# Quark tensor and axial charges within Schwinger-Dyson formalism

**Collaboration with T. M. Doi, S. Imai, H. Suganuma**

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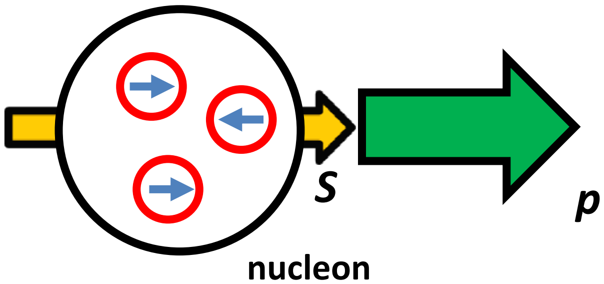
**Les Houches**

# Quark axial and tensor charges

## Axial charge:

**Nucleon axial charge probes the quark longitudinal polarization (helicity)**

$$\langle N(p,S)|\bar{q}\gamma^\mu\gamma_5 q|N(p,S)\rangle = S^\mu \Delta q$$




**Important problem:**

**Proton spin crisis**

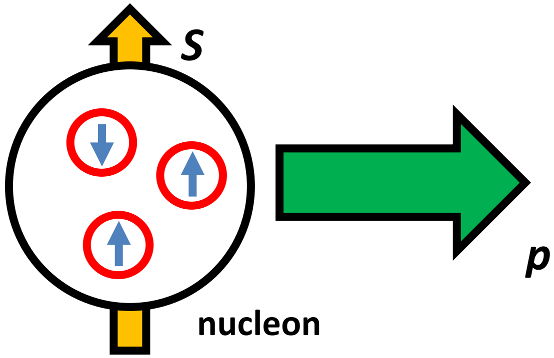
**⇒ Why quark spin fraction so small ?**

$$\left(\sum_q \Delta q \sim 0.3 \neq 1\right)$$

## Tensor charge:

**Nucleon tensor charge probes the quark transverse polarization (transversity)**

$$\langle N(p,S)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q|N(p,S)\rangle = 2(S^\mu p^\nu - S^\nu p^\mu)\delta q$$




**Why important:**

- **Spin structure of the nucleon**
- **Related to the quark EDM contribution to the nucleon EDM**

**(EDM is a powerful probe of new physics beyond standard model)**

# Axial & tensor charges as a probe of relativistic quark

**Relativistic effect of polarized quarks is probed by comparing tensor and axial charges:**

**Tensor charge:**

$$\langle N(p,S)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q|N(p,S)\rangle = 2(S^\mu p^\nu - S^\nu p^\mu)\delta q$$

**μ=0, ν=3** ➡ $\delta q \sim \langle N(p,S)|\bar{q}\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}q|N(p,S)\rangle$ **(Dirac representation)**

Same Nonrelativistic limit
(quark spin)

Relativistic component
with different sign

**Axial charge:**

$$\langle N(p,S)|\bar{q}\gamma^\mu\gamma_5 q|N(p,S)\rangle = S^\mu \Delta q$$

**μ=3** ➡ $\Delta q \sim \langle N(p,S)|\bar{q}\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix}q|N(p,S)\rangle$ **(Dirac representation)**

➡ **Difference of axial and tensor charges**
**probes how relativistic the polarized quarks are in nucleon**

M. Gockeler *et al.*, Nucl. Phys. Proc. Suppl. **53**, 315 (1997).

# Quark electric dipole moment and tensor charge

**Neutron EDM is a powerful probe of new physics beyond standard model**

**$d_n < 2.9 \times 10^{-26}$e cm**

C. A. Baker *et al*., Phys. Rev. Lett. **97**, 131801 (2006).

**Neutron EDM is sensitive to the quark EDM** $-\frac{i}{2} d_q \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}$

**EDM = first order coefficient of the multipole expansion**

$$\langle n \mid J_\mu^{\mathrm{EM}} \mid n \rangle \big|_{\not{CP}} = \frac{F_3(q^2)}{2M_n} \bar{n} q_\nu \sigma^{\mu\nu} \gamma_5 n \qquad d_n = \lim_{q^2 \to 0} \frac{F_3(q^2)}{2M_n}$$

**where** $\langle n | J_\mu^{\mathrm{EM}} | n \rangle |_{\mathrm{CPV}} = \sum_q d_q q^\nu \langle n | \bar{q} \sigma_{\mu\nu} q | n \rangle$

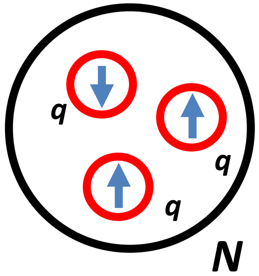
➡ **Quark EDM contribution to the neutron EDM is given by the quark tensor charge (first order coefficient)**

$$d_n = d_q \langle n | \sigma^{\mu\nu} | n \rangle$$

T. Bhattacharya, V. Cirigliano and R. Gupta, Pos LATTICE**2012**, 179 (2012).

# Quark axial & tensor charges in the quark model

**Nonrelativistic limit of axial & tensor charges ⇒ quark spin**




**Quark axial & tensor charge in the NR constituent quark model:**

$$\Delta u = \delta u = \frac{4}{3} \qquad \Delta d = \delta d = -\frac{1}{3}$$

**(in the proton)**

**We have assumed:**

- **Nucleons are made of three massive (nonrelativistic) constituent quarks**
- **S-wave system**
- **No spin-dependent interactions between constituent quarks**
- **(dressed quark axial/tensor charge) = (bare quark axial/tensor charge)**

**This assumption is not obvious!**

**⇒ CVC does not work for the axial/tensor current**
**(like for the vector current)**

**… Exp. data , lattice QCD analysis give smaller result …**

# Experimental data

## Quark axial charge:

- **Isovector axial coupling:**

**$g_A$ = 1.27 (exp)** — UCNA Coll., PRC **87**, 032501 (2013)

**$g_A$ ~ 1.2 (lattice QCD)** — T. Bhattacharya *et al.*, arXiv:1306.5435

**$g_A$ = 1.67 (NR quark model)**

- **Isoscalar axial coupling:**

**$\Delta\Sigma = 0.32 \pm 0.04$ (exp)** — COMPASS, PLB **693**, 227 2010

**$\Delta\Sigma$ ~ 0.6 (lattice QCD)** — T. Bhattacharya *et al.*, arXiv:1306.5435

**$\Delta\Sigma$ = 1 (NR quark model)**

**⇒ Proton spin crisis**

## Quark tensor charge:

- **Extraction from experiment:**

$$\begin{cases} \delta u &= 0.860 \pm 0.248 \\ \delta d &= -0.119 \pm 0.060 \end{cases}$$

**(renormalized at $\mu$ = 4 GeV)**

**⇒ Extracted from exp data of deeply virtual $\pi^0$ photoproduction**

I. Bedlinskiy et al. (JLAB), Phys. Rev. Lett. **109**, 112001 (2012).

G. R. Goldstein, J. O. Gopnzalez Hernandez, S. Liuti, arXiv:1401.0438.

- **Lattice QCD result:**

$$\begin{cases} \delta u &= 0.839 \pm 0.060 \\ \delta d &= -0.231 \pm 0.055 \end{cases}$$

**(renormalized at $\mu$ = 2 GeV)**

S. Aoki, M. Doui, T. Hatsuda and Y. Kuramashi, Phys. Rev. D **56**, 433 (1997);
T. Bhattacharya et al., arXiv:1306.5435.

# Object of study

The nucleon axial/tensor charges in the constituent quark model is in disagreement with experiment, lattice QCD.

Two sources of deviation can be inferred:

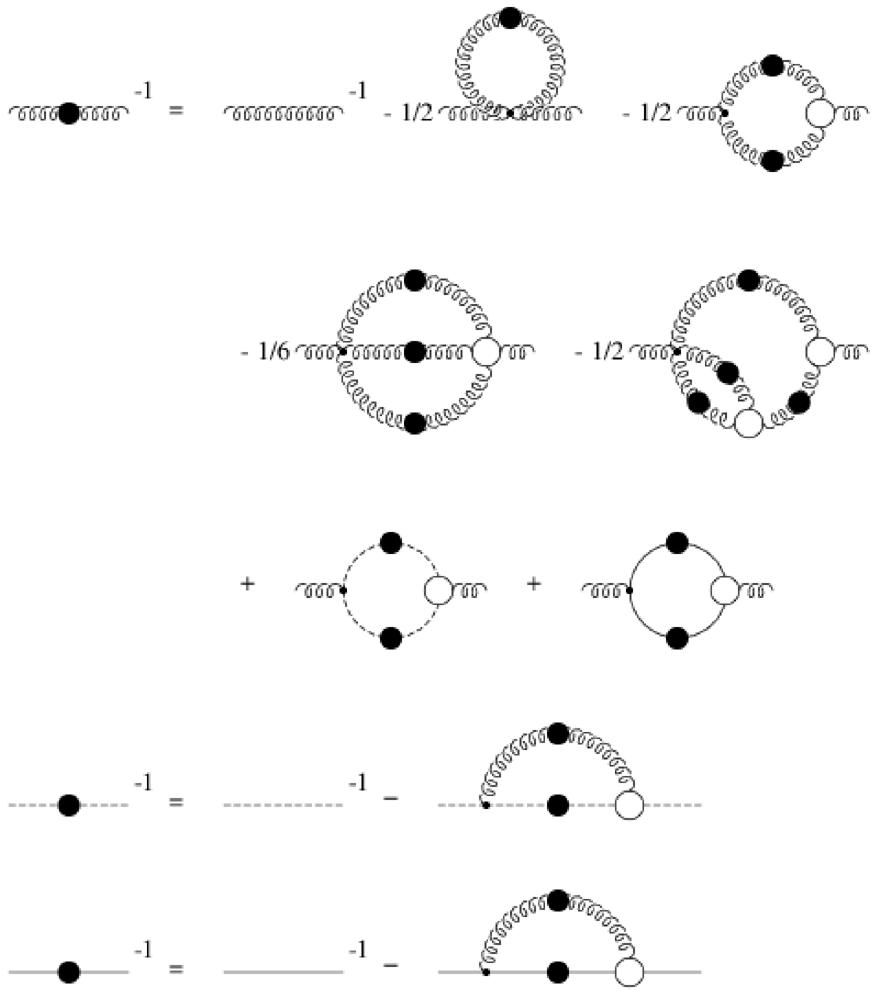
- Dressing of the single quark axial/tensor charges by gluons
  ($\delta q^{(bare)} = \delta q^{(dressed)}$ ? )
- Many-body interactions between constituent quarks

Gluon dressing of quark axial/tensor charges can be evaluated in the Schwinger-Dyson formalism

**Object of study:**

**Evaluate the gluon dressing effect to the single quark axial/tensor charges in the Schwinger-Dyson formalism.**

# *Full Schwinger-Dyson equation*




**Black blobs : full propagator**
**White blobs : full 1PI vertex**

**If the 1PI vertices are full,**
**then exact solution of QCD**
**(gluon, ghost, quark propagators)**

**To obtain the full solution,**
**we need to solve a tower**
**of infinite set of self-consistent**
**equation (n-point vertices)**

**⇒ Need to truncate the SDE**
**⇒ Ansatze and approximations for vertices & dressing functions**

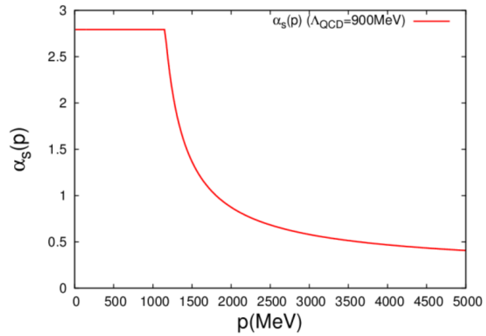
R. Alkofer and L. von Smekal, Phys. Rep. **353**, 281 (2001).

# Setup of the SD formalism

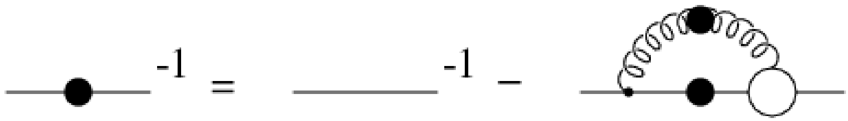
- **QCD scale parameter: $\Lambda_{QCD}$ = 900 MeV**
- **Landau gauge**
- **Gluon dressing function:**

  **⇒ RG improved strong coupling**

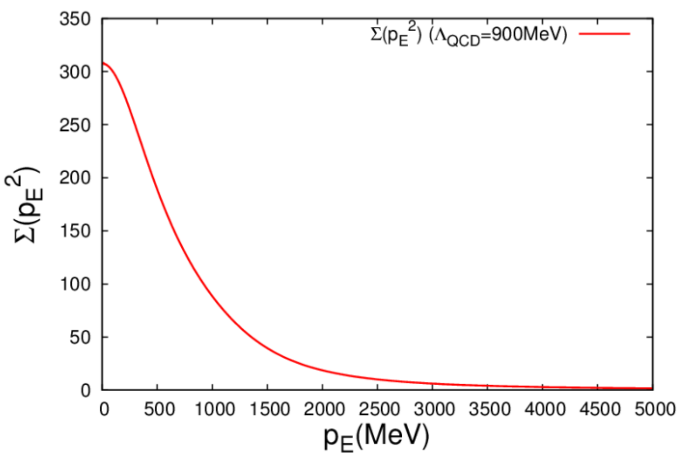
  K. Higashijima, Phys. Rev. D **29**, 1228 (1984).




- **Quark propagator Schwinger-Dyson equation**

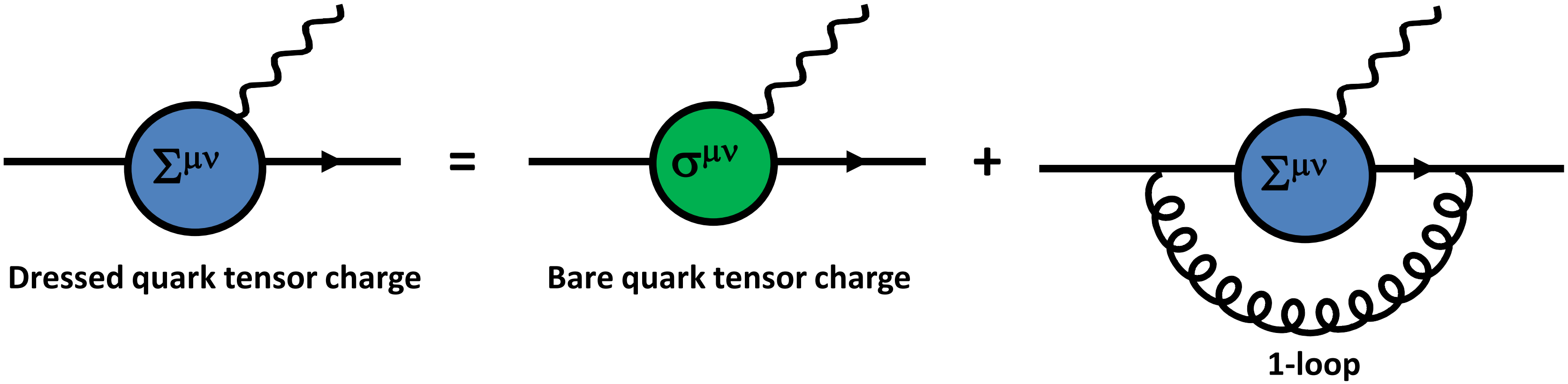



- **Dynamical quark mass:**




**The dynamical quark mass is generated with the Schwinger-Dyson equation:**

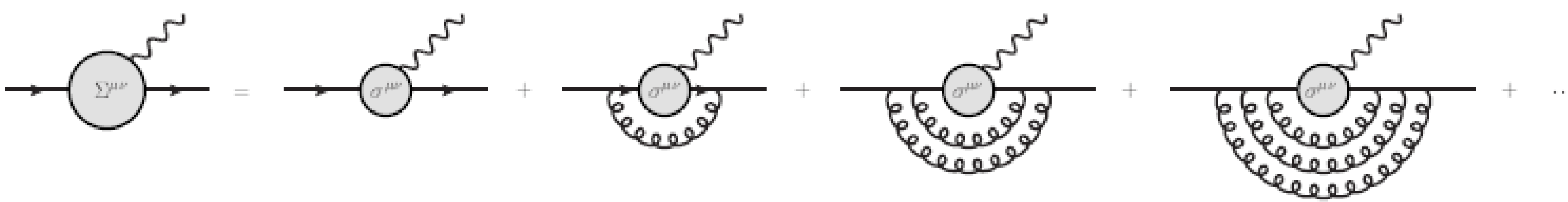
# Quark tensor charge SDE




**SDE is self-consistent equation :**

**⇒ Can consider the infinite sum of rainbow-ladder diagrams**

**Works as**



# Tensor charge SDE

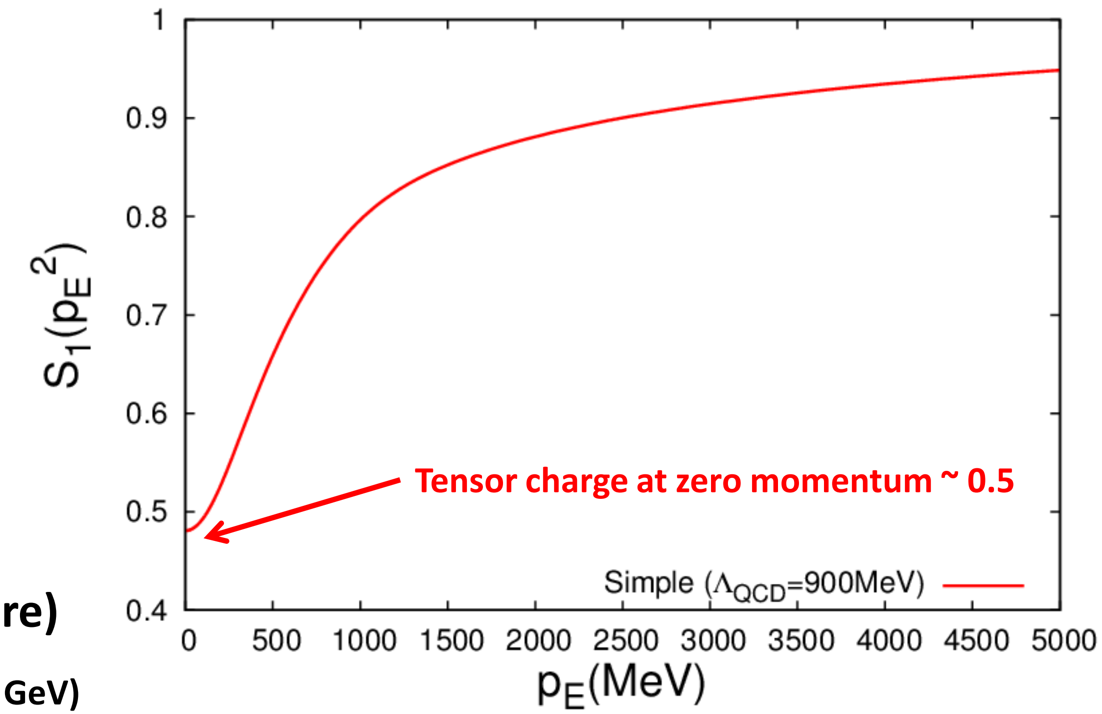
$$
\begin{aligned}
S_1(p_E^2) &= 1+\frac{C_2(N_c)}{3\pi^2}\int_0^\Lambda k_E^3 dk_E \int_0^\pi \sin^2\theta d\theta \frac{\alpha_s[(p_E-k_E)^2]}{[k_E^2+\Sigma^2(k_E^2)]^2} Z^2(k_E^2) \\
&\quad \times\Bigg\{ S_1(k_E^2)\left[\left(\frac{\Sigma^2(k_E^2)}{p_E^2}-1\right)\left(1+\frac{(p_E^2-k_E^2)^2}{(p_E-k_E)^4}\right)+\frac{\frac{\Sigma^2(k_E^2)}{p_E^2}(p_E^2-2k_E^2)+2p_E^2-k_E^2}{(p_E-k_E)^2}\right] \\
&\quad +2S_2(k_E^2)\Sigma(k_E^2)\left[-\left(1+\frac{k_E^2}{p_E^2}\right)\left(1+\frac{(p_E^2-k_E^2)^2}{(p_E-k_E)^4}\right)+2\frac{p_E^2-k_E^2+\frac{k_E^4}{p_E^2}}{(p_E-k_E)^2}\right] \\
&\quad -\frac{1}{2}S_3(k_E^2)\left[k_E^2+\Sigma^2(k_E^2)\right]\left[\left(\frac{k_E^2}{p_E^2}-1\right)\left(1+\frac{(p_E^2-k_E^2)^2}{(p_E-k_E)^4}\right)+2\frac{p_E^2-\frac{k_E^4}{p_E^2}}{(p_E-k_E)^2}\right]\Bigg\}, \\
S_2(p_E^2) &= \frac{C_2(N_c)}{3\pi^2 p_E^2}\int_0^\Lambda k_E^3 dk_E \int_0^\pi \sin^2\theta d\theta \frac{\alpha_s[(p_E-k_E)^2]}{[k_E^2+\Sigma^2(k_E^2)]^2} Z^2(k_E^2)\cdot\left[2-\frac{5}{2}\frac{p_E^2+k_E^2}{(p_E-k_E)^2}+\frac{1}{2}\frac{(p_E^2-k_E^2)^2}{(p_E-k_E)^4}\right] \\
&\quad \times\Big\{\Sigma(k_E^2)S_1(k_E^2)-\left[k_E^2-\Sigma^2(k_E^2)\right]S_2(k_E^2)\Big\}, \\
S_3(p_E^2) &= \frac{C_2(N_c)}{3\pi^2 p_E^2}\int_0^\Lambda k_E^3 dk_E \int_0^\pi \sin^2\theta d\theta \frac{\alpha_s[(p_E-k_E)^2]}{[k_E^2+\Sigma^2(k_E^2)]^2} Z^2(k_E^2)\cdot\left[1+\frac{p_E^2-2k_E^2}{(p_E-k_E)^2}+\frac{(p_E^2-k_E^2)^2}{(p_E-k_E)^4}\right] \\
&\quad \times\left\{2\frac{\Sigma^2(k_E^2)}{p_E^2}S_1(k_E^2)-4\Sigma(k_E^2)\frac{k_E^2}{p_E^2}S_2(k_E^2)-\left[k_E^2+\Sigma^2(k_E^2)\right]\frac{k_E^2}{p_E^2}S_3(k_E^2)\right\}.
\end{aligned}
$$

NY, T. M. Doi, S. Imai, H. Suganuma, Phys. Rev. D**88**, 074036 (2013).

# Tensor charge: result

**tensor charge at zero momentum:**

**~ 0.5** **(not renormalized)**




$\delta q^{(dressed)} \sim 0.6\ \delta q^{(bare)}$

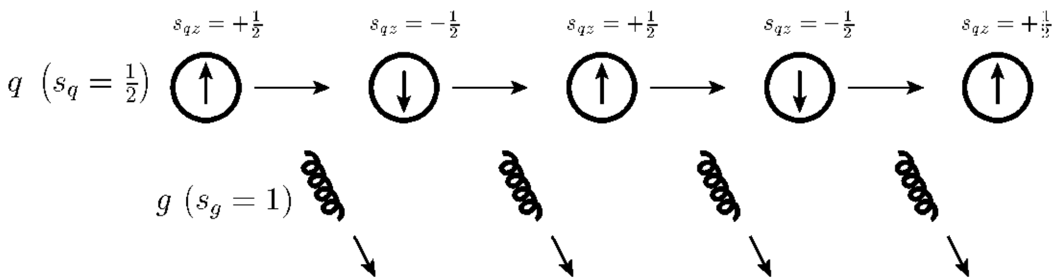
**(Renormalization scale $\mu$ = 2 GeV)**

**⇒ The bare quark tensor charge is significantly suppressed by the gluon dressing**

NY, T. M. Doi, S. Imai, H. Suganuma, Phys. Rev. D**88**, 074036 (2013).

# *Interpretation: superposition of quark spin flip*

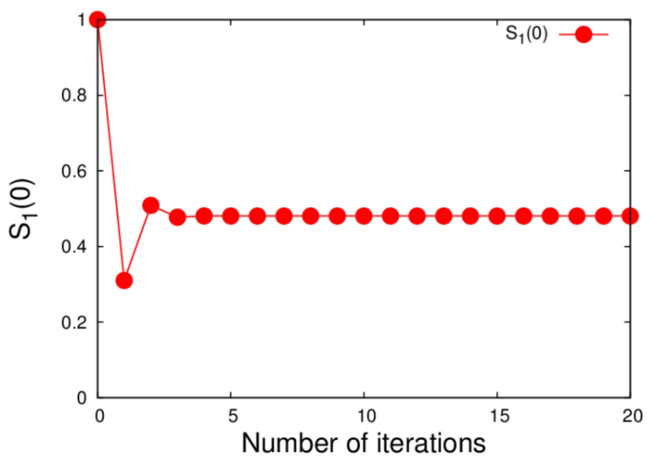
**Quark spin (1/2) flips after each gluon emission/absorption (spin 1)**




**⇒ Sum (infinite) of contribution is always smaller than the bare one**

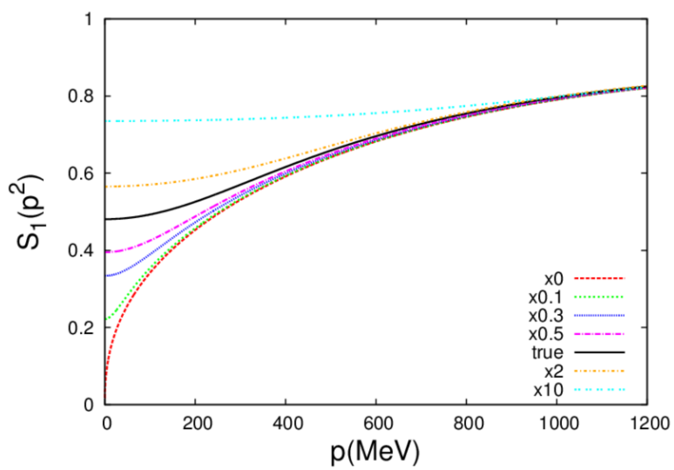
**$\delta q^{(\text{dressed})} < \delta q^{(\text{bare})}$**

**Iteration ≈ ladder expansion**



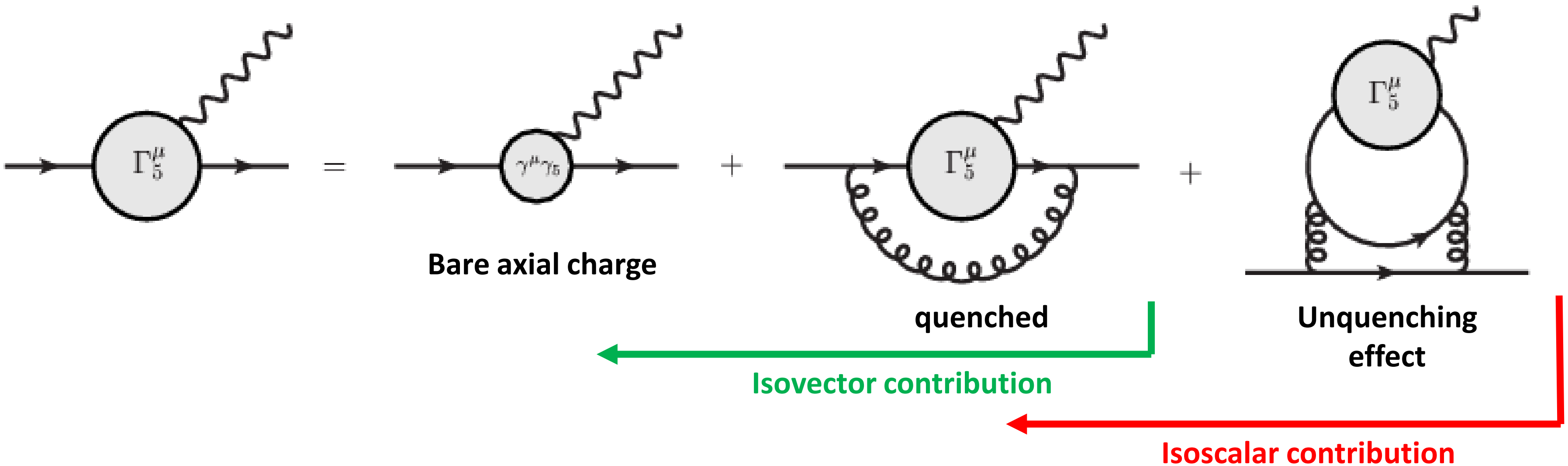

**⇒ oscillation = sum of spin flip**

**Resizing dynamical quark mass**



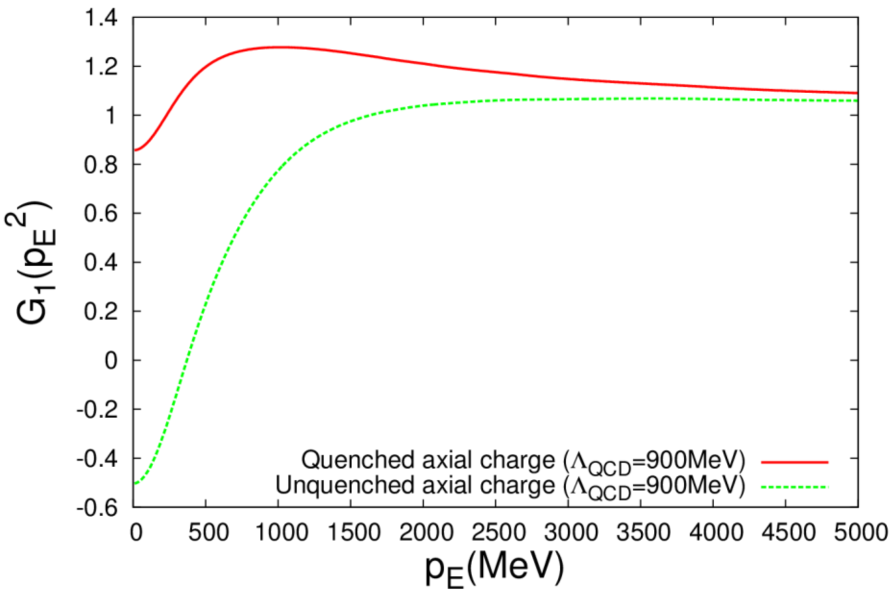

**⇒ light = easy to flip**

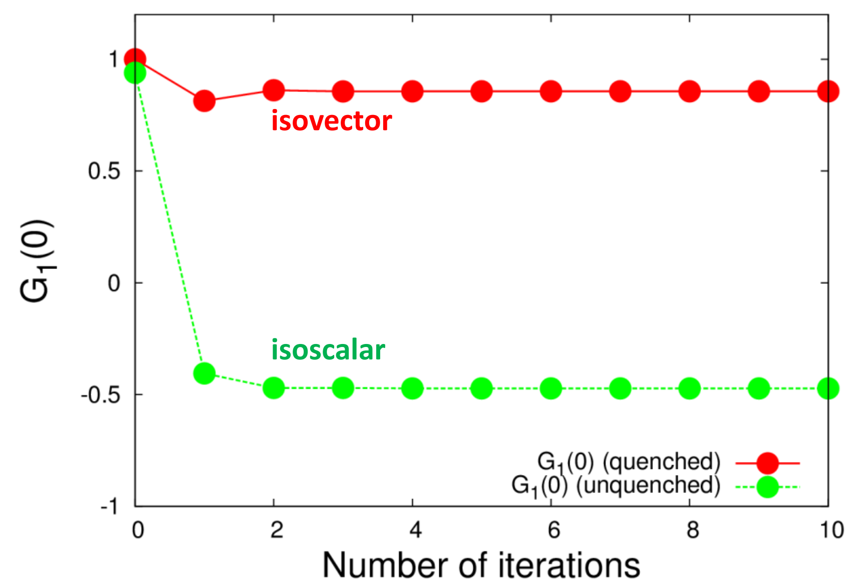
# Quark axial charge SDE




NY, S. Imai, T. M. Doi, H. Suganuma, submitted in PRD

# *Axial charge: result*




- **Isovector axial vector coupling:**

  **$g_A$ ~ 1.4** **Suppression (c.f. NRQM) due to gluon emission/absorption (like tensor charge)**

- **Isoscalar axial vector coupling:**

  **$\Delta\Sigma$ ~ - 0.47** **Additional suppression due to ABJ axial anomaly**

  **⇒ Anomaly effect is too large**

NY, S. Imai, T. M. Doi, H. Suganuma, submitted in PRD

# *Flavor singlet axial current*

**Our result (single quark):** $\Delta\Sigma \sim -0.5$

**Exp data:** $\Delta\Sigma \sim 0.3$

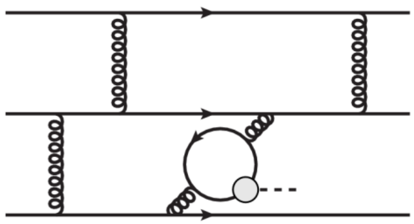
**Anomaly effect of the single quark SDE too large ~ O(1)**

**Phenomenologically, the total (single quark + many-body) anomaly effect not large:**

$\Delta\Sigma$ **(anomaly) ~ O(0.1)** T. P. Cheng and L.-F. Li, Phys. Rev. Lett. **62**, 1441 (1989).

## Where is the problem?

- **We have not considered the many-body-effect (exchange current)**
  **⇒ Exchange current effect may interfere destructively**



- **Inner loop may be sensitive to the IR region**
  **⇒ We must improve gluon sector SDE, unquenching effect, beyond rainbow effect, …**

## Summary:

- We have calculated the quark axial & tensor charges in the Schwinger-Dyson formalism with a simple setup.
- The quark tensor and isovector axial charges are suppressed by the gluon dressing, due to the spin flip of the quark after gluon emission/absorption.
- The quark isoscalar axial charge is additionally suppressed by the axial anomaly effect… Too large.

## Future subjects:

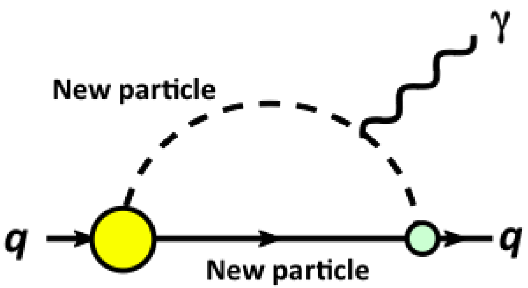
- Improvement of SDE: gluon, unquenching, beyond rainbow…
- Many-body effects of anomaly contribution.
- Calculation of the orbital angular momentum of the single quark in the SD formalism.

# ***Backup slides***

# *Scale of new physics probed by the EDM*

**Naïve estimation (example of neutron EDM):**

- **Coupling of new physics ~ O(1) (naturalness assumption)**
- **Contribute from one-loop graph**
- **1 Yukawa coupling (required for chirality flip)**
- **$d_n/d_q$ ~ O(1) (hadron level analysis)**




$$\Rightarrow \quad d_n = \frac{Y_q e}{4\pi^2 M_{\rm NP}} \sim \frac{10^{-21}}{M_{\rm NP}/{\rm GeV}} e\, cm$$

**Exp data: $d_n < 2.9 \times 10^{-26}$ e cm**

C. A. Baker *et al*., Phys. Rev. Lett. **97**, 131801 (2006).

**⇒ Current exp data of Neutron EDM can probe $M_{NP}$ ~ 10TeV!**

**(for $d_n < 10^{-28}$ e cm , $M_{NP}$ ~ 1000TeV can be probed: well beyond reach of LHC!)**

**(Don't forget that naturalness was assumed!)**

# Quark wave function renormalization