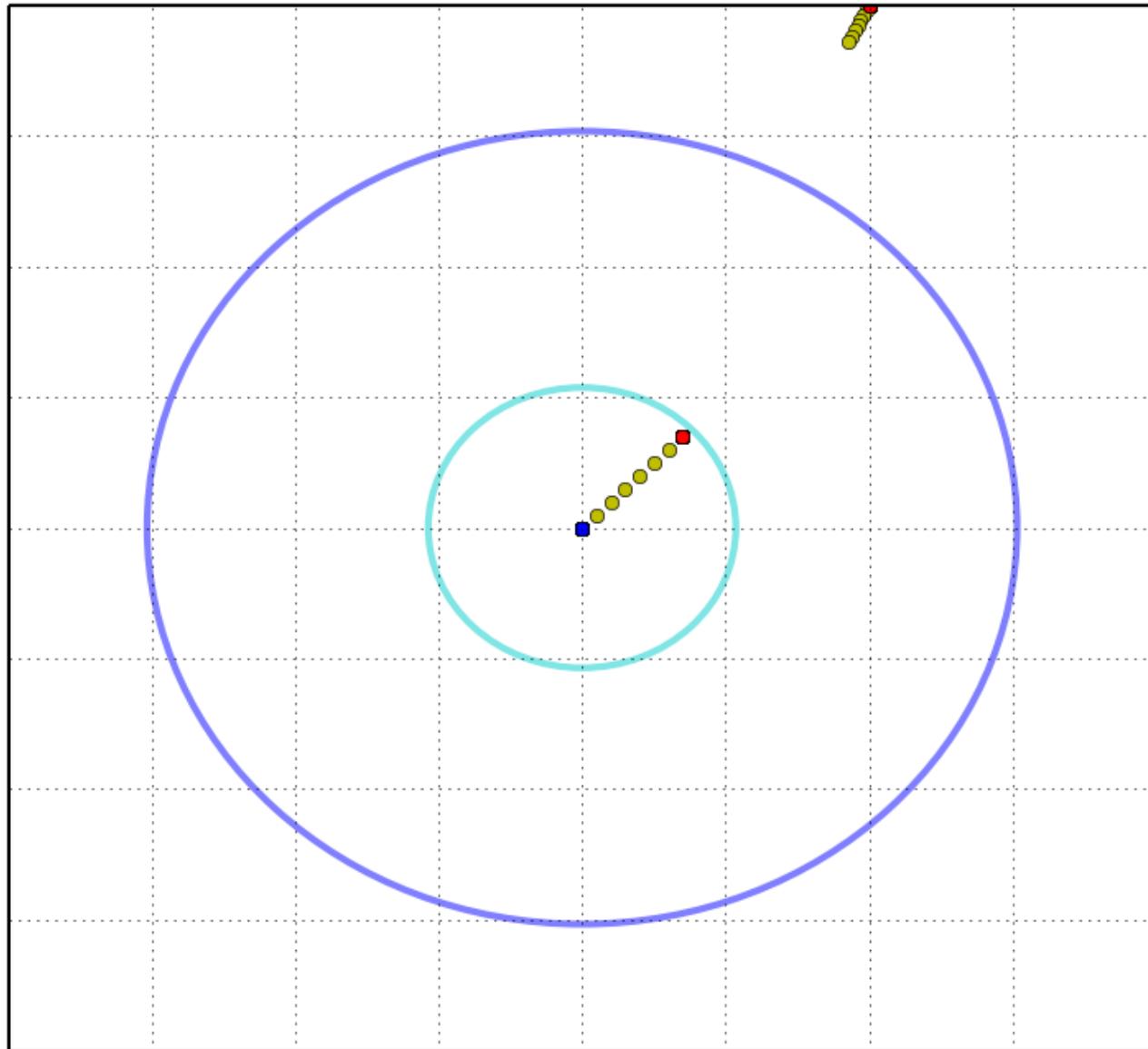
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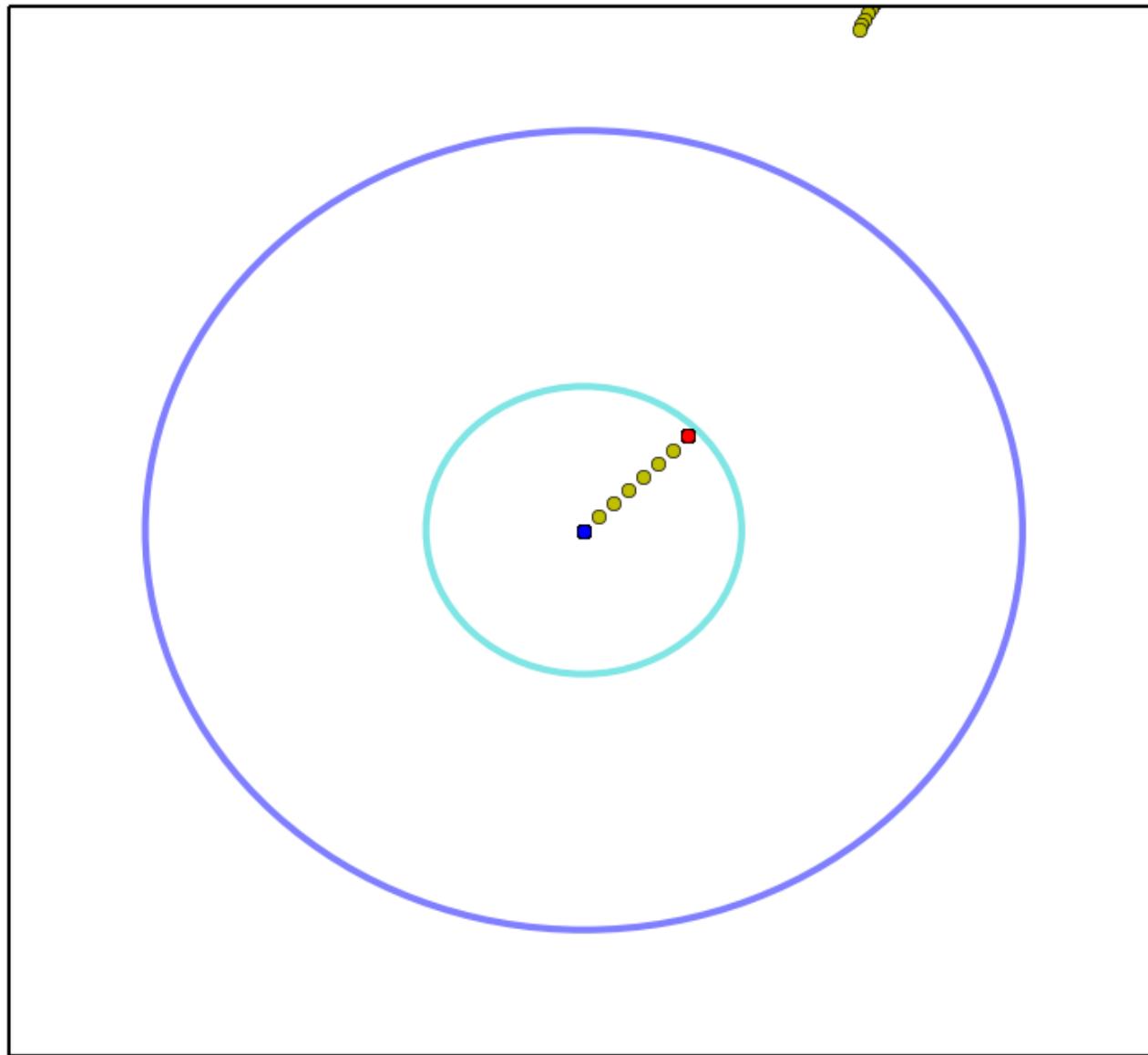


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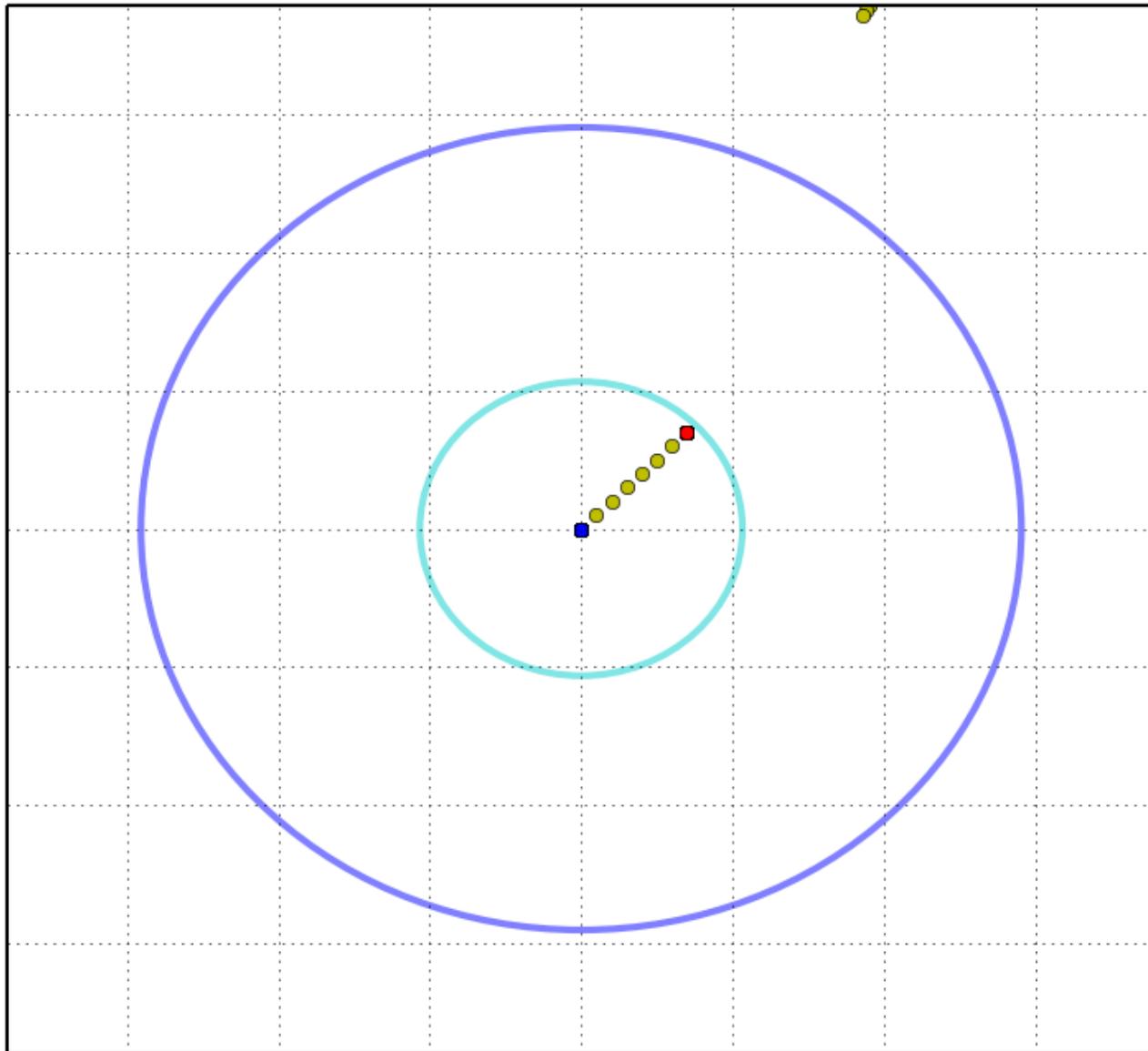


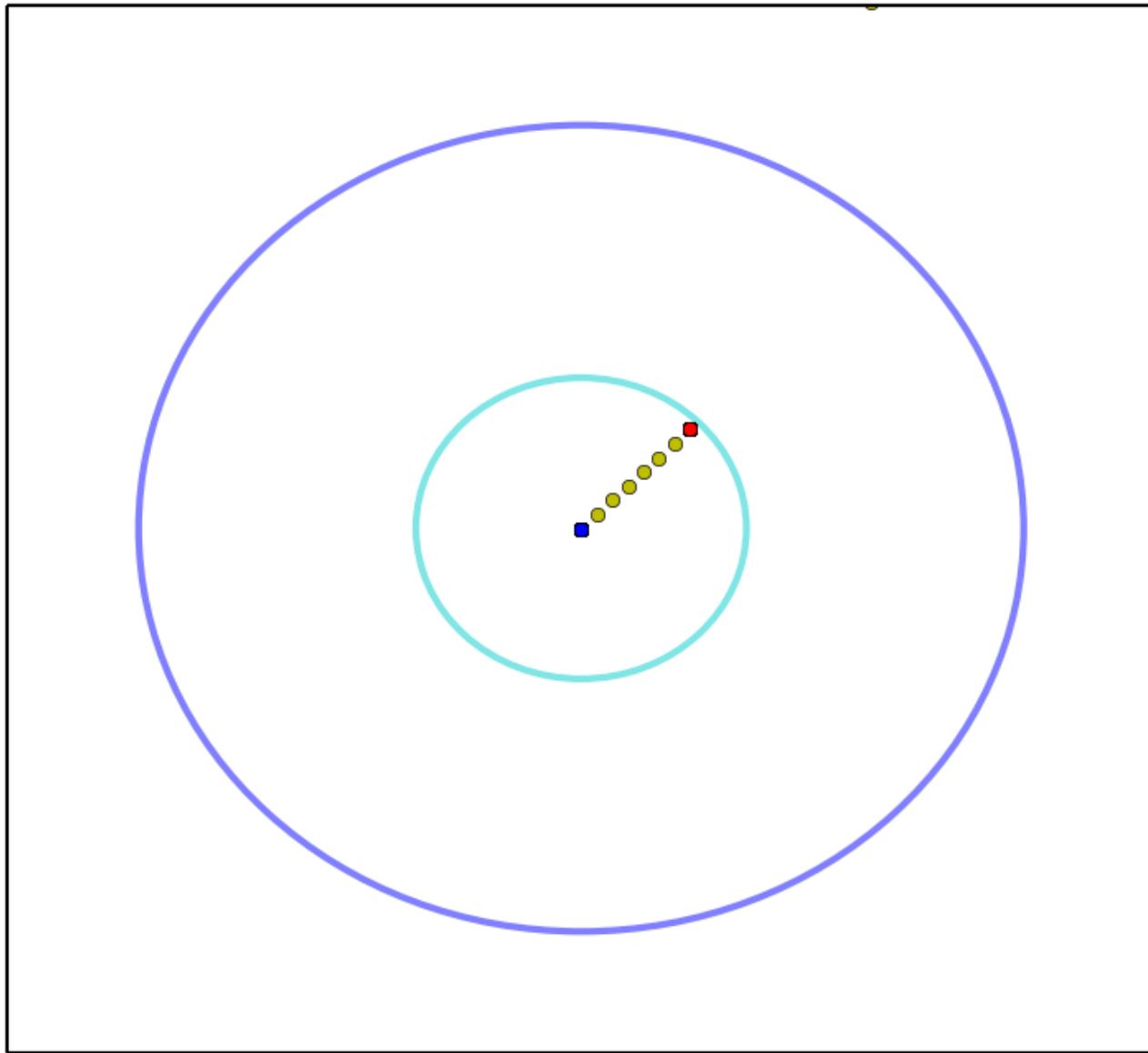


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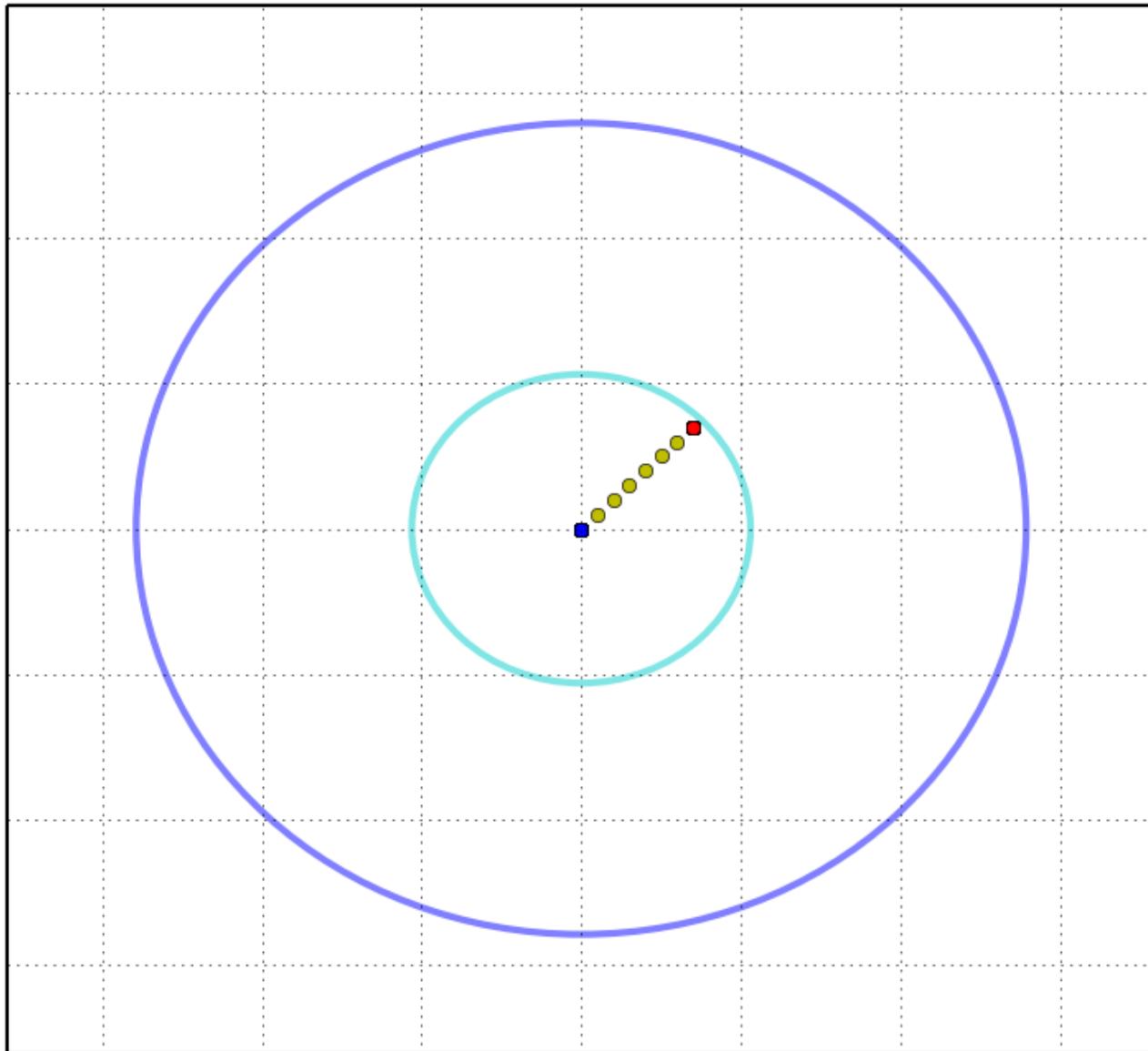


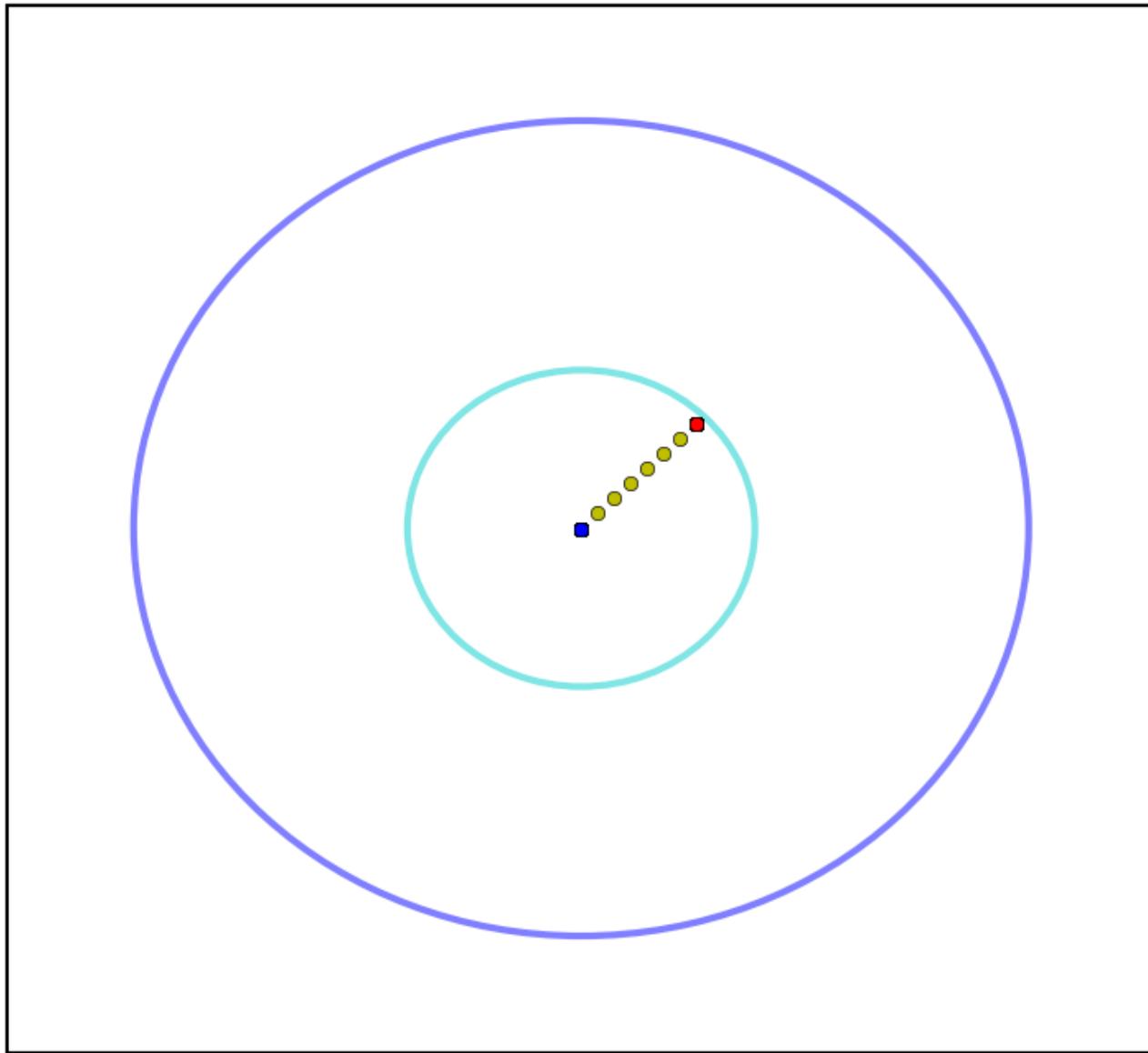
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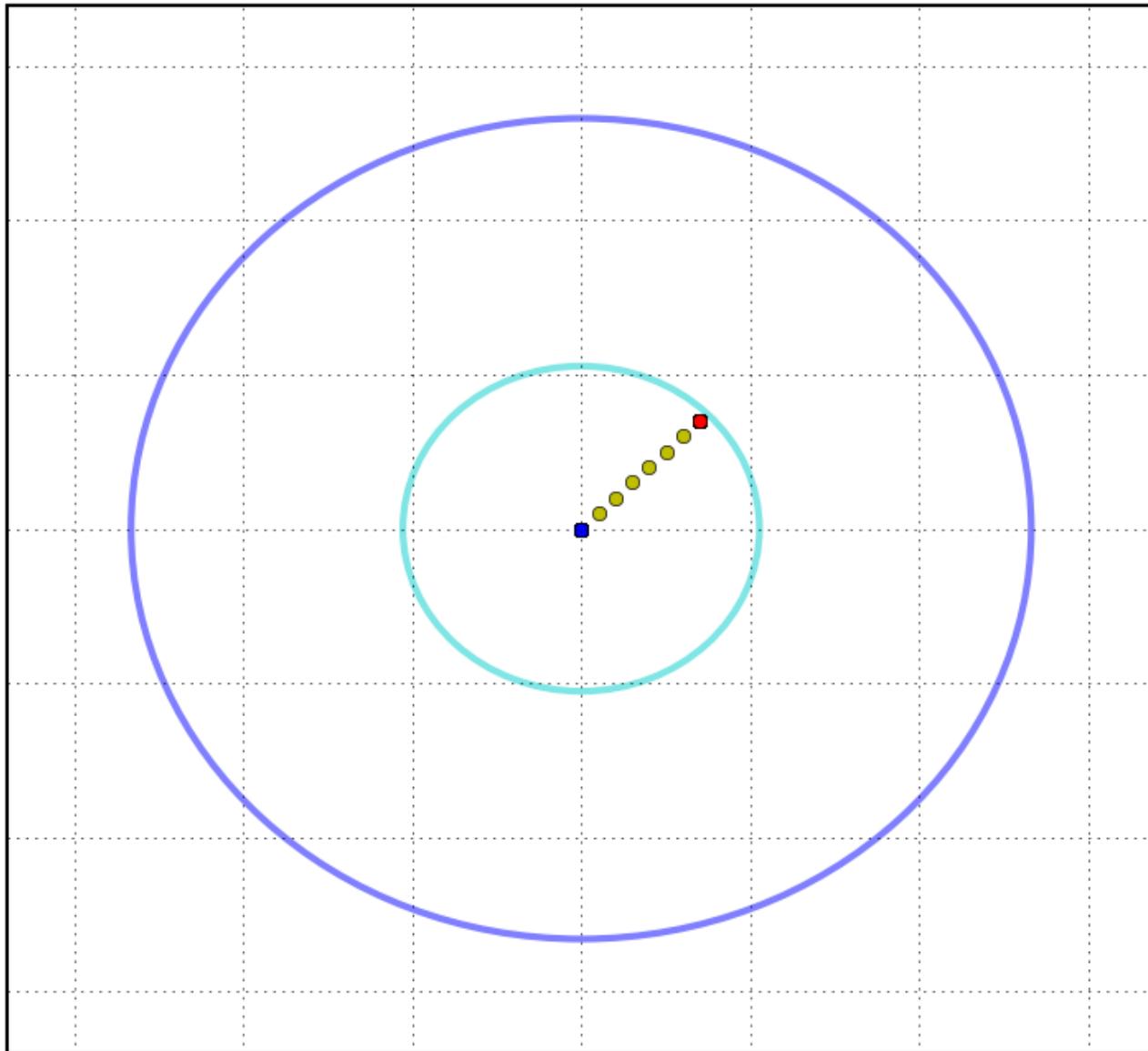


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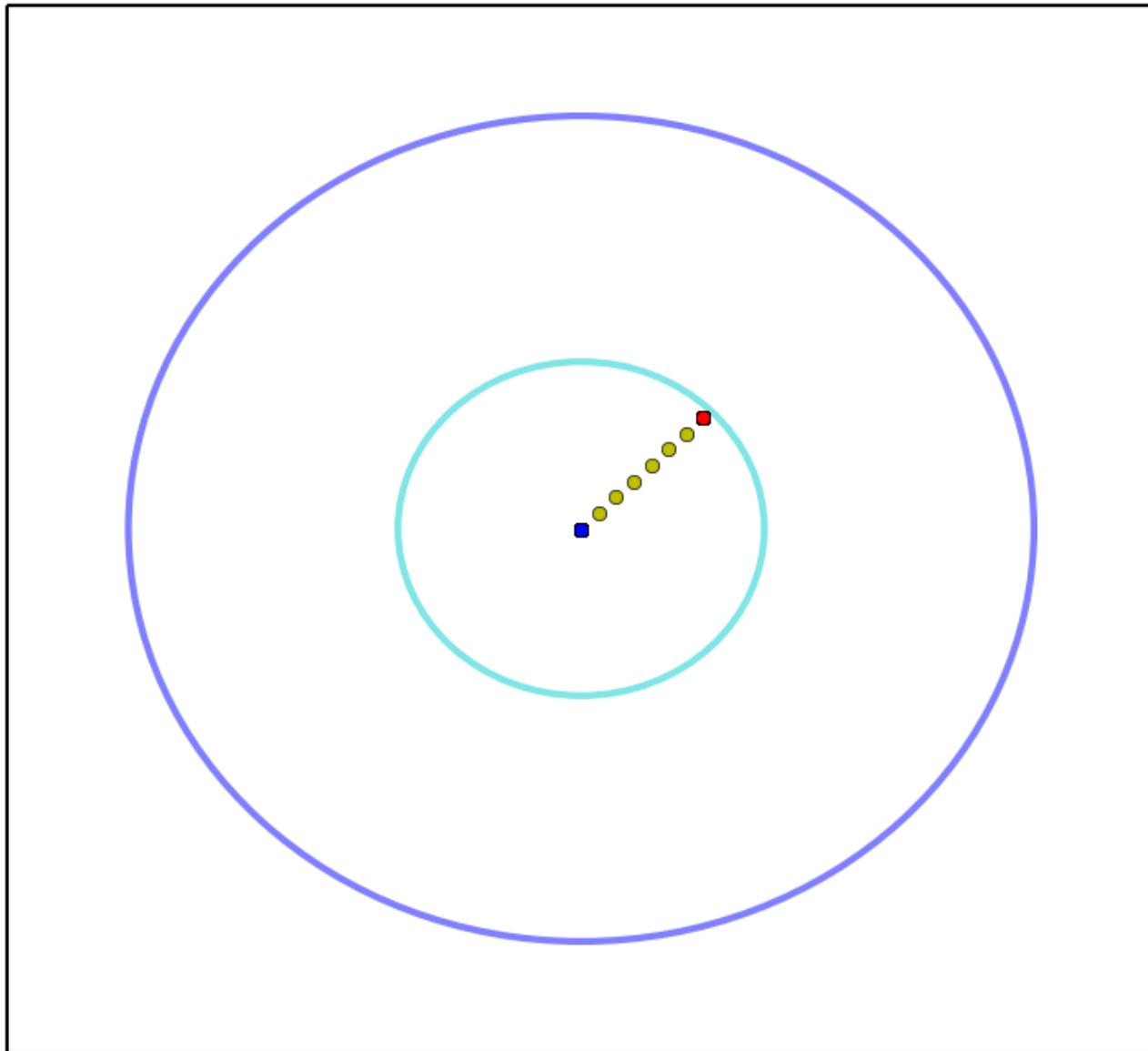


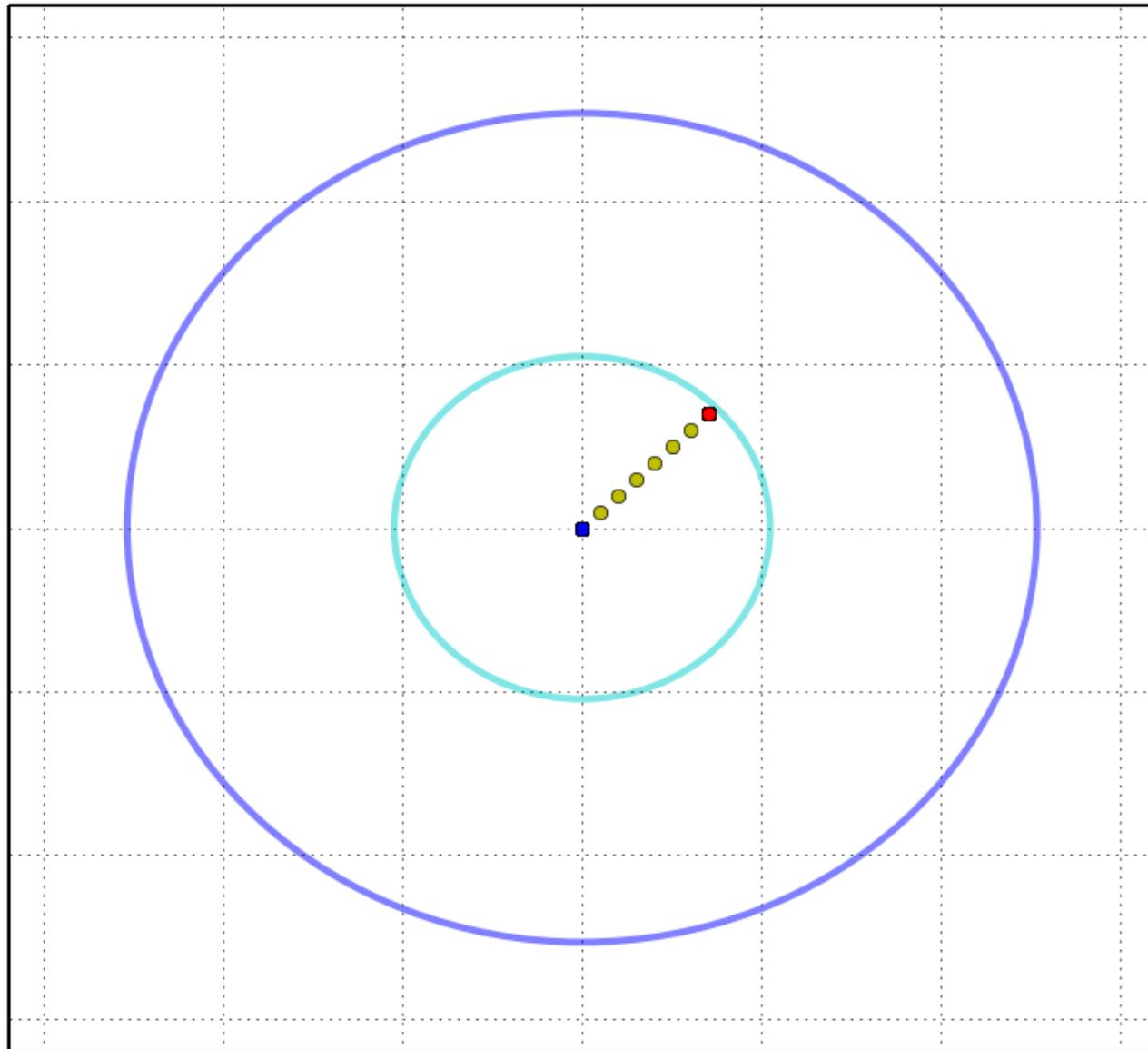


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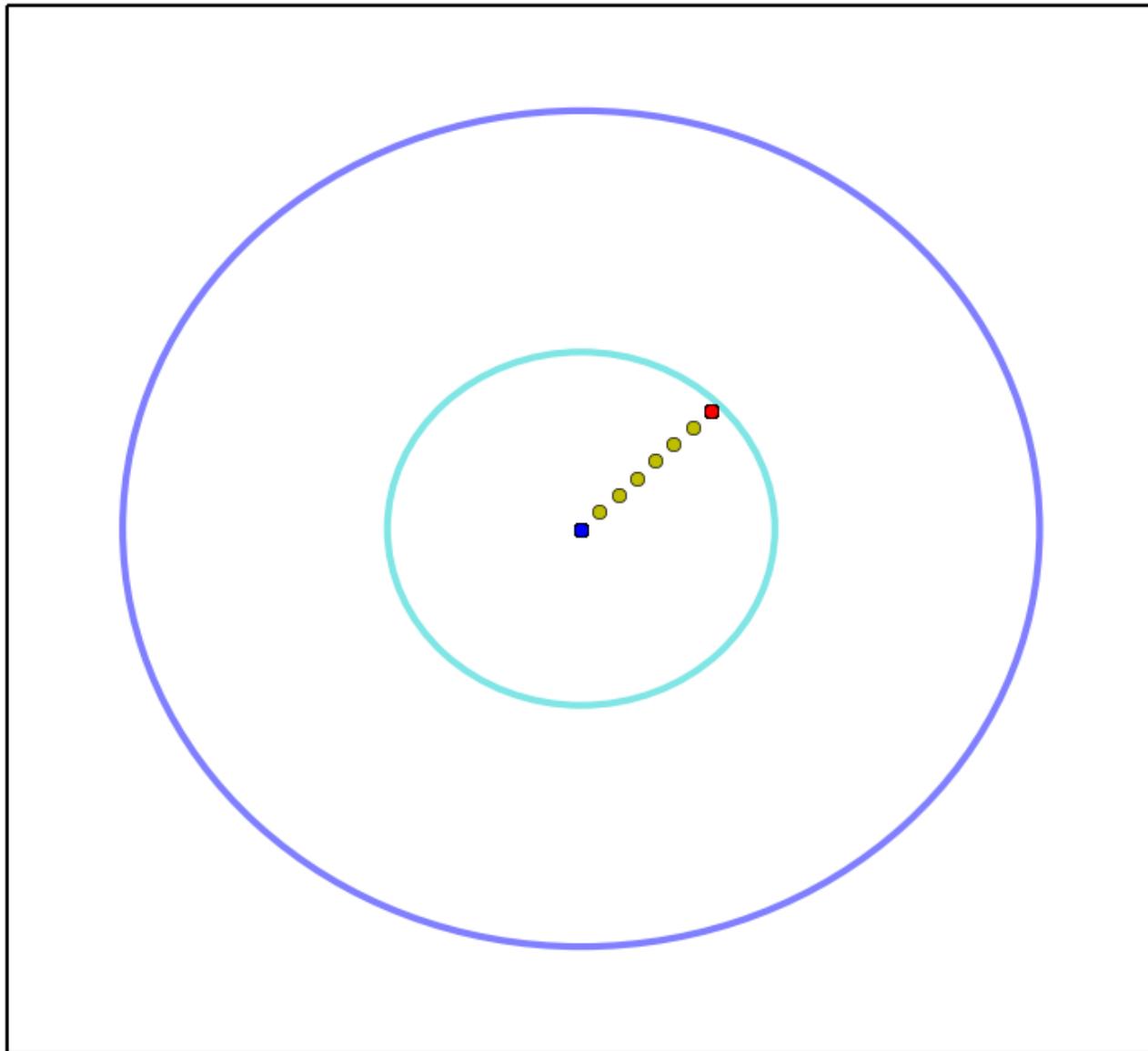


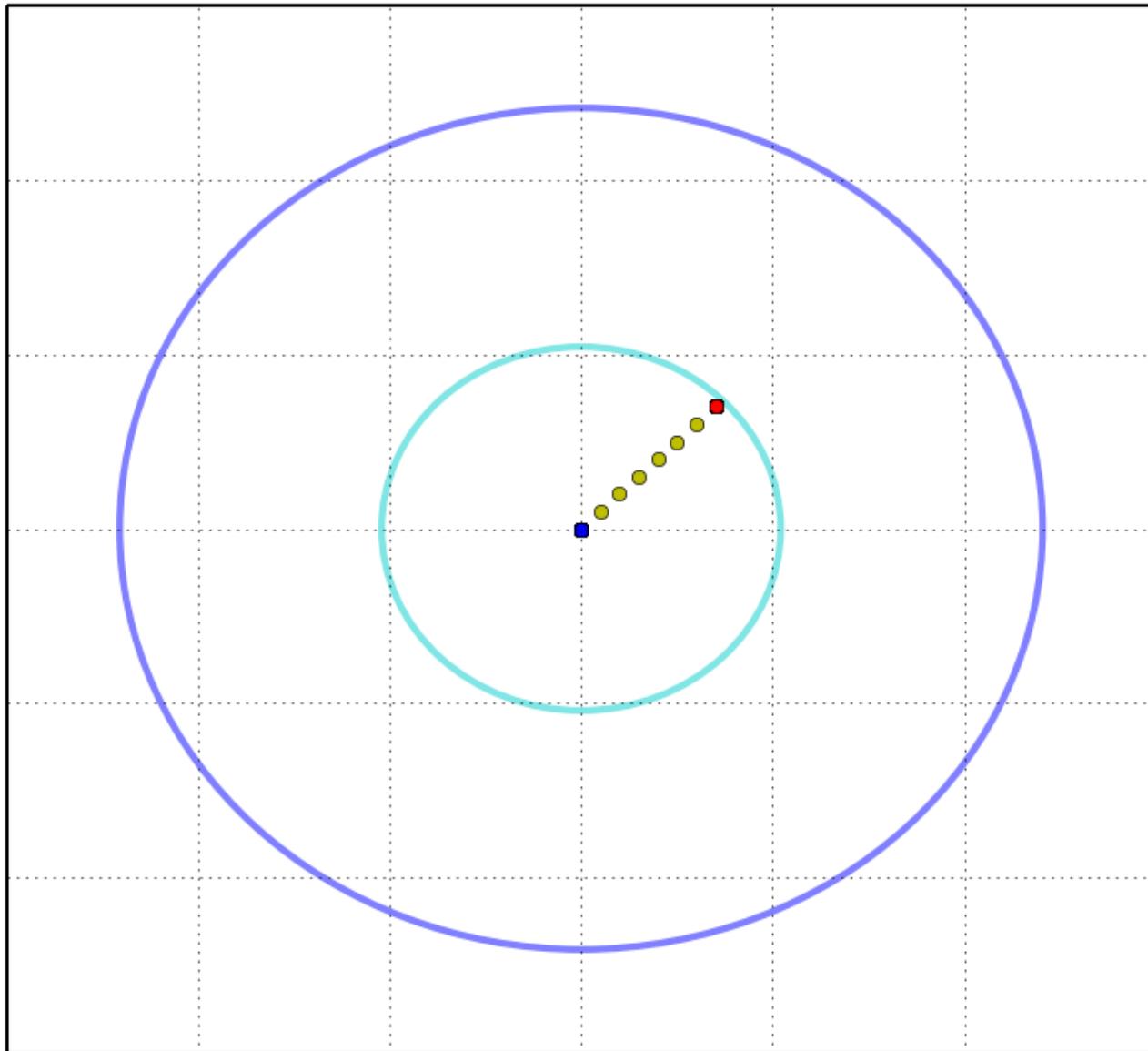
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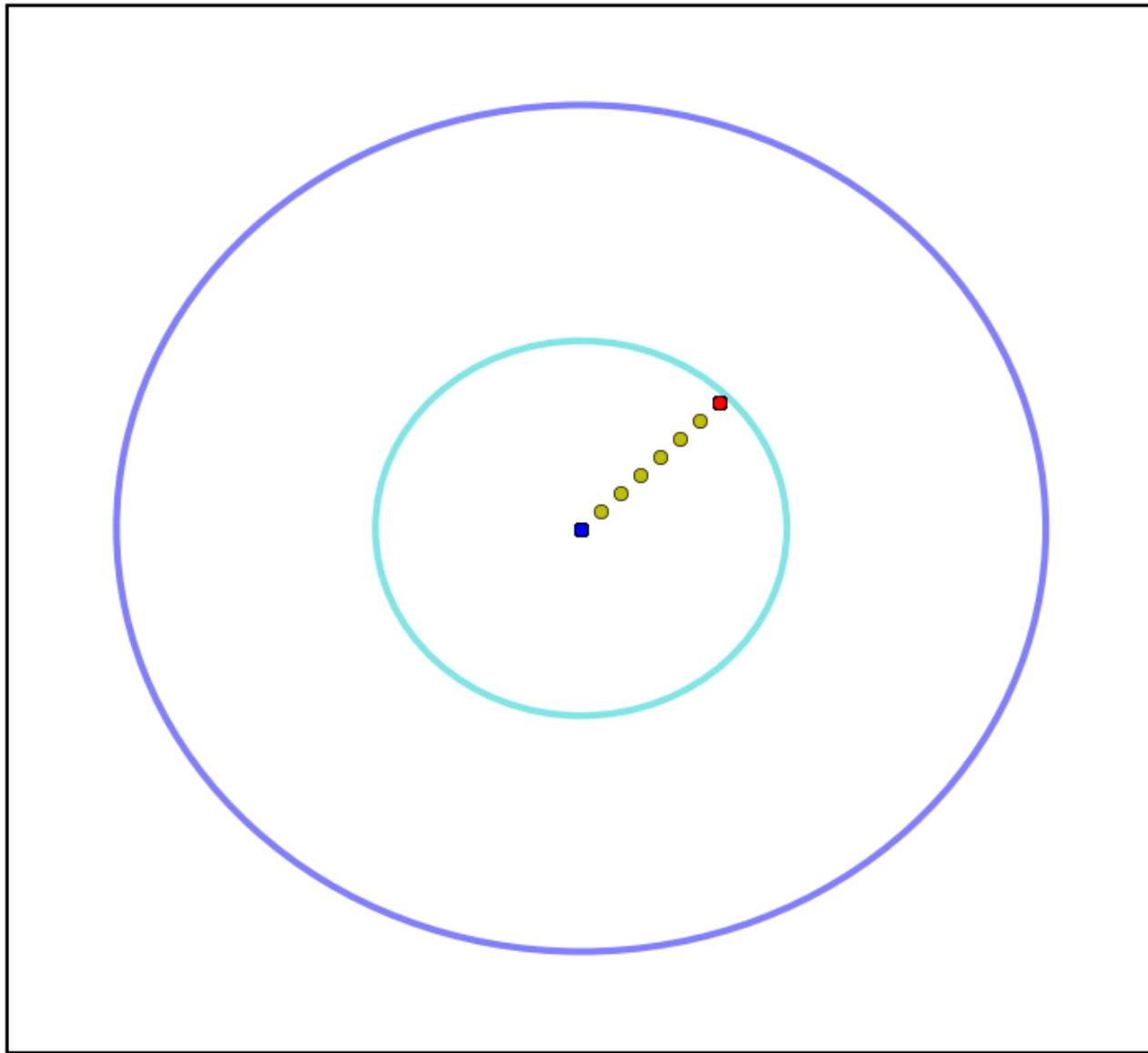




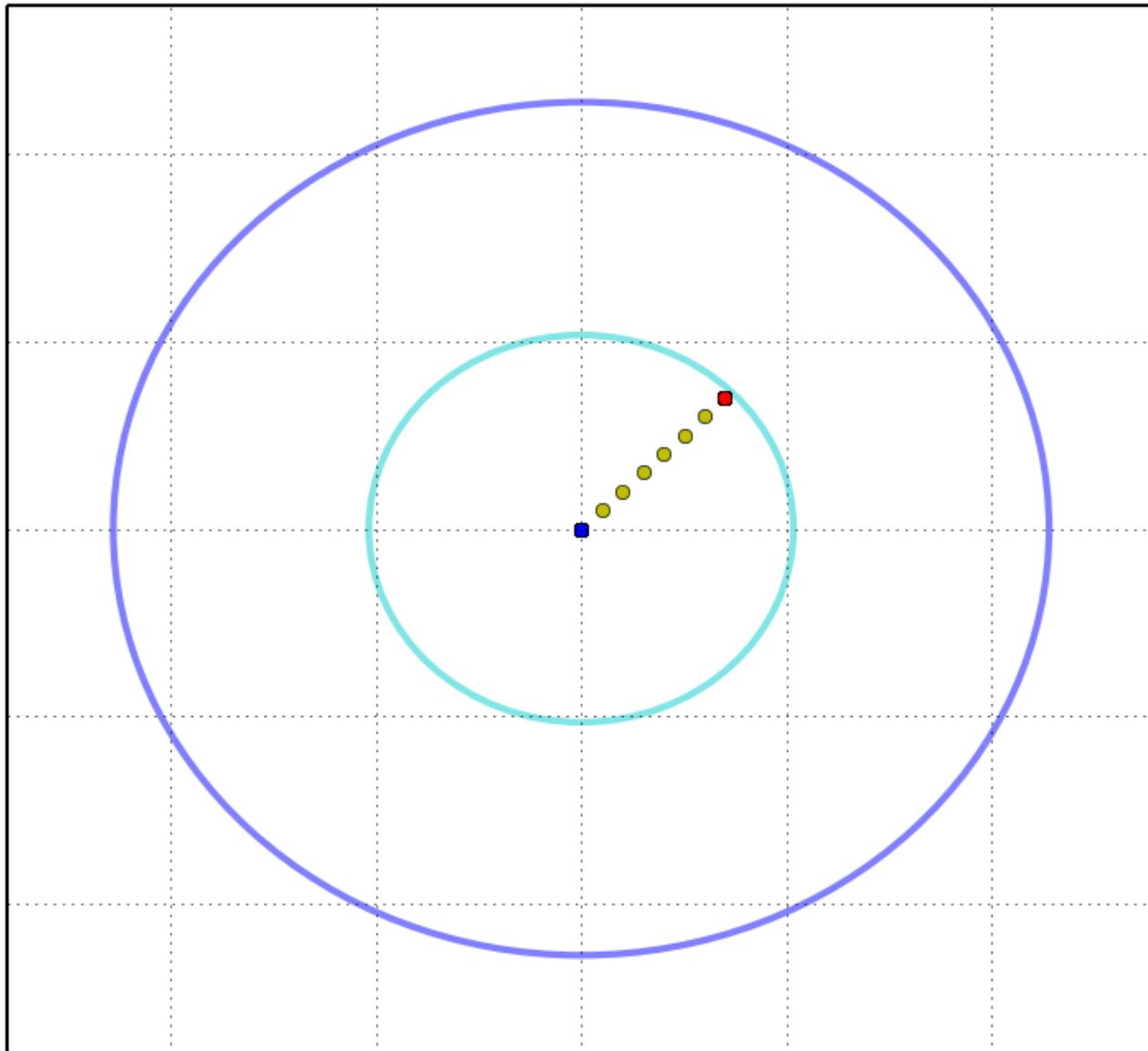
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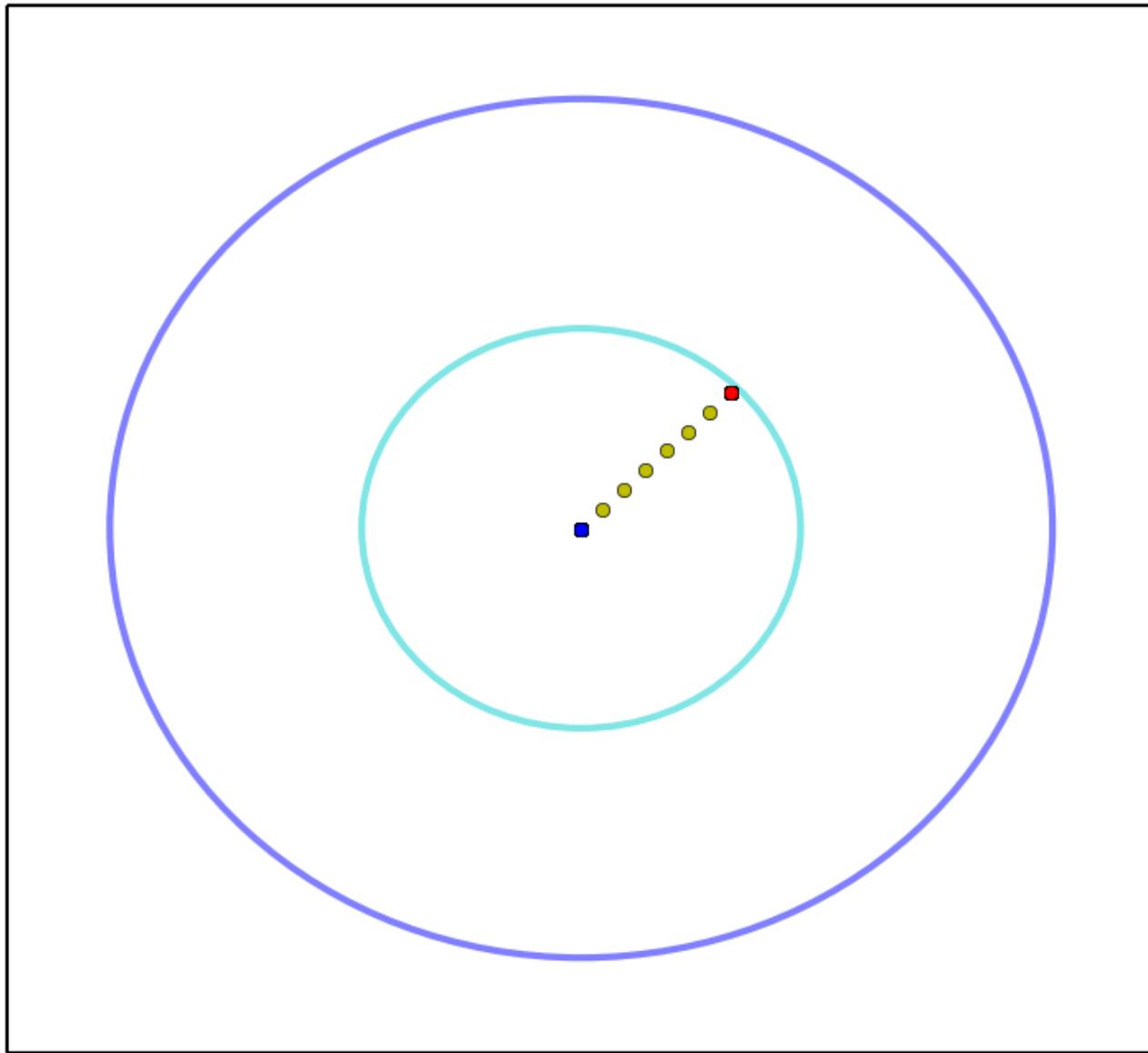


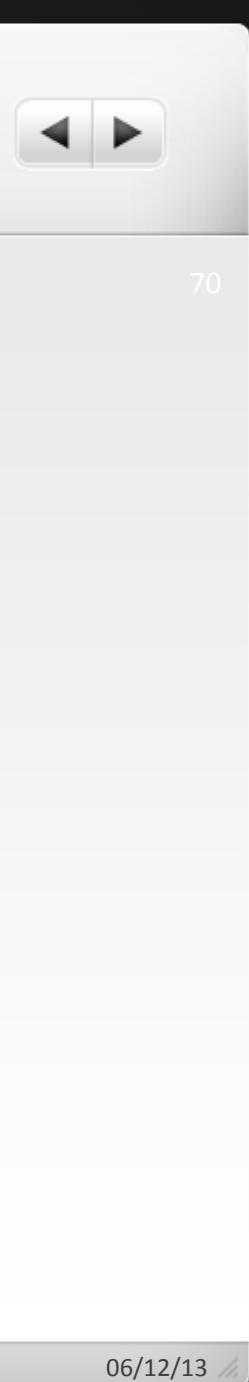
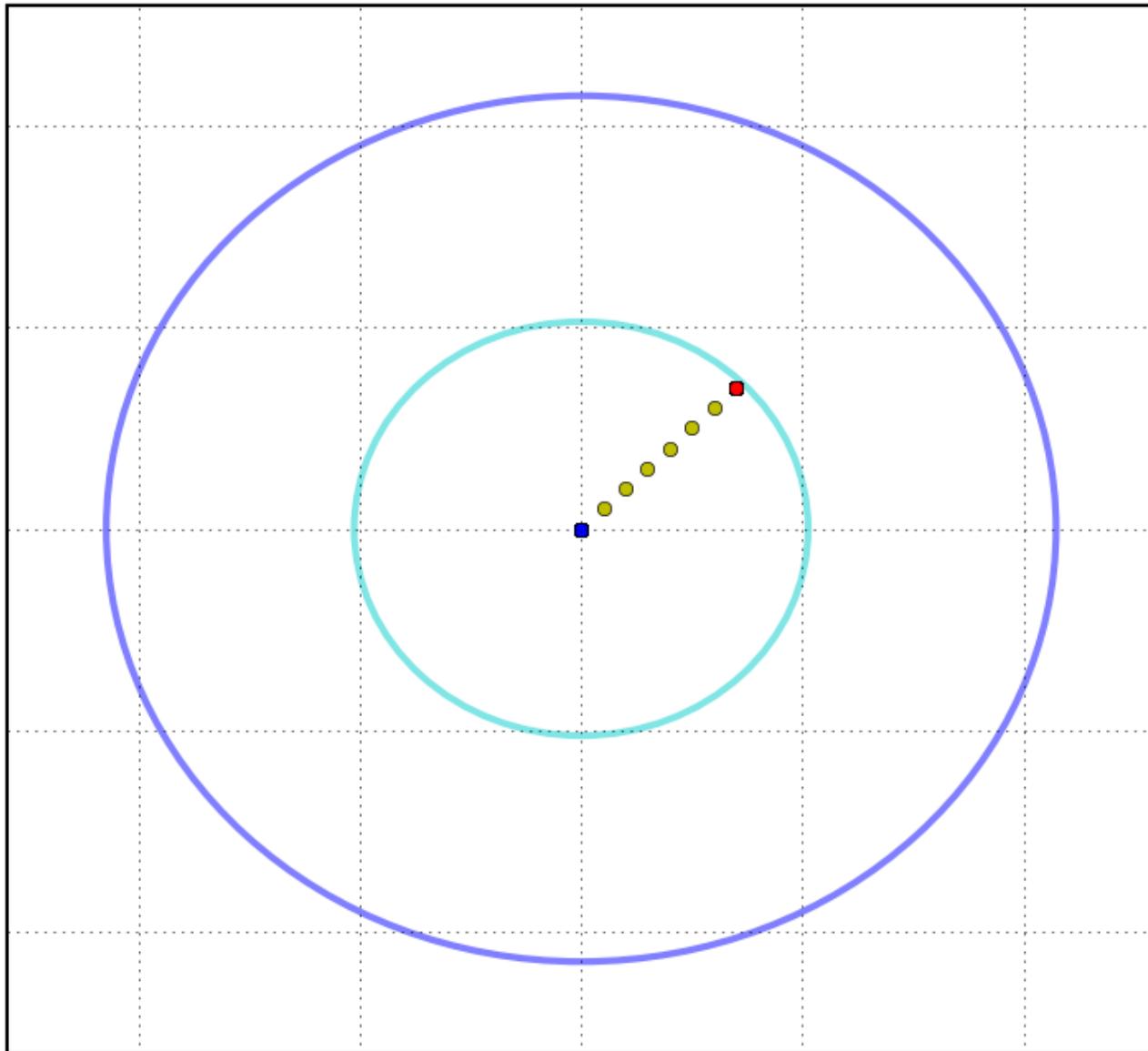


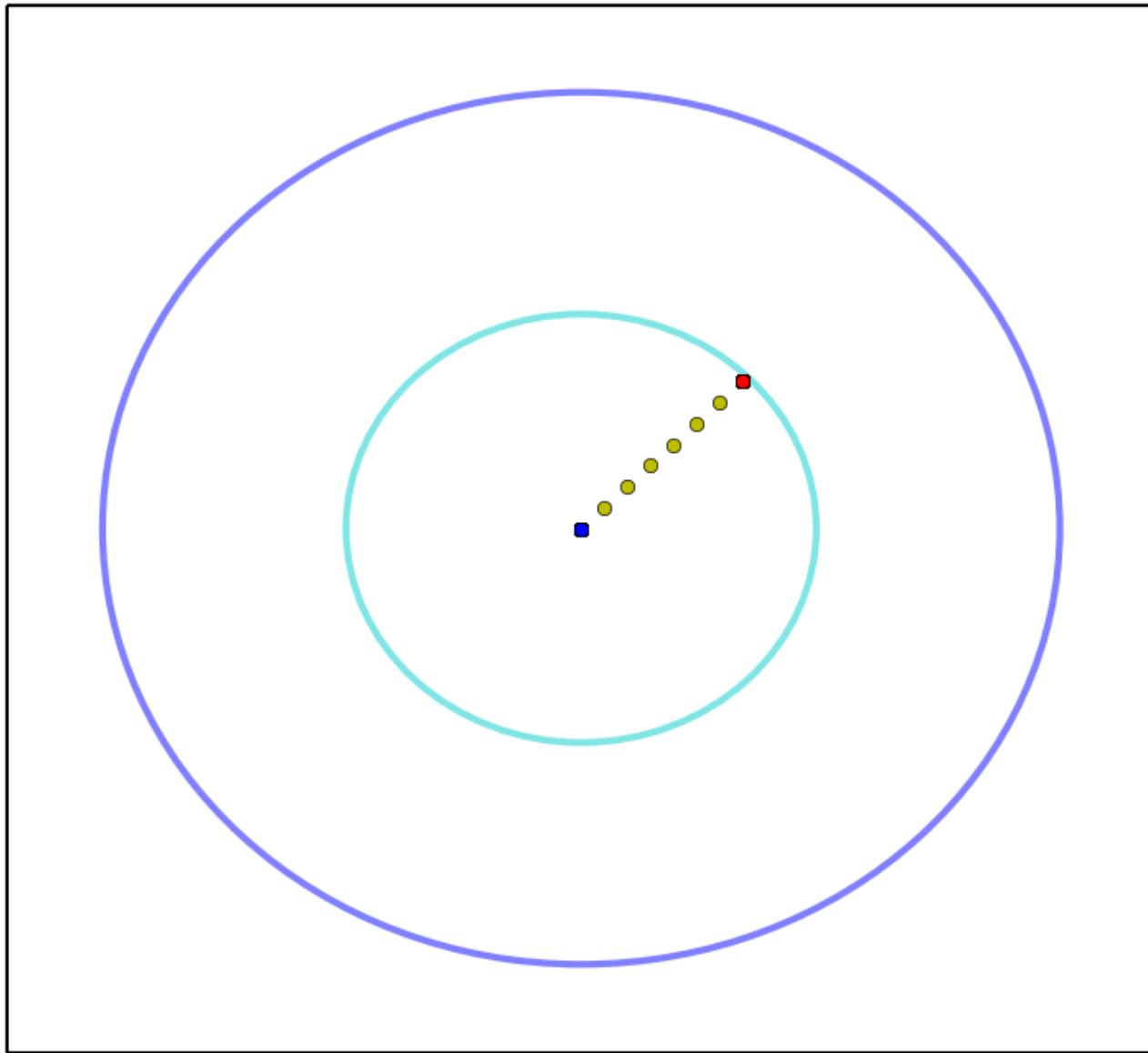
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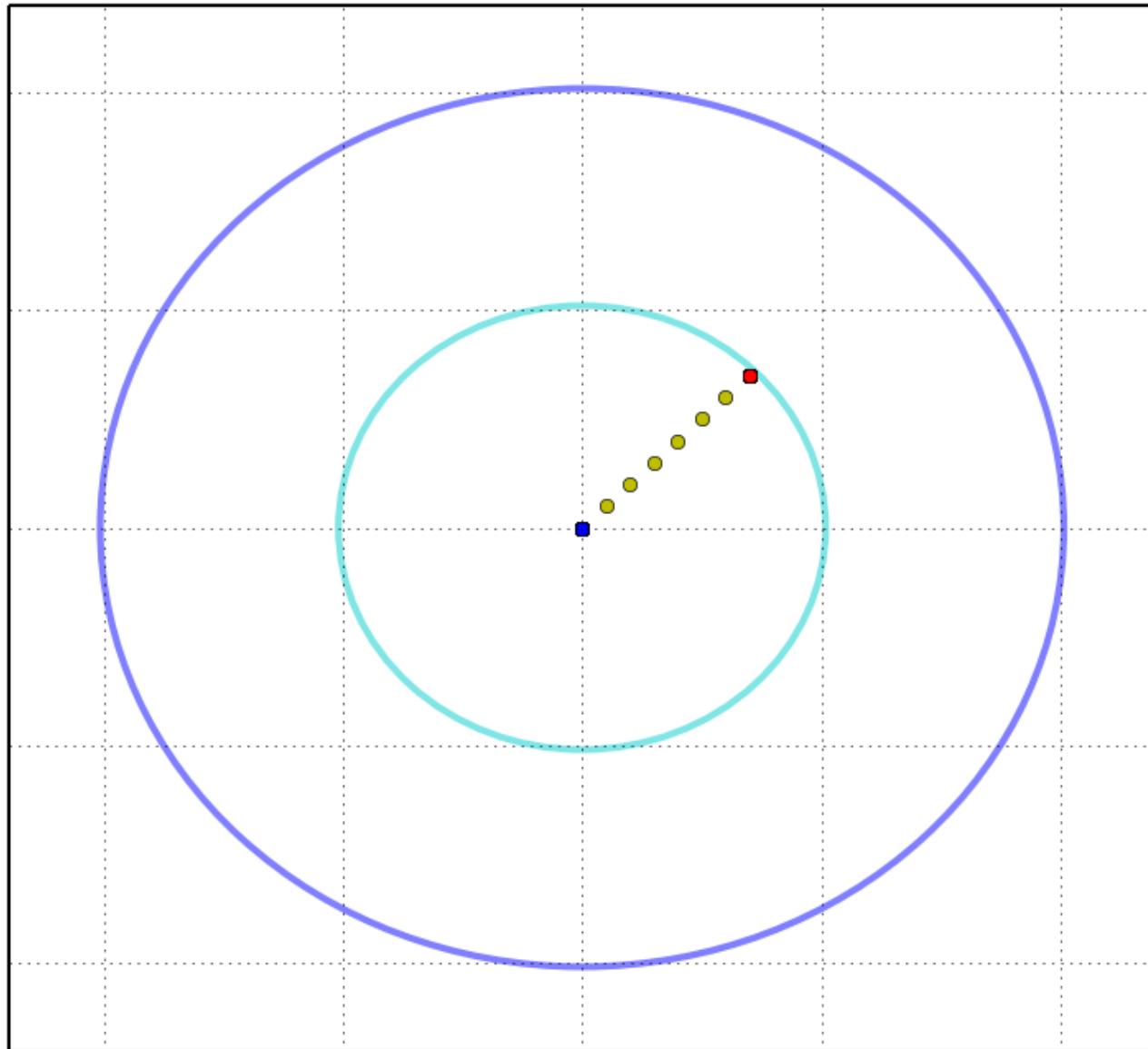
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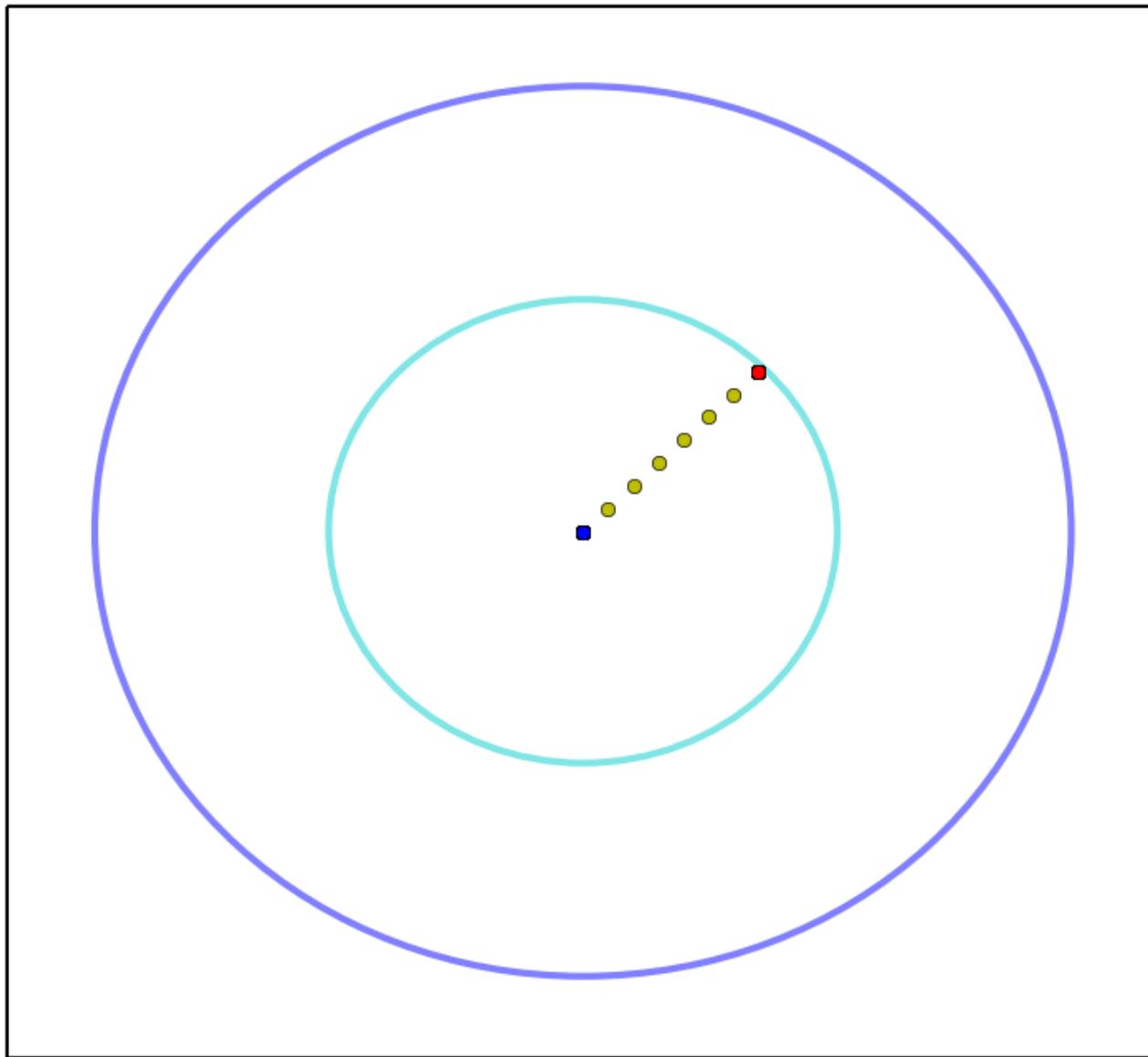


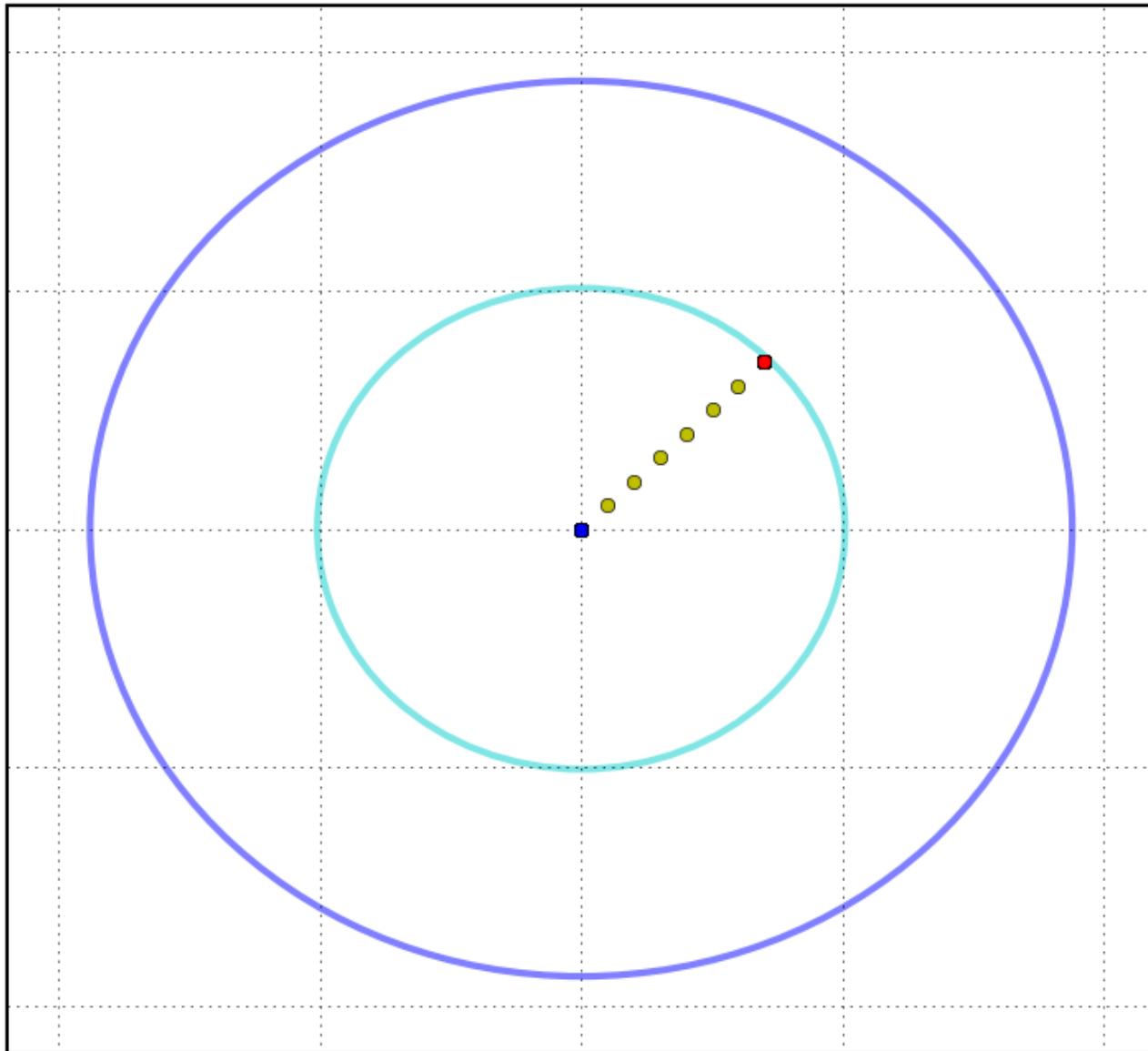


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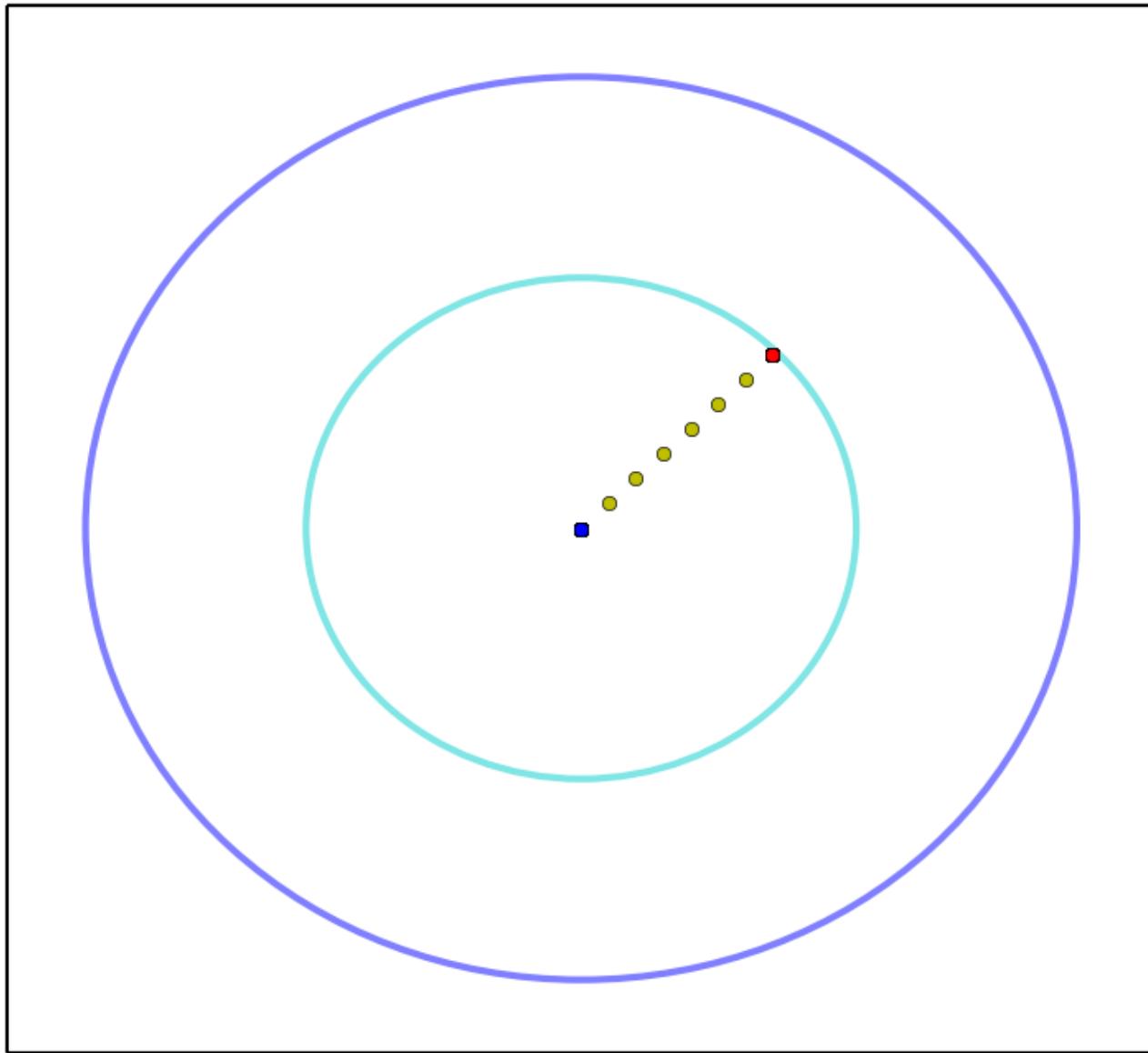


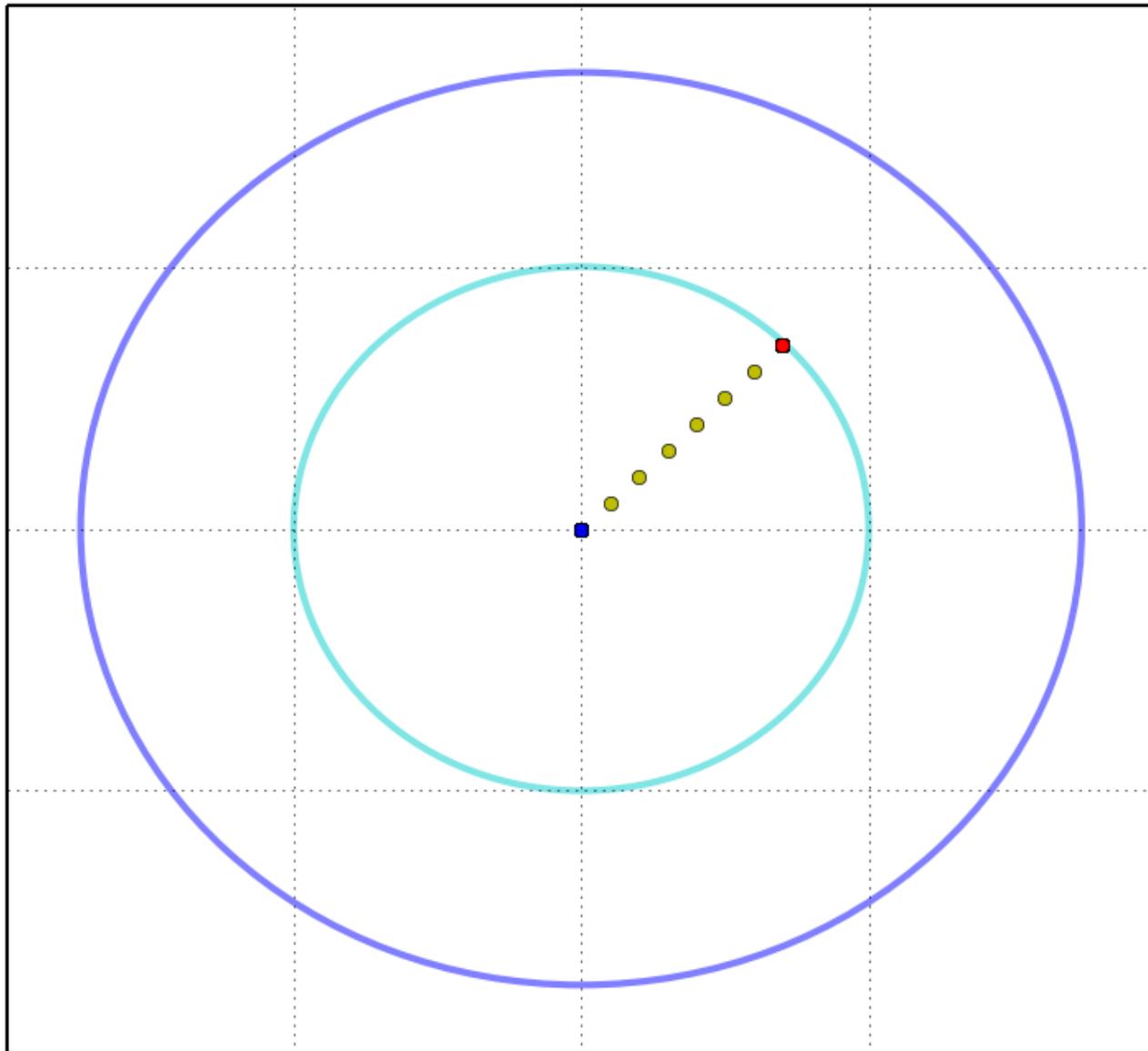
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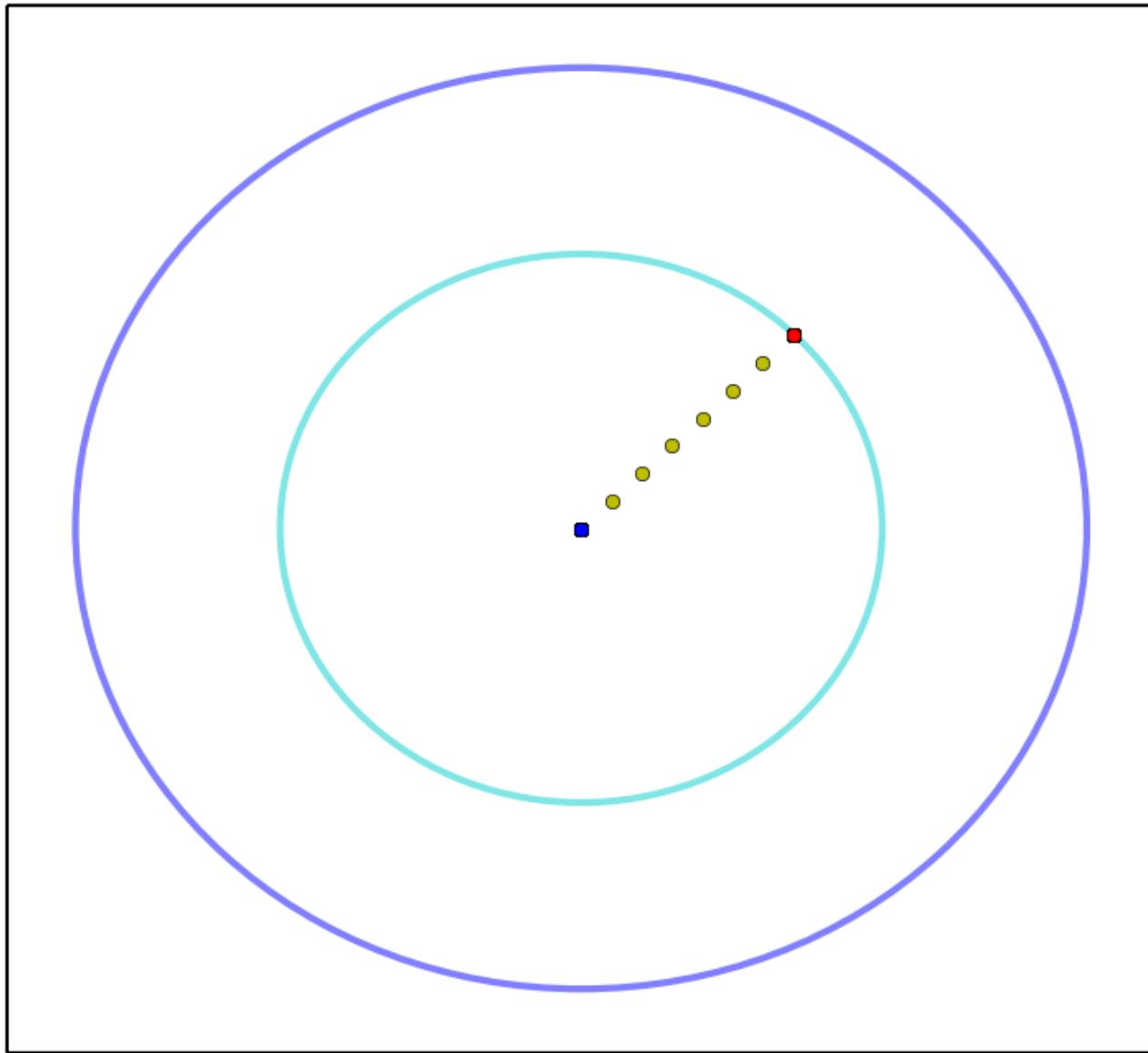


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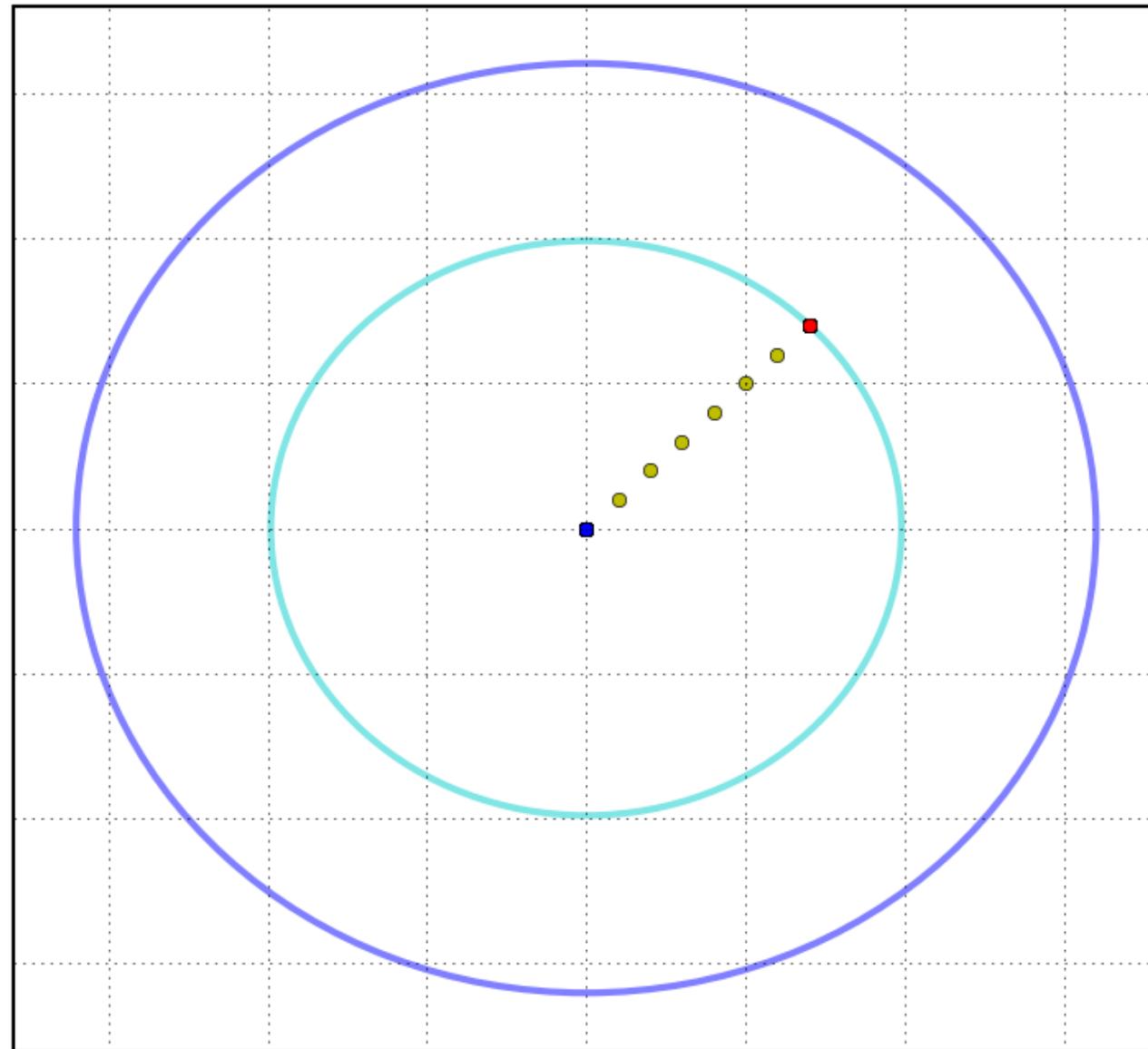




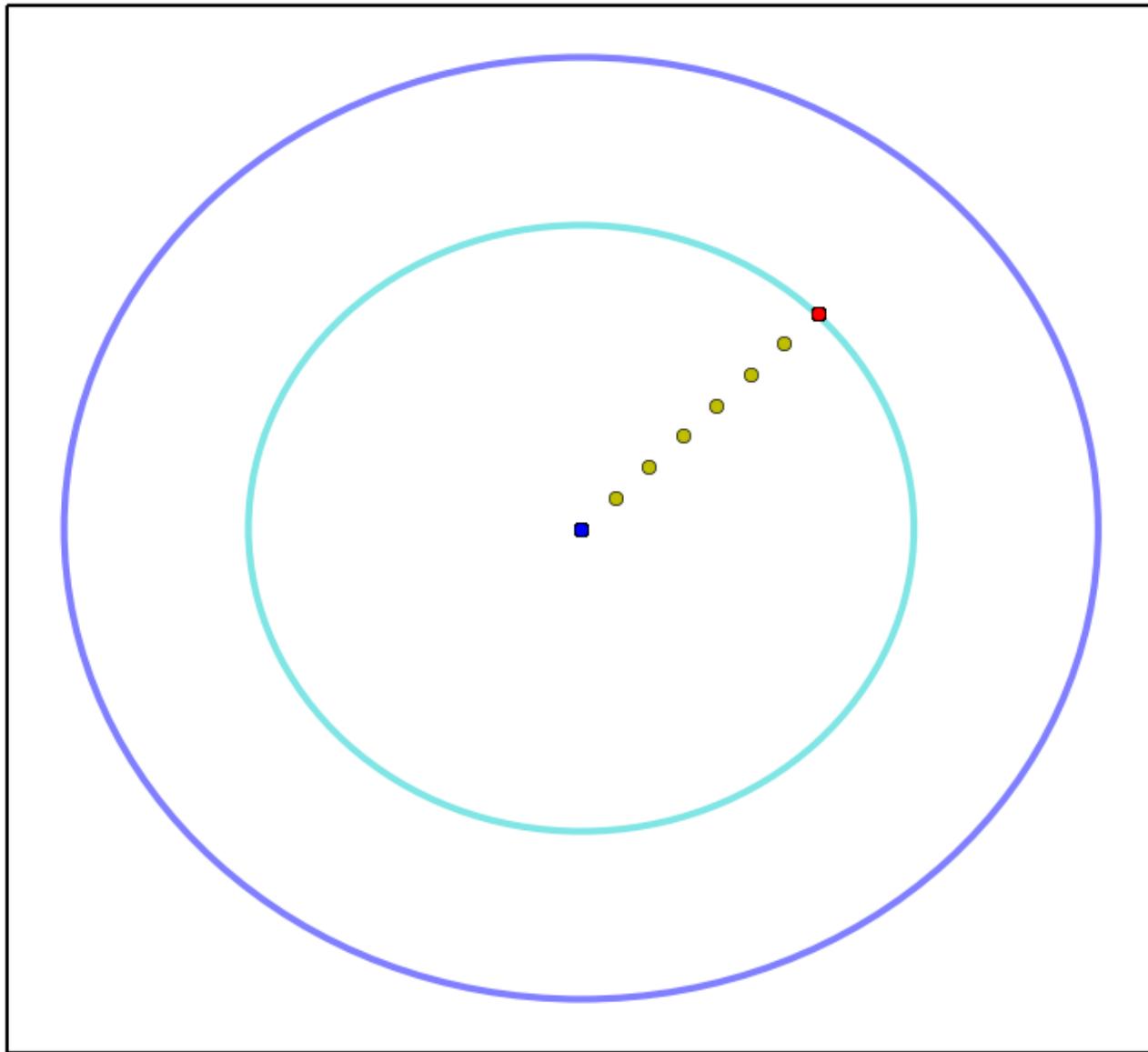
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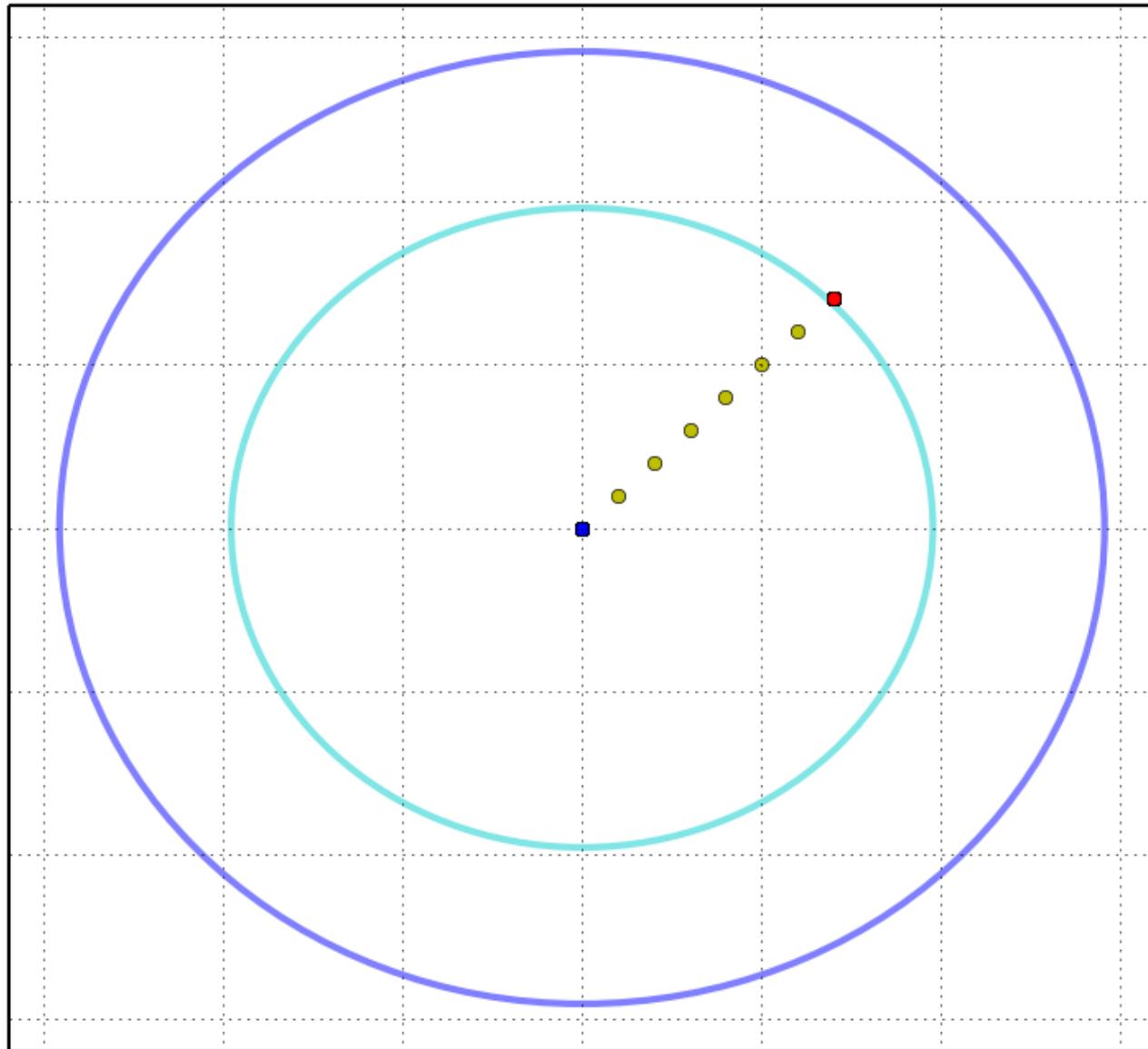
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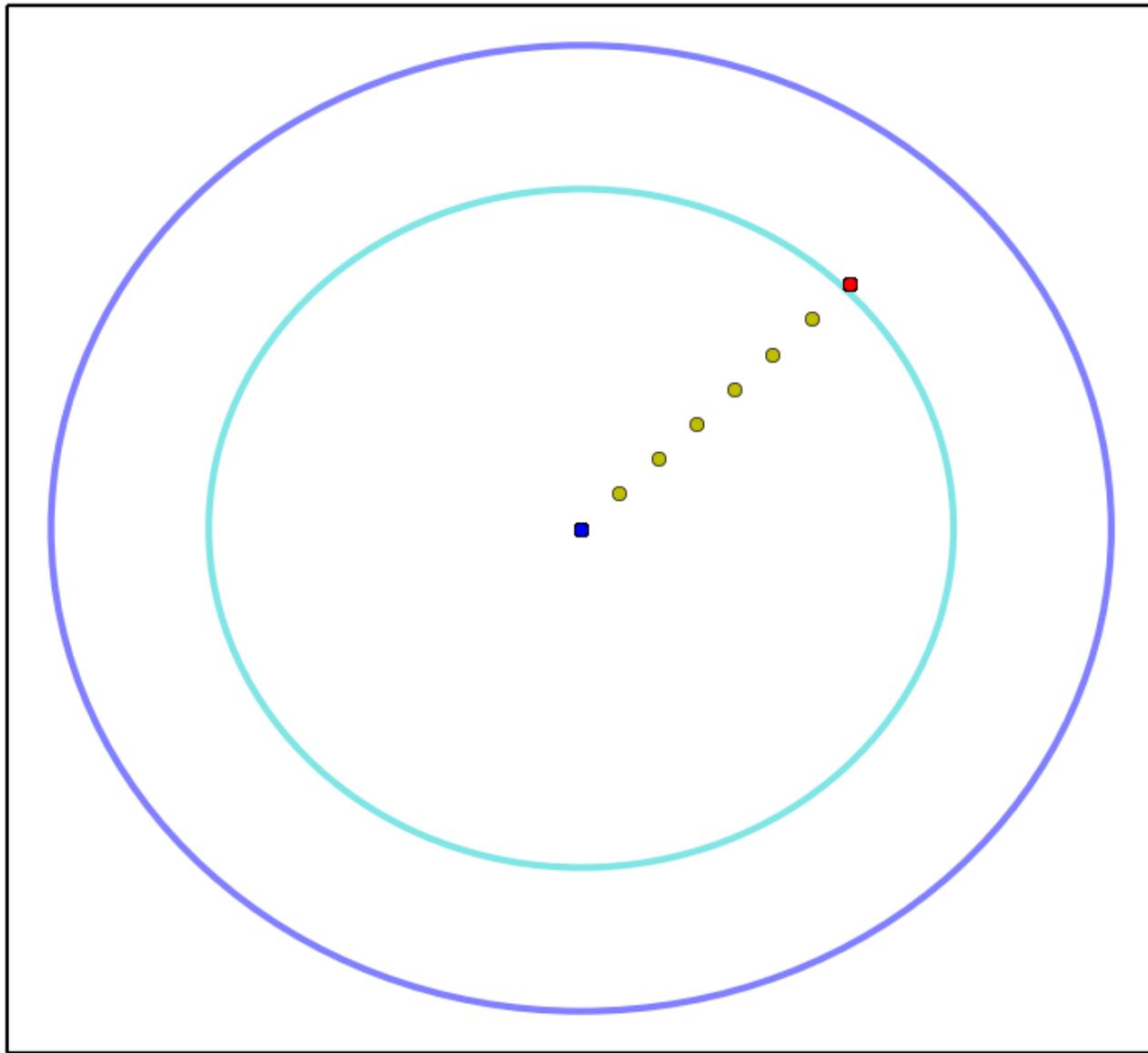


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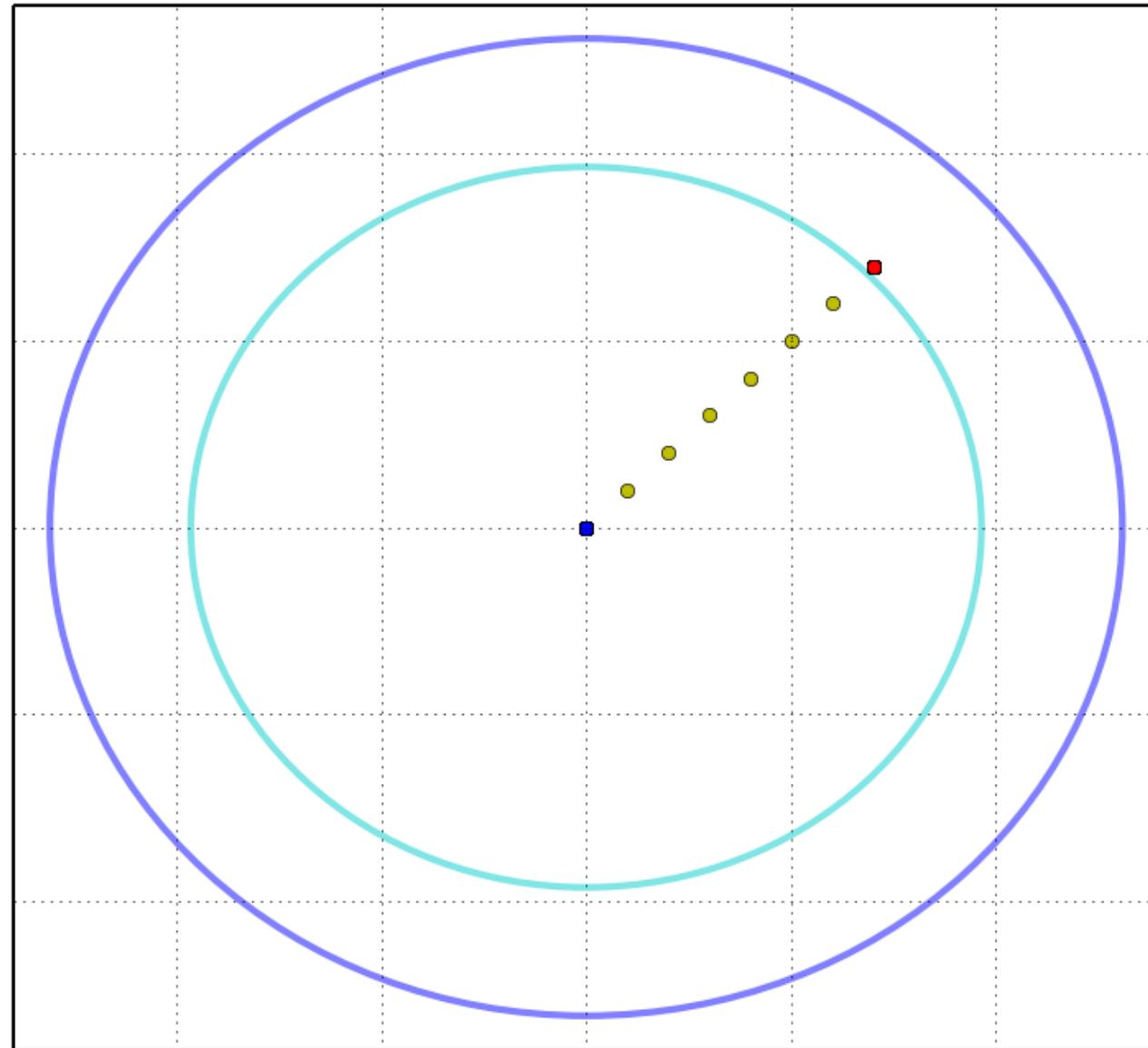


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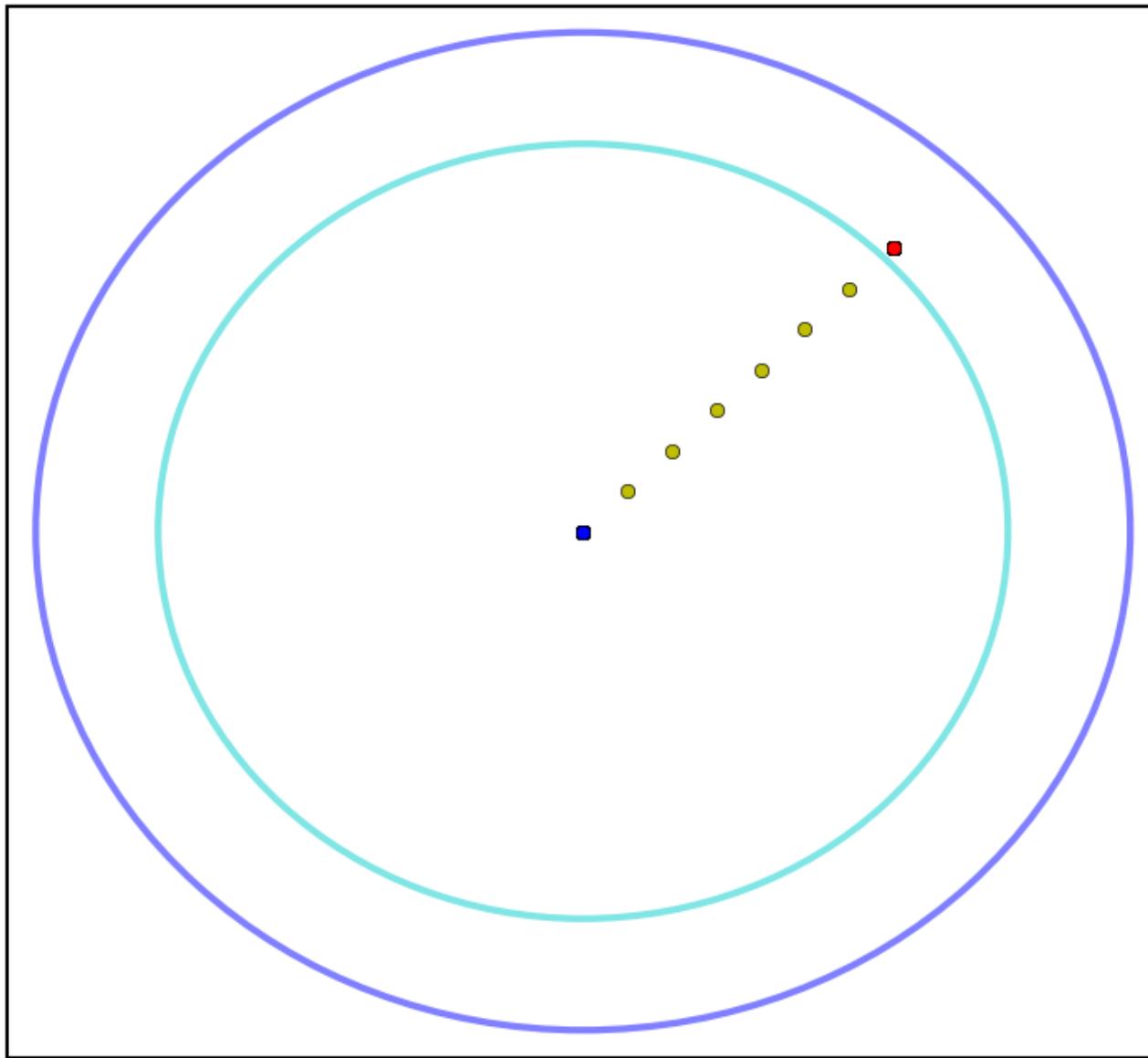


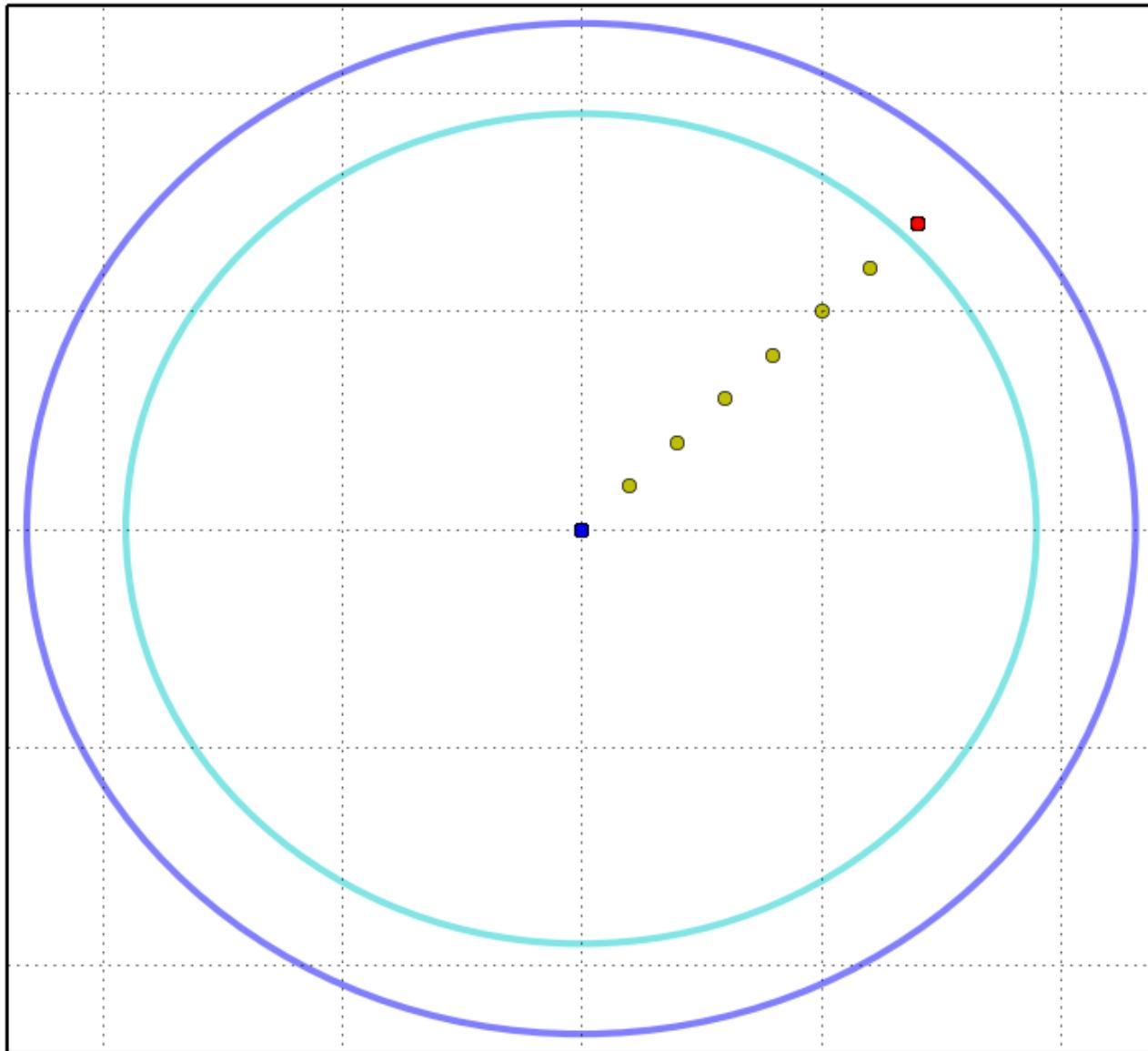


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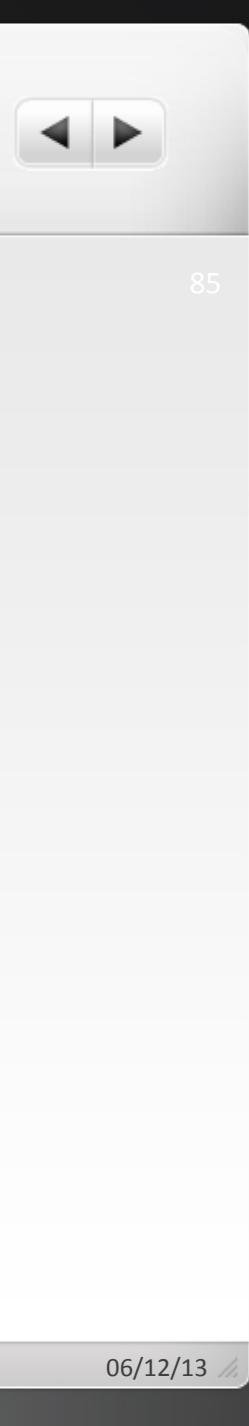
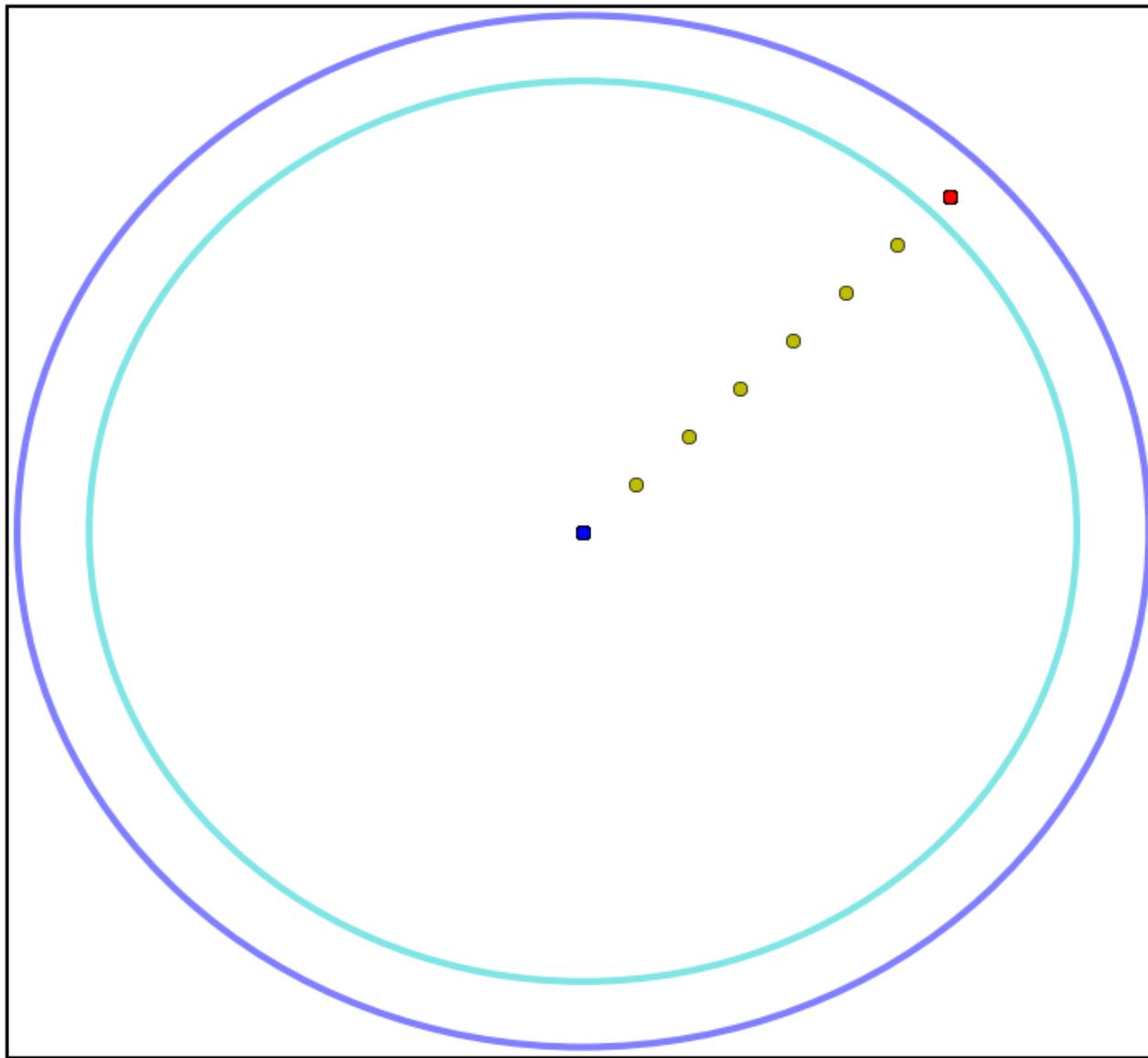


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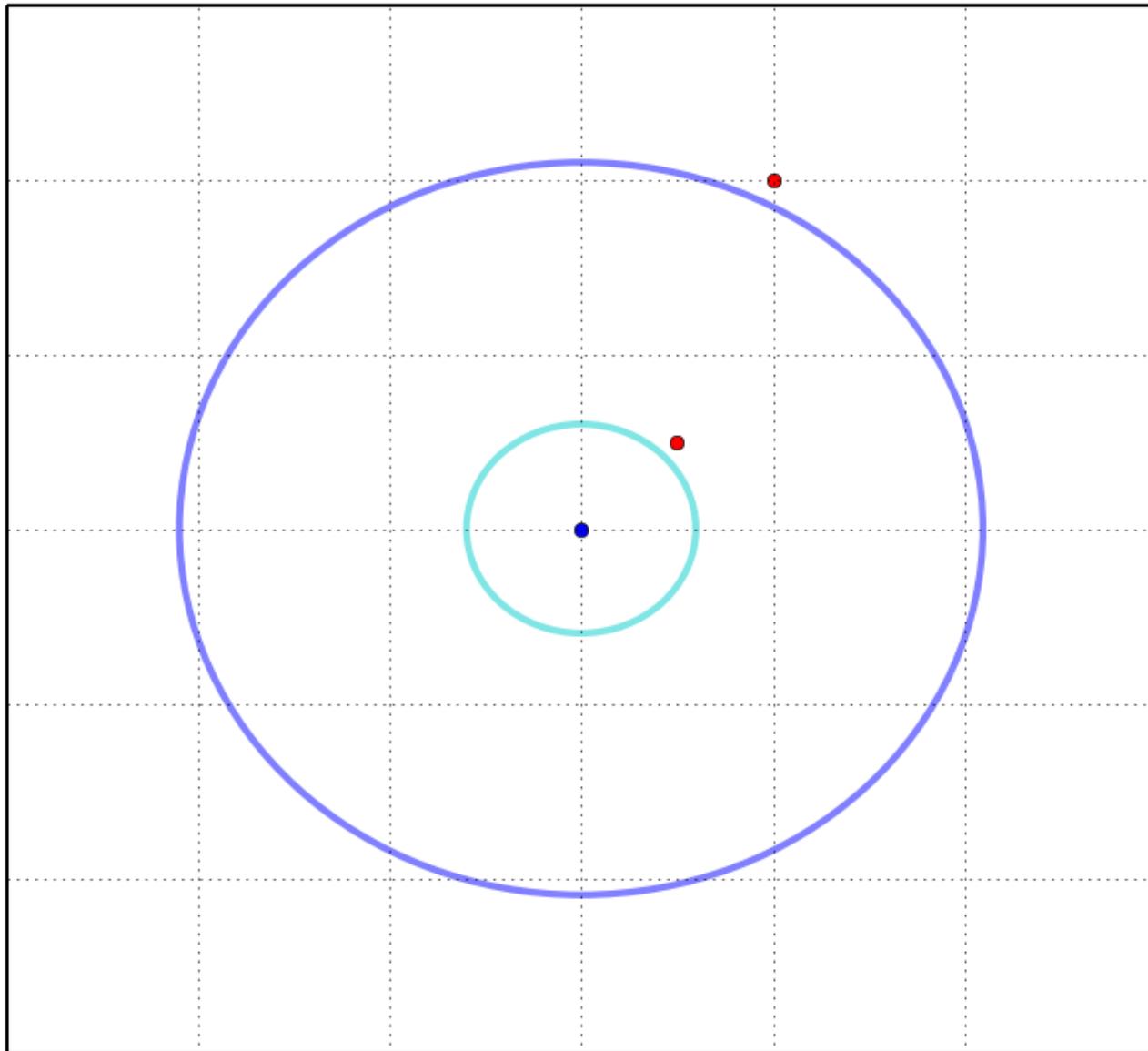
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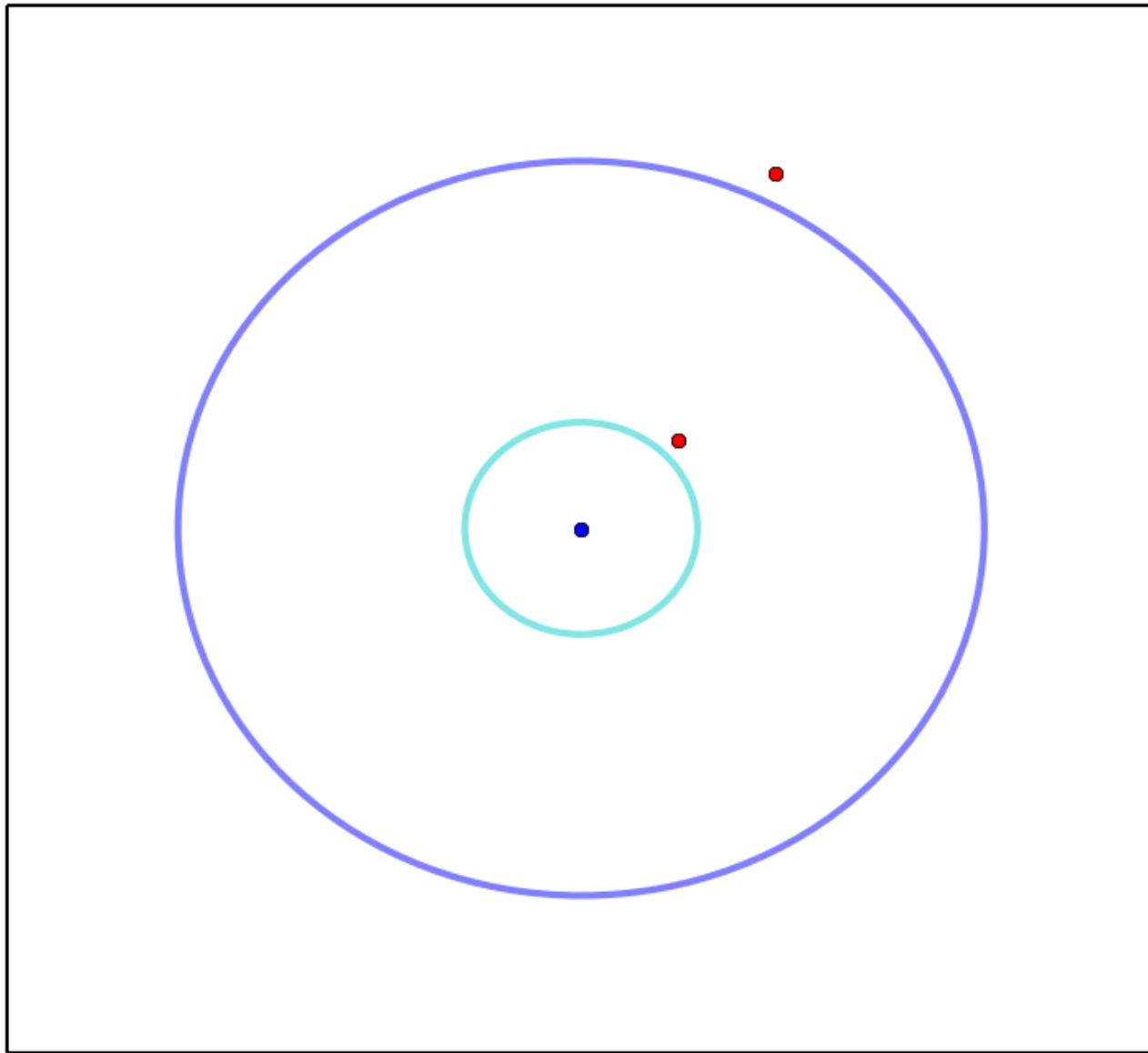


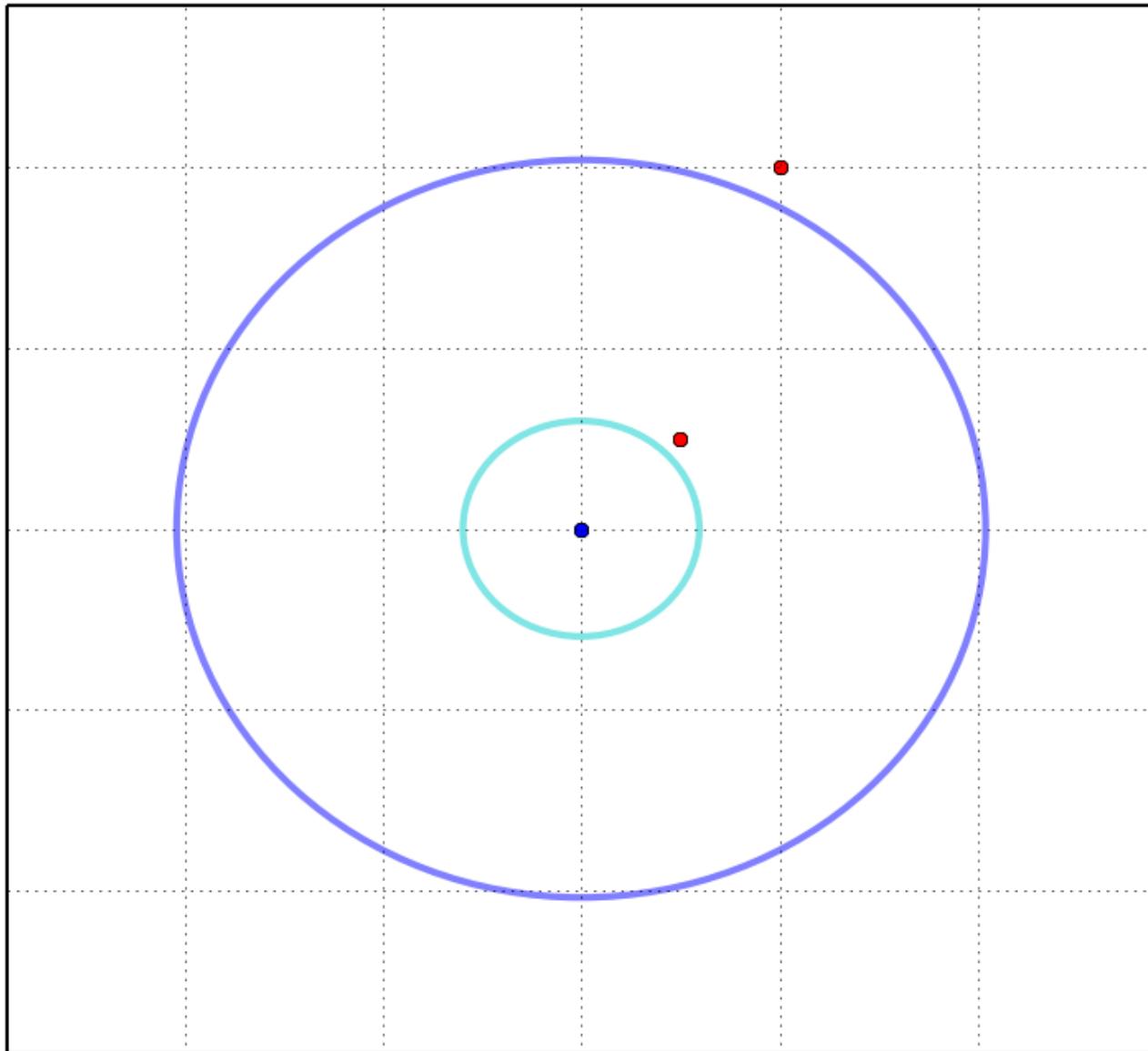
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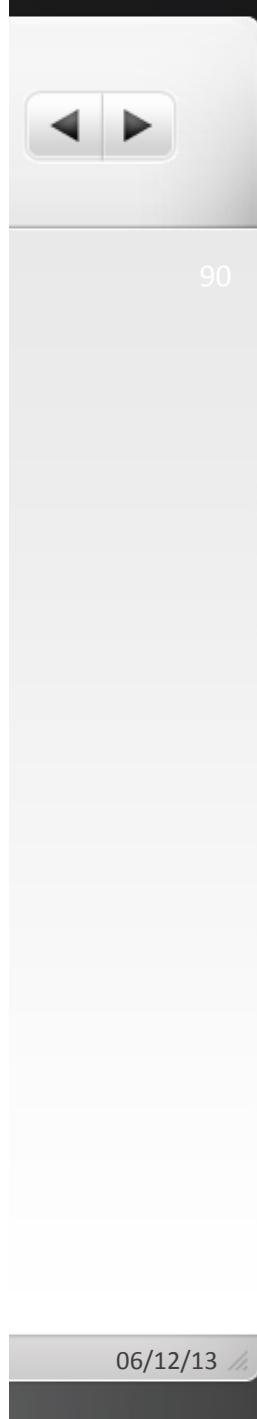
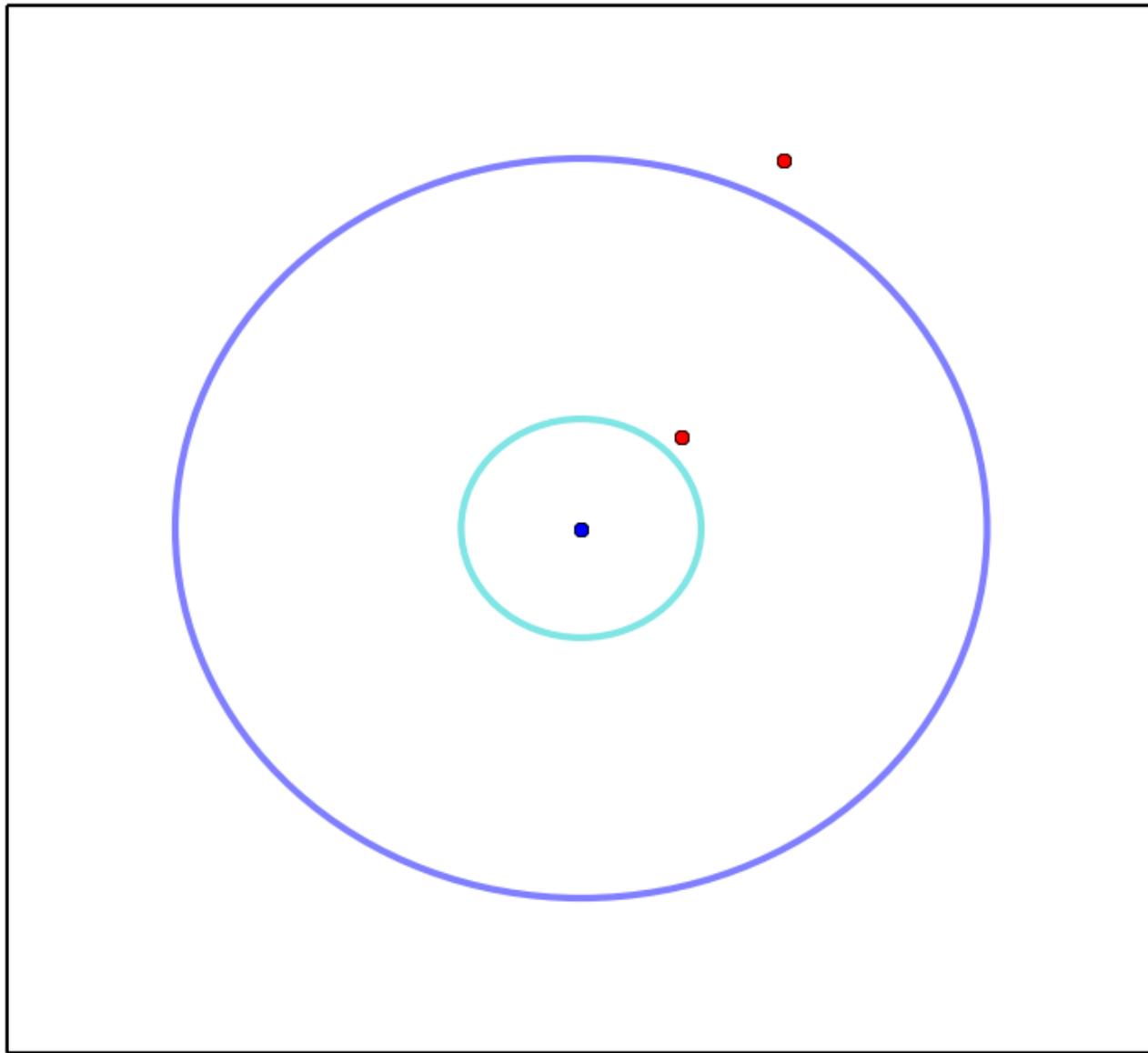


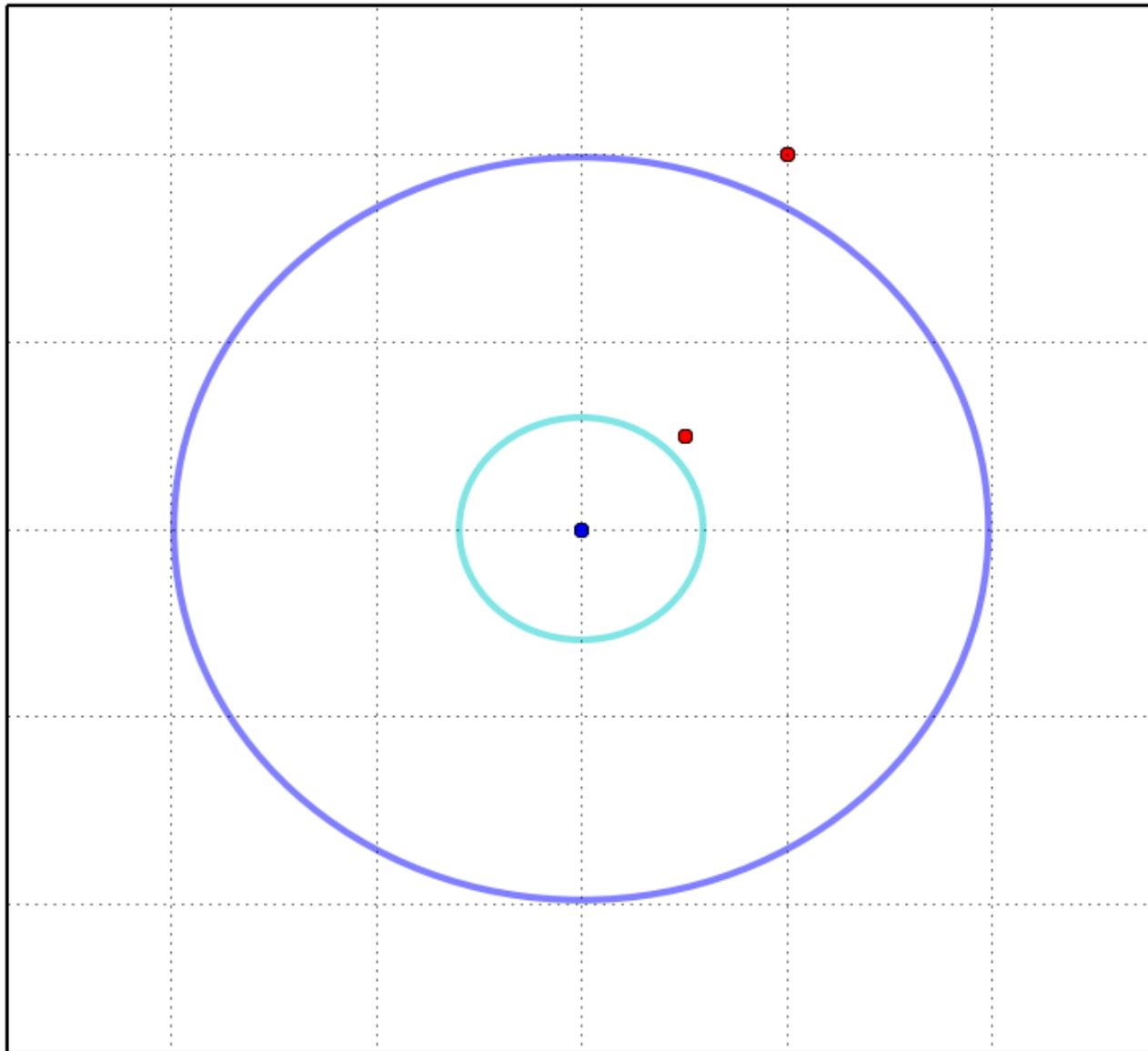
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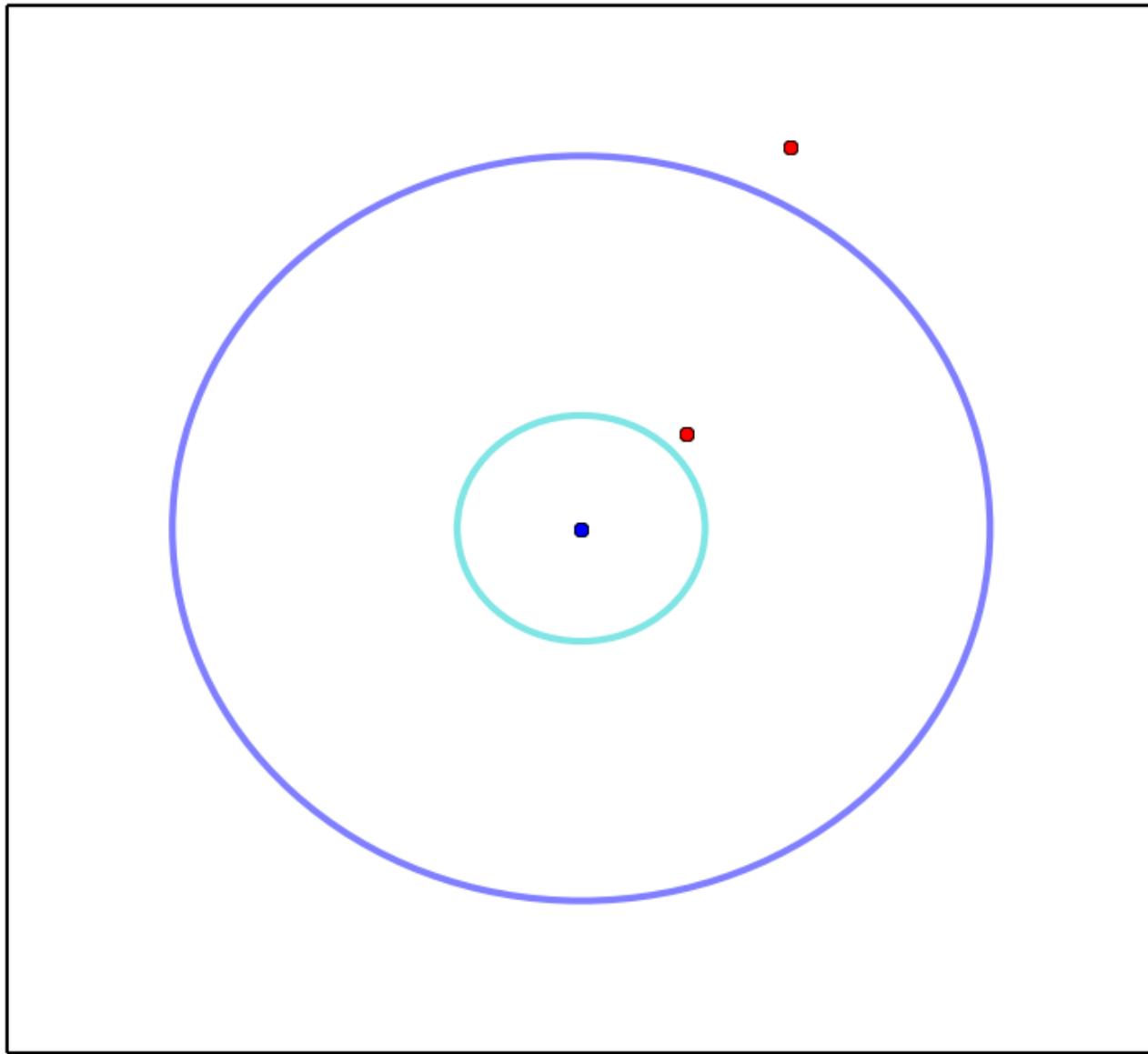


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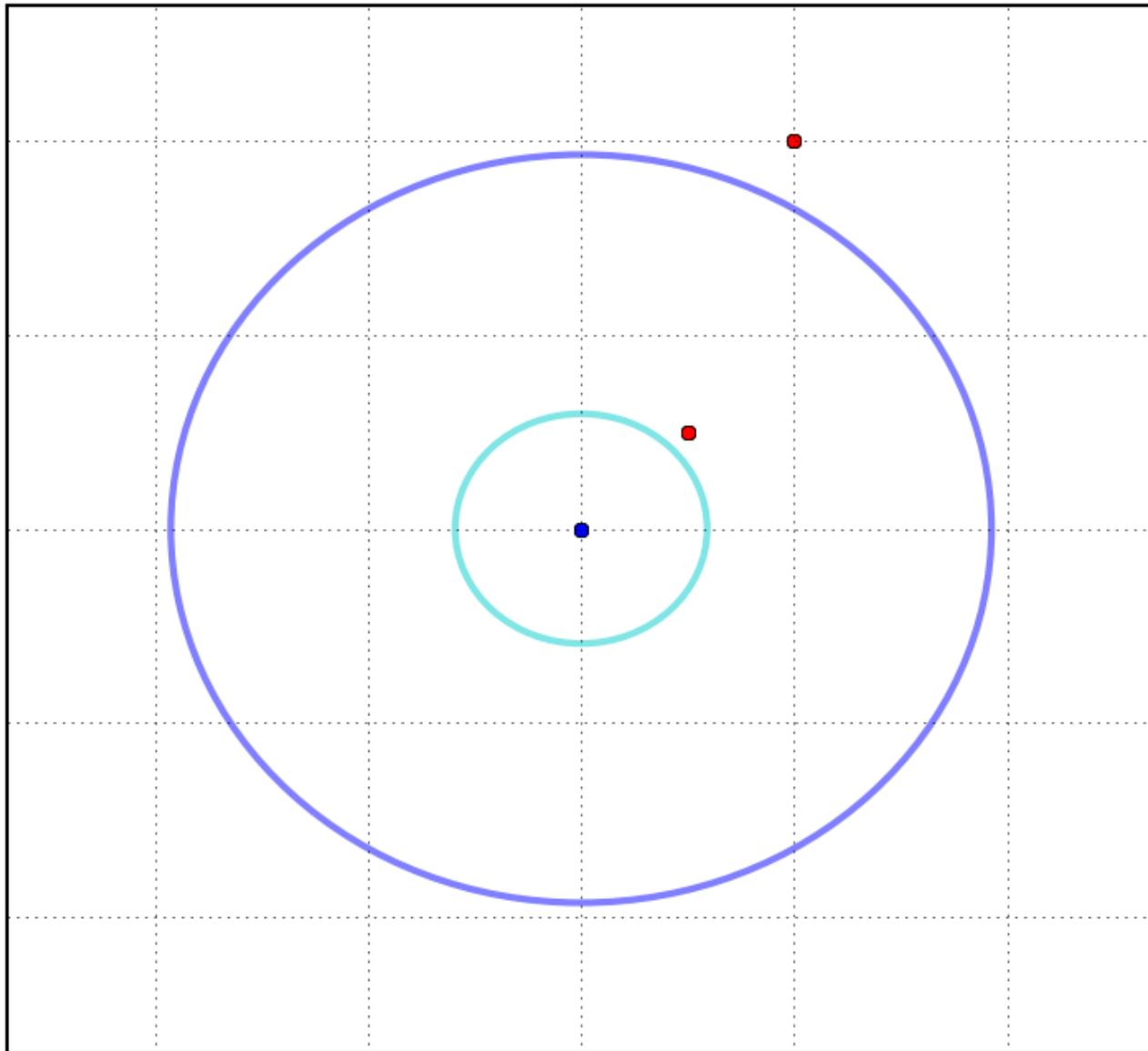




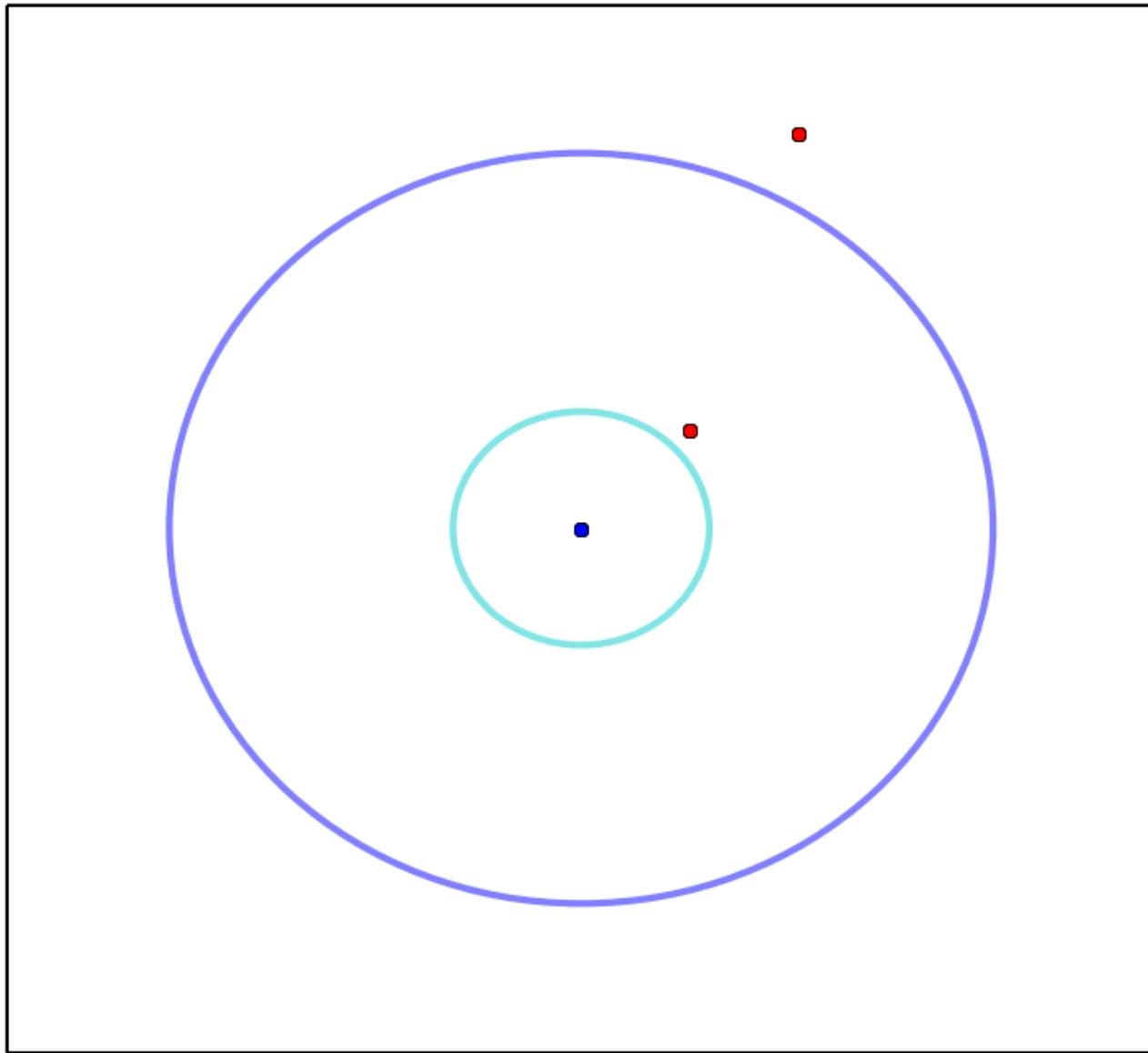
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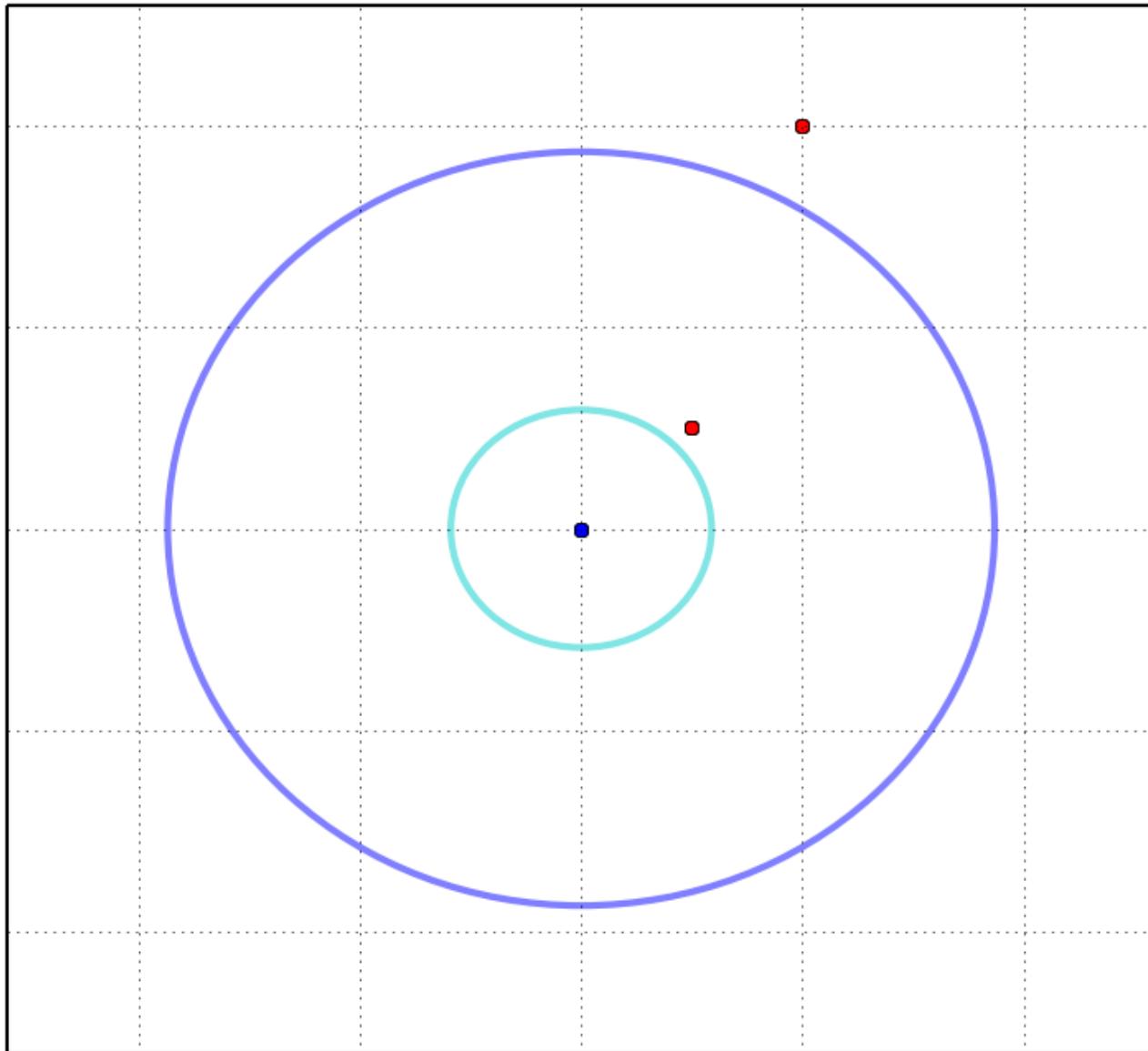
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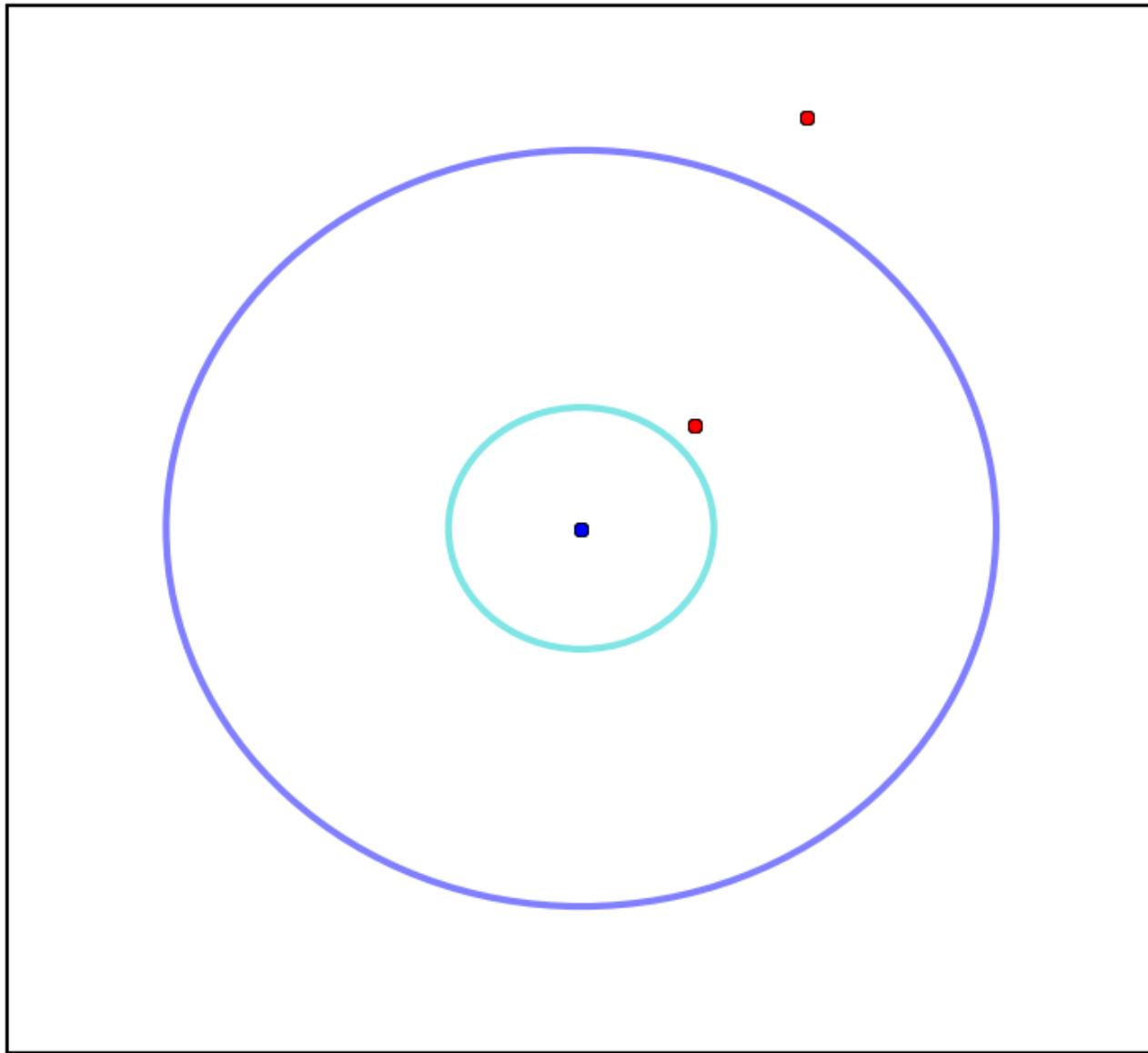
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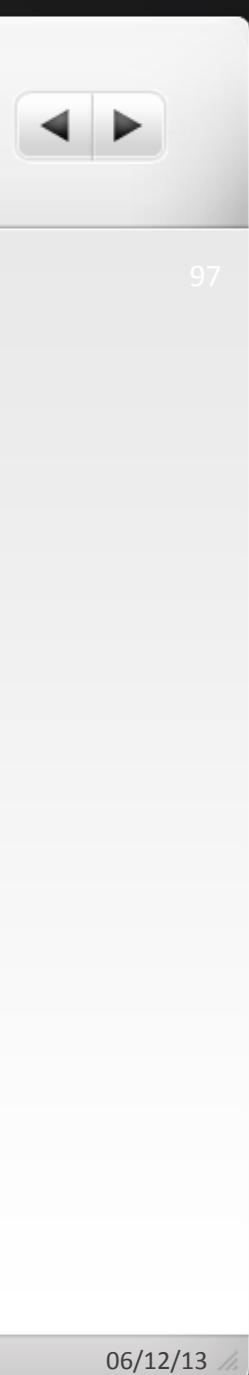
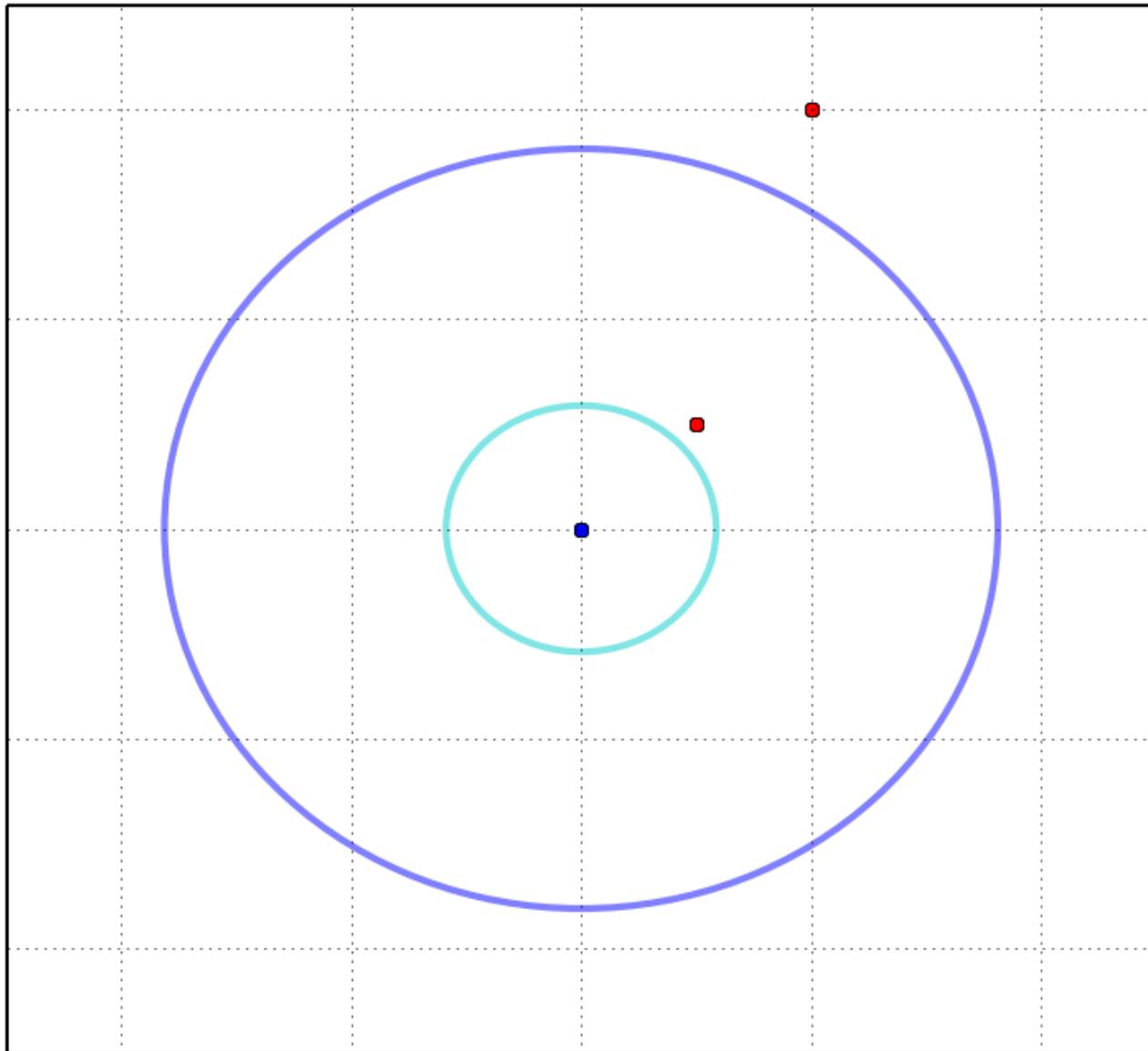


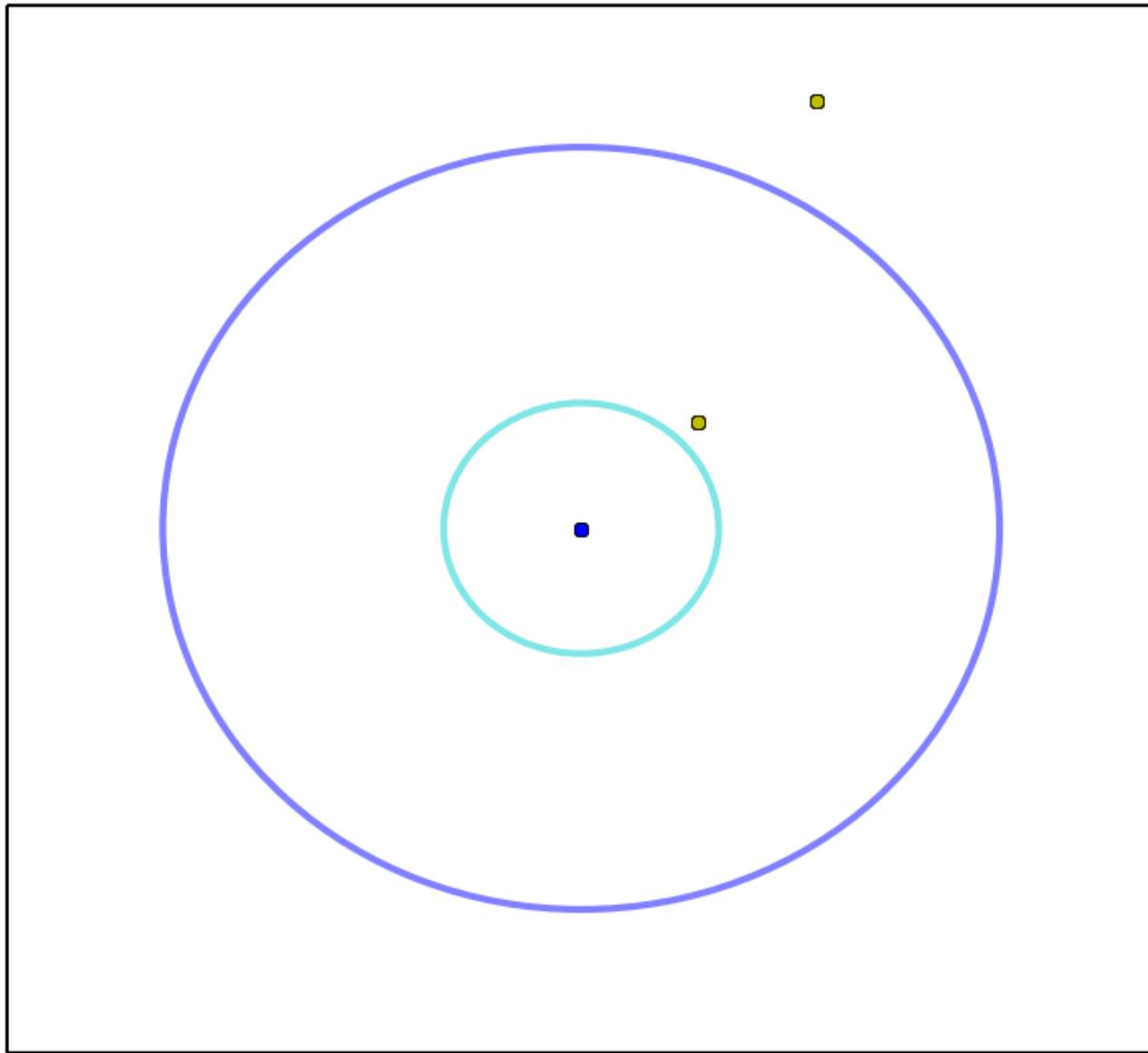
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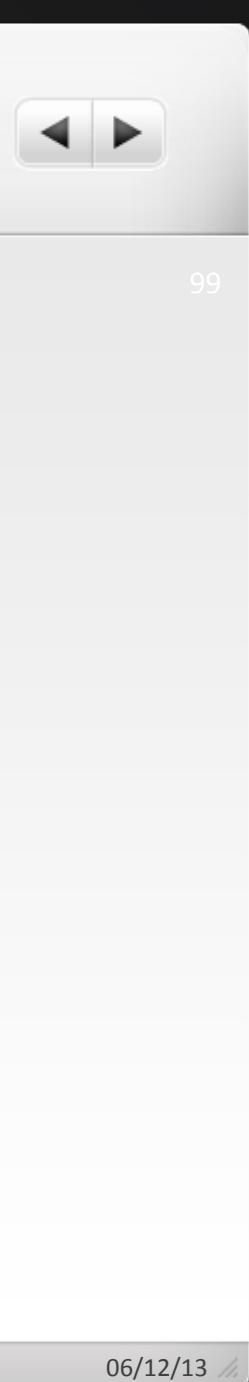
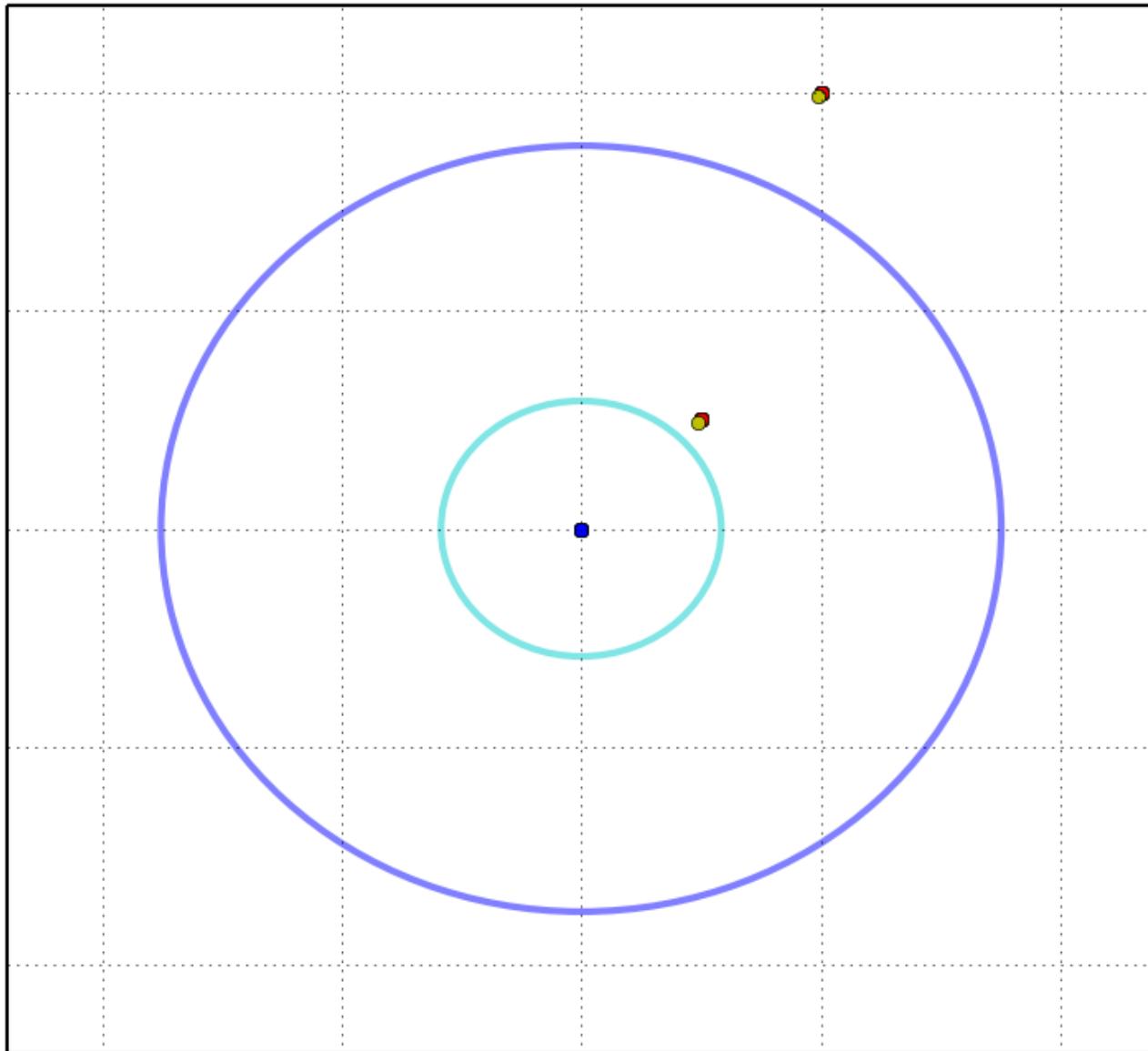
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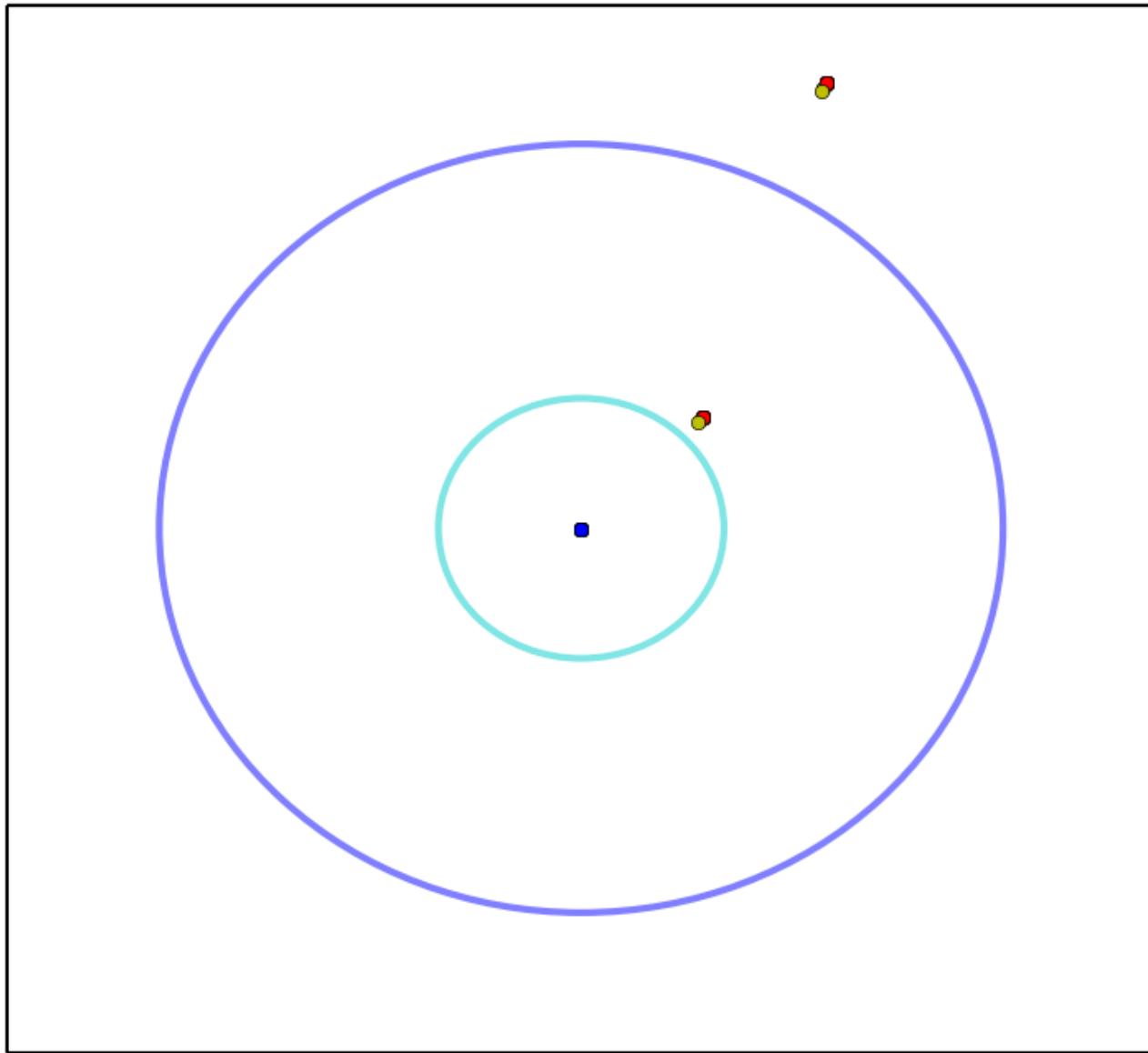




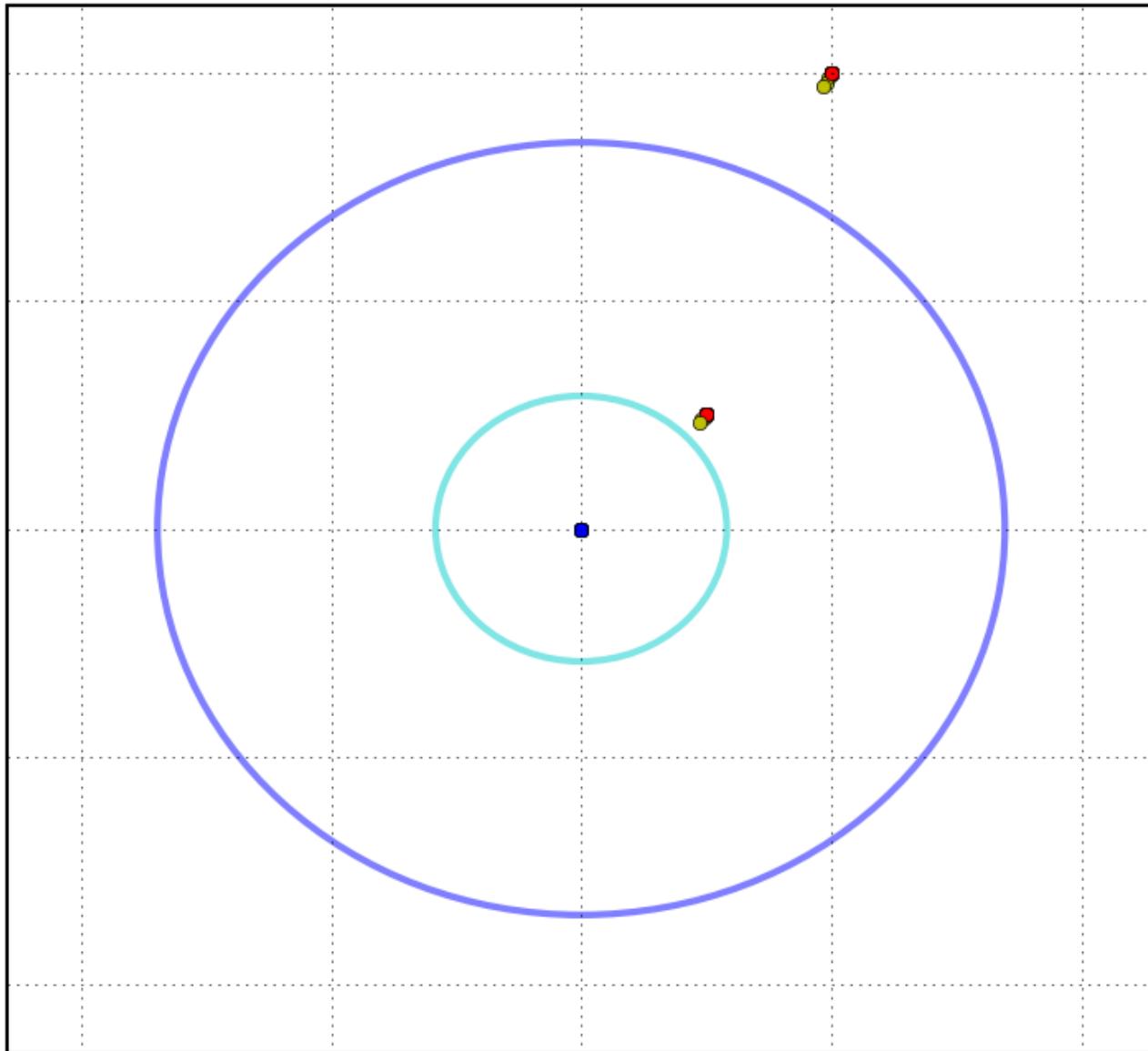


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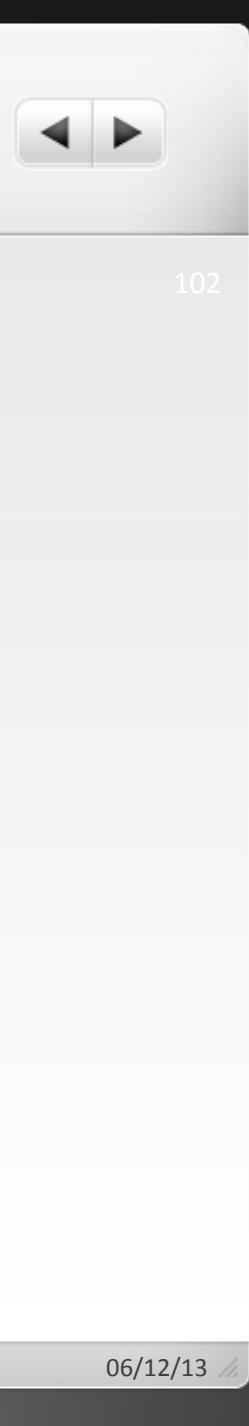
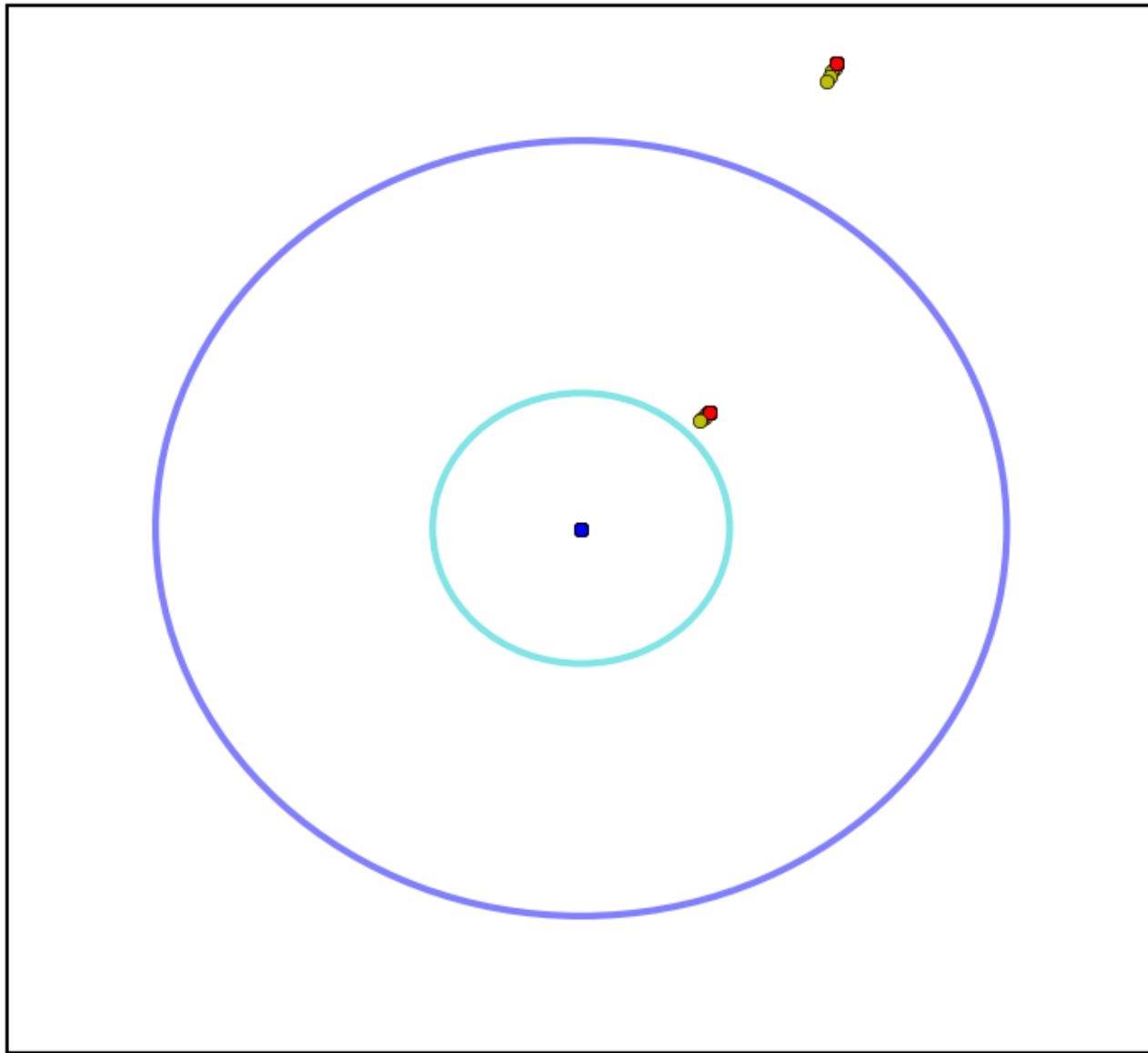


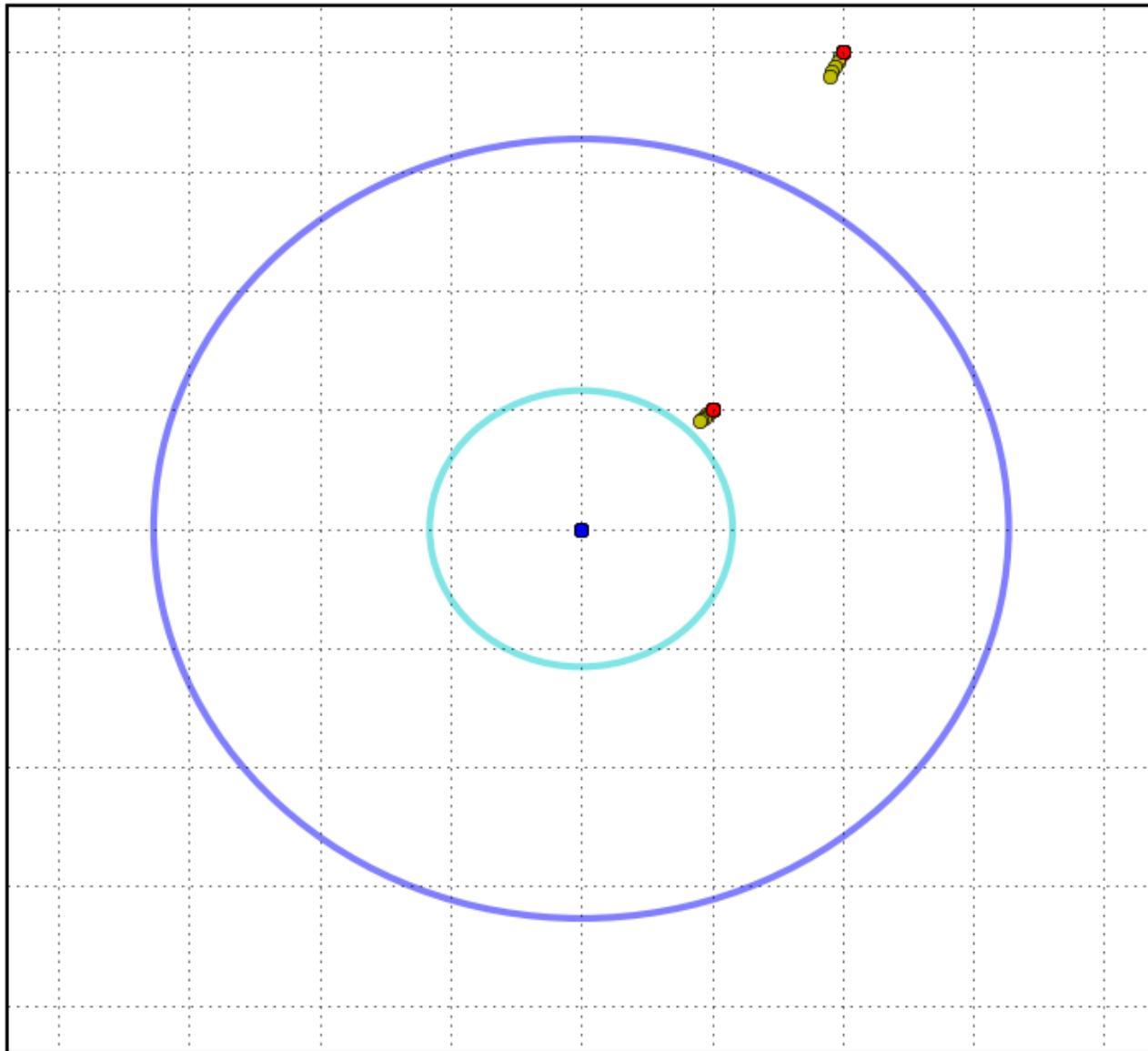


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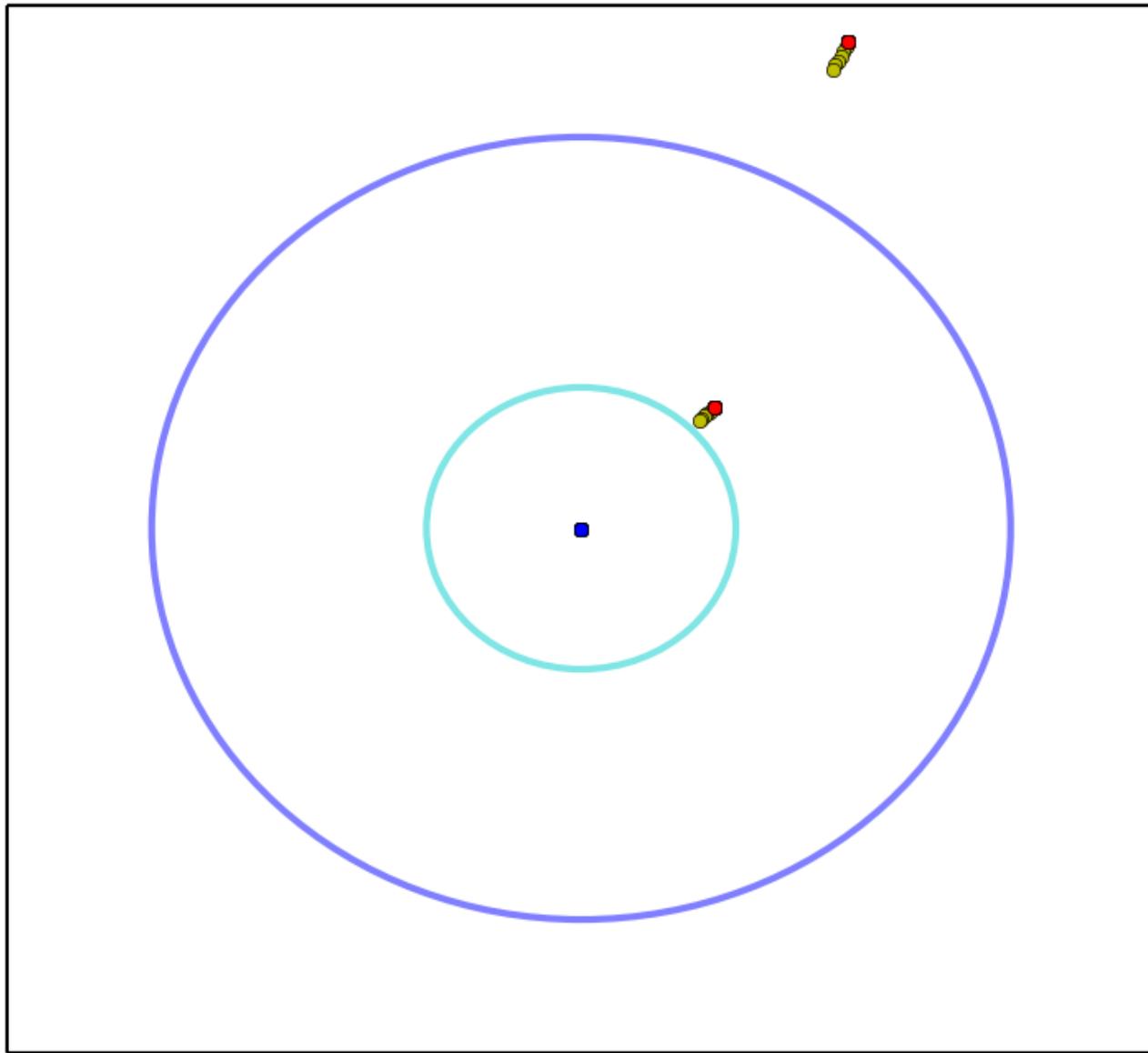


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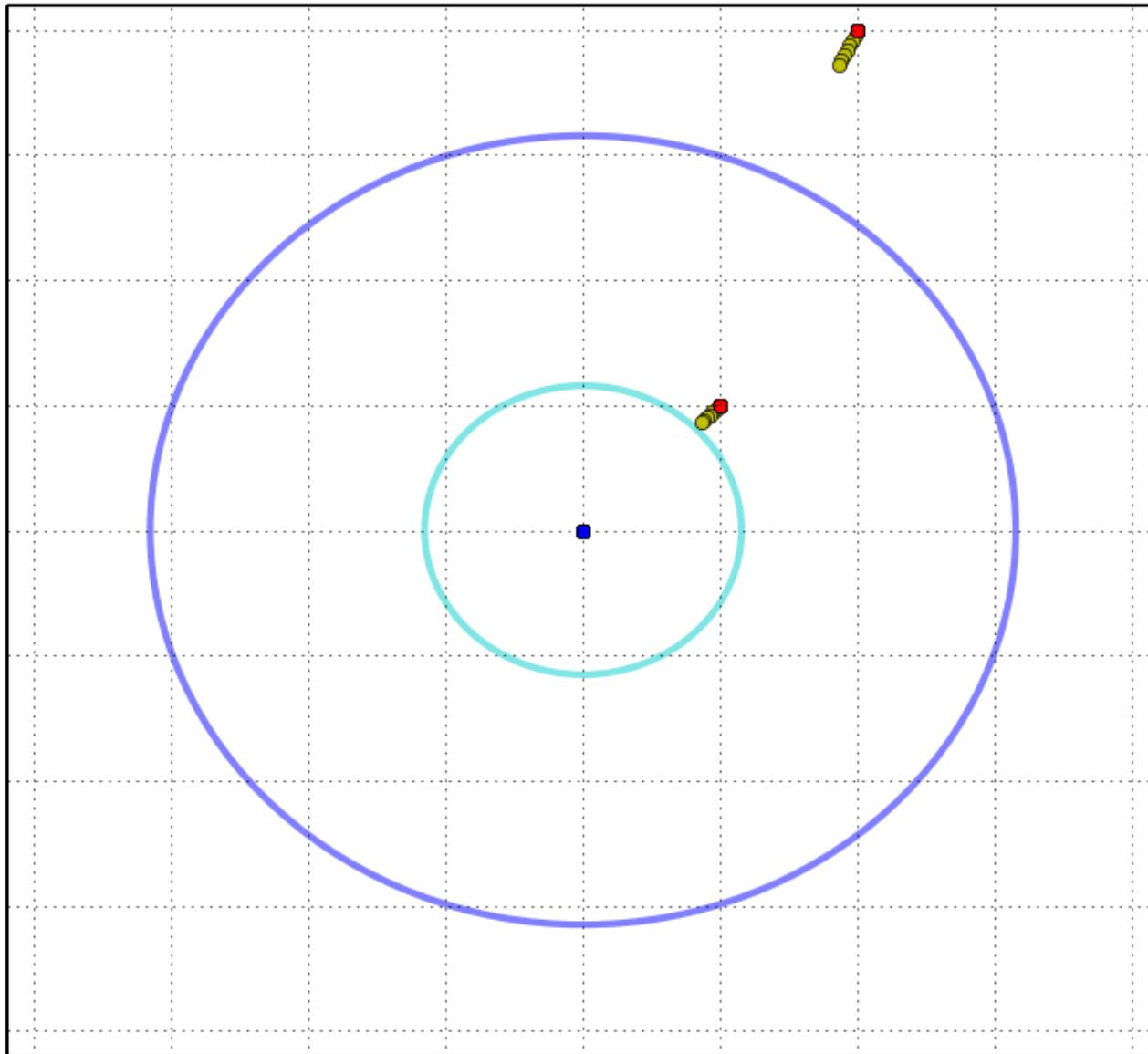




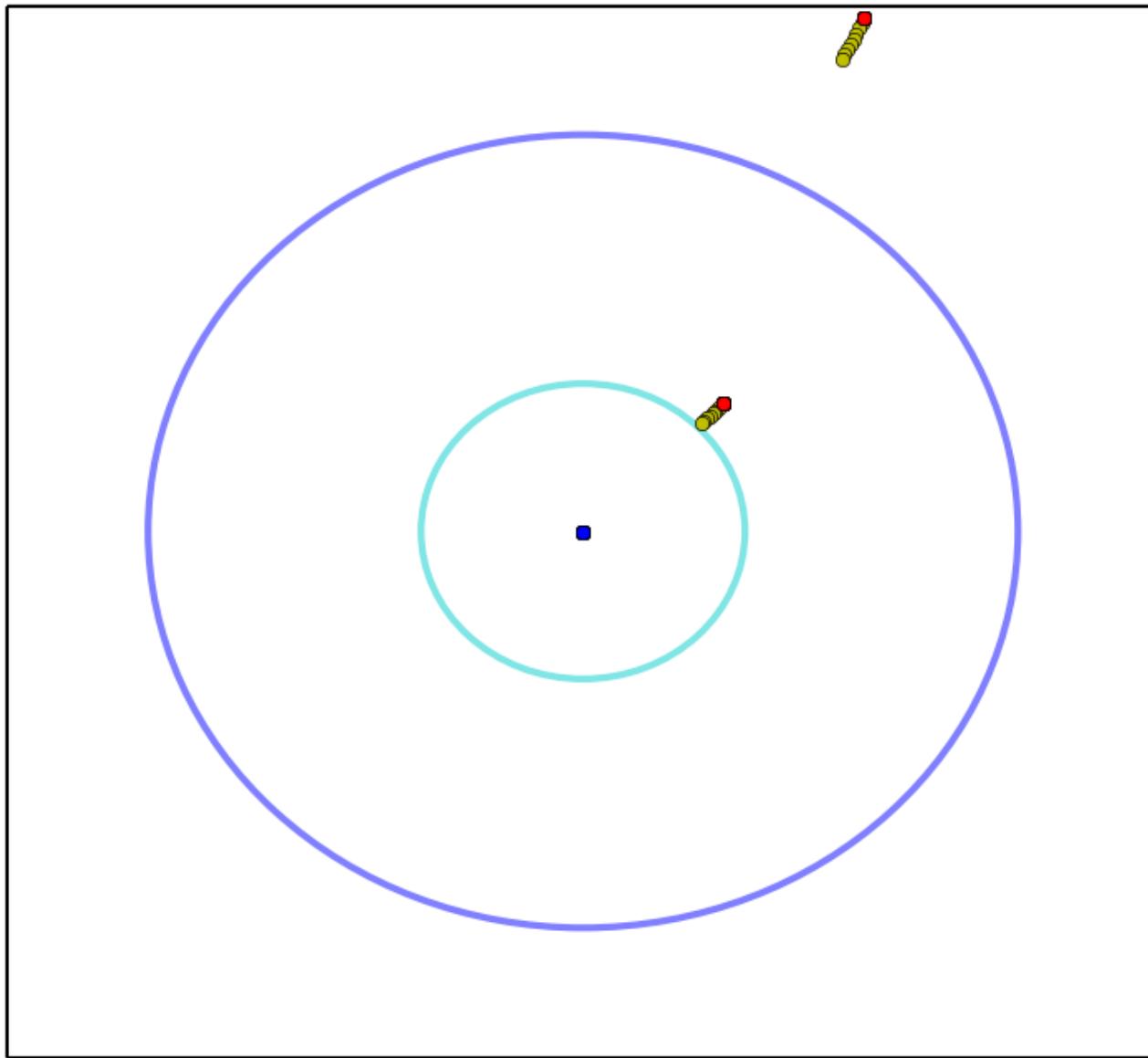
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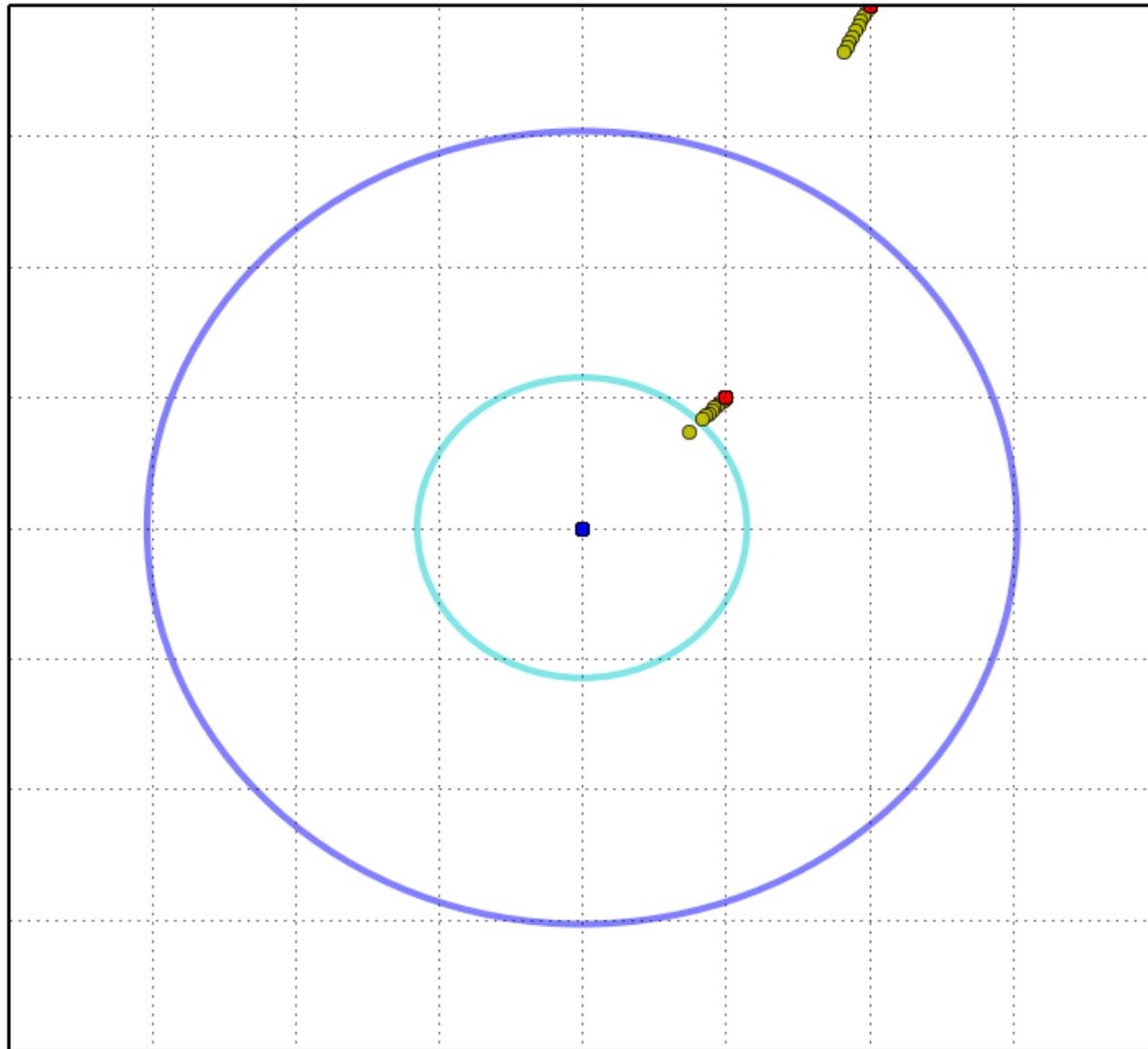
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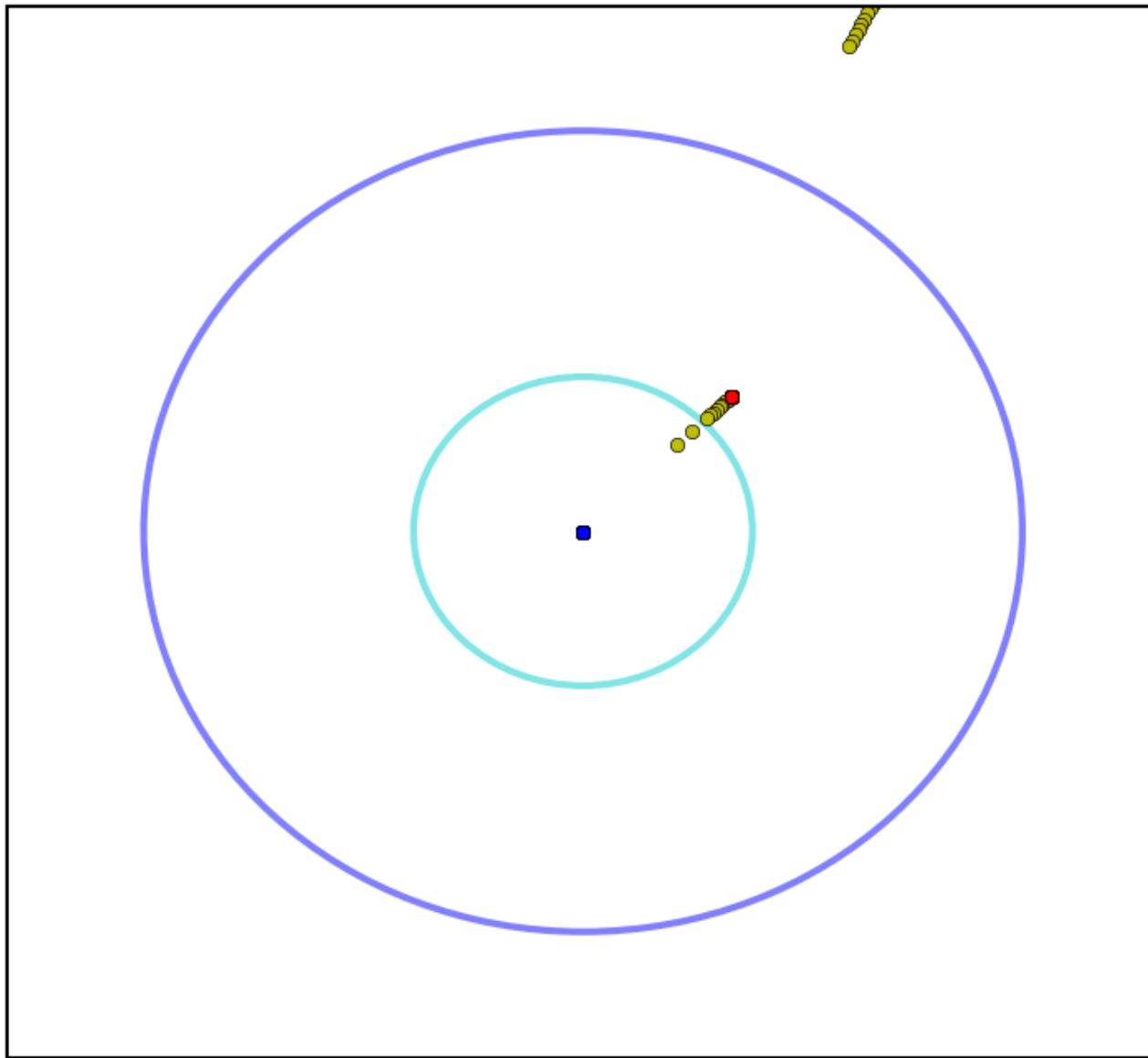
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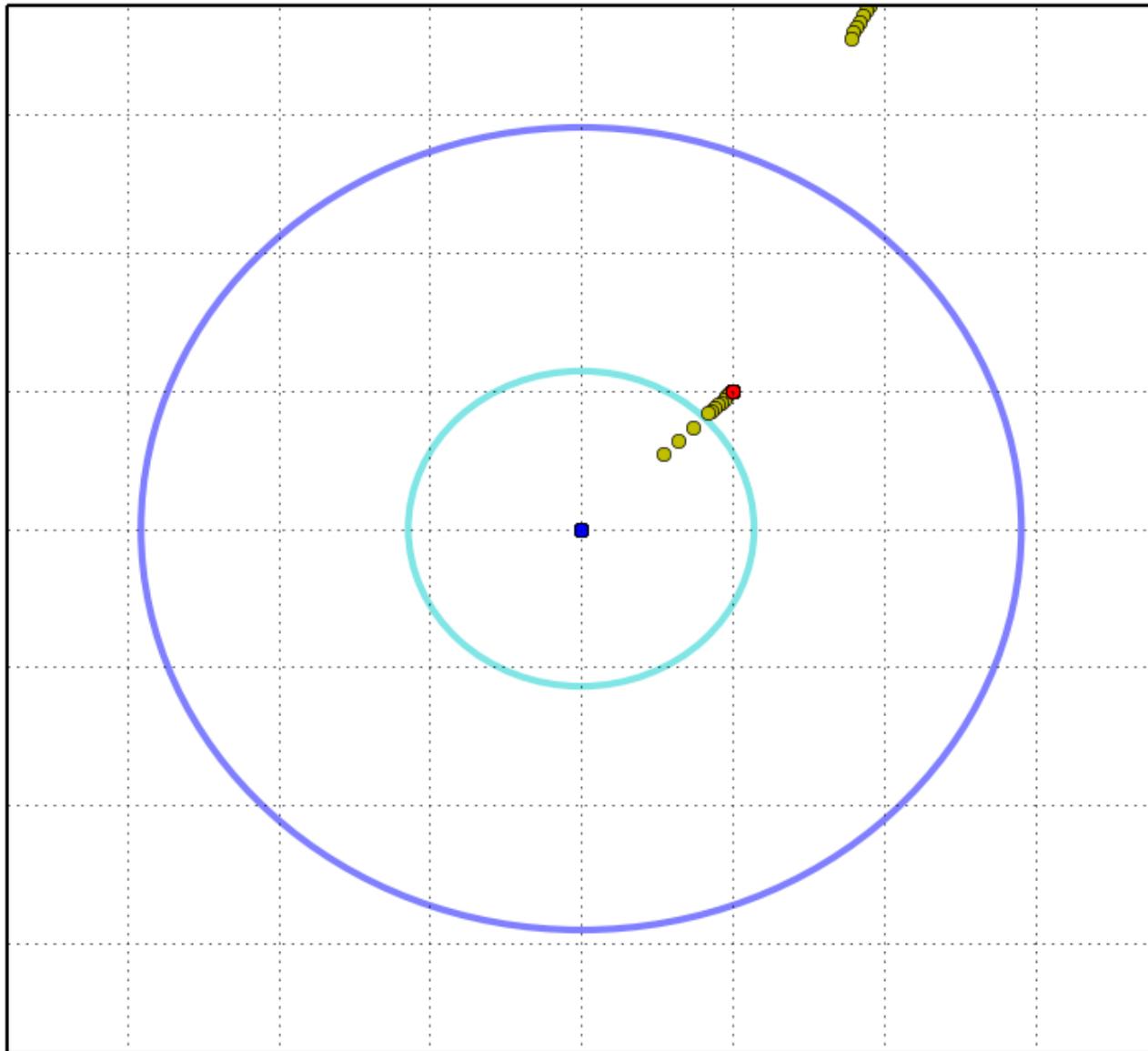
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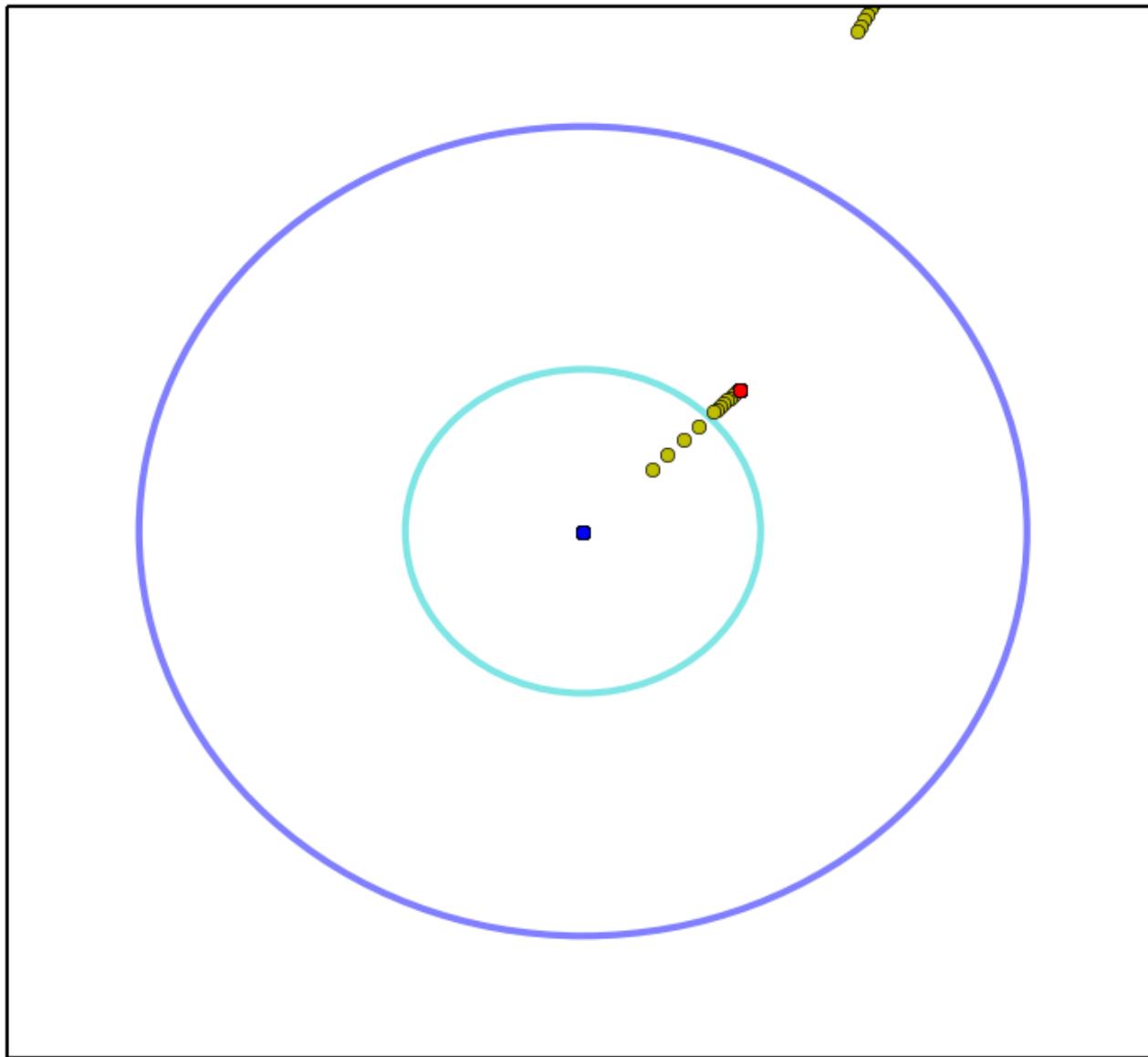
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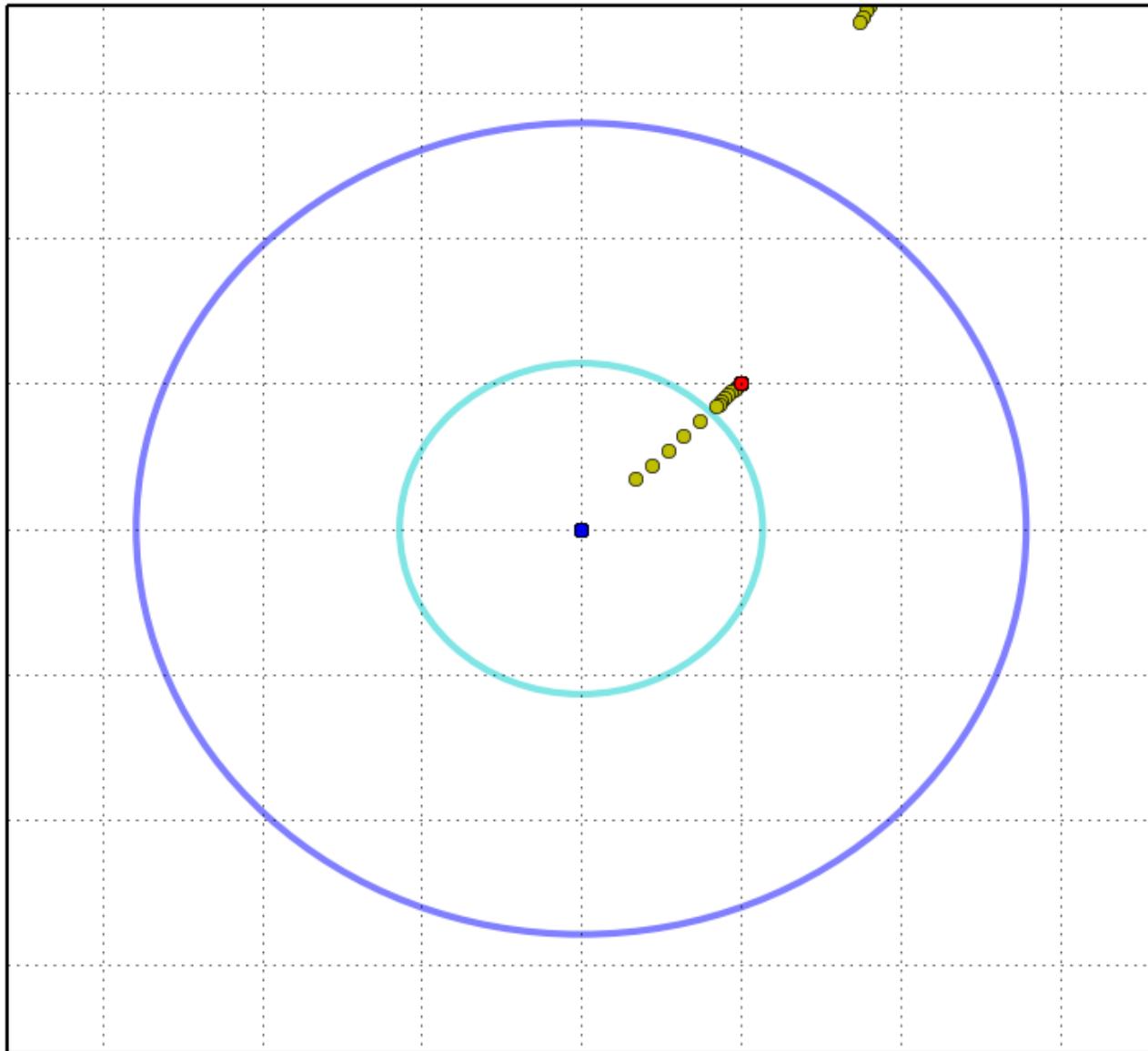


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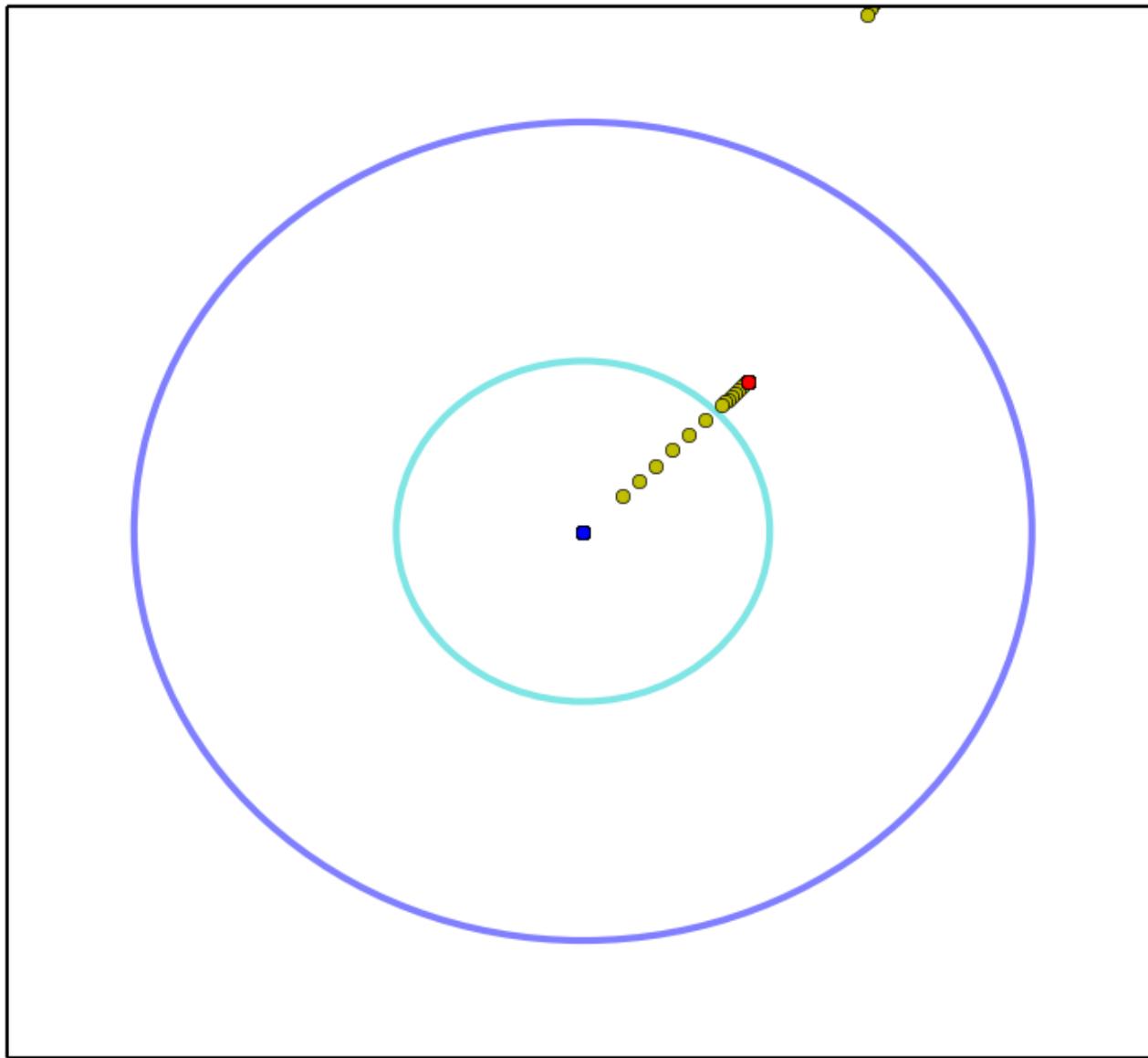


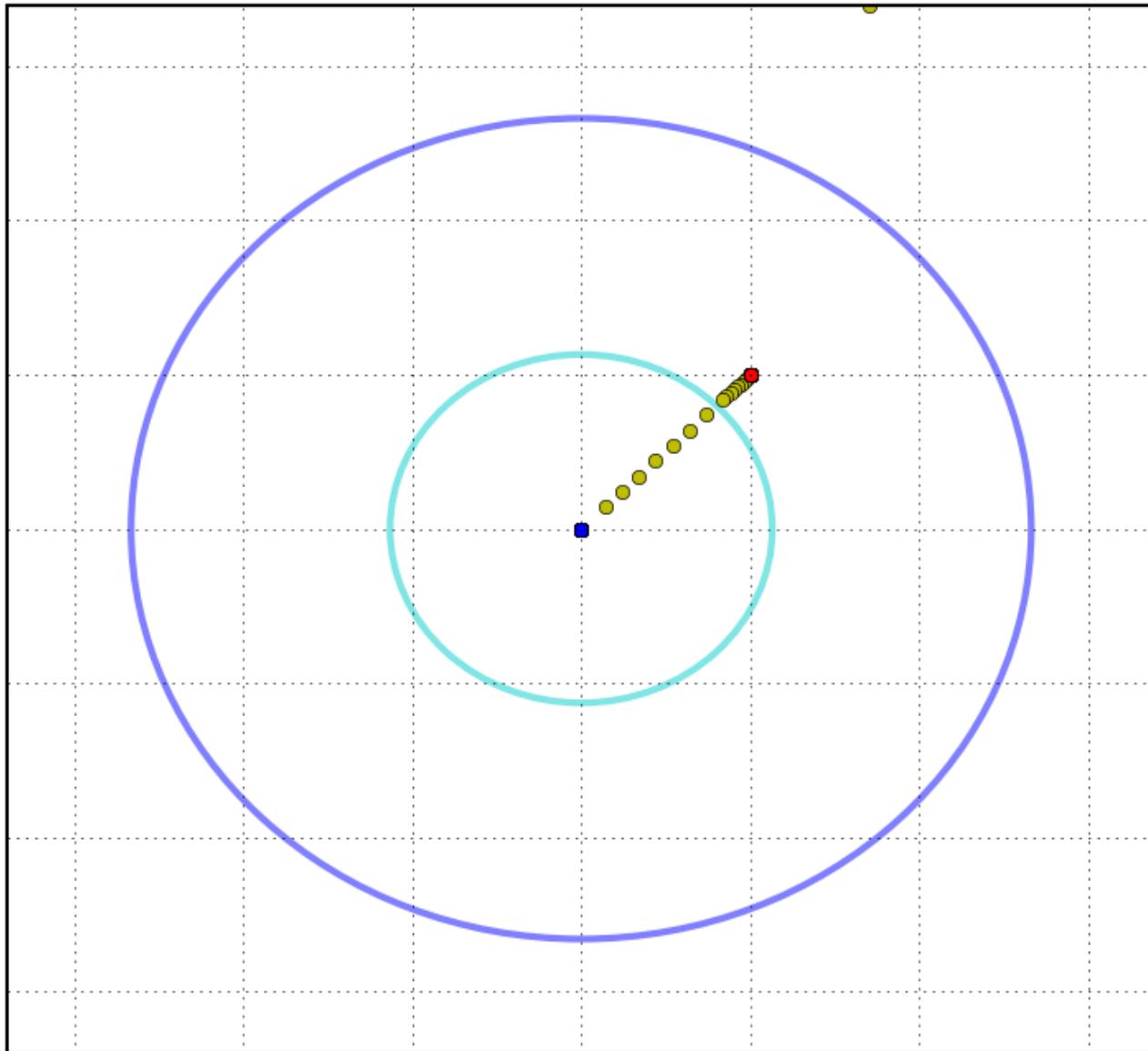
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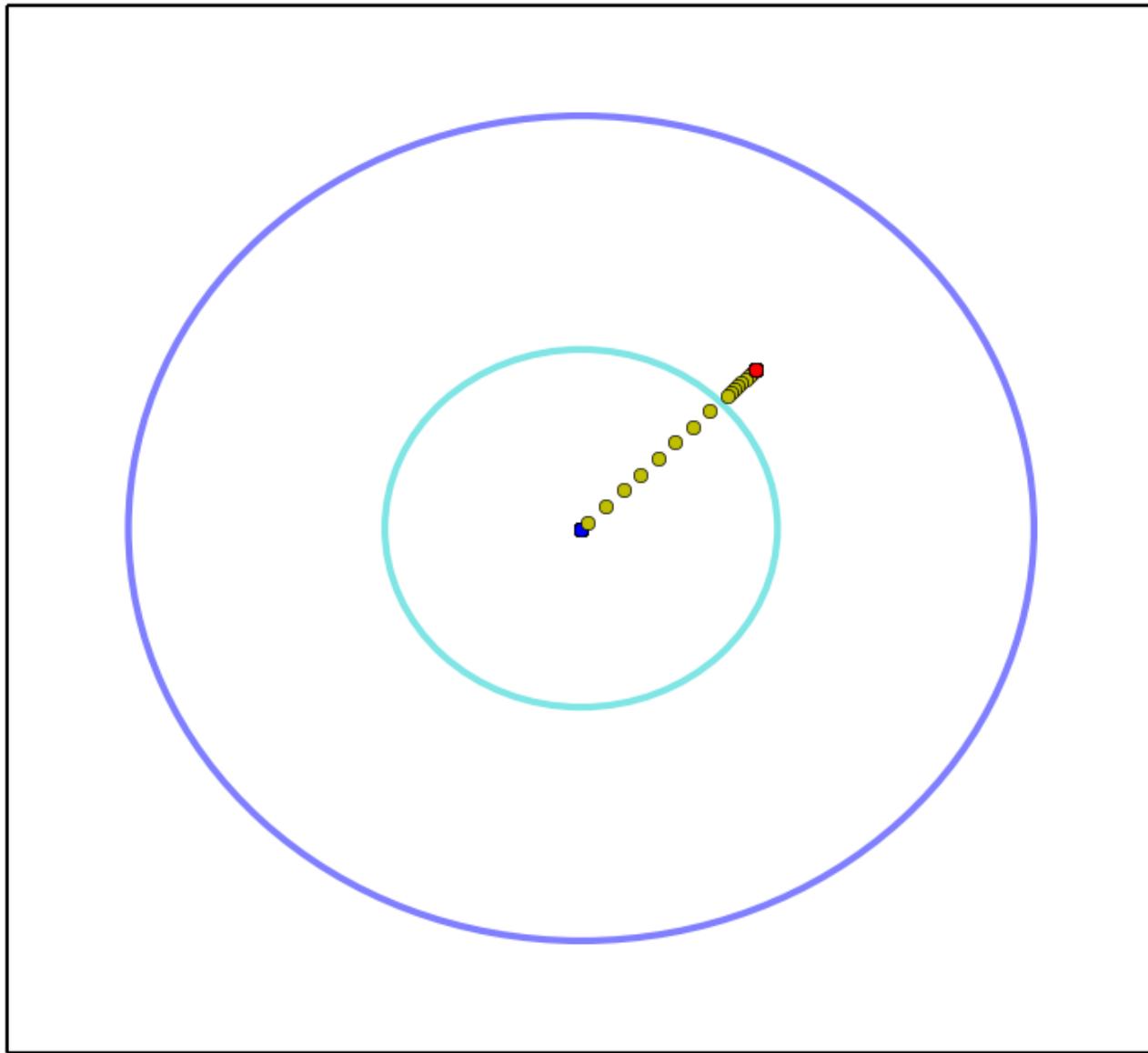


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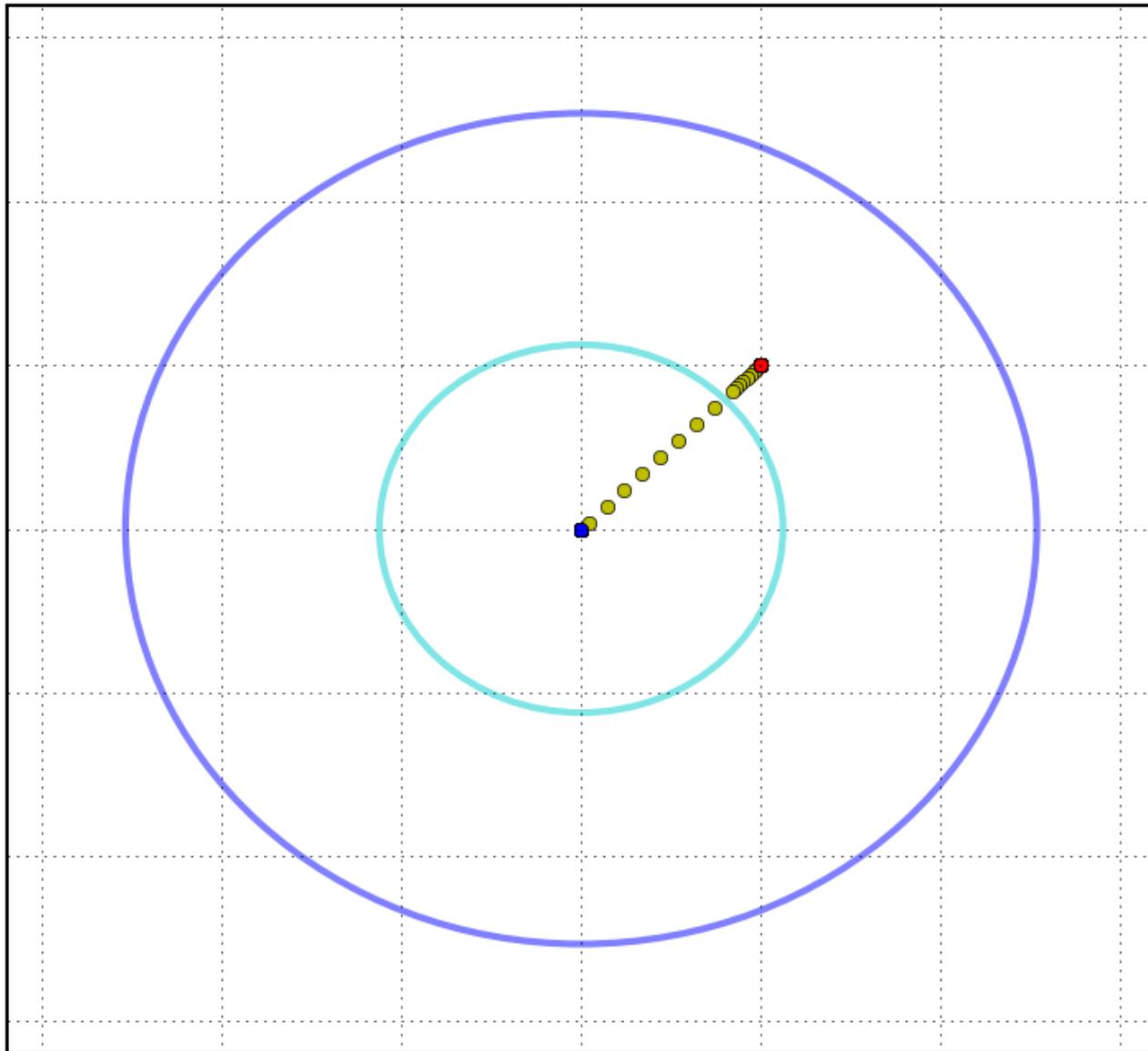




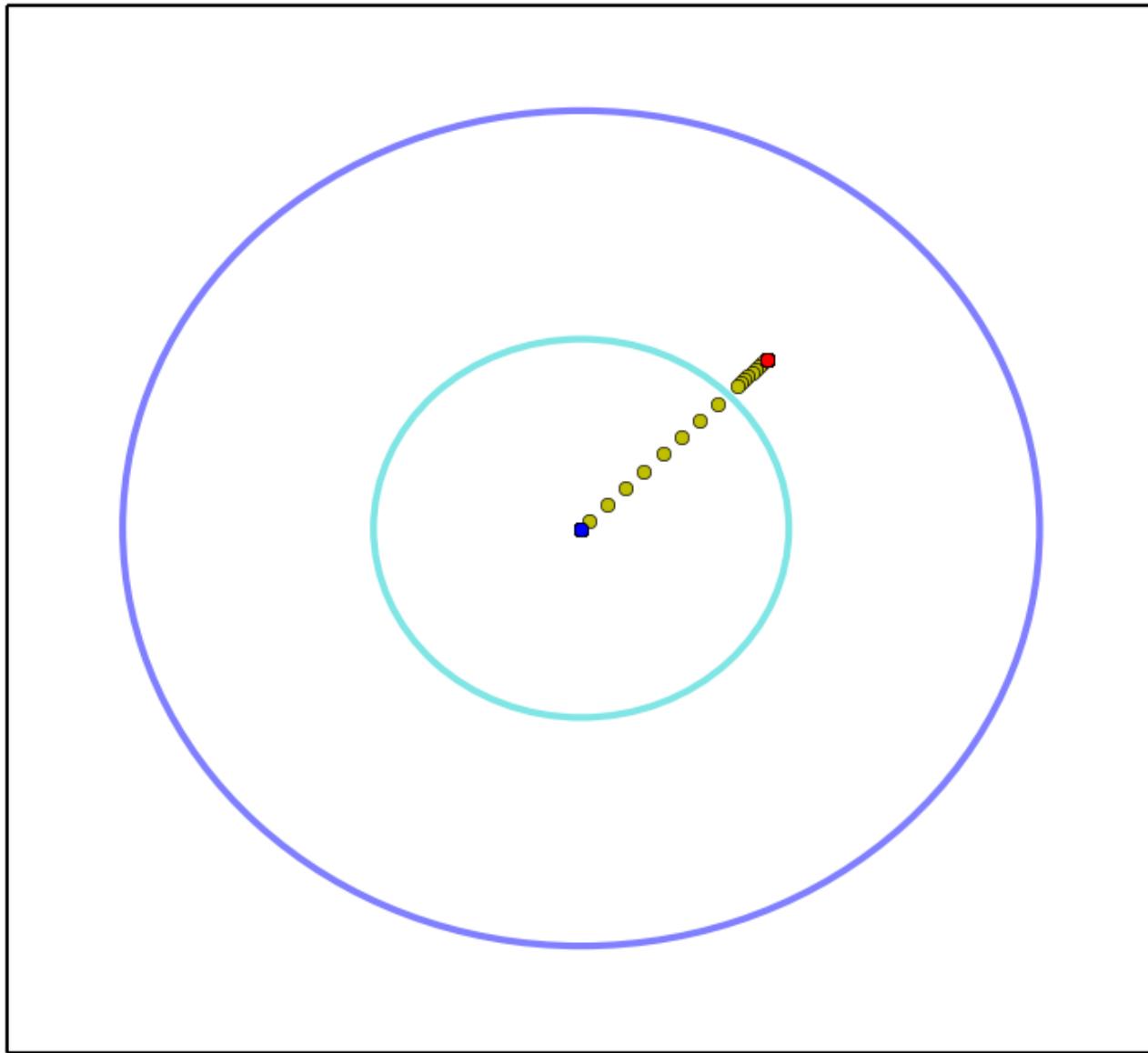
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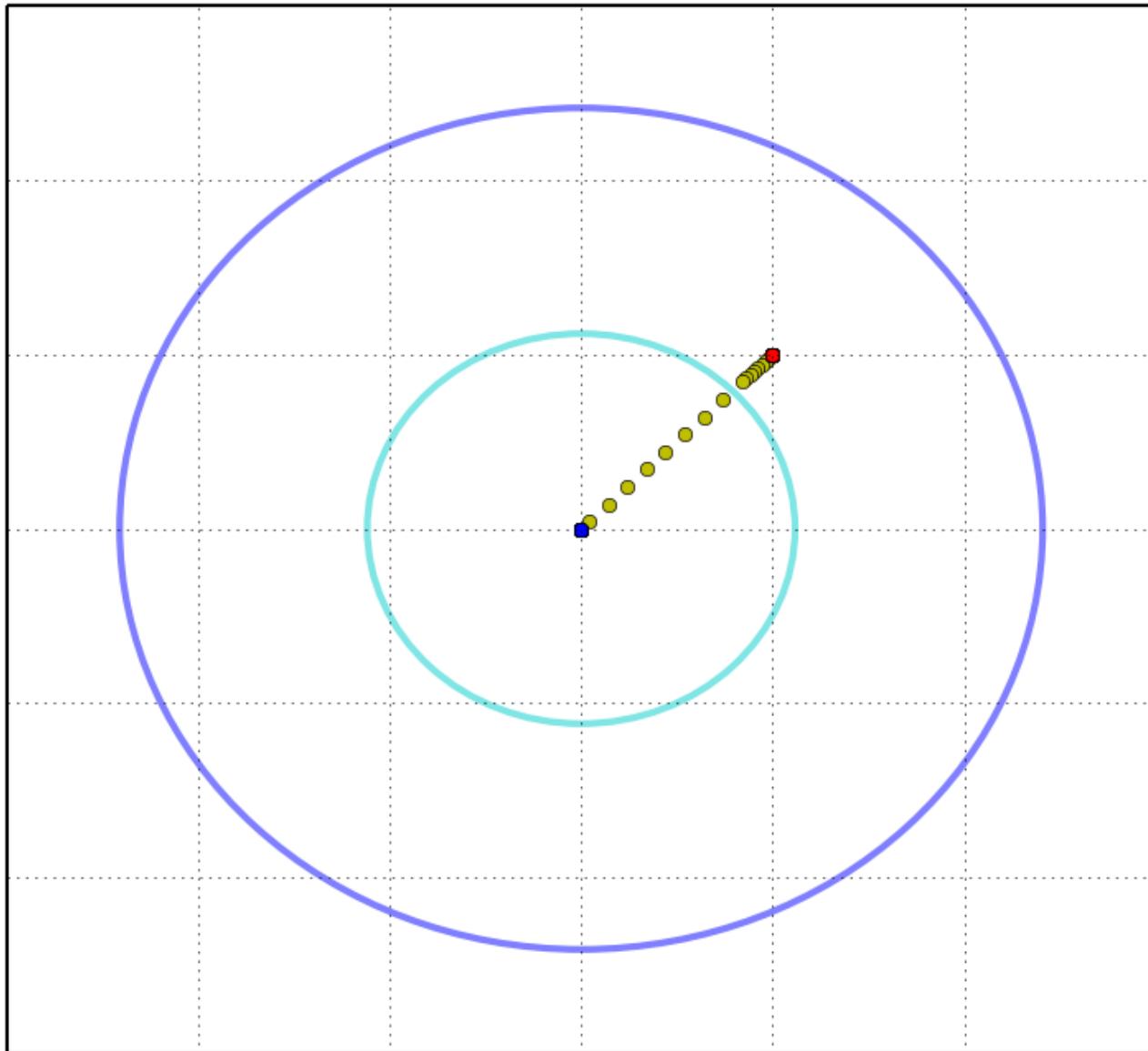
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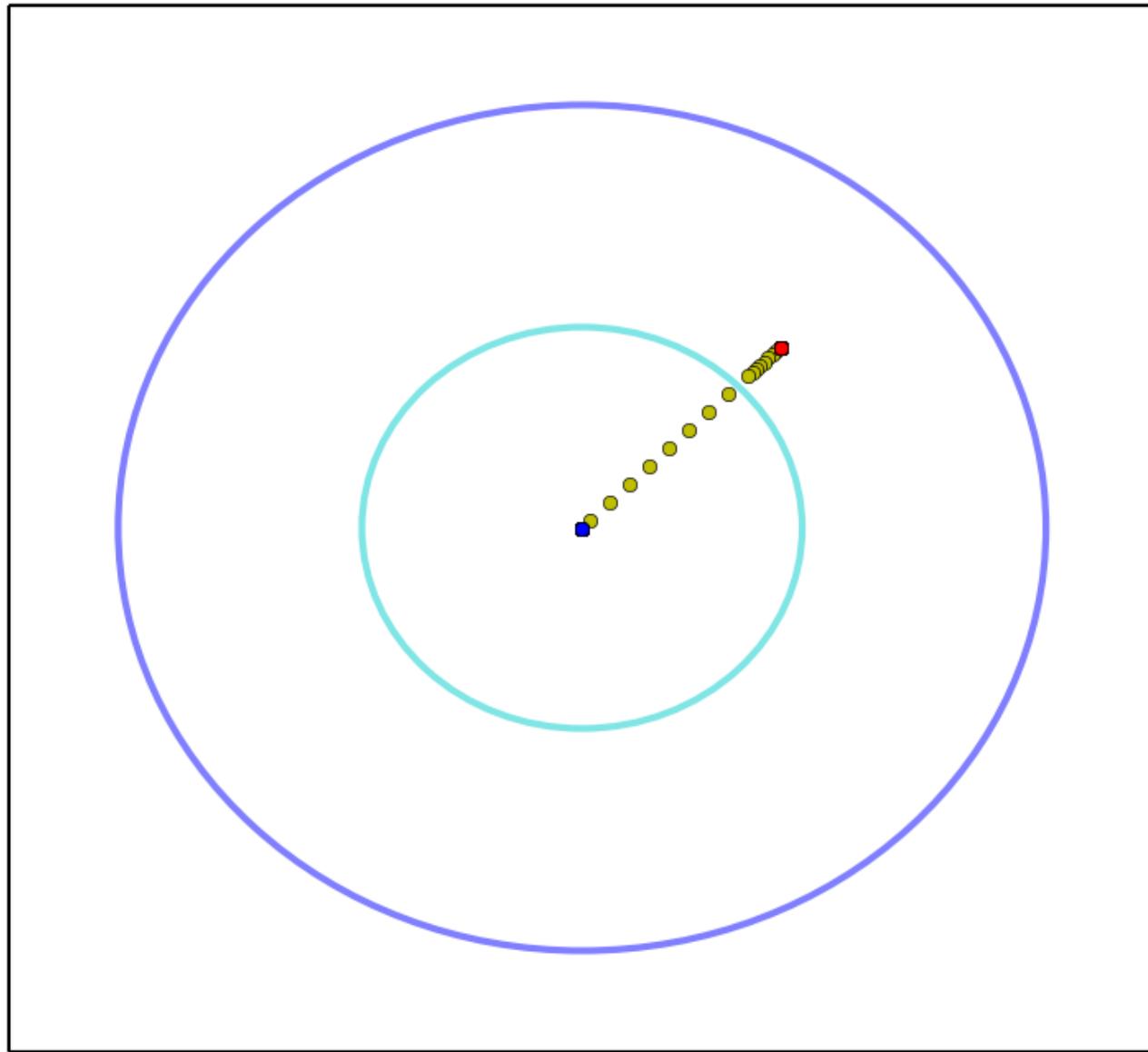
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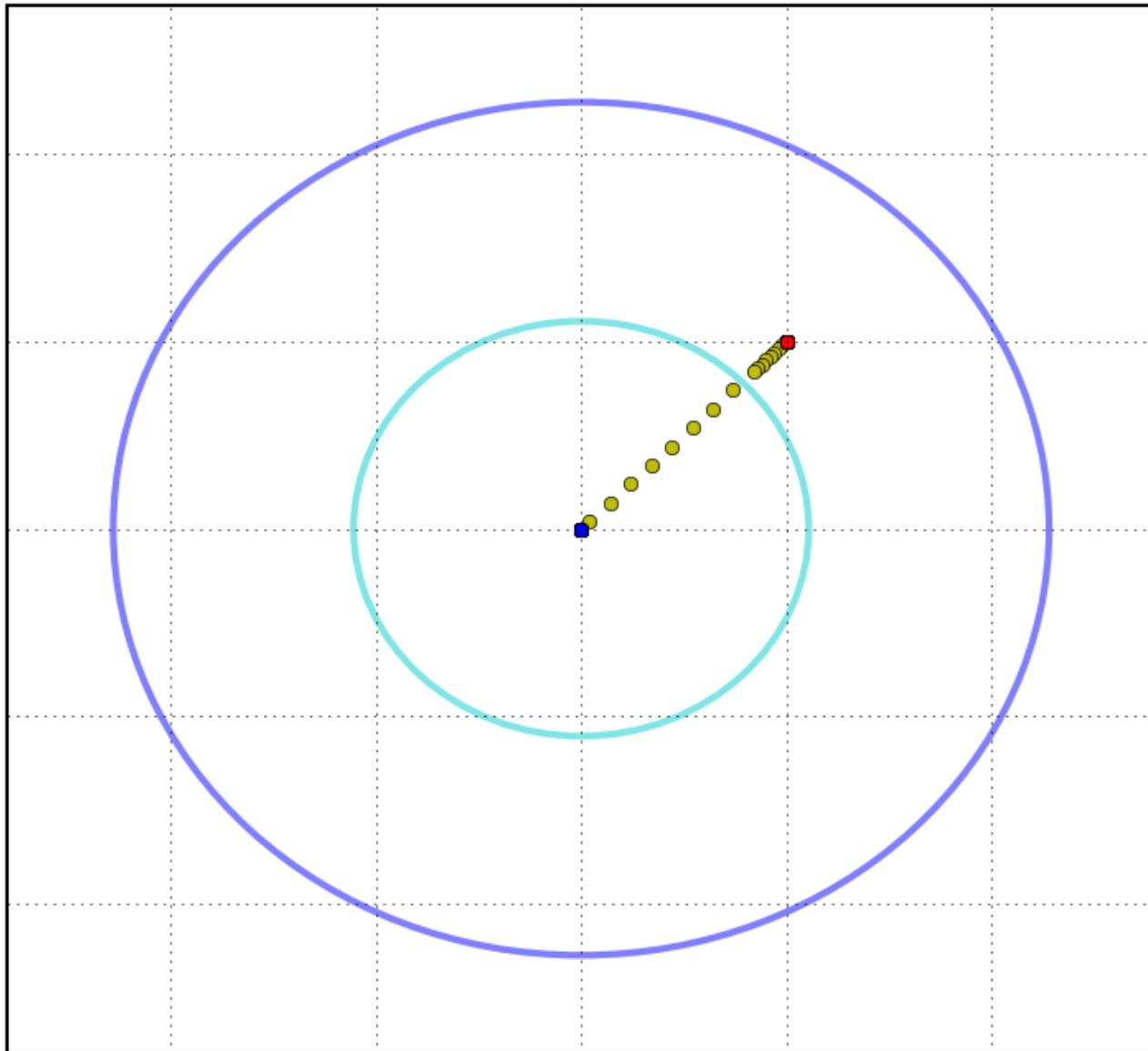
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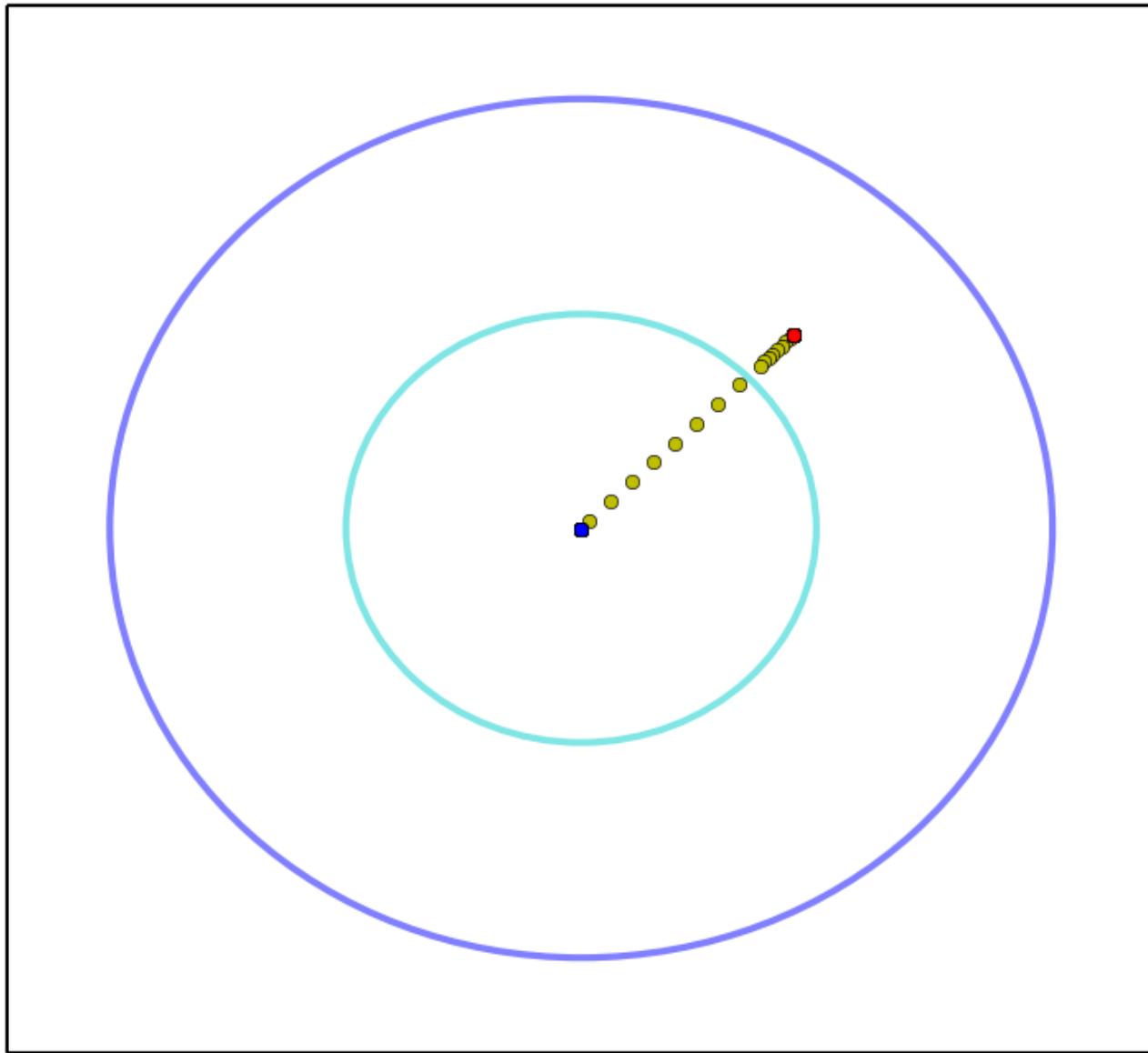
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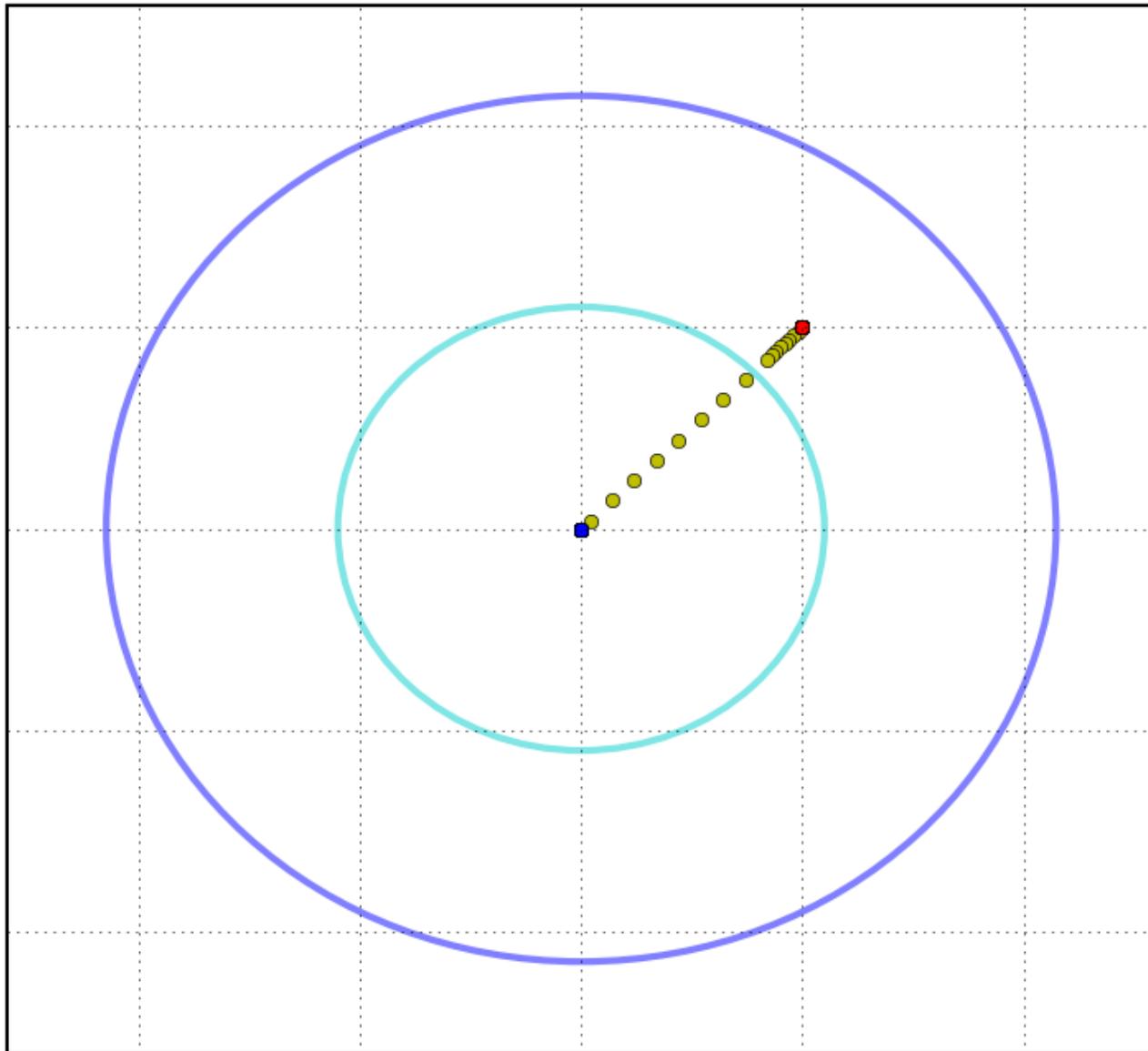


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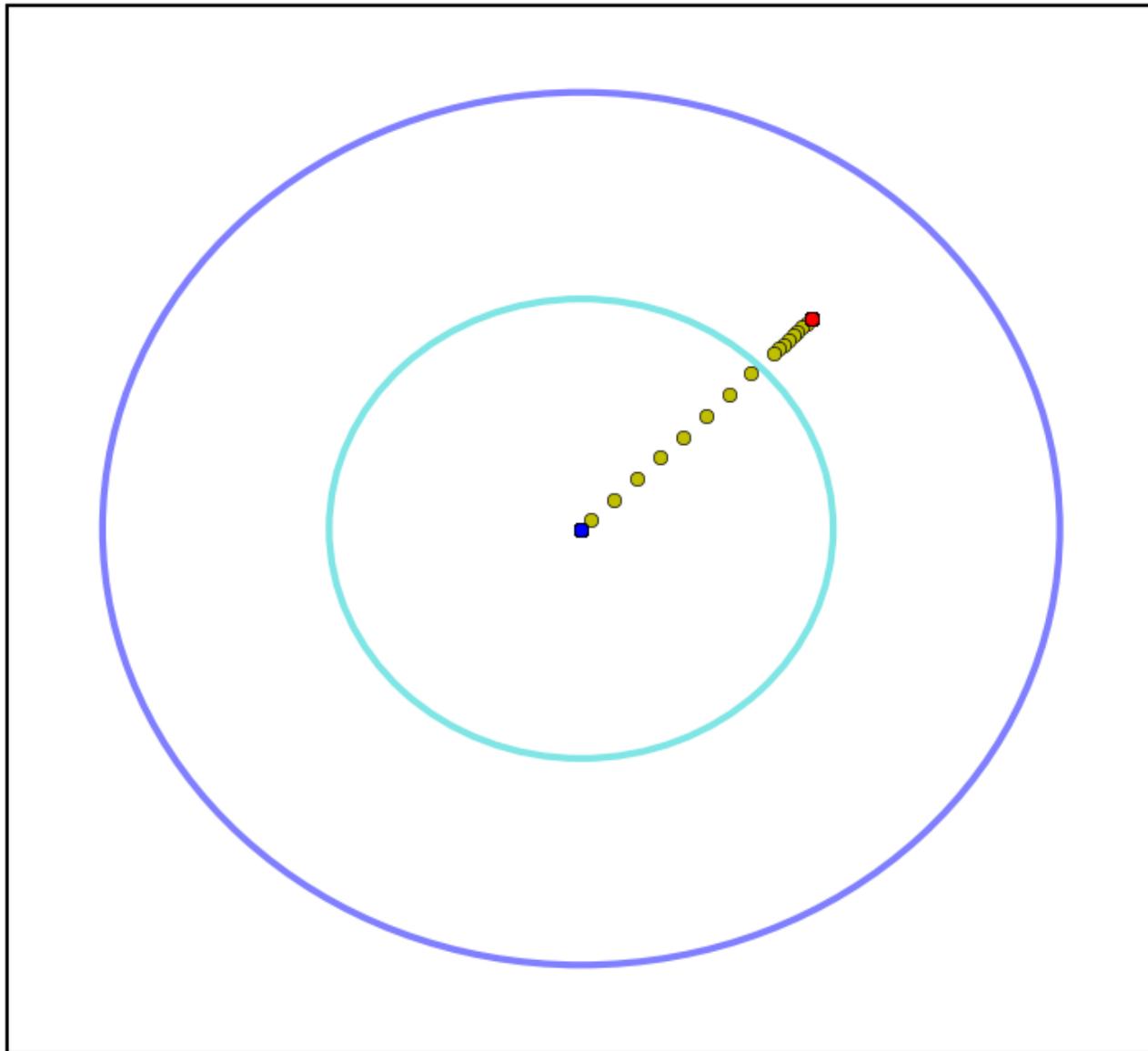


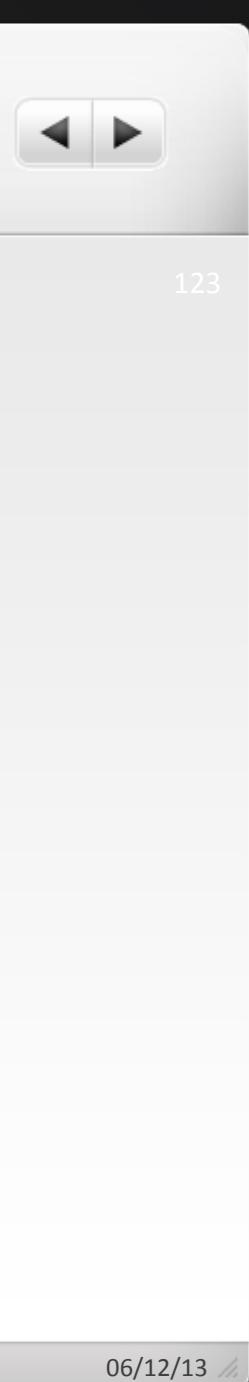
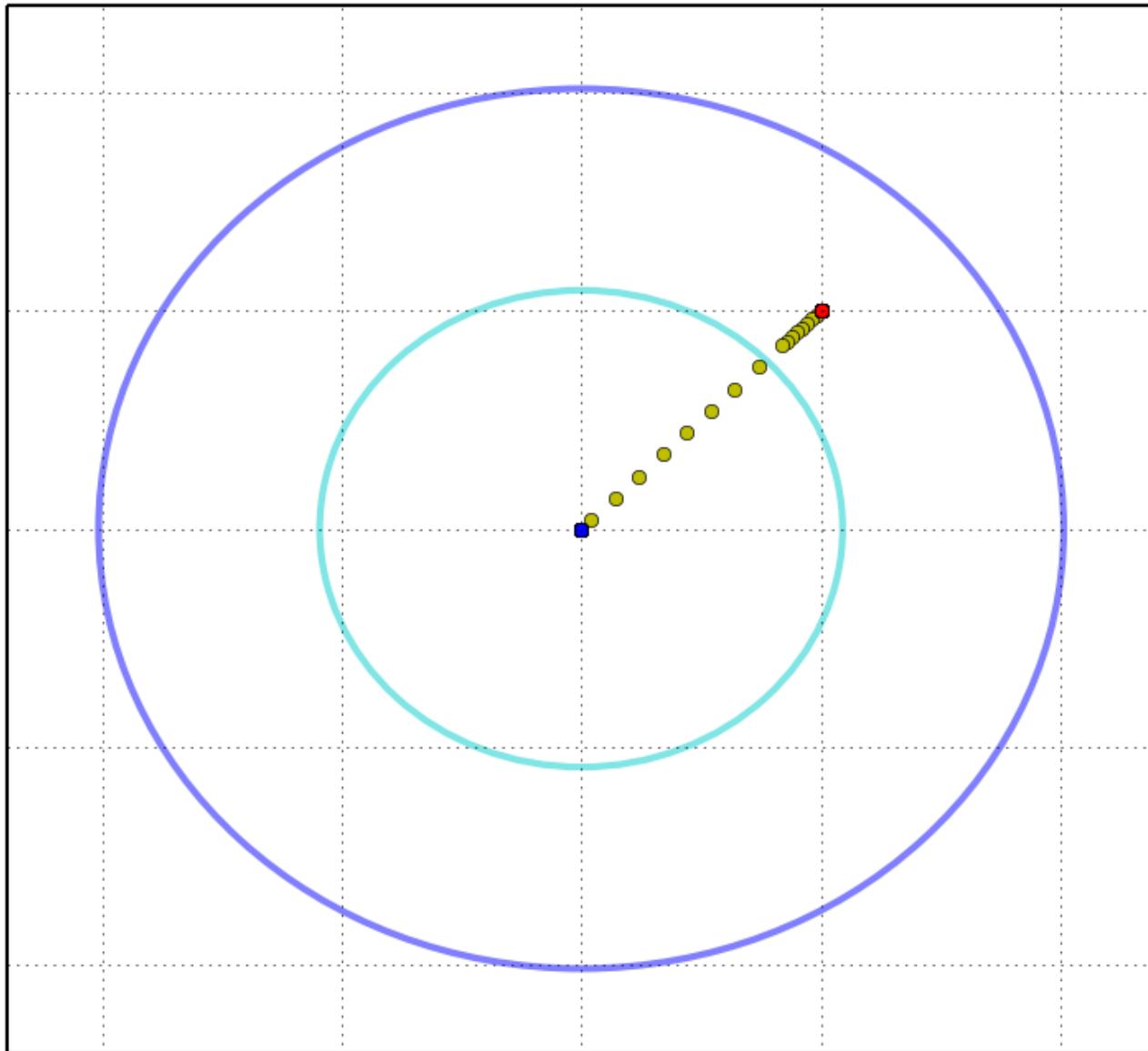
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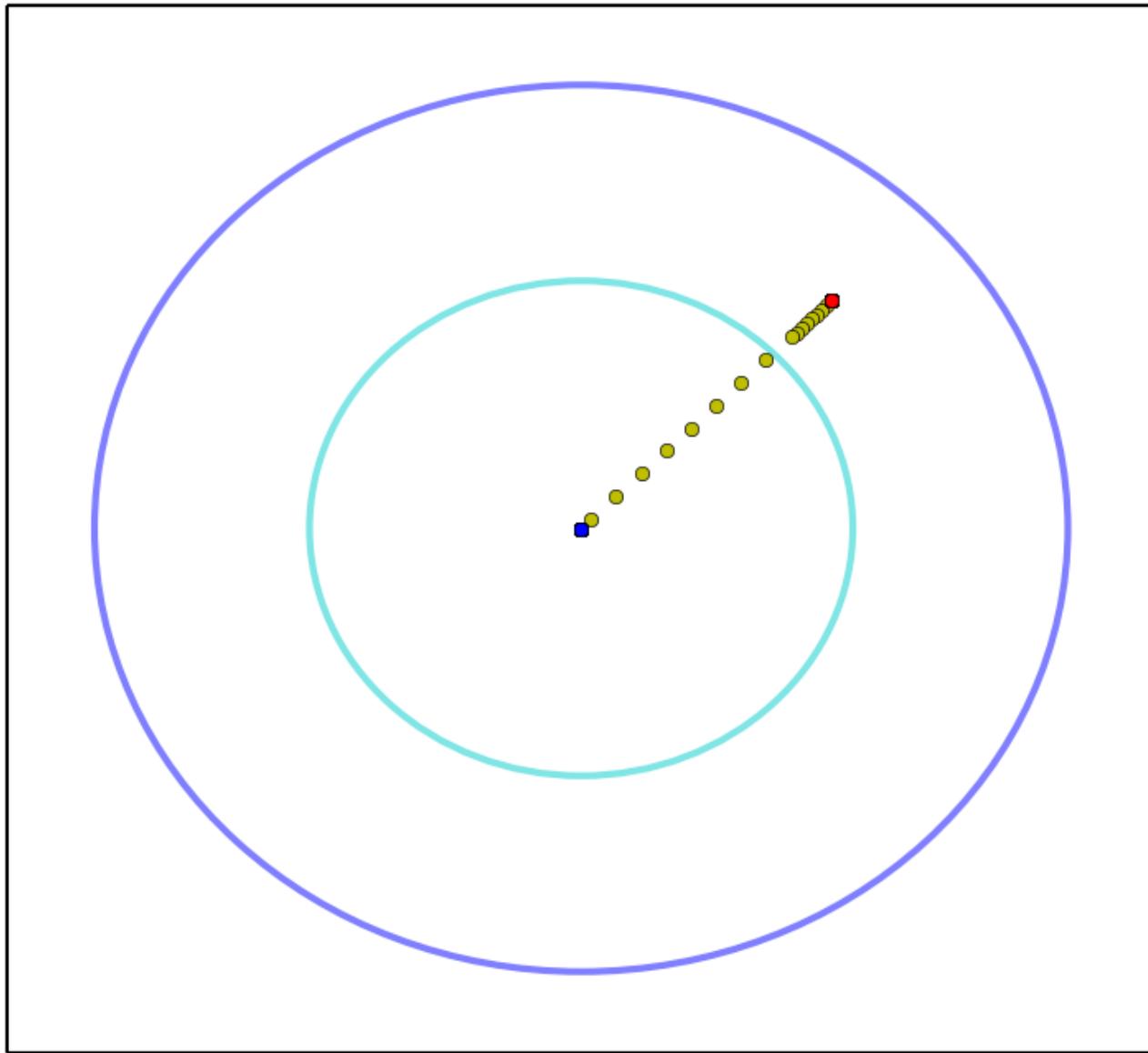




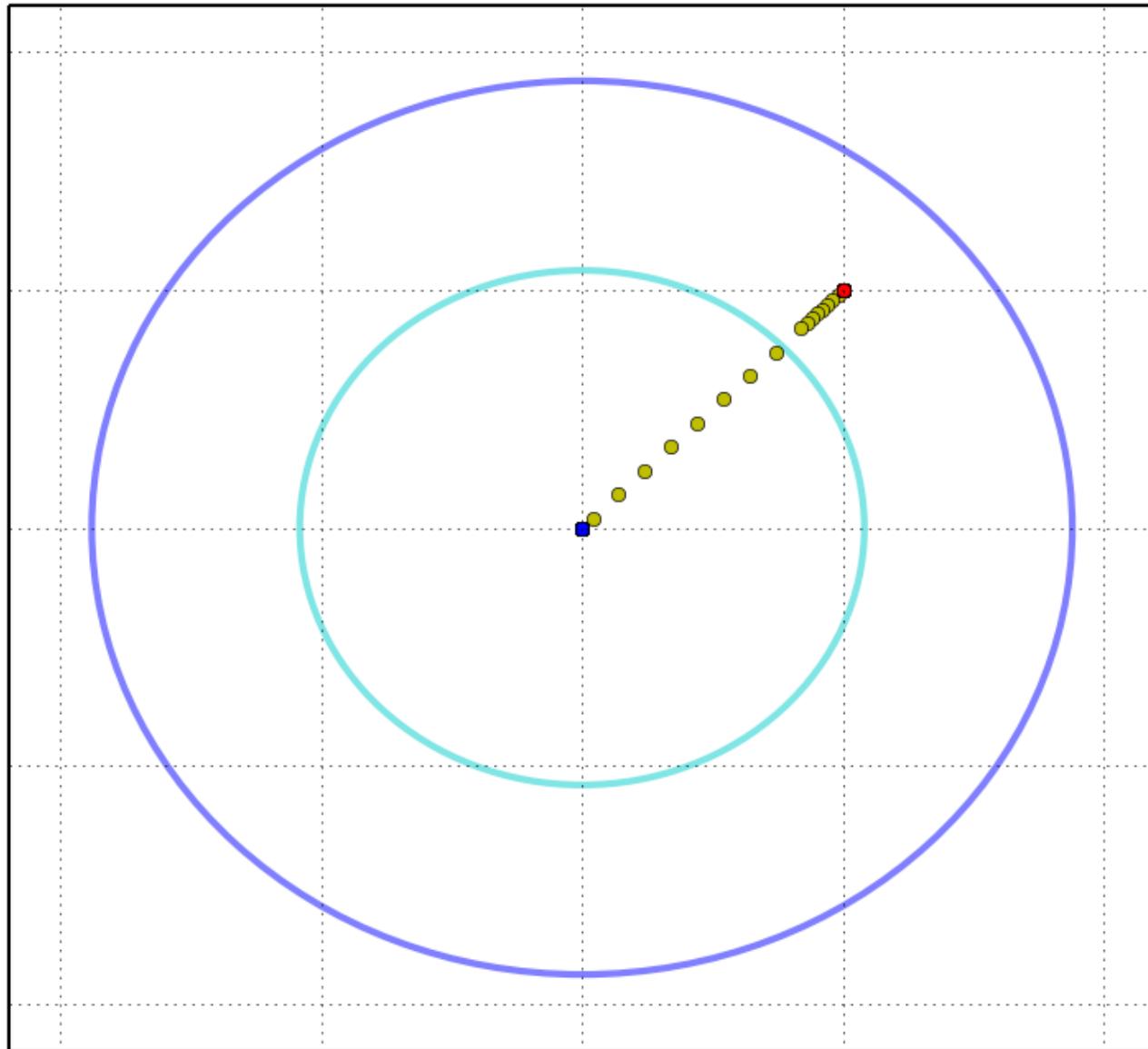
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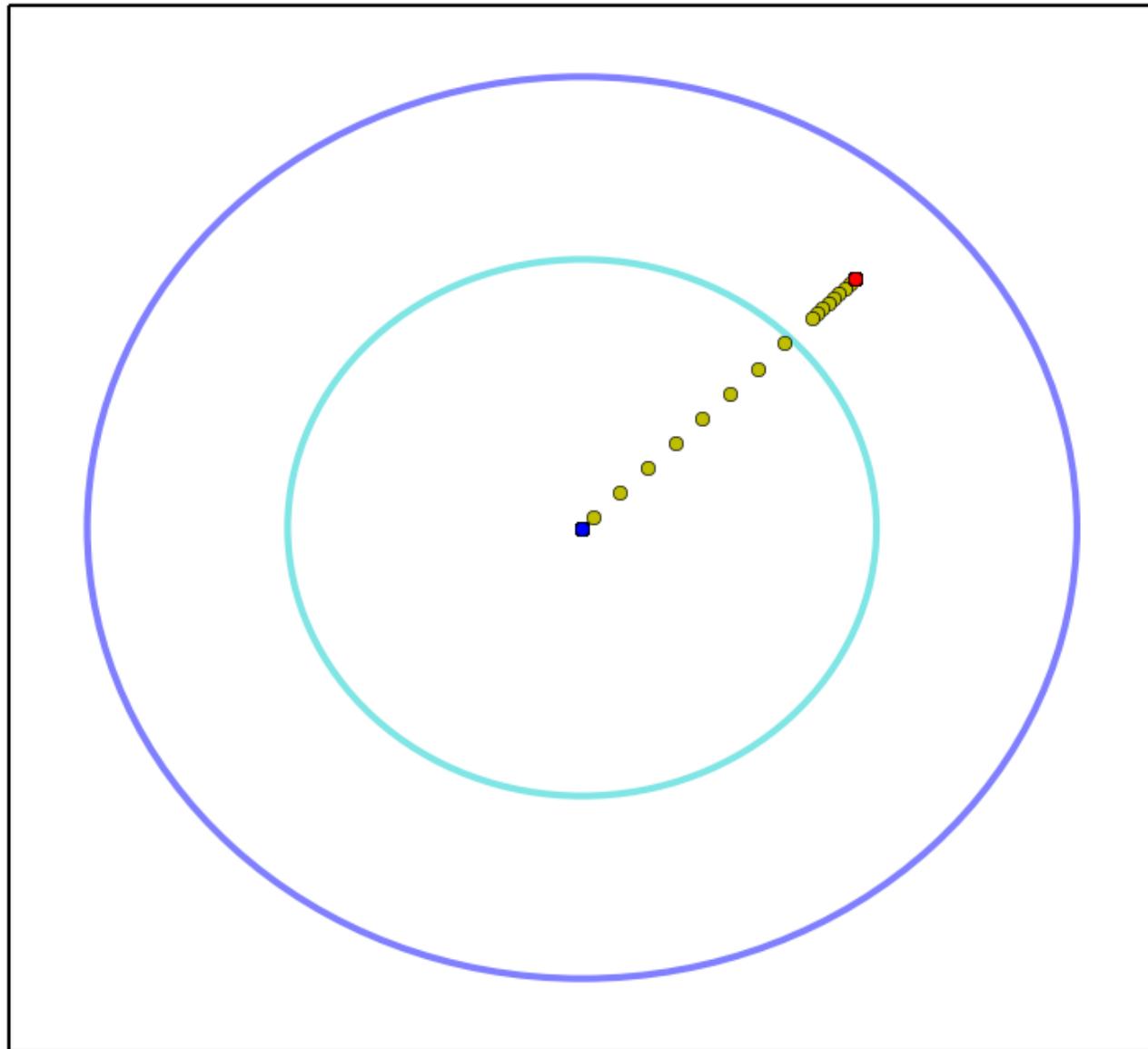


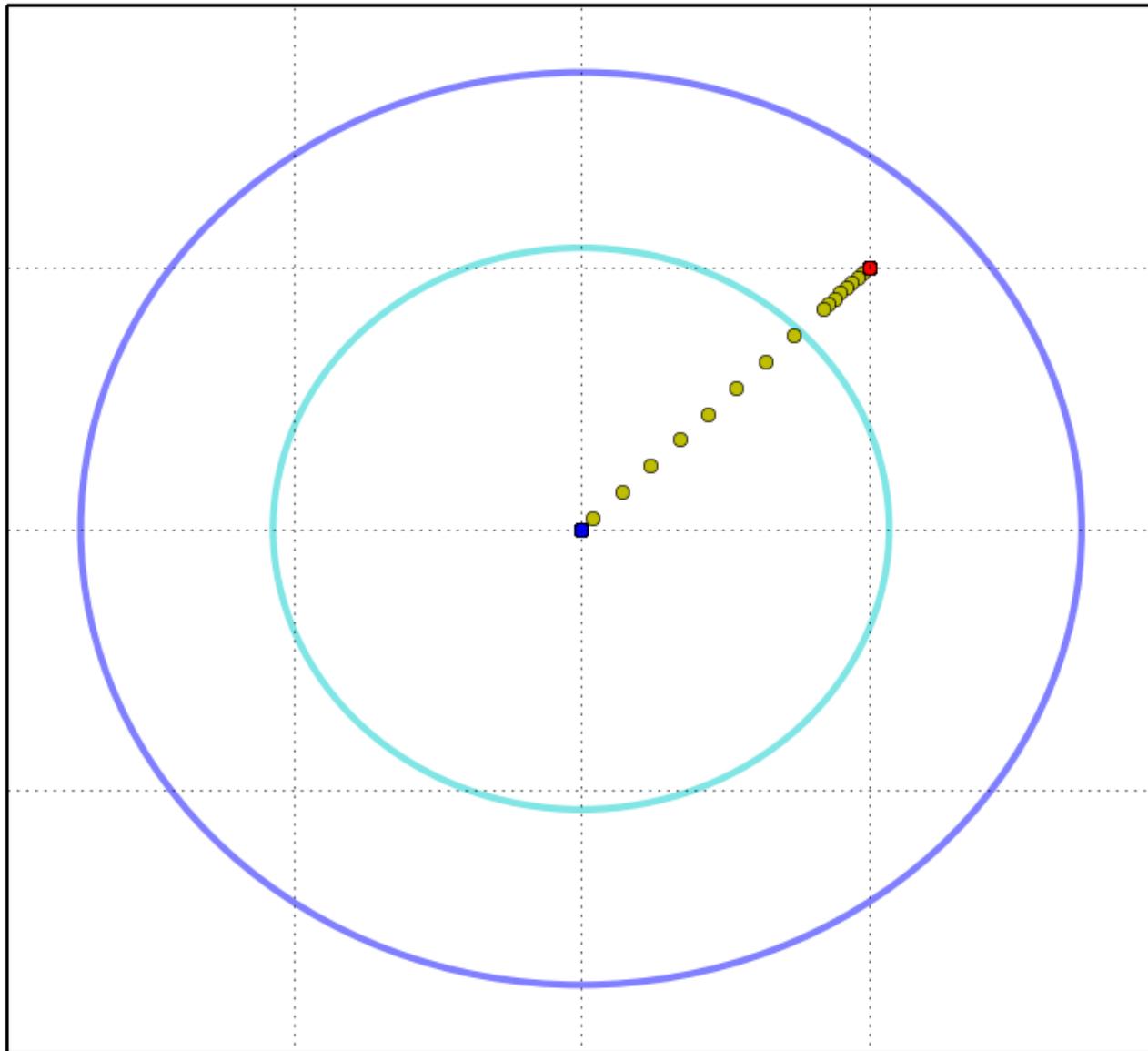


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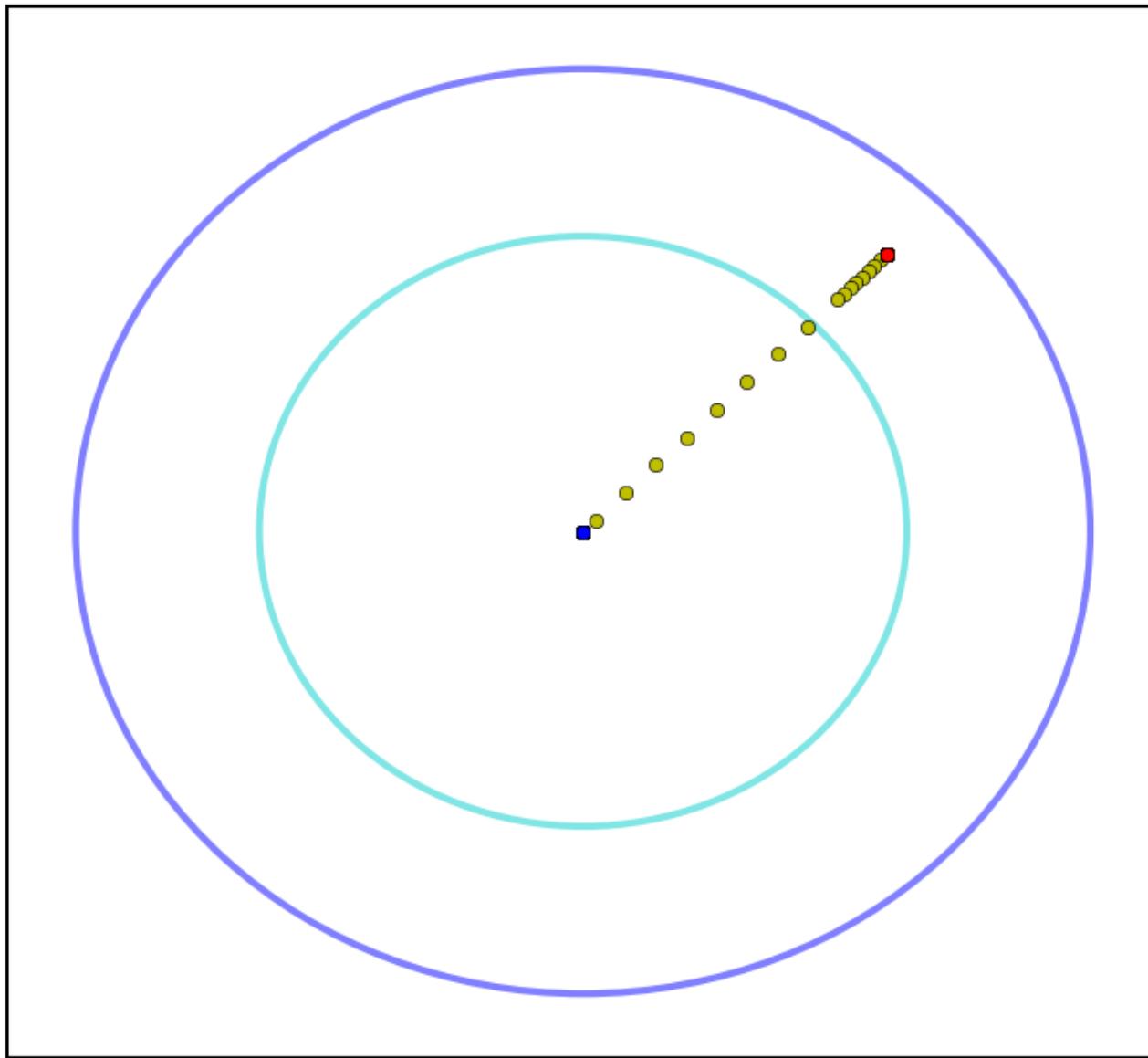


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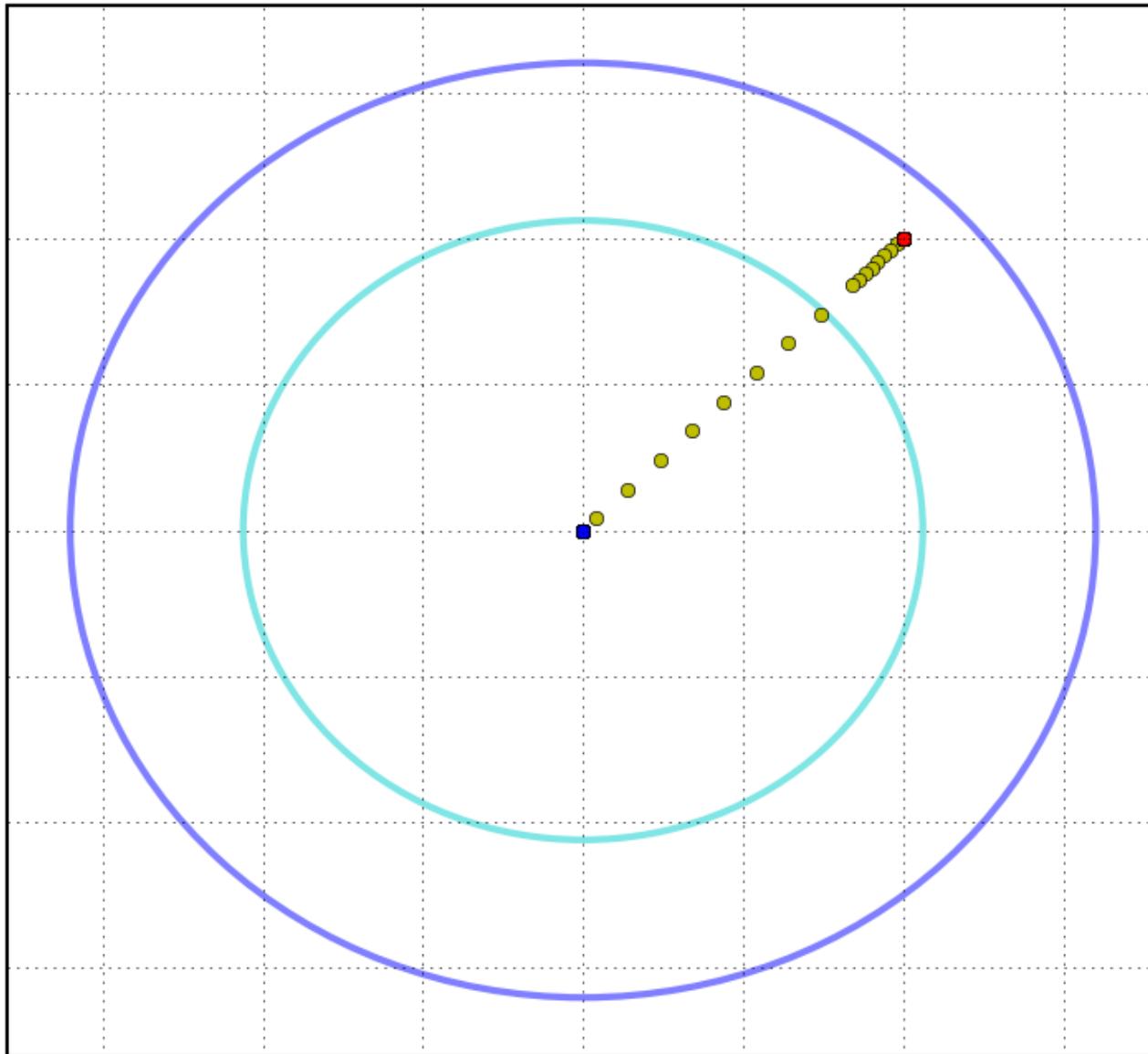




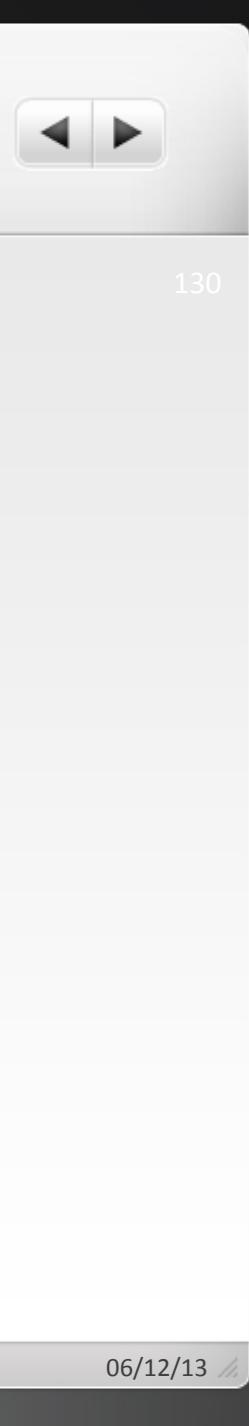
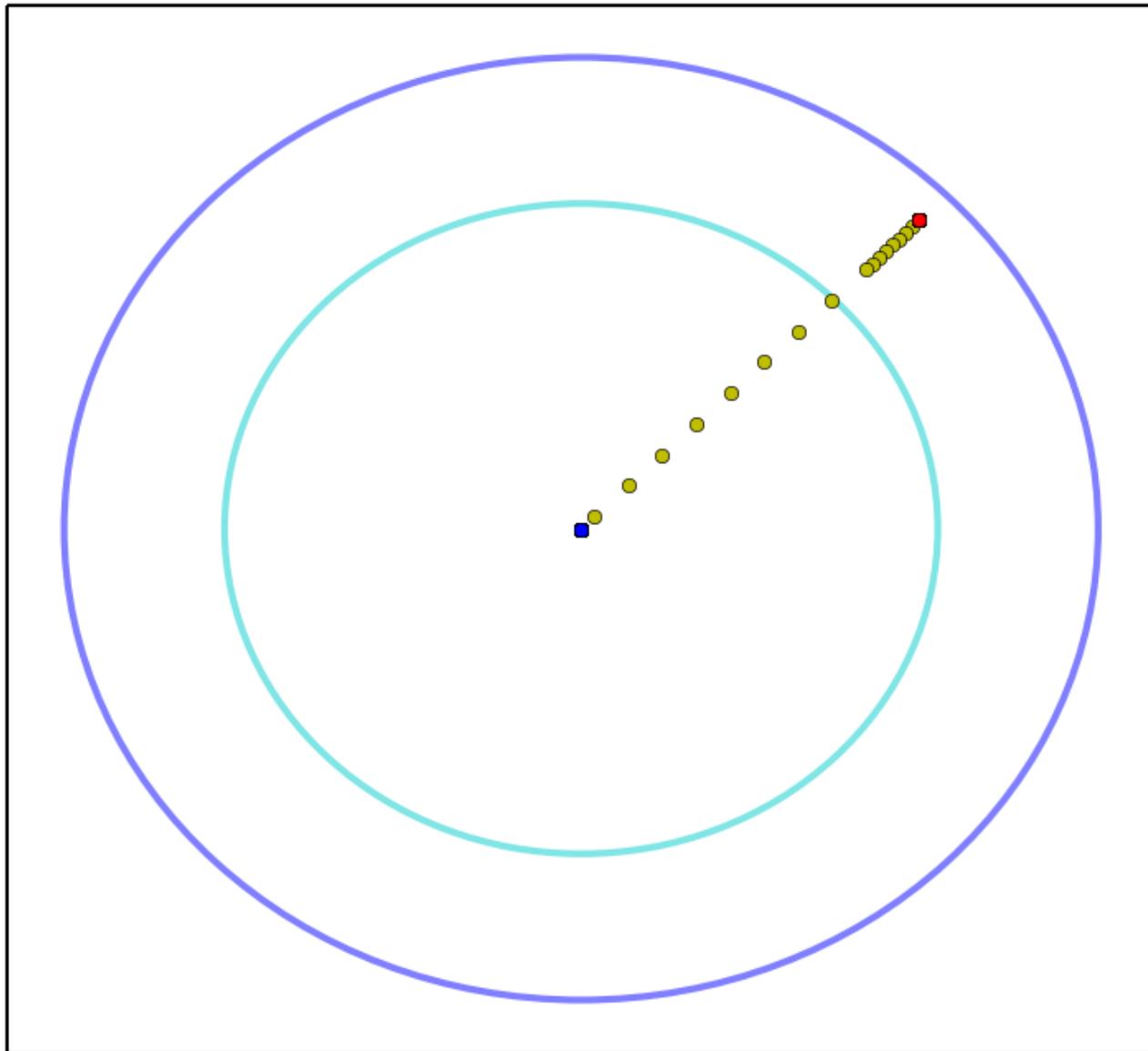
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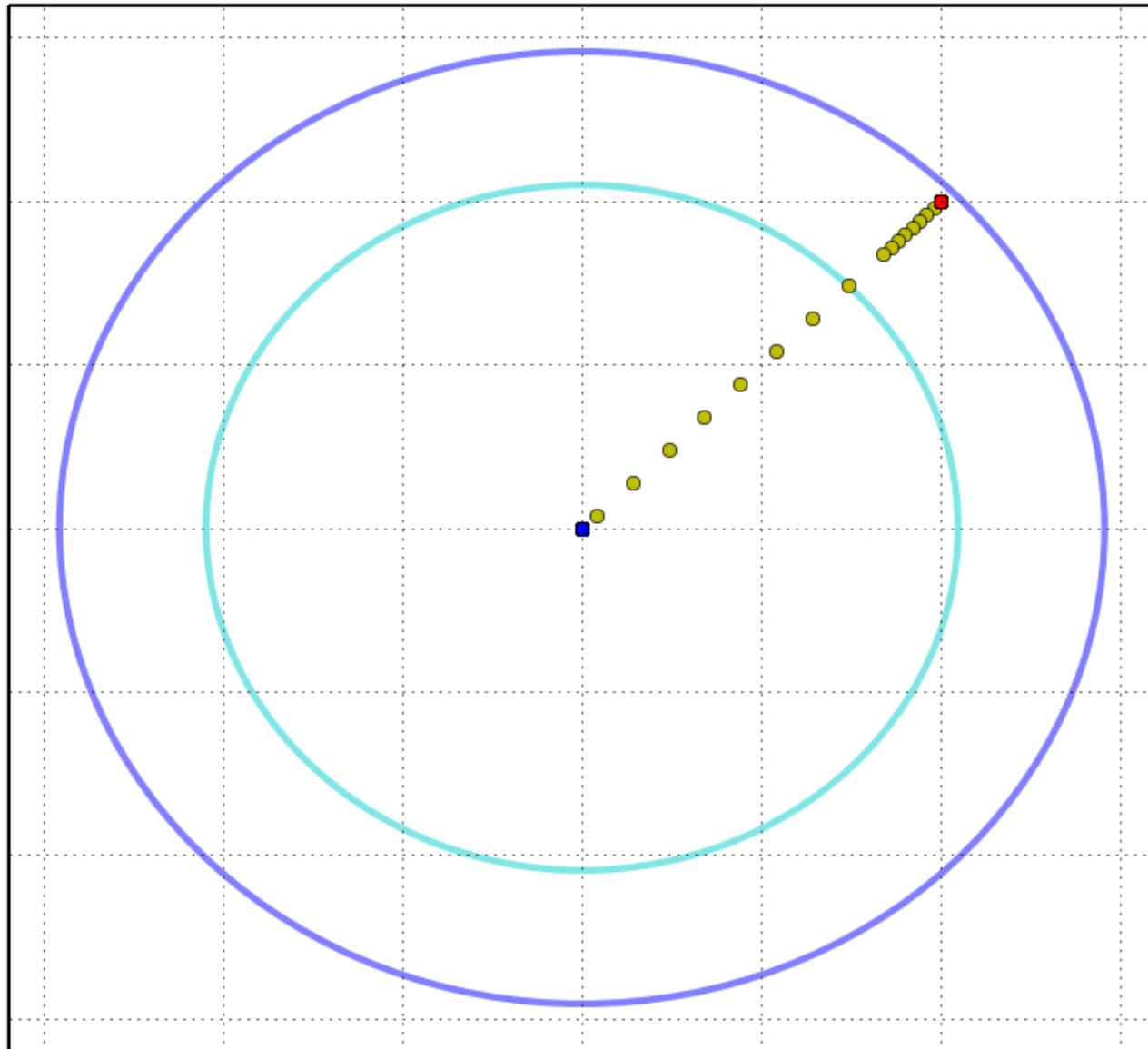


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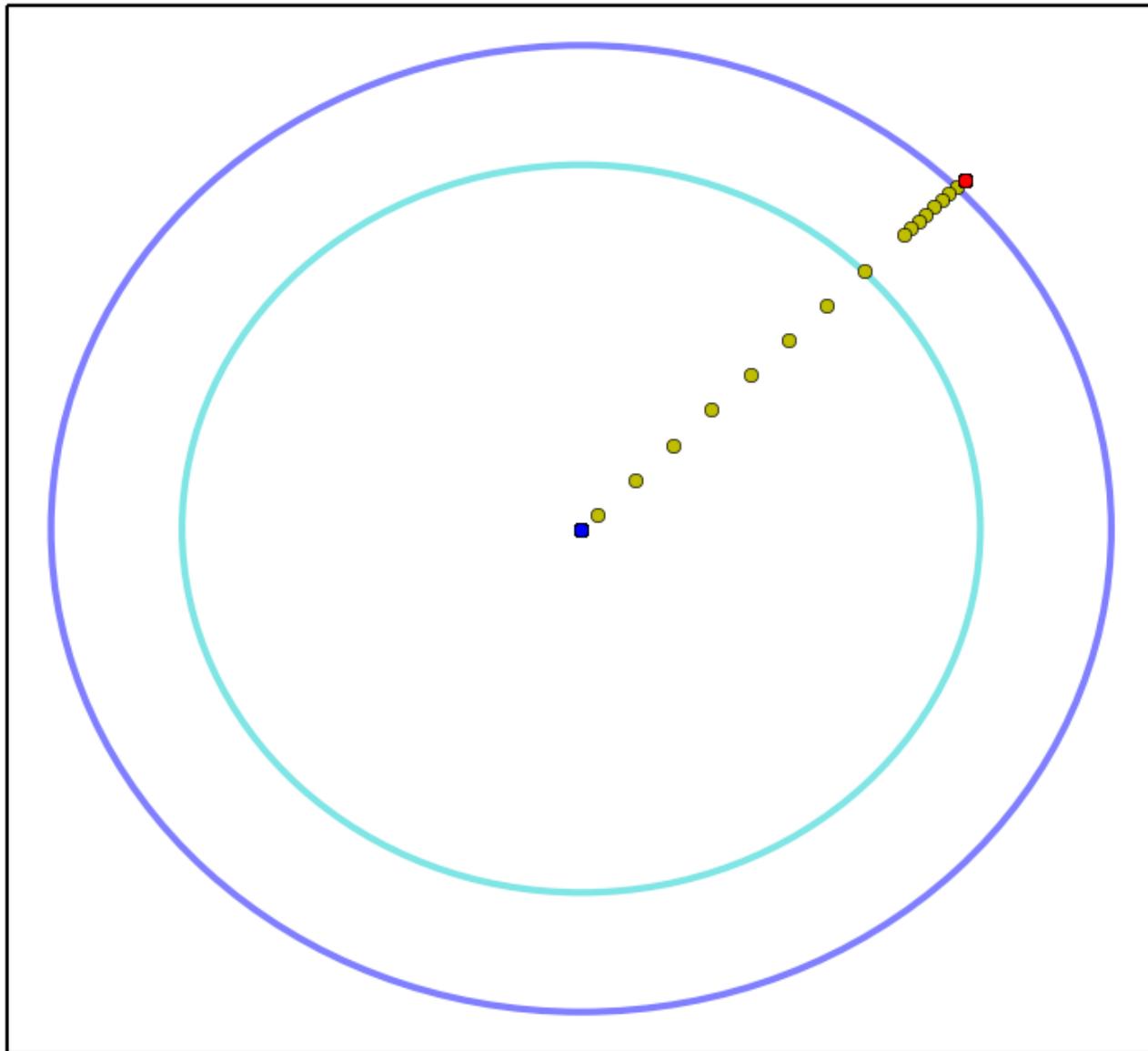


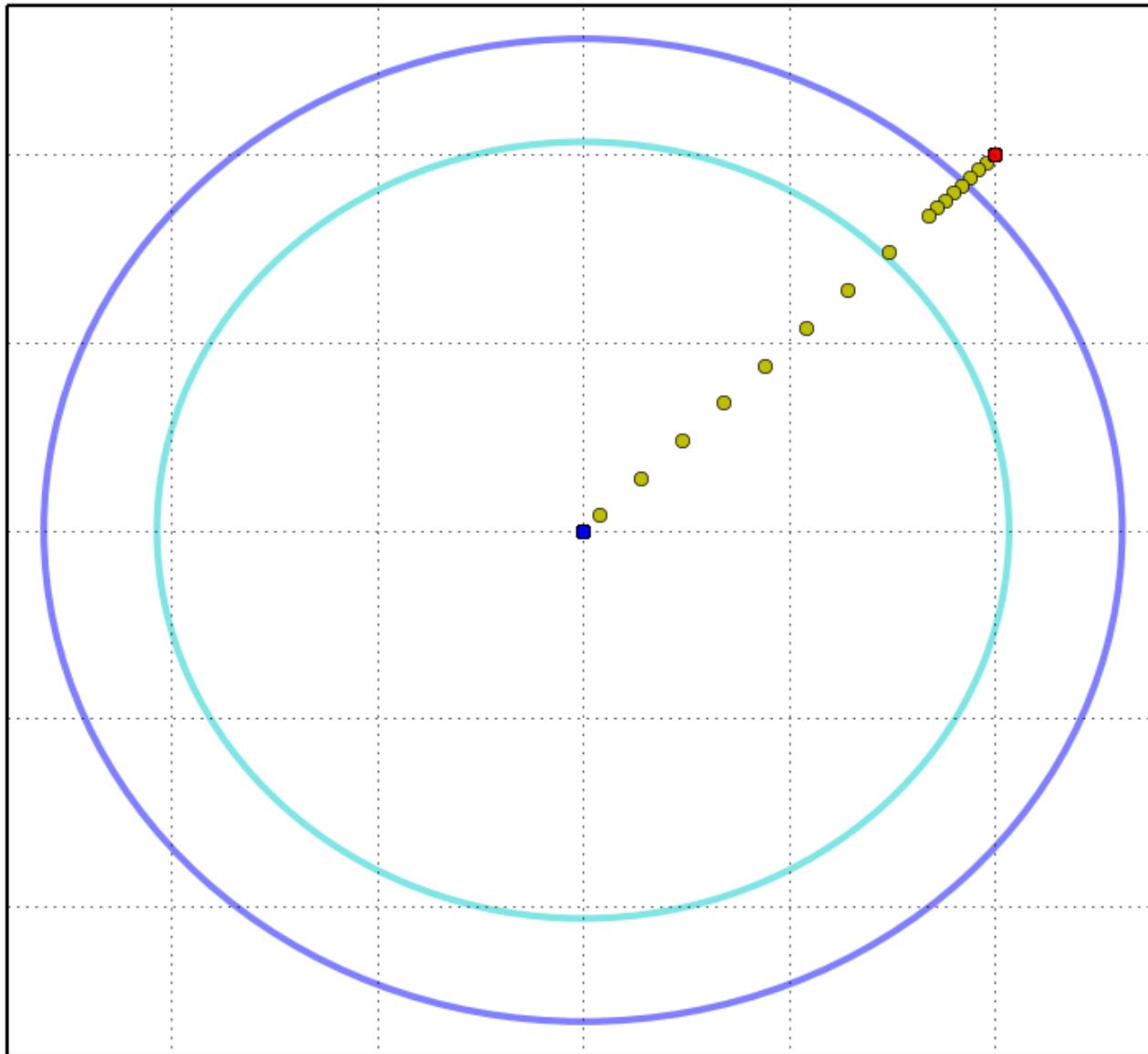
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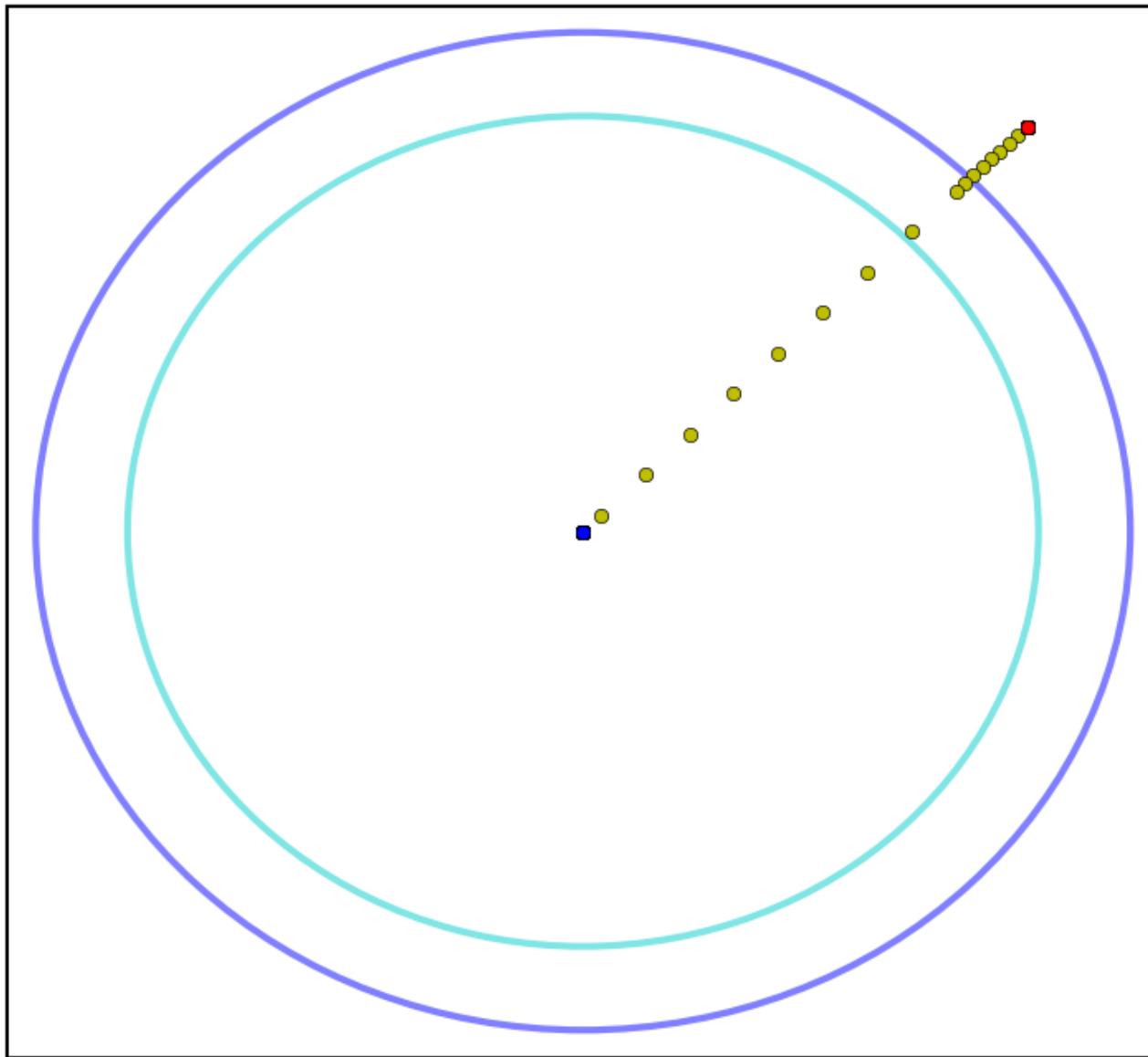


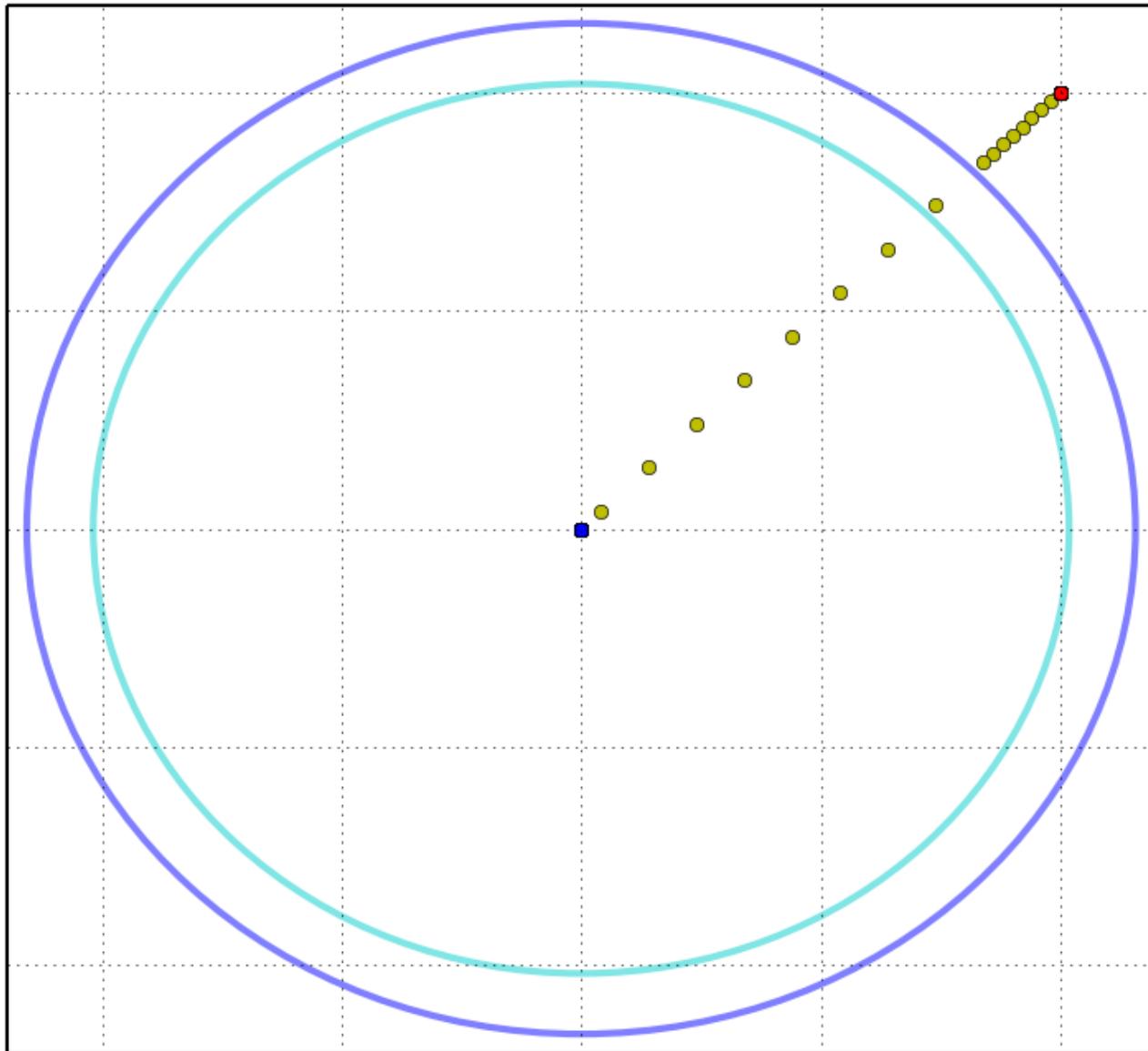
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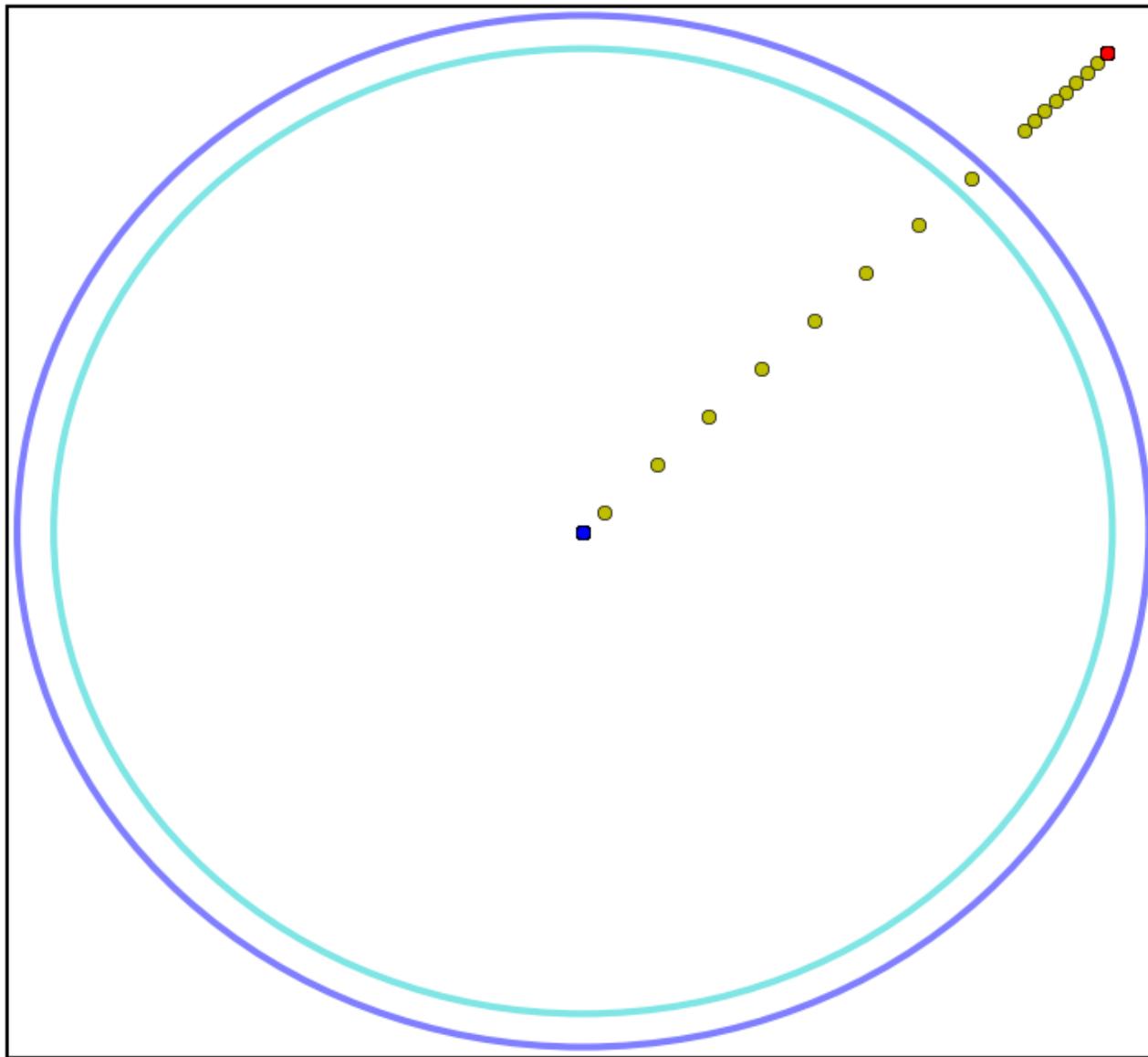




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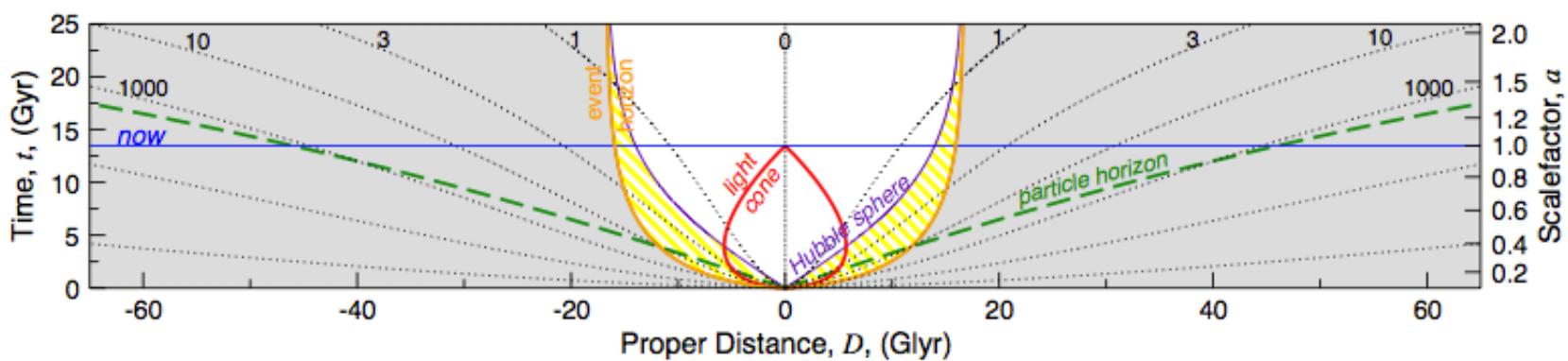






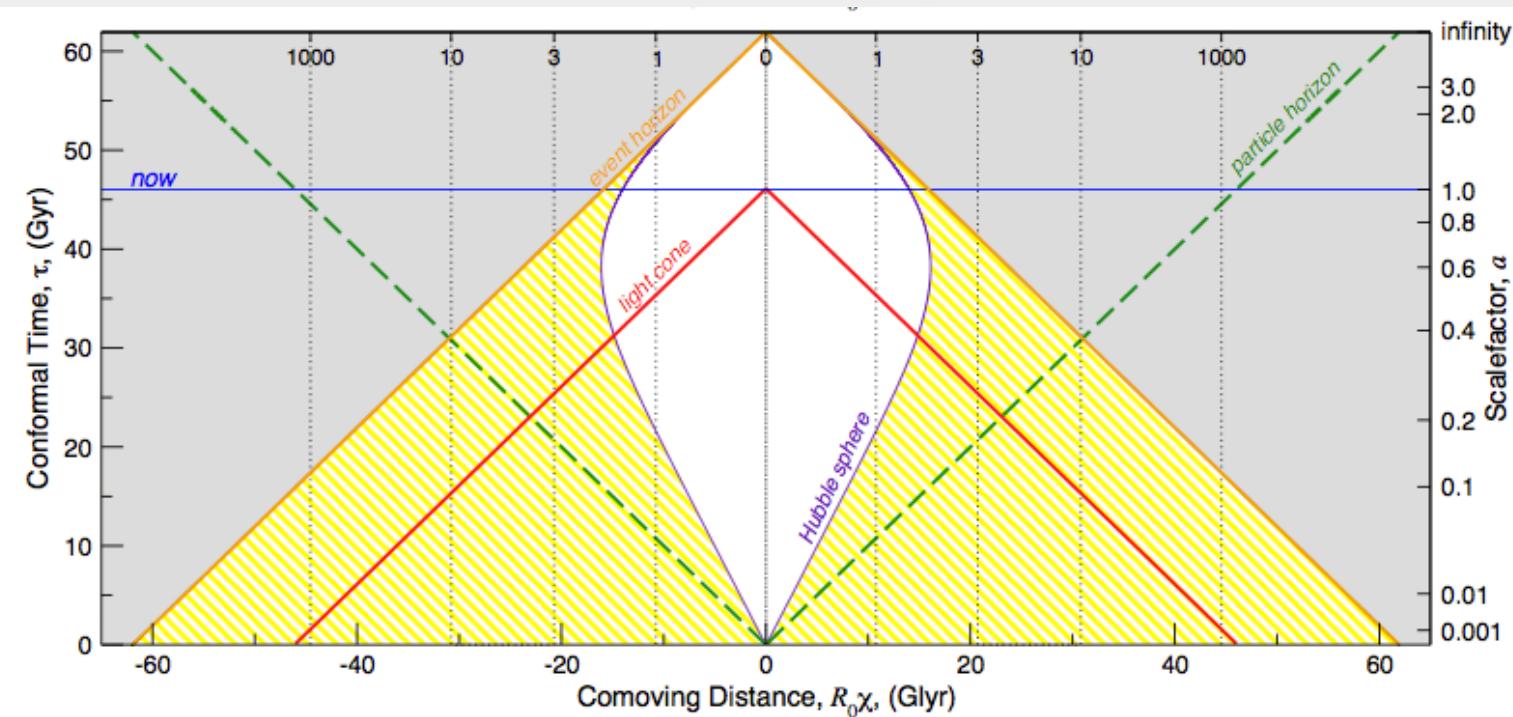


### III) Mind-blowing animations





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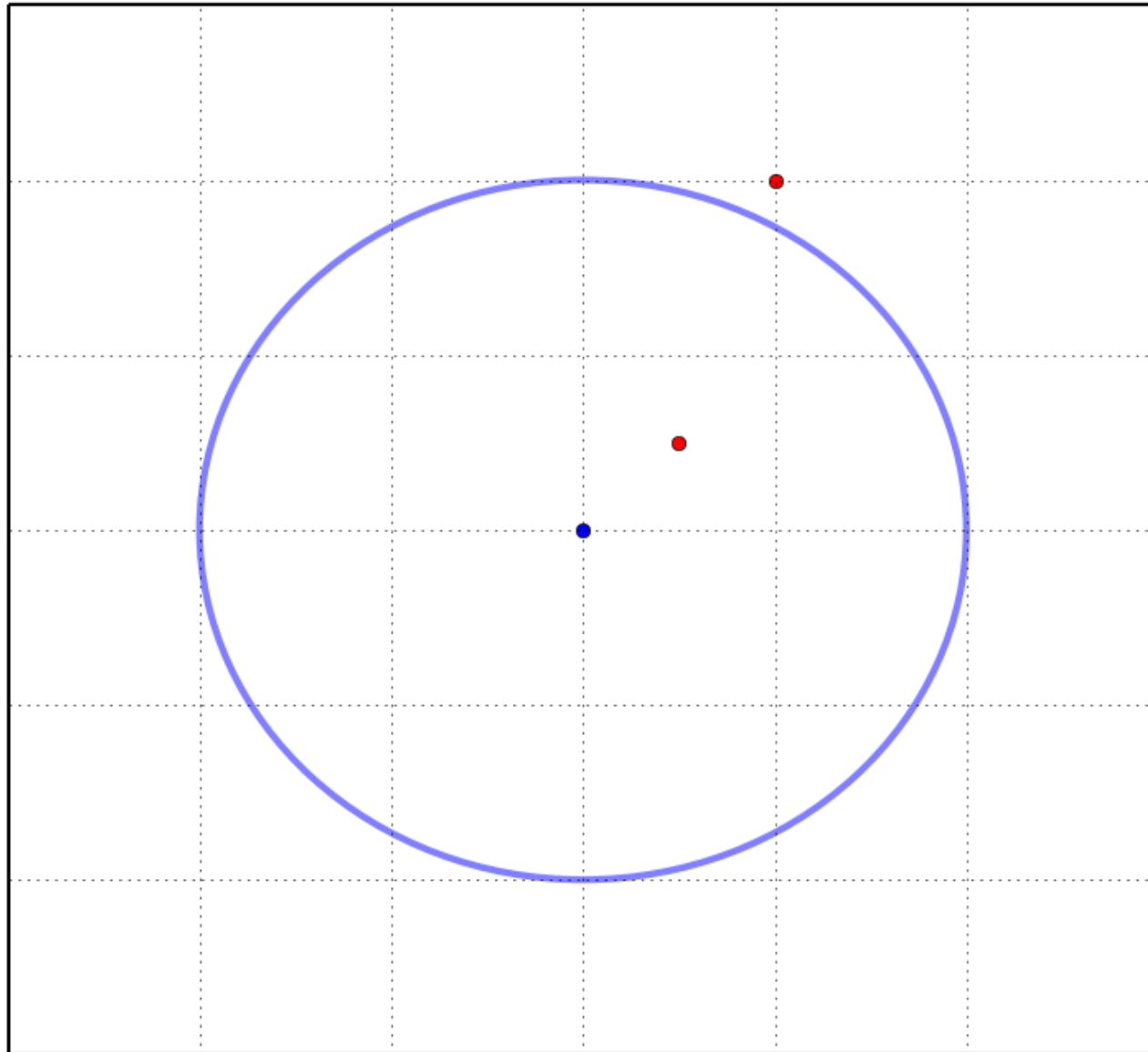




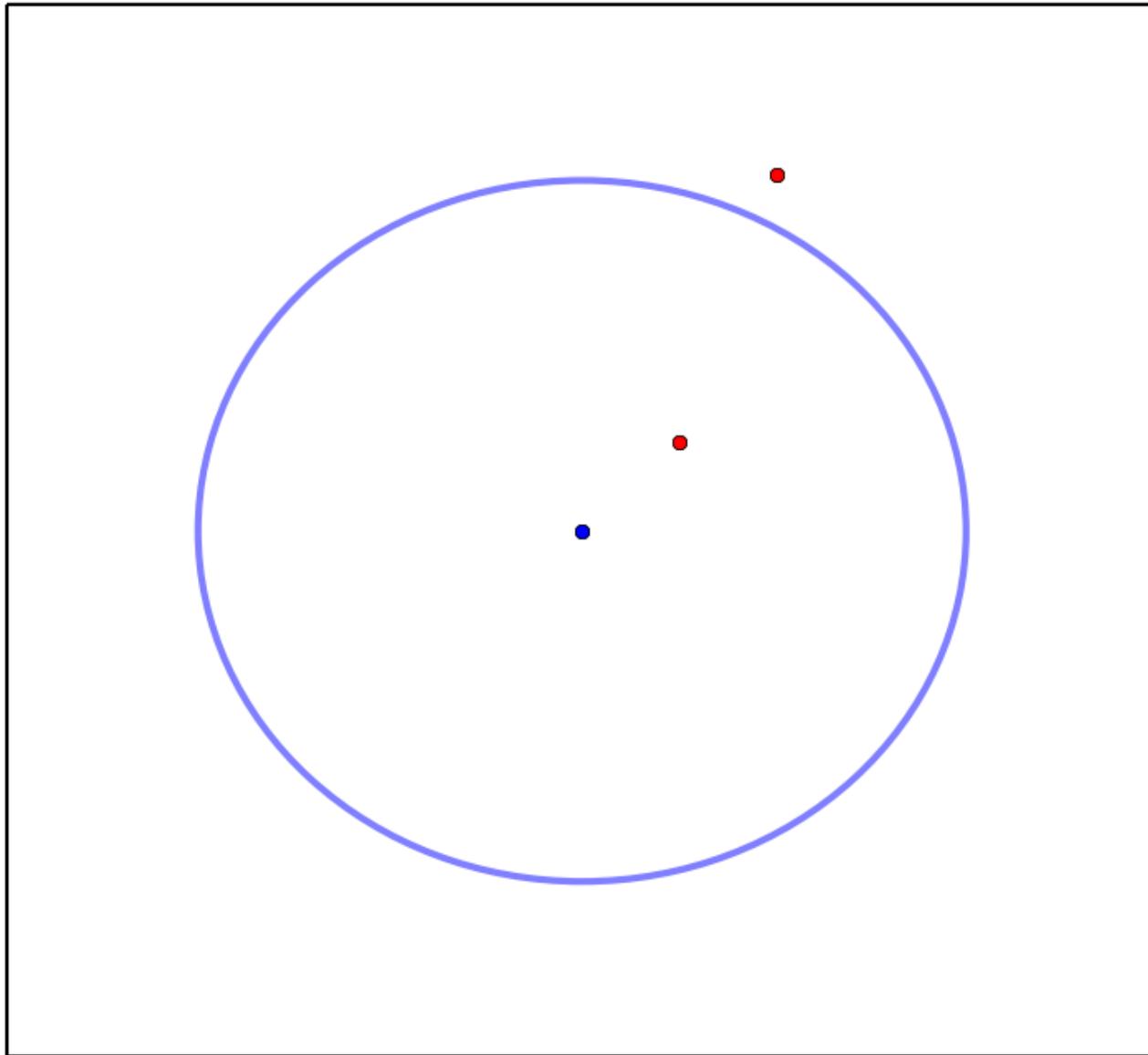
## An example: de Sitter Universe

- de Sitter spacetime = empty Universe with only a positive cosmological constant
- Exponential scale-factor
- Accelerated expansion
- $H = \text{constant}$
- Hubble Sphere = Event Horizon, fixed in proper distance

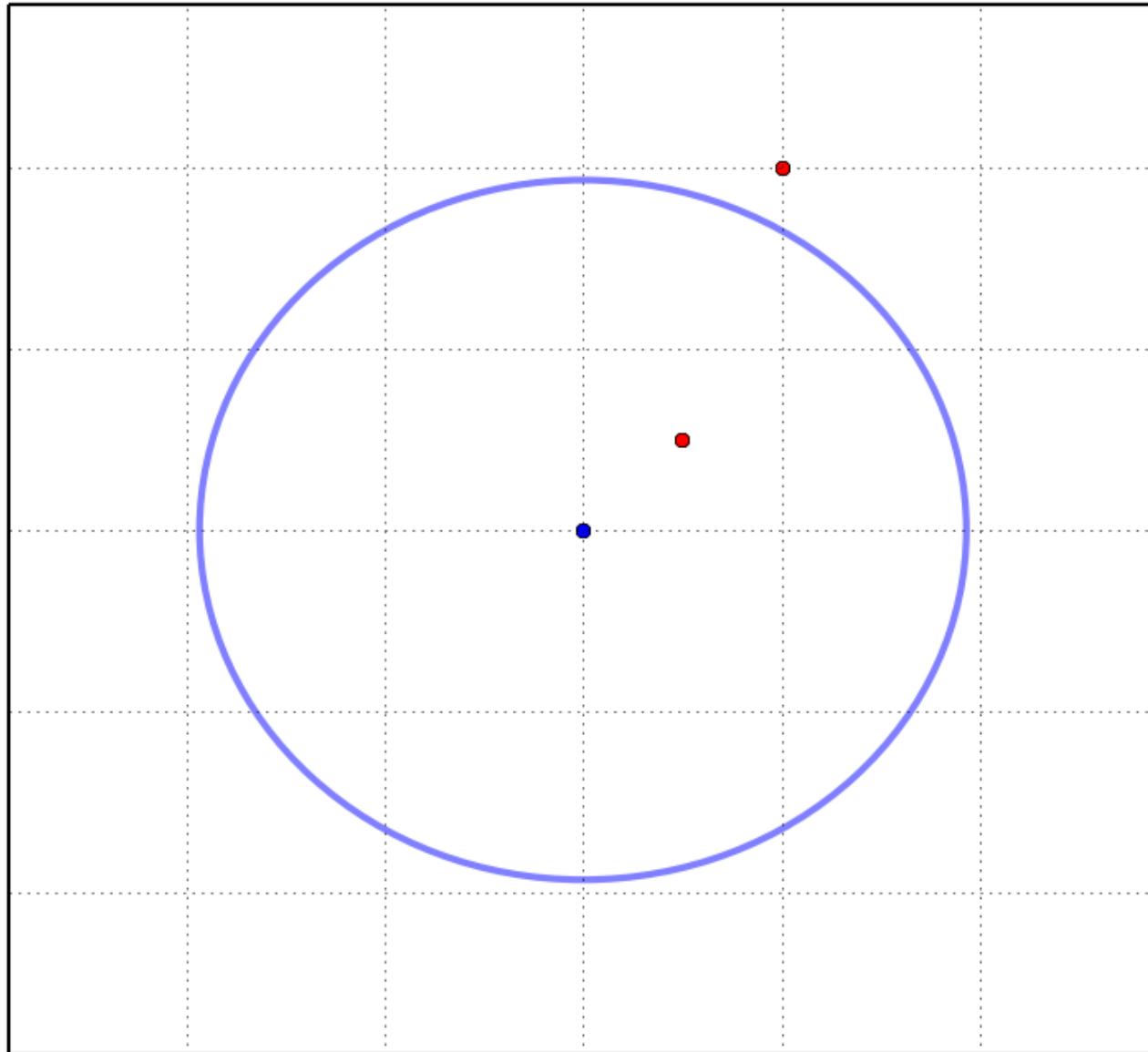
$$a(t) \propto e^{Ht}$$



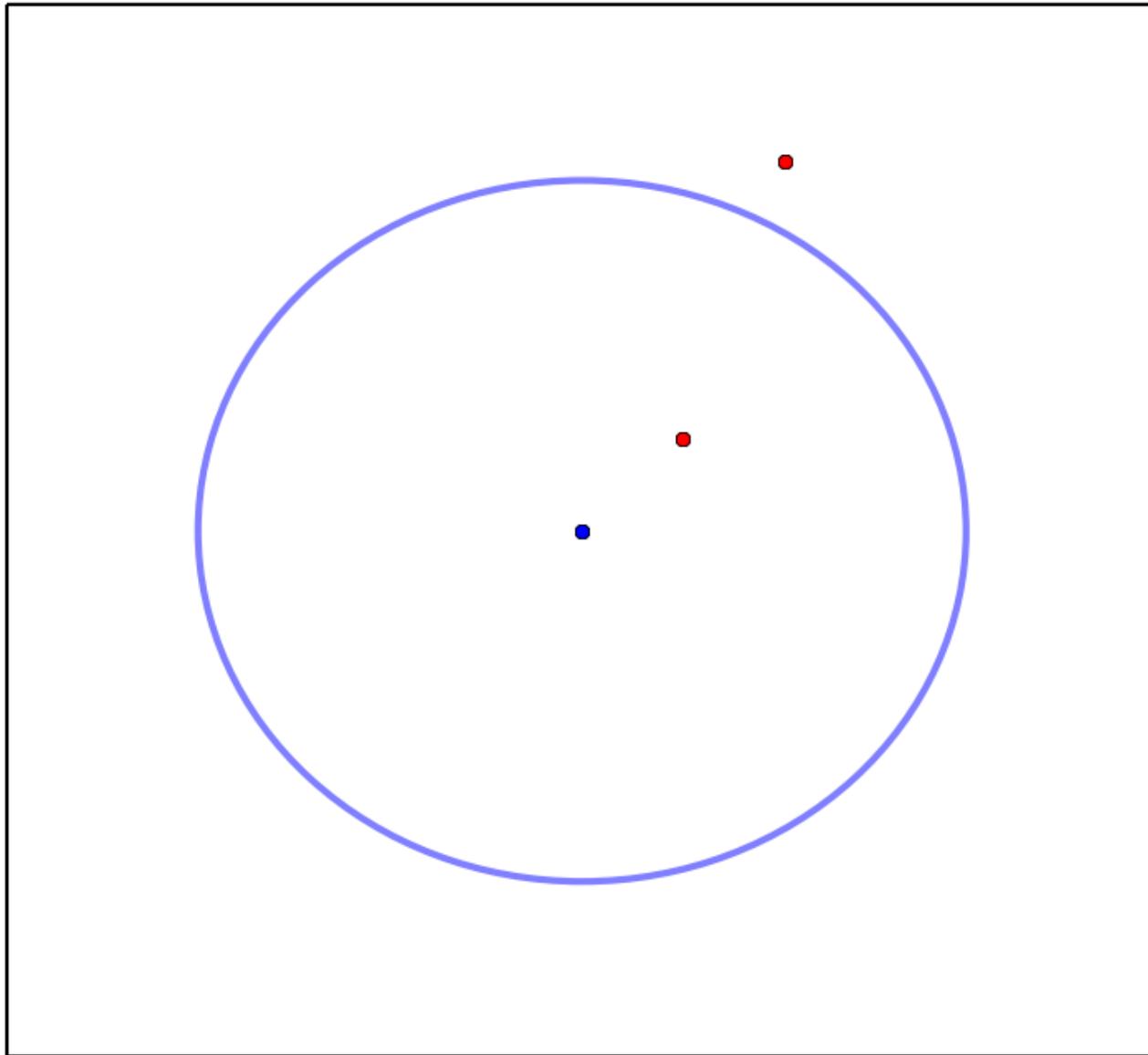
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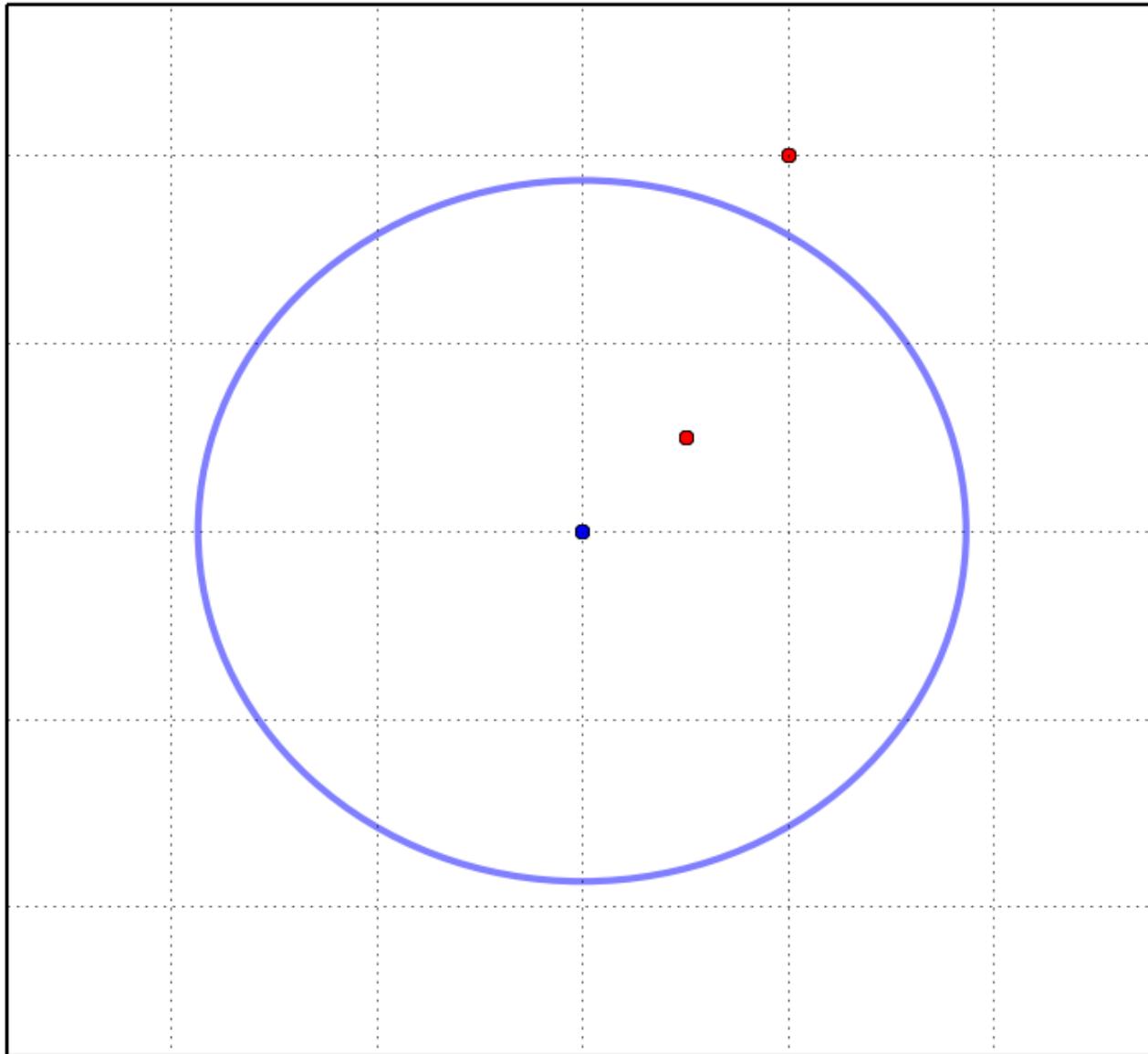
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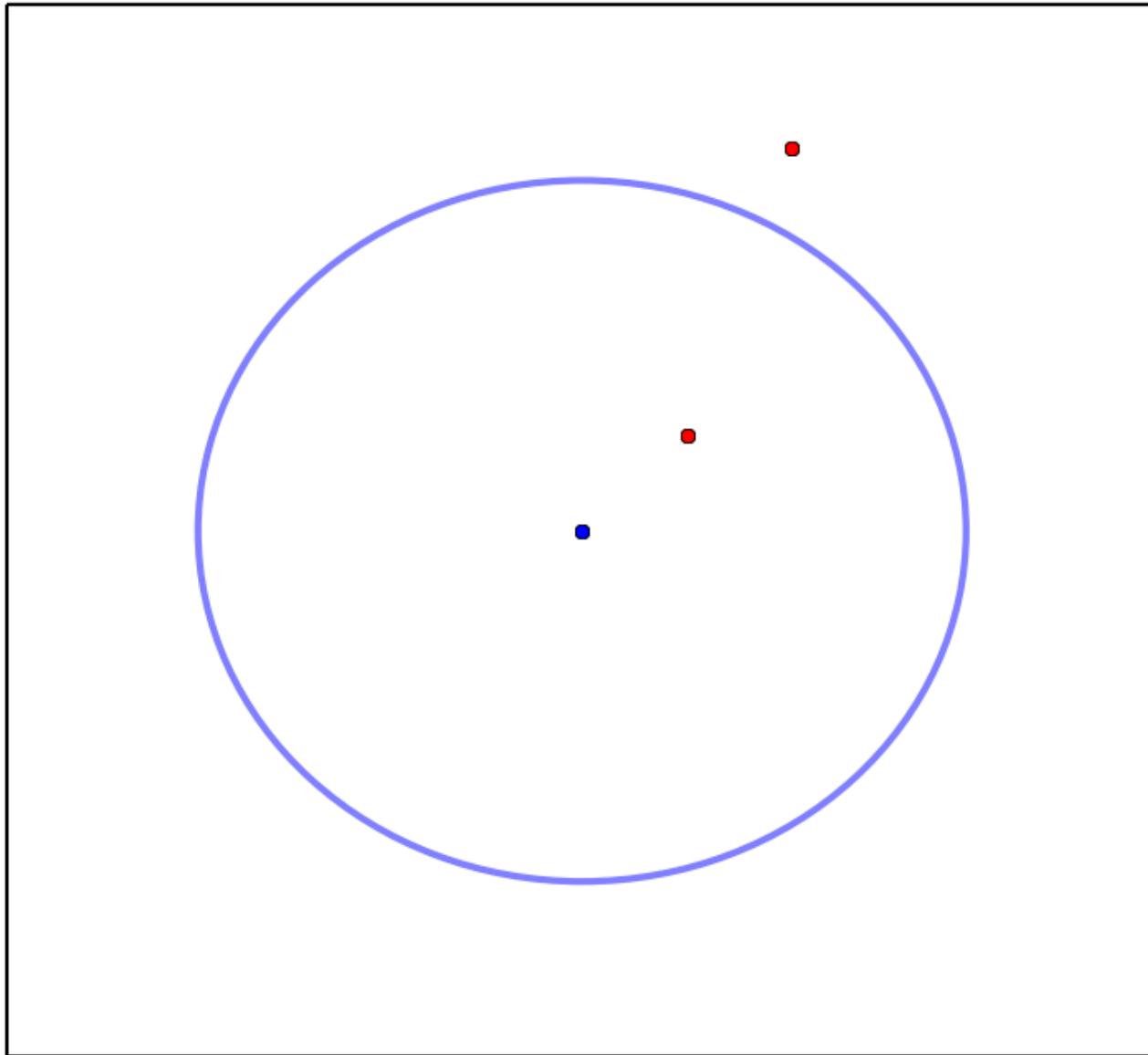
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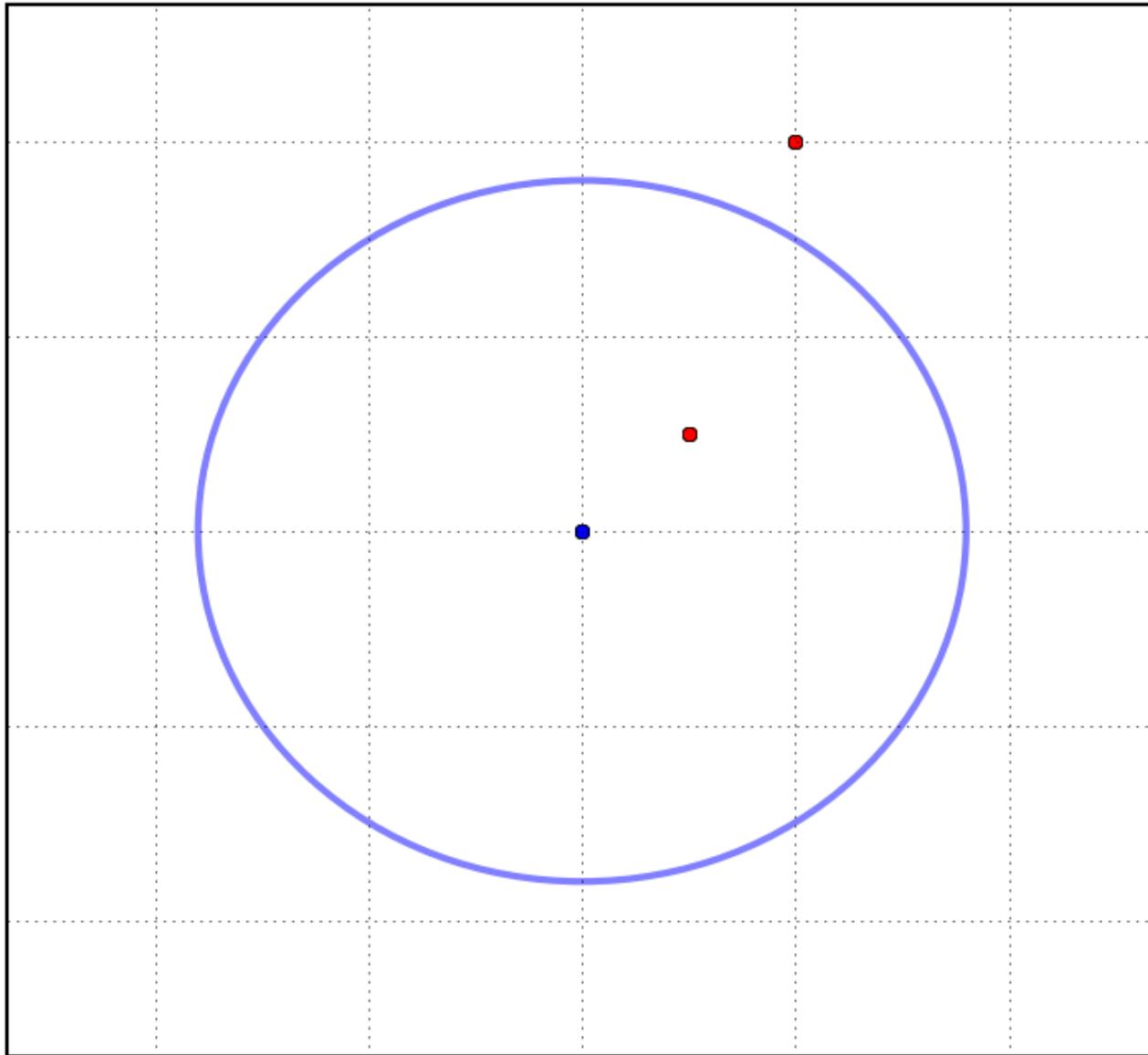
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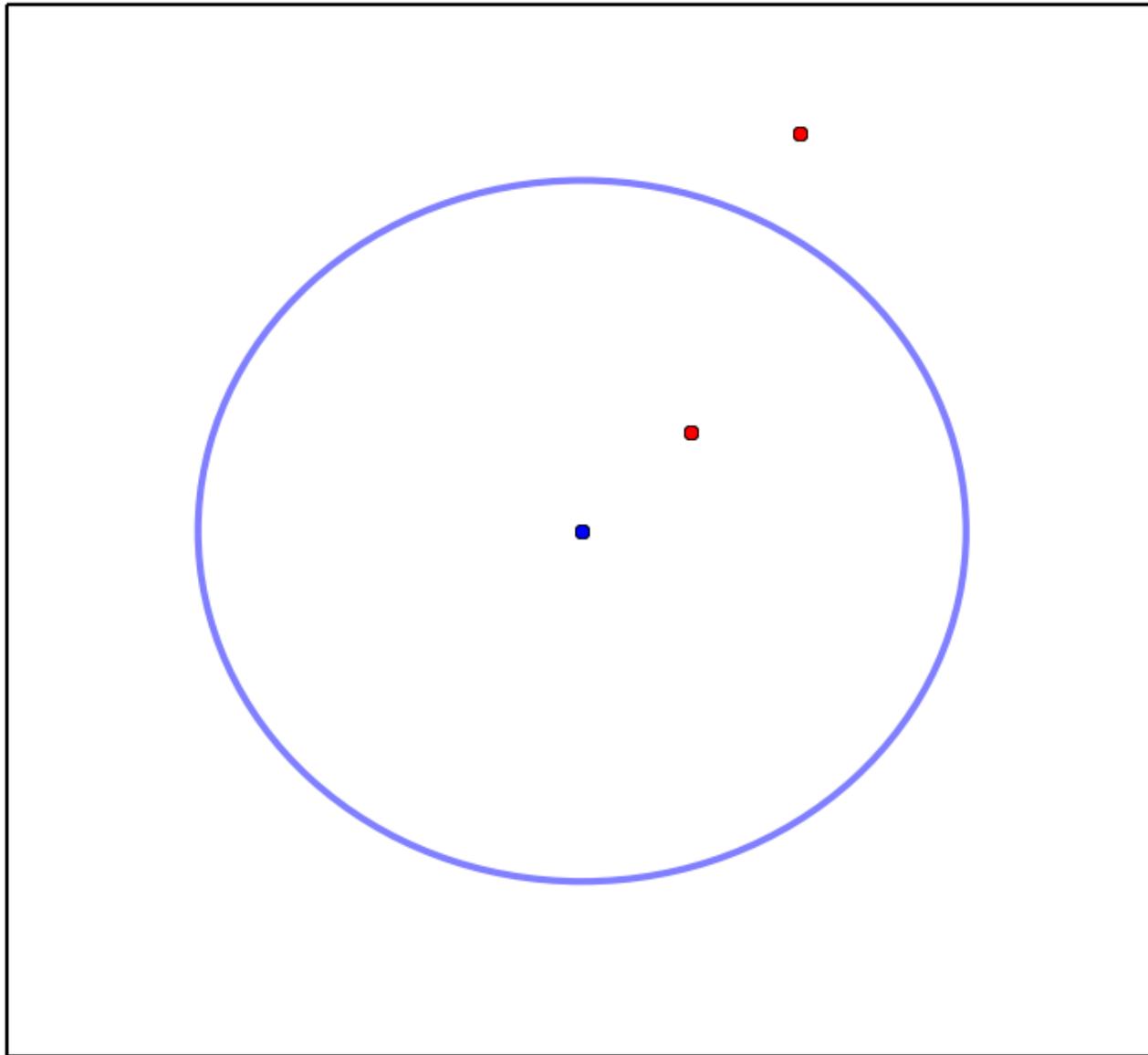
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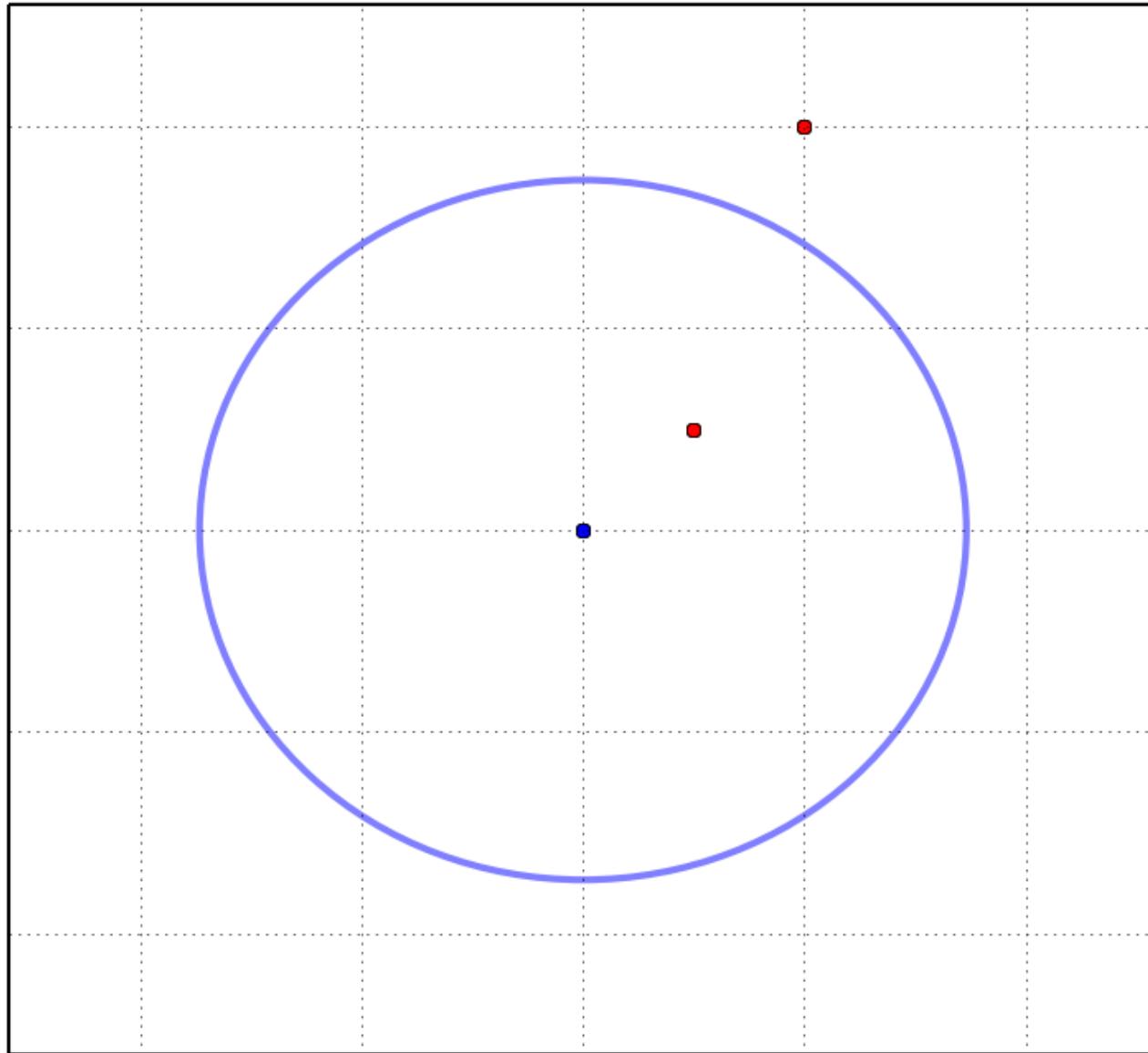
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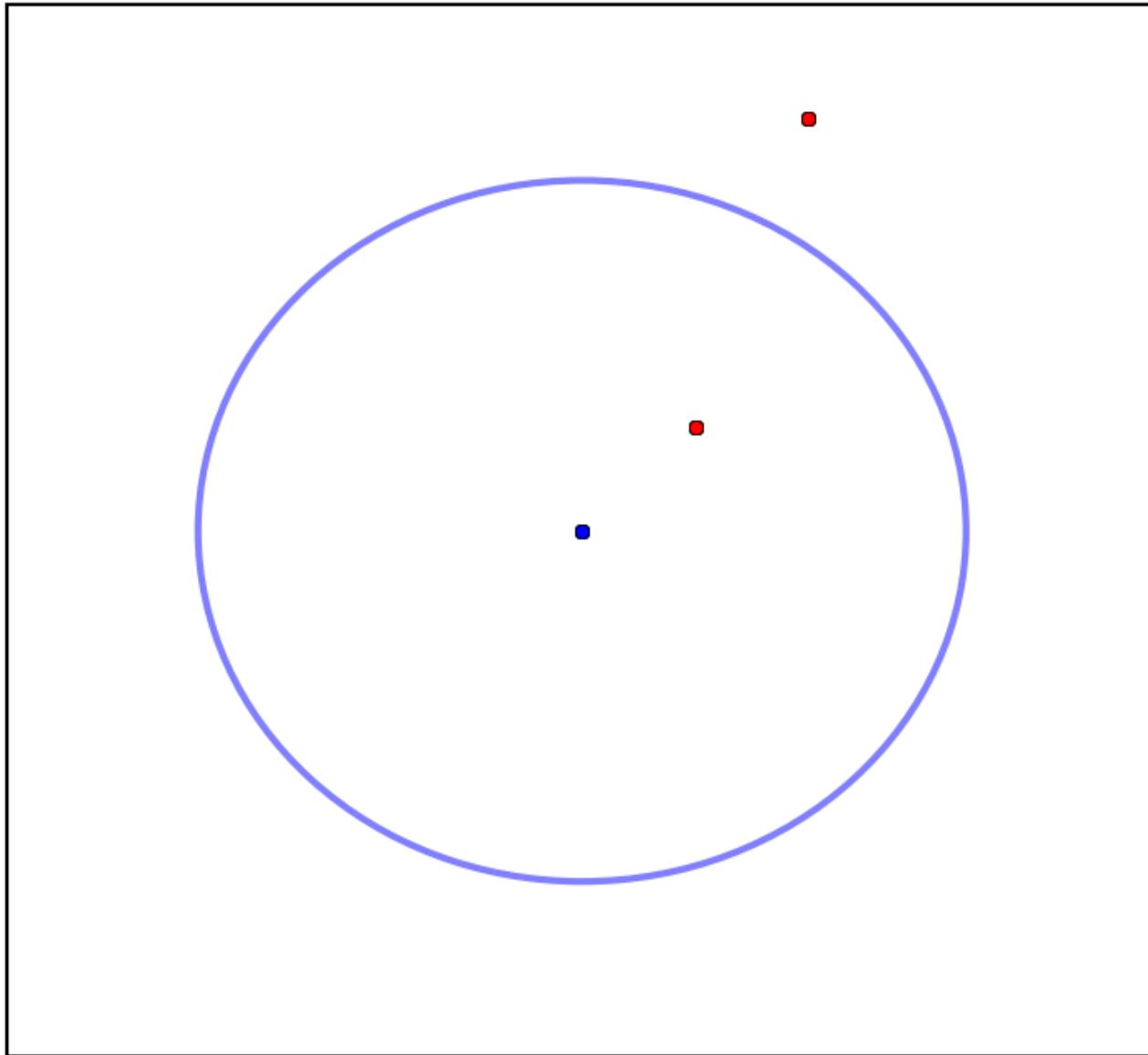
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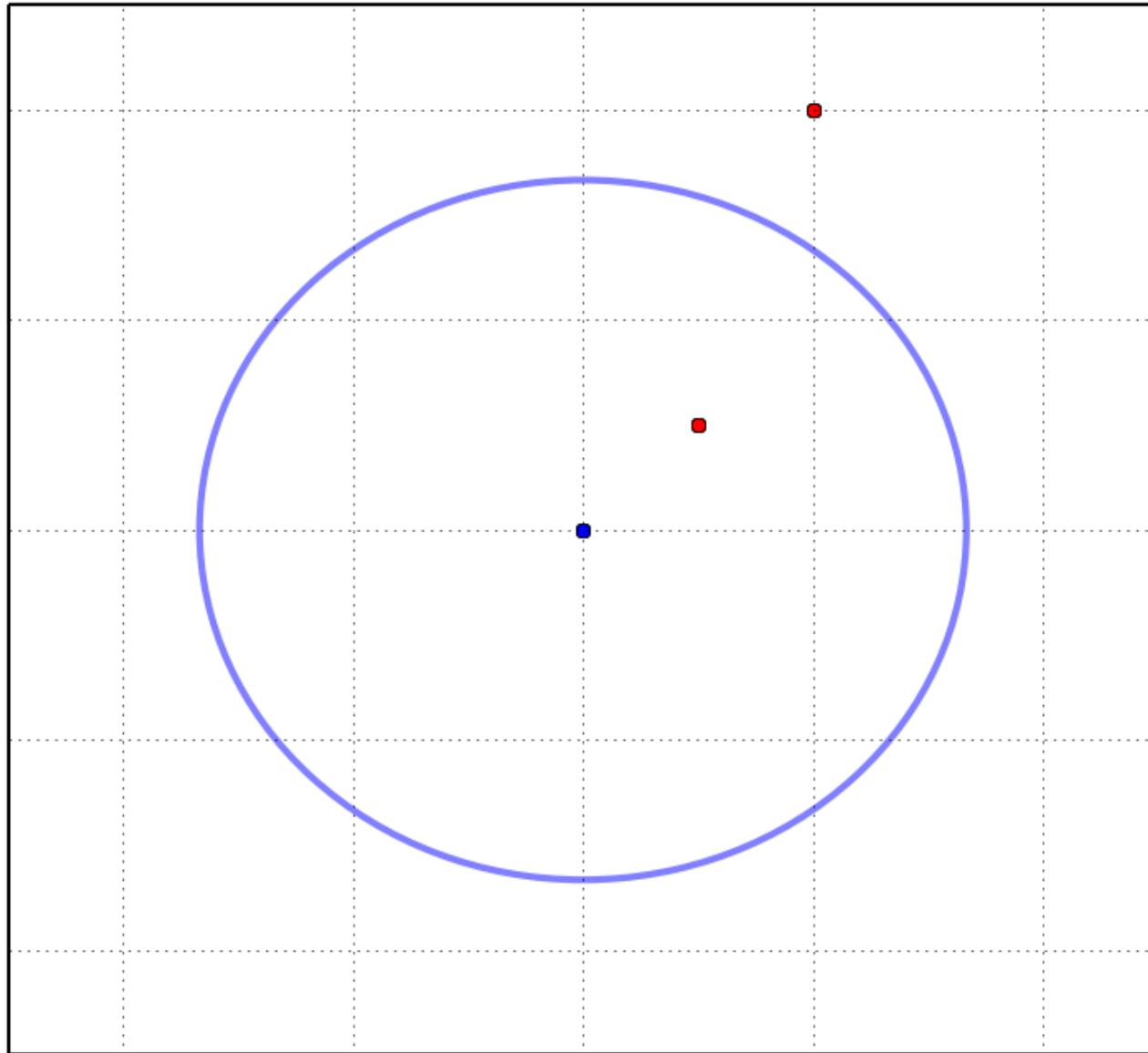
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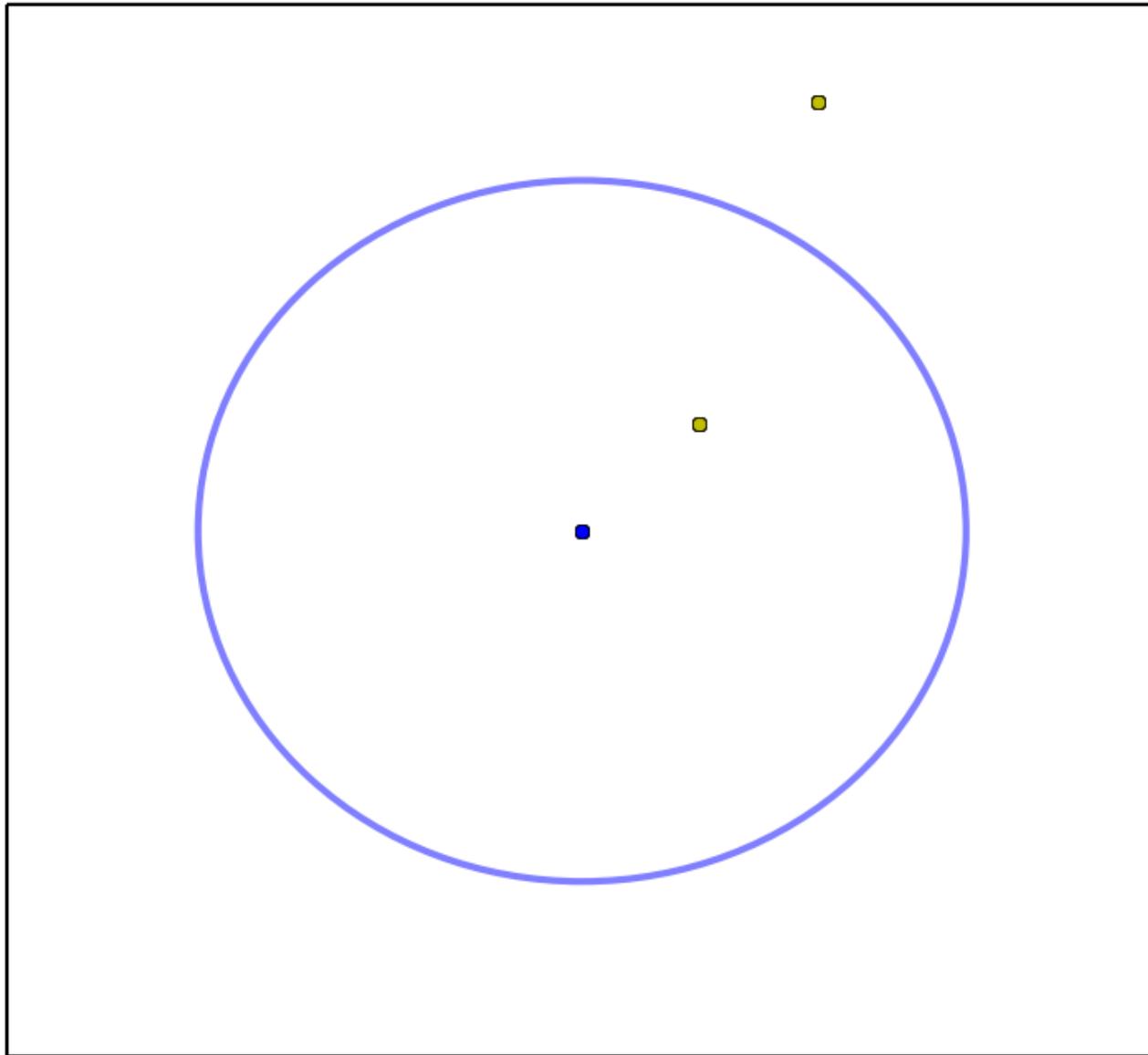
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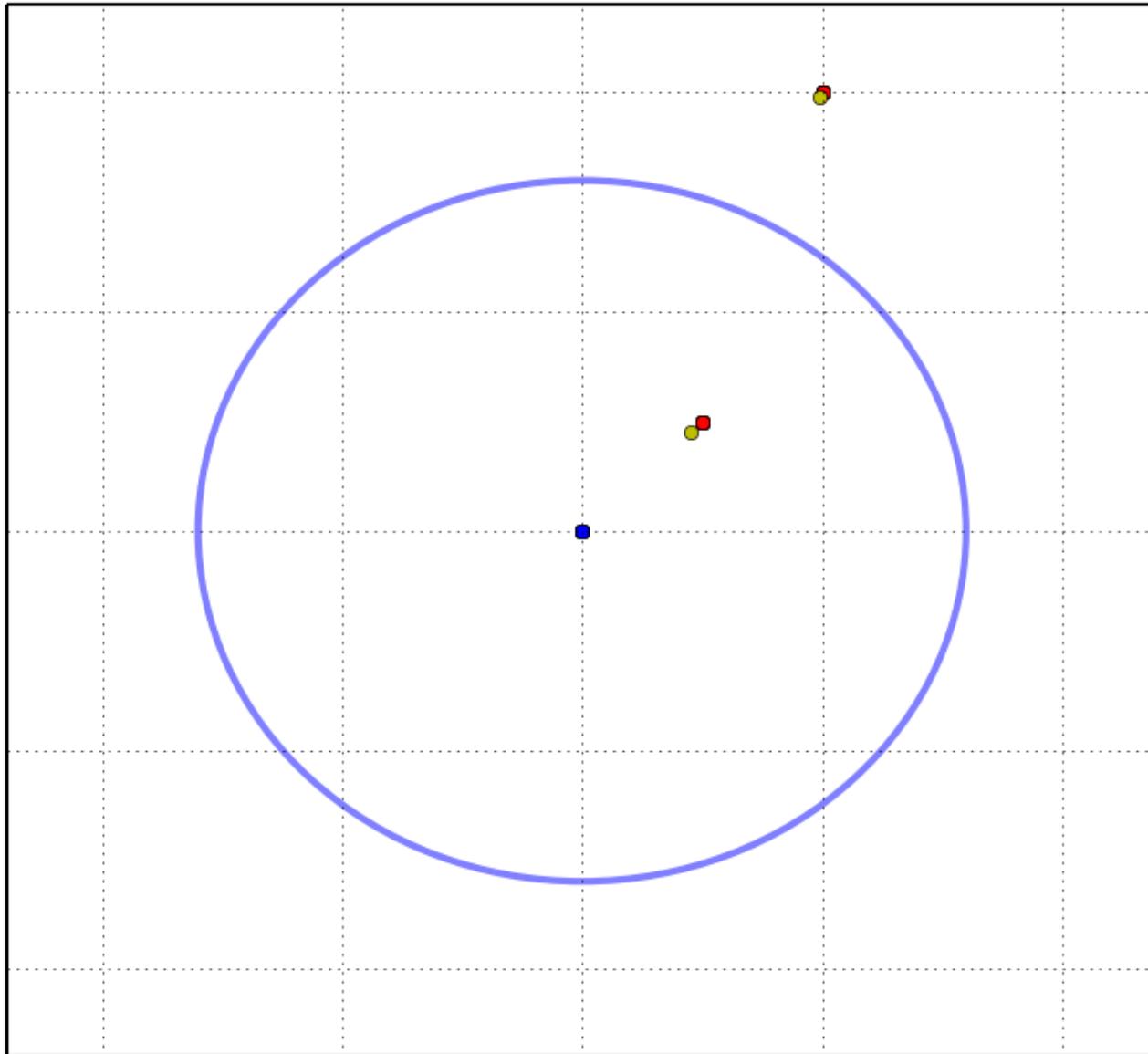
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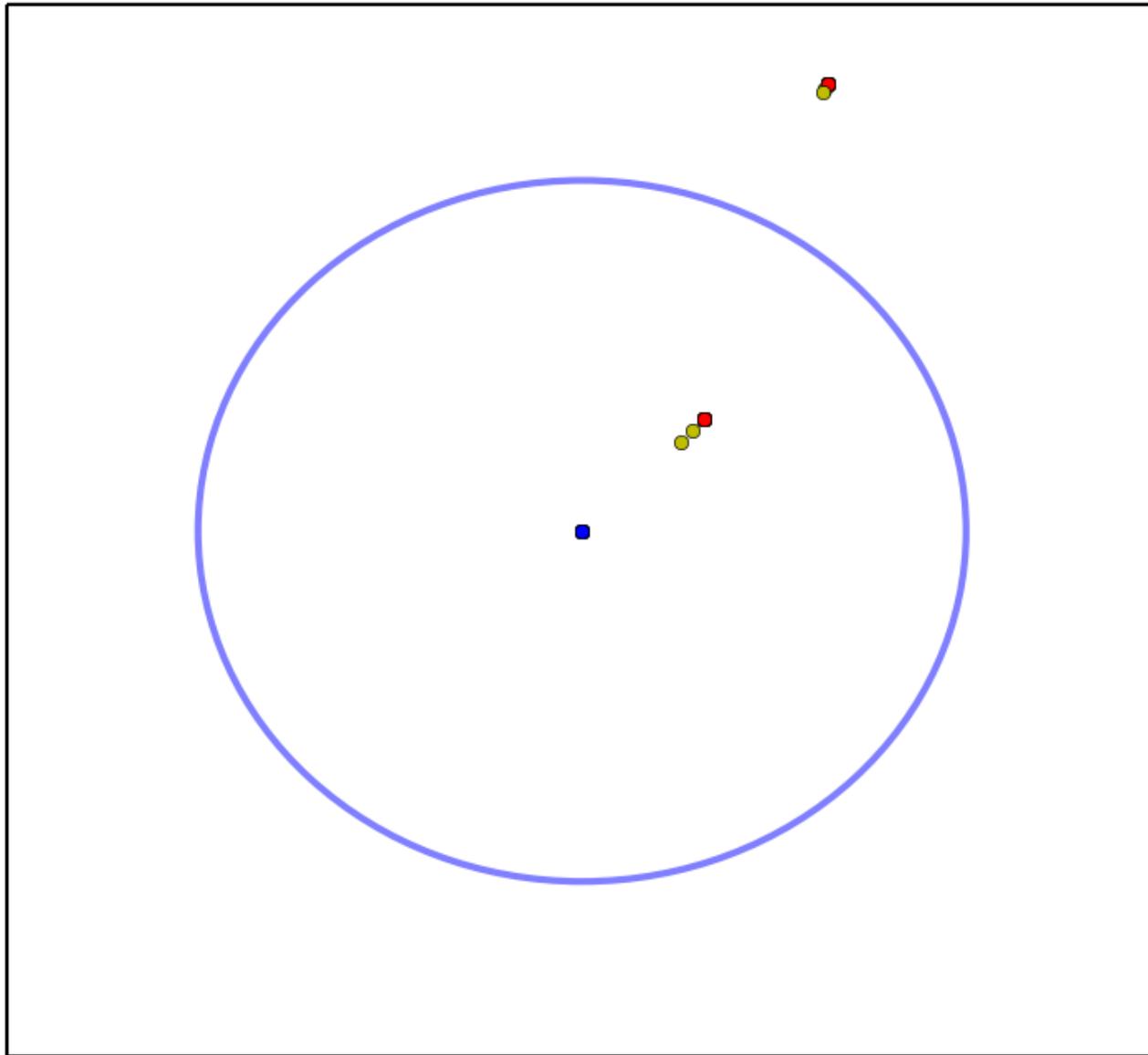
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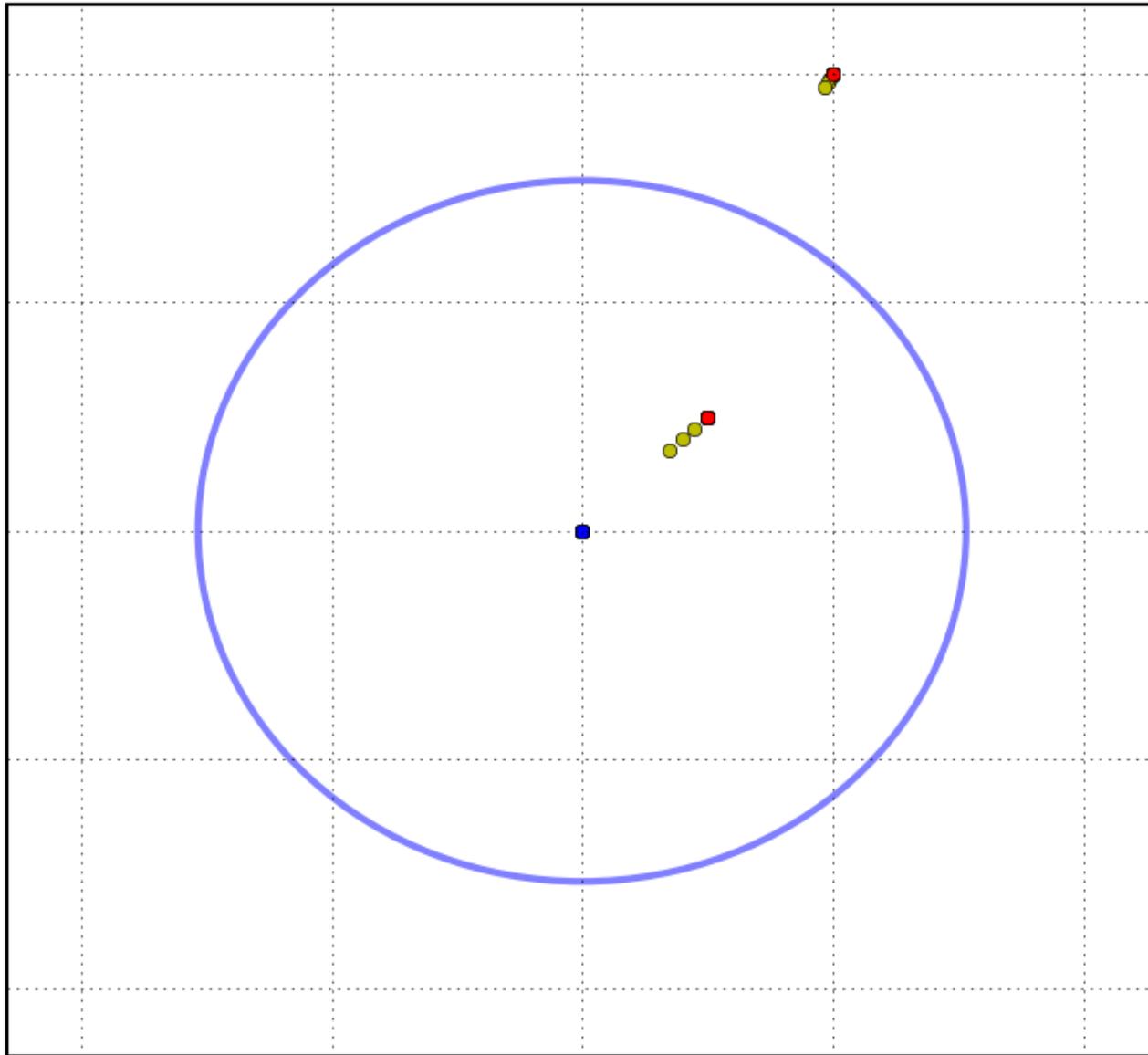
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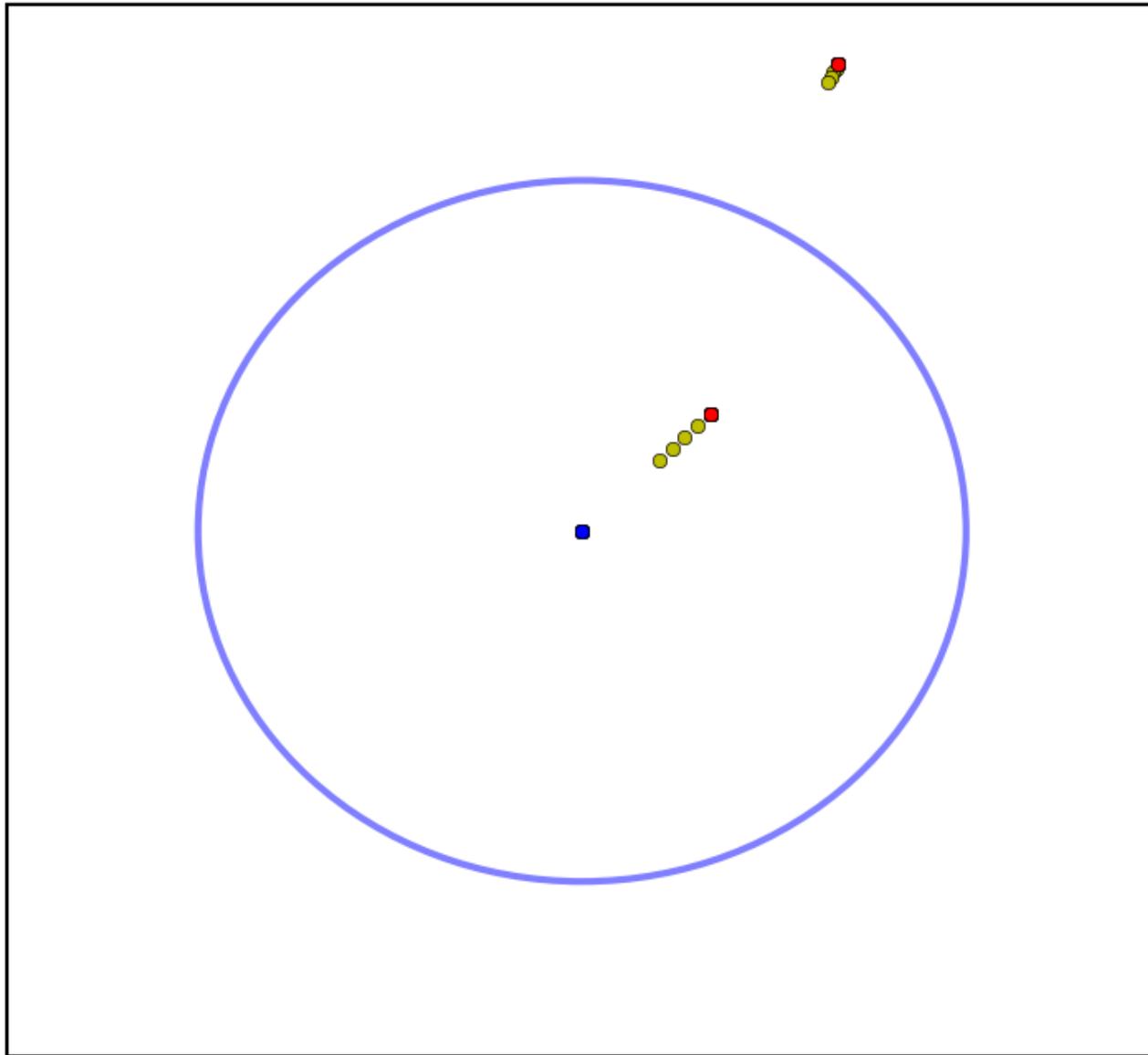
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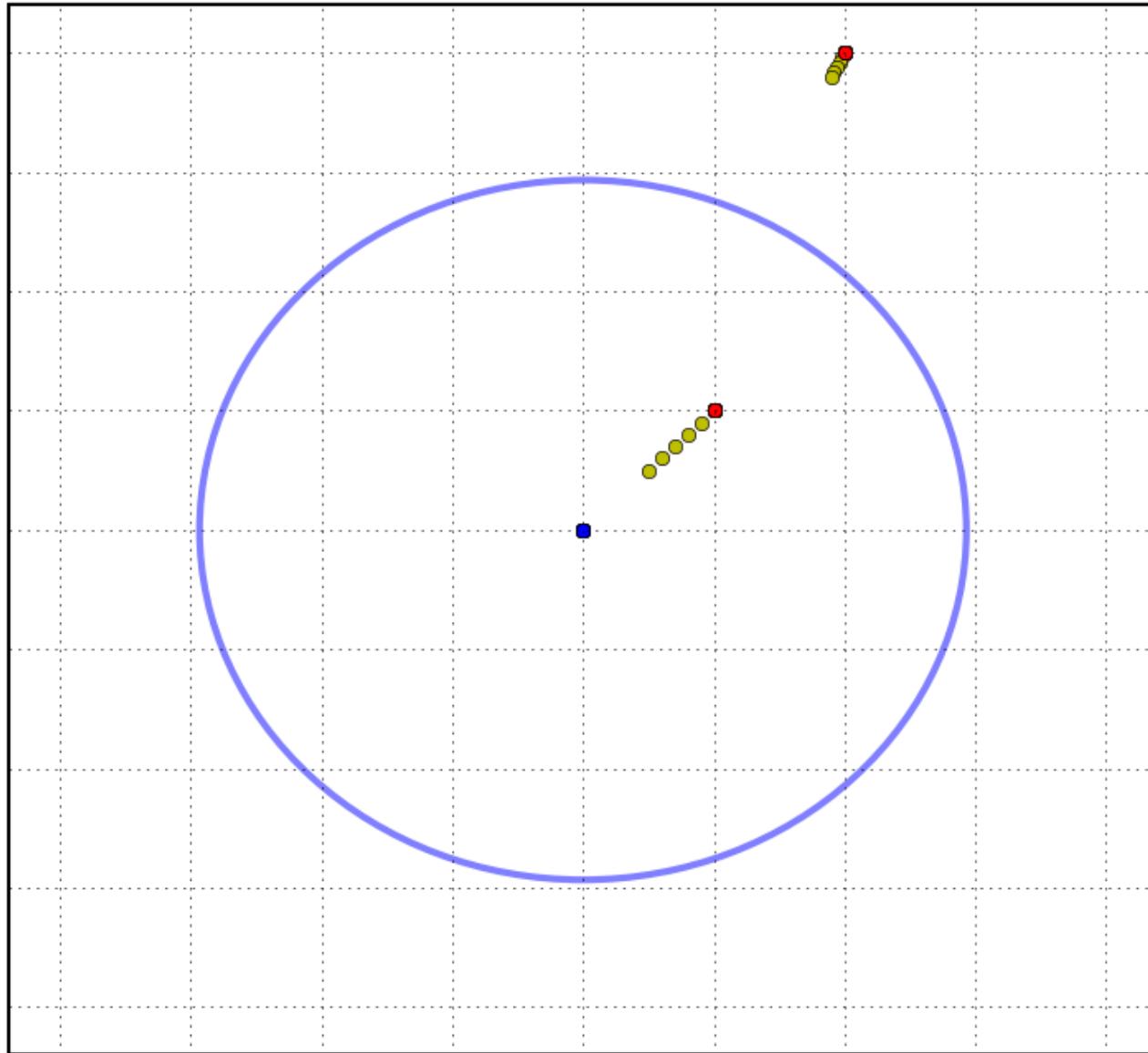
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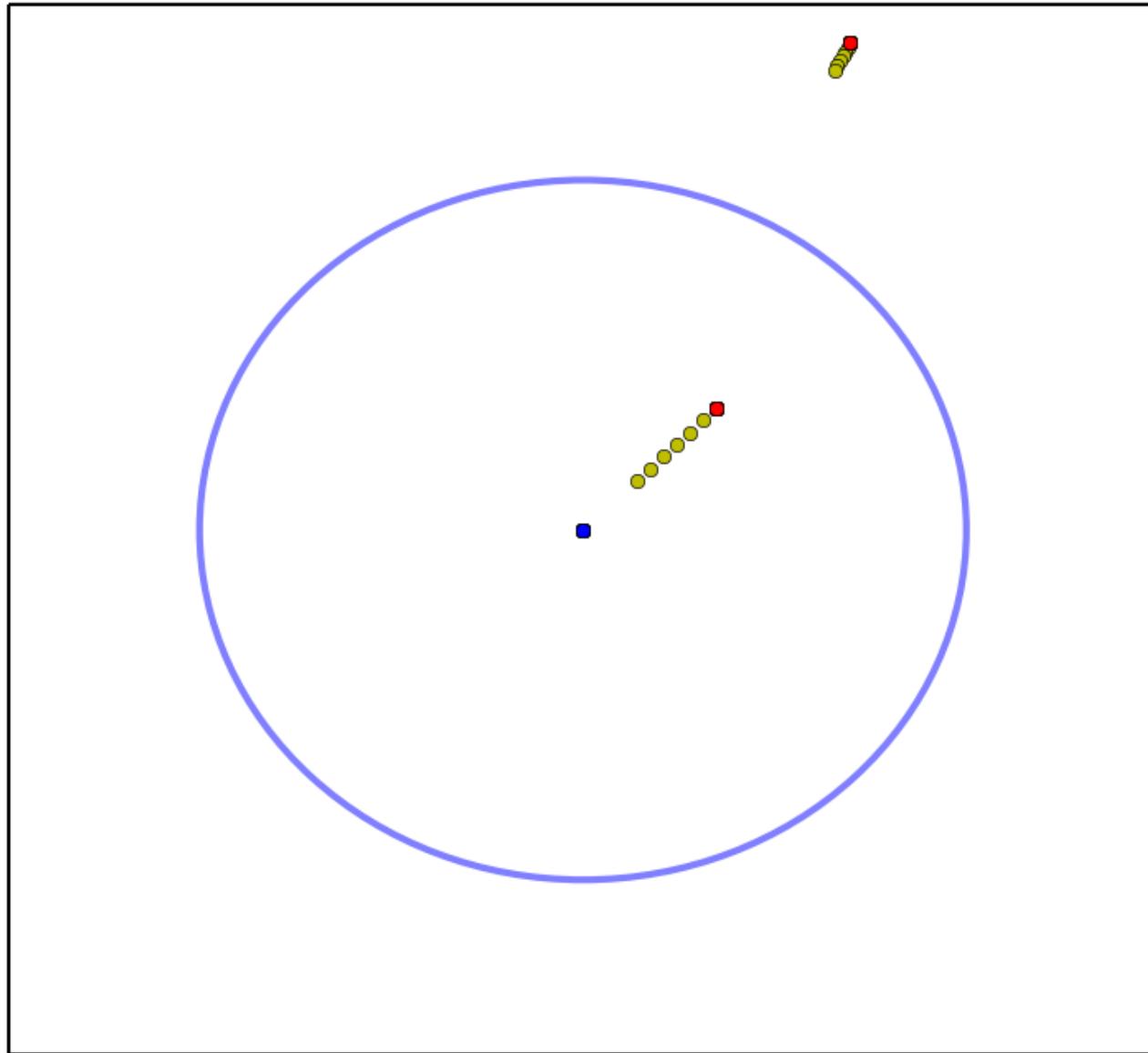
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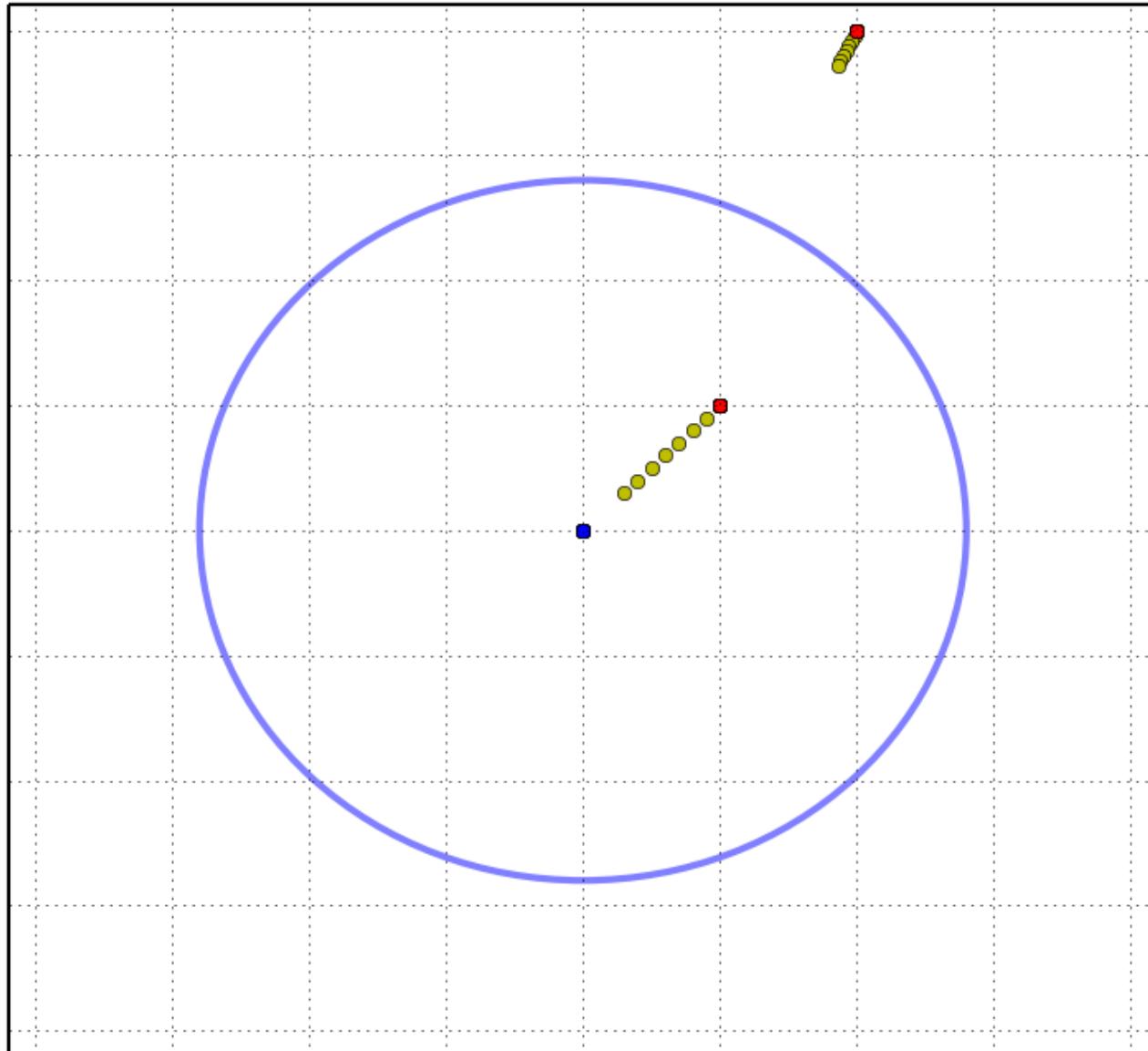
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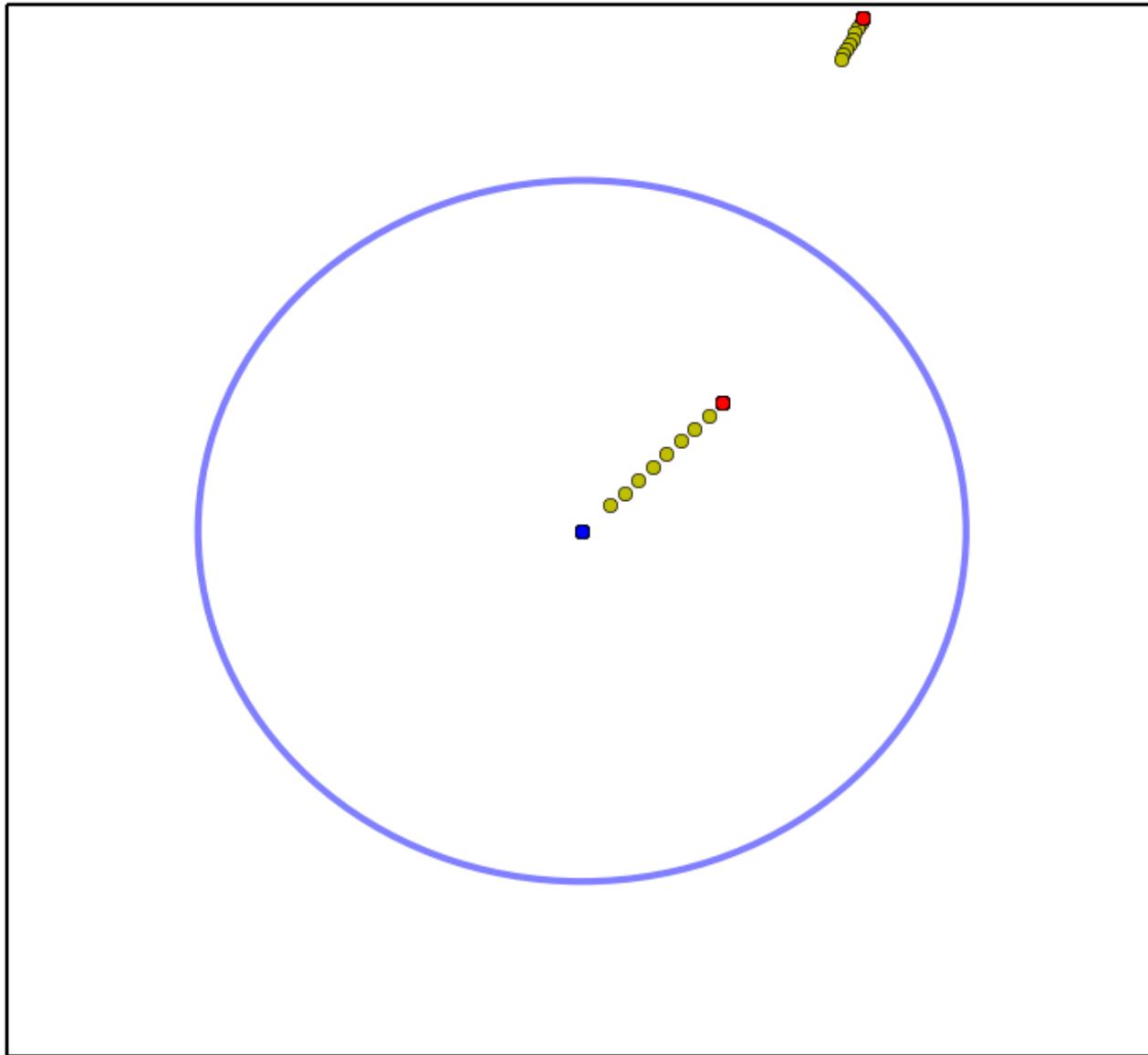
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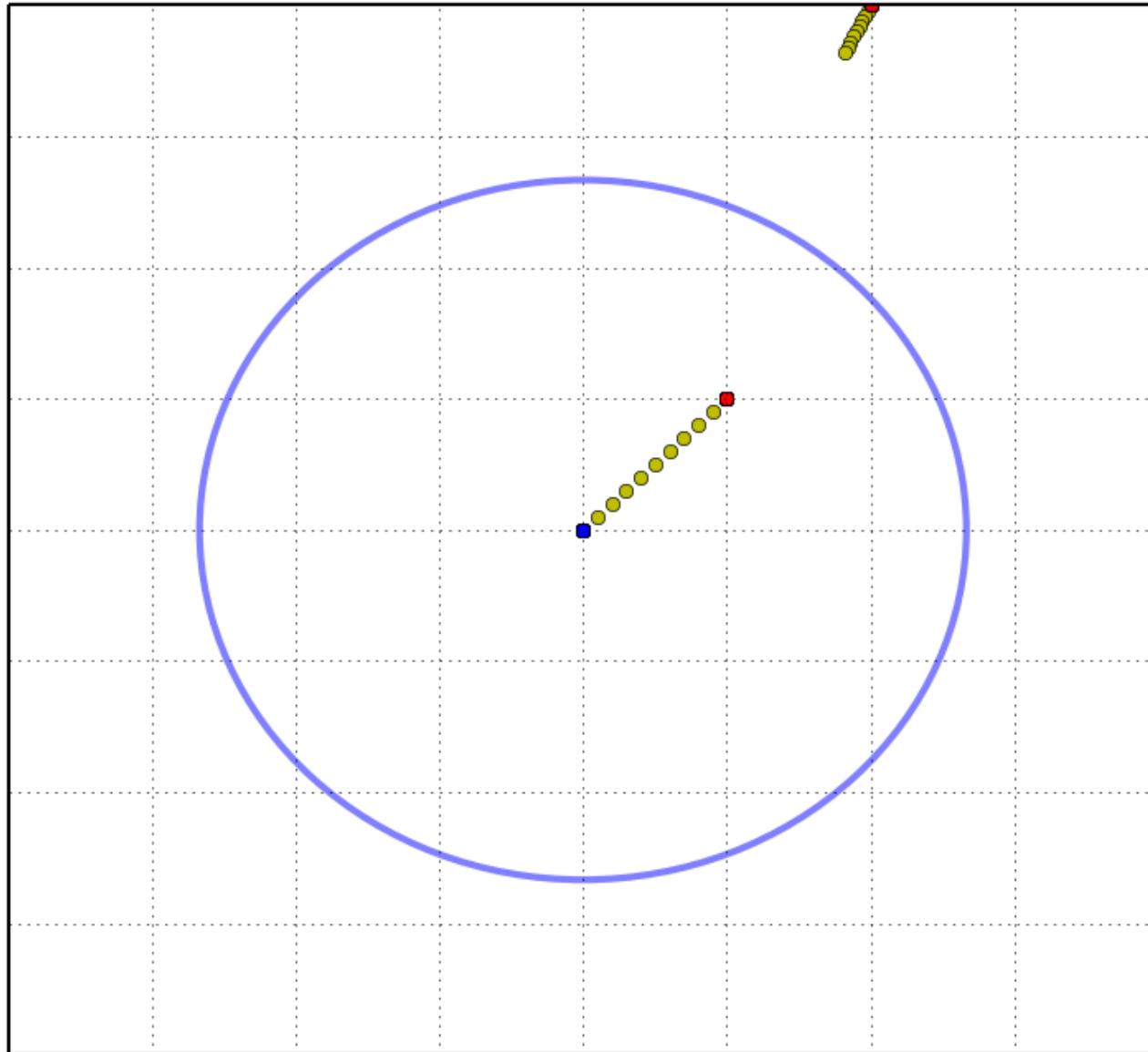
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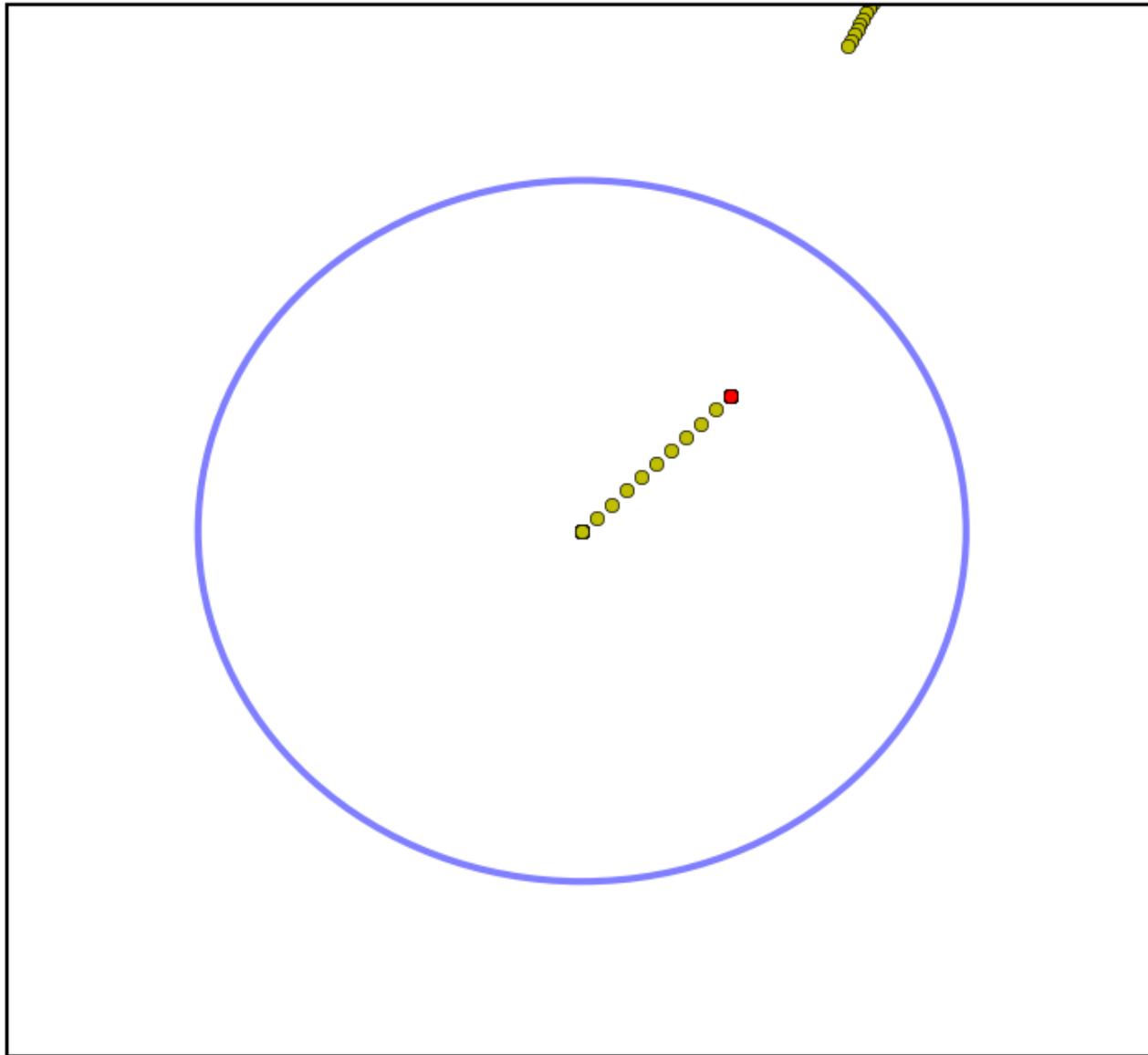
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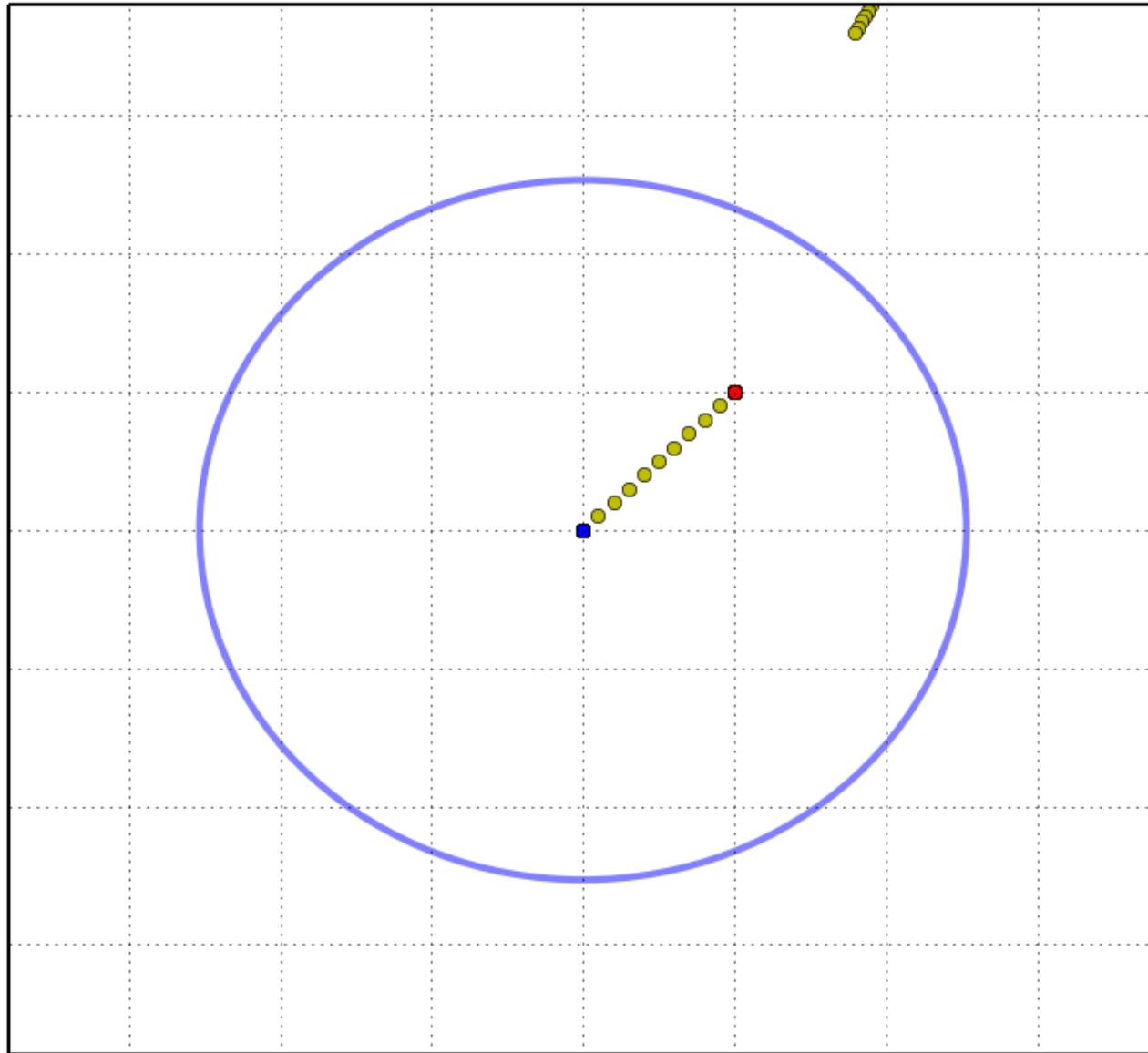
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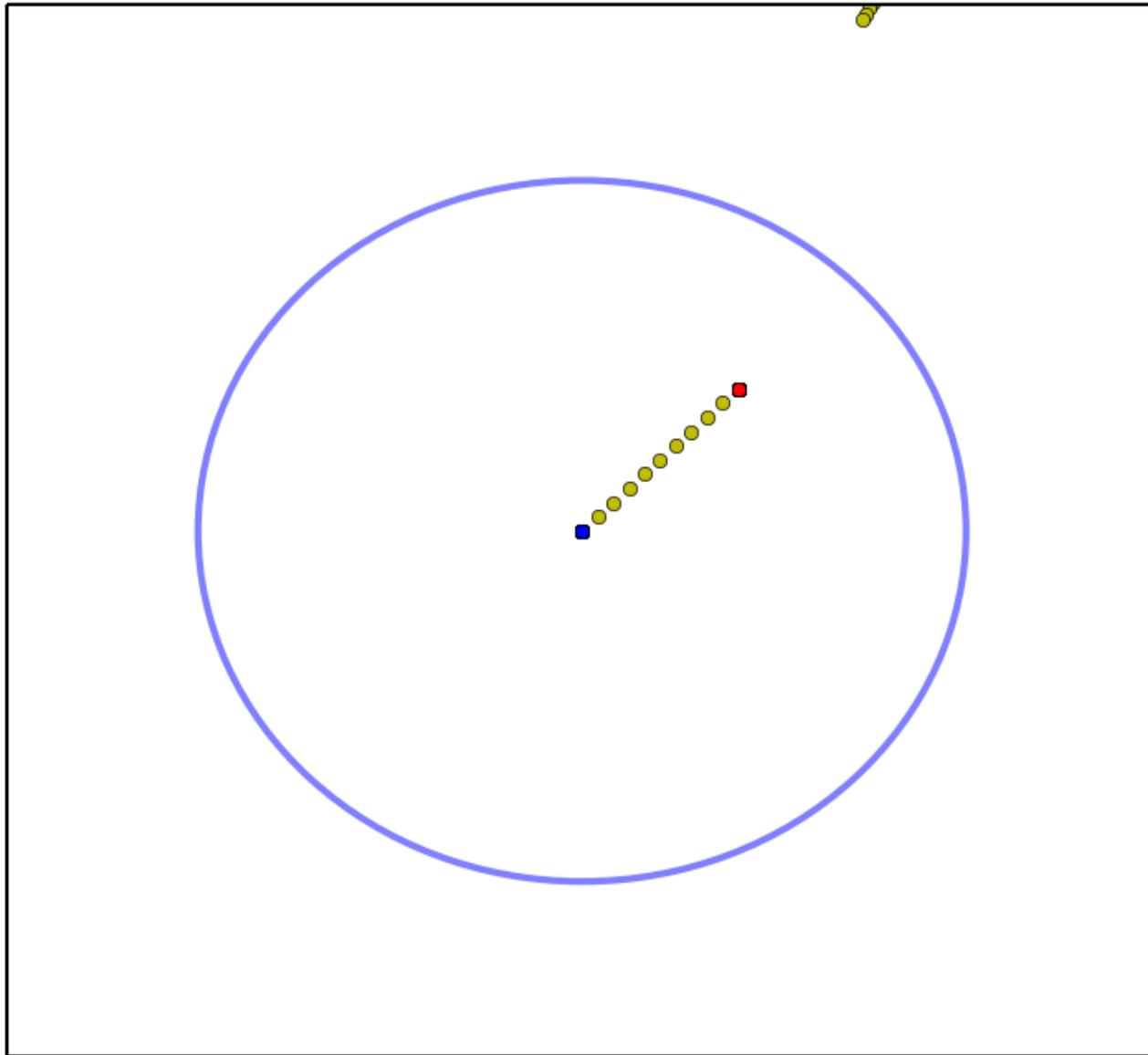
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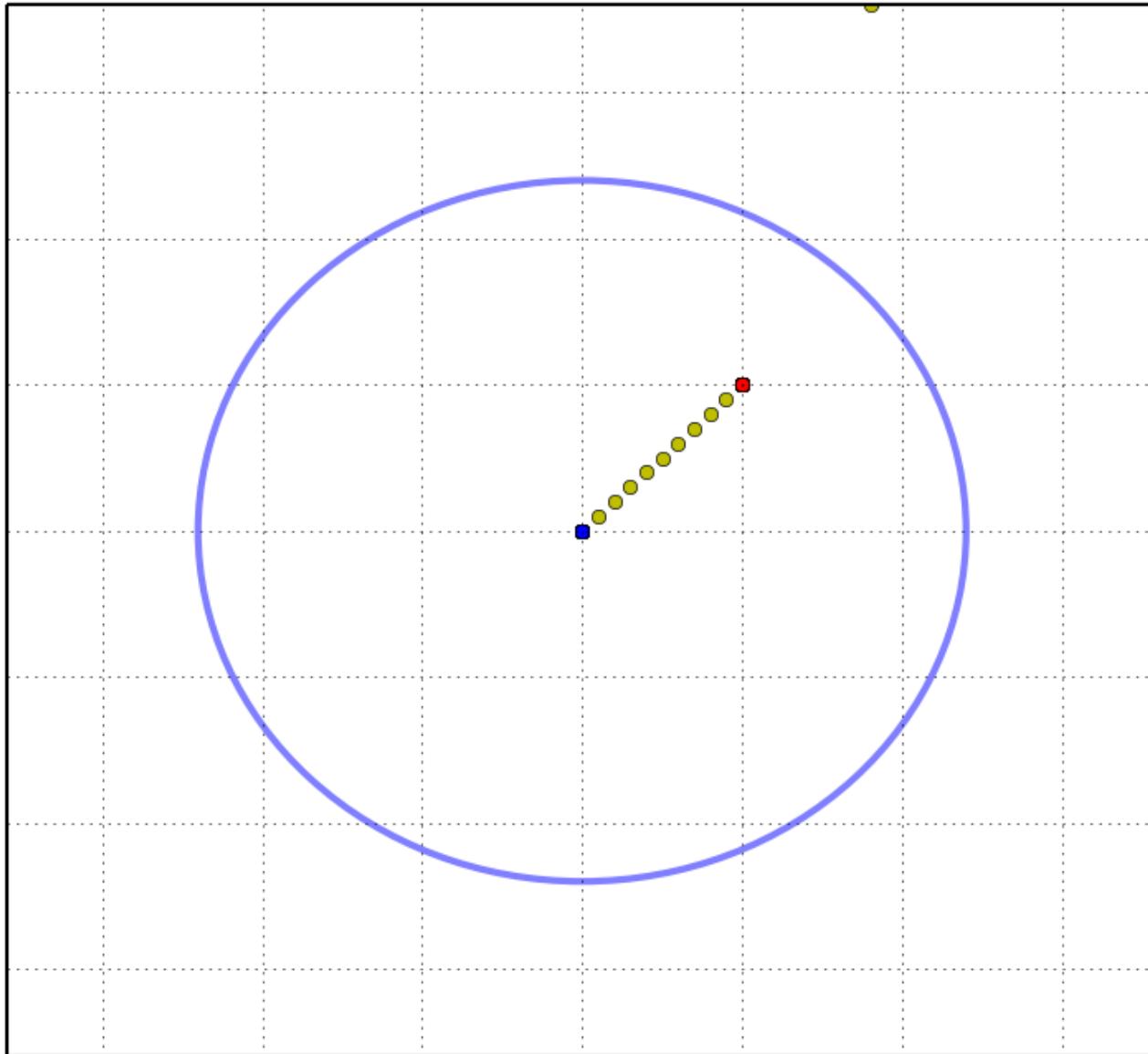
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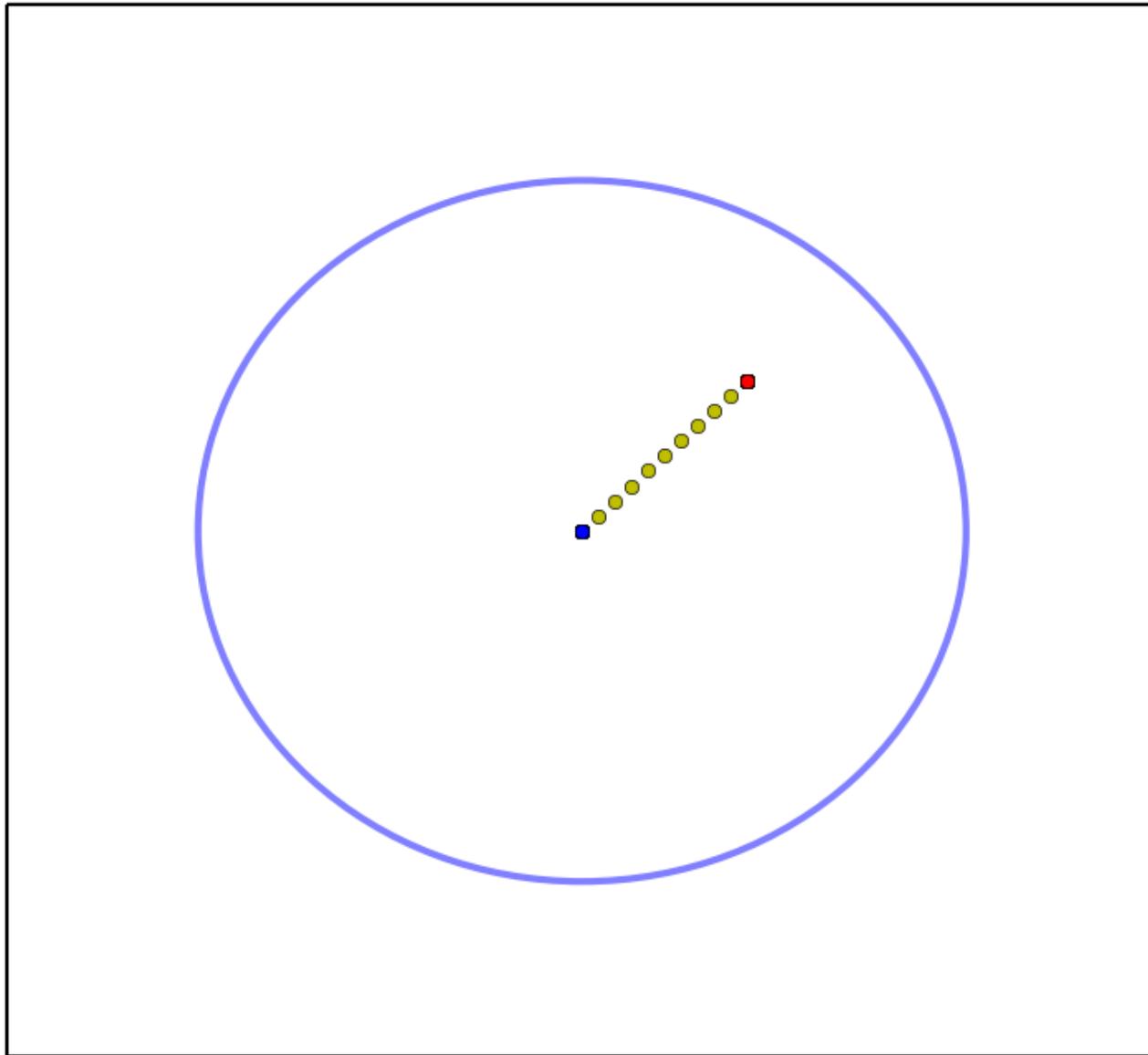
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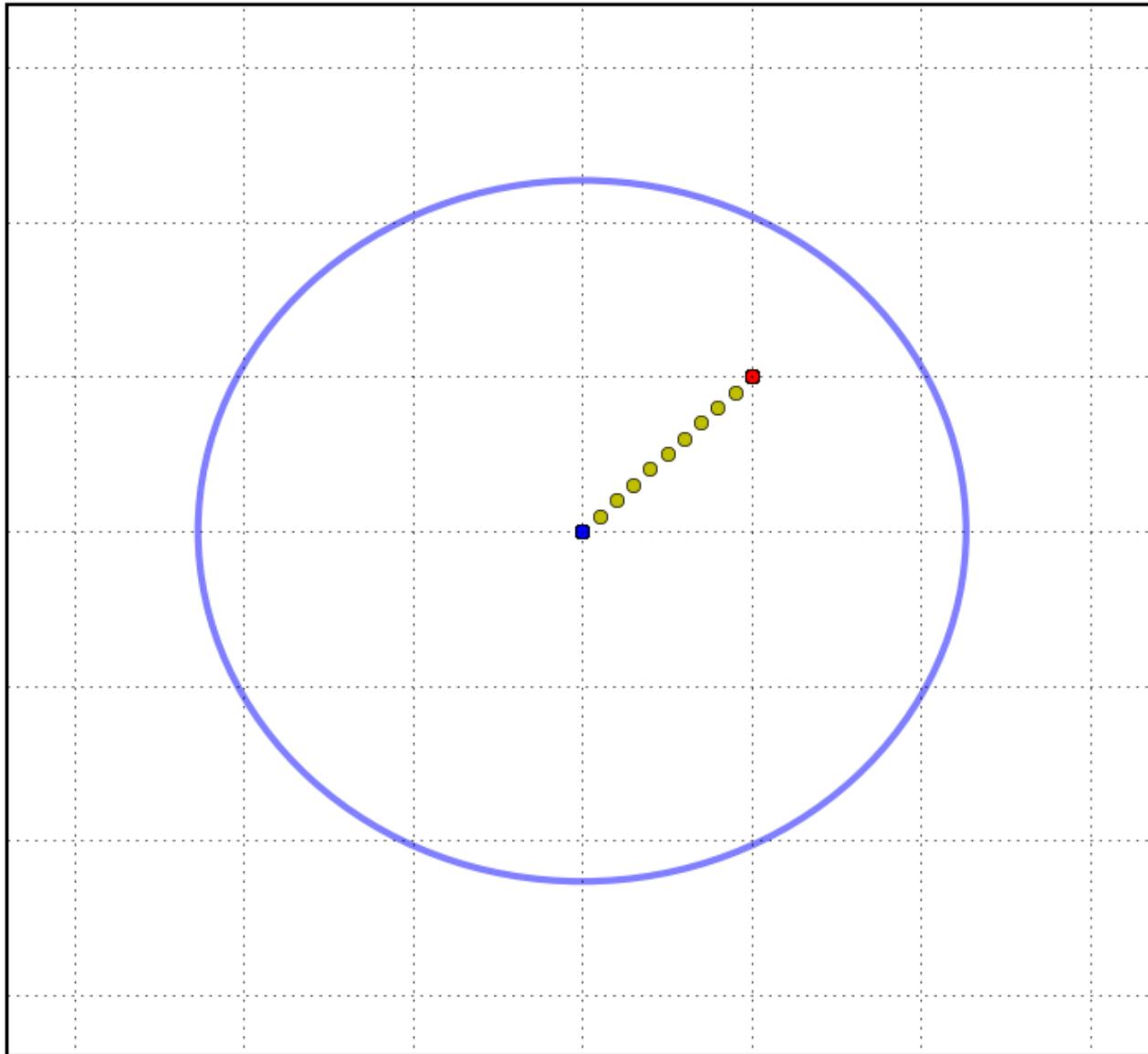
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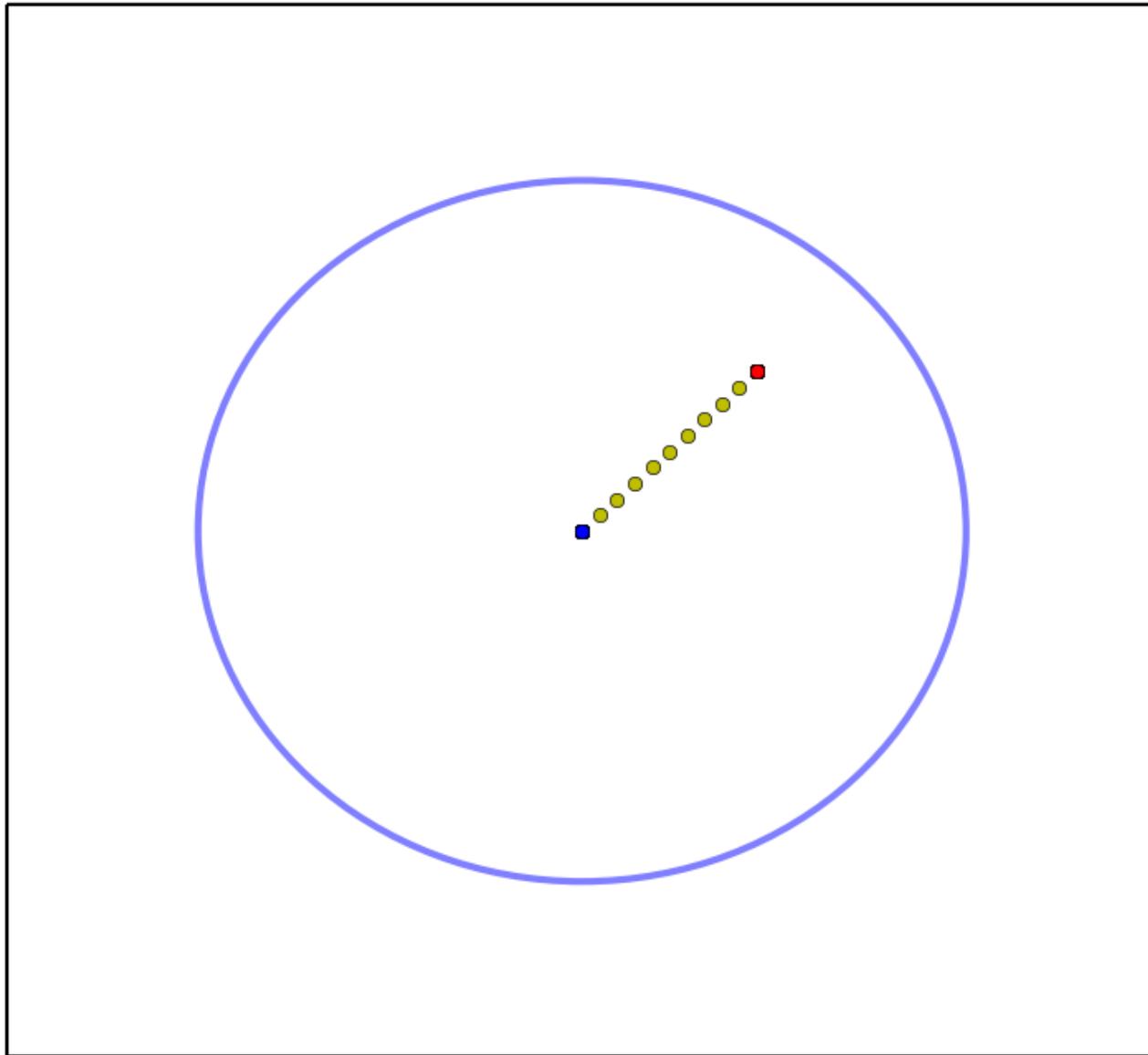
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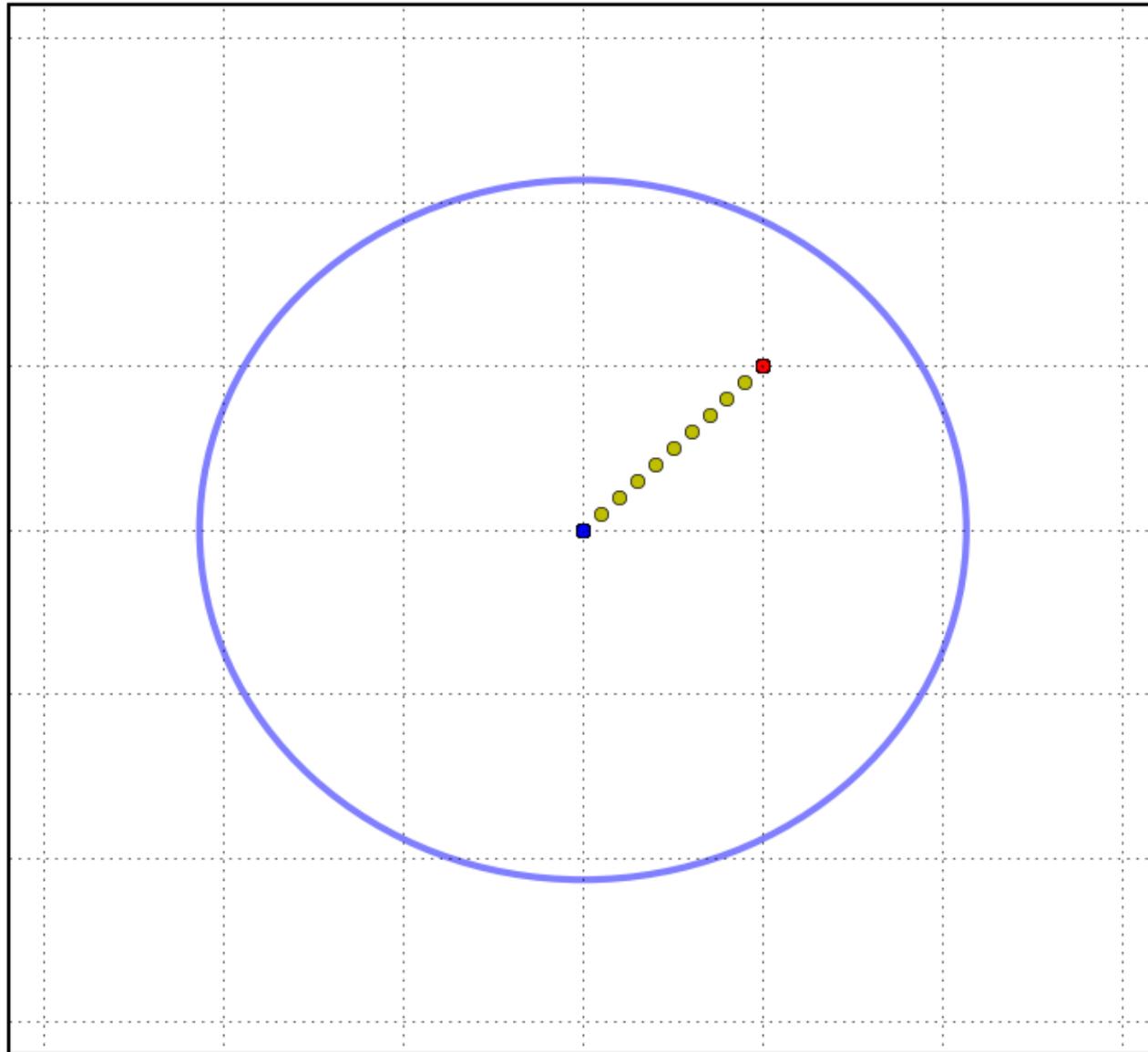
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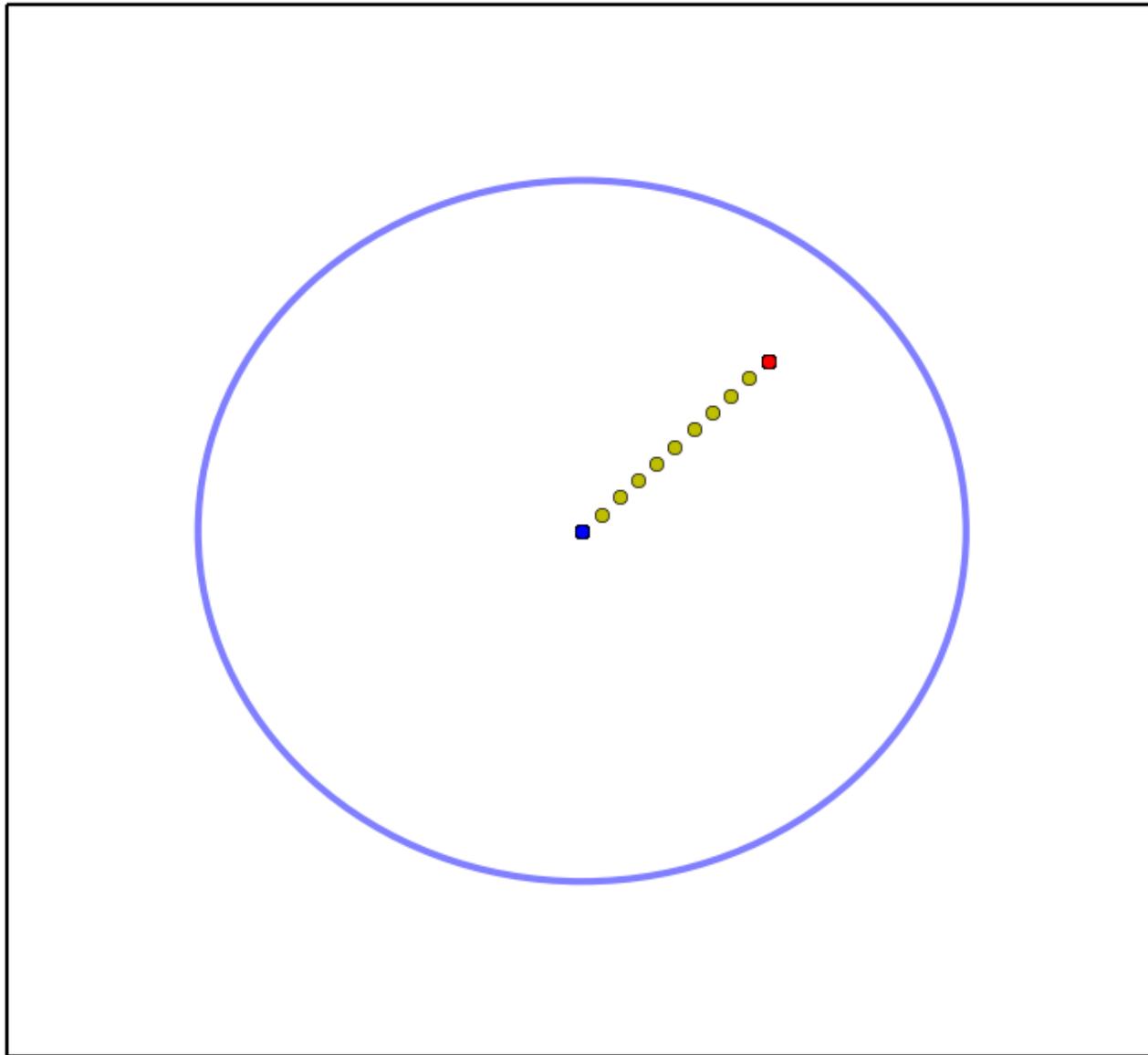
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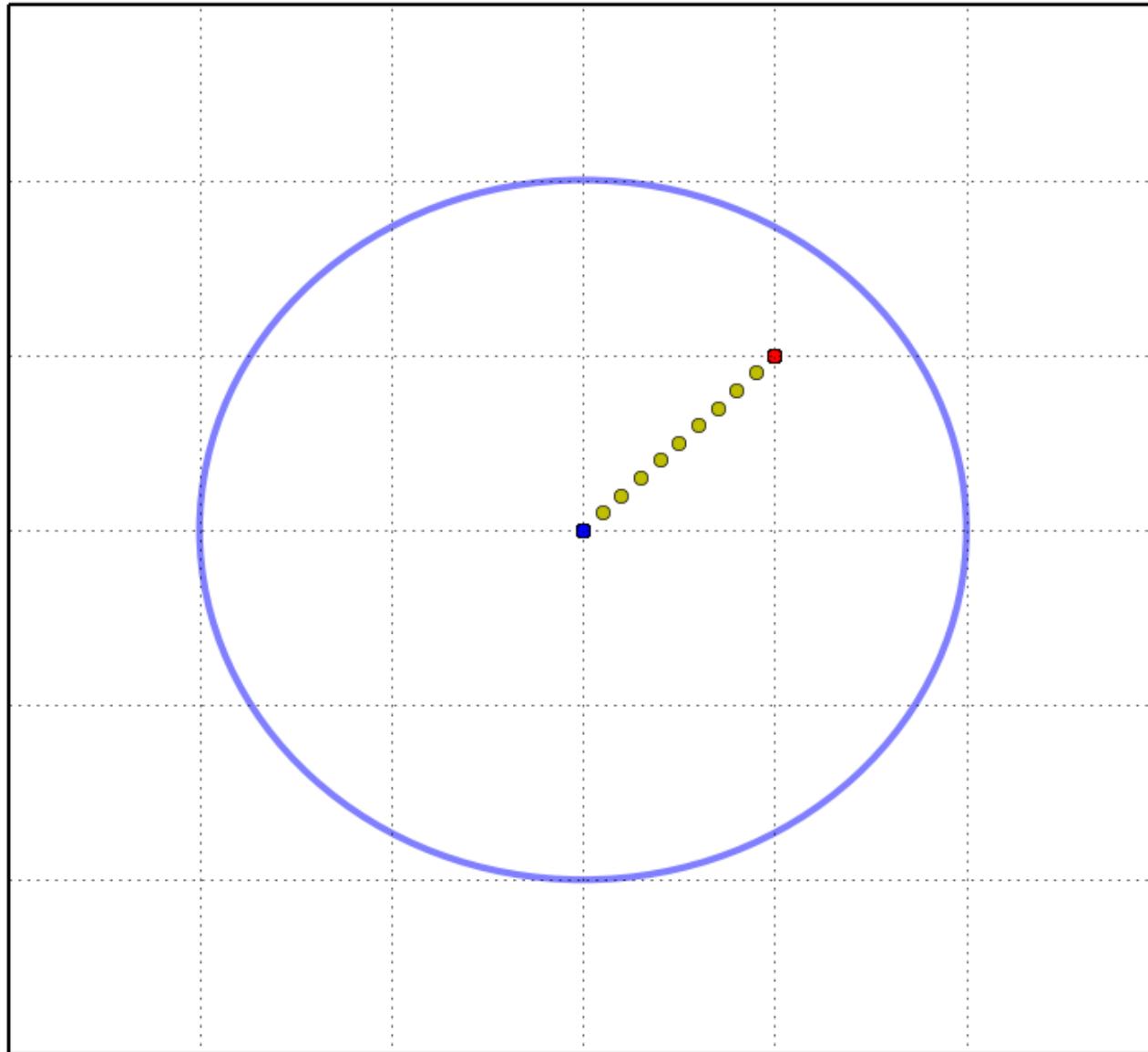
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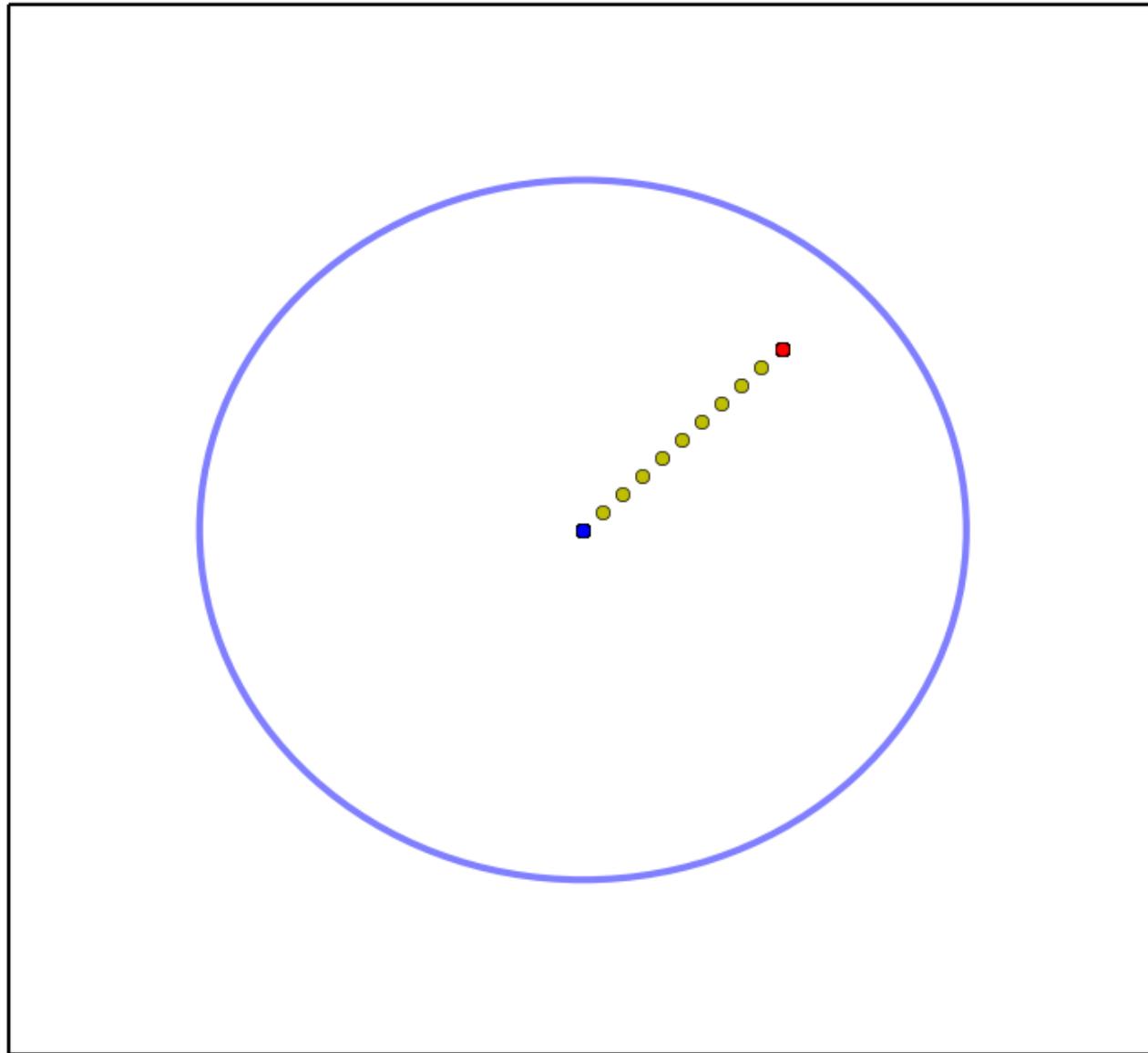
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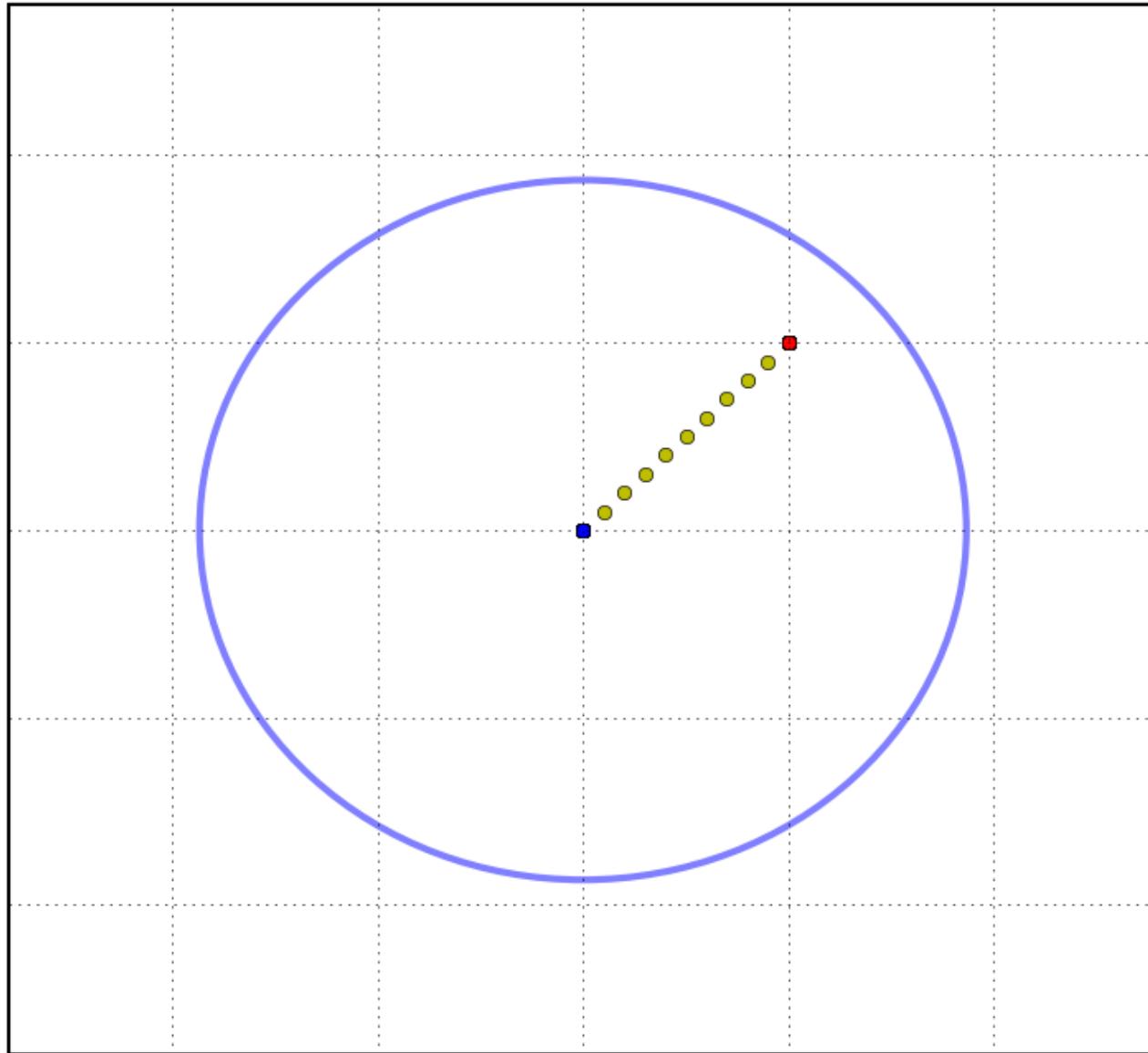
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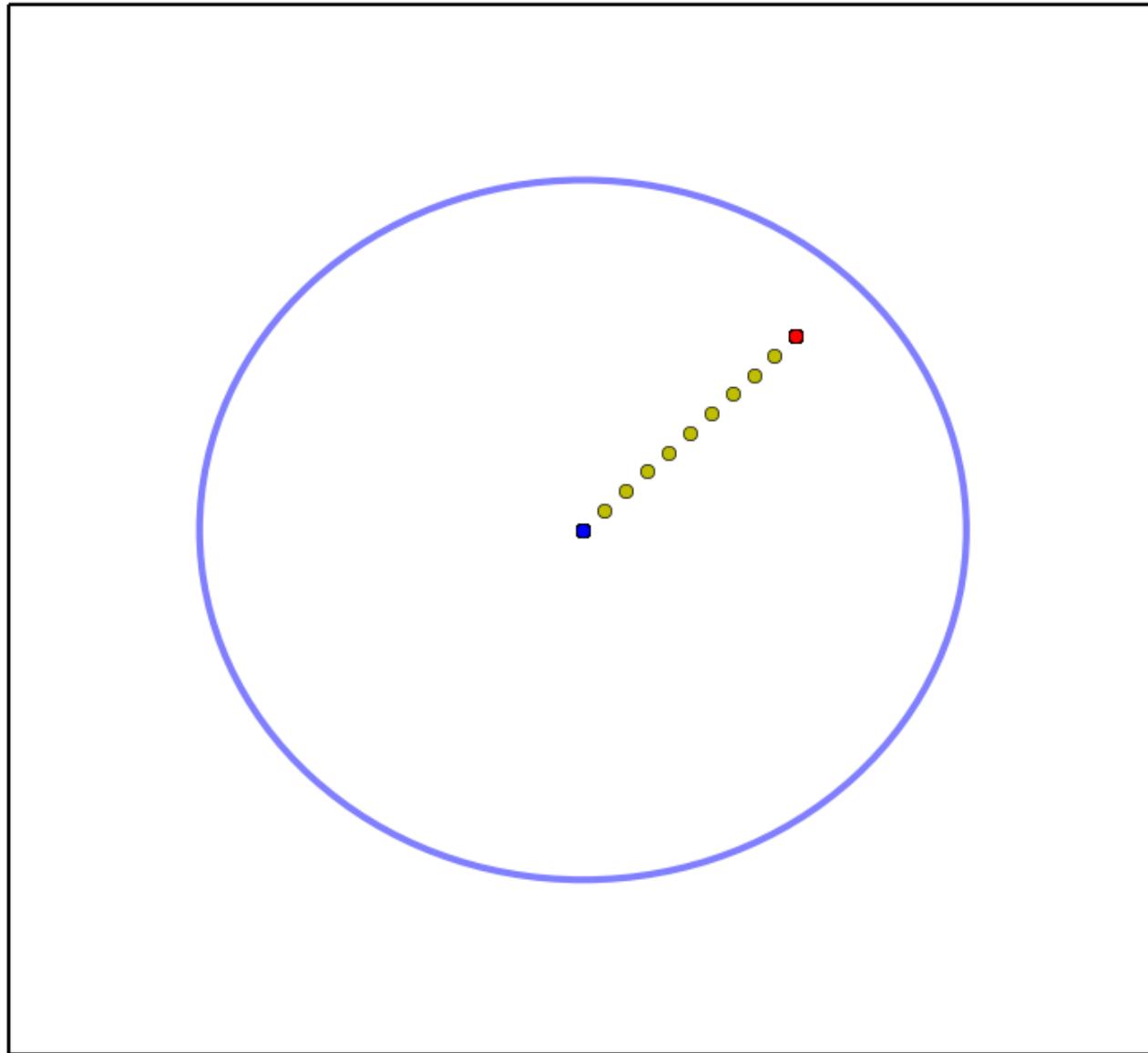
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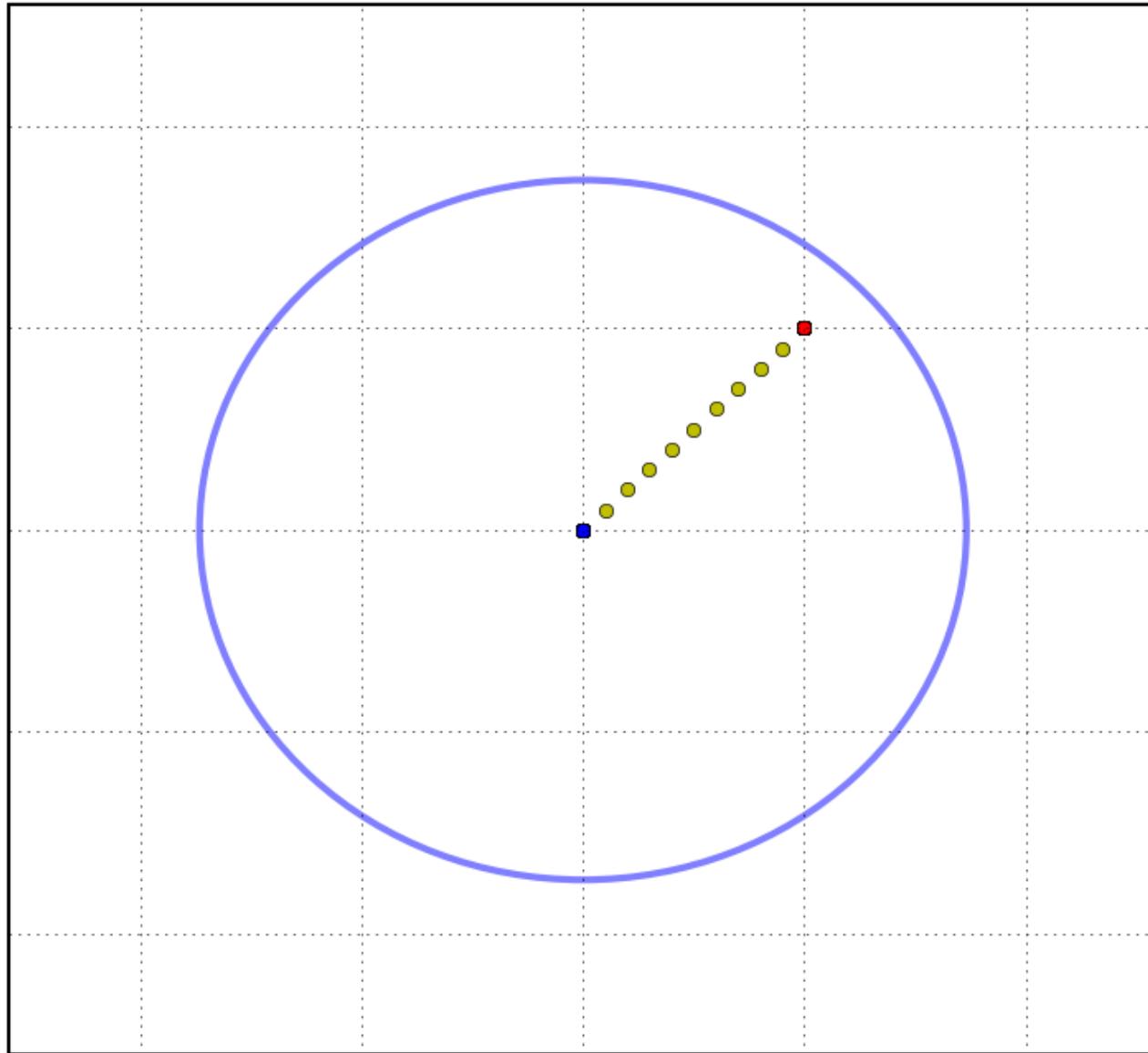
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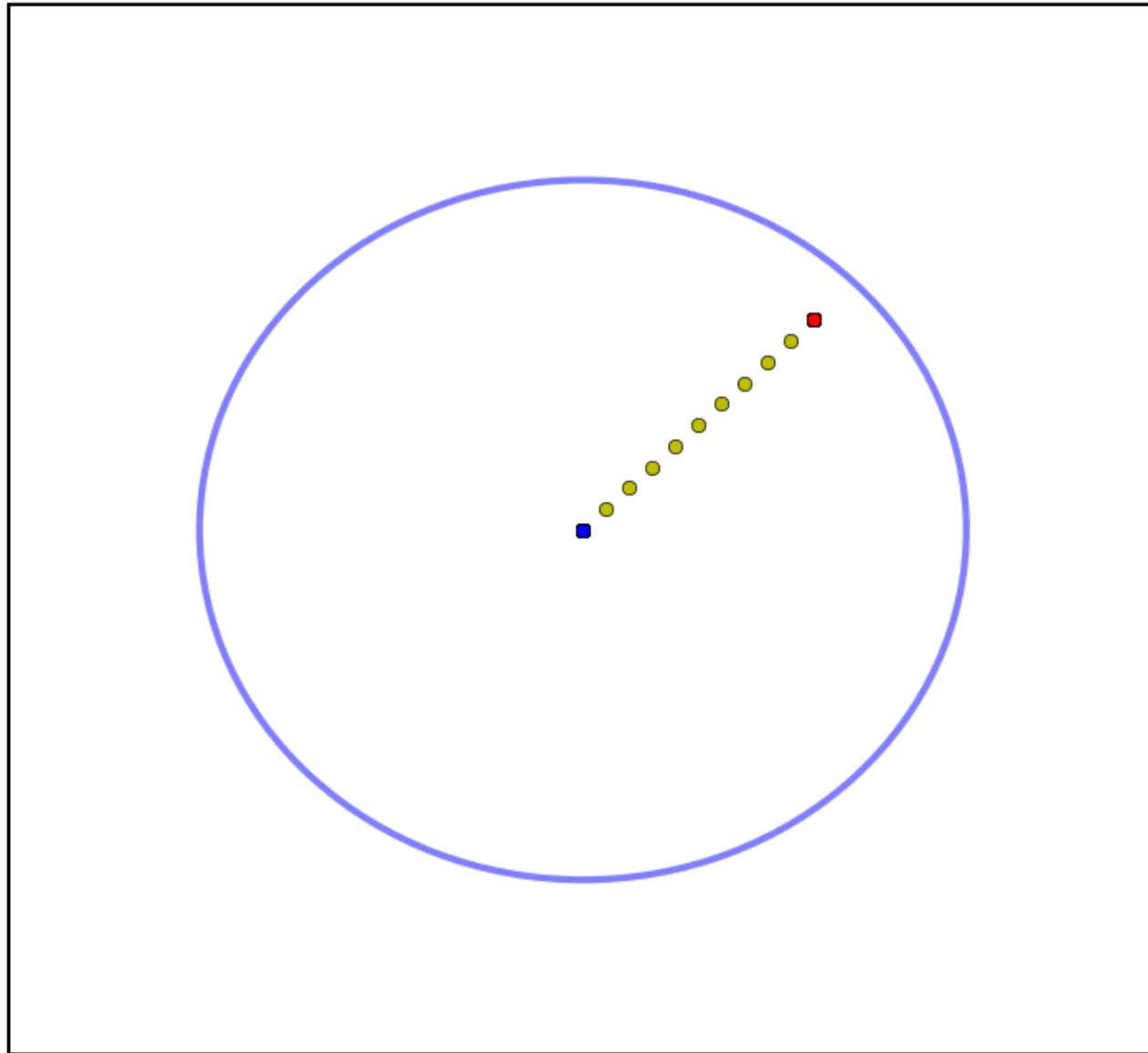
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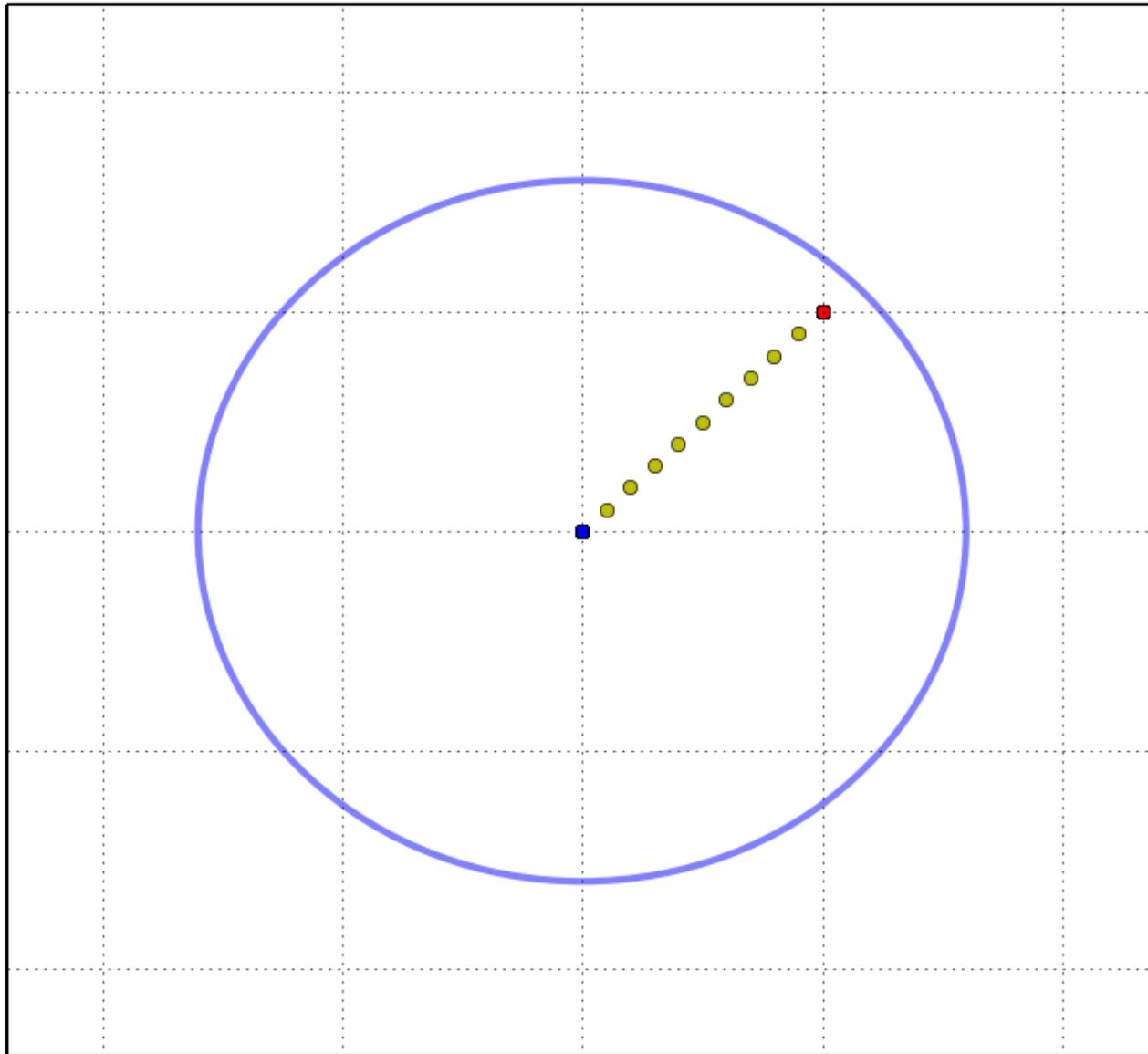
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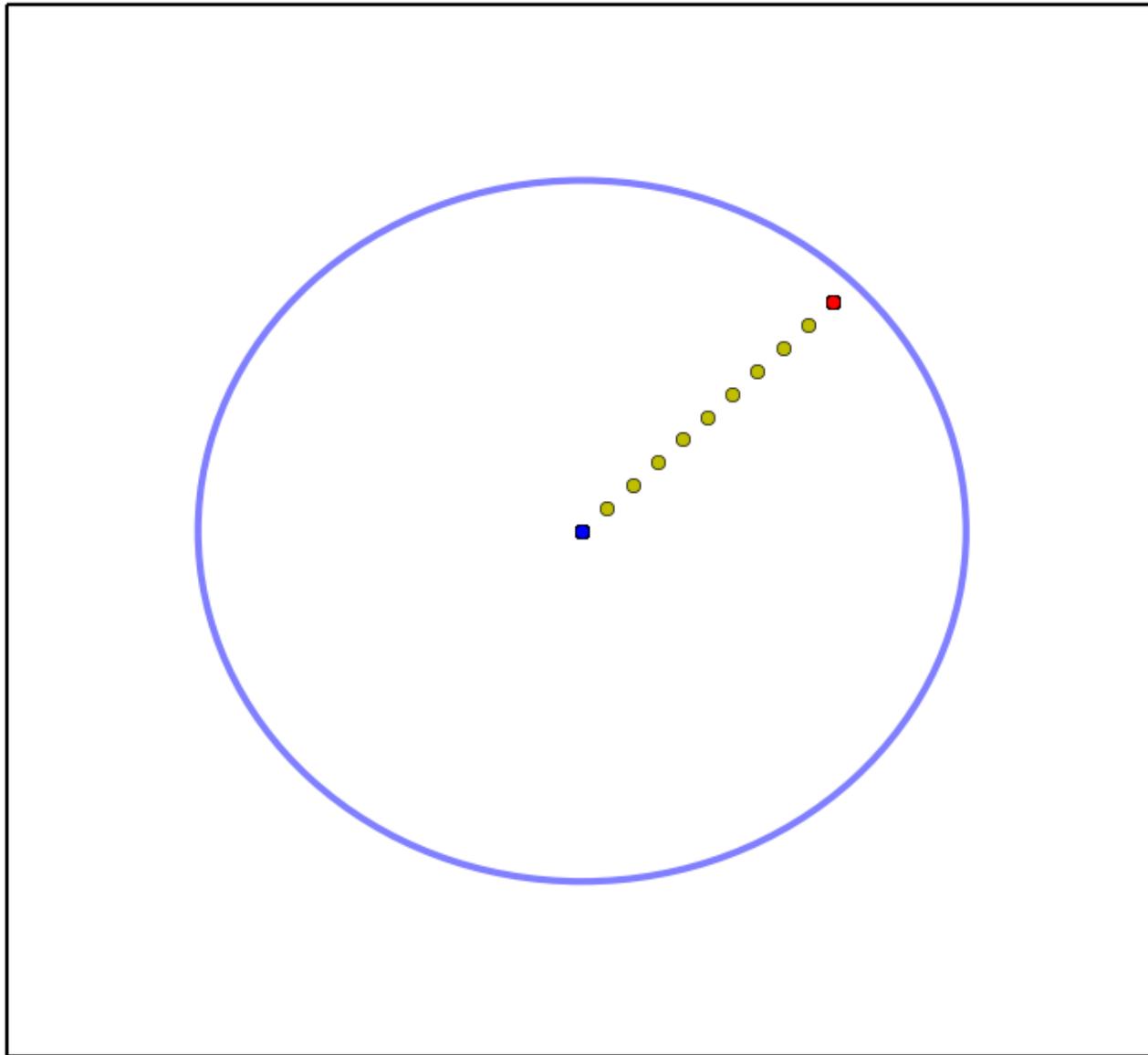
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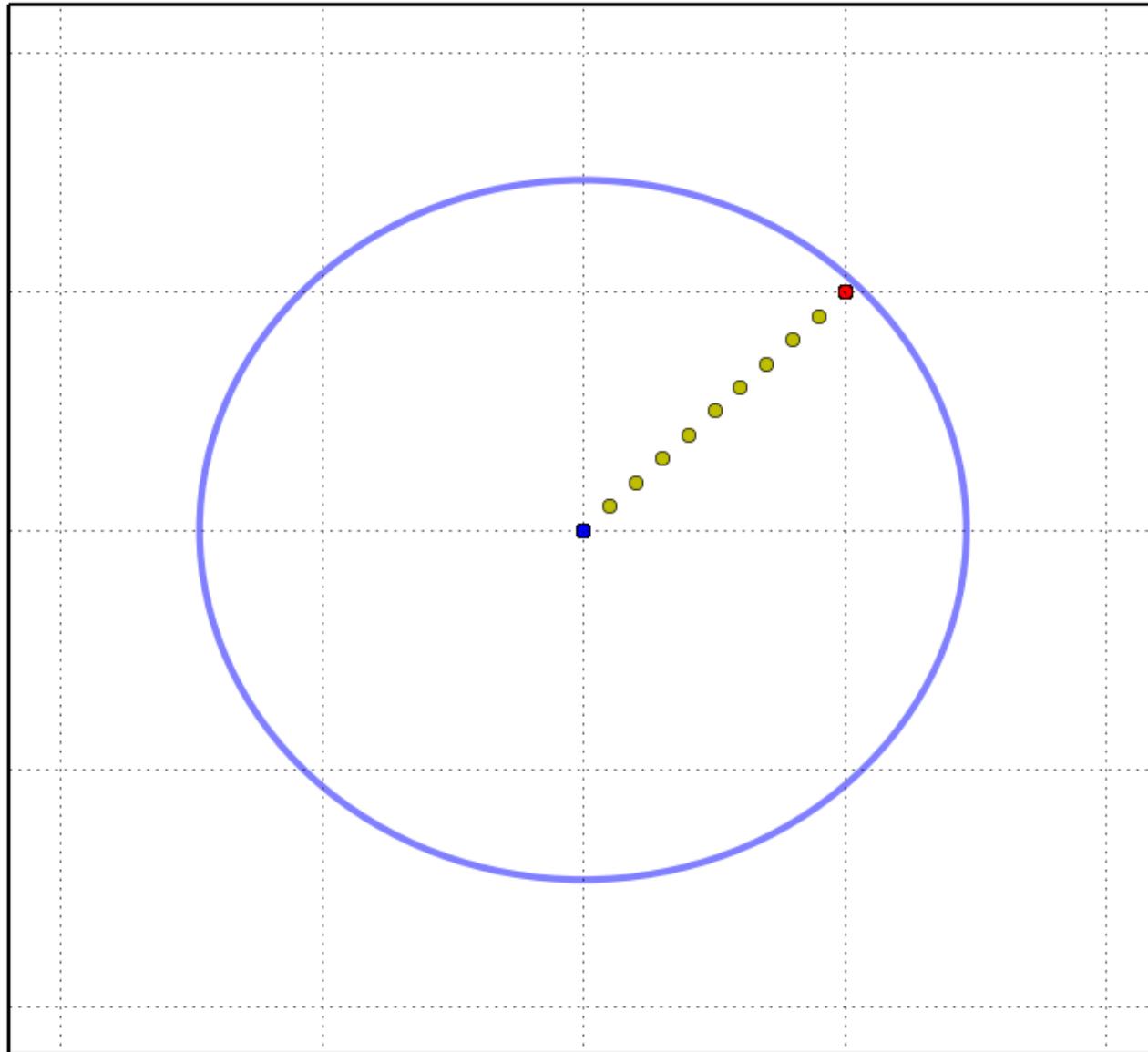
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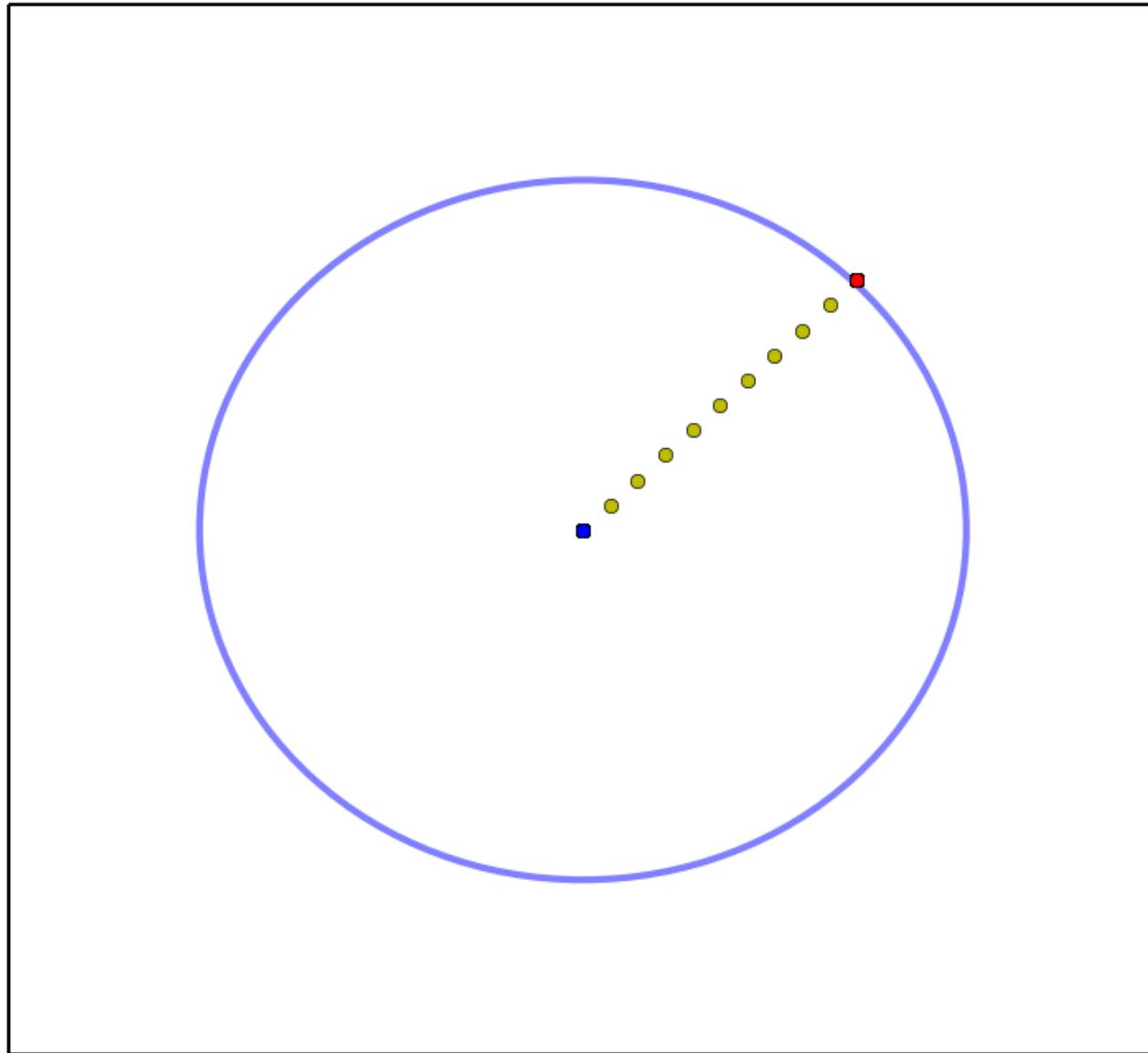
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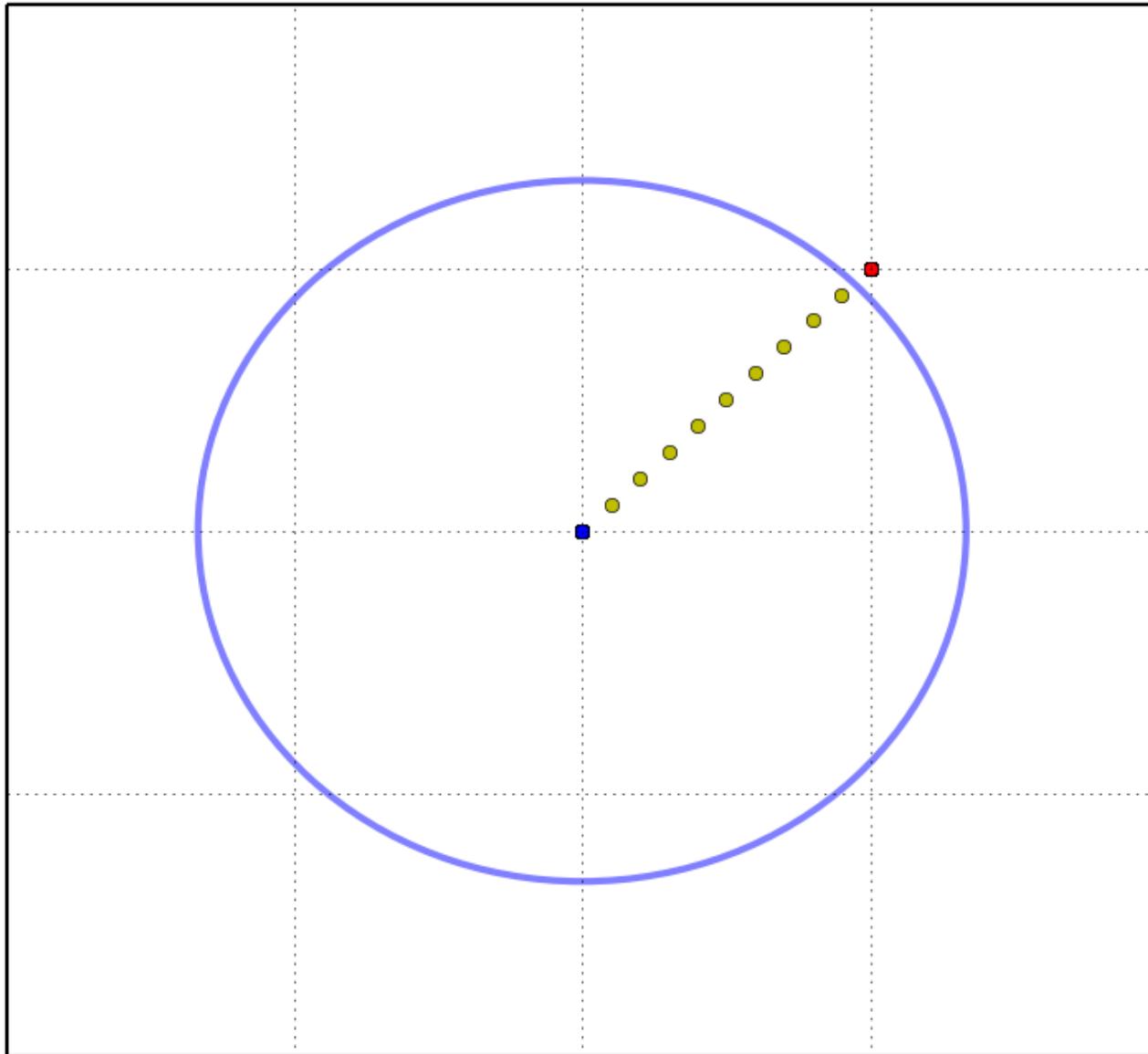
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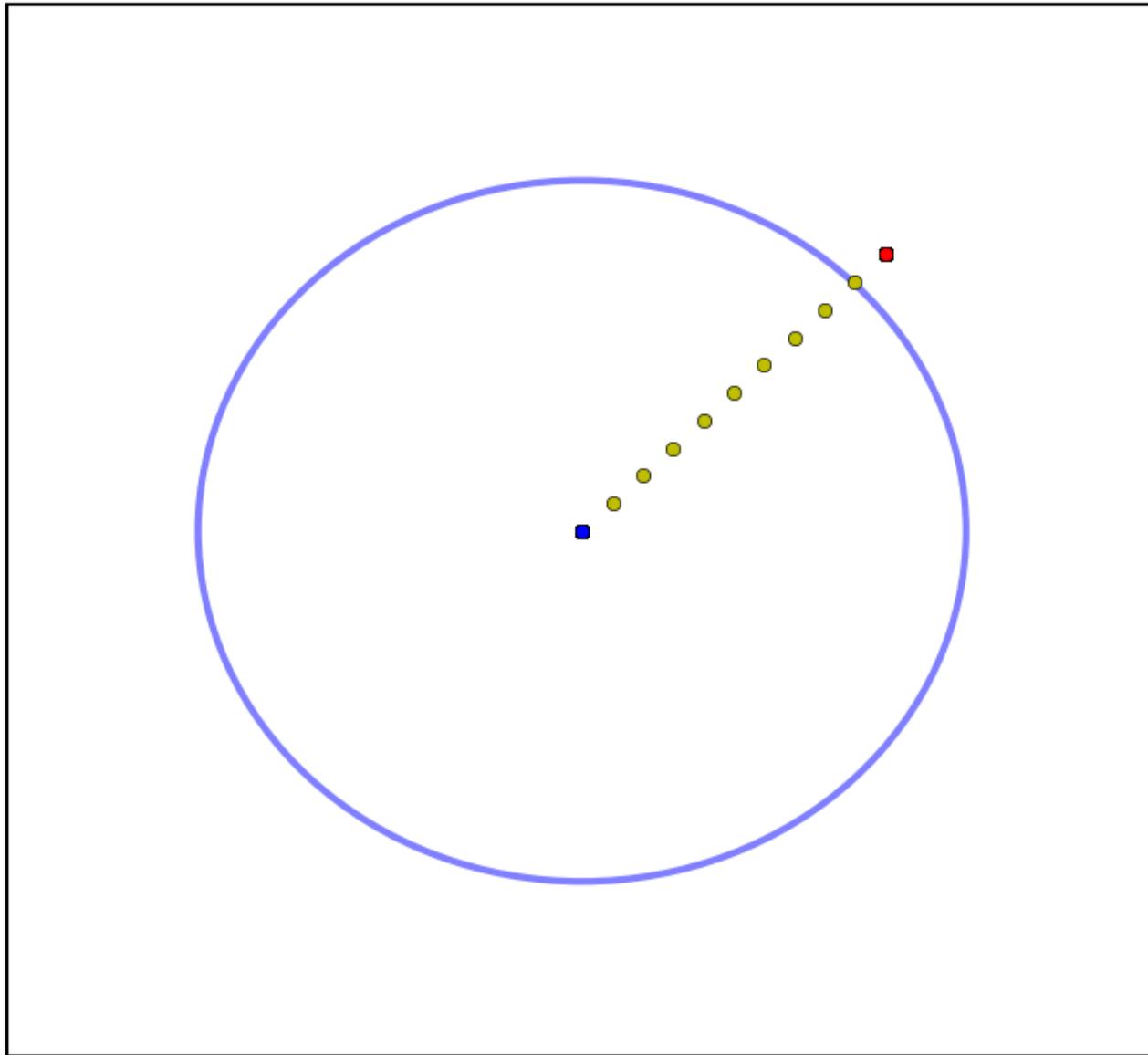
178



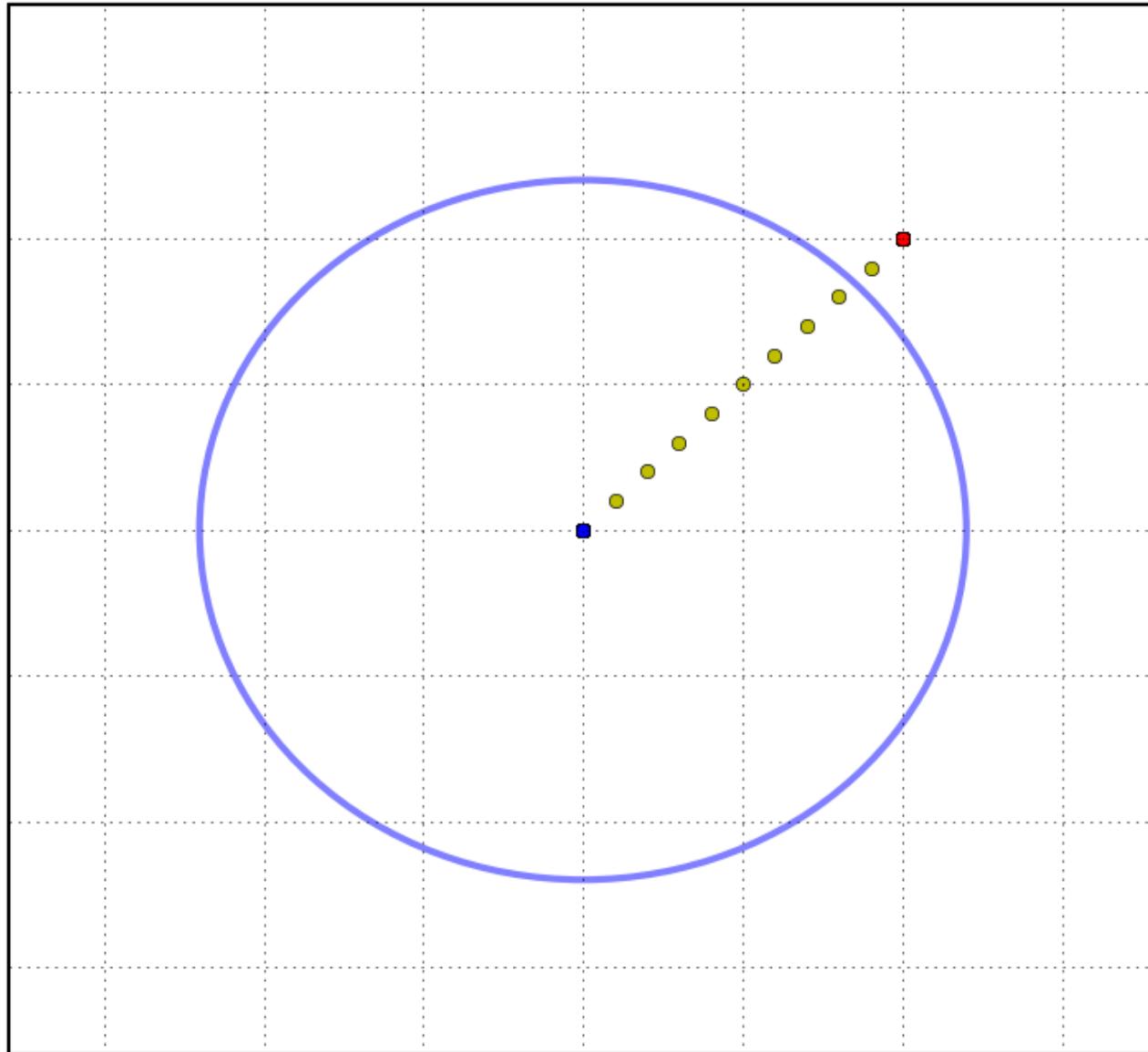
179



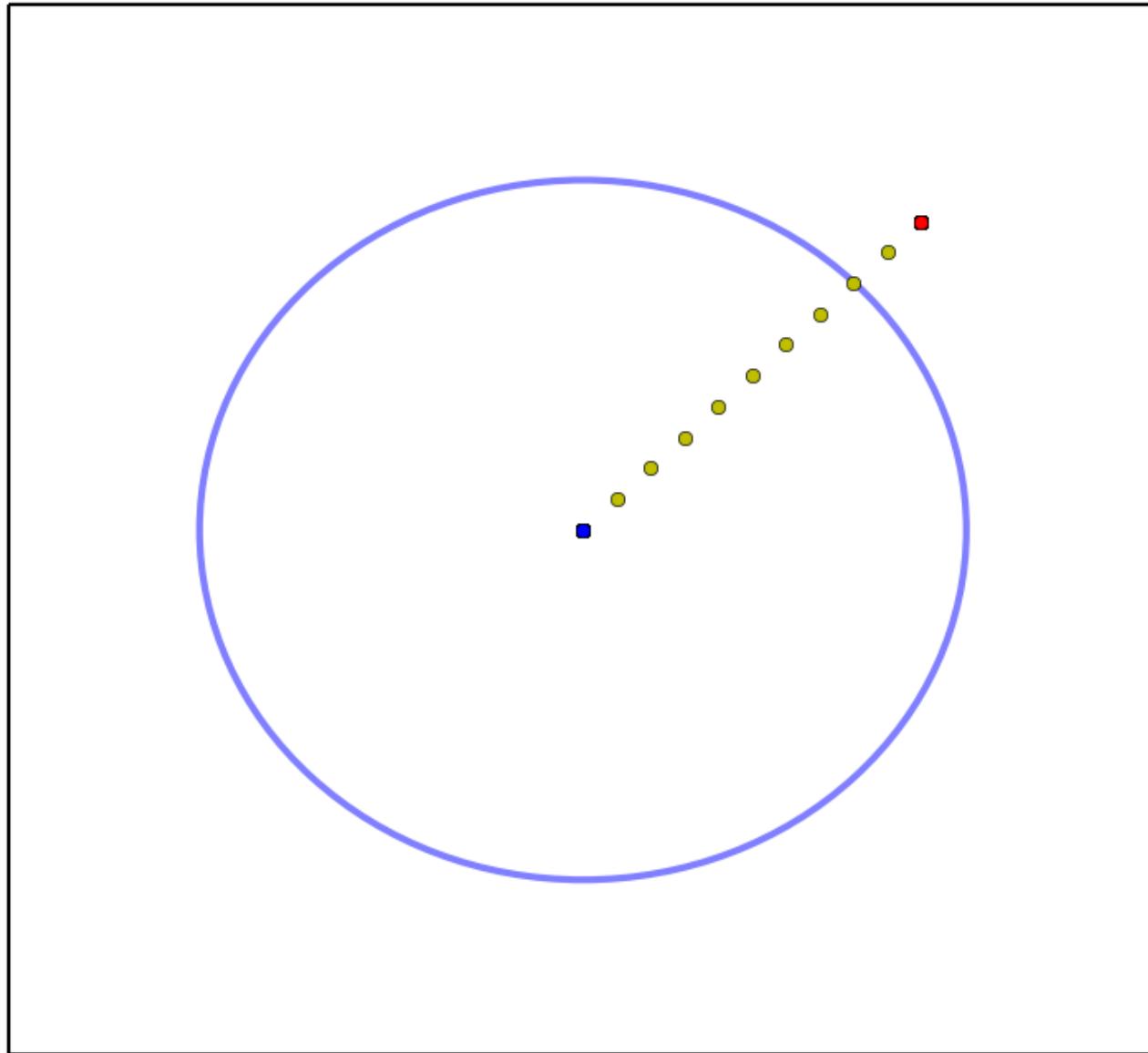
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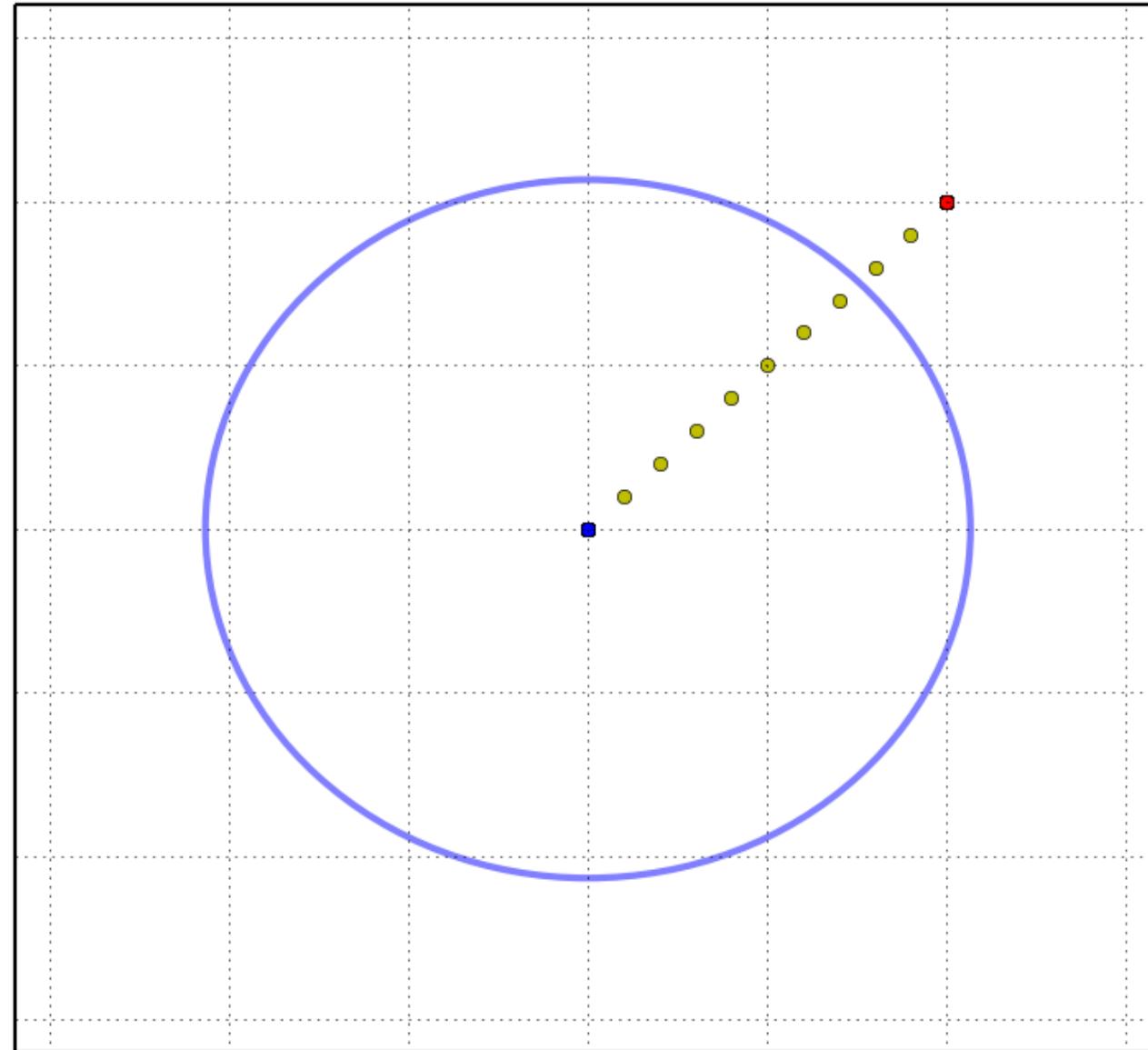
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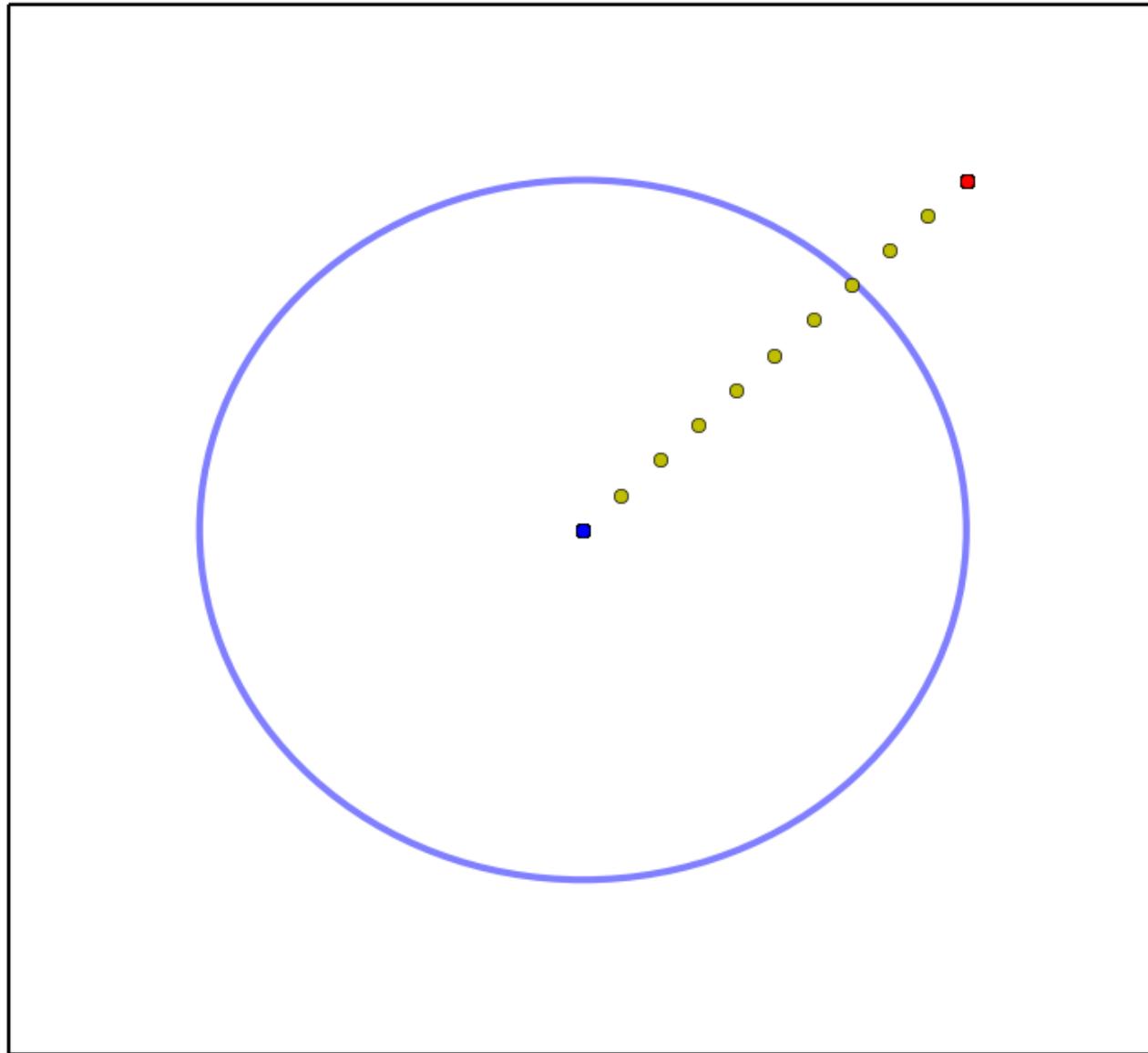
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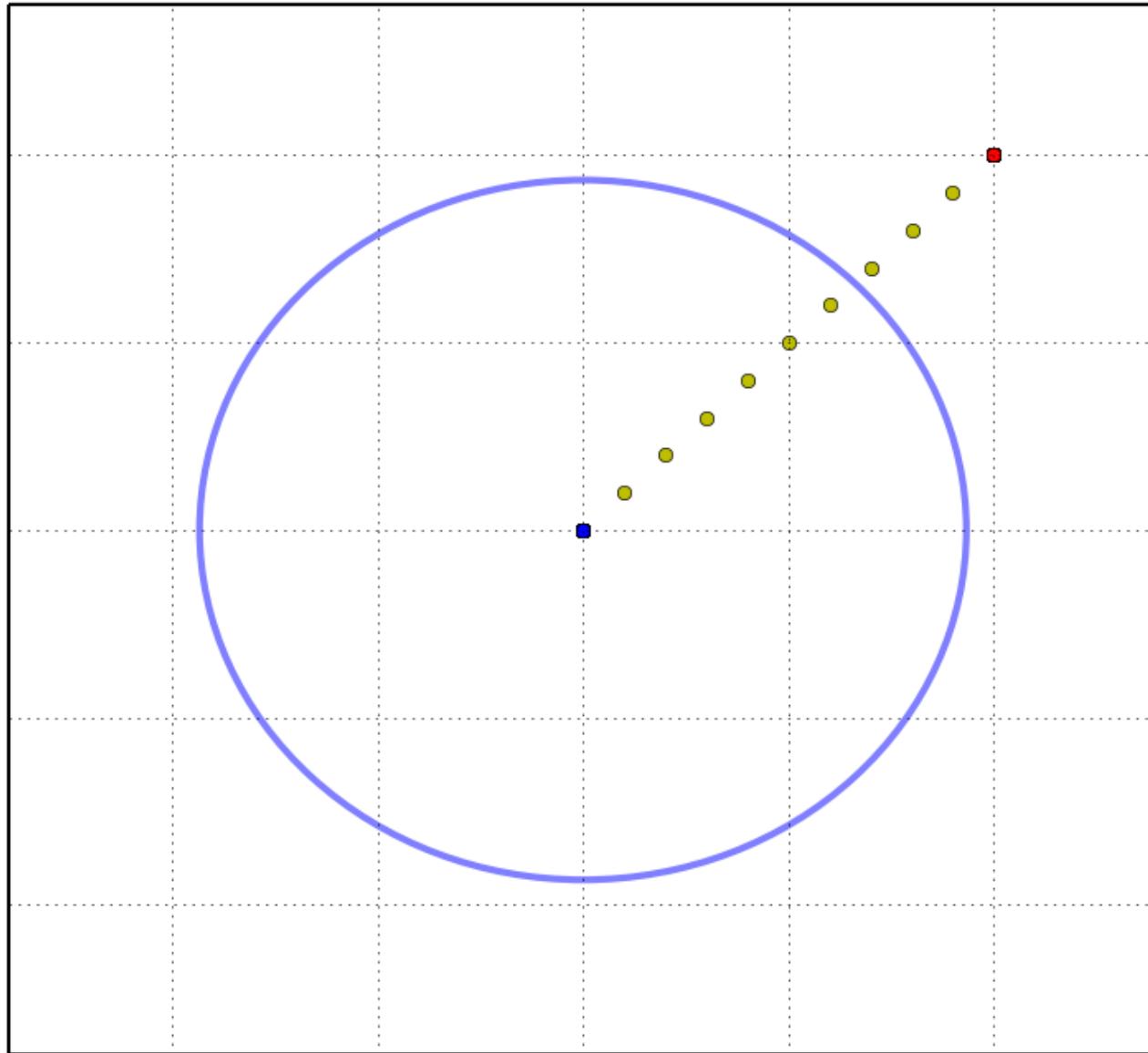
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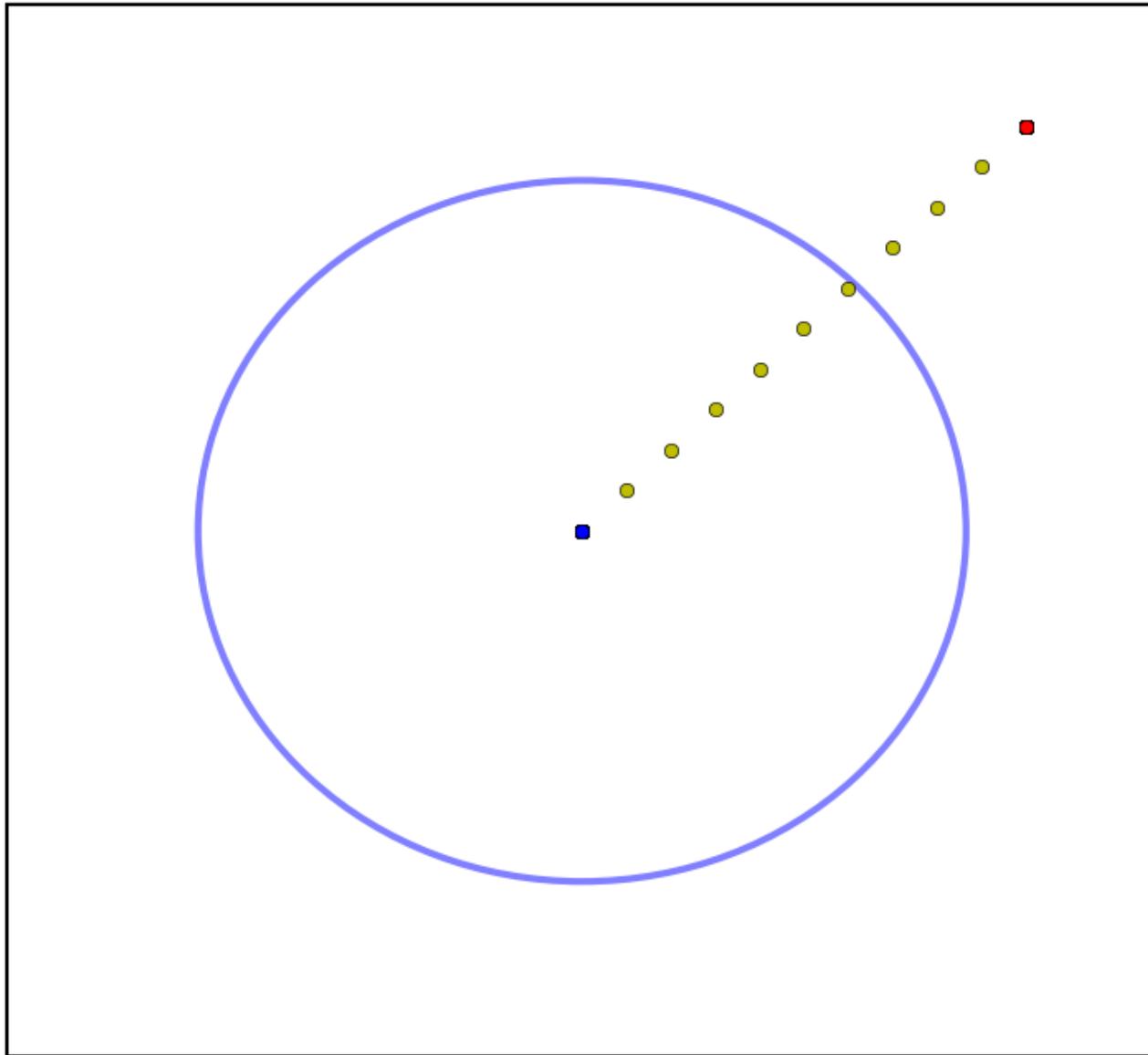
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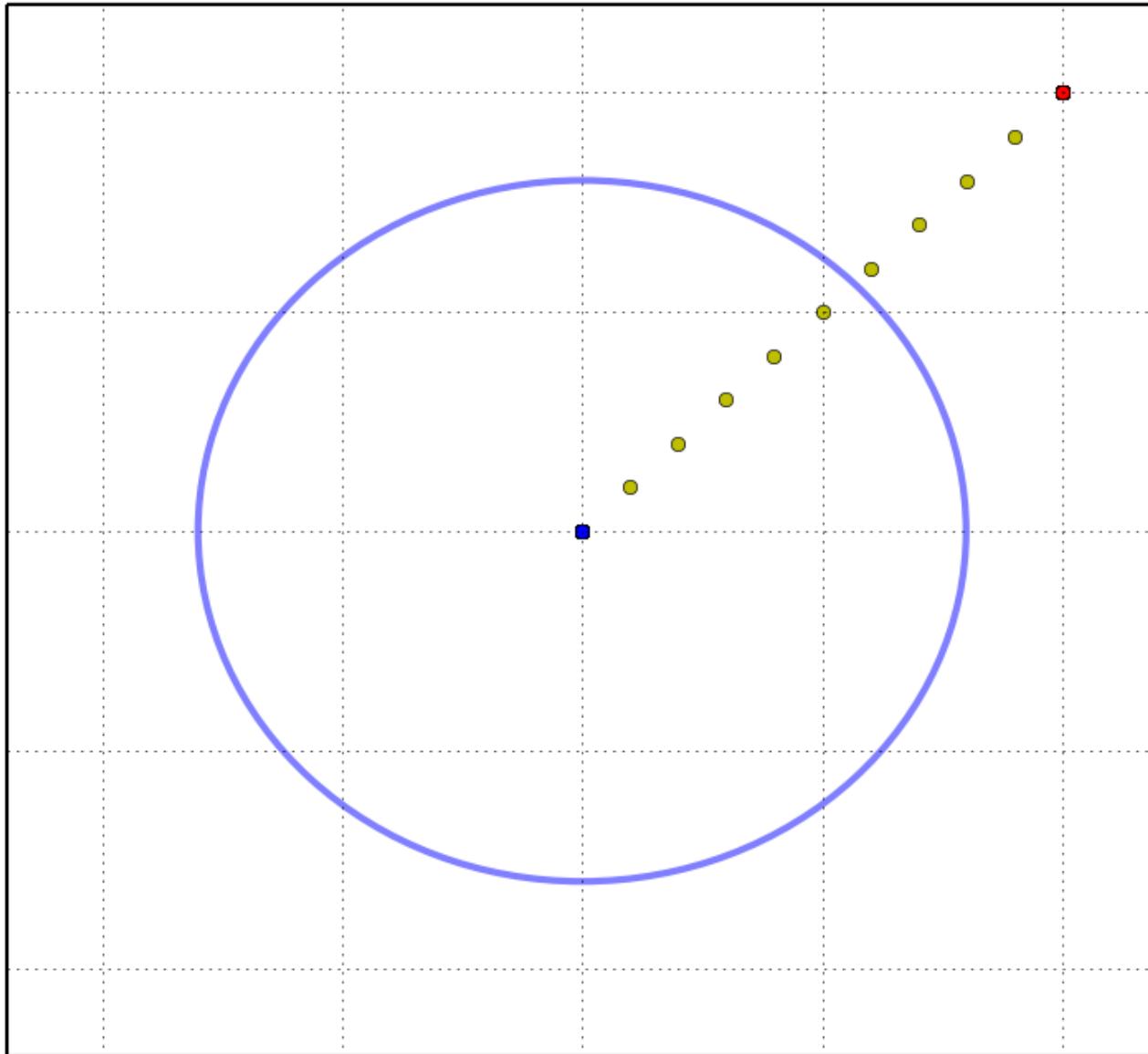
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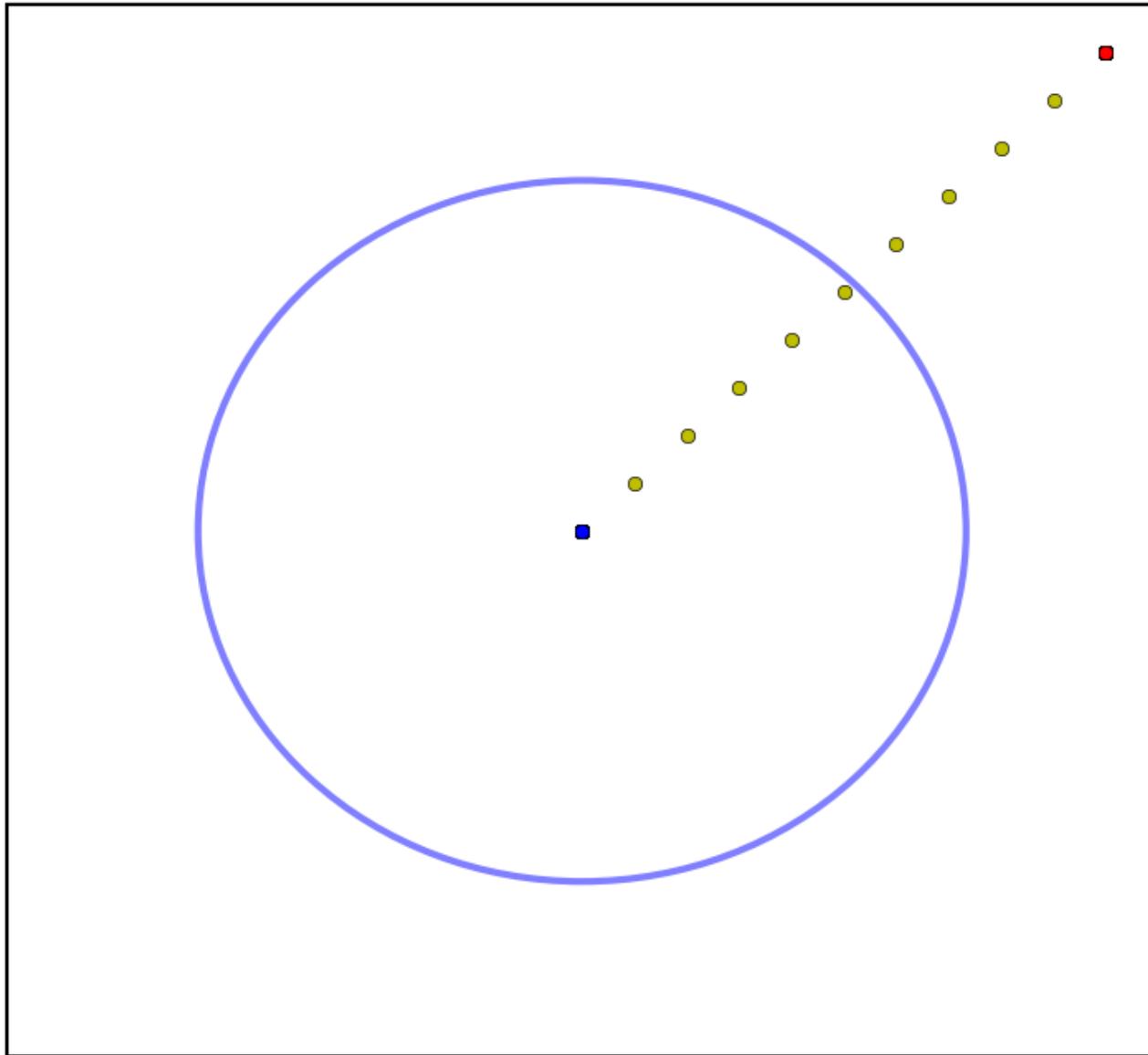
186



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# Outline

- I) Concepts to understand by the end of this talk
- II) Hubble Law, Hubble Sphere and Event Horizon
- III) Mind-blowing animations
- IV) Concepts not to understand (i.e. my real work)**



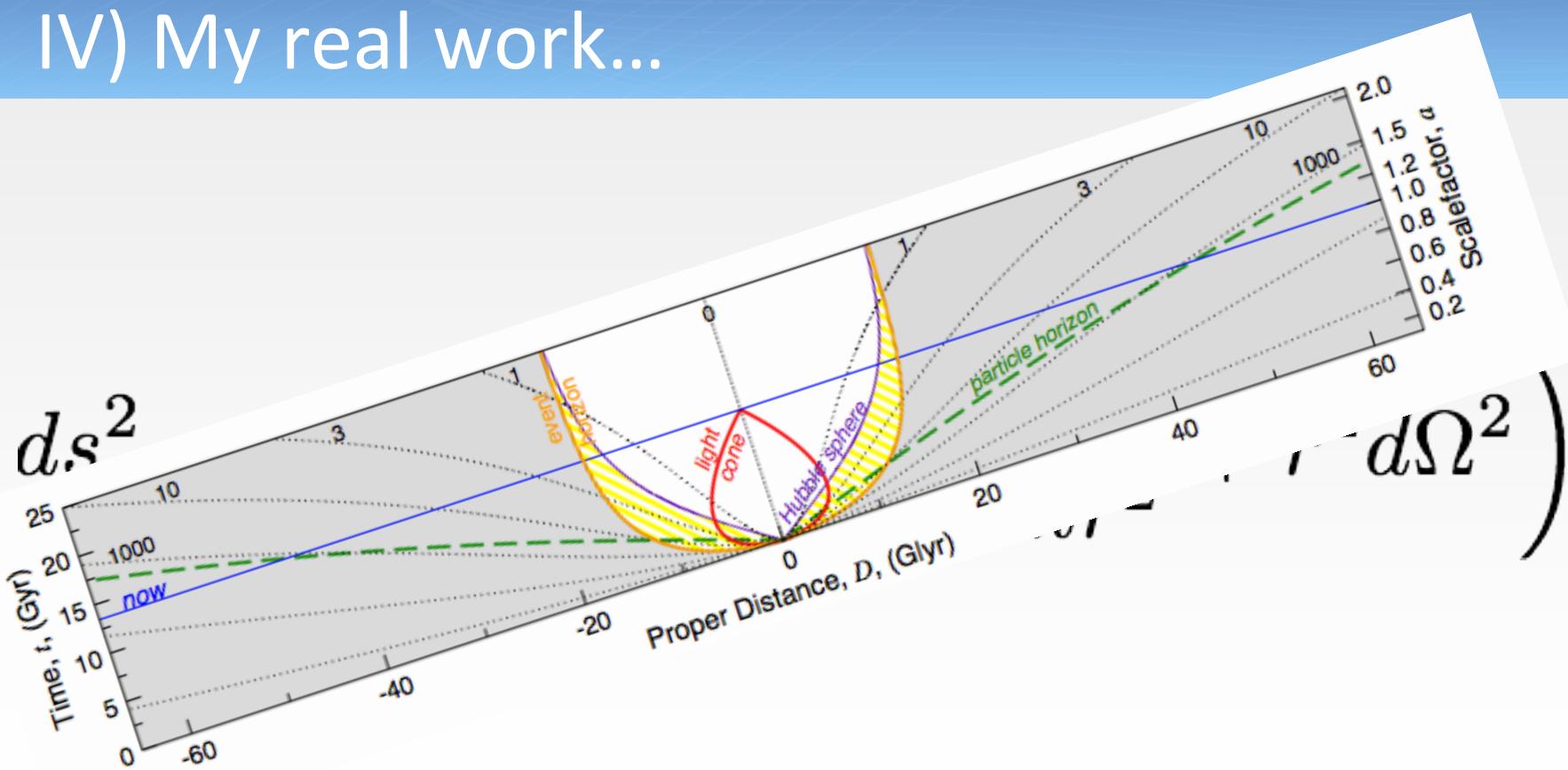
192

## IV) My real work...

$$ds^2 = c^2 dt^2 - R(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

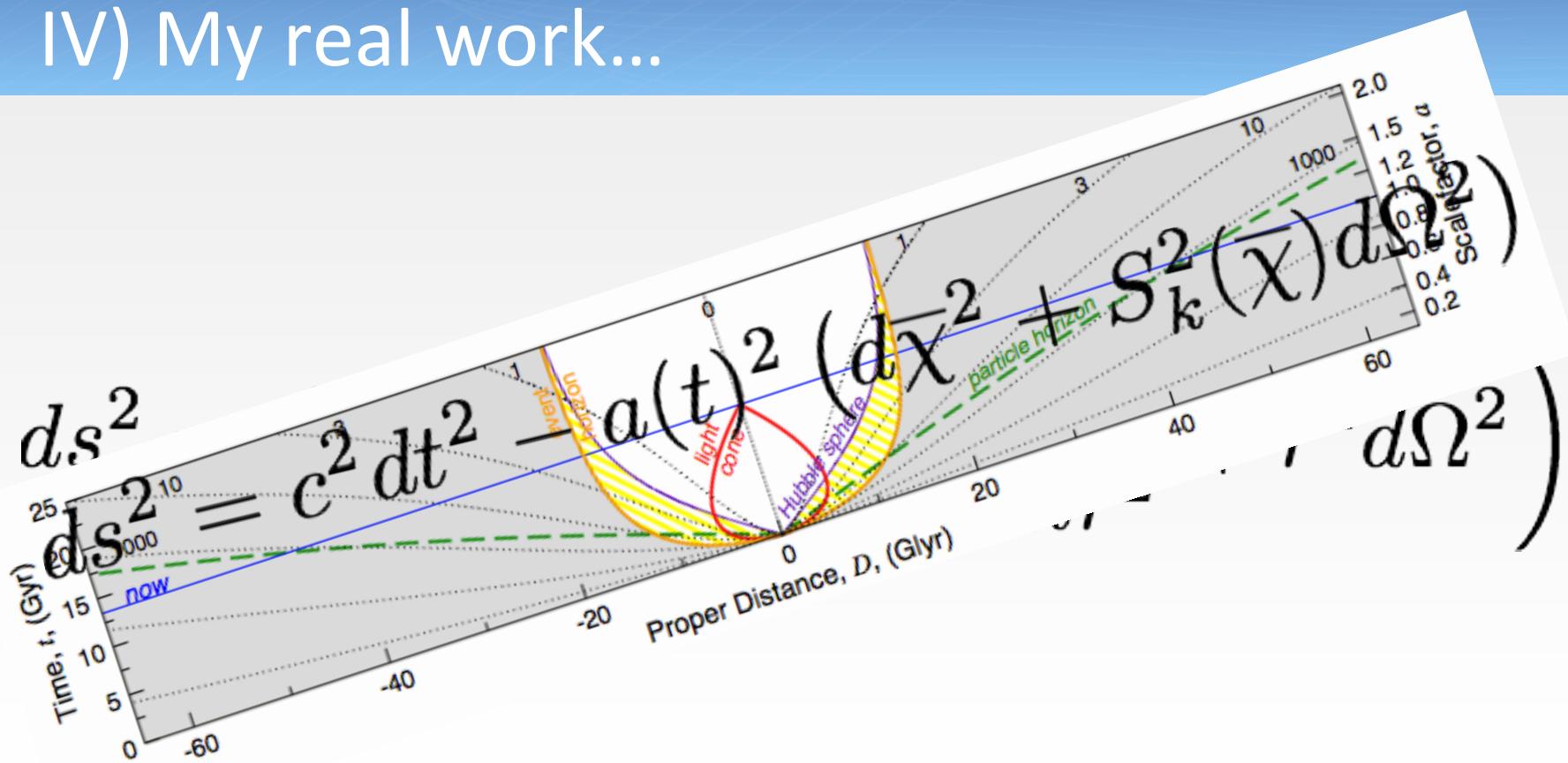


## IV) My real work...



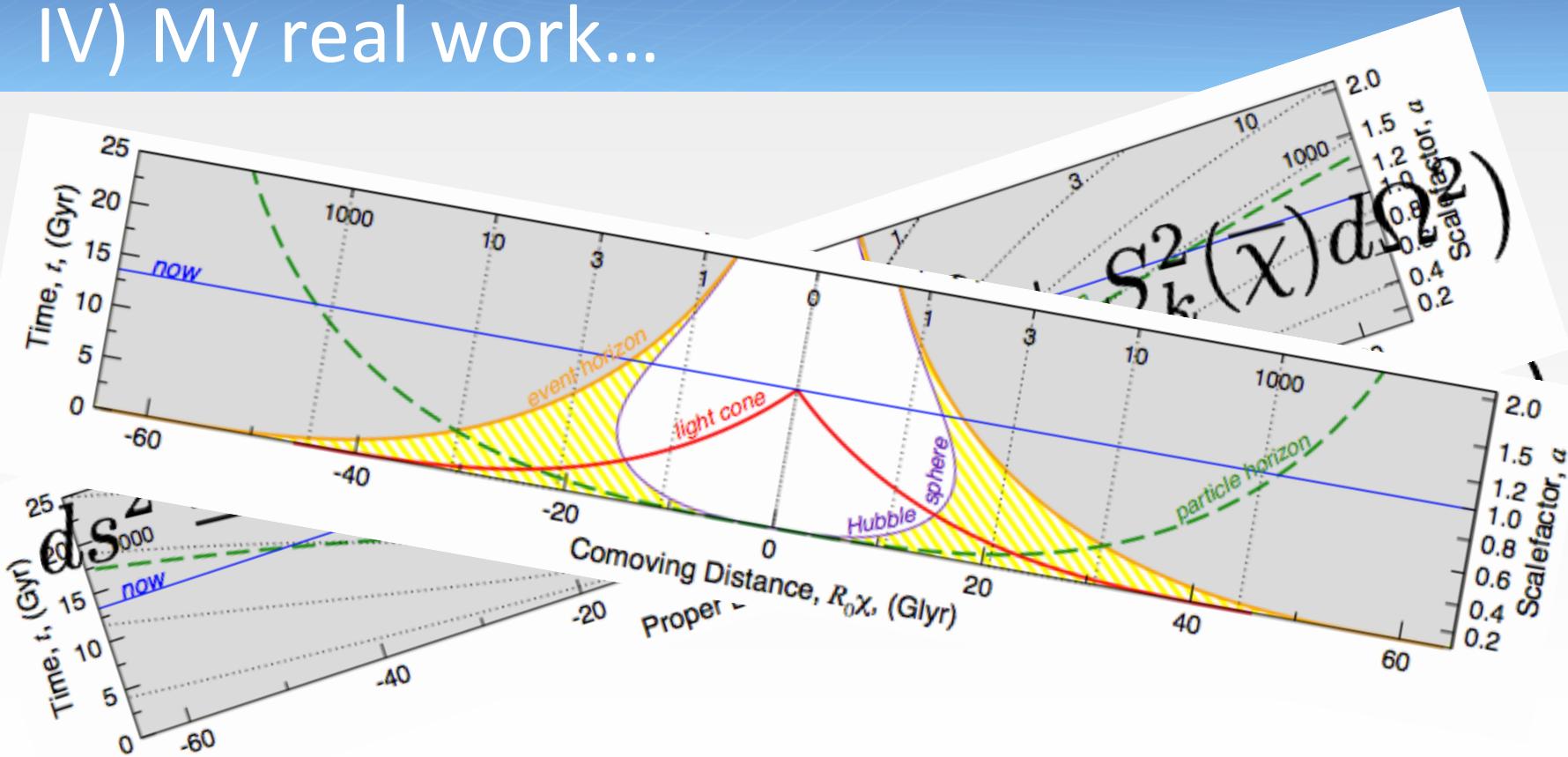


## IV) My real work...



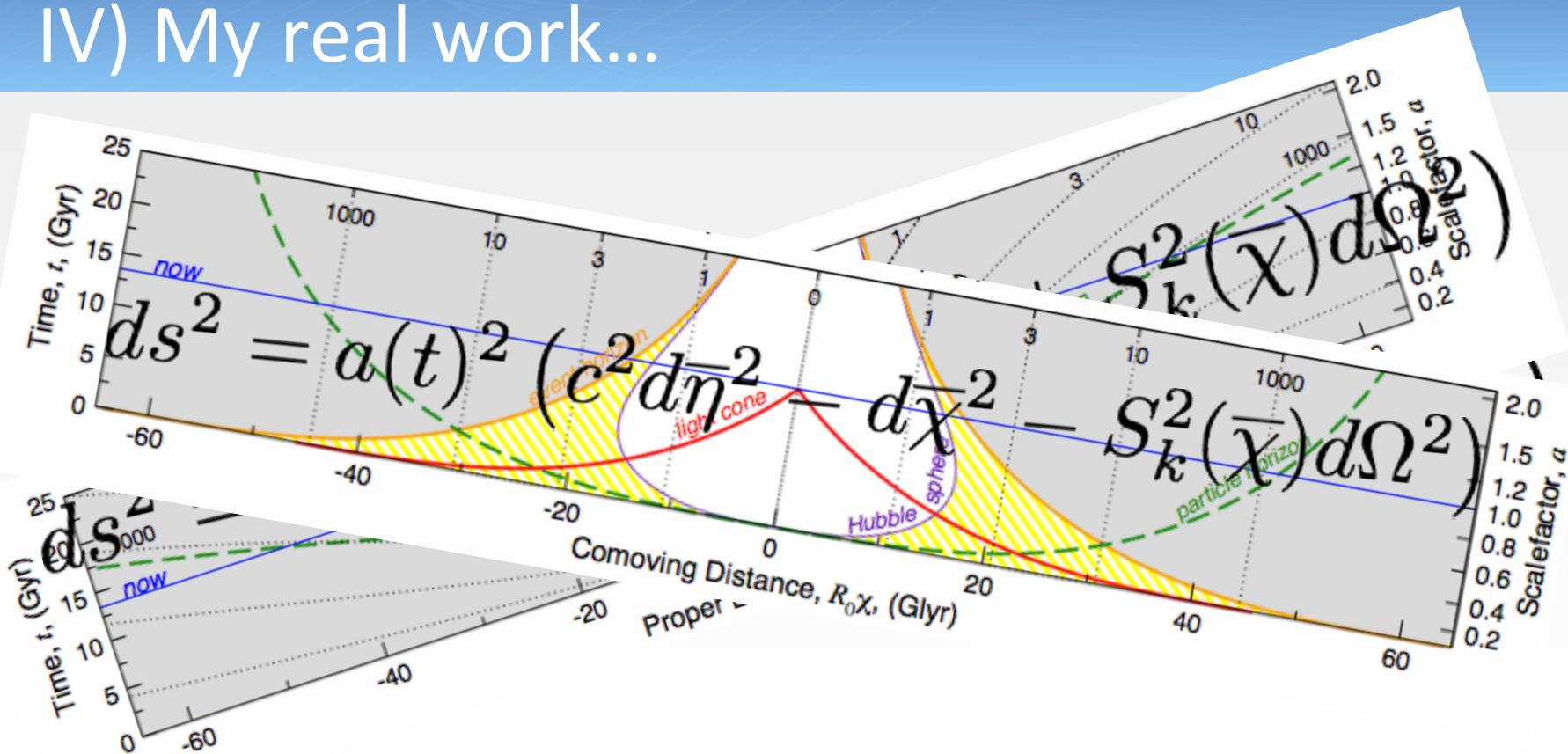


## IV) My real work...





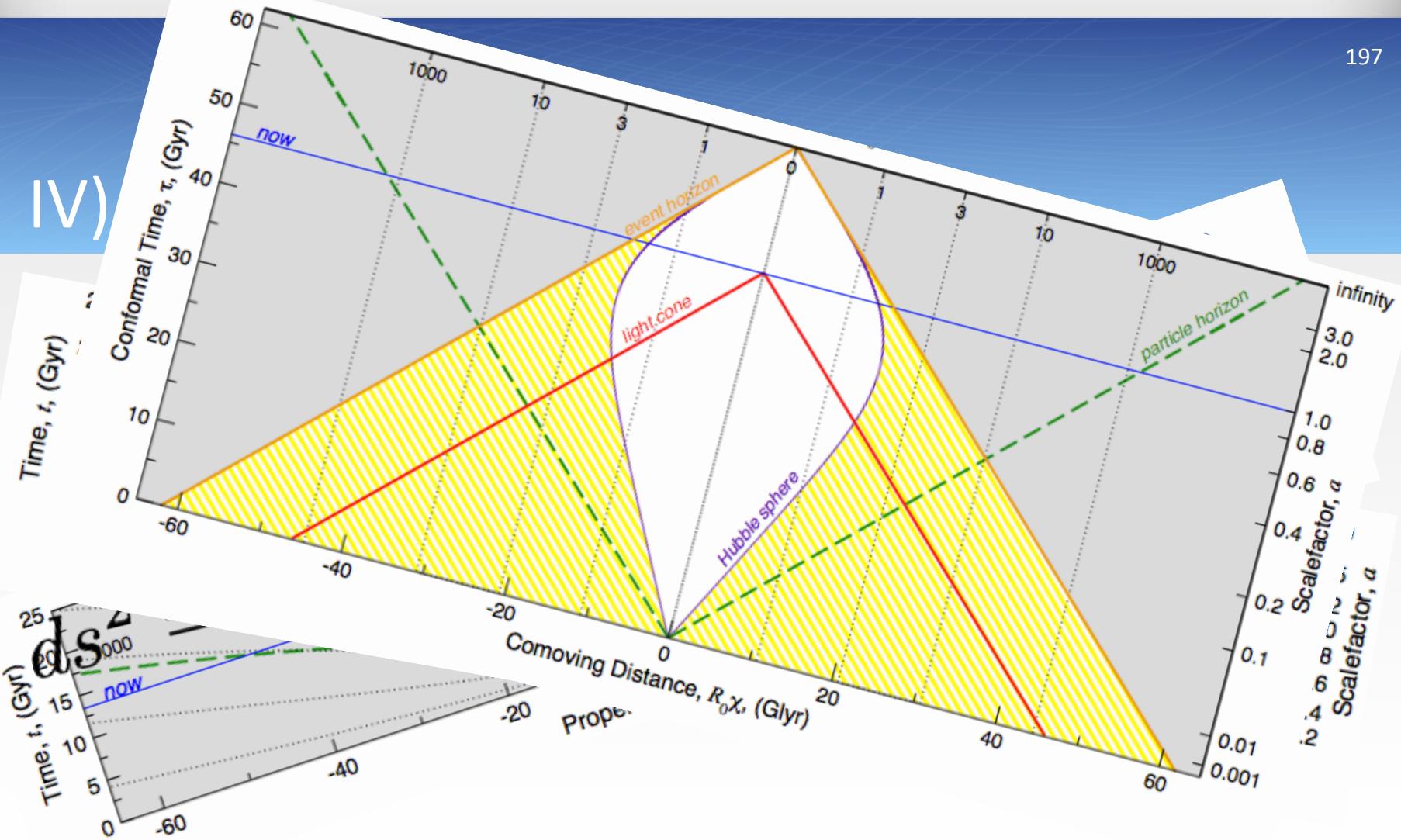
## IV) My real work...





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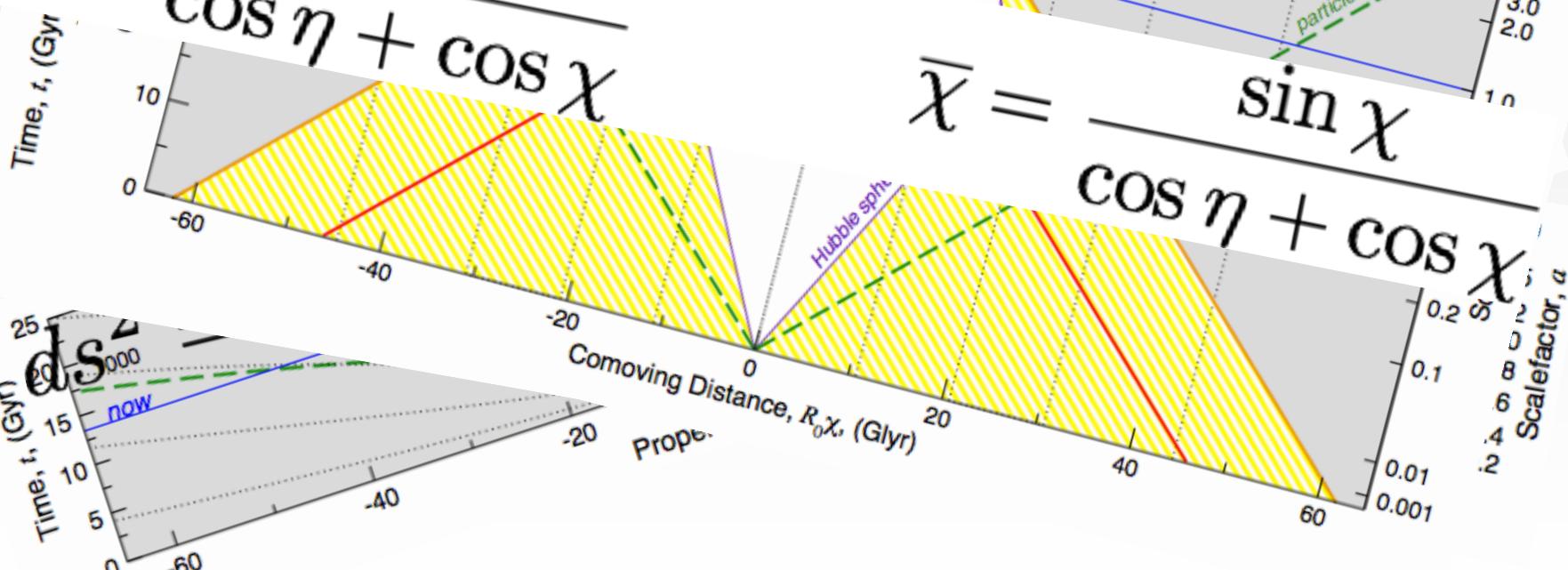
IV)

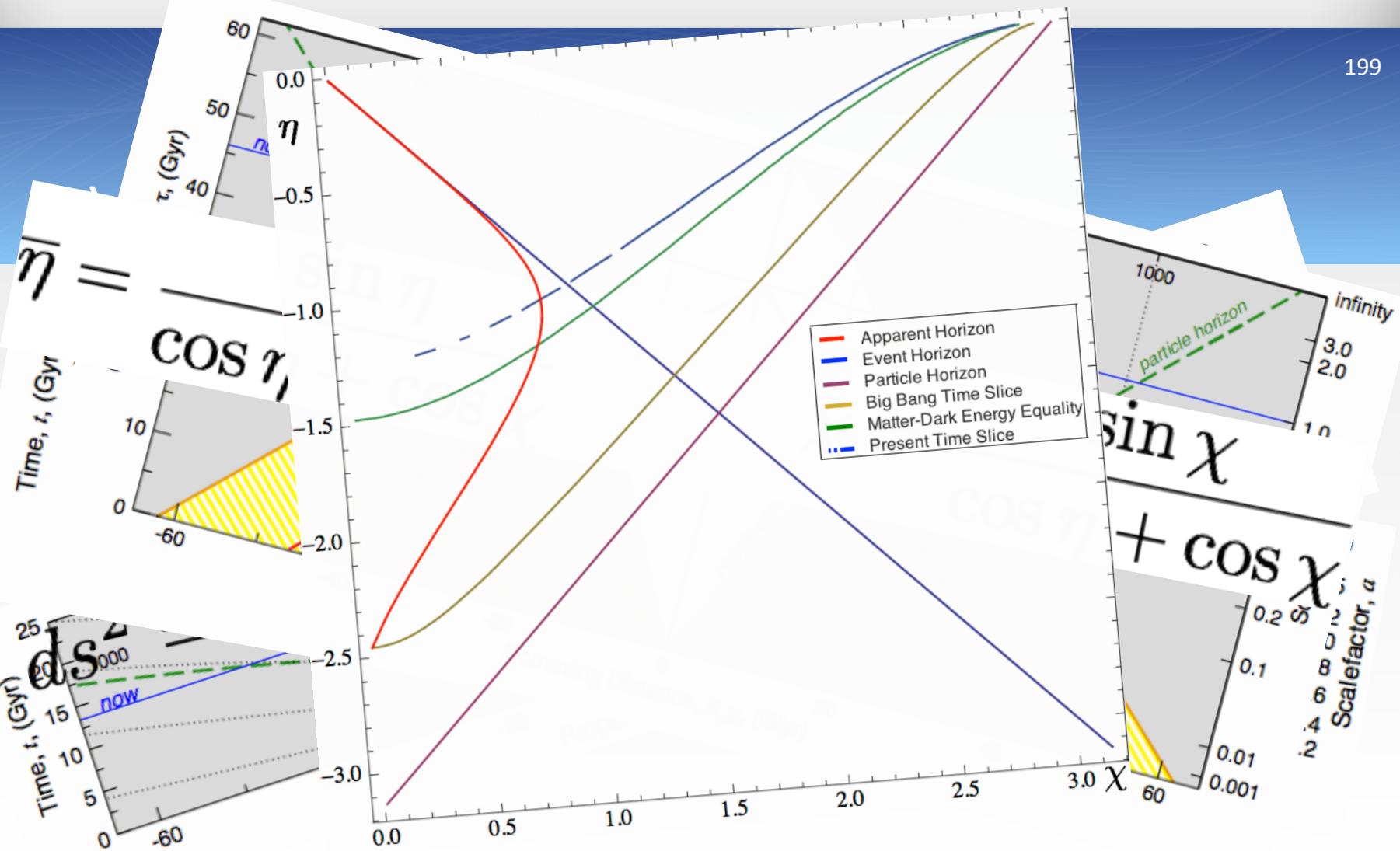


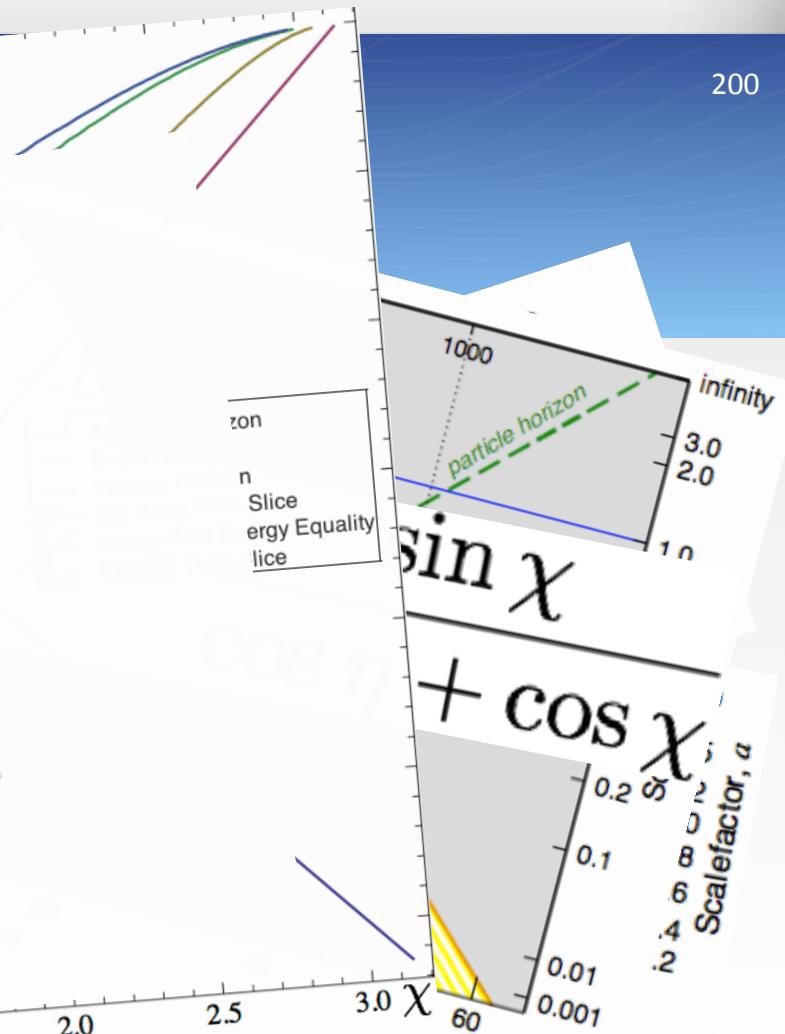
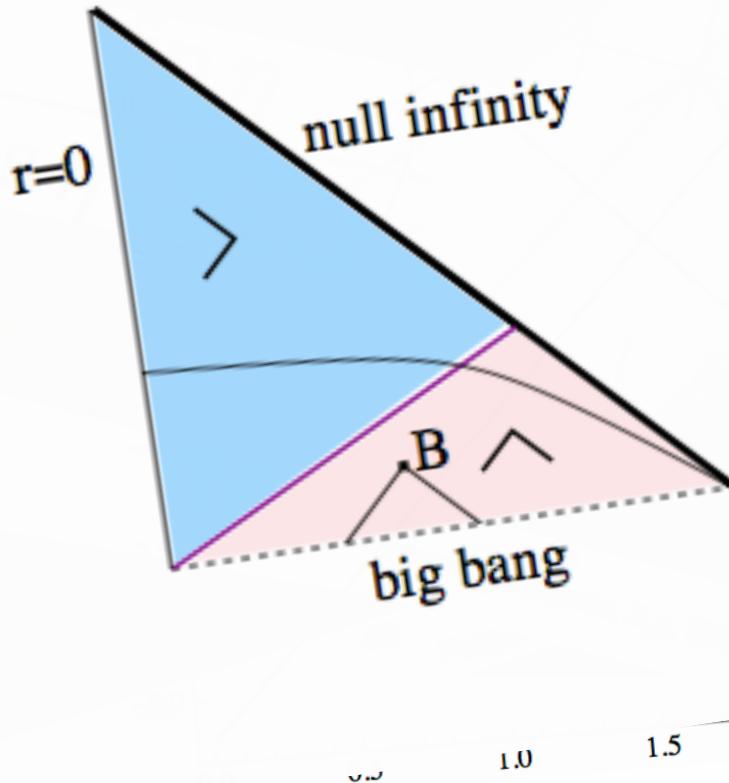
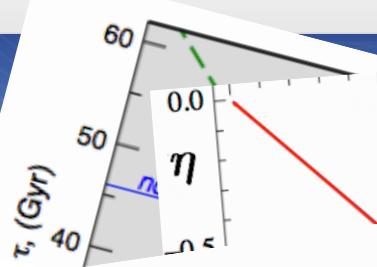


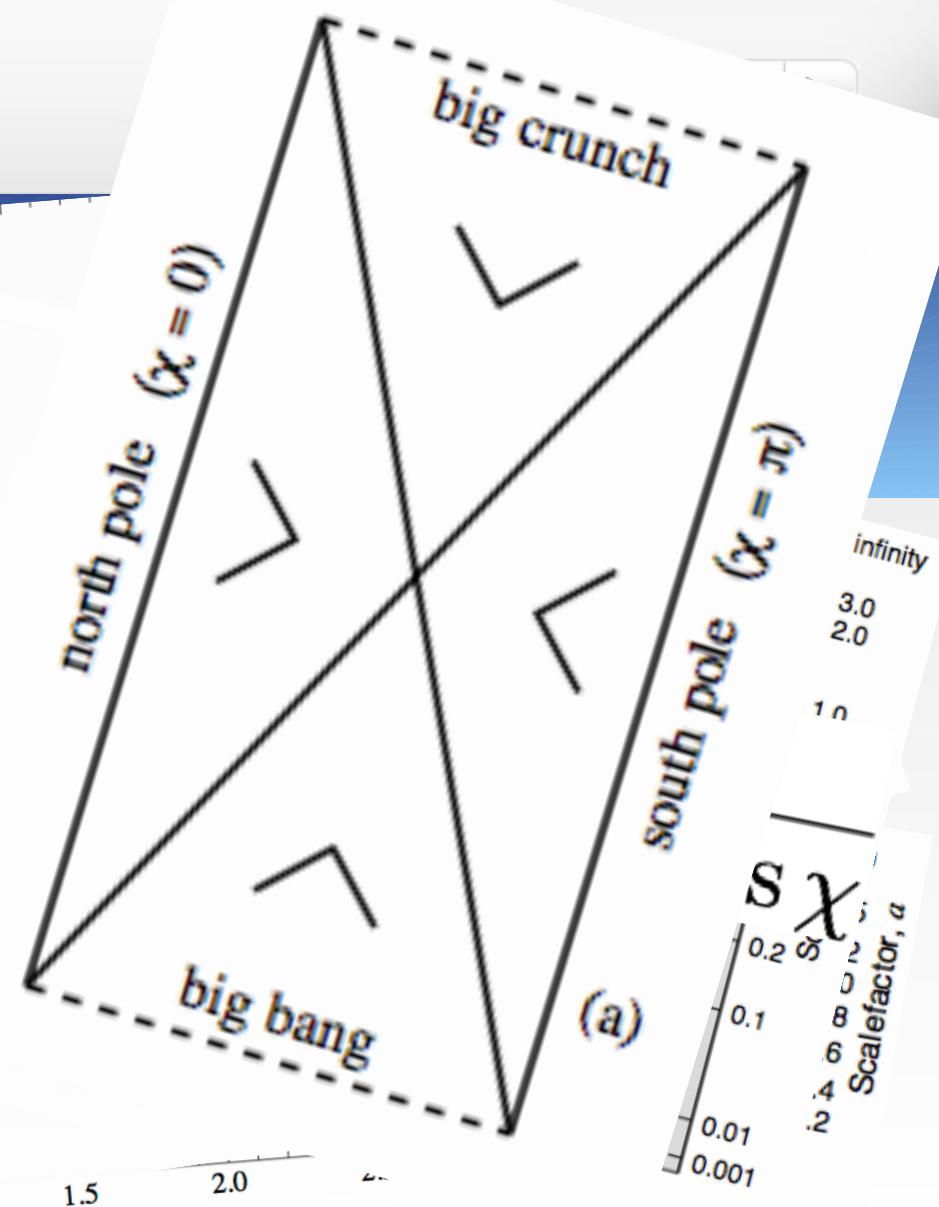
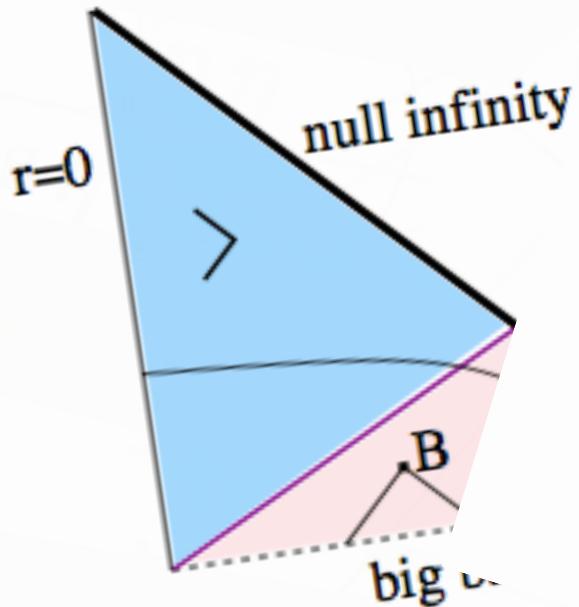
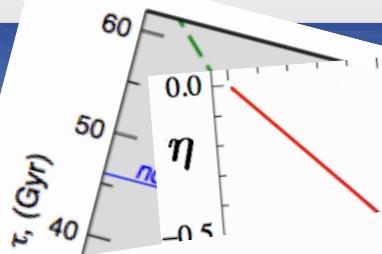
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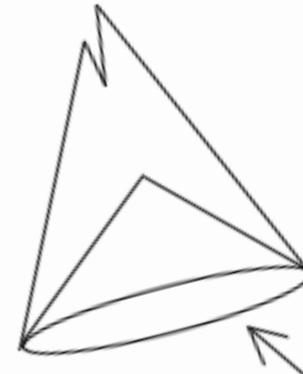
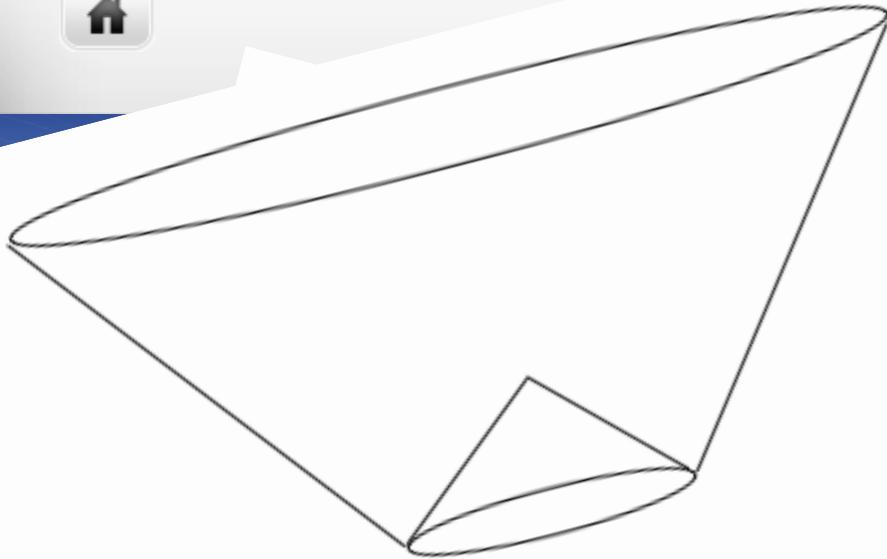
$$\bar{\eta} = \frac{\sin \eta}{\cos \eta + \cos \chi}$$





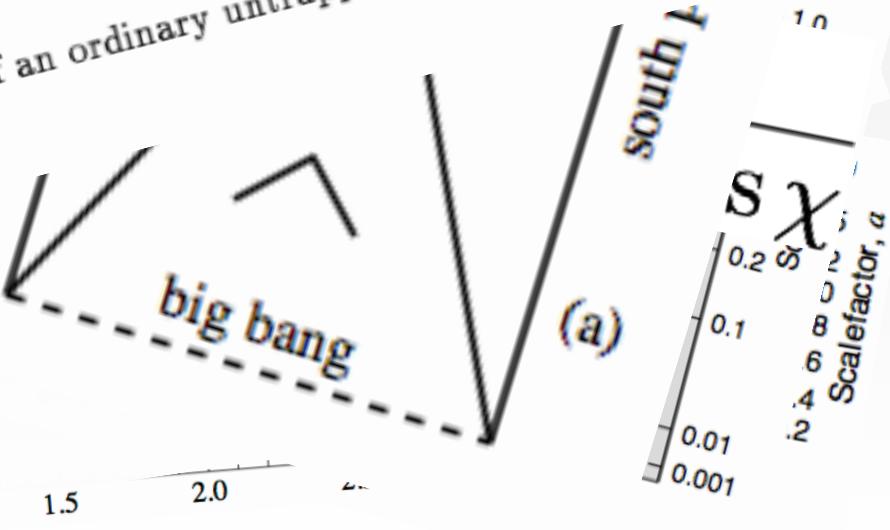
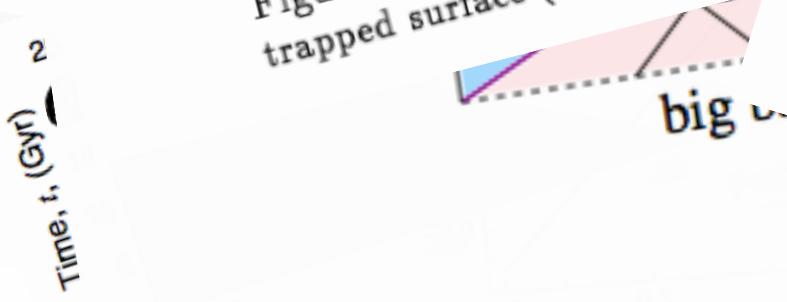






trapped surface

Figure 1.3: Boundary of the future of an ordinary untrapped surface (left) and a trapped surface (right).



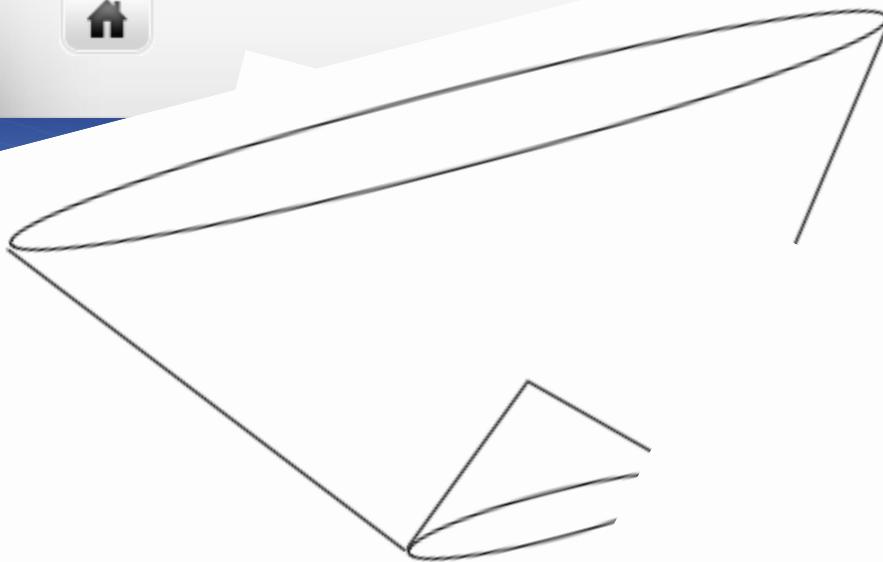
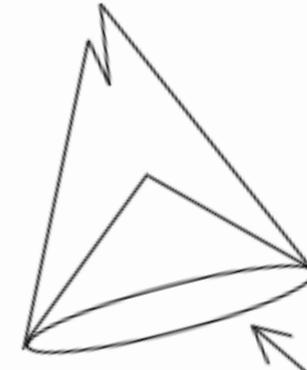


Figure 1.3:  
trapped sv



trapped surface

(left) and a

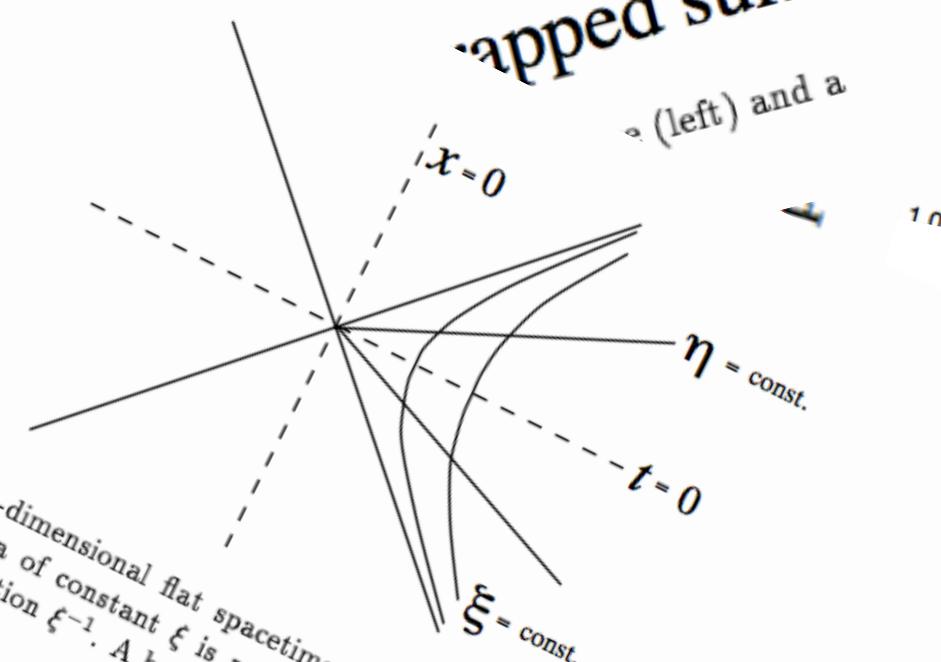
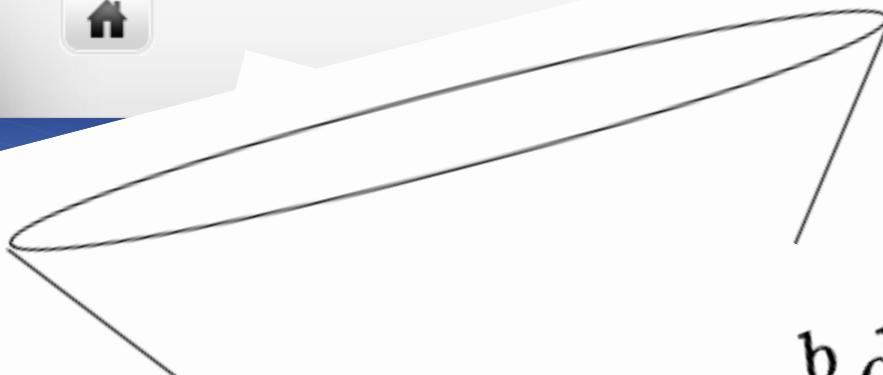
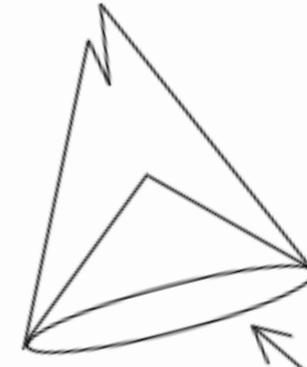


Figure 3.2: Two-dimensional flat spacetime in Minkowski and Rindler coordinates. A hyperbola of constant  $\xi$  is a uniformly accelerated timelike worldline with proper acceleration  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .



$$\delta Q \propto \int H T_{ab} \chi^a \chi^b d\lambda dA$$



trapped surface

(left) and a

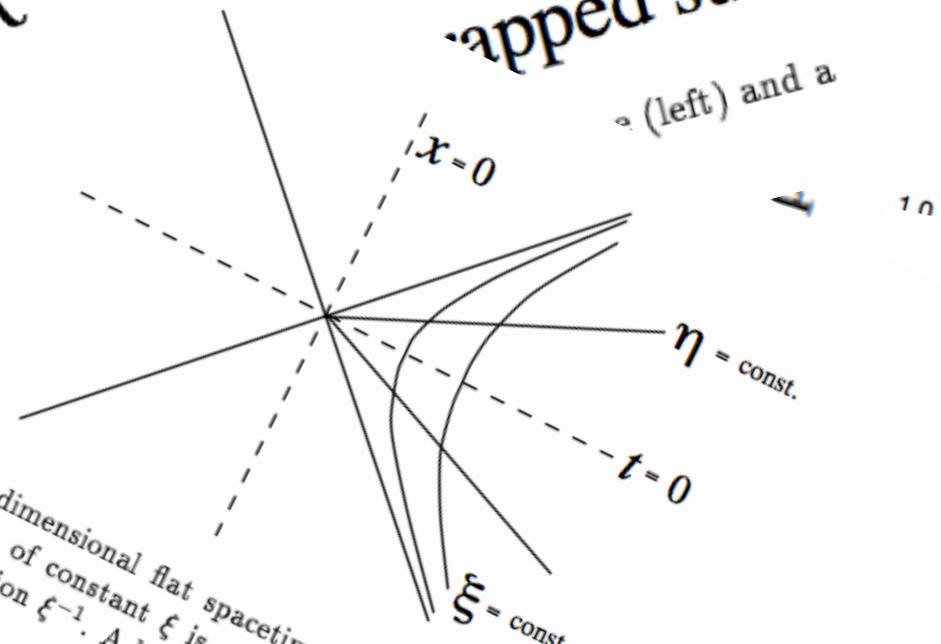
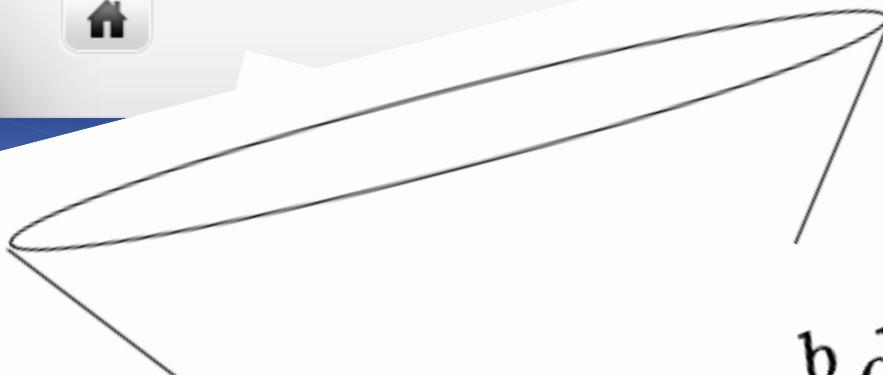


Figure 1.3:  
trapped sv

Figure 3.2: Two-dimensional flat spacetime in Minkowski and Rindler coordinates. A hyperbola of constant  $\xi$  is a uniformly accelerated timelike worldline with proper acceleration  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .



$$\delta Q \propto \int_H T_{ab} \chi^a \chi^b d\lambda dA$$

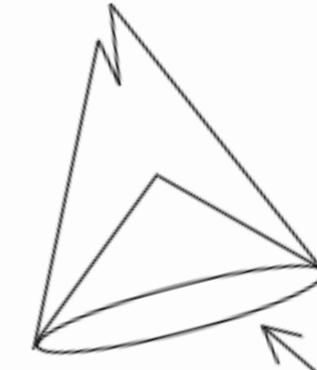
$$dS \propto \int_H \theta d\lambda dA \propto \int_H \xi d\lambda dA$$

Time,  $t_1$  (Gyr)

Figure 1.5:  
trapped sv

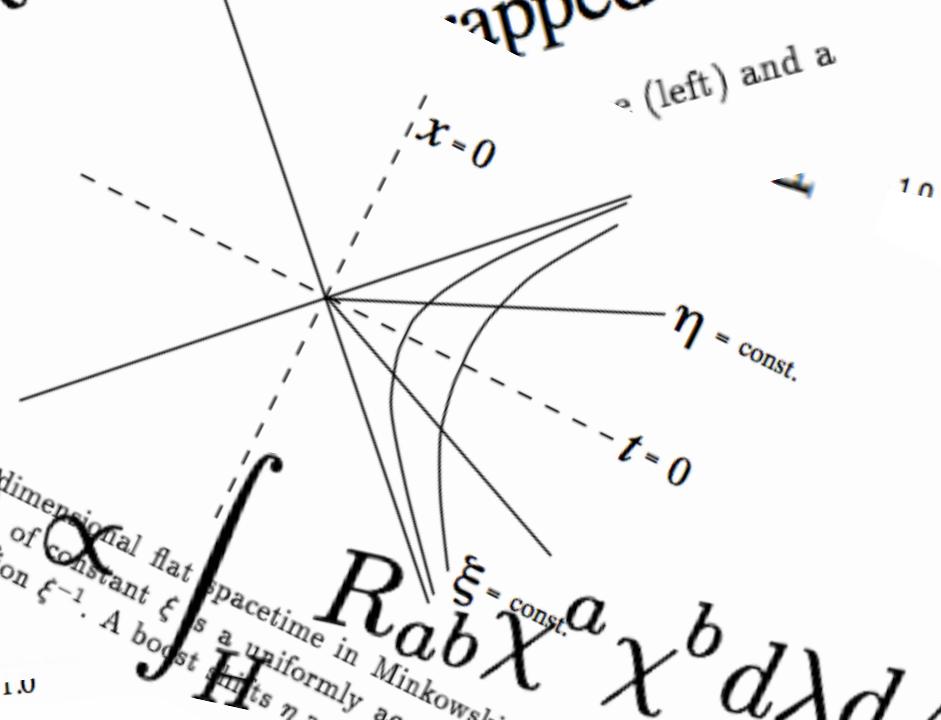
Figure 3.2:

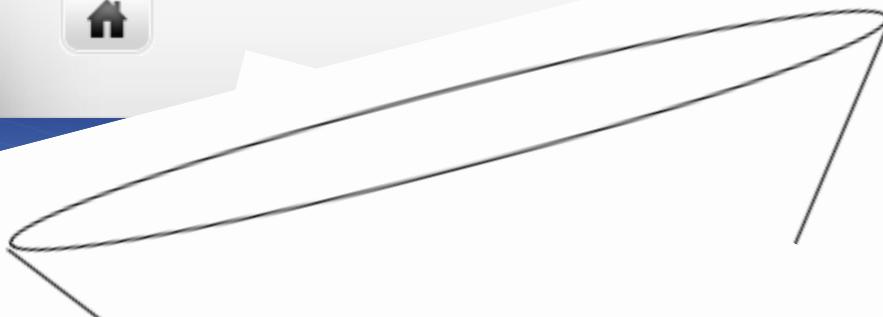
nates A hyperbol of constant  $\xi$  is a uniformly accelerated timelike worldline with proper acceleration  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .



trapped surface

(left) and a





$$\xi^\mu = (1, -\hat{H}r, 0, 0)$$

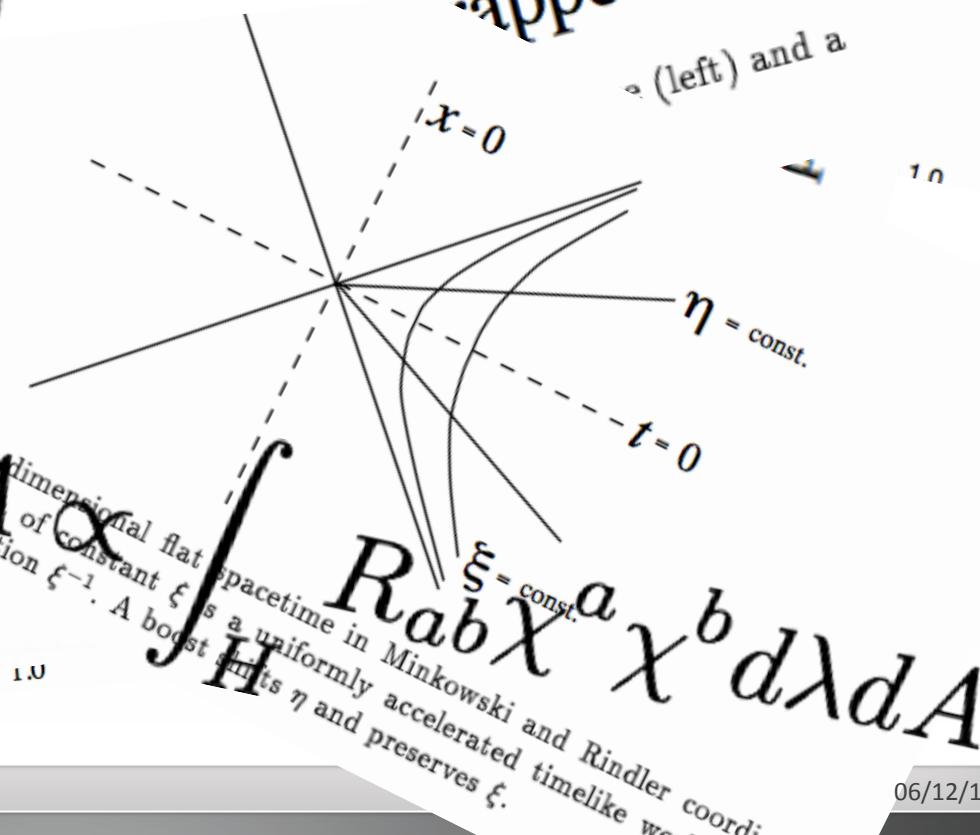
apparent surface

$$\delta Q \propto \int H$$

$$dS \propto \int_H \theta d\lambda dA$$

Figure 1.5:  
trapped sv

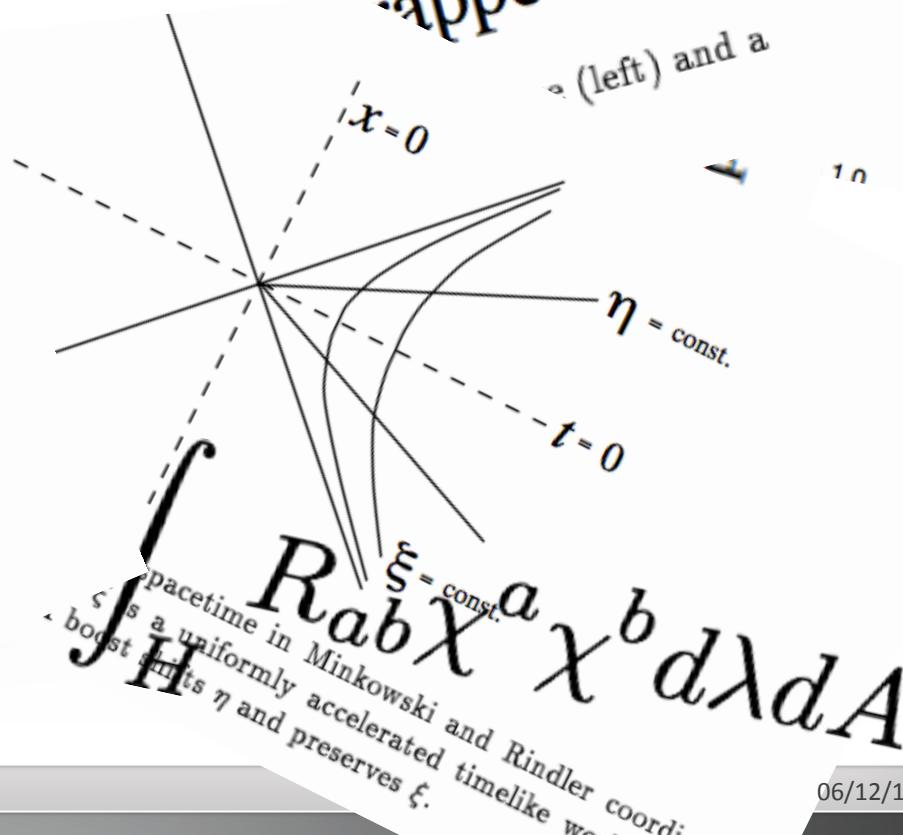
Time,  $t_1$  (Gyr) 2  
2





$$\xi^\mu = (1, -\hat{H}r, 0, 0)$$

apparent surface





$$\xi^\mu = (1, -\hat{H}r, 0, 0)$$

apparent surface

$$\delta Q \propto \int_H^H \delta E = \delta \int A dt \xi_\mu \xi^\nu T_\nu^\mu$$

(left) and a

$$dS \propto \int_H^H \theta d\lambda dA$$

$\xi = \text{const.}$

$t=0$

$x=0$

$R_{ab} \chi^a \chi^b d\lambda dA$

Figure 1.5:  
trapped sv

Figure 3.2:  
nates  
with proper acceleration  $\xi^{-1}$ . A boost with its  $\eta$  and preserves  $\xi$ .

$$TdS = \frac{H}{2\pi} \left( \frac{-2\pi \dot{H}}{G} dt \right)_{(0,0)} \text{apparent surface}$$

$$\xi^\mu = (1,$$

$$\delta Q \propto \int_H^H \delta E = \delta \int A dt \xi_\mu \xi^\nu T_\nu^\mu$$

$\xi = \text{const.}$

$x=0$  (left) and a

$t=0$

$dS \propto \int_H^H \theta d\lambda dA$

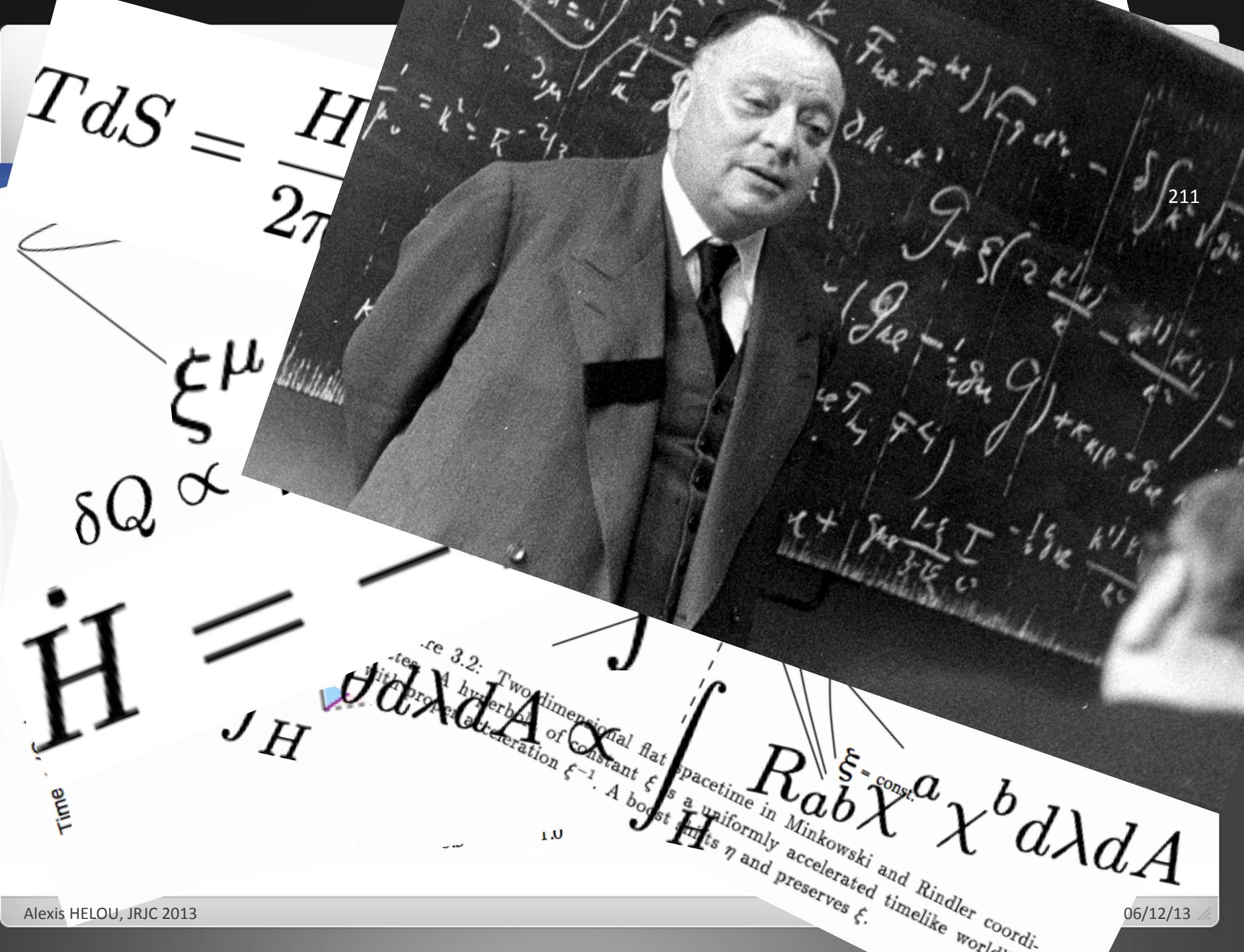
Figure 3.2: Two-dimensional flat spacetime in Minkowski and Rindler coordinates. A hyperboloid of constant  $\xi$  is a uniformly accelerated timelike worldline with proper acceleration  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .

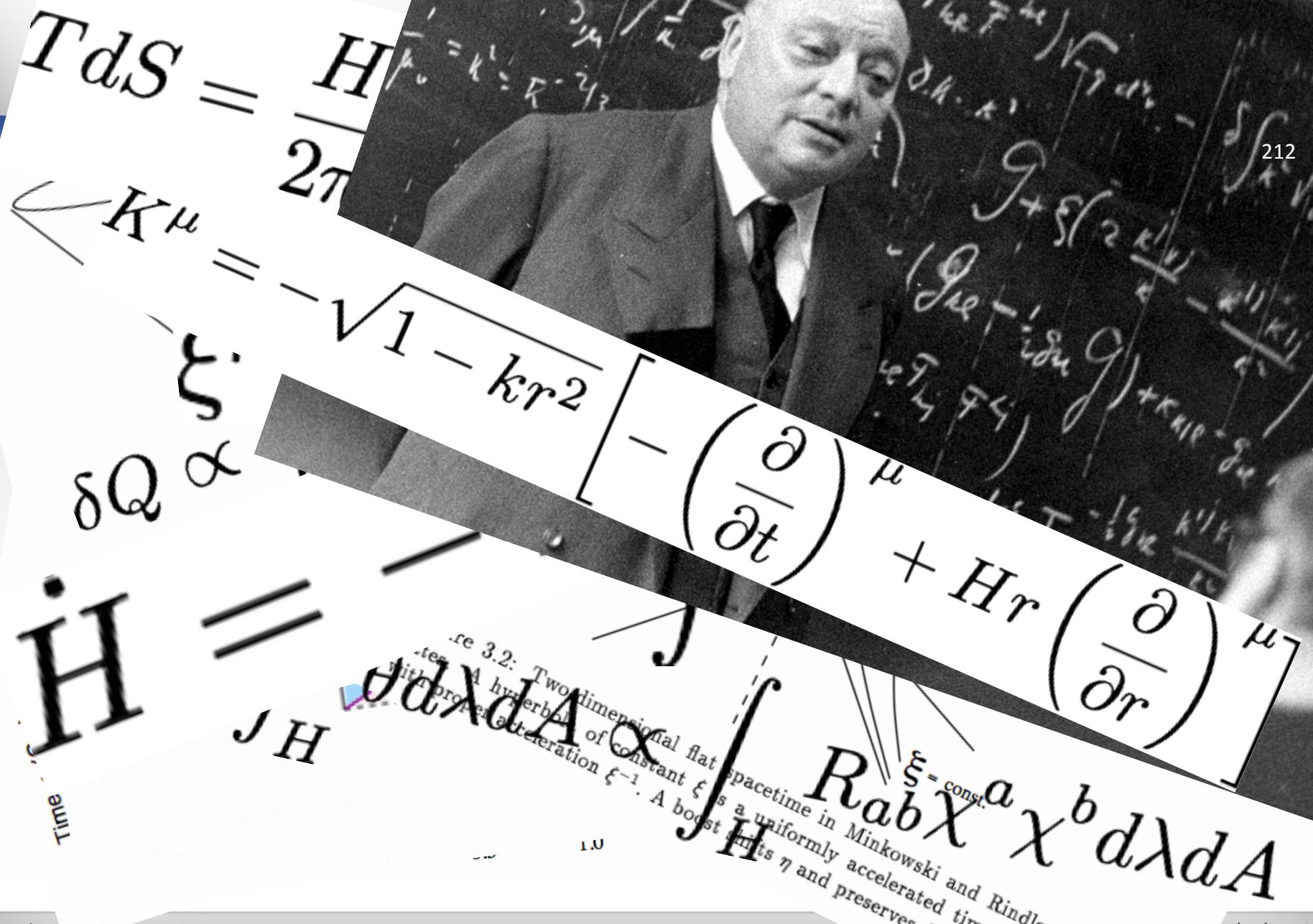
$$TdS = \frac{H}{2\pi} \left( -\frac{2\pi}{G} \frac{\dot{H}}{H^3} dt + \right) \text{ (left) and a } \xi^\mu = (1, \delta Q, \dot{r}, -A\pi G(\rho + p))$$

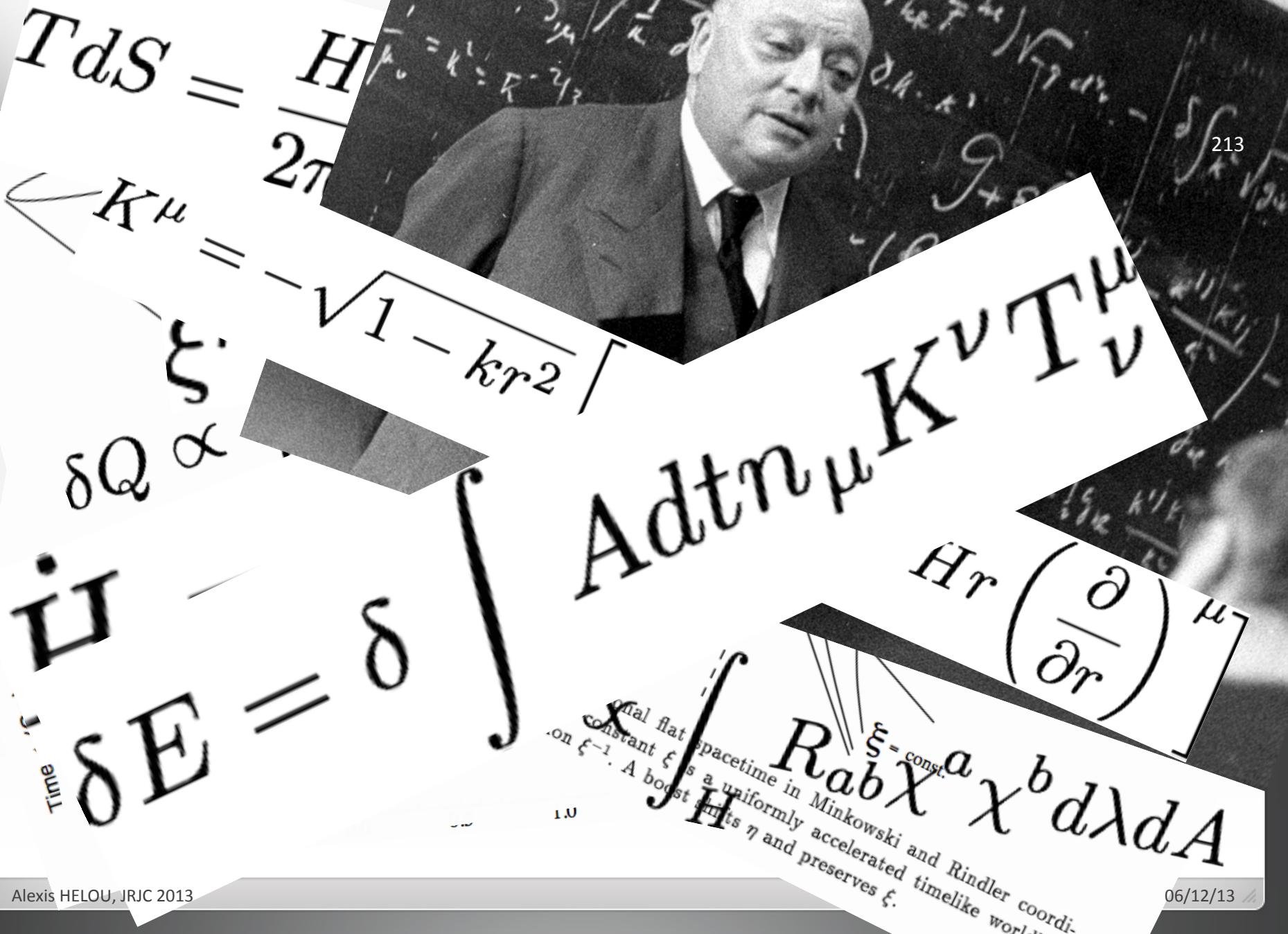
$$\dot{H} = \int_H A dt \xi_\mu \xi^\nu T_\nu^\mu$$

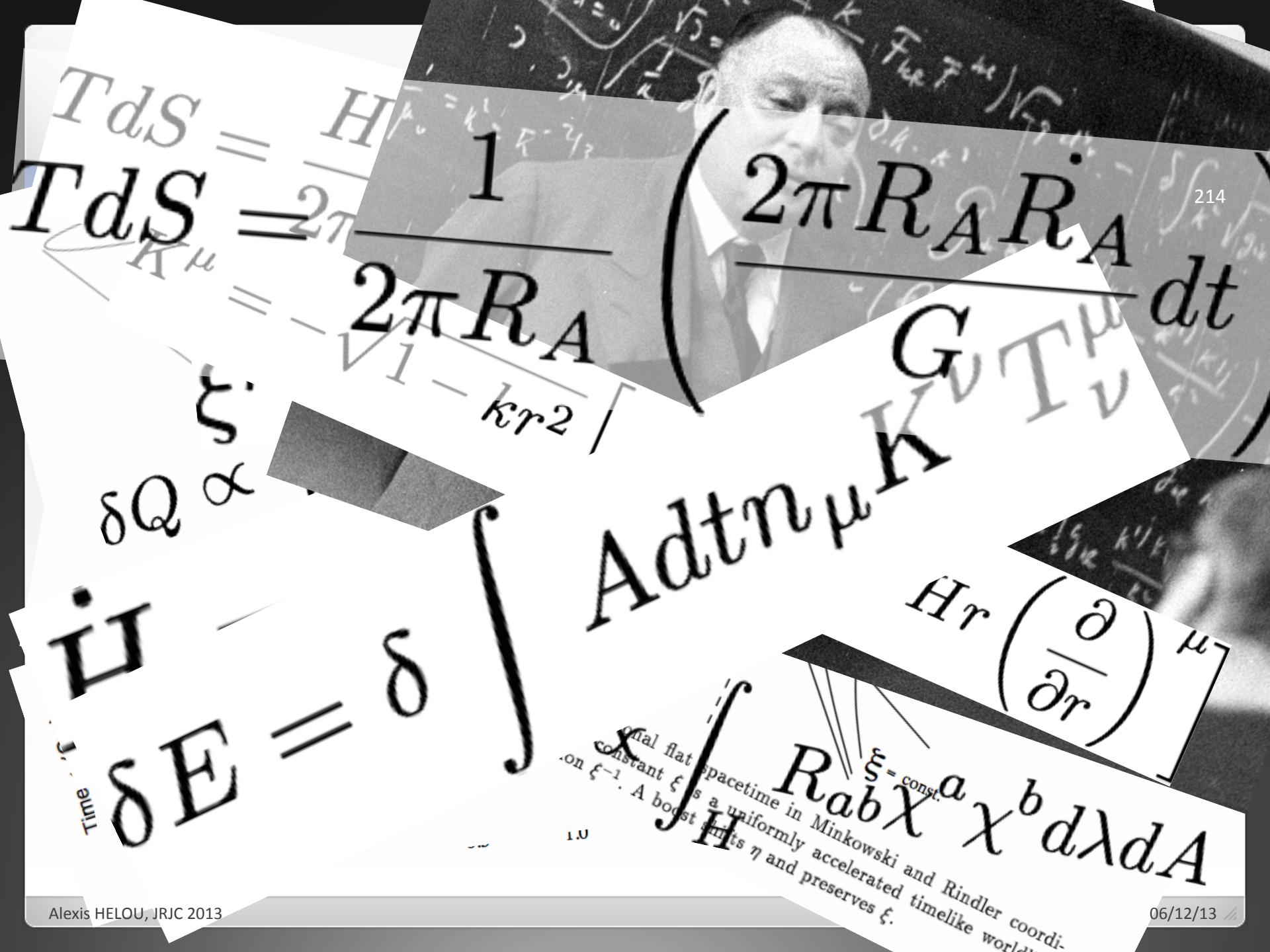
Time

Figure 3.2: Two-dimensional flat spacetime in Minkowski and Rindler coordinates. A hyperboloid of constant  $\xi$  is a uniformly accelerated timelike worldline with proper acceleration  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .











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$$\frac{R_A R_A}{G} dt$$

$$H_r \left( \frac{\partial}{\partial r} \right)^\mu$$

$$R_{ab} \chi^a \chi^b d\lambda dA$$

$\xi = \text{const.}$   
radial flat spacetime in Minkowski and Rindler coordinates  
 $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p)$$



$\int_{\Sigma} R_{ab} \chi^a \chi^b d\lambda dA$

$H_r \left( \frac{\partial}{\partial r} \right)^\mu$

$\xi = \text{const}$

Euclidean flat spacetime in Minkowski and Rindler coordinates on  $\xi^{-1}$ . A boost shifts  $\eta$  and preserves  $\xi$ .

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p)$$

$$\rho = \frac{E}{V} = \frac{H_0^{-1}/2G}{\frac{4}{3}\pi H_0^{-3}} = \frac{1}{\frac{8\pi}{3}GH_0^{-2}} = \frac{\frac{8\pi}{3}L_P^2H_0^{-2}}{}$$

$H_r \left( \frac{\partial}{\partial r} \right)^\mu$

$\int_{\Sigma} R_{ab} \chi^a \chi^b d\lambda dA$

On  $\xi^{-1}$ . A boost with  $\eta$  and preserves  $\xi$ .

$\xi = \text{const.}$

Global flat spacetime in Minkowski and Rindler coordinates.

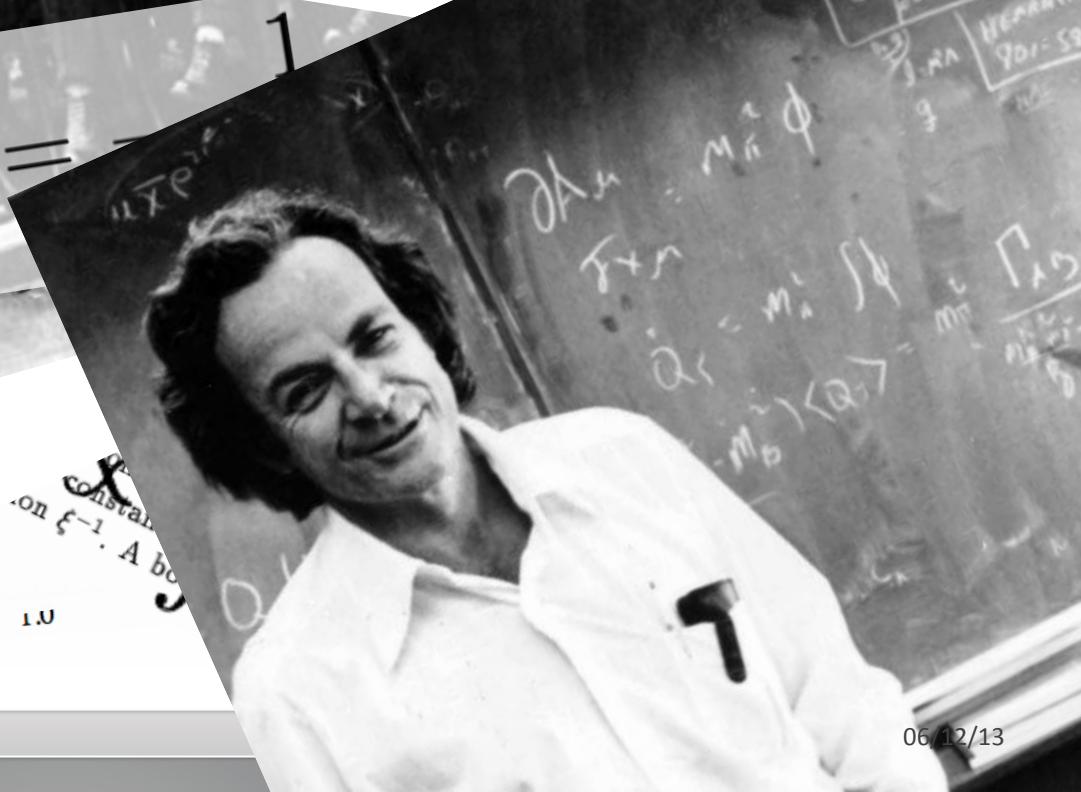
$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho)$$

$$\rho = \frac{E}{V} = \frac{H_0^{-1}/2G}{\frac{4}{3}\pi H_0^{-3}} = -$$

$$J$$

on  $\xi^{-1}$ . A  $b$

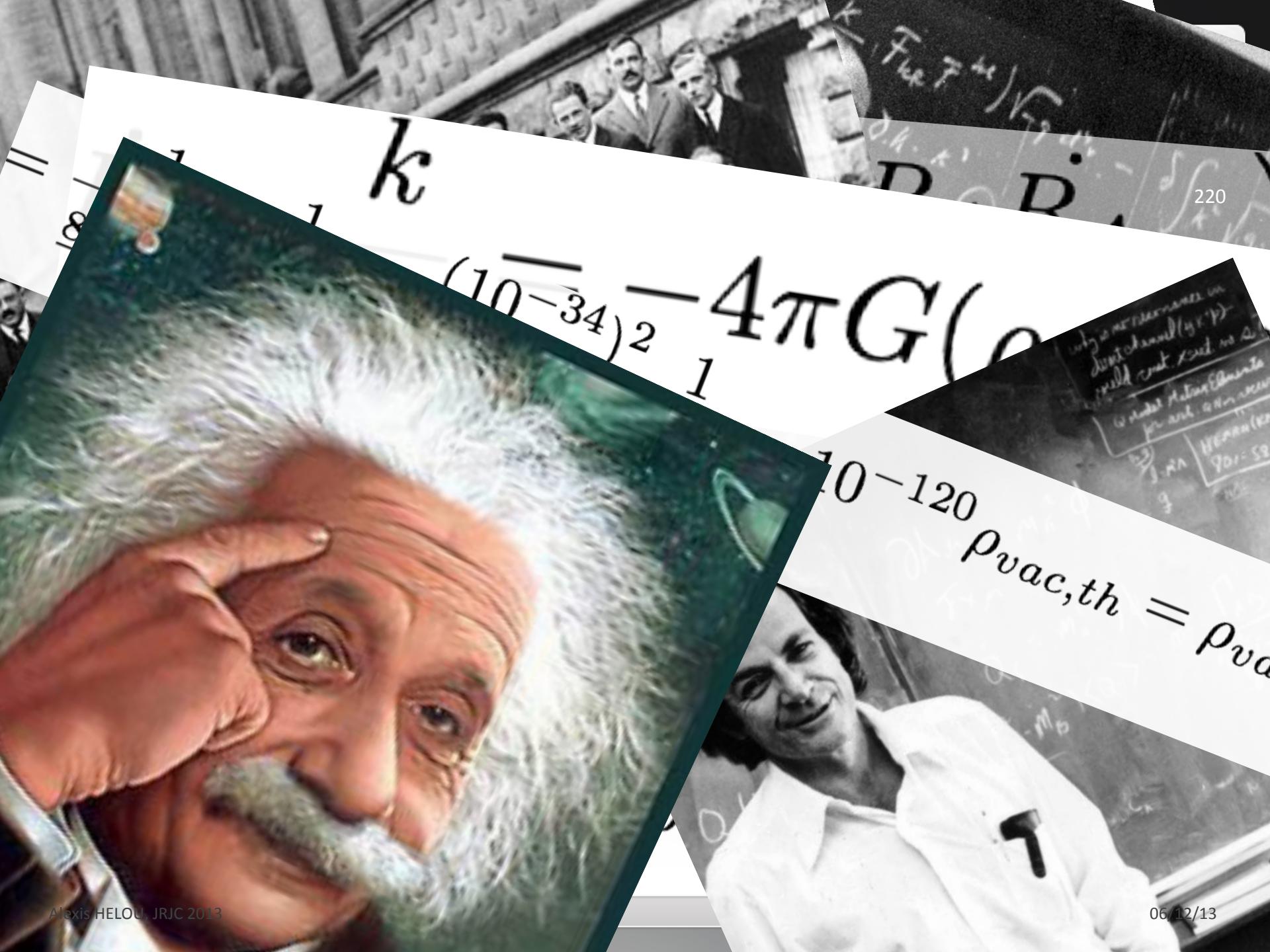
1.0



$$= \frac{1}{\frac{8\pi}{3} \frac{H_0^{-2}}{L_P^2}} \frac{1}{L_P^4} \approx \frac{(10^{-34})^2}{(10^{26})^2} \frac{1}{L_P^4} = 10^{-120} \rho_{vac,th} = \rho_{vac}$$

$$\rho = \frac{E}{V} = \frac{H_0^{-1}/2G}{\frac{4}{3}\pi H_0^{-3}} =$$

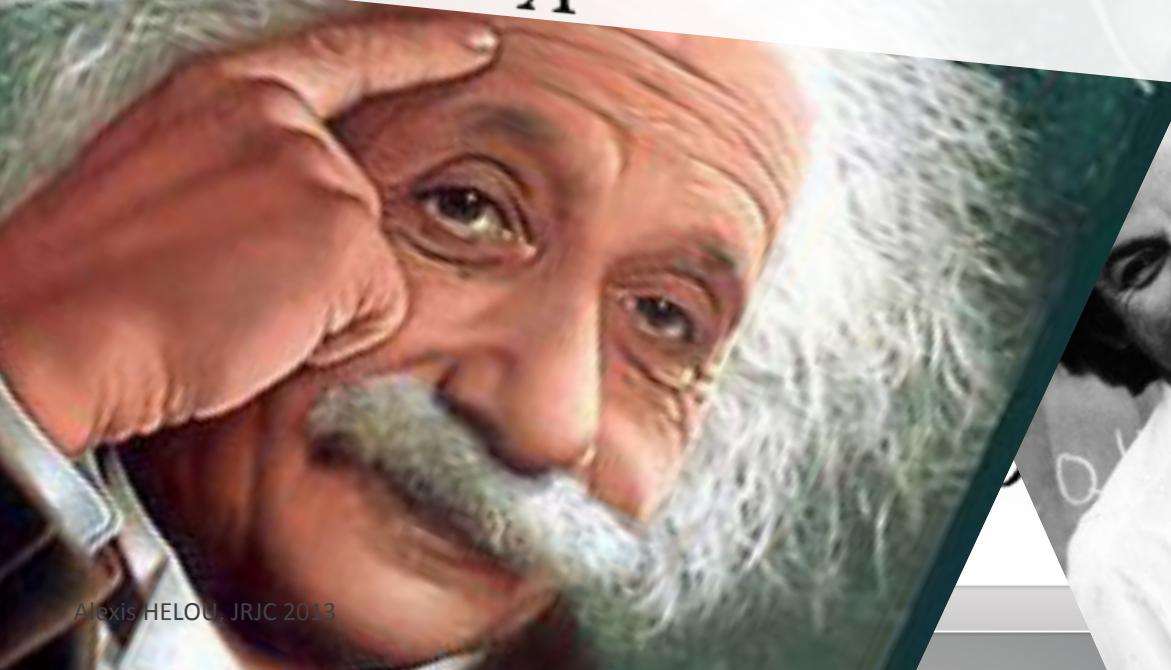




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 $k$  $(10^{-34})^2$  $1$  $-4\pi G(\rho)$  $10^{-120}$  $\rho_{vac,th} = \rho_{vac}$

$$\rho = \frac{1}{\frac{8\pi}{3} G R_A^2}$$



Alexis HELOU, JRJC 2013

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho$$



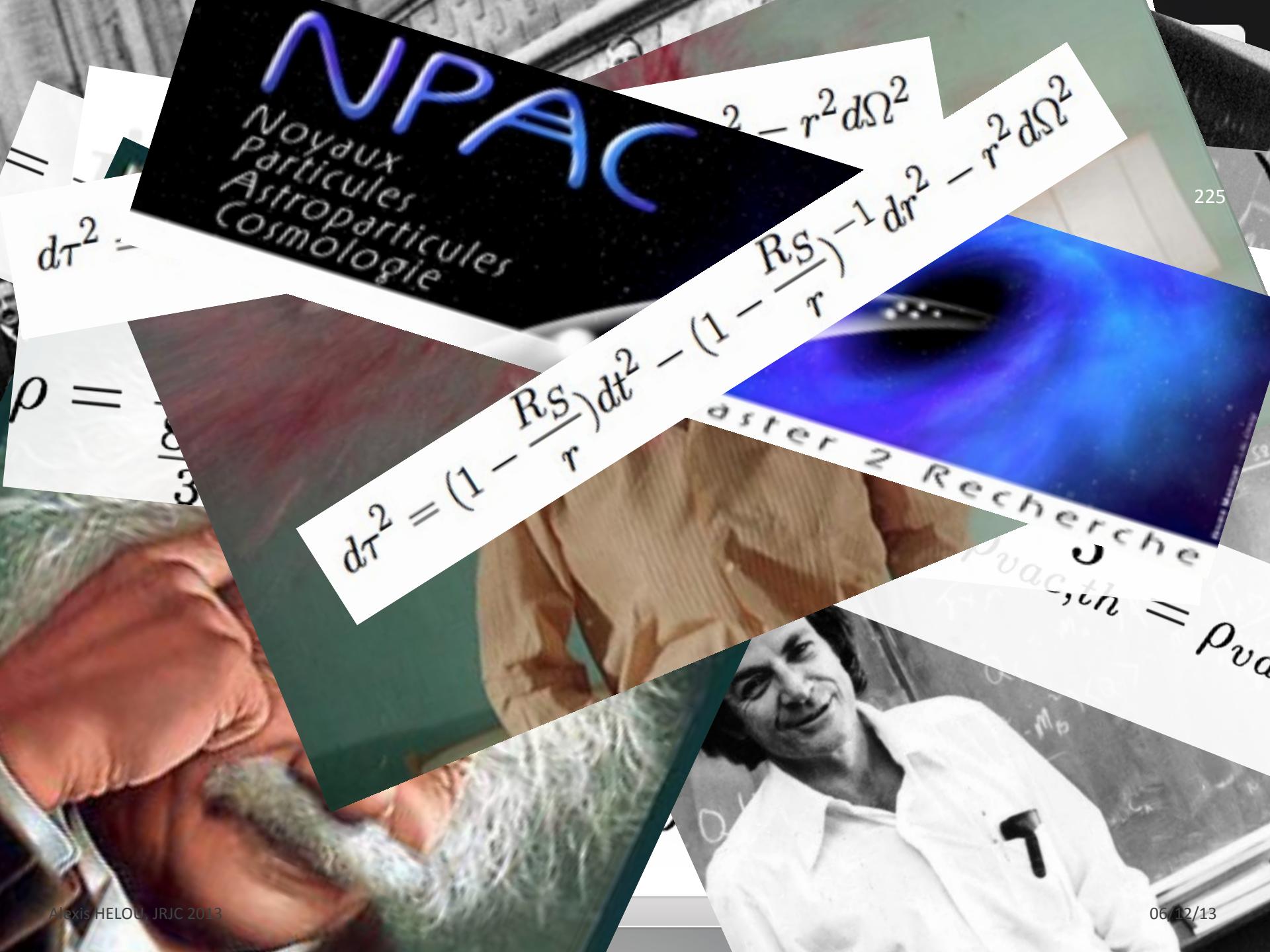
06/12/13

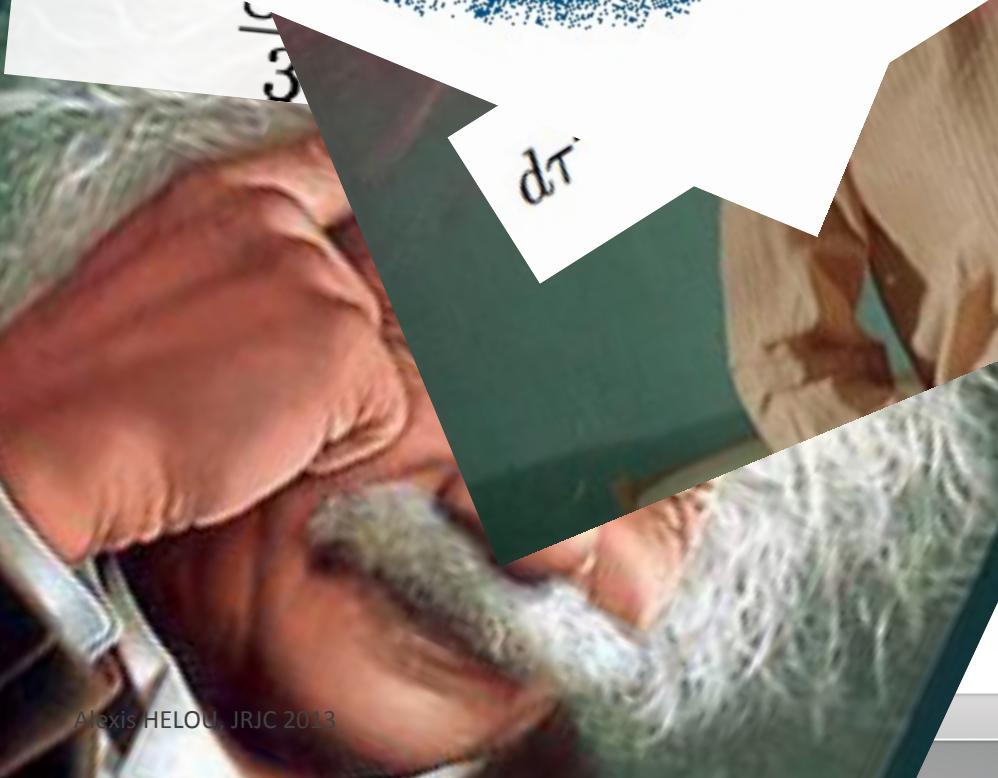
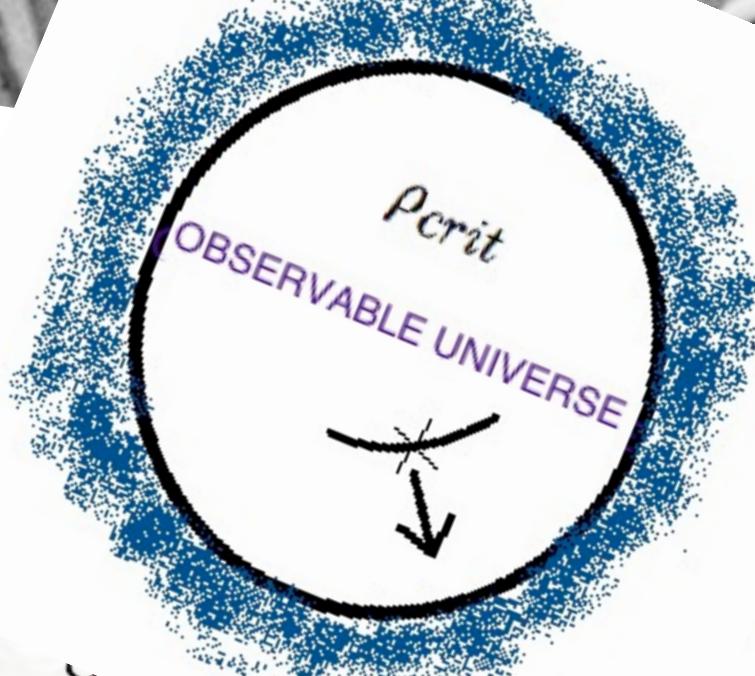


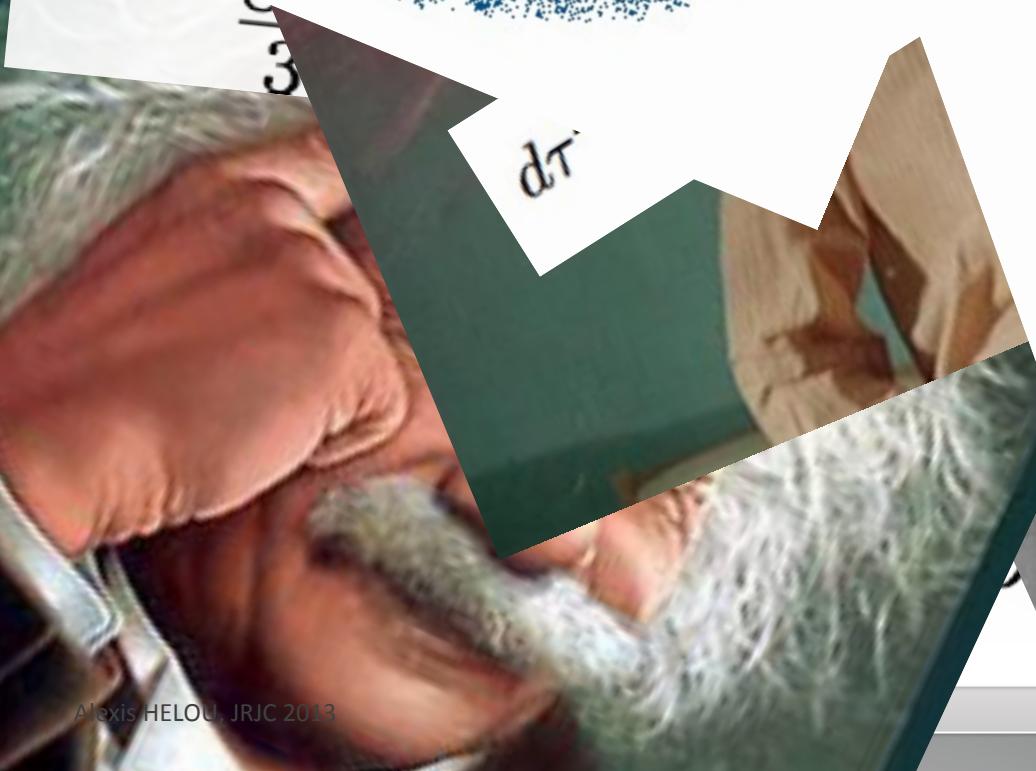
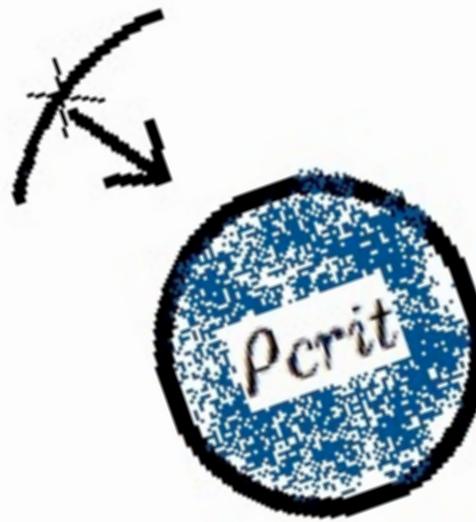
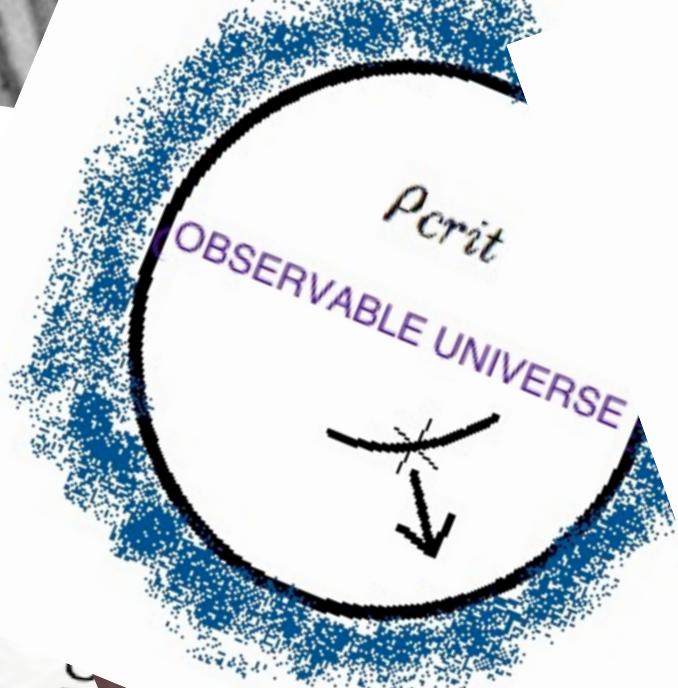
$$d\tau^2 = \left(1 - \frac{r^2}{R_H^2}\right)dt^2 - \left(1 - \frac{r^2}{R_H^2}\right)^{-1}dr^2 - r^2 d\Omega^2$$

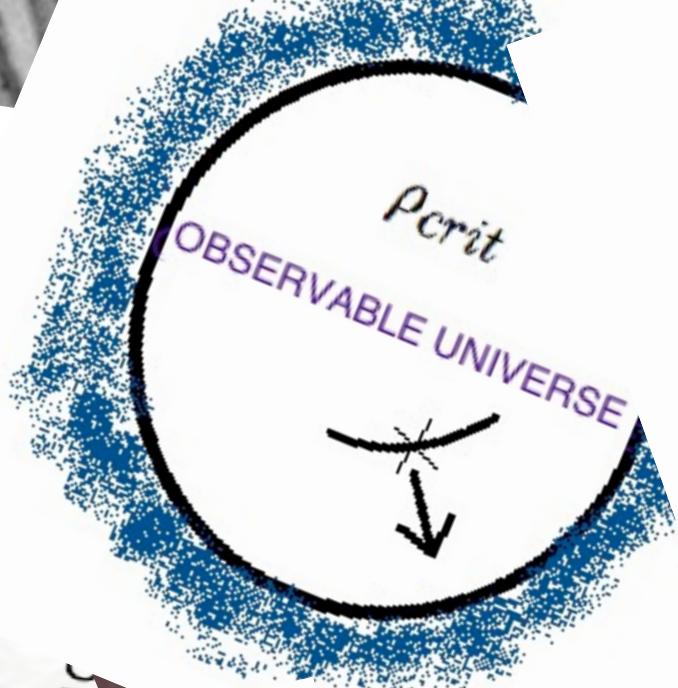




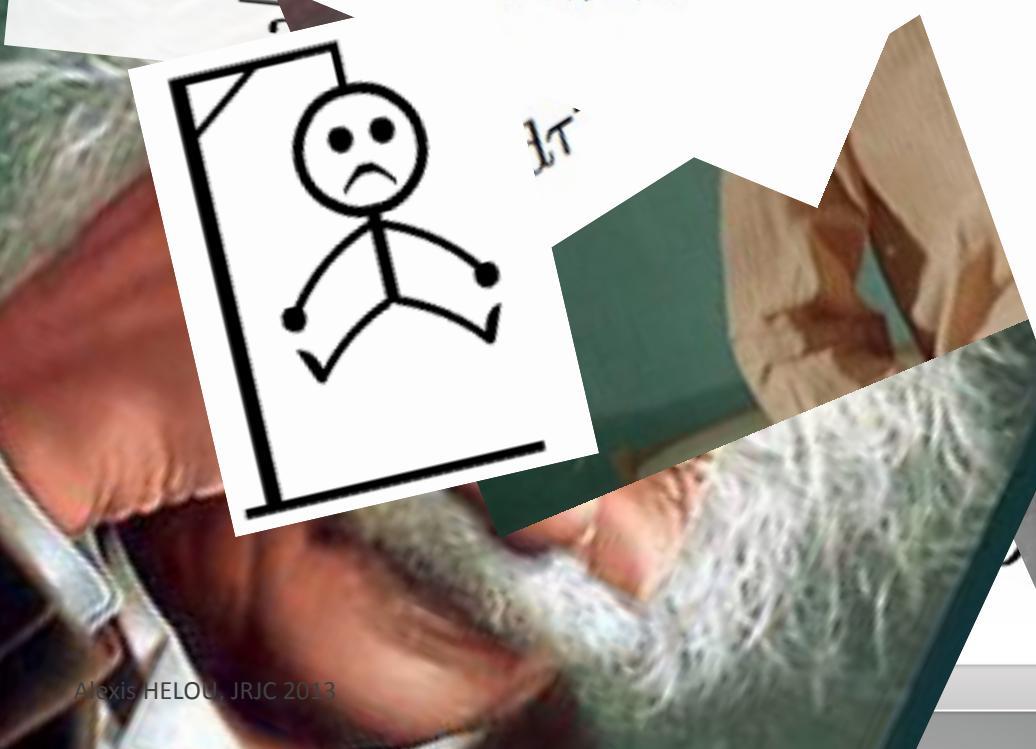


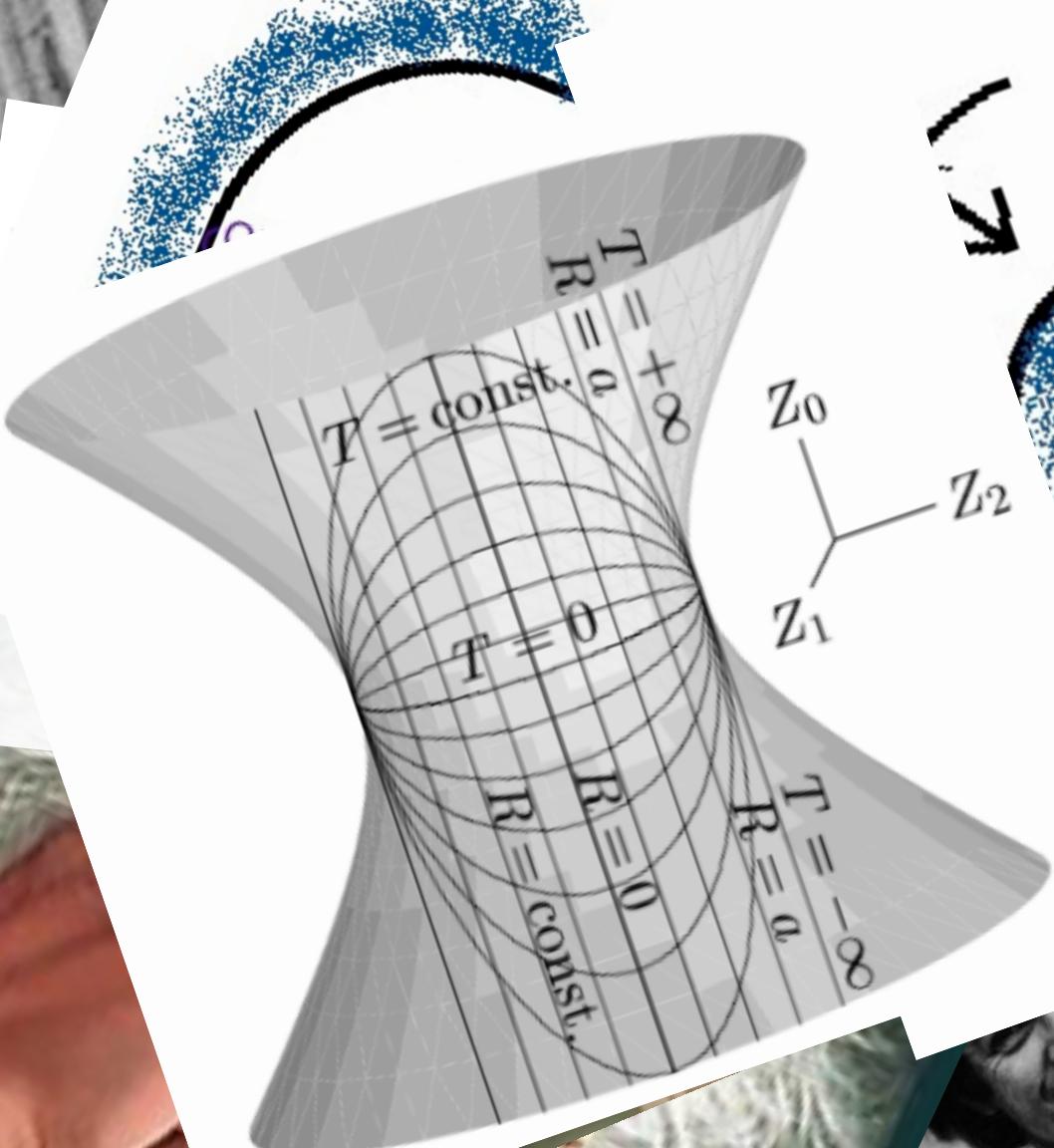






BLACK HOLE





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$$\rho_{c,th} = \rho_{vac}$$







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# CONCLUSIONS

- Some galaxies recede from us at superluminal speed.



## CONCLUSIONS

- Some galaxies recede from us at superluminal speed.
- Let us consider a galaxy beyond our Hubble Sphere, but within our Event Horizon. Therefore it is receding from us FASTER than the speed of light. However, we will be able to see it as it is right now, in the future!



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- The Hubble Sphere **IS NOT** a horizon!!!
- It is doable to present 238 slides in 20 minutes.



## Acknowledgements

- B. Racine for helping me with python and giving away his codes
- All the organisers of JRJC, for this awesome week
- Chouchou for his bad jokes (and bad music)
- Very interesting reference:  
[arXiv:astro-ph/0310808v2](https://arxiv.org/abs/astro-ph/0310808v2), Davis & Lineweaver



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# Thank you for your attention (...)

zzzzzz...





## Question pour homonyme

- The formula for the event horizon:

$$\eta_{eh} = \int_{t_0}^{+\infty} \frac{dt}{a(t)}$$

- In a matter-dominated Universe (not Dark Energy-dominated), we have:

$$a(t) \propto t^{2/3}$$

- Then how far will we be able to see in this decelerated Universe?