

Accessing the gluon TMDs with $J/\psi + \gamma$ SSA at AFTER

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University of Tübingen

in collaboration with W. den Dunnen, J.-P. Lansberg, C. Pisano

Probing the Strong Interactions at AFTER using the LHC beams,
Ecole de Physique des Houches, Jan.13, 2014

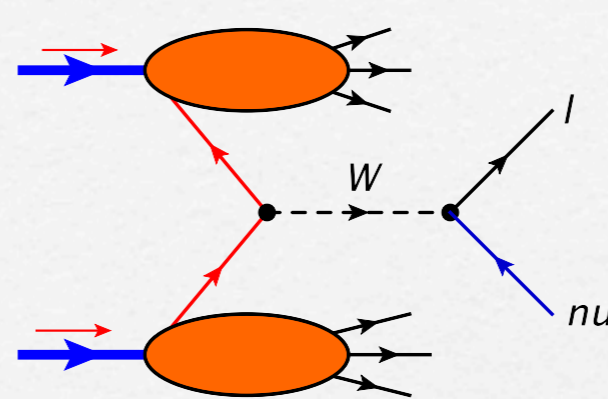
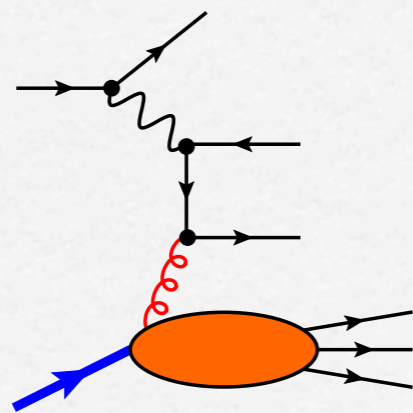
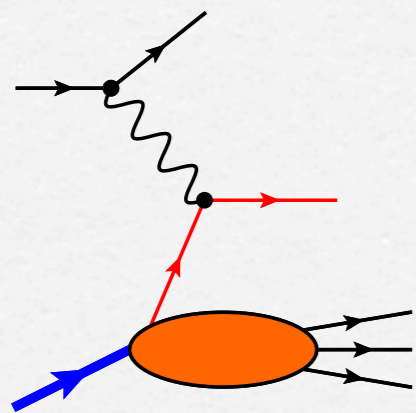
Collinear Factorization

vs.

TMD Factorization

Collinear factorization in pQCD

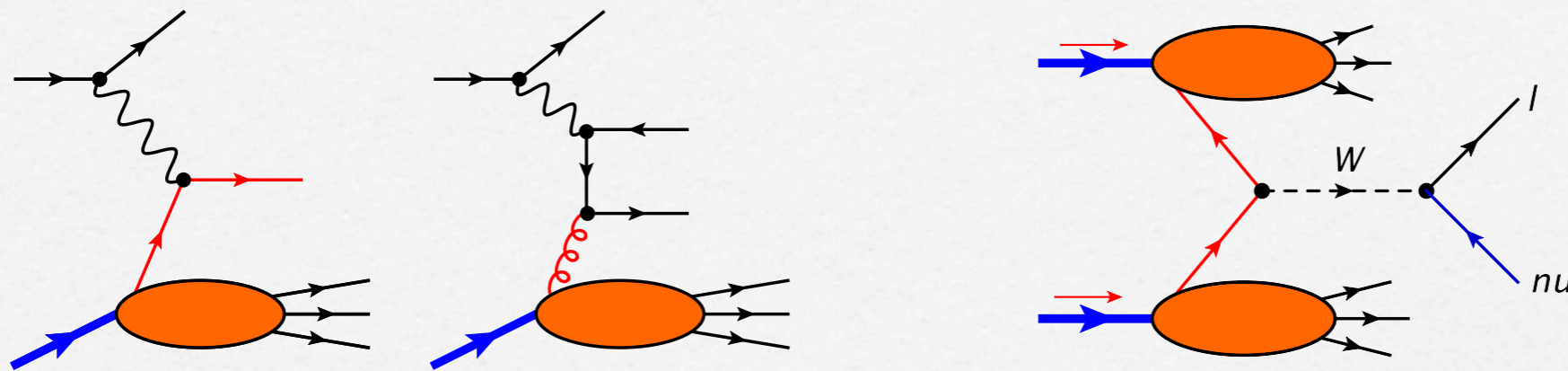
- applicable to one-scale processes, e.g. **1-particle inclusive processes**



$$\frac{d\sigma}{dx dQ^2}$$

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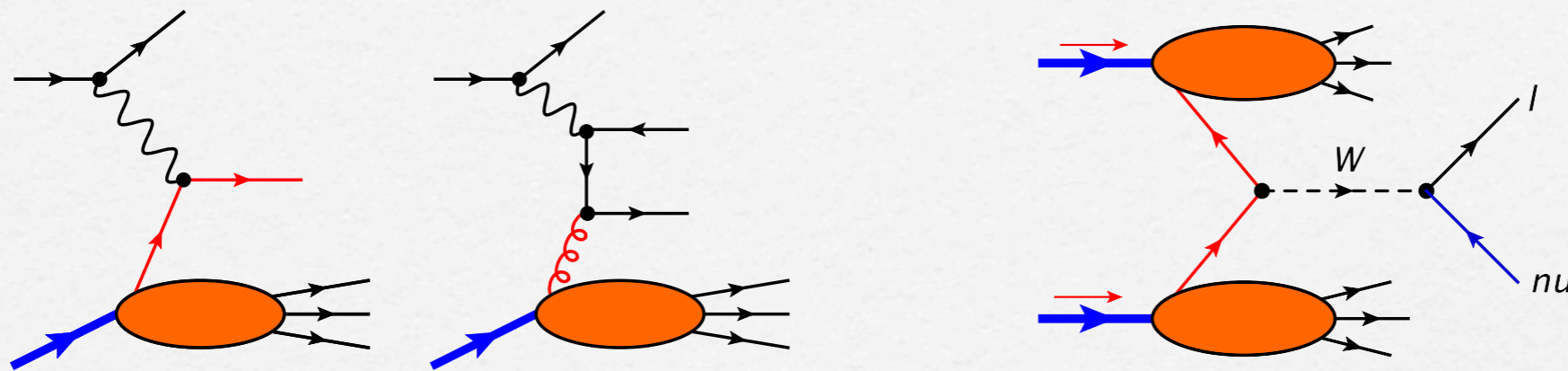


$$\frac{d\sigma}{dx dQ^2}$$

- Cross sections at high energies \rightarrow (hard part) \times (soft parts)
- hard part \rightarrow pQCD (NLO, NNLO, ...) ; soft parts \rightarrow universal, 1-dim

Collinear factorization in pQCD

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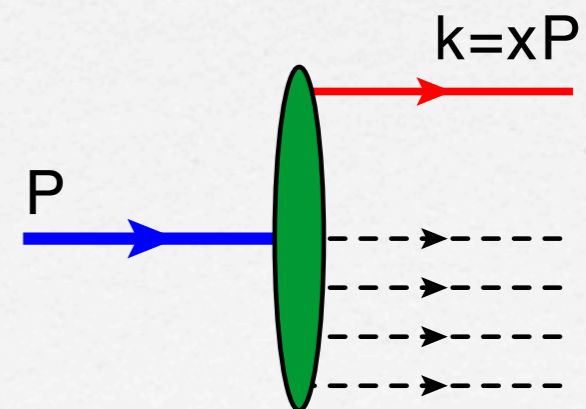
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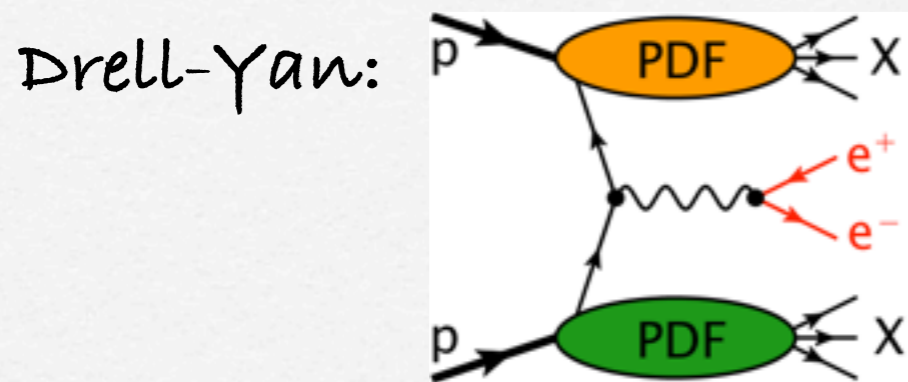
collinear parton distributions

$$q(x, \mu) = \int \frac{d\lambda}{4\pi} e^{i\lambda x(P \cdot n)} \langle P | \bar{\psi}(0) \not{n} W \psi(\lambda n) | P \rangle$$

$$G(x, \mu) = -\frac{1}{x(P \cdot n)} \int \frac{d\lambda}{2\pi} e^{i\lambda x(P \cdot n)} \langle P | F^{n\alpha}(0) W F^n_{\alpha}(\lambda n) | P \rangle$$



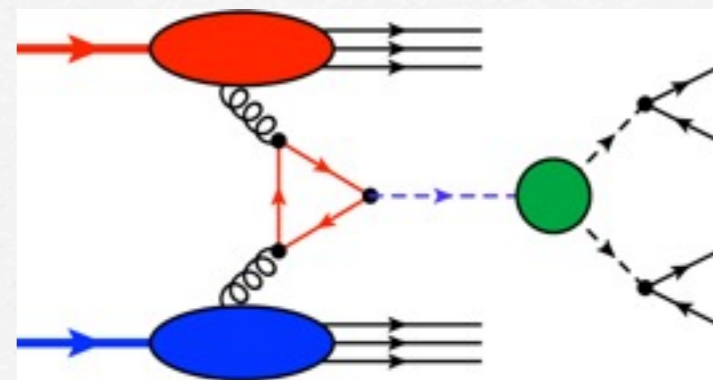
□ Collinear factorization: 2 (or more...)-particle inclusive processes



two scales: hard scale Q +

→ integrated observables

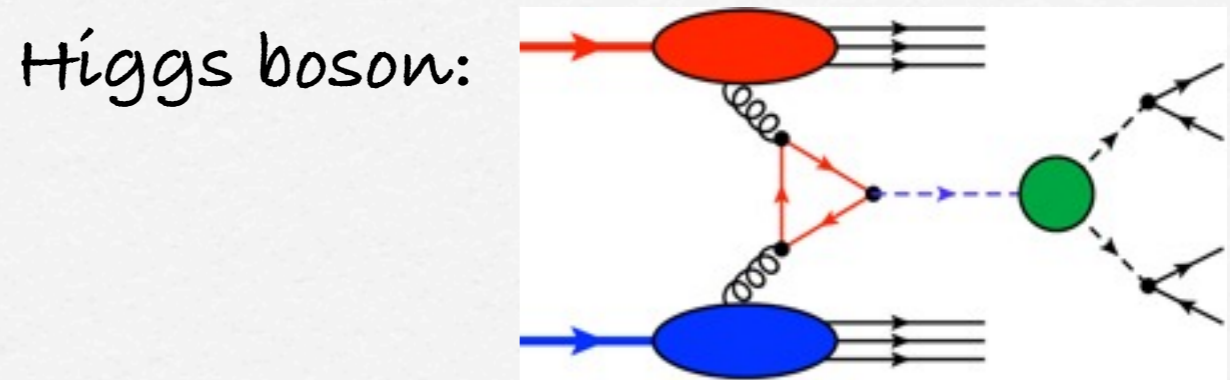
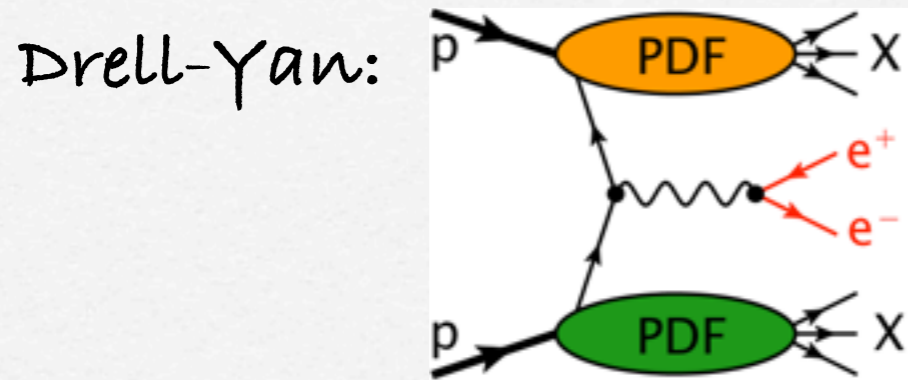
Higgs boson:



final state transverse momentum q_T

$$\int d^2 q_T w(q_T) \frac{d\sigma}{dx dQ^2 dq_T} \equiv \langle w(q_T) \rangle$$

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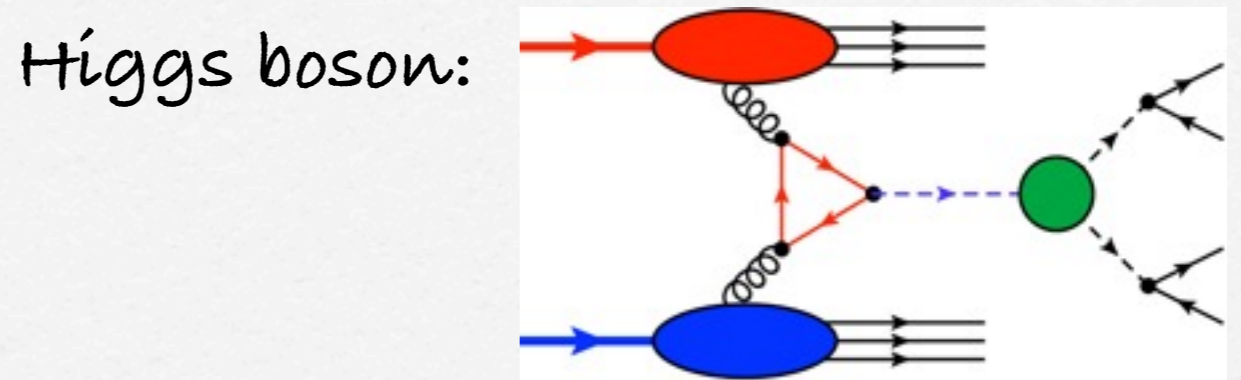
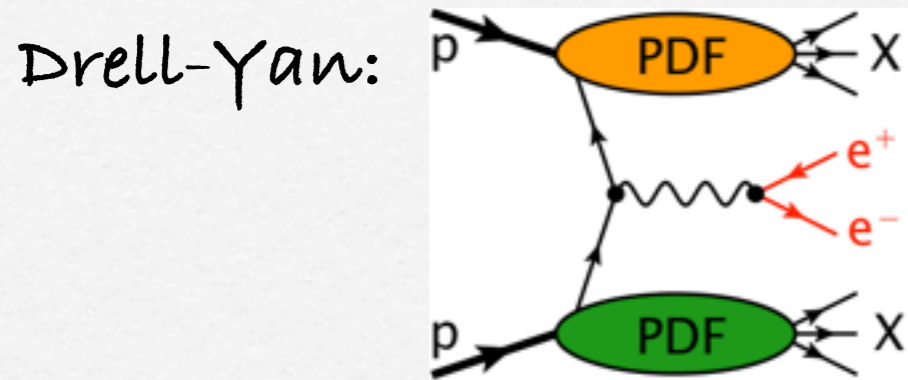
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□ q_T -dependence:

$$\frac{d\sigma}{dq_T} (q_T \sim Q)$$

one scale → collinear factorization ok,
transverse momentum generated perturbatively in hard part

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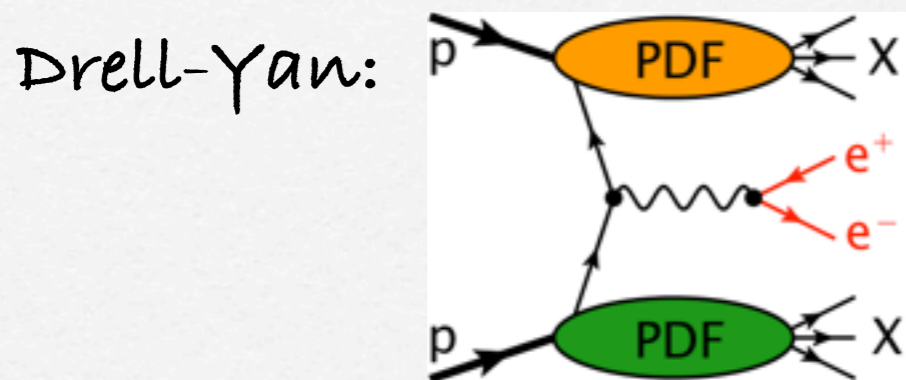
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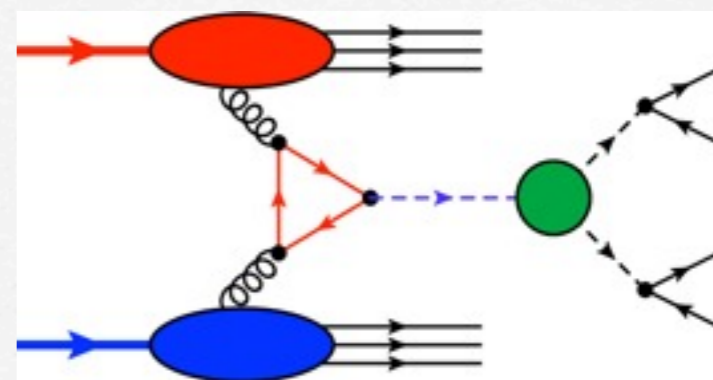
$$\frac{d\sigma}{dq_T} (\Lambda_{\text{QCD}} \ll q_T \ll Q)$$

large logs in the hard part (gluon radiation) $\log^n(q_T/Q)$
→ CSS-resummation → coll. fact. still applicable

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$$\frac{d\sigma}{dq_T} (\Lambda_{\text{QCD}} \sim q_T \ll Q)$$

→ Transverse momentum dependent (TMD) factorization!

Problem:

Description of q_T - distributions in collinear factorization at $q_T \ll Q$

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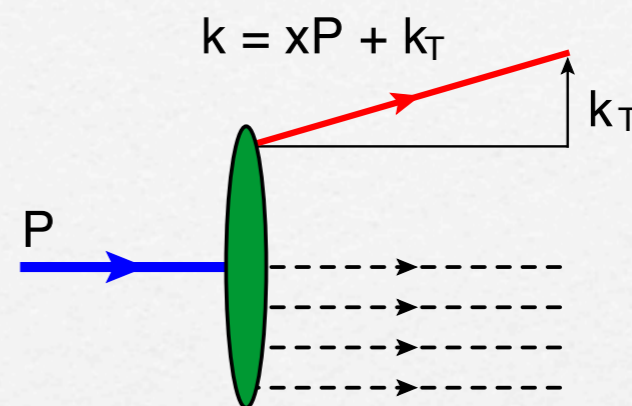
Idea of TMD factorization:

small transverse momentum q_T from

"intrinsic" transverse parton momentum k_T

→ different kind of factorization

→ additional degree of freedom of partonic motion



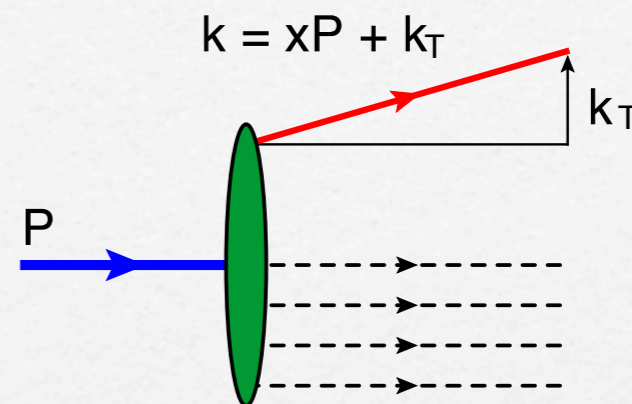
Problem:

Description of q_T - distributions in collinear factorization at $q_T \ll Q$

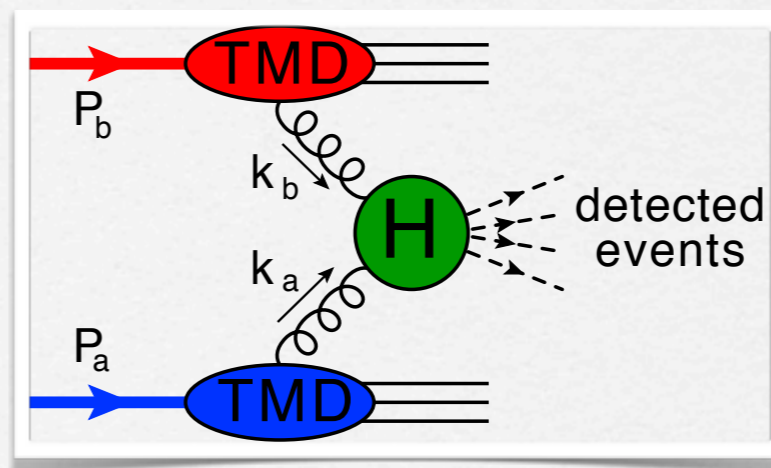
$$\frac{d\sigma}{d^2q_T}$$

Idea of TMD factorization:

- small transverse momentum q_T from "intrinsic" transverse parton momentum k_T
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- TMD factorization theorem (gluon-gluon) $q_T \ll Q$

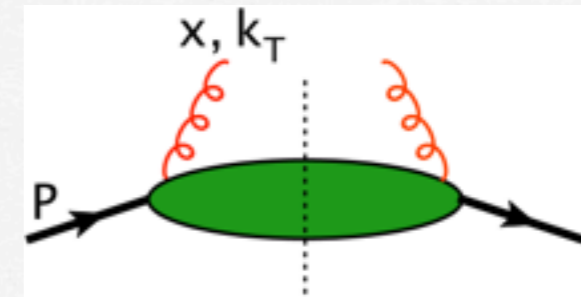


$$d\sigma \propto dPS |H|^2 \int d^2k_{aT} \int d^2k_{bT} \delta^{(2)}(k_{aT} + k_{bT} - q_T) \Gamma(x_a, k_{aT}) \Gamma(x_b, k_{bT}) + \mathcal{O}(q_T/Q)$$

- proven for pp - collisions with color singlet final states

[Collins; Ji, Ma, Yuan; Qiu; Rogers, Mulders; ...]

TMD gluonic matrix element



$$\Gamma^{\alpha\beta}(x, \vec{k}_T) = \frac{1}{x^2(P \cdot n)} \int \frac{d\lambda d^2 z_T}{(2\pi)^3} e^{i\lambda x(P \cdot n) + i\vec{k}_T \cdot \vec{z}_T} \langle P | F^{n\alpha}(0) \mathcal{W} F^{n\beta}(\lambda n + \vec{z}_T) | P \rangle$$

$\Gamma^{[T-even]}(x, \vec{k}_T)$		$\Gamma^{[T-odd]}(x, \vec{k}_T)$	
		flíp	flíp
u	f_1^g	$h_1^{\perp g}$	
L	$g_{1L}^{\perp g}$		$h_{1L}^{\perp g}$
T	$g_{1T}^{\perp g}$	$f_{1T}^{\perp g}$	h_1^g $h_{1T}^{\perp g}$

[Mulders, Rodrigues, PRD 63,094021]

* unpolarized & linearly polarized gluons:
 helicity flíp TMDs \rightarrow azimuthal modulations
 \rightarrow talk C. Pisano

$$\Gamma^{\alpha\beta}(x, k_T) = \frac{1}{2x} \left[-g_T^{\alpha\beta} f_1^g(x, k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2} k_T^2 g_T^{\alpha\beta}}{M^2} h_1^{\perp g}(x, k_T^2) \right]$$

* unpolarized gluons in transversely pol. proton: gluon Sivers function (T-odd)

$$\Gamma_T^{\alpha\beta}(x, k_T) = \frac{1}{2x} \left[g_T^{\alpha\beta} \frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp g}(x, k_T^2) + \dots \right]$$

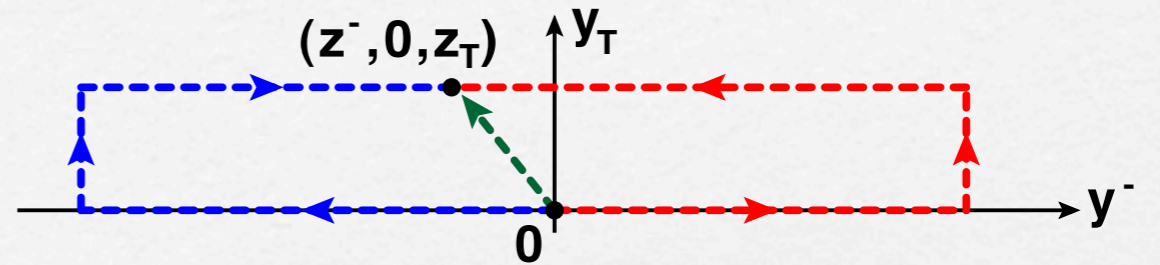
\rightarrow gluonic Spin - Orbit correlation

Wilson line for TMDs

Color gauge invariant definition of TMDs \rightarrow Wilson line

$$\mathcal{W}[0; (0, z)] = \mathcal{P} \exp \left[-ig \int_0^z ds \cdot A(s) \right]$$

\rightarrow Wilson line for TMD:
nontrivial, process dependent:

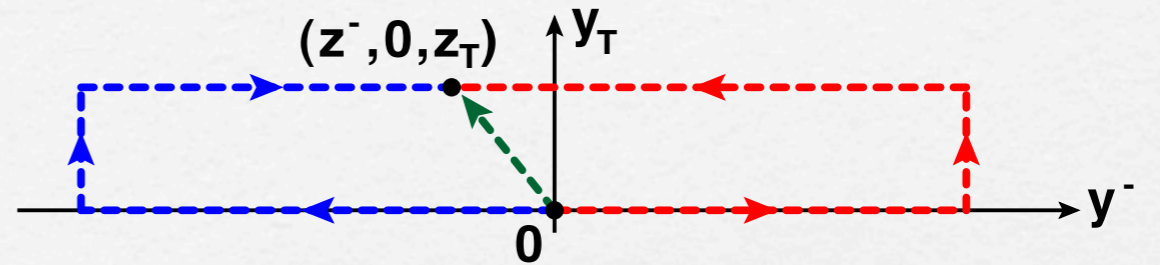


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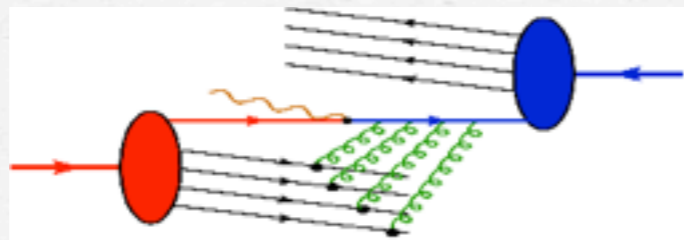
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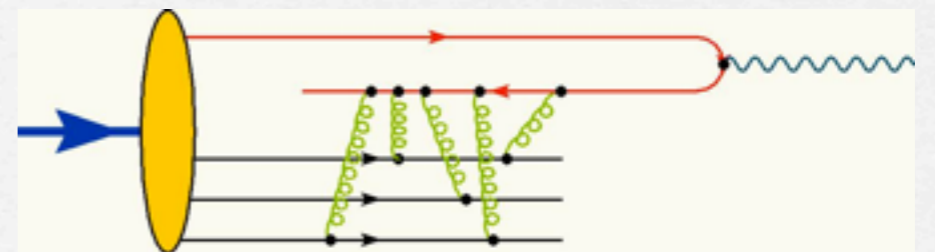
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Initial State Interactions: $pp \rightarrow$ color singlet + X



Final State Interactions: SIDIS

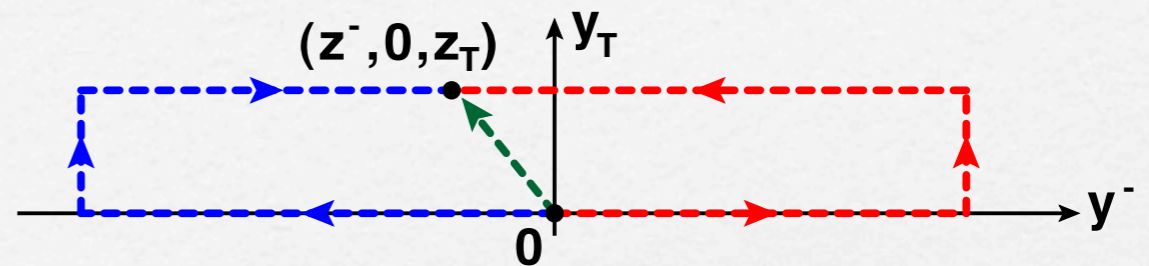


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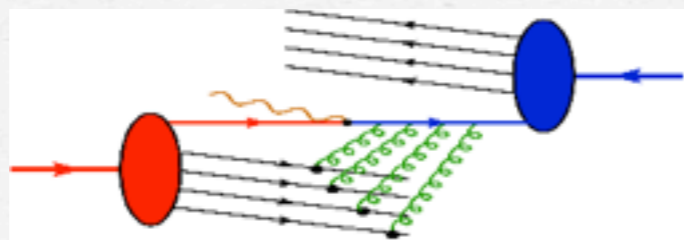
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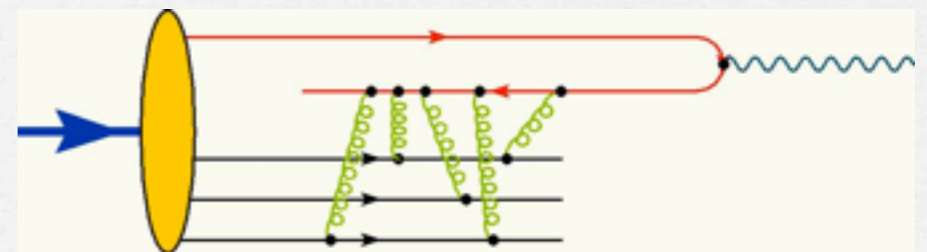
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Time-reversal odd (τ -odd) TMDs \rightarrow sign change

$$f_{1T}^{\perp g}(x, k_T^2) \Big|_{\text{FSI}} = -f_{1T}^{\perp g}(x, k_T^2) \Big|_{\text{ISI}}$$

"Color Entanglement"

TMD factorization problematic in pp - collisions with a colored final state!

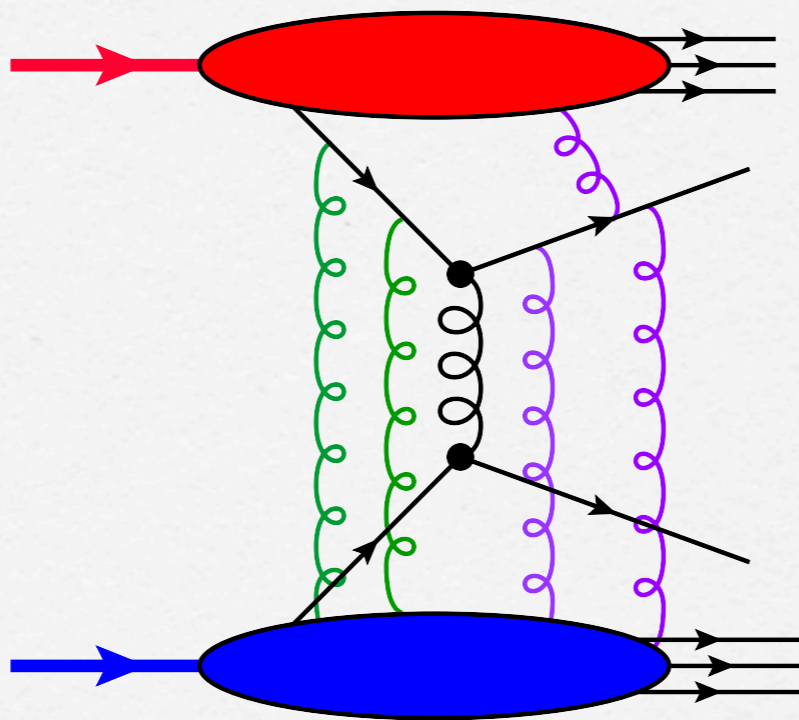
[Collins, Qiu; Rogers, Mulders]

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Example: dijet production $pp \rightarrow \text{jet} + \text{jet} + X$



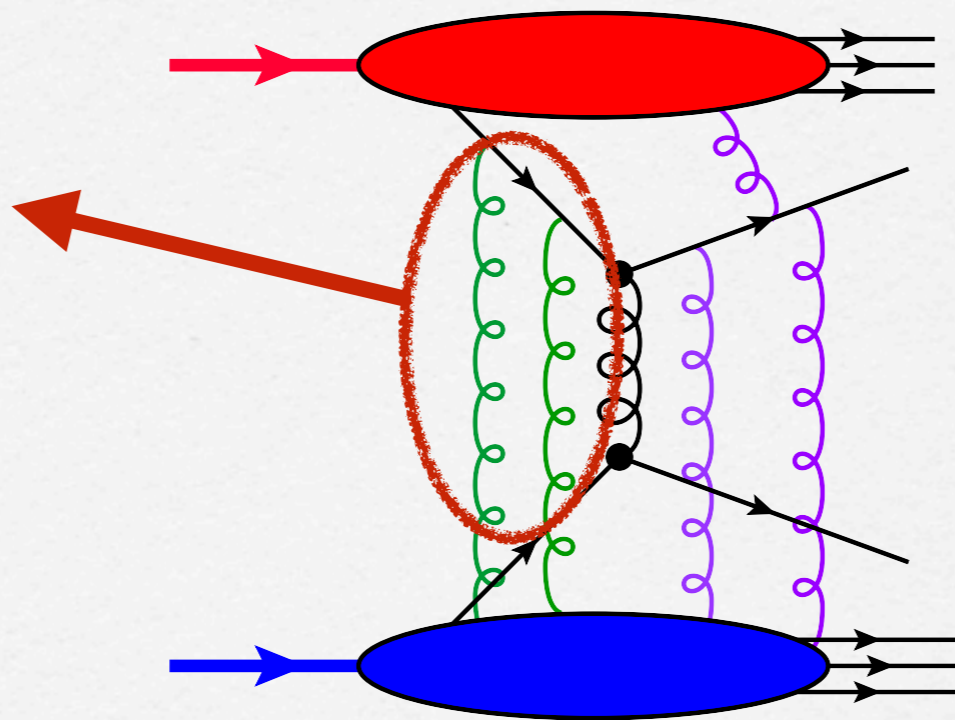
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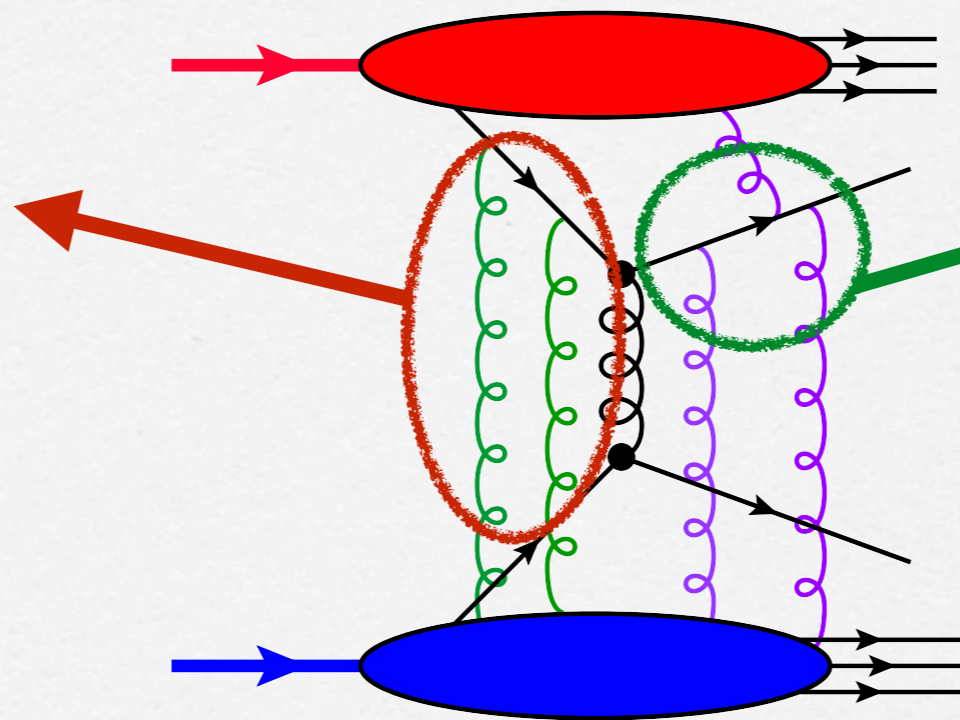
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Final State Interactions
from both nucleons are
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→ cannot define Wilson lines
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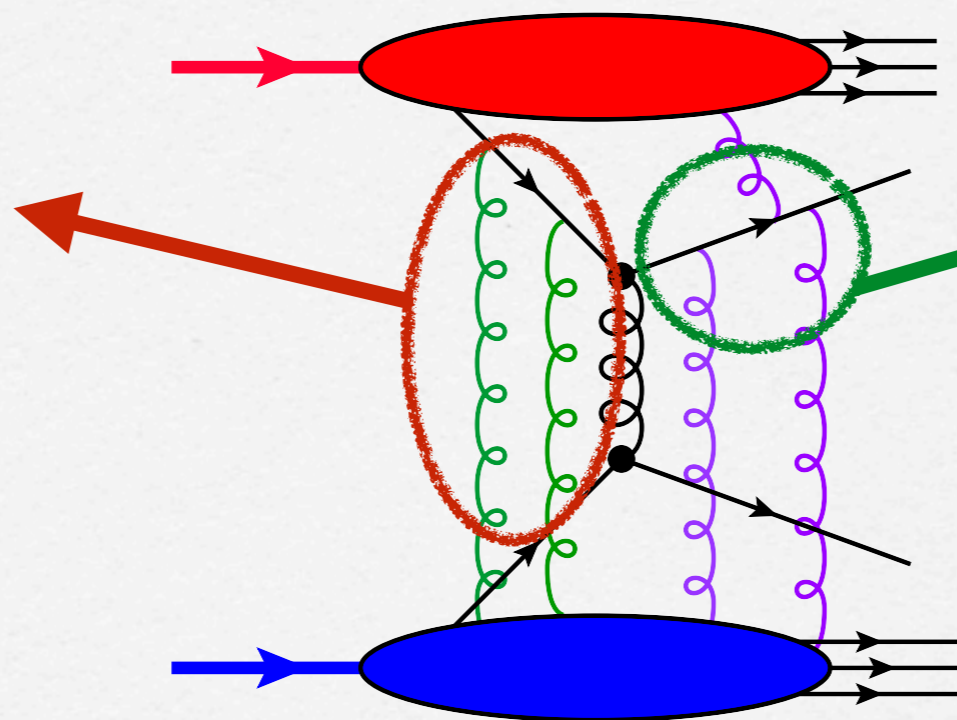
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TMD factorization in pp - collisions only for color singlet final states:

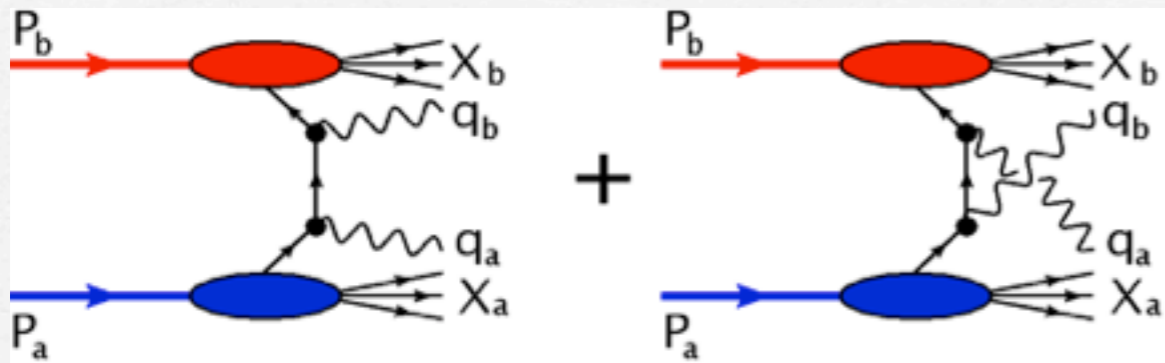
$p + p \rightarrow$ leptons, isolated photons, isolated quarkonia in a color singlet state

**Gluon TMDs from *SSAs*
in pp-collisions
(AFTER, RHIC)**

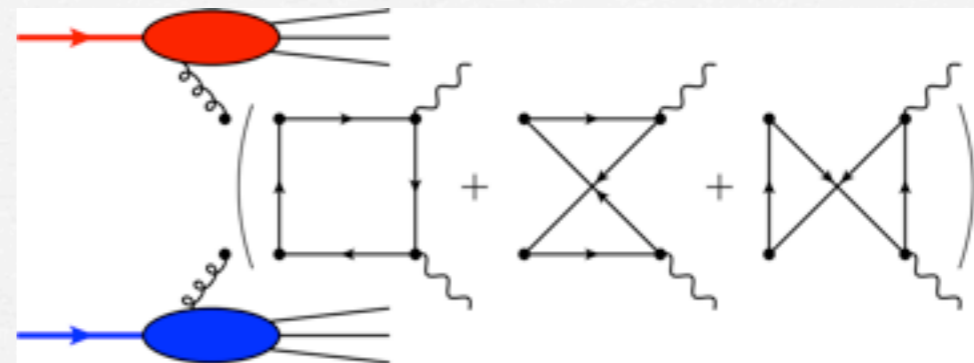
Photon Pair production

[Qiu, M.S., Vogelsang, PRL 107, 062001 (2011)]

quark TMDs



gluon TMDs at $O(\alpha_s^2)$



- no colored final state ($\gamma\gamma, \gamma Z, ZZ$) \Rightarrow TMD factorization ok
- contaminations from quark contributions:
- $\gamma\gamma$ - production: huge background from π^0 - decays, need isolated photons: isolation cuts
- γZ or ZZ - production: statistics (?)

Single Quarkonium - production in pp - collisions

[LO: Boer, Pisano, PRD86, 094007; NLO: Ma, Wang, Zhao, PRD88, 014027]

exclusive production of a heavy quarkonium state (color singlet model):

$$p + p \rightarrow (\eta, \chi, \dots) + X$$

$$\begin{aligned} \text{S-waves: } L=0, J=0: & \quad \eta : {}^1S_0^{(1)} \\ & \quad \chi_{0,2} : {}^3P_{0,2}^{(1)} \\ \text{P-waves: } L=1, J=0,2: & \quad \chi_{0,2} : {}^3P_{0,2}^{(1)} \end{aligned} \quad 2S+1L_J$$

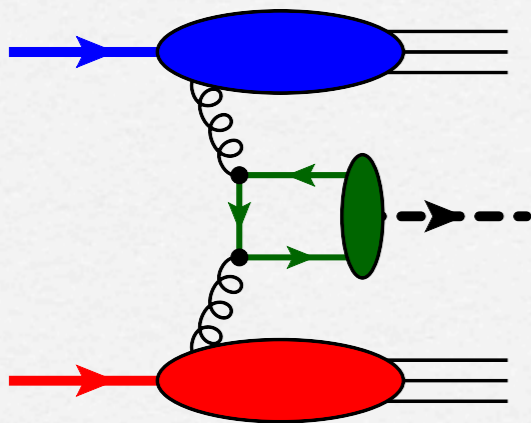
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QQ - rest frame: non-relativistic approach

neglect relative quark momenta in hard part

$$\int \frac{d^3 \vec{k}}{(2\pi)^3} \psi_{00}(\vec{k}) = \frac{1}{\sqrt{4\pi}} R_0(0) \quad \int \frac{d^3 \vec{k}}{(2\pi)^3} k^\alpha \psi_{1L_z}(\vec{k}) = -i \varepsilon_{L_z}^\alpha(q) \sqrt{\frac{3}{4\pi}} R'_0(0)$$

no contamination from quark sector (at LO)

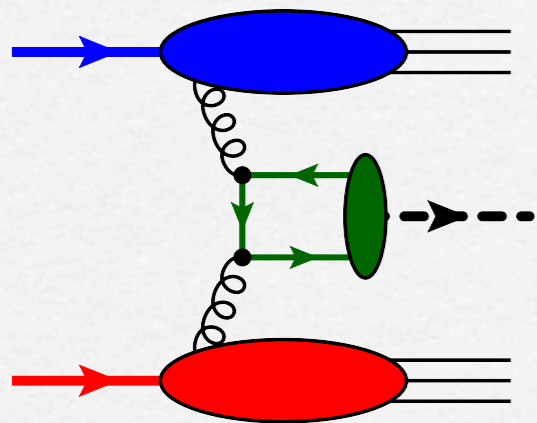
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TMD - formalism ($q_T \sim \Lambda, Q = M_Q$):

$$\frac{d\sigma(\eta)}{dyd^2q_T} = C_\eta ([f_1^g \otimes f_1^g] - [h_1^g \otimes h_1^g])$$

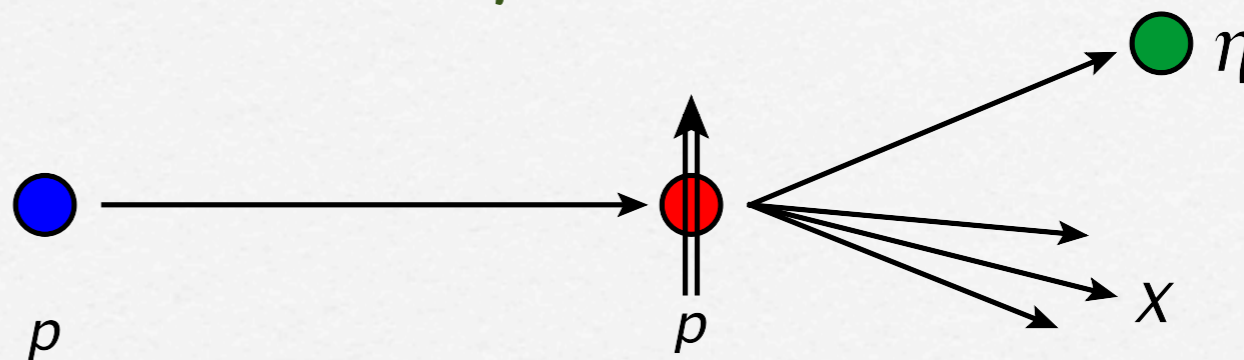
$$\frac{d\sigma(\chi_0)}{dyd^2q_T} = C_{\chi_0} ([f_1^g \otimes f_1^g] + [h_1^g \otimes h_1^g])$$

$$\frac{d\sigma(\chi_2)}{dyd^2q_T} = C_{\chi_2} ([f_1^g \otimes f_1^g])$$

- (in principle) possible to extract both TMD - structures!
- Not possible to tune the hard scale, $Q = M_Q$ not that large!
- Transv. Momentum q_T must be very small, difficult to implement isolation

SSA for isolated η -production

SSA at AFTER:

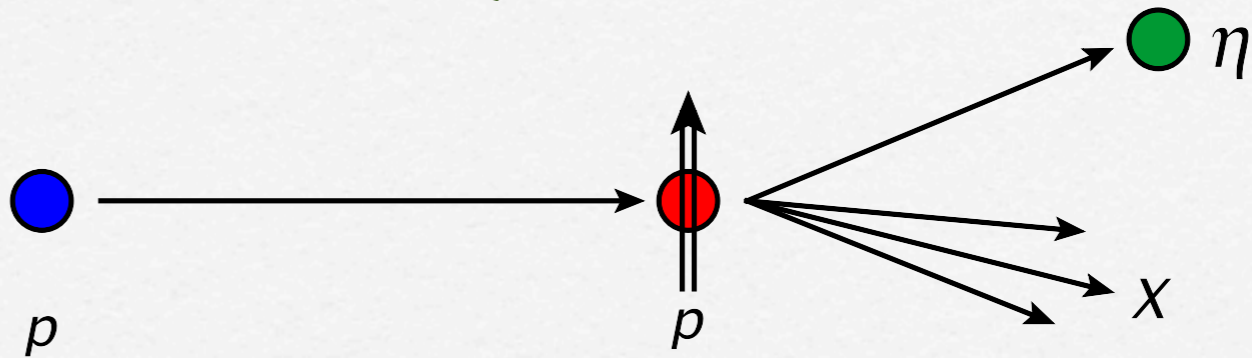


Long. pol. target:

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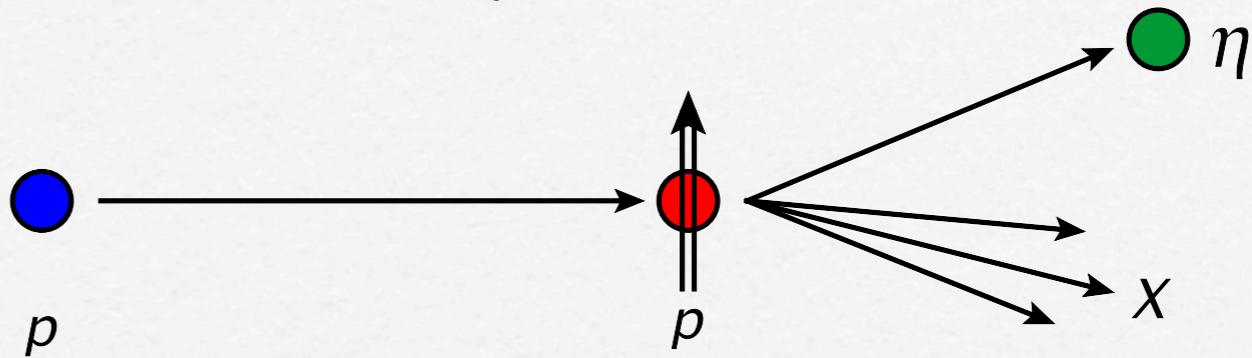
gluon Sivers

gluon TMD Transversity

gluon "pretzelosity"

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Transverse SSA at AFTER (in principle) sensitive to polarized gluon TMDs at large x !

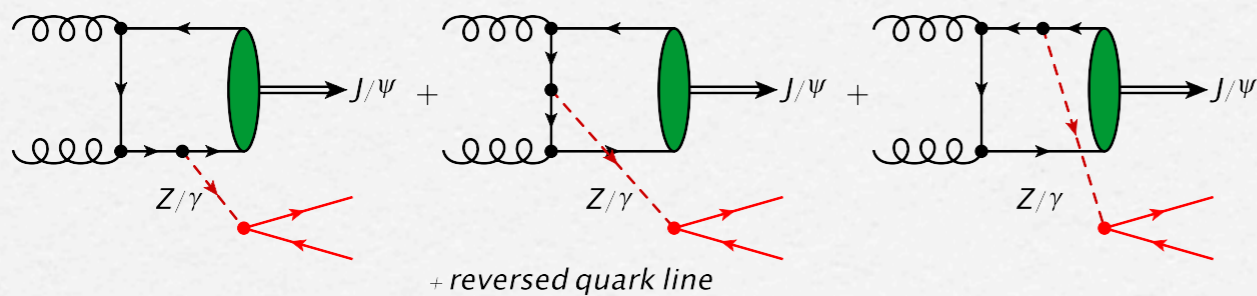
J/ψ + γ (μ⁺μ⁻) - production

[Iden Dunnen, Lansberg, Pisano, M.S., in preparation]

→ less statistics, but more kinematic "freedom"...

→ J/ψ + γ: both J/ψ and photon need to be isolated;

J/ψ + dilepton: only isolation for J/ψ, but cross section much reduced...



$$\begin{aligned}
 q^\mu &= P_{J/\psi}^\mu + P_\gamma^\mu [P_{J/\psi}^\mu + l^\mu + \bar{l}^\mu] \\
 Q^2 &= (P_{J/\psi} + P_\gamma)^2 [(P_{J/\psi} + l + \bar{l})^2] \\
 q_T &= P_{J/\psi,T} + P_{\gamma,T} [P_{J/\psi,T} + l_T + \bar{l}_T] \\
 M_B^2 &= (l + \bar{l})^2
 \end{aligned}$$

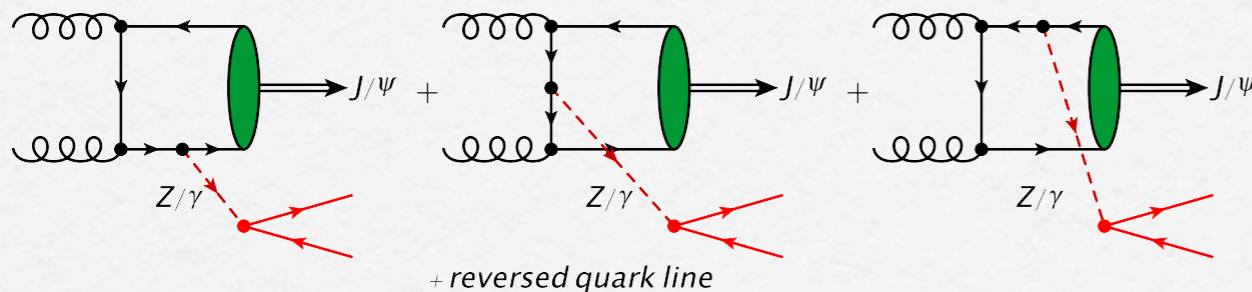
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J/ψ + dilepton: only isolation for J/ψ, but cross section much reduced...



$$\begin{aligned}
 q^\mu &= P_{J/\psi}^\mu + P_\gamma^\mu [P_{J/\psi}^\mu + l^\mu + \bar{l}^\mu] \\
 Q^2 &= (P_{J/\psi} + P_\gamma)^2 [(P_{J/\psi} + l + \bar{l})^2] \\
 q_T &= P_{J/\psi,T} + P_{\gamma,T} [P_{J/\psi,T} + l_T + \bar{l}_T] \\
 M_B^2 &= (l + \bar{l})^2
 \end{aligned}$$

TMD result at $q_T \ll Q$

$$\begin{aligned}
 \frac{d\sigma_{UU}^{qq \rightarrow J/\psi(\mu^+\mu^-)}}{dY dQ d^2q_T dM_B^2 d\Omega} &= \hat{F}_1(Q, \cos\theta, M_B^2) [f_1^g \otimes f_1^g] + \hat{F}_2(Q, \cos\theta, M_B^2) [h_1^{\perp g} \otimes h_1^{\perp g}] \\
 &+ \hat{F}_3(Q, \cos\theta, M_B^2) \cos(2\phi) ([f_1^g \otimes h_1^{\perp g}] + \{x_a \leftrightarrow x_b\}) \\
 &+ \hat{F}_4(Q, \cos\theta, M_B^2) \cos(4\phi) [h_1^{\perp g} \otimes h_1^{\perp g}] + \mathcal{O}(q_T/Q)
 \end{aligned}$$

- Factors F_1, F_2, F_3, F_4 perturbatively at LO → NLO: future work...
- J/ψ + γ: $F_2 = 0$ → pure f_1^g - extraction from q_T - distribution possible
- 2-particle final state: azimuthal $\cos(2\phi)$ and $\cos(4\phi)$ - modulation

→ talk C. Pisano

SSA in isolated $J/\psi + \gamma [\mu^+\mu^-]$ - production

Long. Pol. Target:

$$\begin{aligned}\Delta\sigma_{UL} &\equiv d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \\ &= \hat{F}_3(Q, \cos\theta, M_B^2) \sin(2\phi) [f_1^g \otimes h_{1L}^{\perp g}] \\ &\quad + \hat{F}_4(Q, \cos\theta, M_B^2) \sin(4\phi) [h_1^{\perp g} \otimes h_{1L}^{\perp g}]\end{aligned}$$

gluonic "wormgear" function:
lin. pol. gluons in long. pol. proton (T-odd)

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Transv. Pol. Target:

$$\begin{aligned} \Delta\sigma_{UT} &\equiv d\sigma^{\uparrow} - d\sigma^{\downarrow} \\ &= \sin\phi_s \left[\hat{F}_1 [f_1^g \otimes f_{1T}^{\perp g}] - \hat{F}_2 ([h_{1L}^{\perp g} \otimes h_{1T}^g] + [h_{1L}^{\perp g} \otimes h_{1T}^{\perp g}]) \right. \\ &\quad \left. + \sin(2\phi) \{ \dots \} + \sin(4\phi) \{ \dots \} \right] + \cos\phi_s \left[\sin(2\phi) \{ \dots \} + \sin(4\phi) \{ \dots \} \right] \end{aligned}$$

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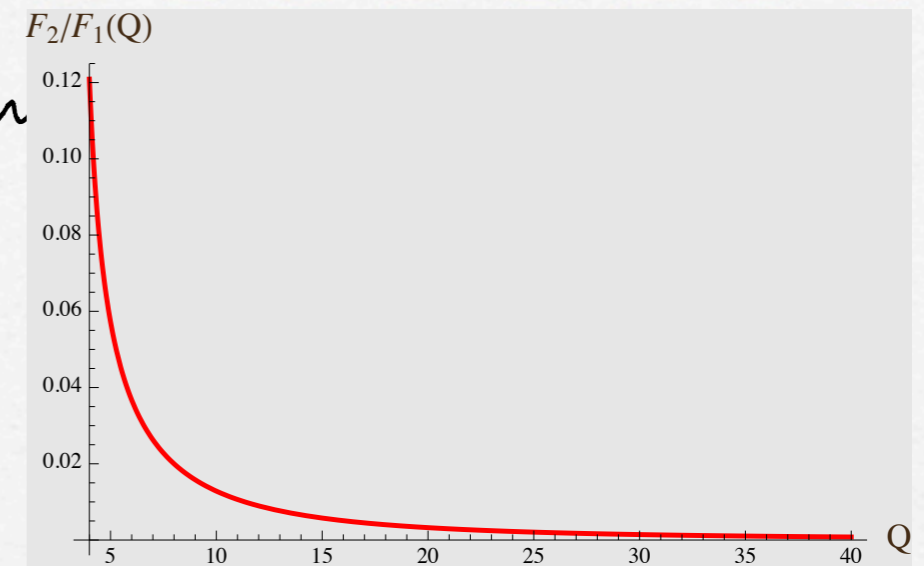
Sivers effect \rightarrow sufficient to integrate out CS - angles θ and ϕ [and dilepton mass M_B^2]

$$\begin{aligned} A_{UT}(Y, Q, q_T) &= \frac{\int d\Omega dM_B^2 (d\sigma^{\uparrow} - d\sigma^{\downarrow})}{\int d\Omega dM_B^2 (d\sigma^{\uparrow} + d\sigma^{\downarrow})} \\ &= \sin\phi_s \left[\frac{[f_1^g \otimes f_{1T}^{\perp g}] - \beta(Q) ([h_1^{\perp g} \otimes h_1^g] + [h_1^{\perp g} \otimes h_{1T}^{\perp g}])}{[f_1^g \otimes f_1^g] + \beta(Q) [h_1^{\perp g} \otimes h_1^{\perp g}]} \right] \end{aligned}$$

- perturbative factor β very small for $J/\psi + \mu^+\mu^-$ - production
- even identical to zero for $J/\psi + \gamma$ - production

$$\beta(Q) = \frac{\int d(\cos \theta) dM_B^2 \hat{F}_2(Q, \cos \theta, M_B^2)}{\int d(\cos \theta) dM_B^2 \hat{F}_1(Q, \cos \theta, M_B^2)}$$

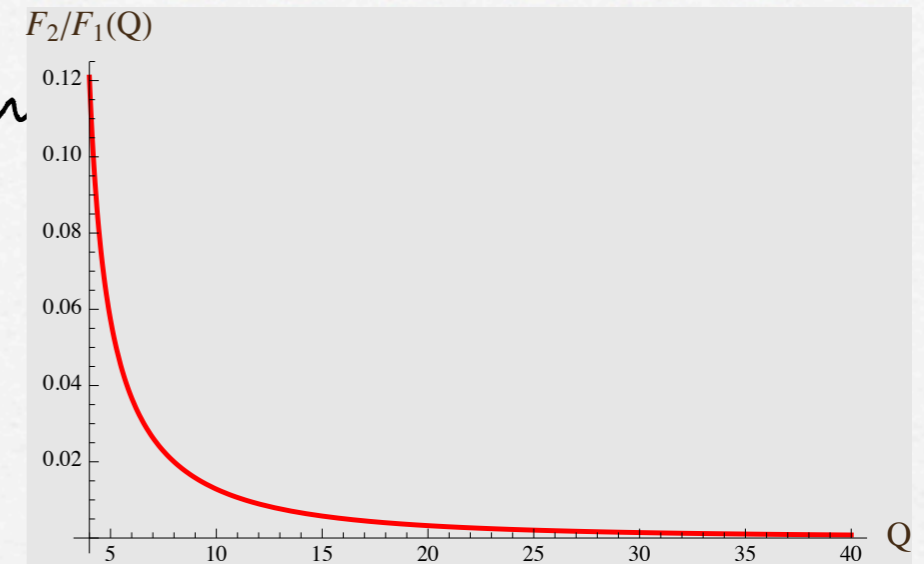
- $\beta < 2\%$ for $Q > 7 \text{ GeV}$



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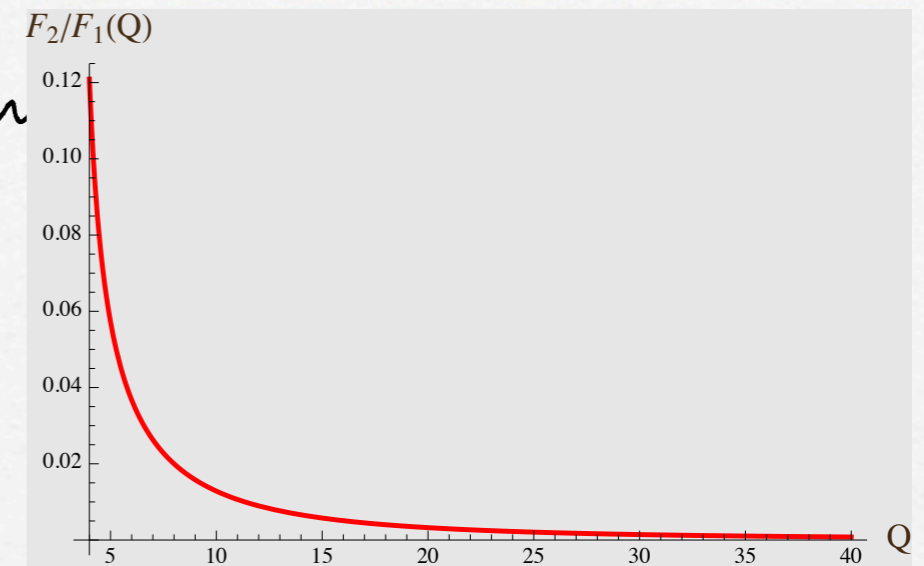
- Hence, we can approximate the SSA:

$$A_{UT}(Y, Q, q_T) \simeq \sin \phi_s \frac{[f_1^g \otimes f_{1T}^{\perp g}]}{[f_1^g \otimes f_1^g]}$$

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Numerical estimates

Not much experimental information on gluon TMDs \rightarrow models, ...

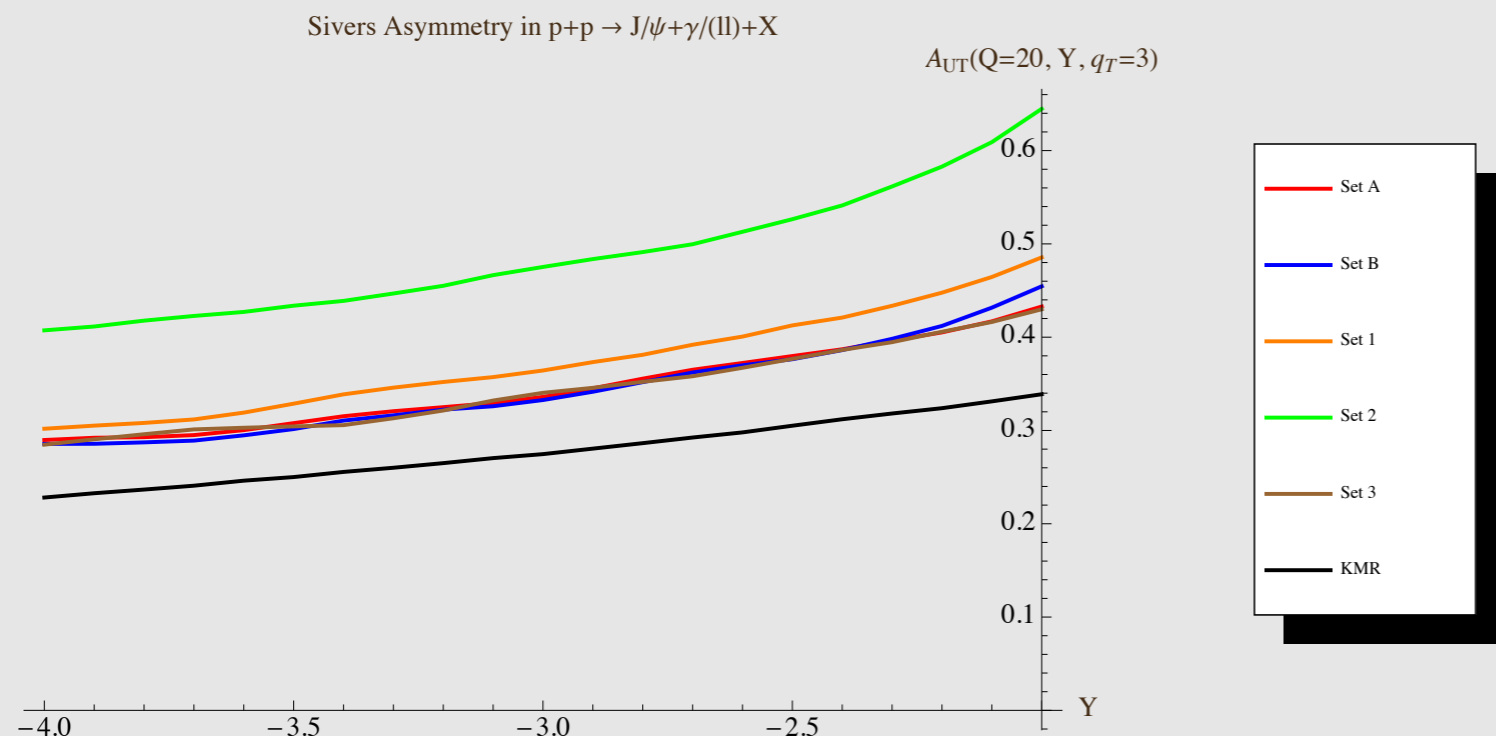
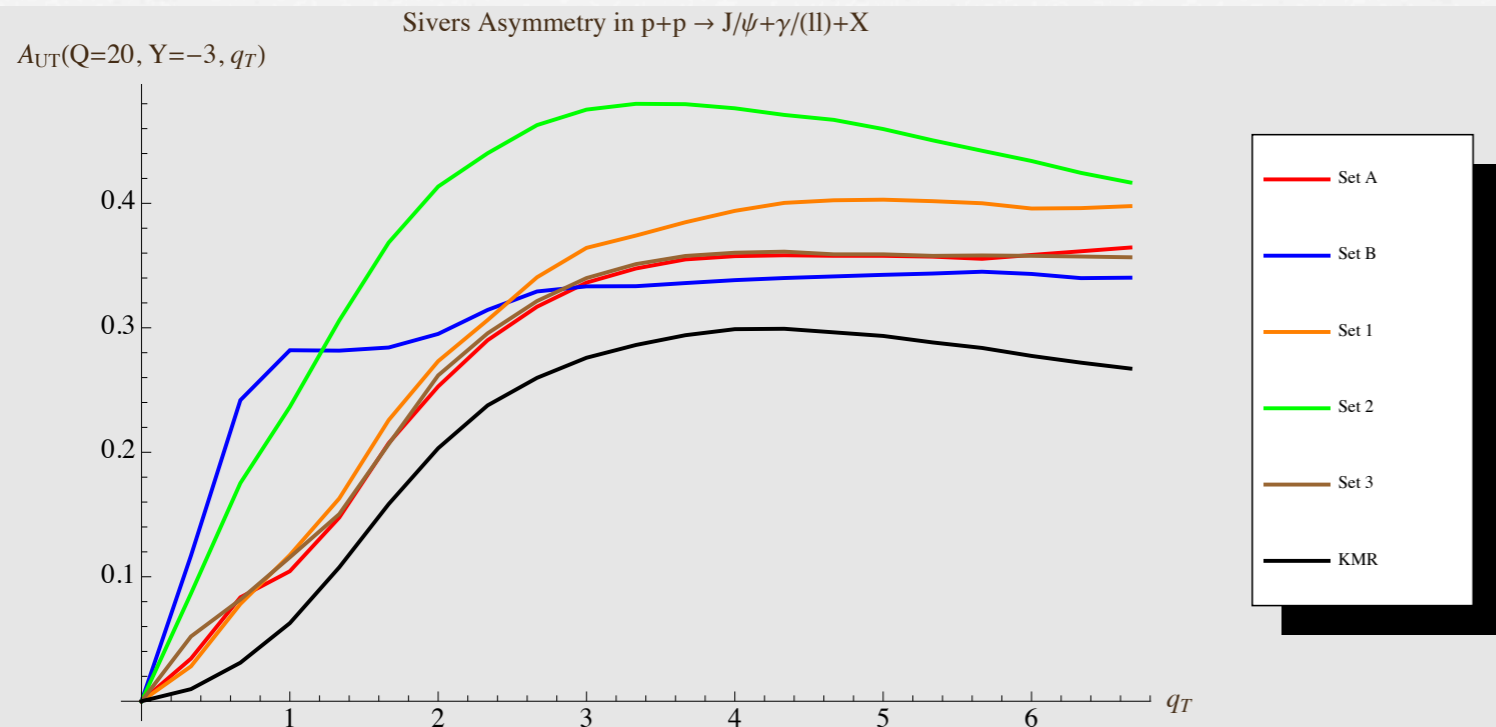
Our approach: Link the unpolarized gluon TMD to "unintegrated parton distribution" used in small-x physics (parameterizations to HERA data)

$$f_1^g(x, k_T) \leftrightarrow \text{uPDF}(x, k_T)$$

Gluon Sivers function: Assume saturation of positivity bound
 \rightarrow upper bounds for SSA

$$f_{1T}^{\perp g}(x, k_T) \leq \frac{M}{k_T} f_1^g(x, k_T)$$

Upper bounds for gluon Sivers asymmetry



Bounds given in AFTER kinematics

$$\sqrt{S} \simeq 120 \text{ GeV}; Y \simeq -4 \text{ to } -2$$

Saturated bounds:
Large effects are predicted
 $SSA = 30\% - 40\%$

even if the Sivers function is
"undersaturated" by a factor 1/100
→ still an effect of about 0.3%

Bounds remain sizeable
at large negative pair rapidities:
 $x_a \ll 1; x_b \rightarrow 1$

Summary

- Isolated quarkonium observables give unique insight into the gluonic structure of the nucleon
- Quarkonium production in combination with TMD factorization: 3-dim momentum picture of (polarized) gluons in the nucleon
- Transverse SSAs at AFTER
→ gluon Sivers effect feasible!

Backup slides

Relations to collinear Parton Densities

"naïve" relation through k_T -integration:

$$\int d^2 k_T f_1^{q/g}(x, k_T^2) = f_1^{q/g}(x) \quad G_F(x, x) = \frac{\pi}{2} f_{1T}^{\perp(1)}(x) = \int d^2 k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2)$$

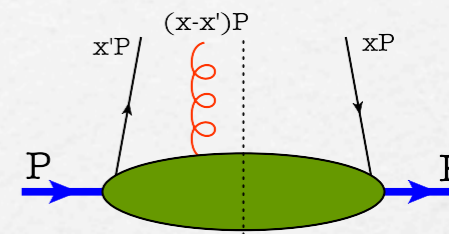
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Quark-Gluon Correlation Functions

(ETQS-matrix element)



$$\frac{M}{2} \epsilon_T^{\alpha\beta} S_{T\beta} G_F^q(x, x') = \int \frac{d\lambda d\eta}{2(2\pi)^2} e^{i(P \cdot n)(x' \lambda + (x - x') \eta)} \langle P, S_T | \bar{q}(0) \gamma^+ g F^{+\alpha}(\eta n) q(\lambda n) | P, S_T \rangle$$

→ Relations work when applying collinear factorization (leading twist, higher twist)

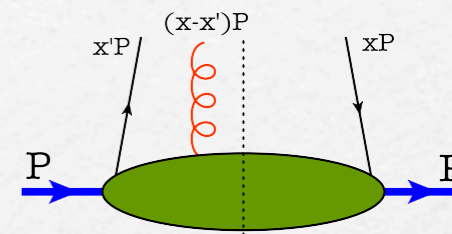
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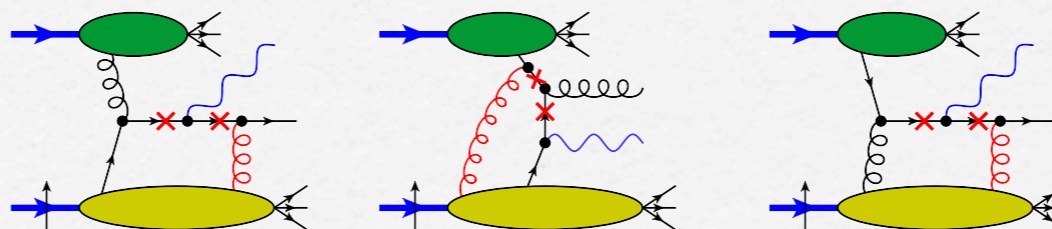


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Example: Collinear Twist-3 factorization for SSA in $pp \rightarrow \gamma X$

[Qiu, Sterman; Kouvaris, Vogelsang, Yuan, Koike, Yoshida, ...]



$$E \frac{d\Delta\sigma}{d^3 q_\gamma} \propto \int \frac{dx'}{x'} \int \frac{dx}{x} f_1^{q/g}(x') [G_F^{q/g}(x, x) - x \frac{d}{dx} G_F^{q/g}(x, x)] \hat{H}^{SGP}(x, x') + \dots$$

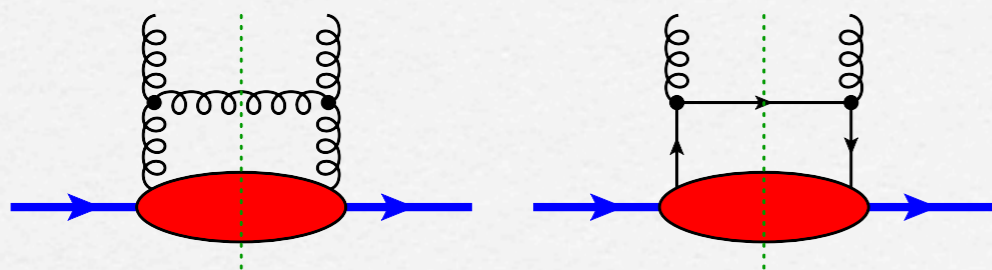
Correct, full definition of TMDs a la Collins, etc.

Relation to collinear Densities through
"perturbative tail" of TMDs (= large k_T behavior)

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Relation to collinear Densities through
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Large transverse momentum behaviour of TMD \rightarrow perturbative calculation:

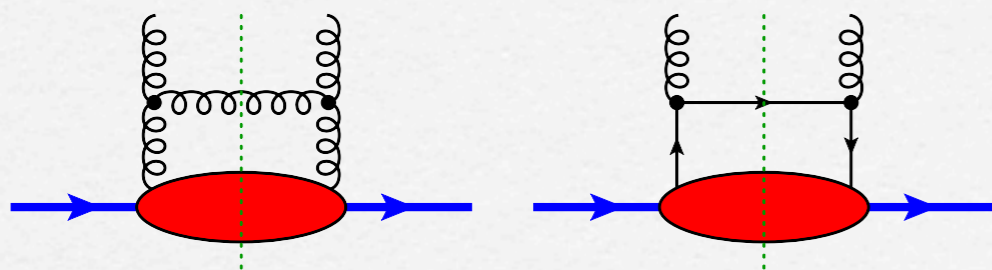


$$f_1^g(x, p_T^2 \gg \Lambda^2) \propto \frac{\alpha_s}{p_T^2} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{ig}(z, \zeta/p_T^2) f_1^i(x/z, \mu) + \mathcal{O}(\alpha_s^2)$$
$$h_1^{\perp g}(x, p_T^2 \gg \Lambda^2) \propto \frac{\alpha_s}{p_T^4} \sum_{i=q,g} \int_x^1 \frac{dz}{z} \frac{1-z}{z} f_1^i(x/z, \mu) + \mathcal{O}(\alpha_s^2)$$

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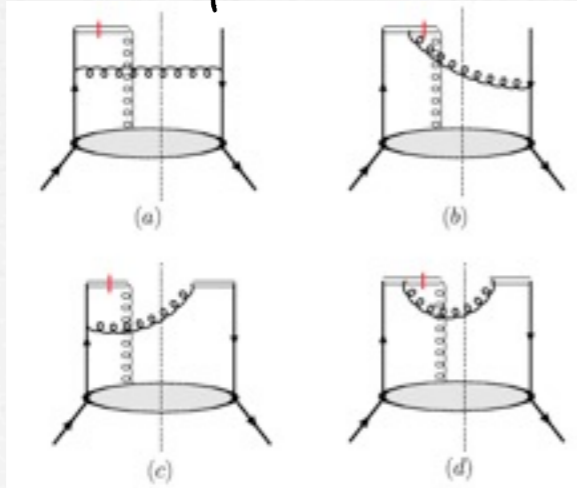
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Sivers function \rightarrow relation to Quark-Gluon correlations at large k_T

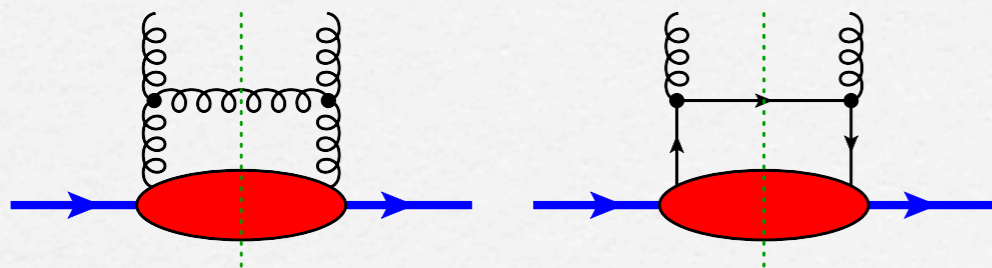


$$f_{1T}^{\perp q}(x, p_T^2 \gg \Lambda^2) \propto \frac{\alpha_s}{p_T^4} \int_x^1 \frac{dz}{z} \left[C_1^{\text{SGP}} G_F\left(\frac{x}{z}, \frac{x}{z}\right) + C_2^{\text{SGP}} \left(\frac{d}{dx} G_F\right)\left(\frac{x}{z}, \frac{x}{z}\right) + C_1^{\text{HFP}} G_F\left(\frac{x}{z}, x\right) \right]$$

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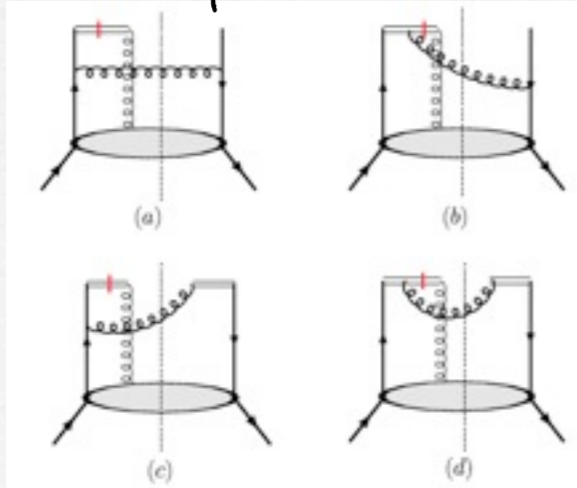
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\rightarrow "TMD fact. = collinear fact."

in overlapping region $\Lambda \ll q_T \ll Q$ in DY/SIDIS

[Ji, Qiu, Vogelsang, Yuan]

Why (transverse) SSA?

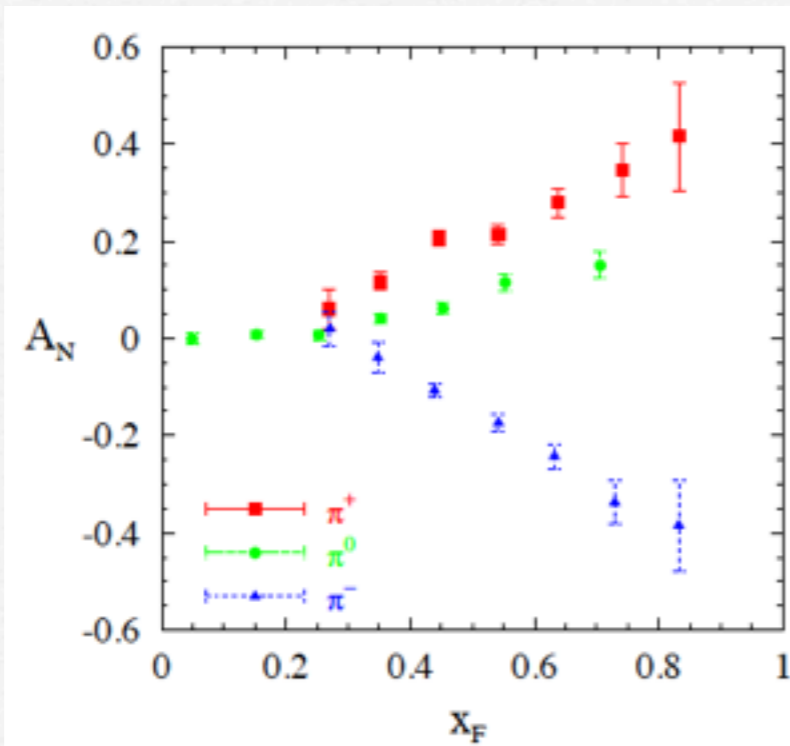
Two important experimental observations for TSSAs

$$A_N = A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

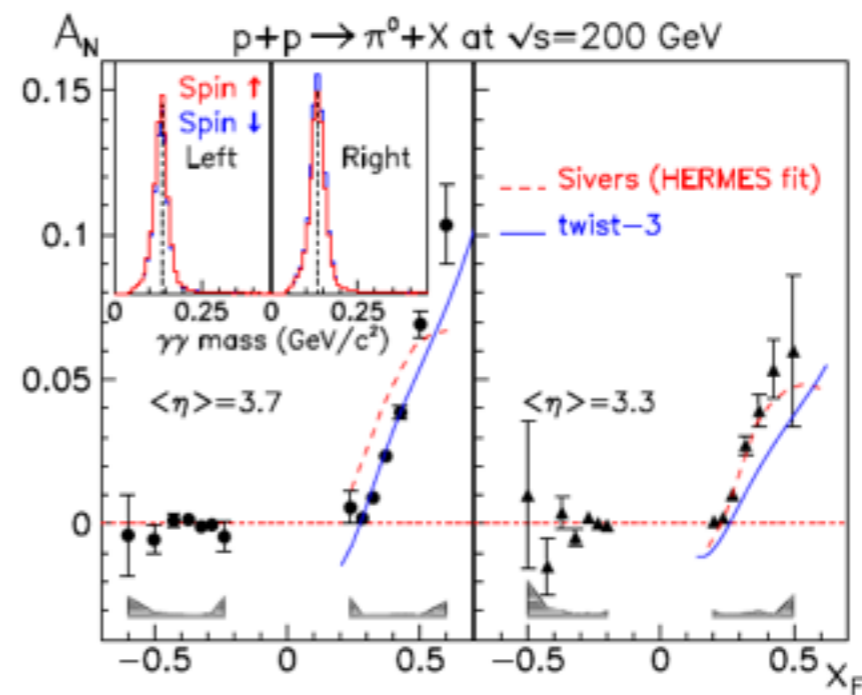
Two important experimental observations for TSSAs

$$A_N = A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

1) Transverse SSA in pion-production $p + p^\uparrow \rightarrow \pi + X$



$\sqrt{s} = 20 \text{ GeV}$ [E704 coll. (1991)]



$\sqrt{s} = 200 \text{ GeV}$ [STAR coll. (2008)]

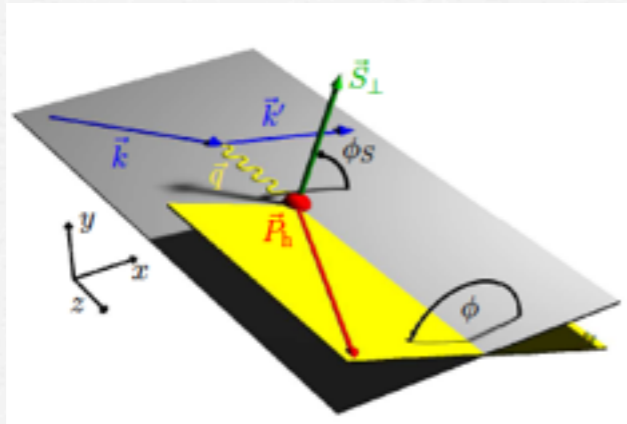
sizeable effect at large x_F (and large $P_T \dots$ (?))

cannot be explained in the naive parton model [Kane, Repko]

→ collinear twist-3 framework / not fully understood to the present day

2) Transverse SSA in semi-inclusive DIS

("Sivers", "Collins" effect...)



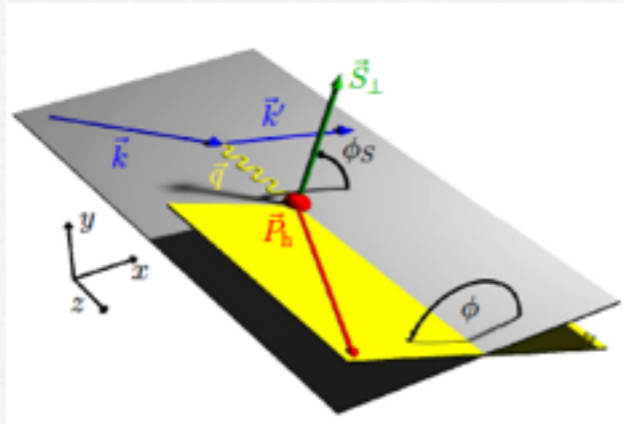
$$e + p^\uparrow \rightarrow e + \pi + X$$

$$d\sigma_{UT}^{\text{SIDIS}} = F_{UT}^{\sin(\phi - \phi_S)} \sin(\phi - \phi_S) + F_{UT}^{\sin(\phi + \phi_S)} \sin(\phi + \phi_S) + \dots$$

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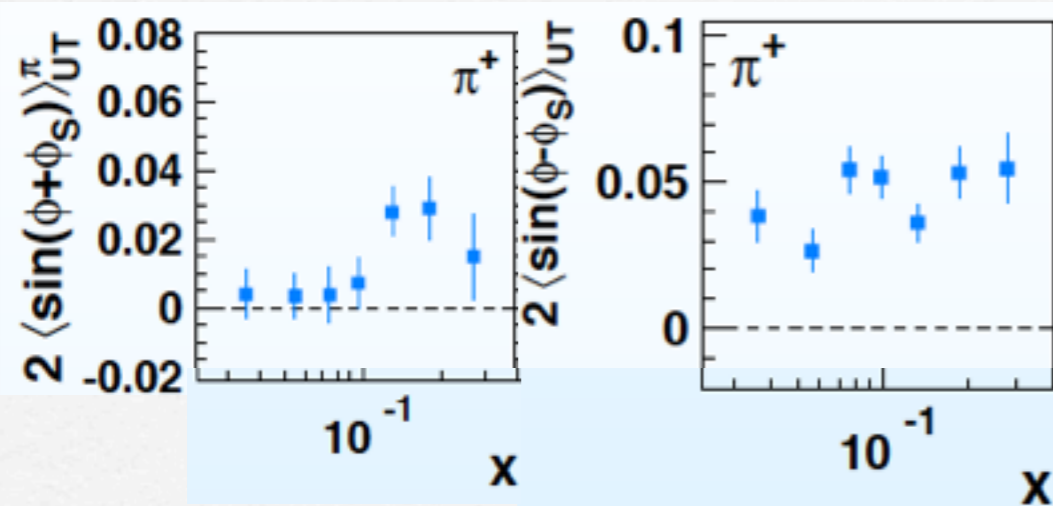
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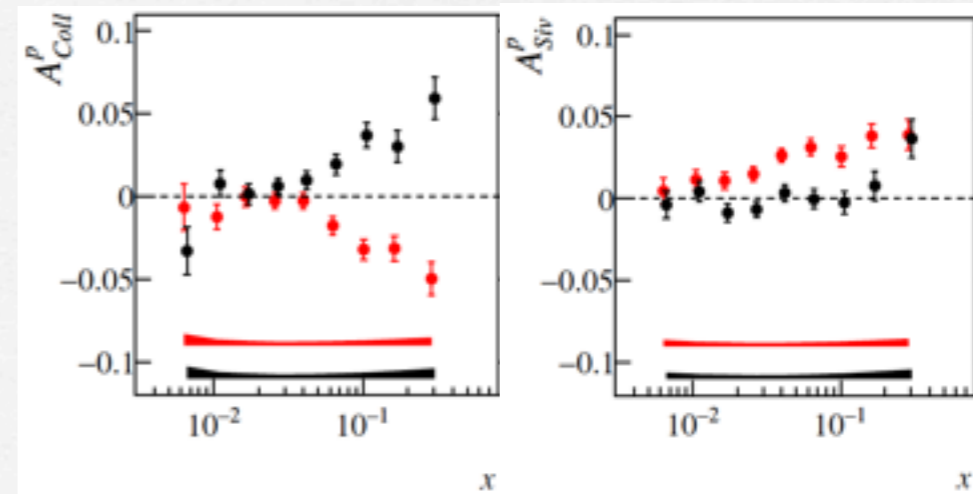


$$d\sigma_{UT}^{\text{SIDIS}} = F_{UT}^{\sin(\phi - \phi_S)} \sin(\phi - \phi_S) + F_{UT}^{\sin(\phi + \phi_S)} \sin(\phi + \phi_S) + \dots$$

HERMES:



COMPASS:



- Effect on the percent-level
- usually discussed in TMD-framework