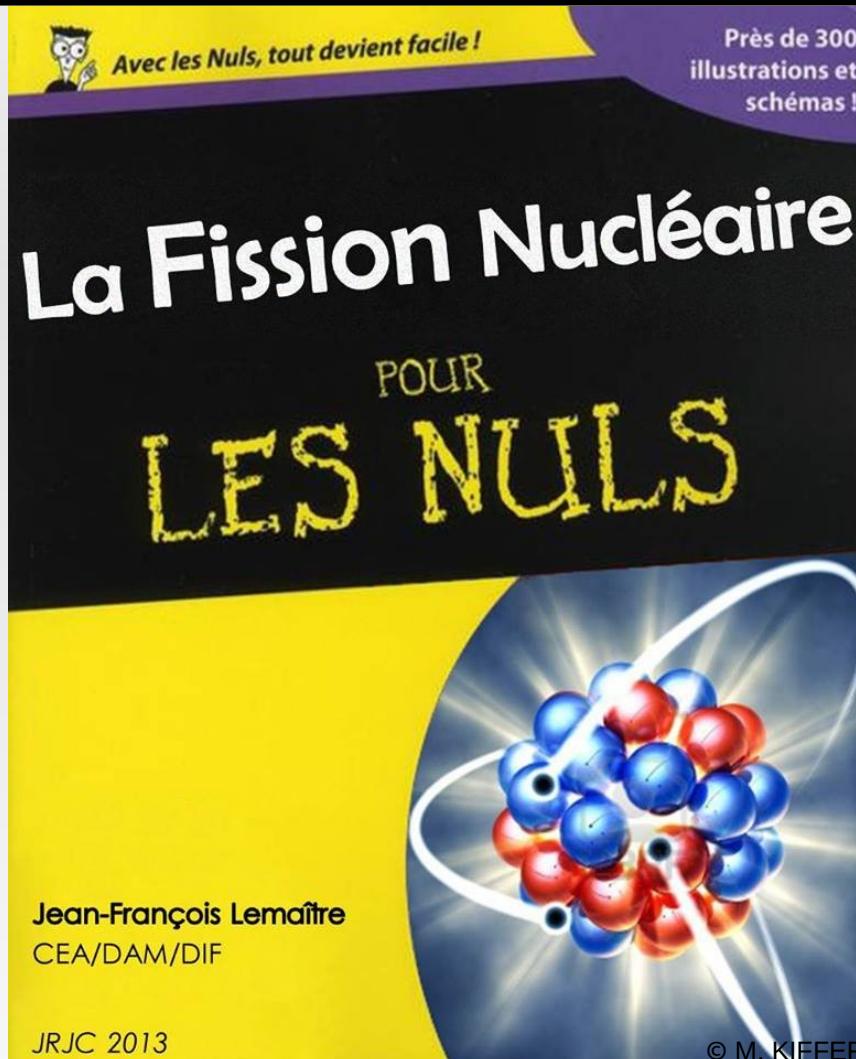


# Modélisation de la fission avec SPY (Scission Point Yields)



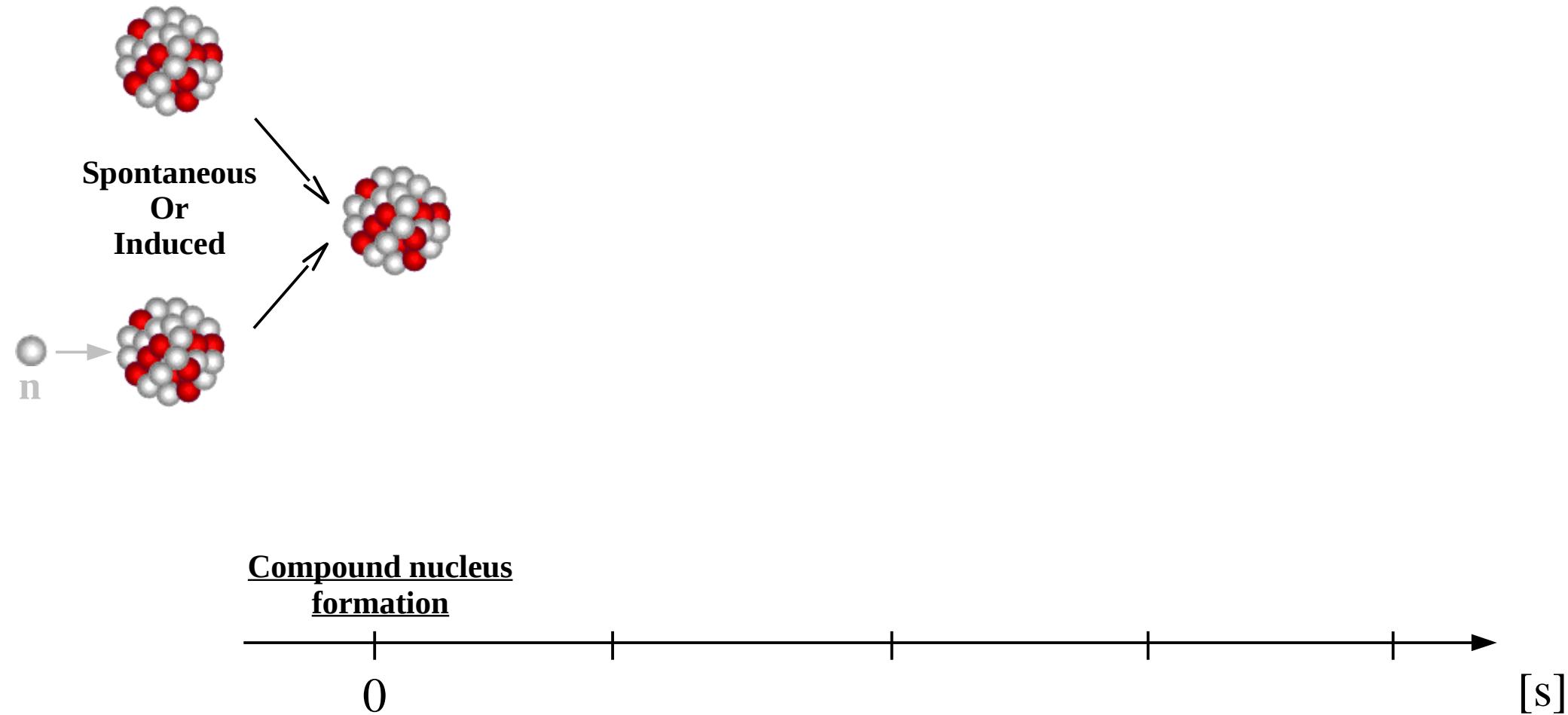
J.-F. Lemaître<sup>1,2</sup>, N. Dubray<sup>2</sup>, S. Hilaire<sup>2</sup>, S. Panebianco<sup>1</sup>, J.-L. Sida<sup>1</sup>

1 : CEA Centre de Saclay, Irfu, 91191 Gif-sur-Yvette, France

2 : CEA, DAM, DIF, 91297 Arpajon, France

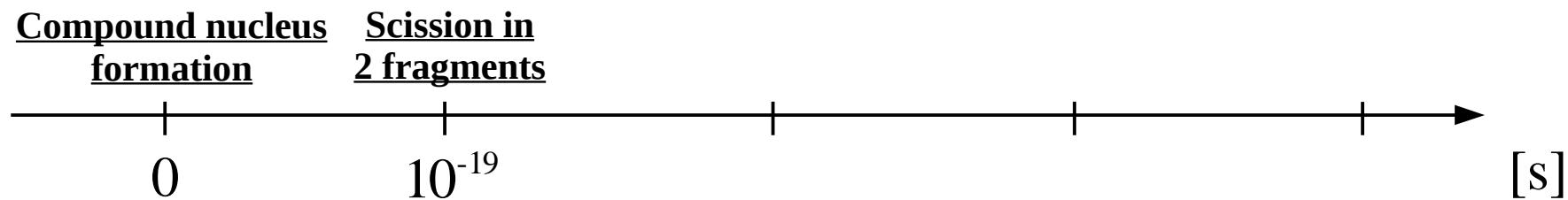
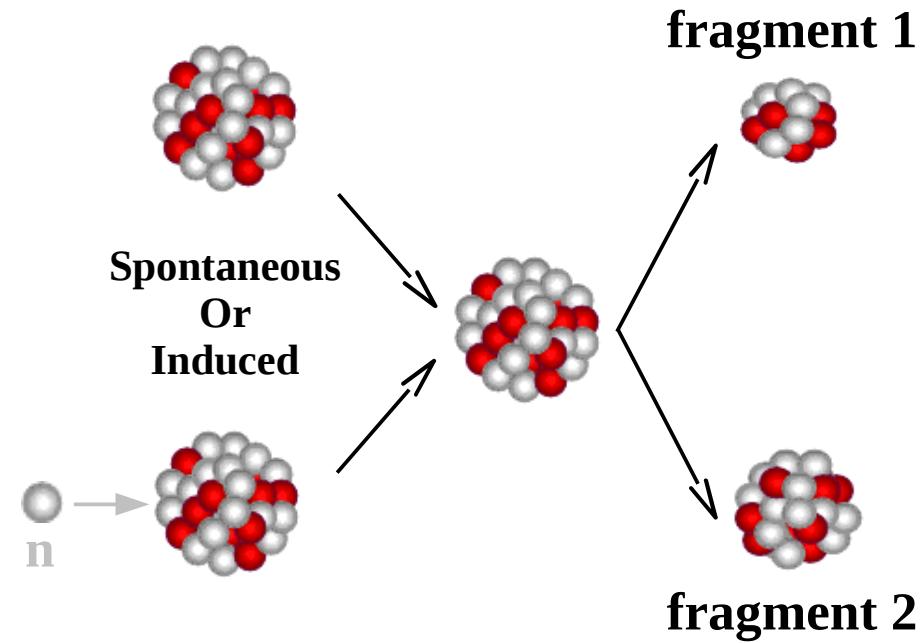
# Fission process

## From mother nucleus to fission products



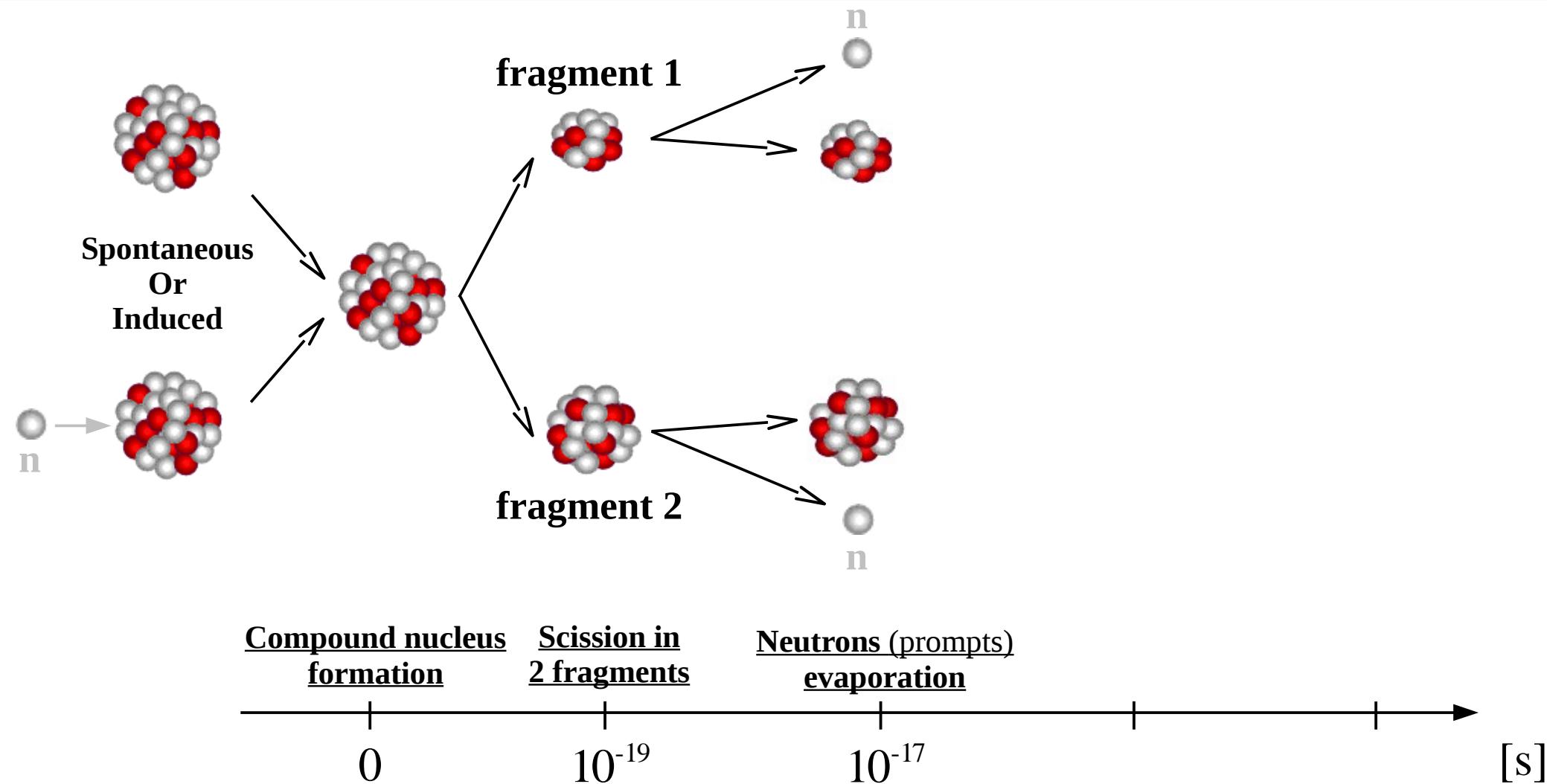
# Fission process

## From mother nucleus to fission products



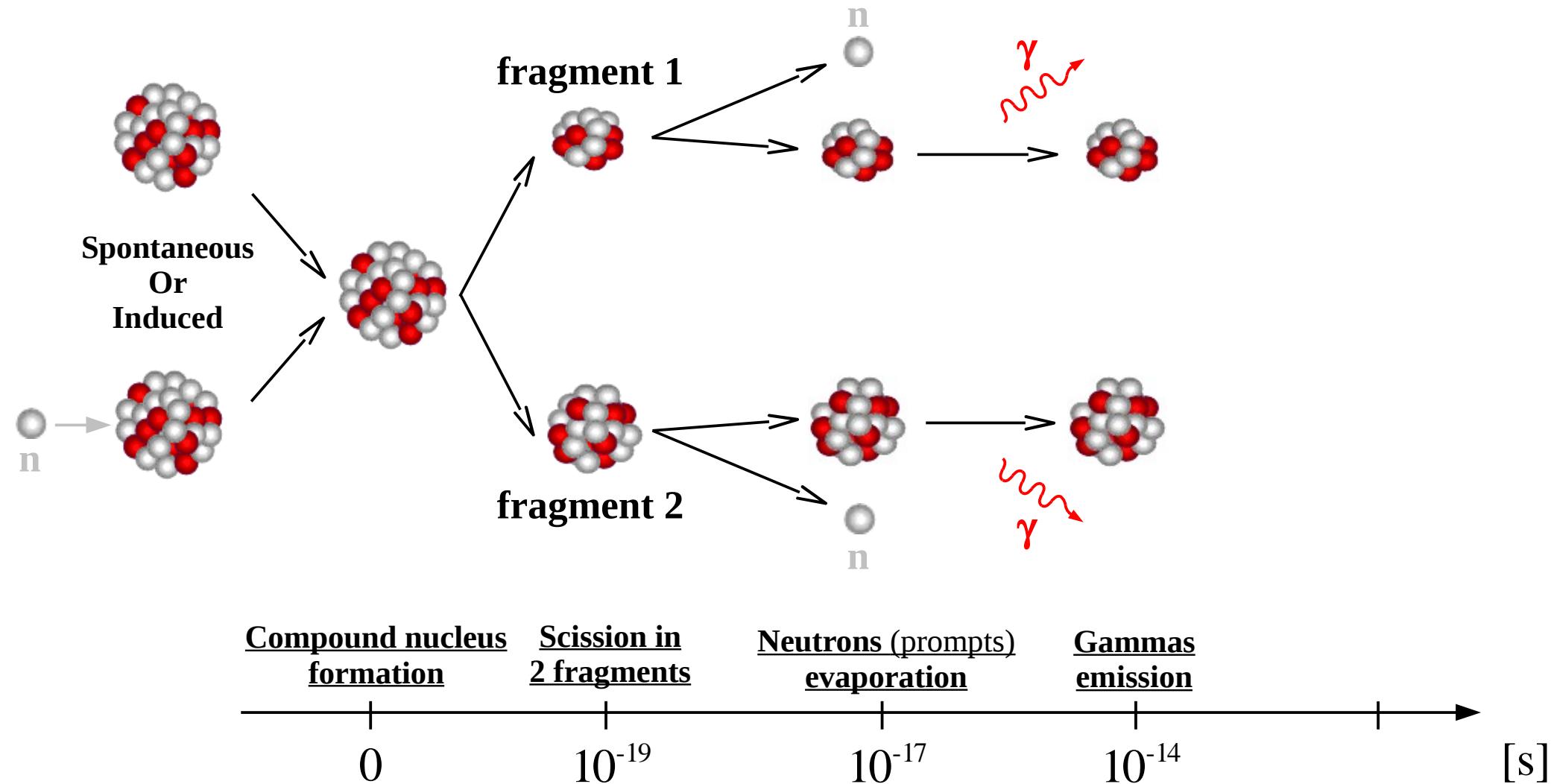
# Fission process

## From mother nucleus to fission products



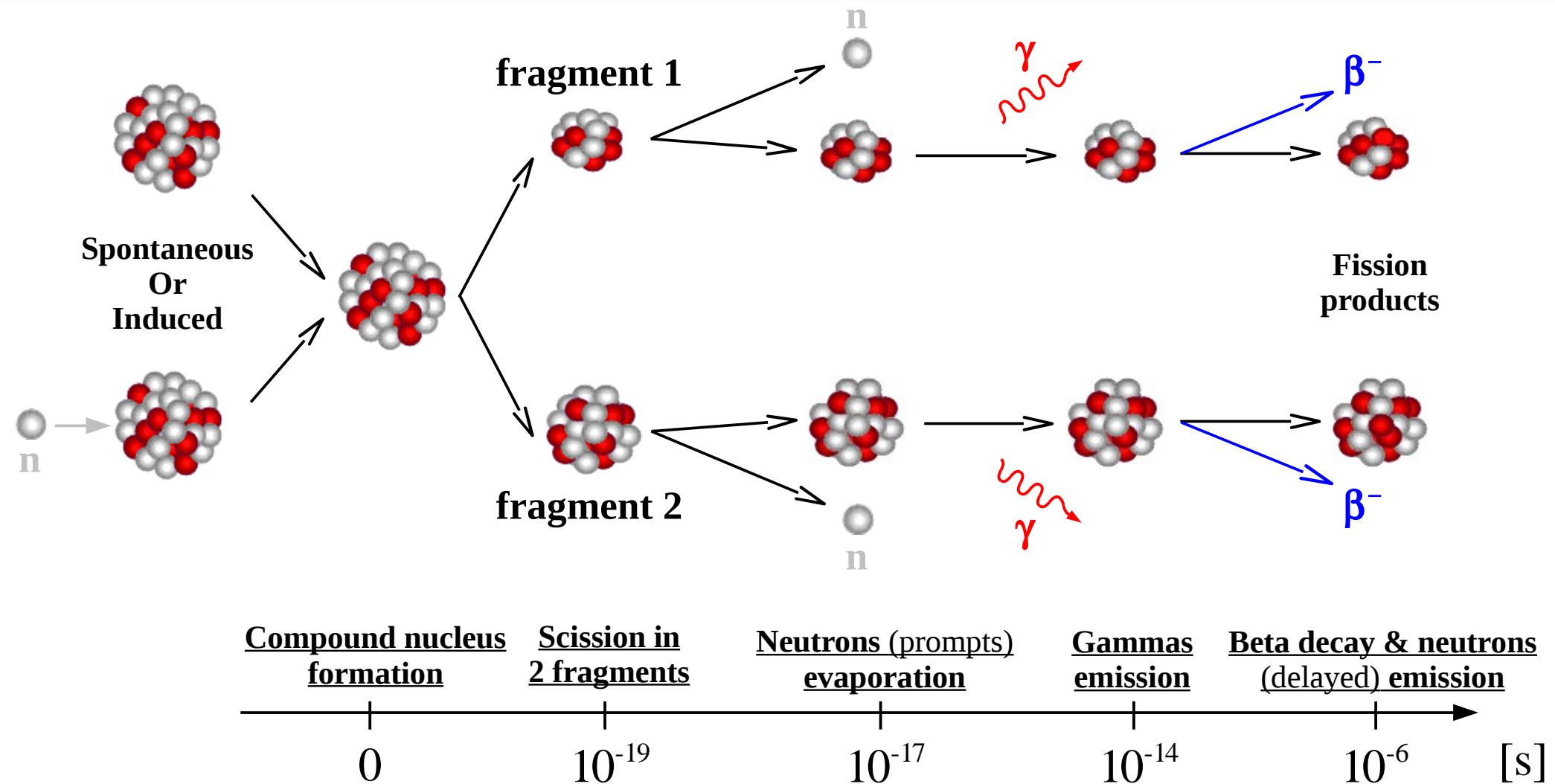
# Fission process

## From mother nucleus to fission products



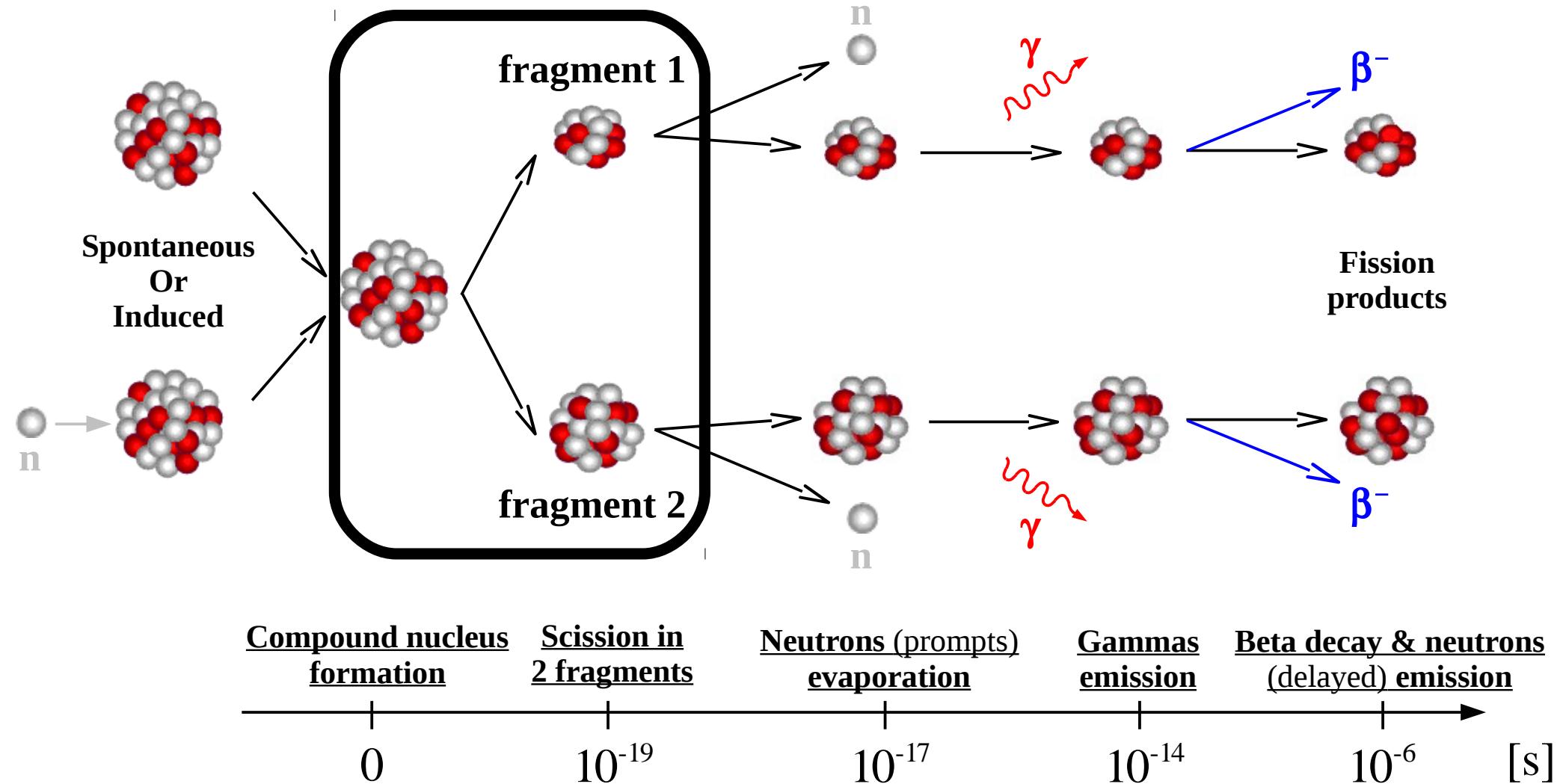
# Fission process

## From mother nucleus to fission products



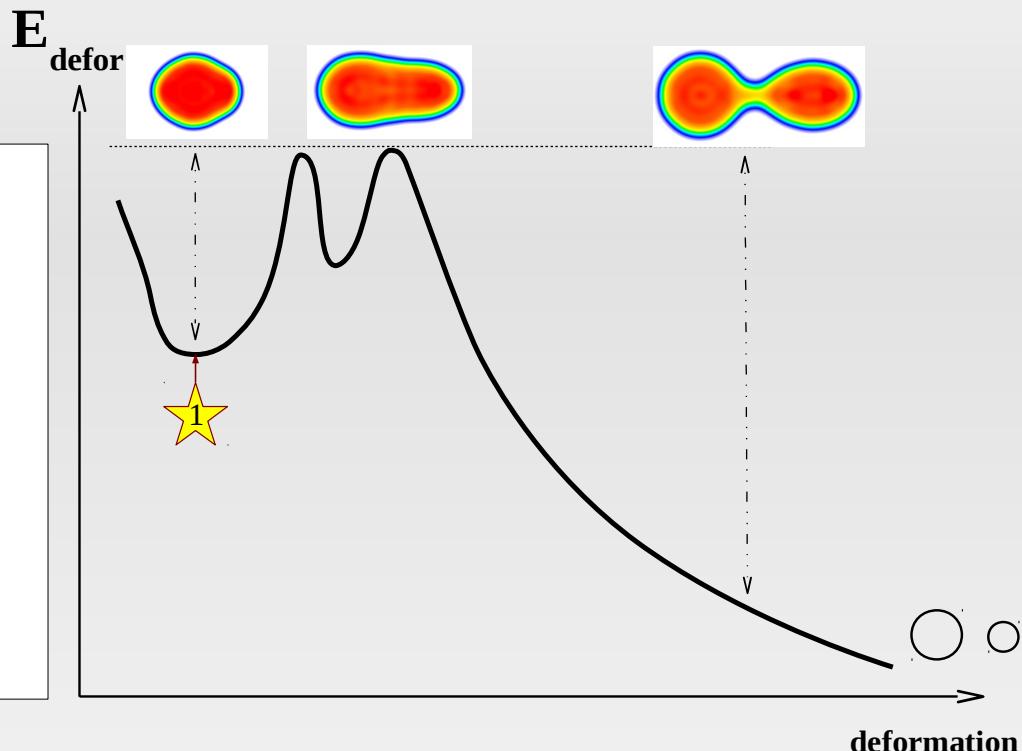
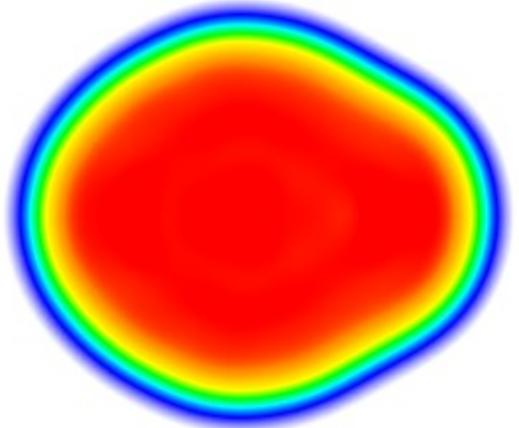
# Fission process

## From mother nucleus to fission products



# From compound nucleus to scission

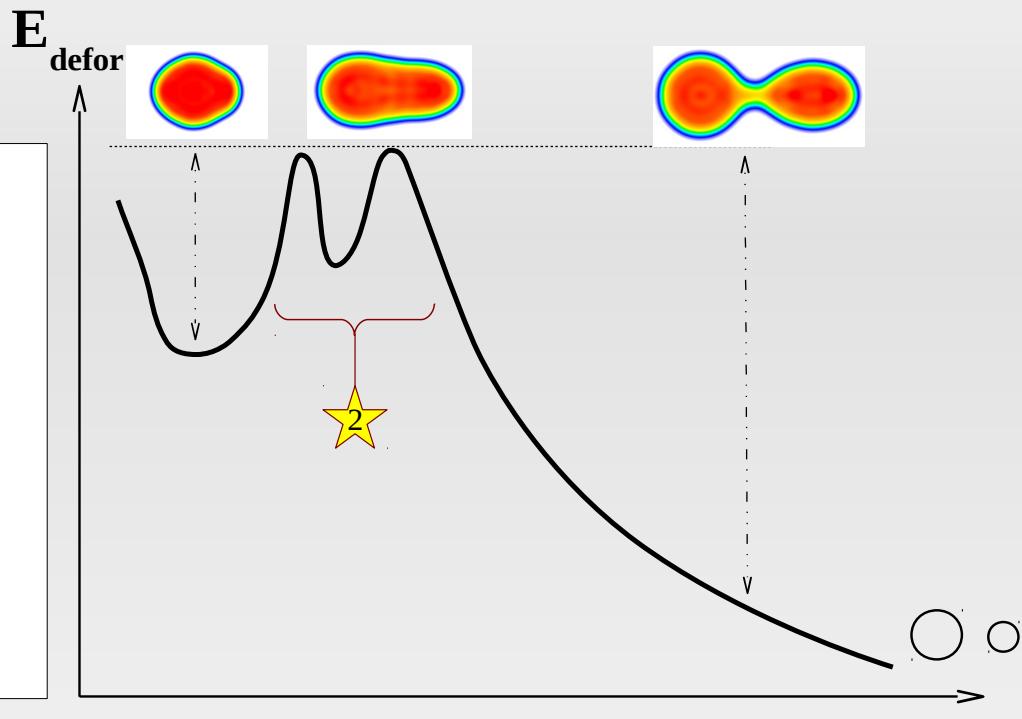
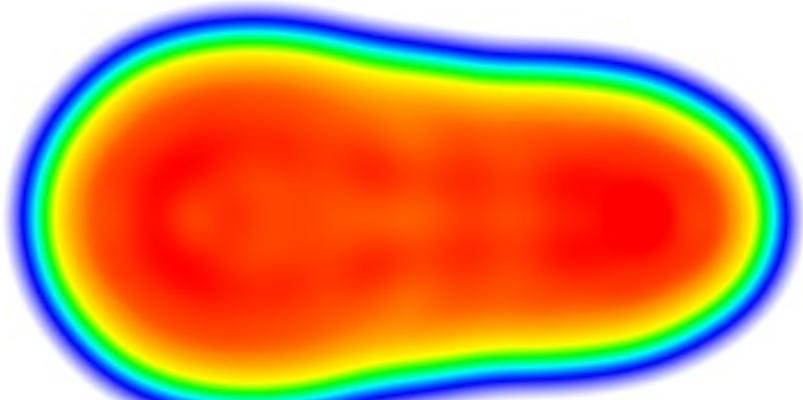
## 1 - Compound nucleus



- Induced fission : excited compound nucleus is formed by neutron capture, Coulomb excitation, photoexcitation . . . on a given nucleus
- Spontaneous fission : nucleus fissions without external excitation

# From compound nucleus to scission

## 2 - Fission barrier – Saddle point

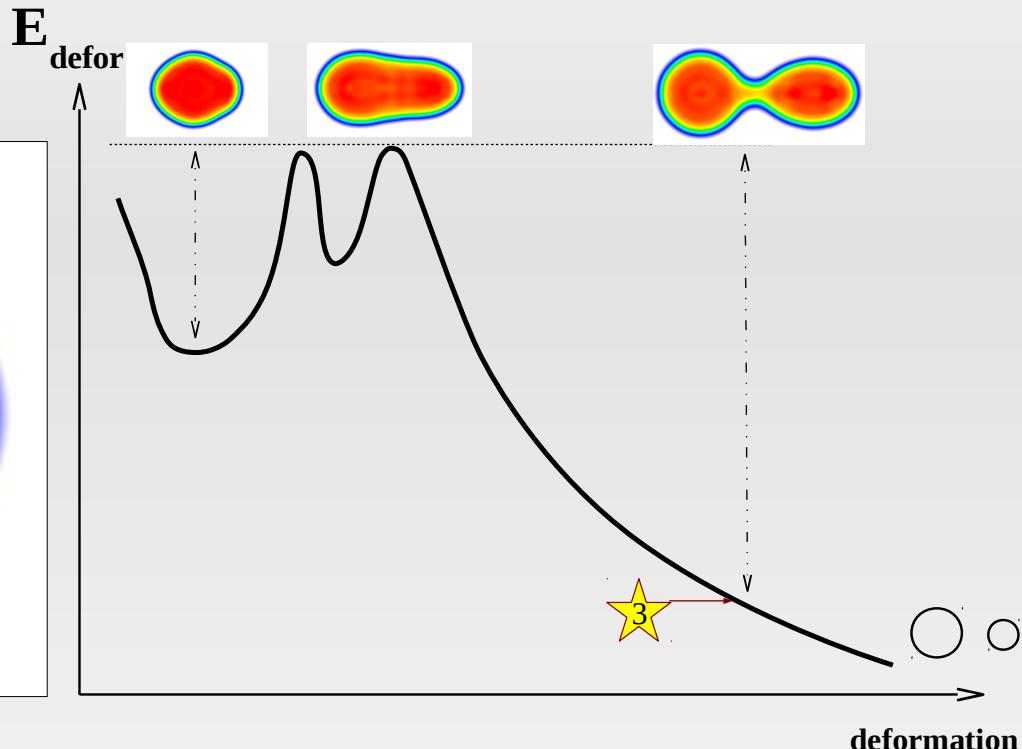
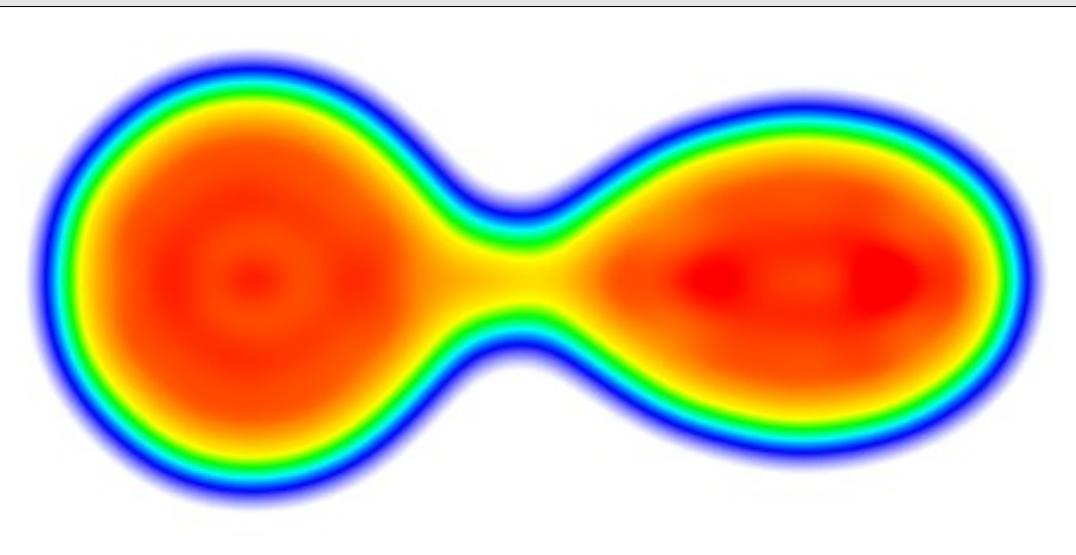


Fission barrier : potential energy of the nucleus from competition between nuclear interaction and Coulomb repulsion.

- Induced fission : excited compound nucleus go over fission barrier
- Spontaneous fission : nucleus "tunnels through" fission barrier

# From compound nucleus to scission

## 3 - Scission point



Scission point corresponds to the system configuration where the properties ( $Z$ ,  $N$ ,  $\beta$ ,  $E^*$ ) of the two fragments (=fragmentation) are fixed

# From compound nucleus to scission

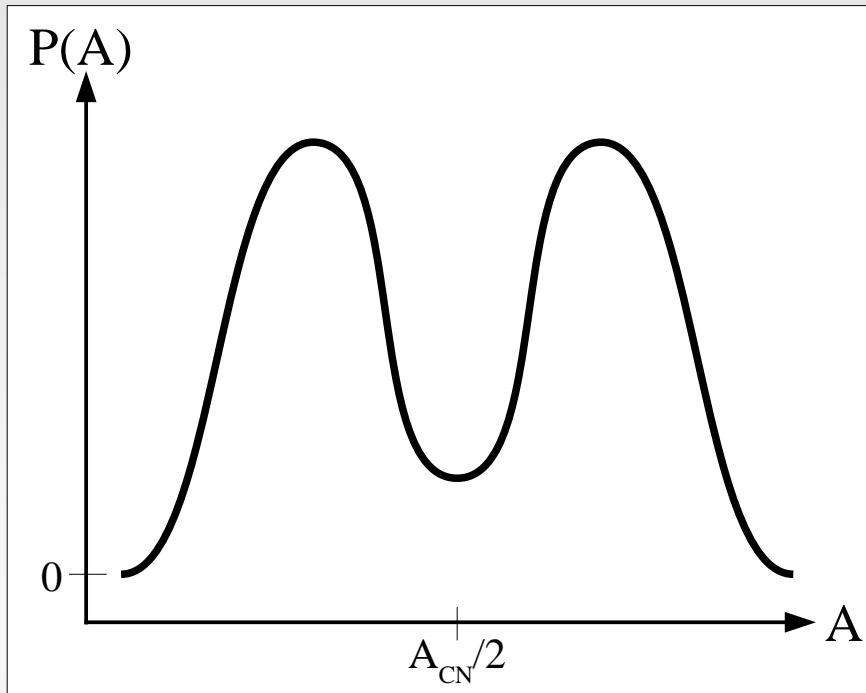
## Fission Yield – Asymmetric / Symmetric fission



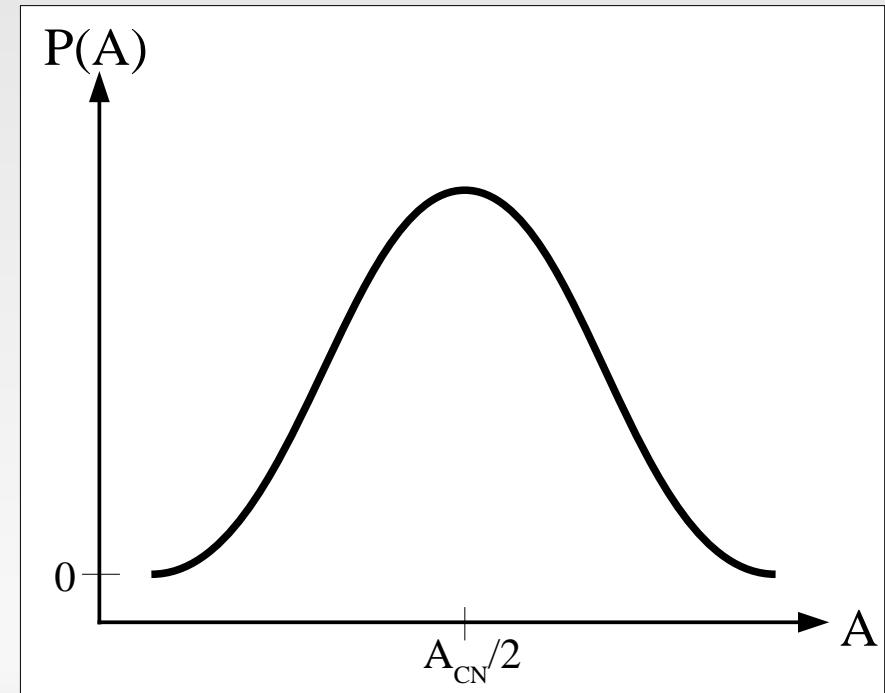
$P(Z_1, N_1, Z_2, N_2)$  = fission yield (normalized to 100%)

$P(A_1) = P(A_2) = 2P(A_1, A_2)$  is isobaric fission yield (normalized to 200%)

Asymmetric fission

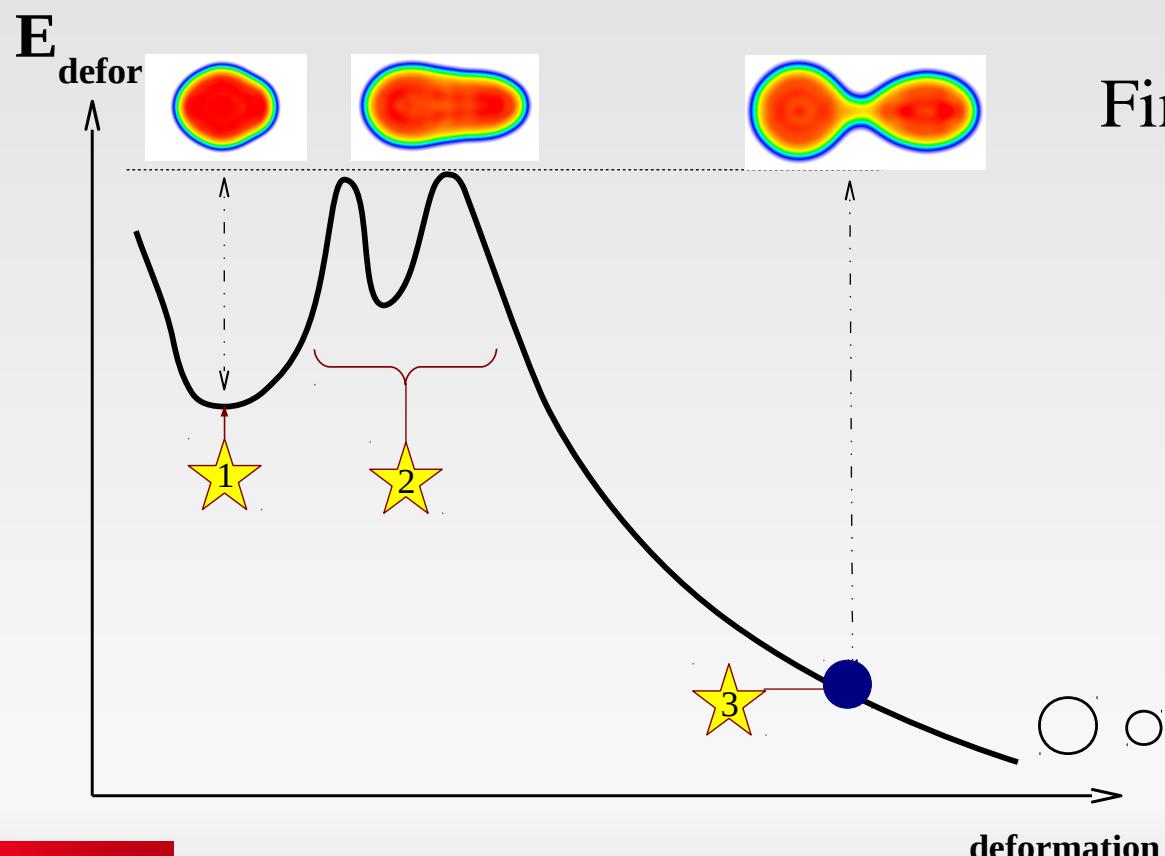


Symmetric fission



# SPY : a new scission point model

## Scission point

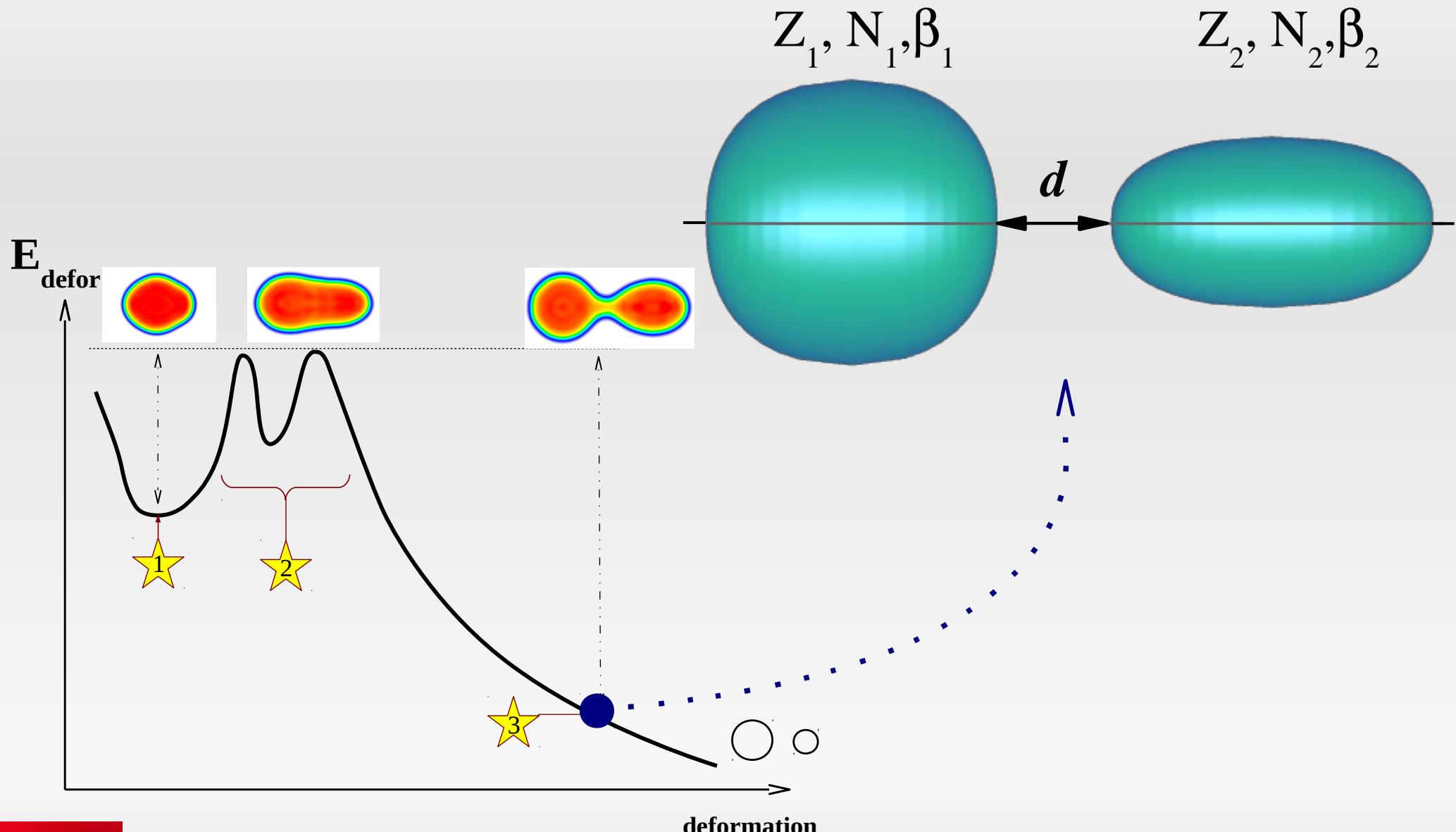


Final state observables :

- yield
- TKE
- TXE
- ...

# SPY : a new scission point model

## System at scission



# SPY : a new scission point model

Between full microscopic & full phenomenological approaches

SPY  
a new scission-point model  
based on microscopic ingredients

# SPY : a new scission point model

Between full microscopic & full phenomenological approaches

SPY

a new scission-point model  
based on microscopic ingredients



Strong hypothesis to model fission process :

- static
- CN formation mechanism neglected
- all fragments properties freezed at scission

# SPY : a new scission point model

Between full microscopic & full phenomenological approaches

SPY

a new scission-point model  
based on microscopic ingredients

Strong hypothesis to model fission process :

- static
- CN formation mechanism neglected
- all fragments properties freezed at scission

Full microscopic data are used  
(Mean field from nucleon-nucleon interaction)

# SPY : model presentation

## Statistical model at scission point

- Thermodynamic equilibrium at scission is supposed
    - statistical equilibrium among system degrees of freedom
  - Isolated fragments
    - microcanonical statistical description
- ⇒ all states at scission are equiprobable

# SPY : model presentation

## Statistical model at scission point

- Thermodynamic equilibrium at scission is supposed
    - statistical equilibrium among system degrees of freedom
  - Isolated fragments
    - microcanonical statistical description
- ⇒ all states at scission are equiprobable

A system configuration is defined by the two fragments DoF :

- proton and neutron numbers ( $Z_1, N_1, Z_2, N_2$ )
- quadrupolar deformations ( $\beta_1, \beta_2$ )
- intrinsic excitation energy ( $E_1^*, E_2^*$ )

# SPY : model presentation

## Statistical model at scission point

- Thermodynamic equilibrium at scission is supposed
    - statistical equilibrium among system degrees of freedom
  - Isolated fragments
    - microcanonical statistical description
- ⇒ all states at scission are equiprobable

A system configuration is defined by the two fragments DoF :

- proton and neutron numbers ( $Z_1, N_1, Z_2, N_2$ )
- quadrupolar deformations ( $\beta_1, \beta_2$ )
- intrinsic excitation energy ( $E_1^*, E_2^*$ )

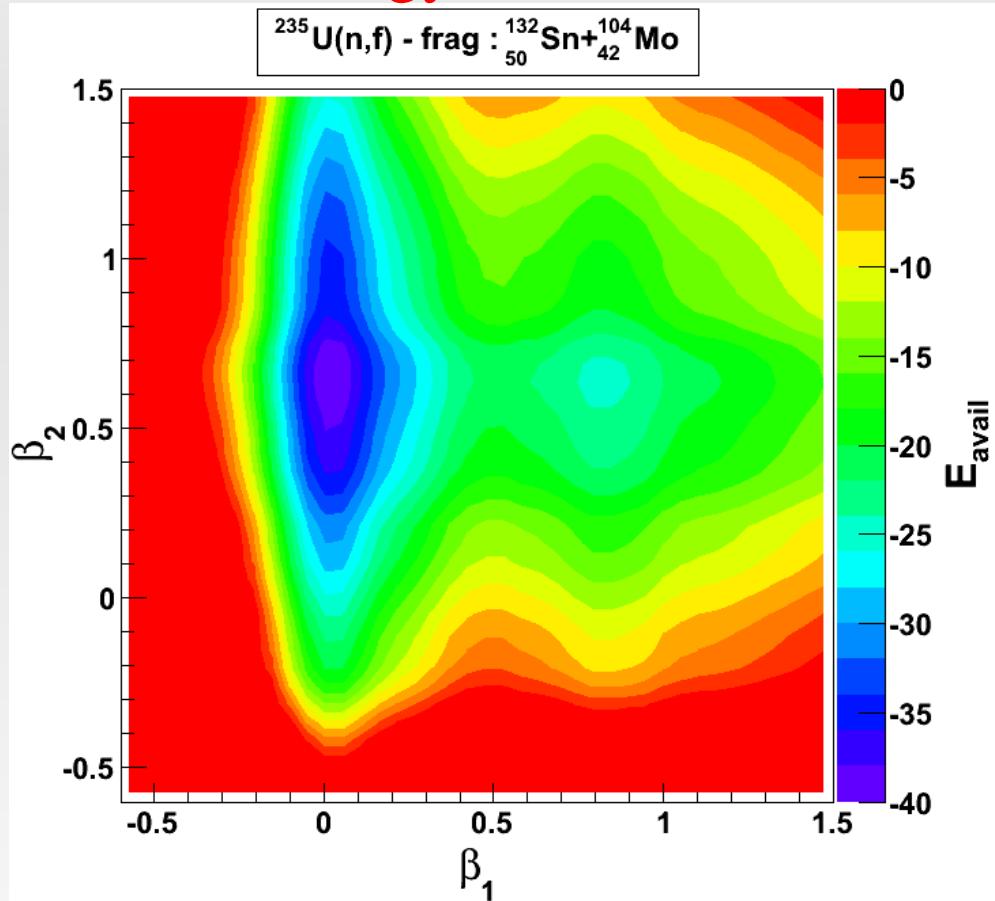
Two quantities are needed to calculate average observables :

- available energy for each configuration :  $E_{\text{avail}}$
- state density of fragments 1 & 2 :  $\rho_1, \rho_2$

# SPY : model presentation

## Available energy

- Available energy balance at scission **for each fragmentation** (500-1000)



→ fragments individual energy  
microscopic calculation HFB - Gogny D1S  
(Amedee data base)  
→ interaction energy  
(nuclear + Couloub energy)

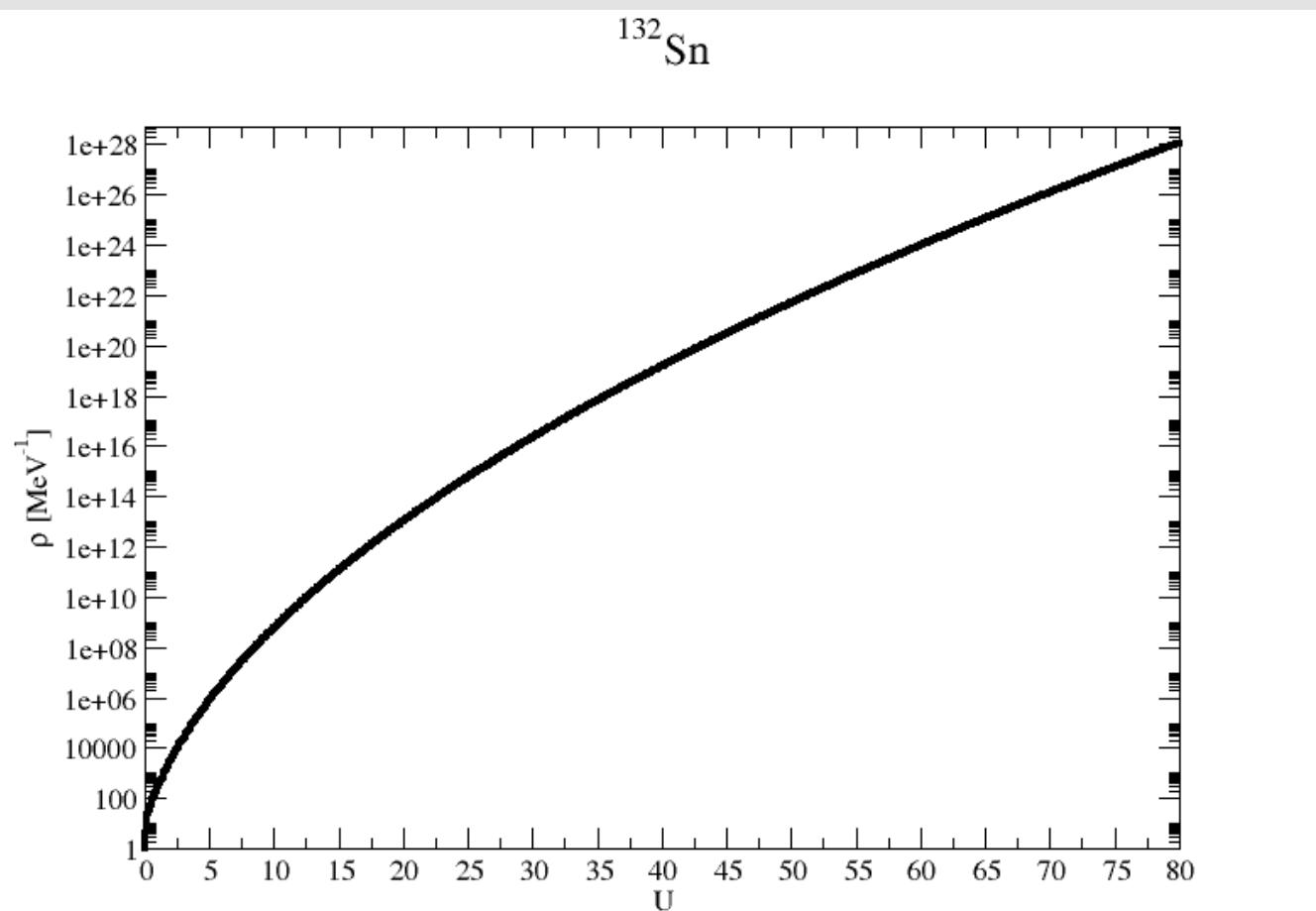
$$\begin{aligned} E_{\text{avail}} = & E_{\text{HFB1}}(Z_1, N_1, \beta_1) + E_{\text{HFB2}}(Z_2, N_2, \beta_2) \\ & + E_{\text{coul}}(d, Z_1, N_1, \beta_1, Z_2, N_2, \beta_2) + E_{\text{nucl}}(d, Z_1, N_1, \beta_1, Z_2, N_2, \beta_2) \\ & - E_{\text{CN}} \end{aligned} \quad \text{if } E_{\text{avail}} < 0: \text{fragmentation is allowed}$$

# SPY : model presentation

## State density

Presently Fermi gas state density type is used (from generalized superfluid model)

$$\rho(U) \propto e^{2\sqrt{a}U} \quad \text{with } a \approx A/8$$



# SPY : model presentation

## Statistical treatment

→ averaged observables : yields,  $\langle \text{TKE} \rangle$ ,  $\langle \text{TXE} \rangle$  . . .

$$\pi(z_1, N_1, z_2, N_2, \beta_1, \beta_2, x) = \rho_1(x | E_{\text{avail}}) \rho_2((1-x) | E_{\text{avail}}) \delta E^2$$

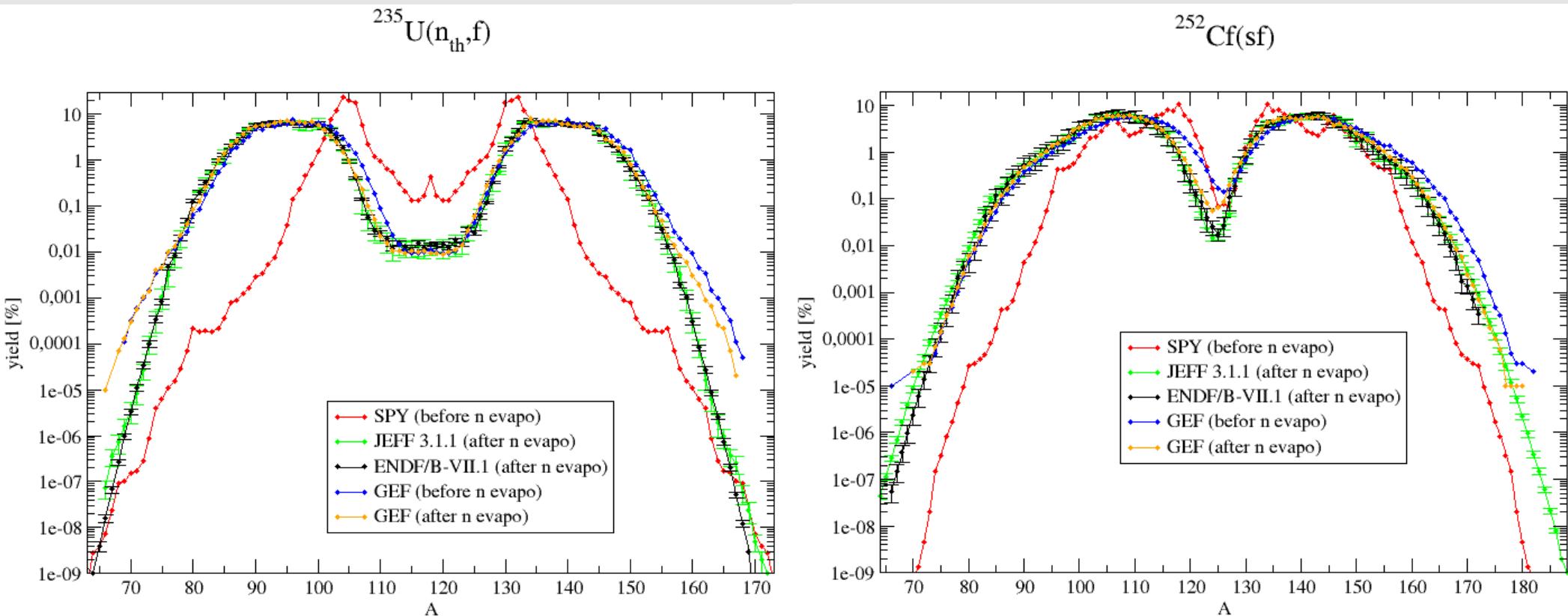
$$\Pi(z_1, N_1, z_2, N_2, \beta_1, \beta_2) = \int_0^1 \pi(z_1, N_1, z_2, N_2, \beta_1, \beta_2, x) dx$$

$$P(z_1, N_1, z_2, N_2) = \int_{-0.6}^{1.3} \int_{-0.6}^{1.3} \Pi(z_1, N_1, z_2, N_2, \beta_1, \beta_2) d\beta_1 d\beta_2$$

- $\pi$  : probability of a given configuration, with  $x$  the fraction of available energy given to excite the fragment 1
- $\Pi$  : probability of a given fragmentation at fixed deformation
- $P$  : yield of a given fragmentation

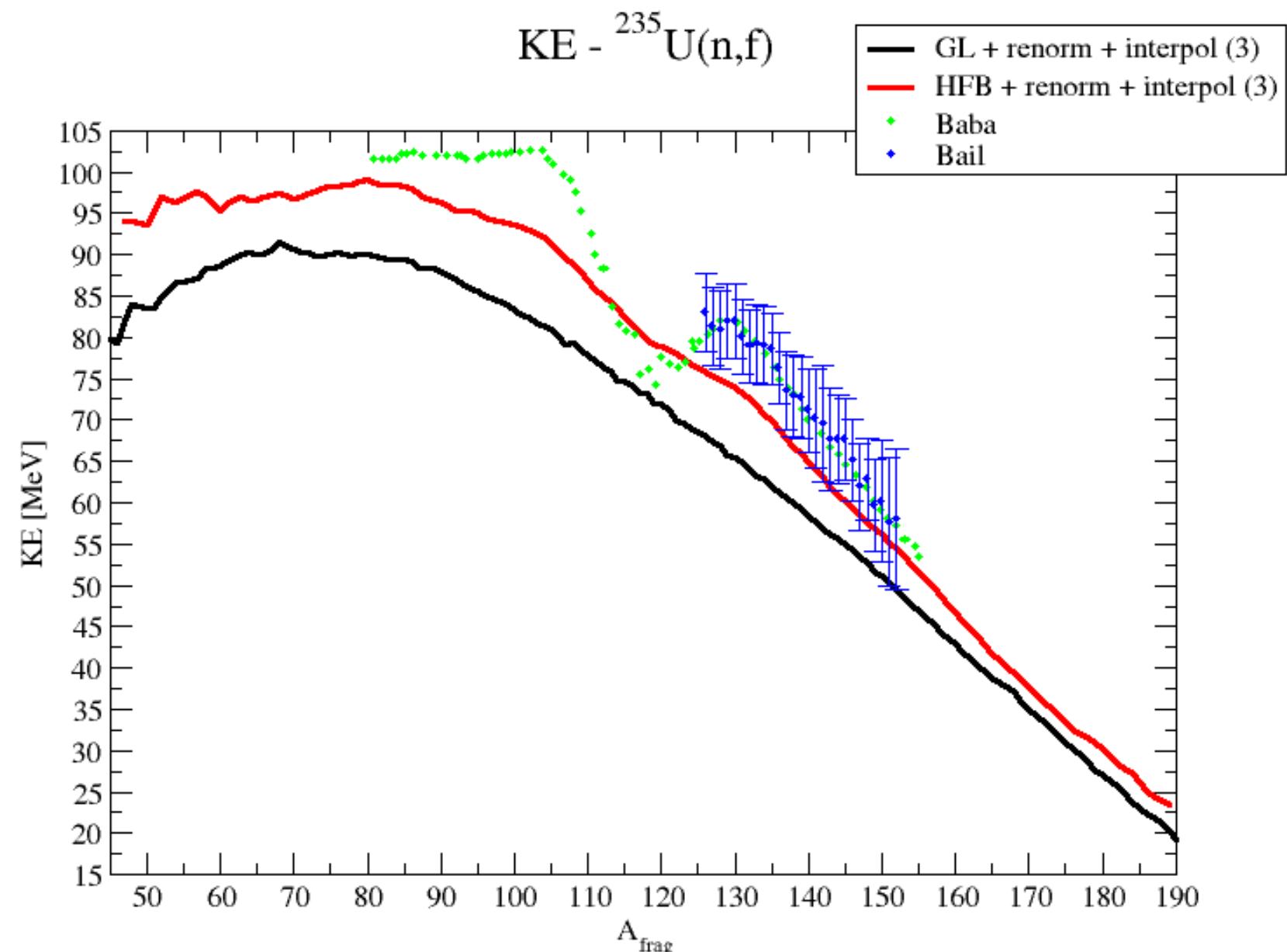
# SPY : results

## Fission yield of $^{235}\text{U}(\text{n},\text{f})$ and $^{252}\text{Cf}(\text{sf})$



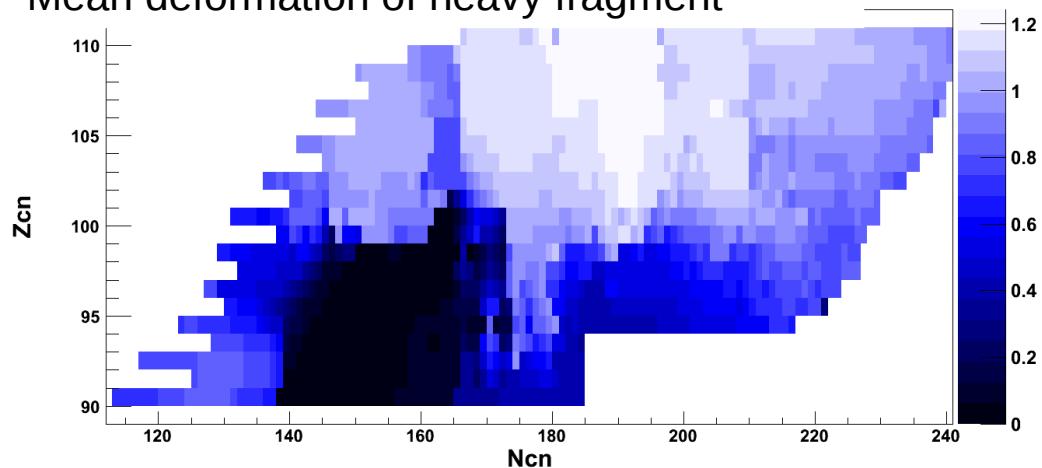
# SPY : results

## Kinetic Energy from $^{235}\text{U}(\text{n},\text{f})$

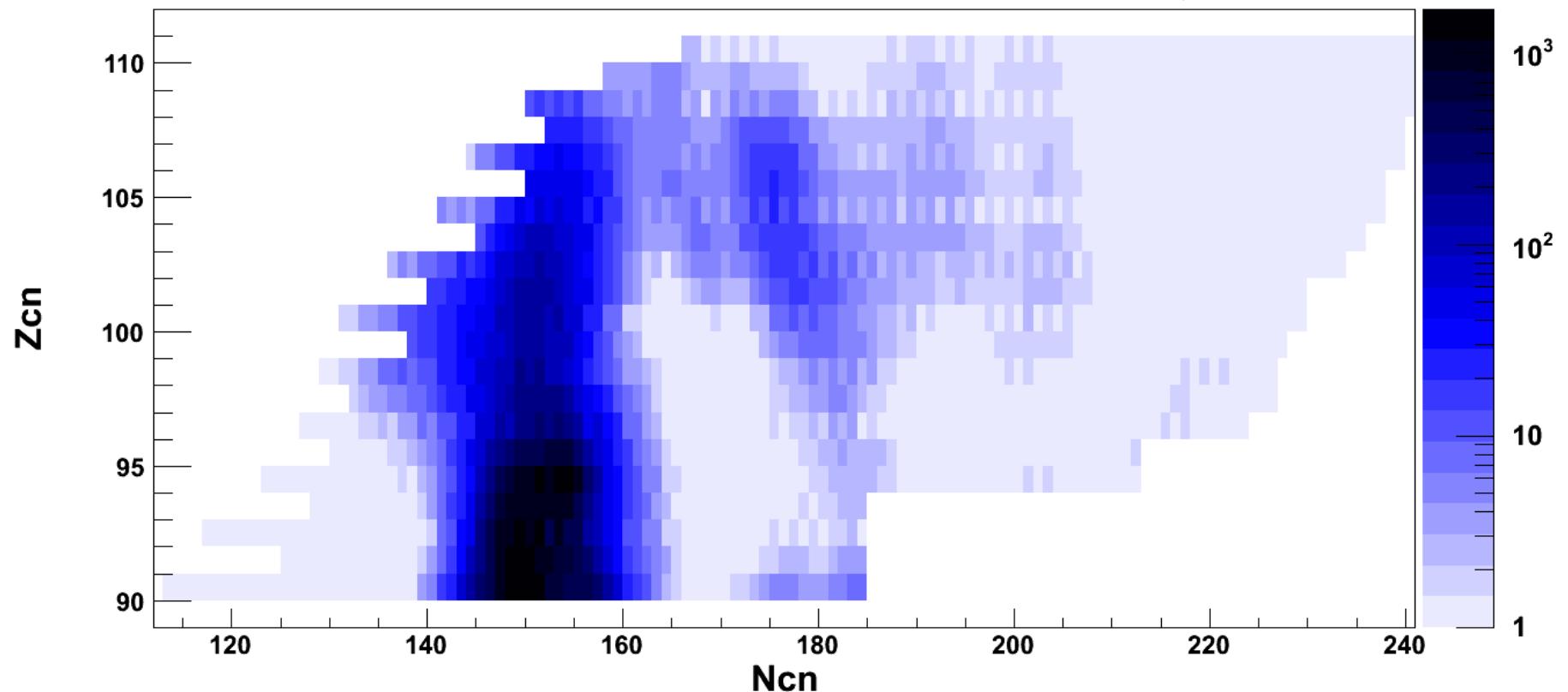
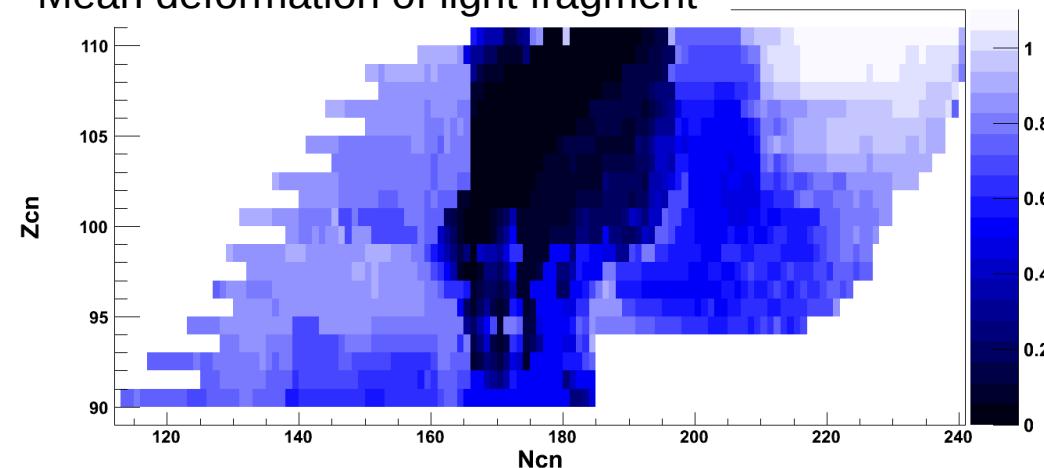


# Chart of Ypeak / Yvalley

Mean deformation of heavy fragment

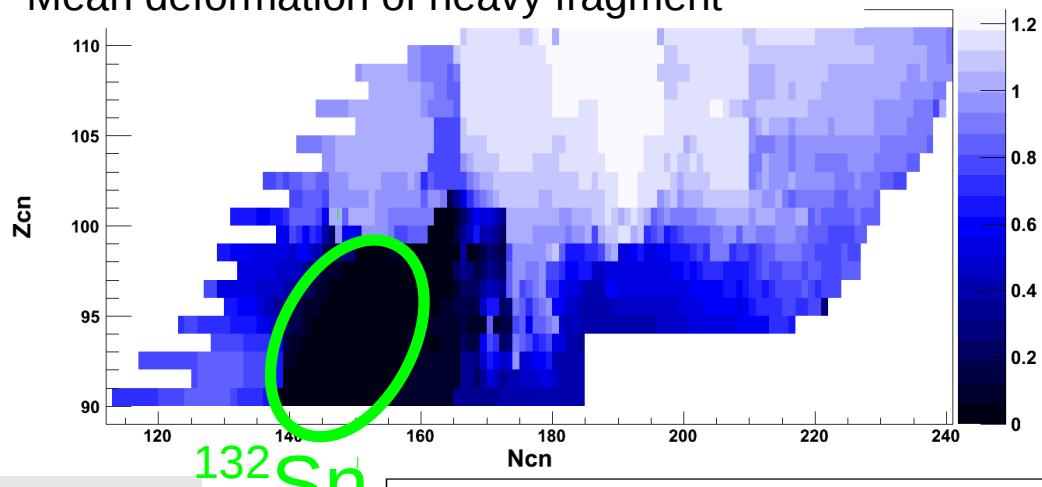


Mean deformation of light fragment

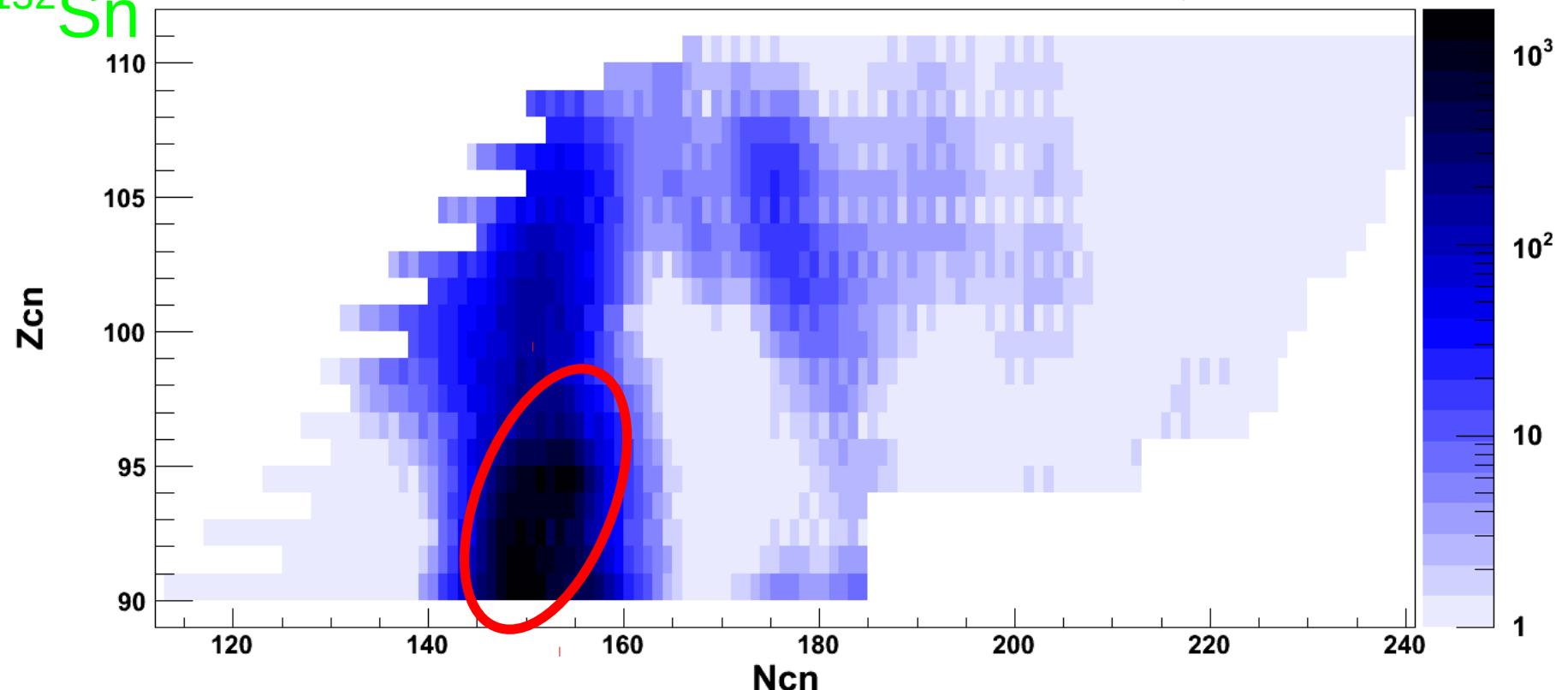
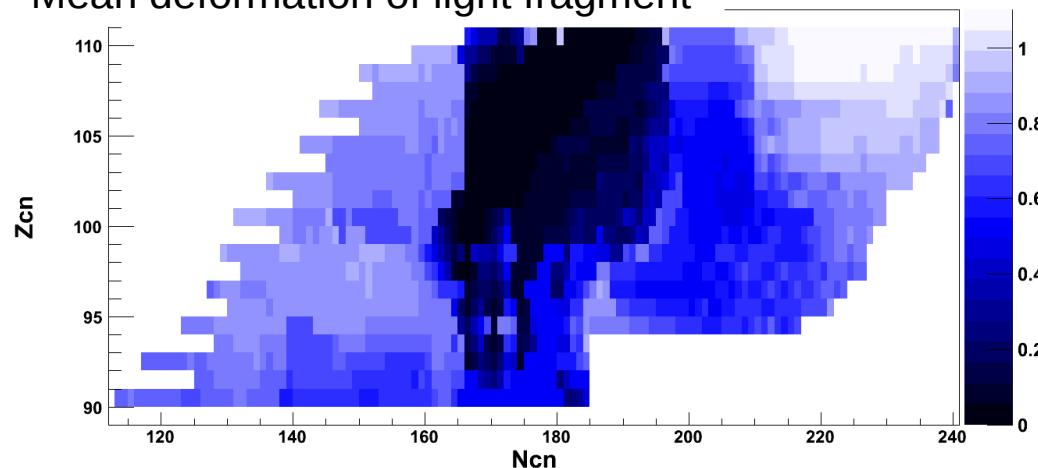


# Chart of Ypeak / Yvalley

Mean deformation of heavy fragment

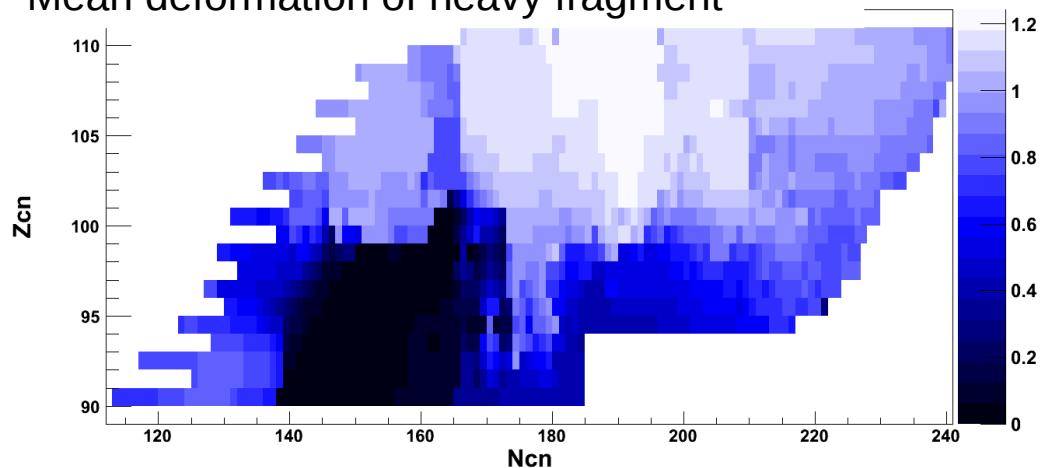


Mean deformation of light fragment

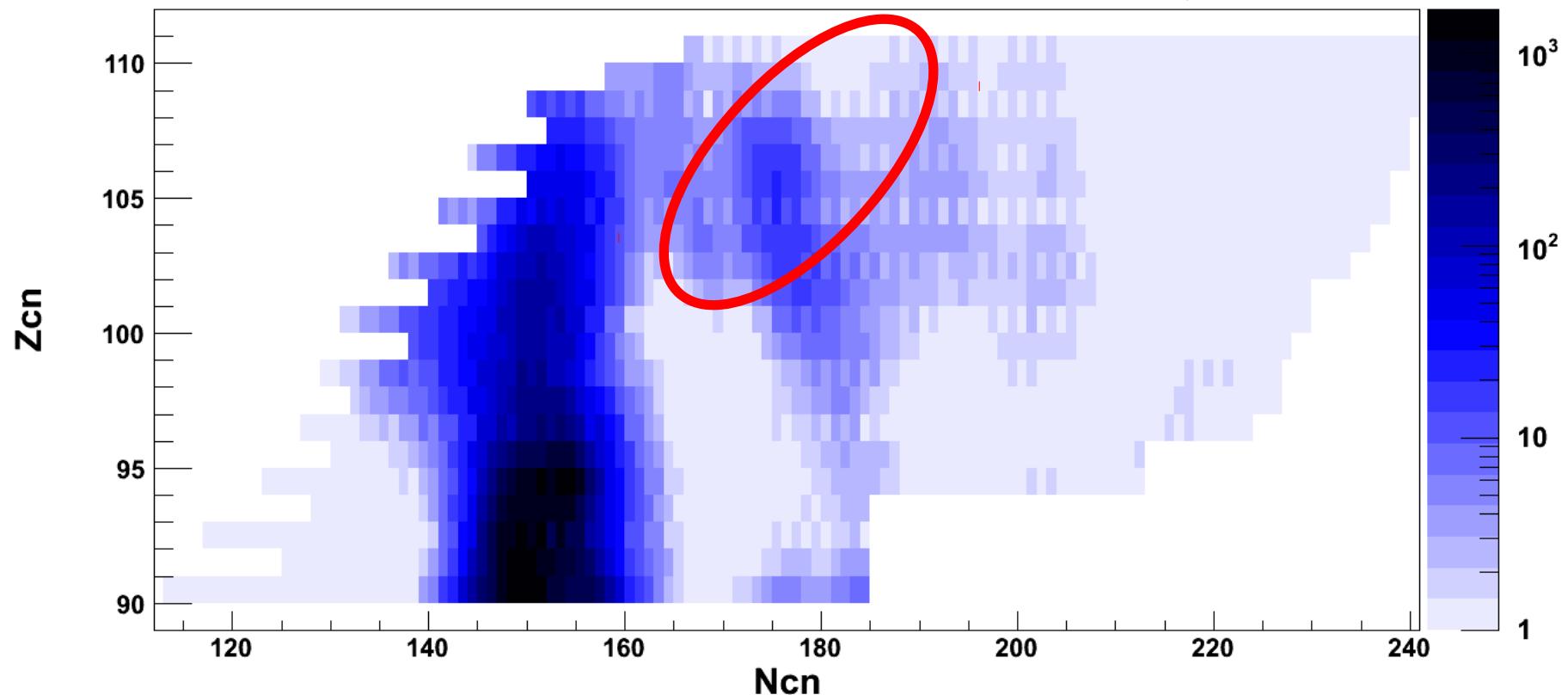
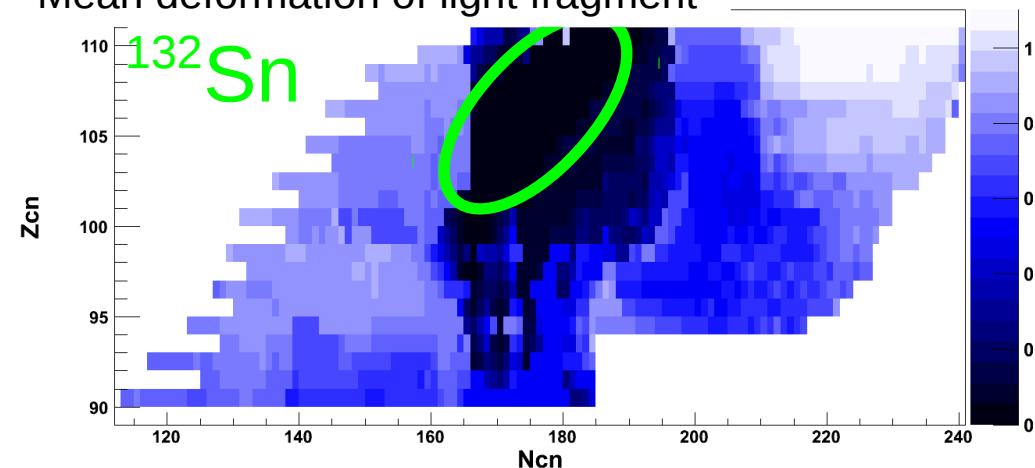


# Chart of Ypeak / Yvalley

Mean deformation of heavy fragment

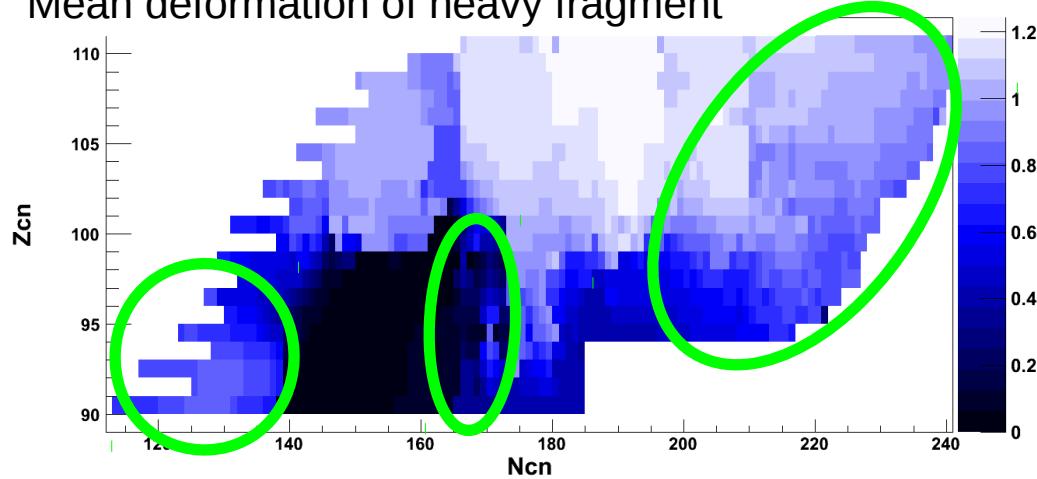


Mean deformation of light fragment

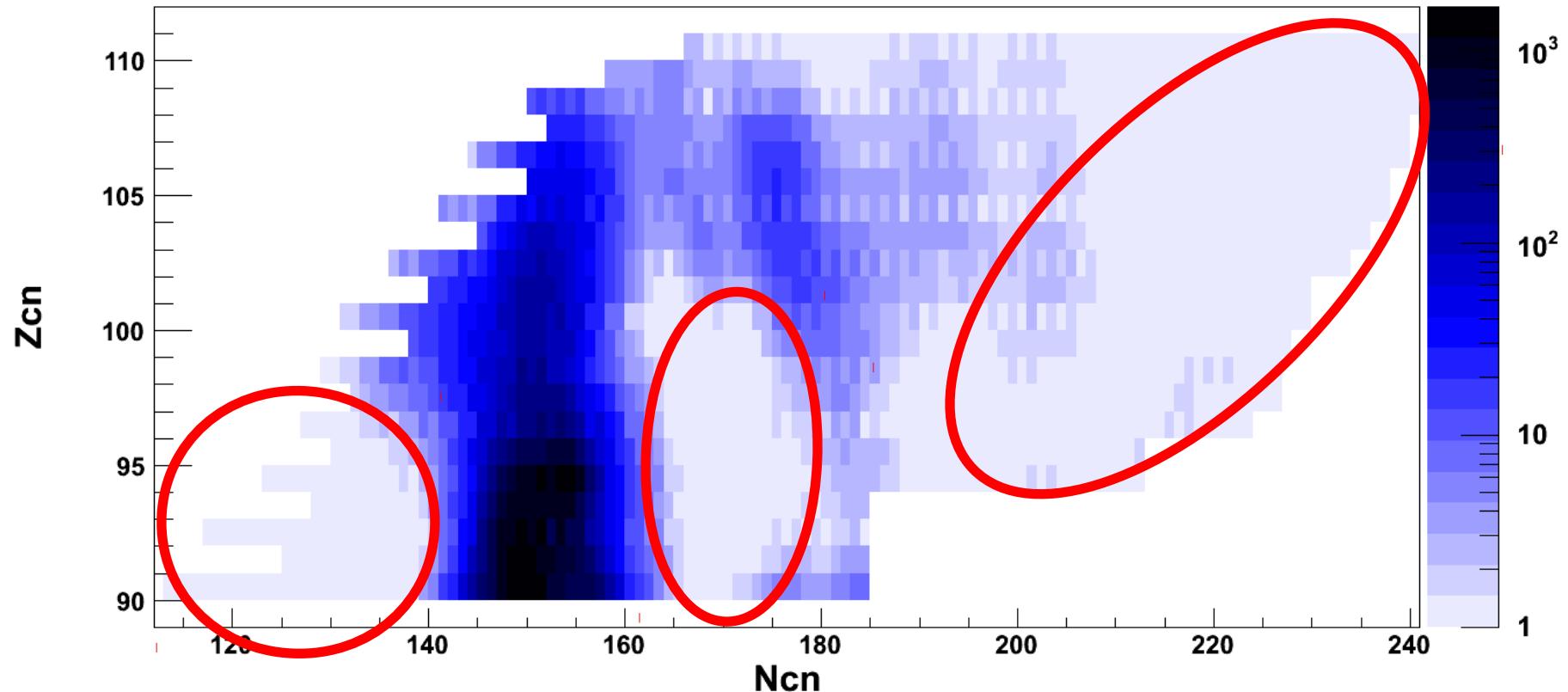
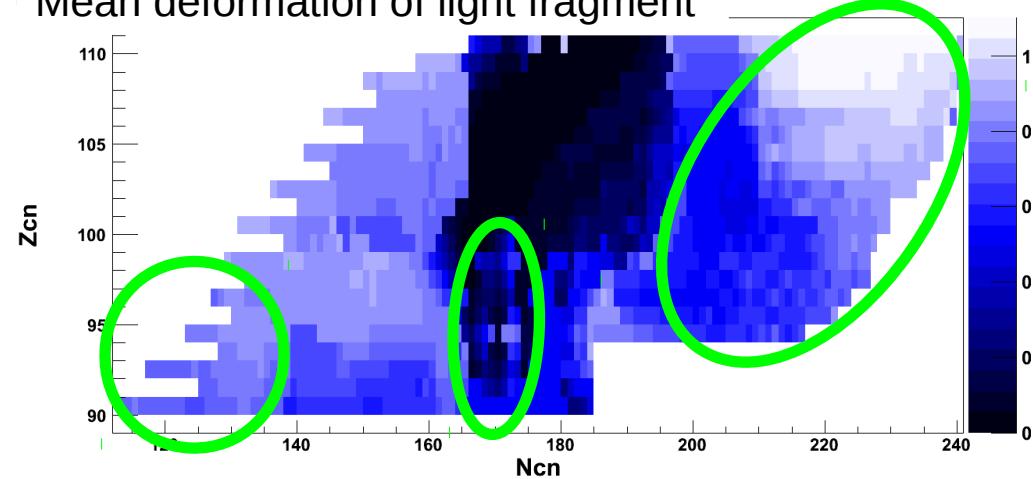


# Chart of Ypeak / Yvalley

Mean deformation of heavy fragment

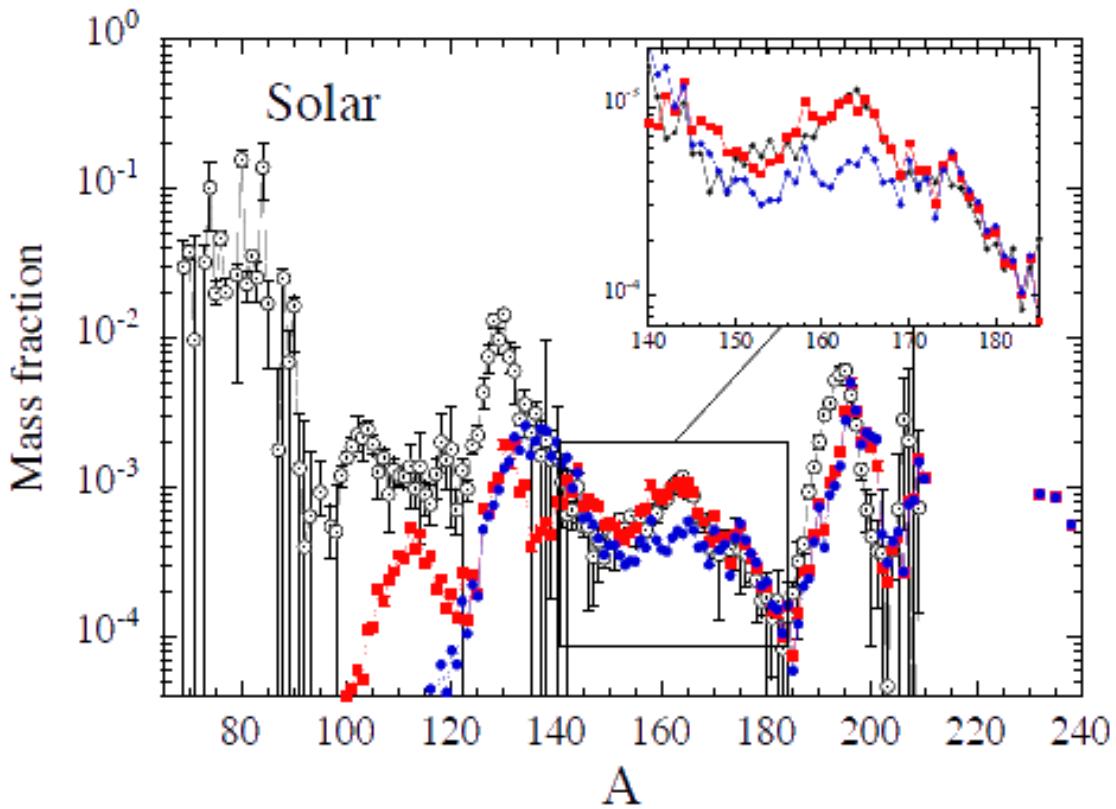


Mean deformation of light fragment



# Solar abundance with SPY calculations

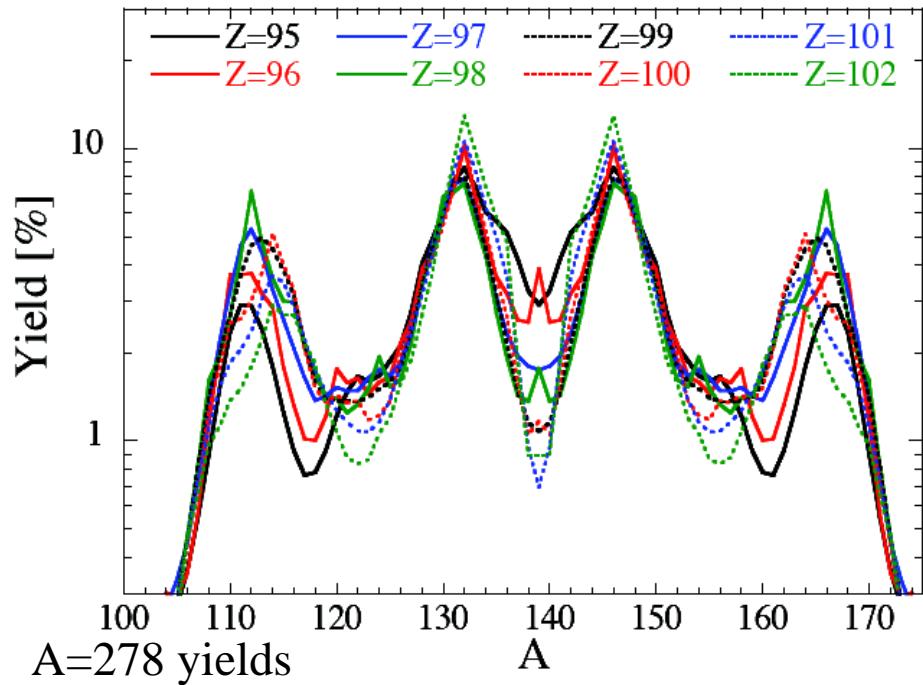
S. Goriely et al accepted in PRL



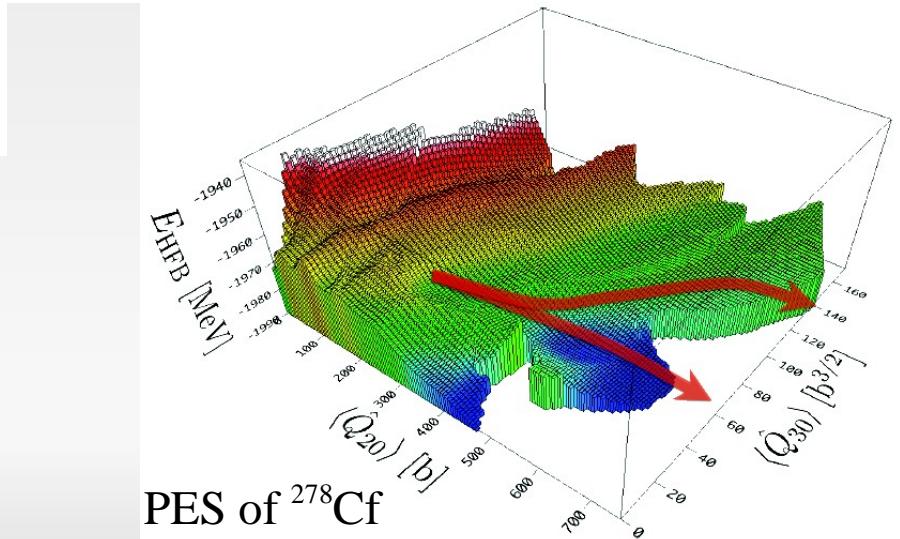
White : Solar r-abundance distribution

Red : fission yields from SPY

Blue : fission yields from GEF model



A=278 yields



PES of  $^{278}\text{Cf}$

# Conclusions

SPY : development of a new statistical scission point model

- integration of microscopic description of the nuclei (**HFB amedee**)
- show that **shell effects** drive the mass asymmetry
  - already applied to  $^{180}\text{Hg}$  fission (*S. Panebianco et al Phys. Rev.C 86, 064601 (2012)*)
- fission fragments predictions over the **whole nuclear chart**
  - already applied to stellar nucleosynthesis (*accepted in PRL*)

# Perspectives

- Calculation & integration of **microscopic state density** in SPY
  - Work started, problem of too high influence of shell effects
- Statistical description of the system (mico-canonical-grand canonical)
- Coulomb interaction calculation method (double folding, diffused edges)
- Integration of Gogny D1M interaction

# Backup slides

## SPY model : energy balance + inputs

$$E_{\text{avail}} = E_{\text{HFB1}} + E_{\text{HFB2}} + E_{\text{coul}} + E_{\text{nucl}} - E_{\text{CN}} \quad \text{if } < 0 : \text{fragmentation is possible}$$

- $E_{\text{HFB}}$  : Individual microscopic energies based on HFB with Gogny D1S interaction
- $E_{\text{coul}} (d = 5 \text{ fm})$  : coulomb interaction based on Cohen Swiatecki formalism  
(Cohen and Swiatecki, *Annals of Physics* 19 (1962) 67)
- $E_{\text{nucl}} (d = 5 \text{ fm})$  : nuclear interaction energy based on the Blocki proximity potential (Blocki et al, *Annals of Physics* 105 (1977) 427)
- $E_{\text{CN}}$  : total energy of fissioning nucleus / compound nucleus
- $\rho_i$  : state density of fragment i (because microcanonical description)

$$\pi(Z_1, N_1, Z_2, N_2, \beta_1, \beta_2) = \int_0^1 \rho_1(x | E_{\text{avail}}|) \rho_2((1-x) | E_{\text{avail}}|) dx$$

$$P(Z_1, N_1, Z_2, N_2) = \int_{-0.6}^{1.3} \int_{-0.6}^{1.3} \pi(Z_1, N_1, Z_2, N_2, \beta_1, \beta_2) d\beta_1 d\beta_2$$

# SPY : model presentation

## Wilkins model VS SPY model

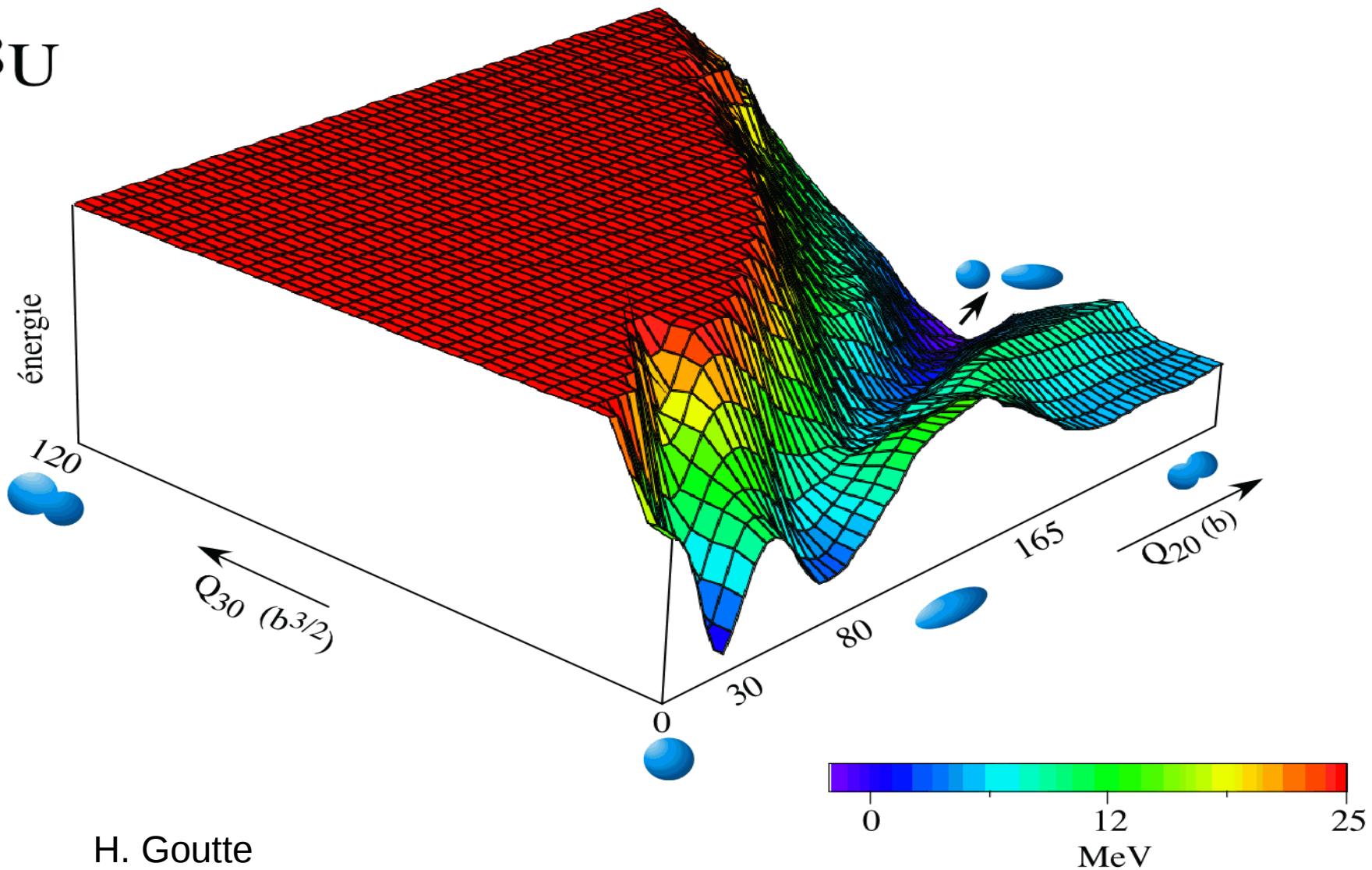
	Wilkins model	SPY model
Individual energy	Liquid drop + Strutinski + pairing	<b>HFB (Gogny D1S)</b> <a href="http://www-phynu.cea.fr/HFB-Gogny.htm">http://www-phynu.cea.fr/HFB-Gogny.htm</a> S. Hilaire and M. Girod, EPJ A <b>33</b> (2007) 237
Energy balance	Relative : $E_{\text{pot}}$	Absolute : $E_{\text{avail}} = E_{\text{pot}} - E_{\text{CN}}$ (if $< 0$ ; fragmentation is allowed)
Temperature parameters	Collective (statistics) + Intrinsic (shell effects)	No temperature <b>States density for statistics</b>
Deformation	Only prolate	Oblate + prolate
Distance $d$	1.2 - 1.4 fm	5 fm
Statistics	Canonical	Microcanonical

$$E_{\text{pot}} = E_1^{\text{ind}} + E_2^{\text{ind}} + E_{\text{coul}} + E_{\text{nucl}}$$

# Backup slides

## PES of U238

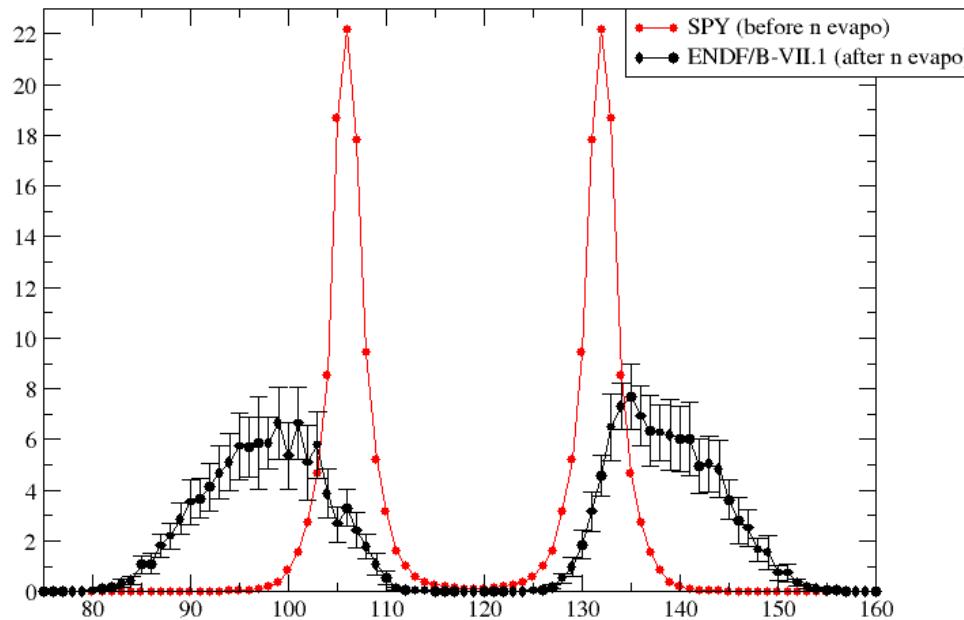
238U



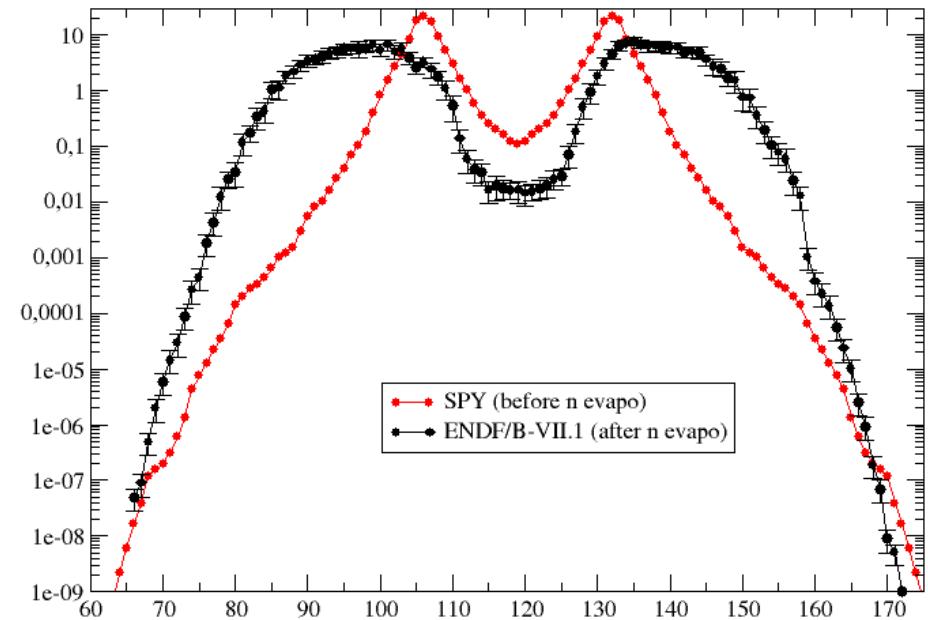
# Backup slides

## Yield of $^{237}\text{Np}$ thermal neutron induced fission

Np237+n\_th yield



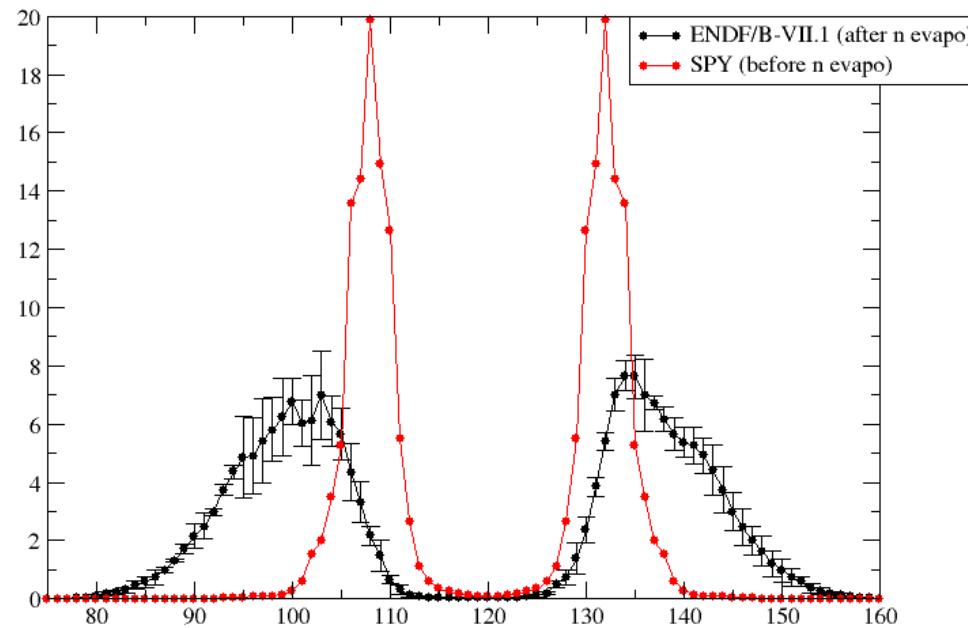
Np237+n\_th yield



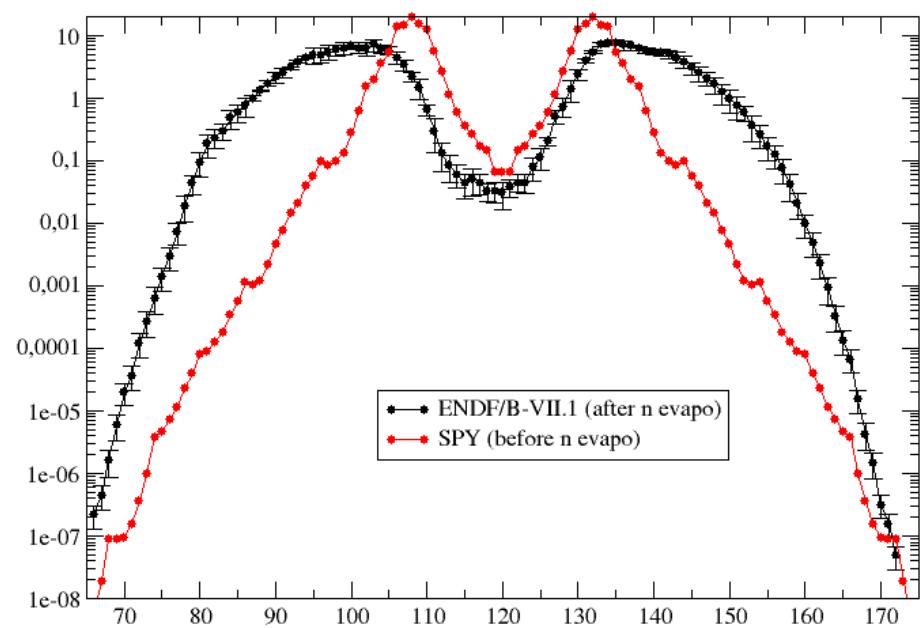
# Backup slides

## Yield of $^{239}\text{Pu}$ thermal neutron induced fission

Pu239+n\_th yield

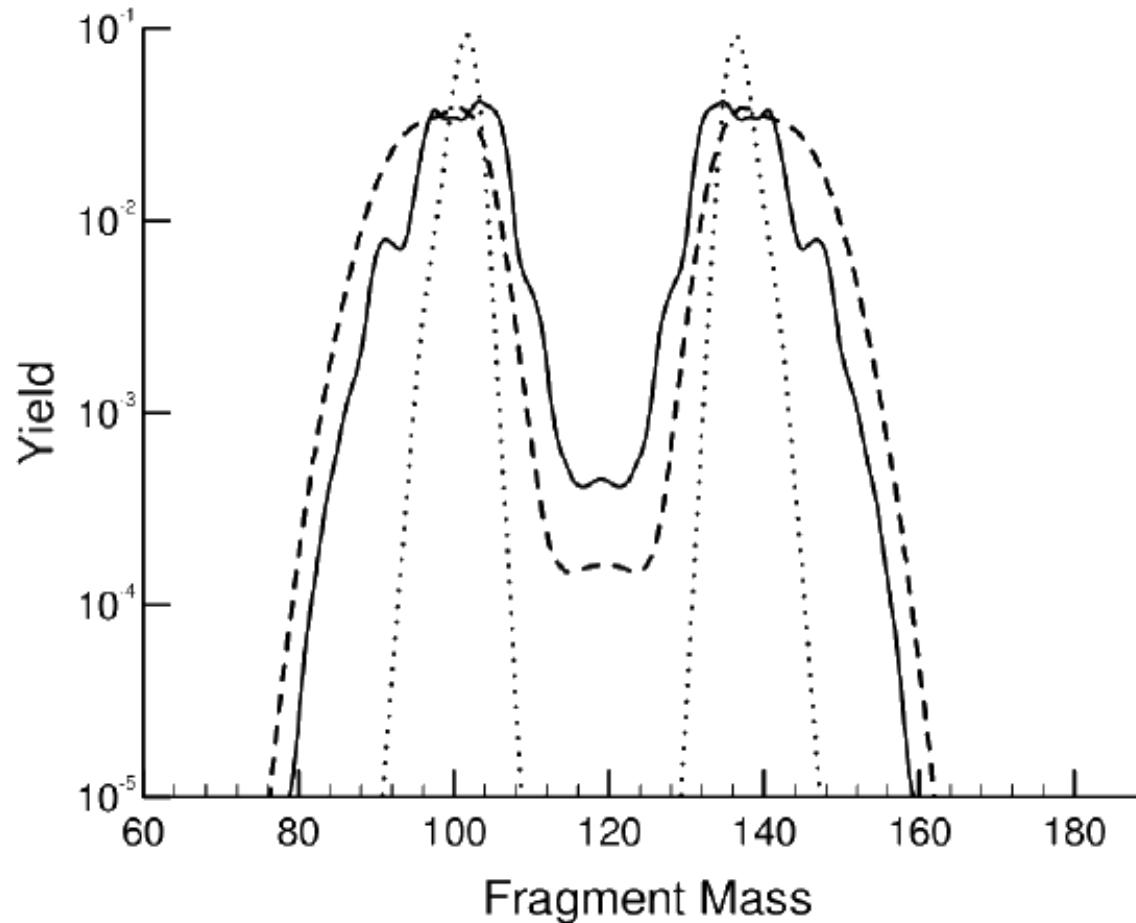


Pu239+n\_th yield



# Backup slides

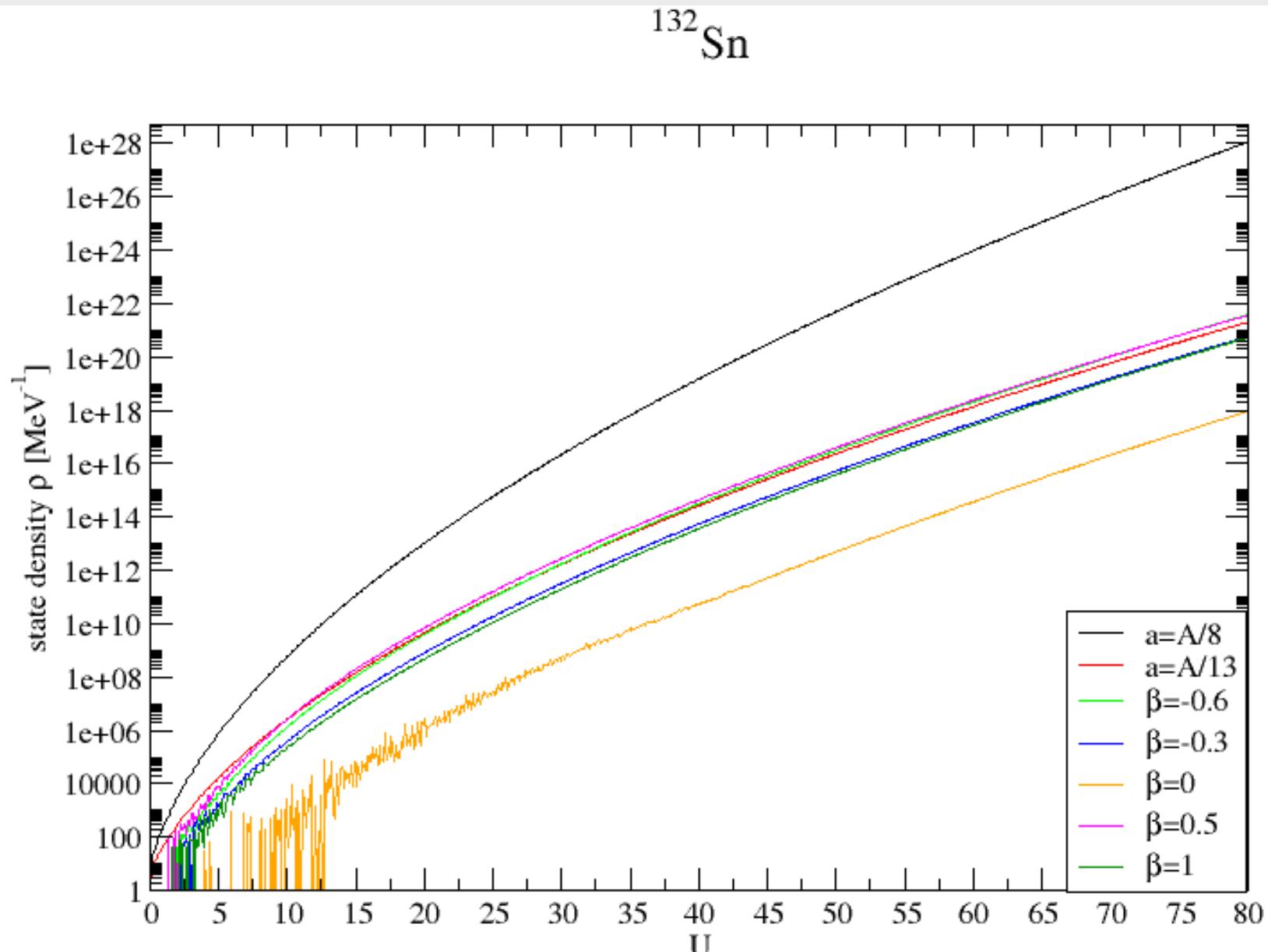
## Yield of $^{238}\text{U}$ from pur microscopic calculations



Rendements en masses. Trait plein : évaluation de Wahl. Pointillés fins : résultats des calculs statiques. Pointillés épais : prise en compte de la dynamique de fission.

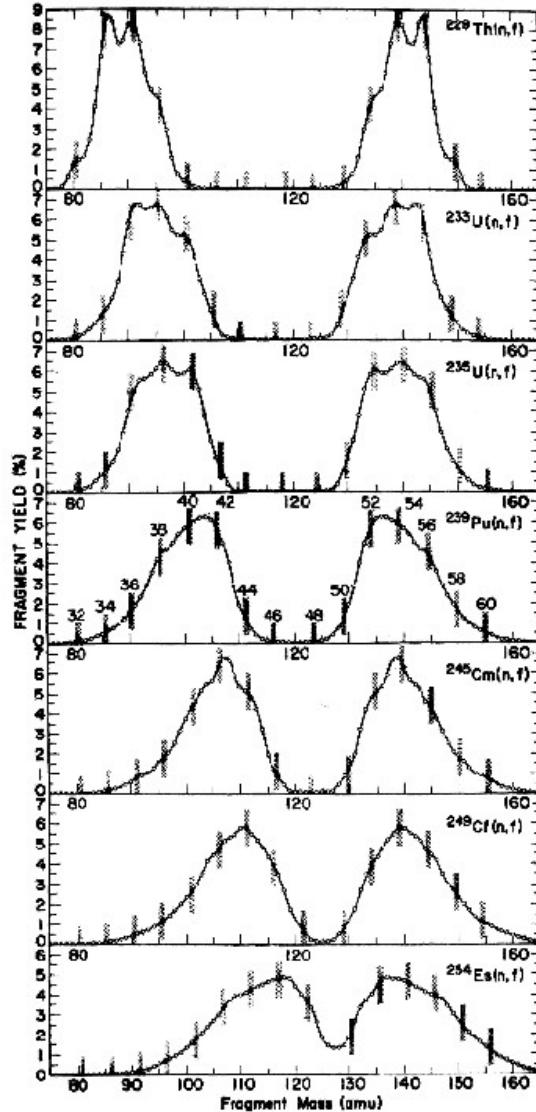
# Backup slides

## Microscopic state density



# Backup slides

## Systematics : mass yields for n-induced fission



nth + 229Th

nth + 233U

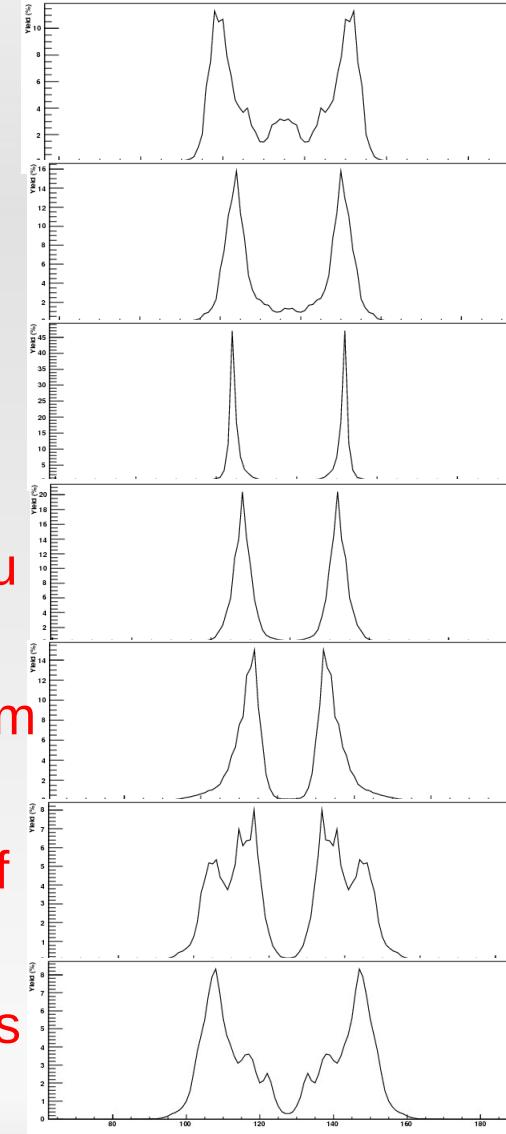
nth + 235U

nth + 239Pu

nth + 245Cm

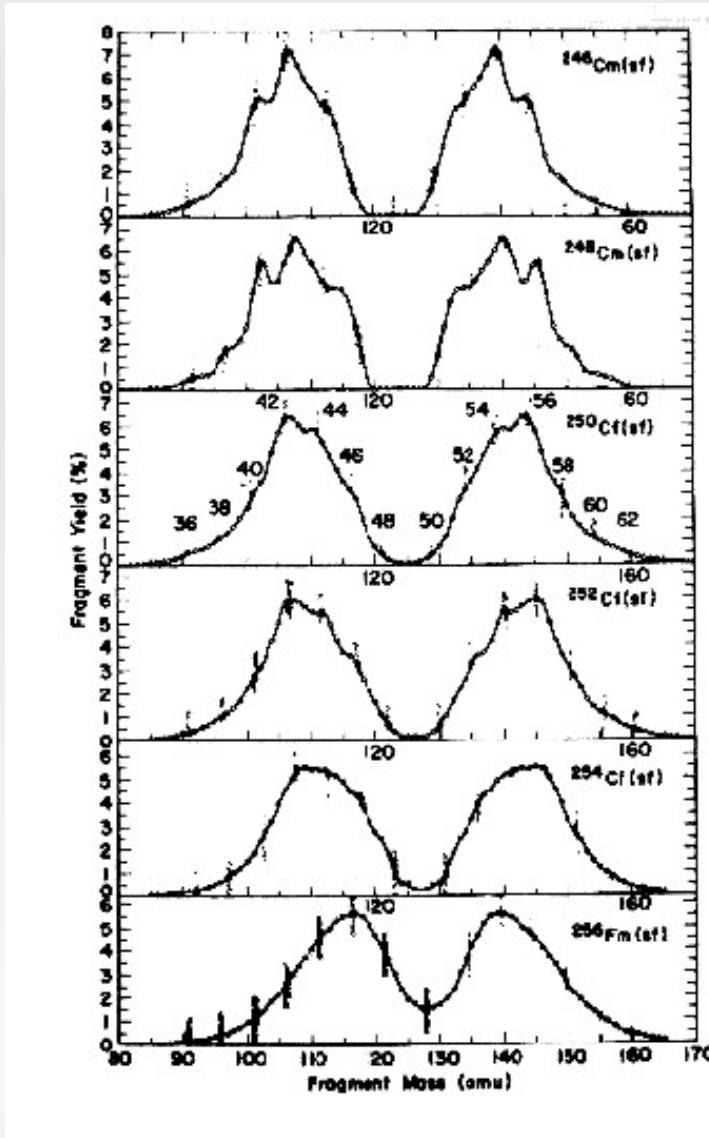
nth + 249Cf

nth + 254Es



# Backup slides

## Systematics : mass yields for spontaneous fission



246Cm(sf)

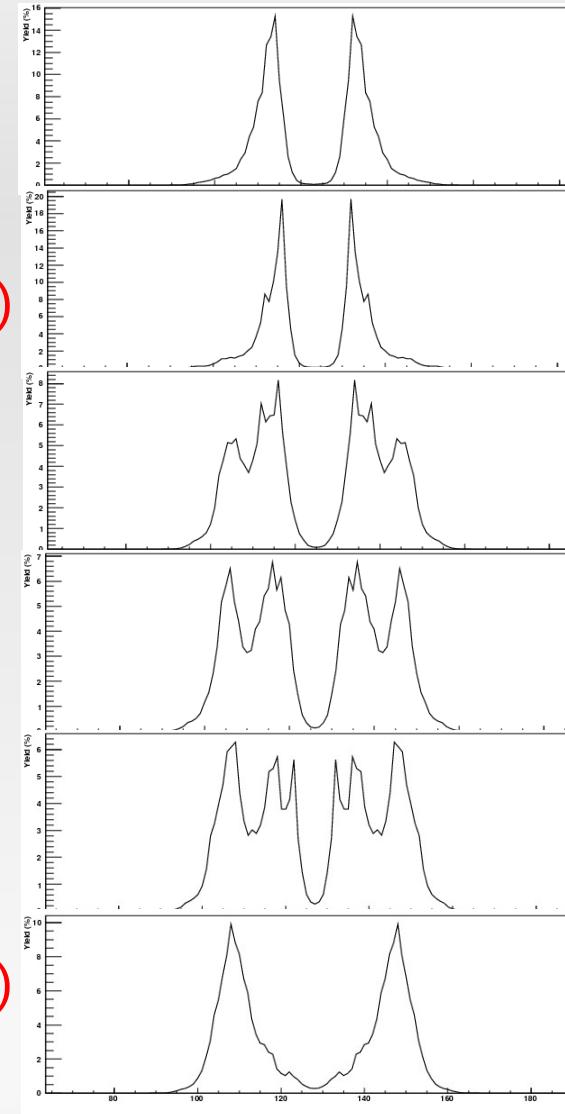
248Cm(sf)

250Cf(sf)

252Cf(sf)

254Cf(sf)

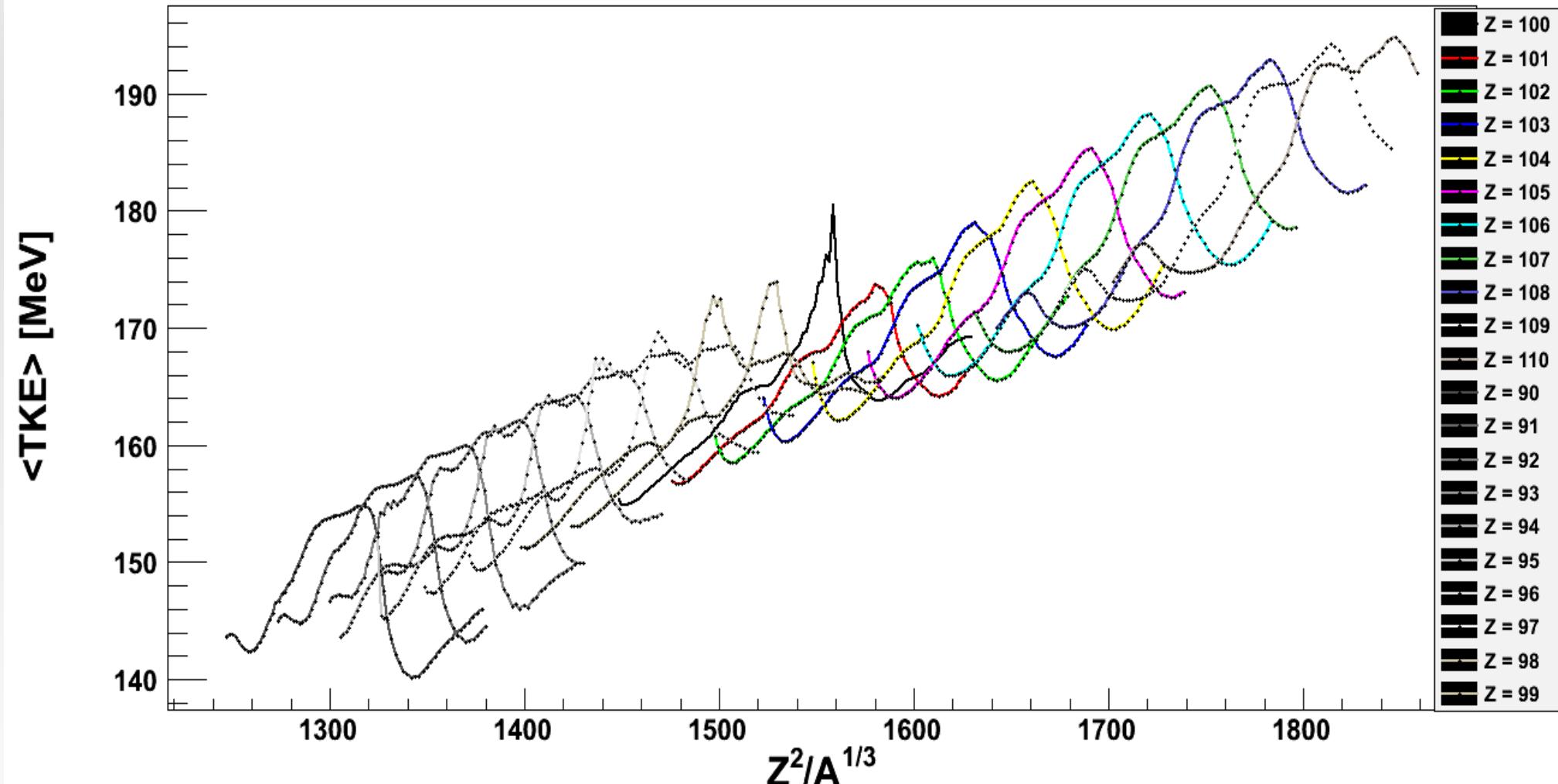
256Fm(sf)



# Backup slides

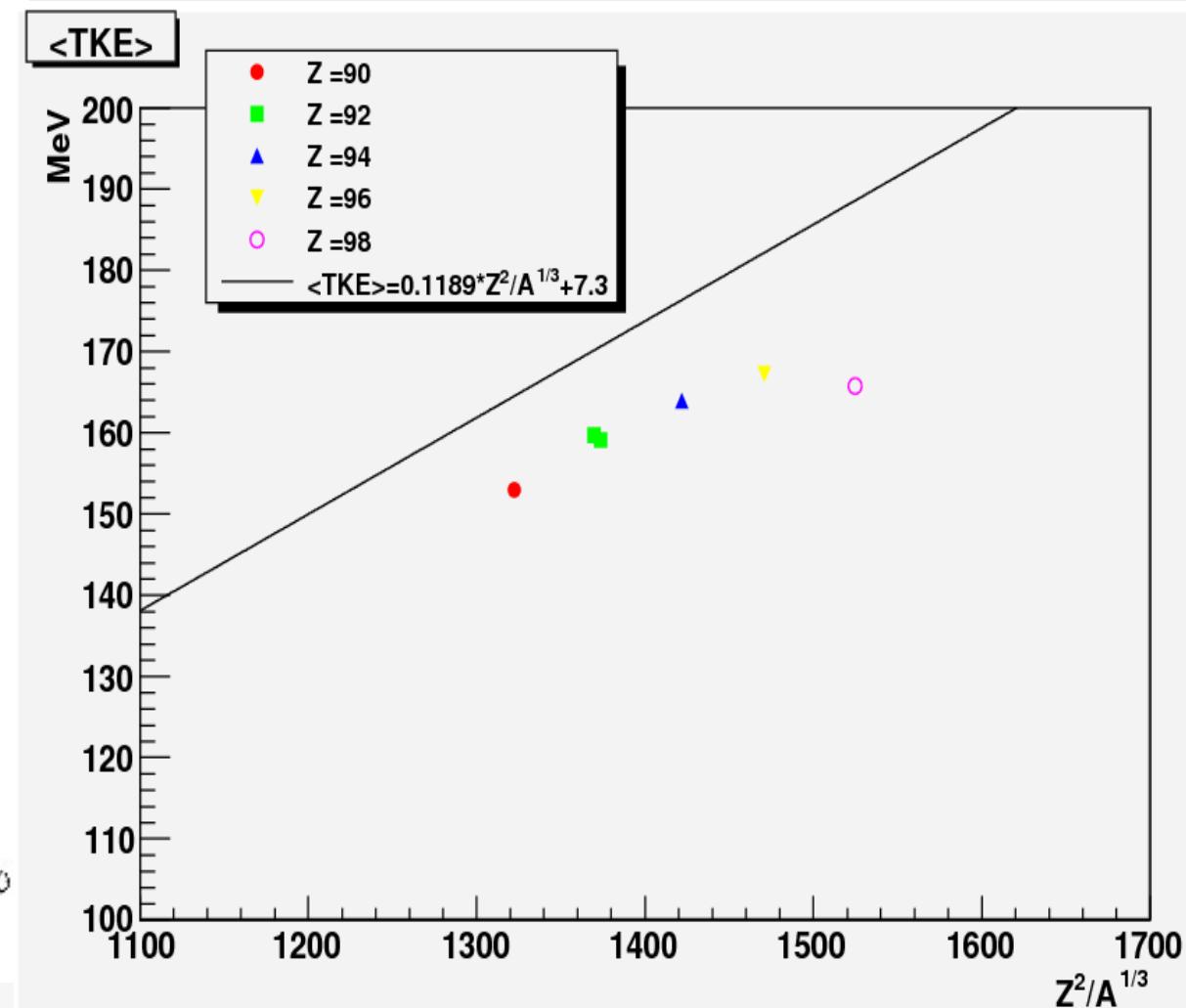
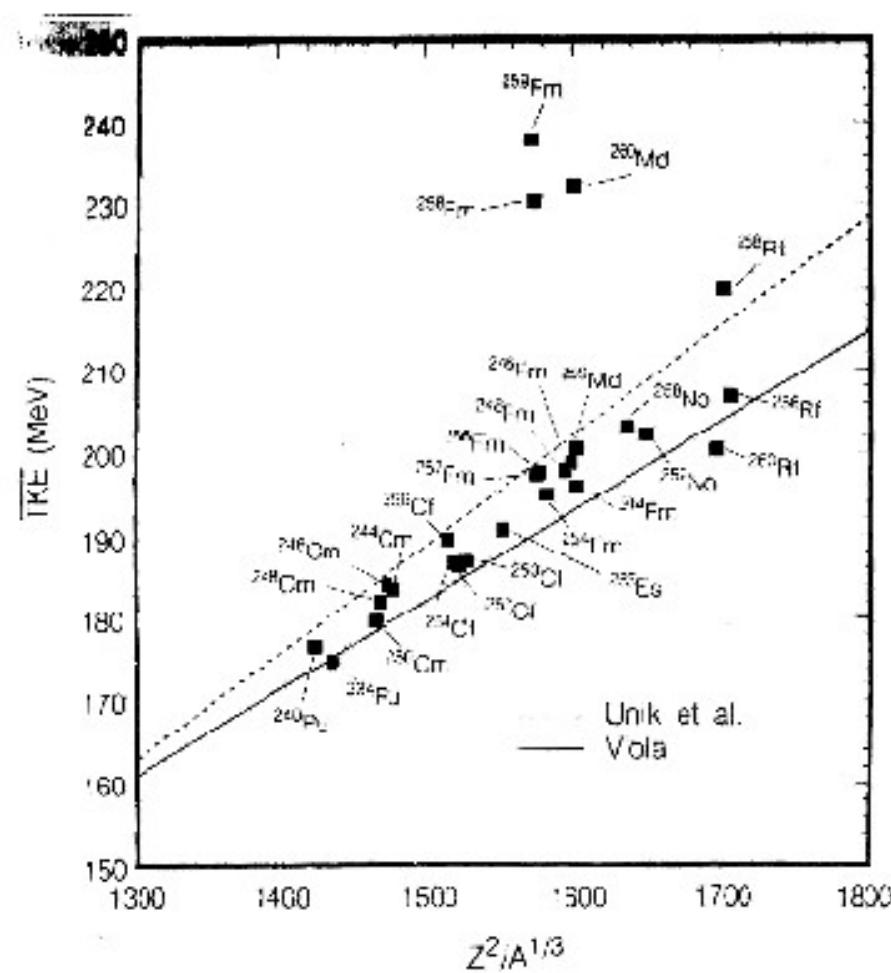
## Systematics : mean TKE

Fit :  $\langle \text{TKE} \rangle = 0.074 Z^2/A^{1/3} + 52$



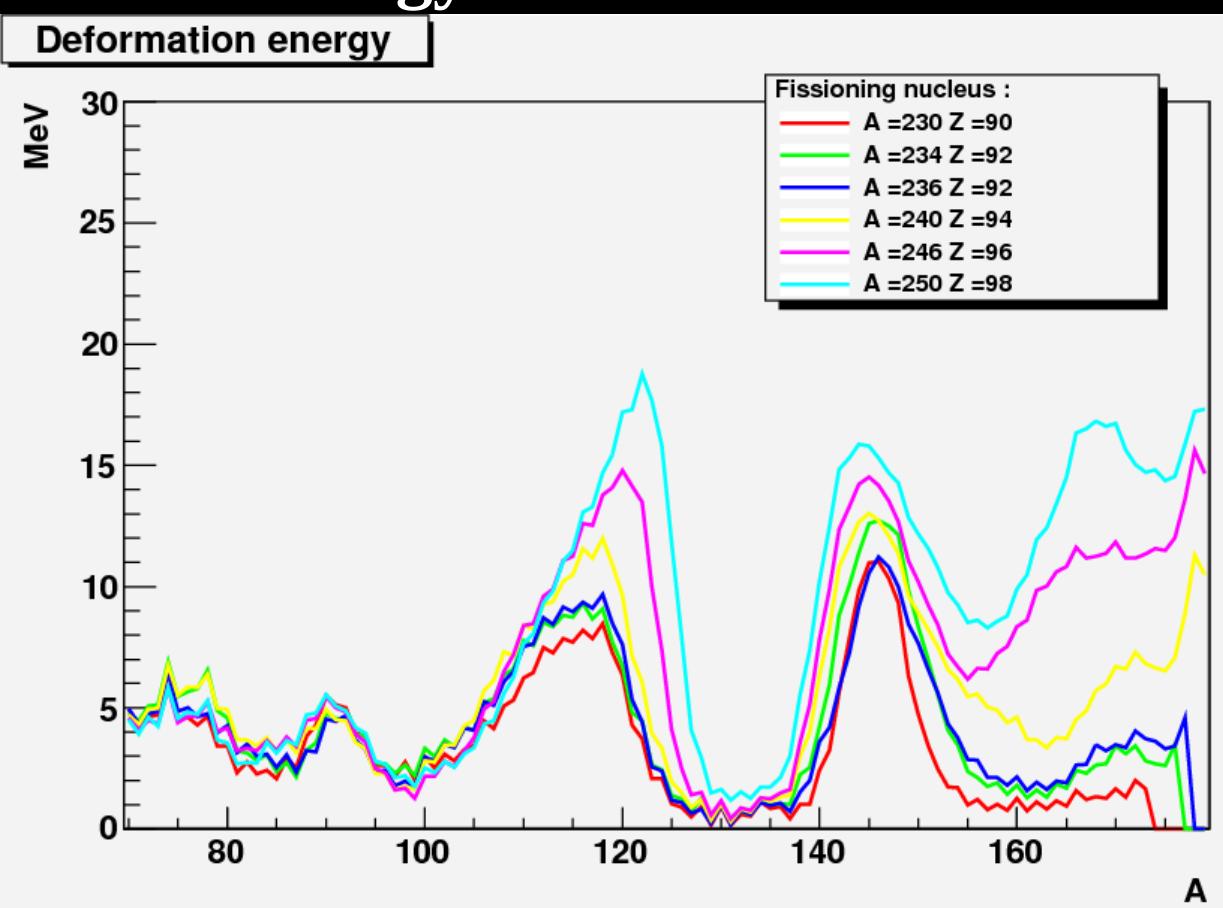
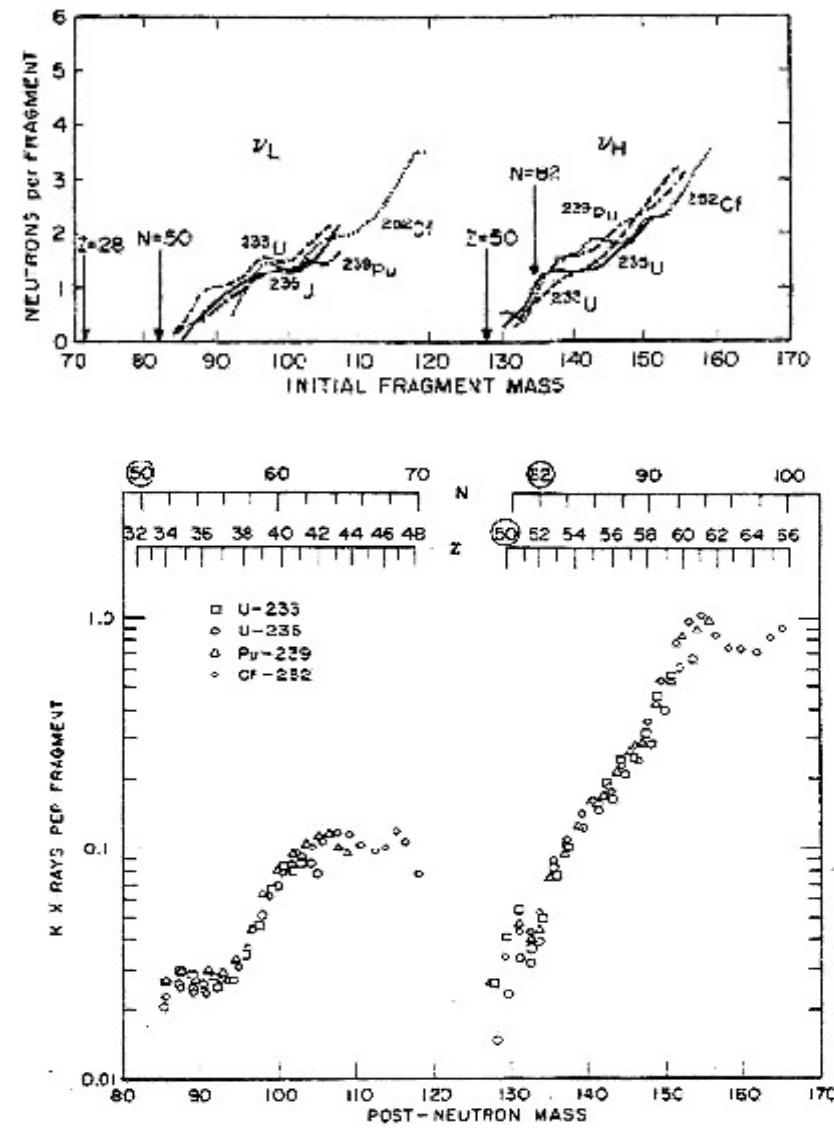
# Backup slides

## Systematics : mean TKE exp VS SPY



# Backup slides

## Systematics : mean deformation energy



The deformation energy is somehow related to the number of emitted particles

# Nuclear fission of $^{180}\text{Hg}$ (ISOLDE)

Is nuclear fission of  $^{180}\text{Hg}$  symmetric ?

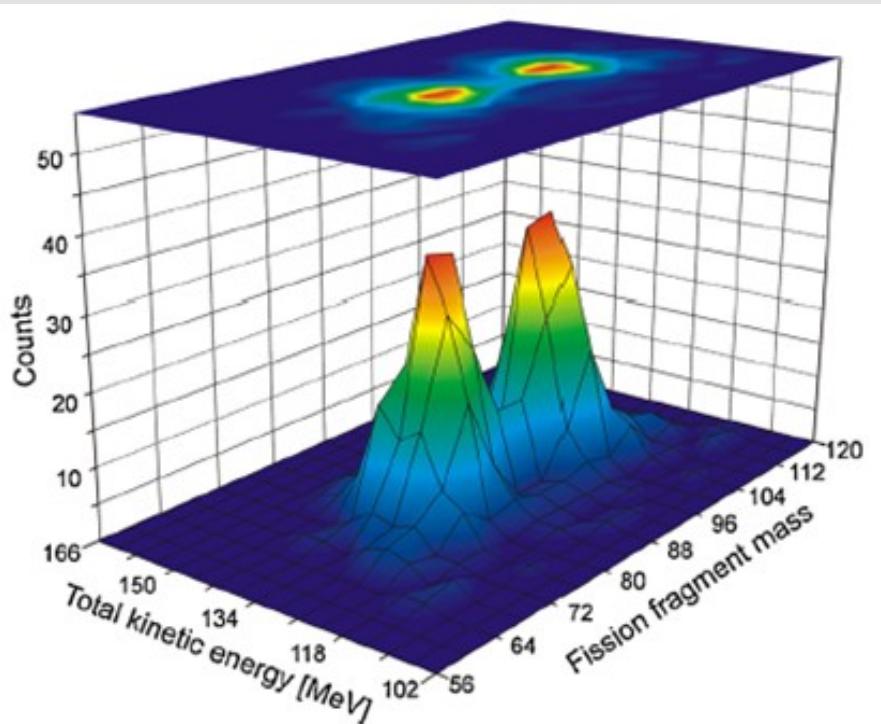
Most likely fragmentation expected :  $^{180}\text{Hg} \rightarrow ^{90}\text{Zr} + ^{90}\text{Zr}$

# Nuclear fission of $^{180}\text{Hg}$ (ISOLDE)

Is nuclear fission of  $^{180}\text{Hg}$  symmetric ?

Most likely fragmentation expected :  $^{180}\text{Hg} \rightarrow ^{90}\text{Zr} + ^{90}\text{Zr}$

Experiment : asymmetric fission !



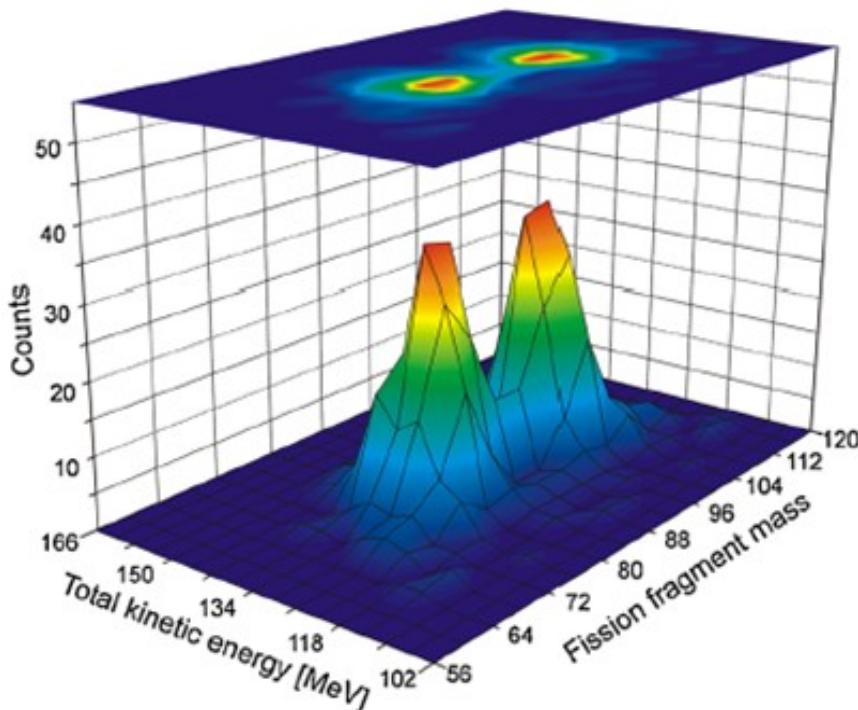
Andreyev et al., PRL 105 (2011) 252502

# Nuclear fission of $^{180}\text{Hg}$ (ISOLDE)

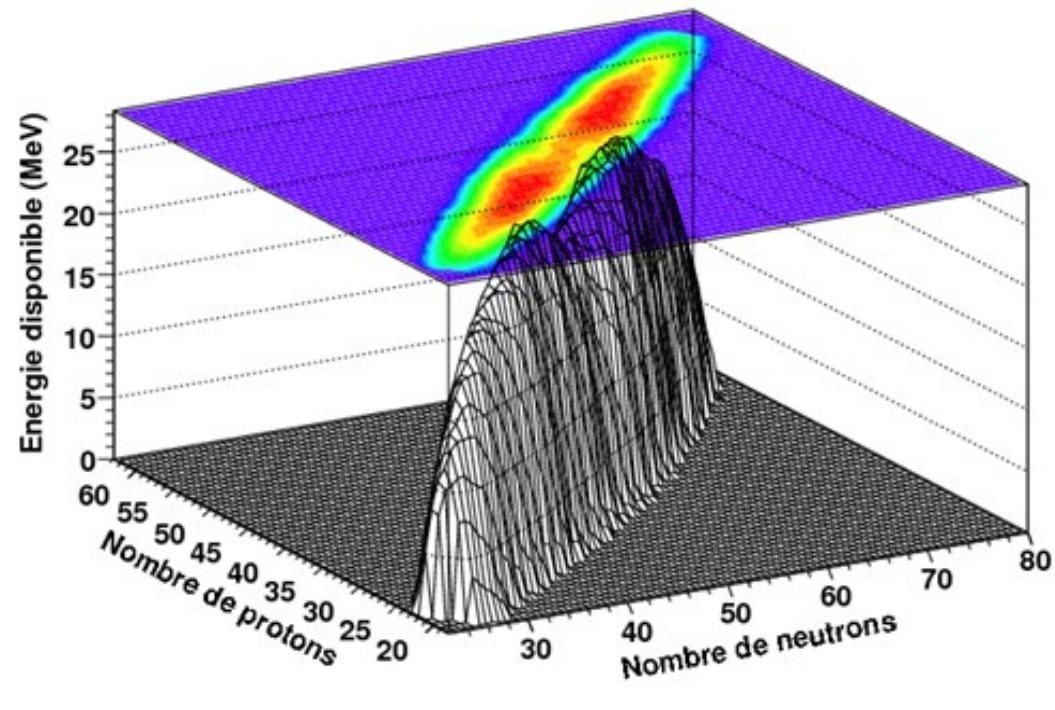
Is nuclear fission of  $^{180}\text{Hg}$  symmetric ?

Most likely fragmentation expected :  $^{180}\text{Hg} \rightarrow ^{90}\text{Zr} + ^{90}\text{Zr}$

Experiment : asymmetric fission !



Andreyev *et al.*, PRL 105 (2011) 252502



S. Panebianco *et al.*, PRC 86 (2012) 064601

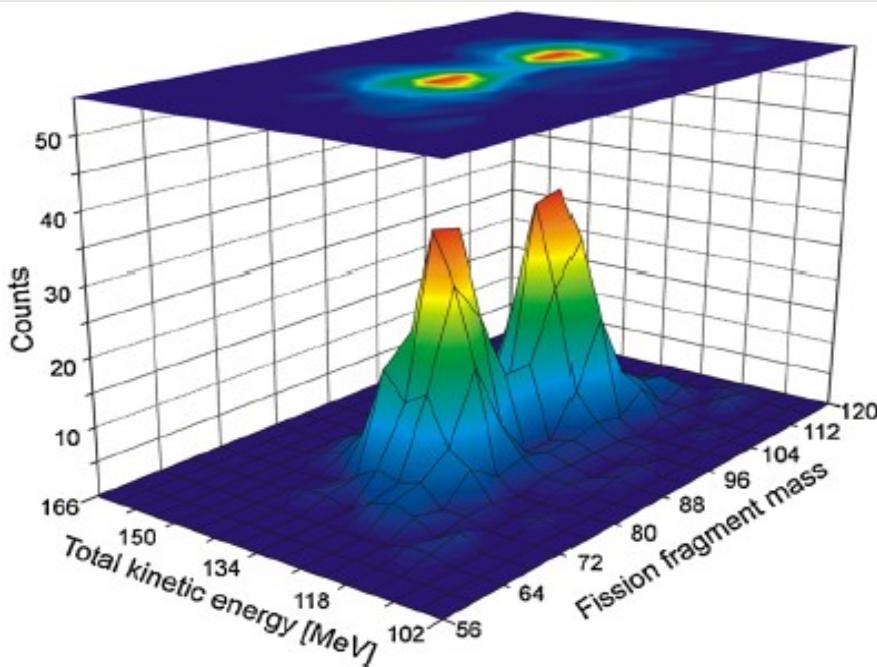
SPY predict an asymmetric fission

# Nuclear fission of $^{180}\text{Hg}$

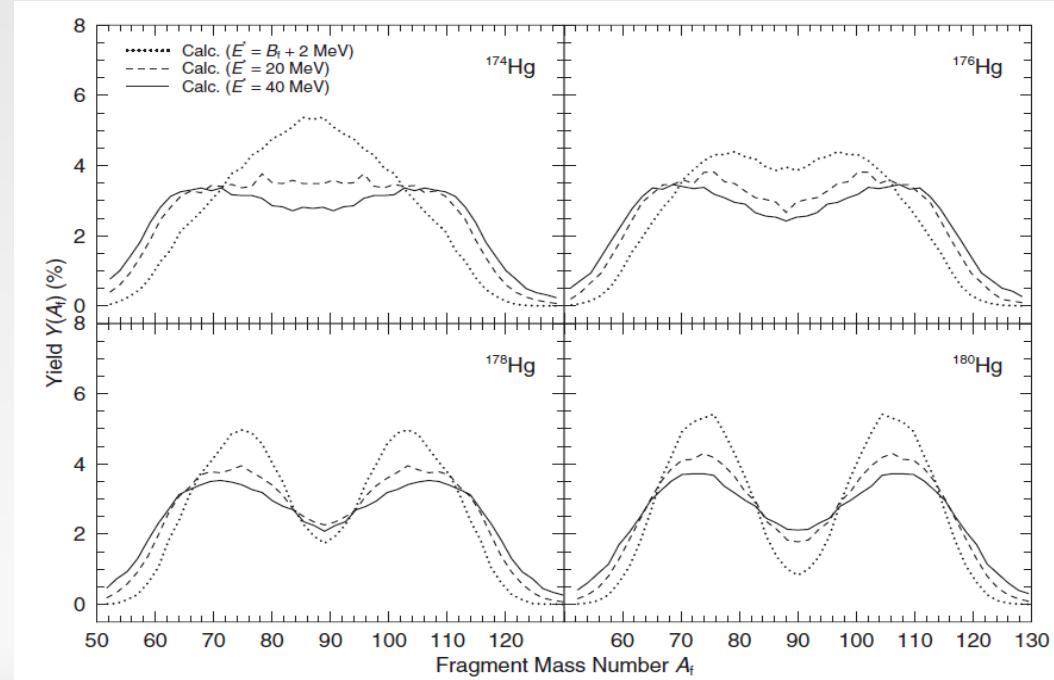
## $\beta$ -delayed fission of $^{180}\text{Hg}$ from $^{180}\text{Tl}$

Surprising asymmetric yields of  $^{180}\text{Hg}$  fission

Due to the nuclear structure of the fissioning nucleus ?



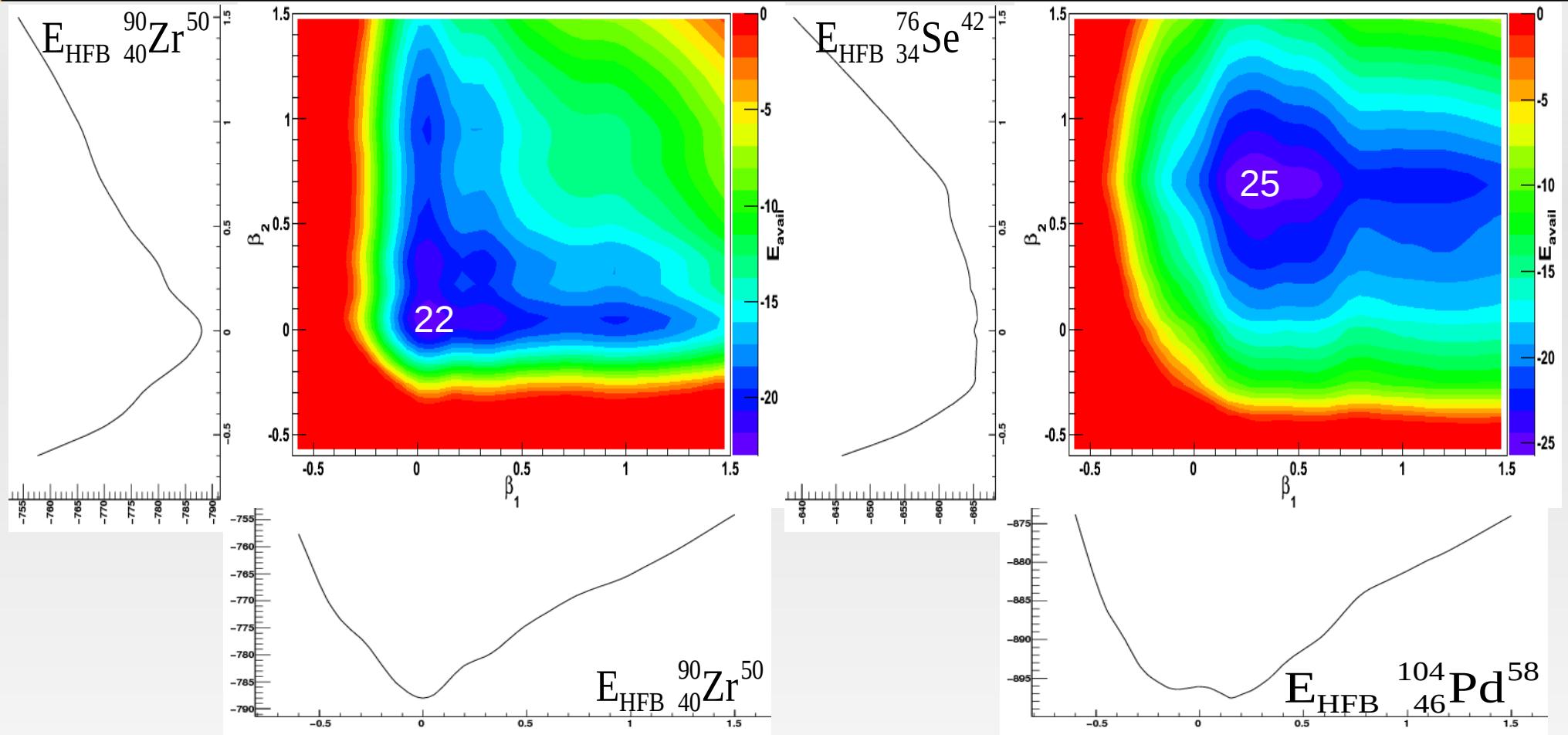
Andreyev et al., PRL 105 (2011) 252502



Möller et al., PRC 85 (2012) 024306

# SPY results

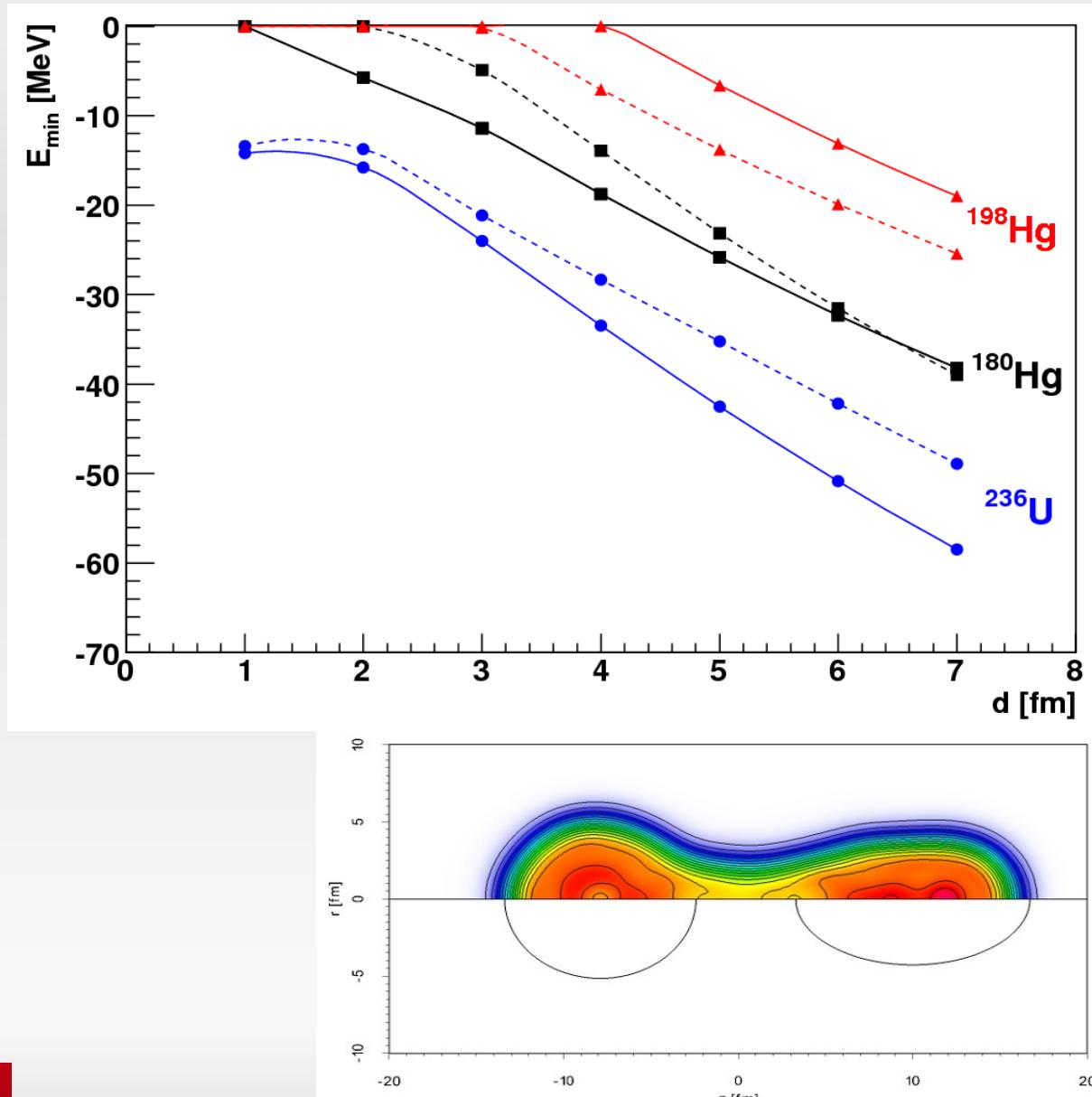
## Available energy for $^{180}\text{Hg}$



$$E_{\text{avail}} = E_{\text{HFB}1} + E_{\text{HFB}2} + E_{\text{coul}} + E_{\text{nucl}} - E_{\text{CN}} \quad \text{if } < 0 : \text{fragmentation is allowed}$$

# SPY results

## Scission point distance $d$ & sym/asym fission

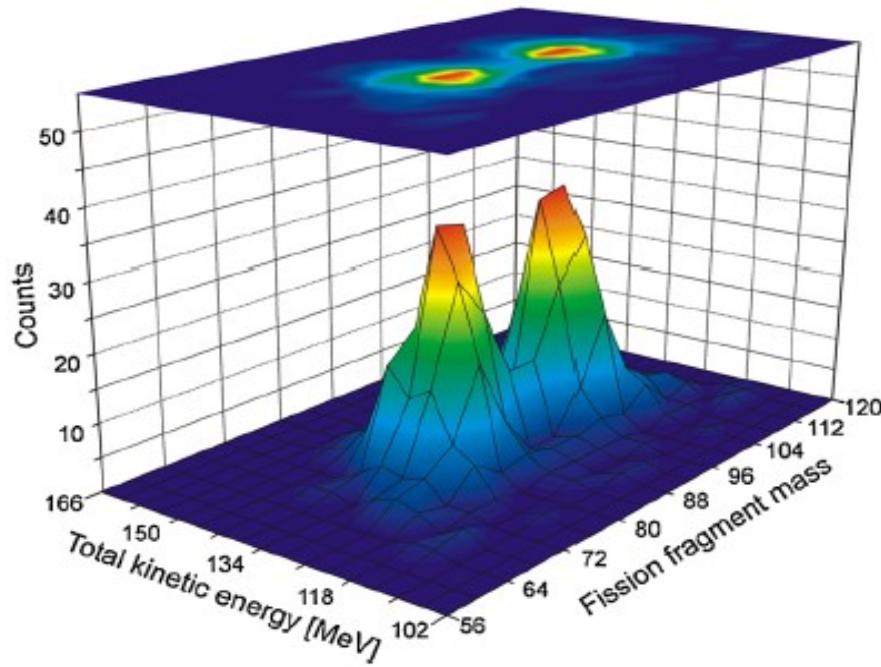


Self-consistent HFB of  $^{180}\text{Hg}$ :  
most probable configuration  
( $q_{20} = 256.12 \text{ b}$  ;  $q_{30} = 33.28 \text{ b}^{3/2}$ )

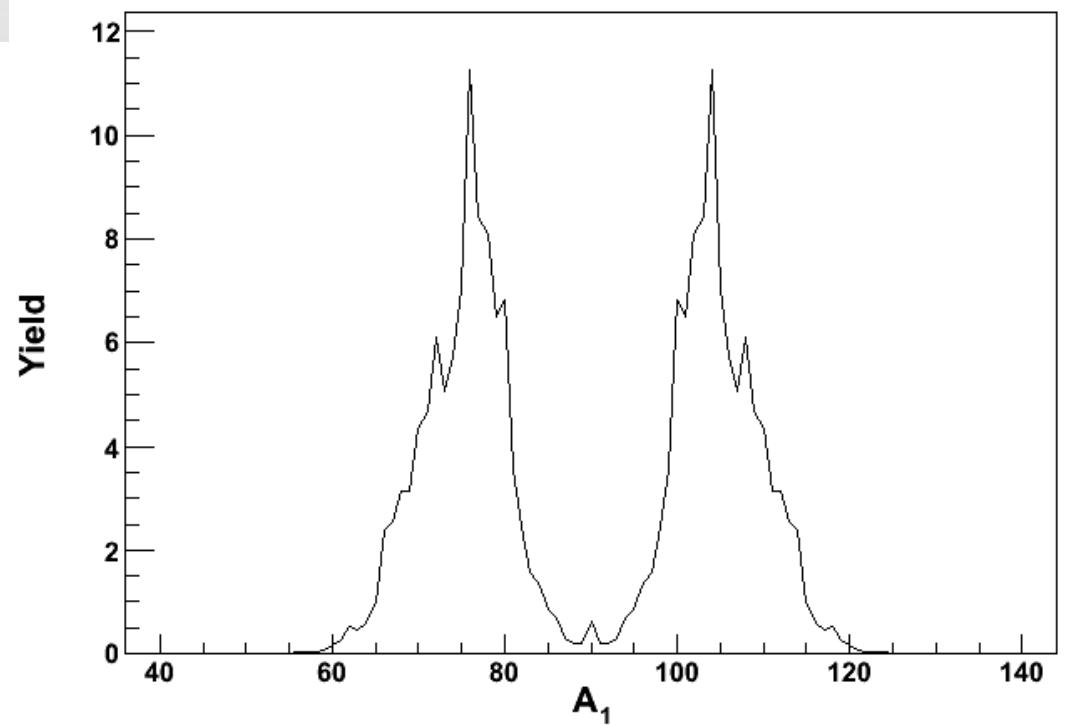
$$d = 5.7 \text{ fm}$$

# Backup slides

## SPY results : Yield for $^{180}\text{Hg}$ case



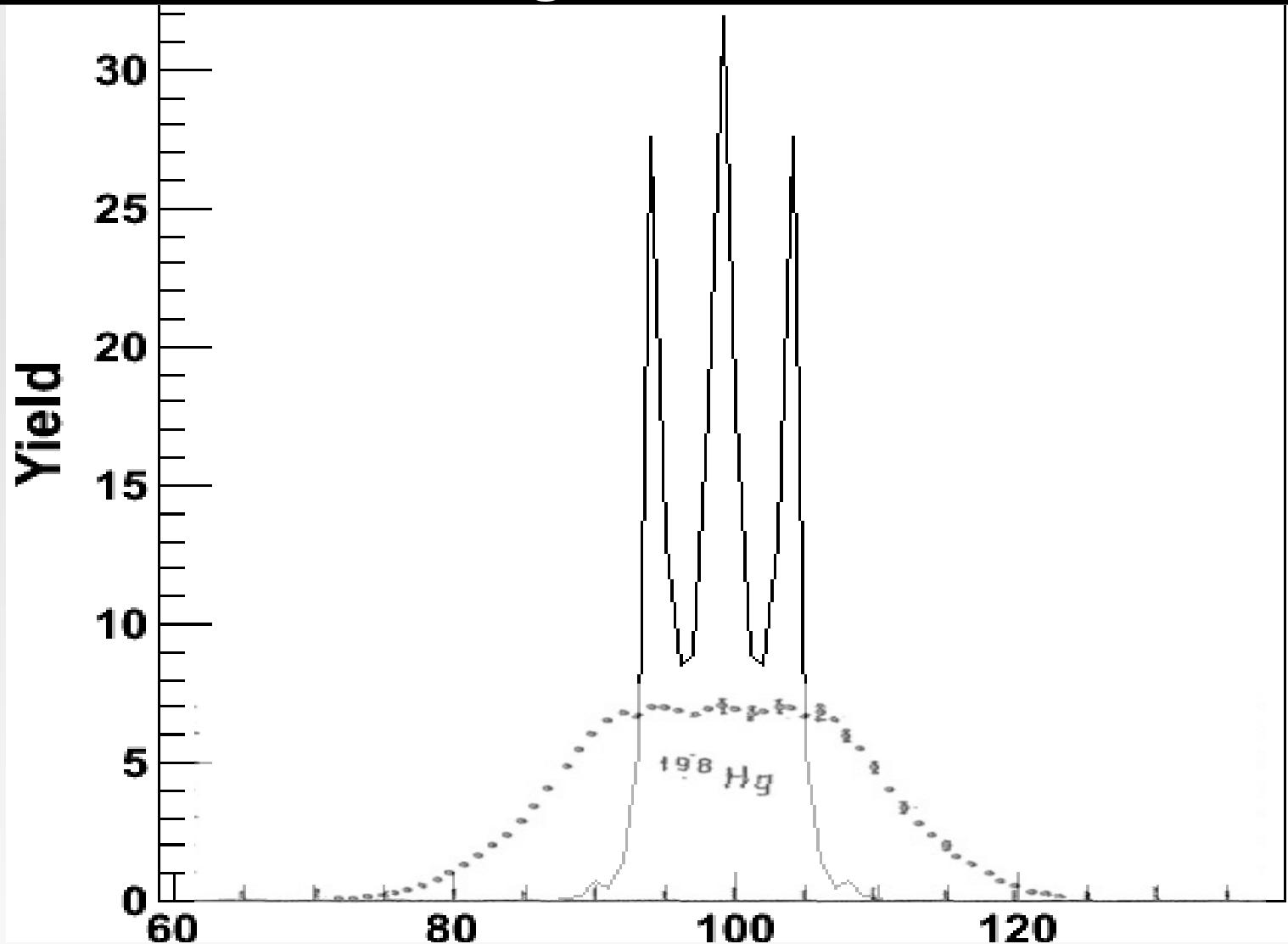
Andreyev *et al.*, PRL 105 (2011) 252502



SPY model with fermi type state density

# Backup slides

## SPY results : Yield for $^{198}\text{Hg}$ case



M. G. Itkis et al. , Yad. Fiz. 53, 1225-1237 (May 1991)

SPY model with fermi type state density

